Container with Most Water

The solution presented in maxArea.cc probably seems questionable at first sight, Here is how I thought about it:

We are tasked to find the two indeces [i, j] (i < j) such that

$$min(height[i] - height[j]) * (j - i)$$

is maximized. Note that the smaller of height[i] and height[j] plays a dominant role in deciding the value of

$$min(height[i] - height[j]) * (j - i)$$

Suppose height[i] < height[j] (for i < j), then the algorithm will perform i++. Note that by incrementing i by 1, we effectly removed [i, j - 1] as a candidate for the final solution (since we either perform i++ or j– in all cases). Observe that

$$\begin{aligned} min(height[i] - height[j]) * (j - i) &= height[i] * (j - i) \\ &> height[i] * (j - 1 - i) \\ &= min(height[i] - height[j - 1]) * (j - 1 - i) \end{aligned}$$

which means the water in the container with vertical lines at indeces [i, j - 1] is less than the water in the container with the vertical lines at indeces [i, j] which means we can garantee that [i, j - 1] is not an optimal solution. Hence we will always reach the optimal indeces.