Examples

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In this file, we run through some examples for ratio 1, 2, 3, and 4 (the code can be found in the following files: 'ratio1_and_ratio.ipynb', 'ratio3.ipynb', and 'ratio4.ipynb'). For outputs of ratio 2, the array represents increasing $kbig$ as specified in each example. For outputs of ratio 3 and ratio 4, the array represents increasing $e = 0, 1, \ldots$. The following runtime may be inaccurate as (for some unknown reason), running Julia in WSL (Windows Subsystem for Linux) can cause inconsistent runtime (as much as a factor of 2!). The examples are as follows,

1. $p = 2, n = 9, M' = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$.
   For $kbig = 10, \ldots, 17$, $e = 0, \ldots, 5$, and $k = n$ for ratio 3, we get

```
ratio 1: 2//1
  0.000167 seconds (1.58 k allocations: 25.297 KiB)
ratio 2 (kbig = 10, ..., 17): 3//2, 3//2, 3//2, 3//2, 3//2, 3//2, 3//2, 3//2
  1.038583 seconds (11.63 M allocations: 189.849 MiB, 41.42% gc time)
ratio 3 (e = 0, ..., 5): 1//2, 3//2, 2//1, 2//1, 2//1, 2//1
  5.483039 seconds (22.42 M allocations: 1.746 GiB, 37.01% gc time, 2.09% compilation time)
ratio 4 (e = 0, 1, 2): 47459//196608, 244067//196608, 73727//49152
ratio 4 (e = 3, 4, 5): 73727//49152, 73727//49152, 73727//49152
  4.015287 seconds (27.18 M allocations: 1.410 GiB, 33.82% gc time, 4.50% compilation time)
```

2. $p = 2, n = 11, M' = \begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}$.
   For $kbig = 12, \ldots, 17$, $e = 0, \ldots, 6$, and $k = n$ for ratio 3, we get

```
ratio 1: 2//1
  0.023624 seconds (8.42 k allocations: 222.844 KiB, 93.94% compilation time)
ratio 2 (kbig = 12, ..., 17): 3//2, 11//8, 11//8, 11//8, 11//8, 11//8
  4.348260 seconds (14.06 M allocations: 363.978 MiB, 43.61% gc time, 3.95% compilation time)
ratio 3 (e = 0, ..., 6): 1//8, 3//8, 3//2, 7//4, 2//1, 2//1, 2//1
151.886204 seconds (286.15 M allocations: 26.290 GiB, 28.96% gc time, 0.19% compilation time)
ratio 4 (e = 0, 1, 2, 3): 94739//3145728, 485111//3145728, 3730397//3145728, 4126457//3145728
ratio 4 (e = 4, 5, 6): 720859//524288, 720859//524288, 720859//524288
106.546226 seconds (294.89 M allocations: 19.330 GiB, 31.73% gc time, 0.36% compilation time)
```

3. $p = 2, n = 11, M' = \begin{pmatrix} 1 & 0 \\ 0 & 9 \end{pmatrix}$.
   For $kbig = 12, \ldots, 17$, $e = 0, \ldots, 7$ and $k = n$ for ratio 3, we get

```
ratio 1: 2//1
  0.001128 seconds (6.26 k allocations: 103.875 KiB)
ratio 2 (kbig = 12, ..., 17): 3//2, 11//8, 43//32, 43//32, 43//32, 43//32
  3.383988 seconds (14.20 M allocations: 339.110 MiB, 48.67% gc time)
ratio 3 (e = 0, ..., 6): 1//32, 3//32, 3//8, 23//16, 7//4, 15//8, 2//1, 2//1
128.349810 seconds (249.19 M allocations: 25.310 GiB, 25.75% gc time)
ratio 4 (e = 0, 1, 2, 3): 11717//3145728, 60677//3145728, 465667//3145728, 1830361//1572864
ratio 4 (e = 4, 5, 6, 7): 169971//131072, 4177625//3145728, 704475//524288, 704475//524288
108.326421 seconds (285.91 M allocations: 18.876 GiB, 32.46% gc time, 0.39% compilation time)
```

4. $p = 3, n = 6, M' = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$.
   For $kbig = 7, \ldots, 12$, $e = 0, \ldots, 6$ and $k = n$ for ratio 3, we get

```
ratio 1: 3//2
  0.022676 seconds (4.47 k allocations: 161.312 KiB, 97.00% compilation time)
ratio 2 (kbig = 7, ..., 12): 7//6, 7//6, 7//6, 7//6, 7//6, 7//6
  4.159131 seconds (21.15 M allocations: 348.741 MiB, 38.84% gc time, 3.43% compilation time)
ratio 3 (e = 0, ..., 6): 1//6, 7//6, 3//2, 3//2, 3//2, 3//2, 3//2
 17.167527 seconds (32.78 M allocations: 2.992 GiB, 28.24% gc time, 1.53% compilation time)
ratio 4 (e = 0, 1, 2, 3): 6137//118098, 124235//118098, 7//6, 7//6
ratio 4 (e = 4, 5, 6): 7//6, 7//6, 7//6
 18.137413 seconds (52.36 M allocations: 2.644 GiB, 36.08% gc time, 2.49% compilation time)
```

5. $p = 3, n = 7, M' = \begin{pmatrix} 1 & 0 \\ 0 & 10 \end{pmatrix}$.
   For $kbig = 8, \ldots, 12$ and for $e = 0, \ldots, 6$, we get

6. $p = 3, n = 7, M' = \begin{pmatrix} 1 & 0 \\ 0 & 28 \end{pmatrix}$.
   For $kbig = 8, \ldots, 12$ and for $e = 0, \ldots, 8$, we get

7. $p = 3, n = 7, M' = \begin{pmatrix} 1 & 6 \\ 3 & 1 \end{pmatrix}$.
   For $kbig = 8, \ldots, 12$, we get

8. $p = 3, n = 7, M' = \begin{pmatrix} 1 & 18 \\ 9 & 1 \end{pmatrix}$.
   For $kbig = 8, \ldots, 12$, we get

9. $p = 5, n = 5, M' = \begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}$.
   For $kbig = 6, \ldots, 9$, we get

10. $p = 5, n = 5, M' = \begin{pmatrix} 1 & 10 \\ 5 & 1 \end{pmatrix}$.
    For $kbig = 6, \ldots, 9$, we get