The Aggregate Effects of the Decline of Disruptive Innovation

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Abstract

This paper proposes a model that explains both recently documented facts about the decline of disruptive innovation and the decline in productivity growth as the result of large firms trying to monopolize technologies by poaching inventors from disruptive activities. To come to this conclusion, the paper builds an endogenous growth model with inventor labor markets on which firms can interact strategically. To inform this model, I perform an event study of the effect of disruptive inventions on their technology fields. I document that technology classes without disruption slowly trend towards incrementalism and that after a disruption, more patents get registered and research becomes less incremental.

1 Introduction

The paper proposes an endogenous growth model where firms' decisions of what type of innovation to pursue creates declining growth. I expand upon the recent empirical findings on disruptive vs. incremental innovation and use these findings, together with my own results, to inform my model. The principal contribution of the paper is to build an endogenous growth model around the difference between radical/disruptive or incremental innovation and the strategic decision that this poses on firms. I show that the efforts of established firms to curtail threatening disruptive innovation can determine the growth path of the economy and that this scenario is consistent with the observed data.

To demonstrate this, I build upon Park et al. (2023) and Funk and Owen-Smith (2017) to measure how disruptive an invention is from patent citation data. I construct an index of how old the work is that citing patents reference. I.e. if a patent's citing patents do not reference older work, I deem it a disruptive innovation that spawns a new literature unconnected to the past. I apply this measure to the international patent data collected by the European Patent office (PATSTAT) between 1980 and 2010 and perform a matching based event study to understand the effect of disruptive invention on technologies. I find that disruptive inventions increase citations, patenting and the chance for a consecutive disruption, but that the effect is decaying over time.

I take this result to construct an endogenous growth model which captures these features of disruptive innovation. The actions of two types of firms drive the fate of the model economy: First, there are disruptive firms. Disruptors do not sell any products at first, but try to invent a fundamentally different technology. Bill Gates and Paul Allen working in a garage to revolutionize home computing were an archetypical disruptive firm. If disruptive inventors are successful, they create a new producing firm with better production technology than that of any currently existing producer. Producing firms, the second firm type, actually earn revenue in the consumer market by selling a product. Producers take an underlying technology invented by disruptive firms and develop it into a product. Producing firms improve their technology incrementally in order to produce a product of higher quality. Steady technological progress requires a mixture of both types of inventions: Disruptive inventions alone never create a consumer product, only ever more advanced production technologies. Incremental inventions alone lead to a slowing rate of technology growth: As incremental inventors strain against the limits of the underlying production technology, the rate of technology growth within each technology declines over time. Every disruptive invention allows incremental inventors to work with a more advanced basic technology and thus increases the value of future incremental improvements by the factor ω. This tension between disruption and incremental growth is the central tradeoff in the model and how well the market economy handles it determines economic growth.

Neither disruptive nor producing firms can conduct research on their own: Firms need inventors to make inventions for them. Firms of both types hire incremental or disruptive inventors on a search and matching labor market. Disruptive and incremental inventors enter the economy and match with firms at fixed rates. The value of each firm is partly determined by the stock of inventors it has hired and those it can hire in the future. Incremental inventors are specialized in their current technology and cannot contribute to other technologies. Thus, whenever a firm switches the technology underlying its products, it effectively loses all incremental inventors it has hired so far. Inserting this labor market into an endogenous growth model is the primary new assumption compared to the literature. This new assumption drives the new findings: Firms try to protect their assets (incremental inventors) from being made obsolete by disruptive innovation.

Successful producing firms can slow down technology disruption by hiring the inventors that disruptive firms would need to innovate. Thus, some firms in the economy actively resist technology growth. Technological progress depends not only on investment in R&D, but also on overcoming this resistance. This is the main mechanism that follows from the introduced assumptions and sets this paper apart from the rest of the endogenous growth literature, which views innovation as the result of investment only.

This paper speaks to the discussion around slowing technology growth, most notably by reconciling a set of seemingly contradictory findings: TFP growth and scientific output per researcher seem to decline, while firms hire an increasing number of researchers for non-decreasing wages (Cowen and Southwood, 2019; Bloom et al., 2017). Likewise, the scientific content of patents is declining (Arora et al., 2019), despite patents with more scientific content being more valuable (Poege et al., 2019). Patents and publications have also become less disruptive (Park et al., 2023; Funk and Owen-Smith, 2017).

A large literature is concerned with the growing dispersion of firm level productivity (Gal, 2017) and declining aggregate productivity growth (Gordon, 2016) throughout the developed world. The literature discusses several different explanations for these phenomena:

Akcigit and Ates (2019) argue that slowing technology diffusion is itself the most likely source of slowing technology growth. Lucking et al. (2019) argue that technology diffusion is still about as fast as it was in the 1980s. However, they do find that technology diffusion was faster during the growth acceleration associated with IT in the 1990s. In my model, growth is driven by disruptive innovation, while incremental inventions (and their diffusion) influence the level of economic activity. However, the model I present also features an inventor-firm labor market, which can serve as micro-foundation for technology diffusion in the endogenous growth model.

Another school of thought argues that ideas are getting harder to find and technology growth thus slows down endogenously. Gordon (2016) makes this point. Bloom et al. (2017) showed that more and more researchers are necessary to double e.g. computing power or crop yields per acre. My paper takes this finding seriously, but offers an alternative interpretation: The very fact that firms invest so many resources in solving the same problems using the same technologies indicates that they are engaged in incremental innovation. Thus, the findings of Bloom et al. (2017) are troublesome because they show a misallocation of inventive talent to incremental innovation with declining returns. Yet, this does not necessarily mean that disruptive ideas are becoming harder to find.

The model explains these trends as outcomes of firms' optimal research strategies: Large firms' profits depend on the fate of their specialty technology. Thus, they cling to incremental innovation and undertake defensive measures to prevent disruption.

A fictitious social planner has to chose between incremental innovation and disruption. Which of the two he picks crucially depends on the weight that he puts on future generations: A disruptive invention will increase economic growth long-term, but the benefits will accrue to future inventors and future firms. In contrast, the current incremental inventors and producing firms unambiguously lose after a disruptive invention. If the current agents die before the growth increase from a disruptive innovation creates value, the social planner cannot compensate them and the low-growth equilibrium with incremental innovations is Pareto-optimal, even though it does not maximize GDP. If people in the model live long enough, the social planner could use the additional GDP to compensate the losers from a disruptive innovation.

My model is built on the framework of Akcigit and Kerr (2018), who assume that firms are proficient in specific technology clusters. I understand technology clusters as more than just one new product, they denote distinct technologies behind multiple individual products, like "telegraphy" or "internal combustion engine". Incremental inventions within these clusters generate higher quality products. In departure from Akcigit and Kerr (2018), firms cannot invent on their own and have to hire inventors specialized in a technology cluster on a search and matching labor market. The labor market for inventors in each cluster corresponds to the results presented in the empirical chapter in section **??**. Specifically, I develop how innovation affects firms' technology $\frac{\partial q}{\partial p}$ and how firms' technology determines profits $\frac{\partial \pi^*}{\partial q}$. Together, these factors determine the value of an invention $V(p)$ in equation (**??**).

My paper also speaks to a larger theoretical literature on market failures that misdirect innovation. Firms under-invest in research that unlocks follow-up inventions, because they cannot profit from the inventions other firms will make, as in Hopenhayn et al. (2006); Denicolò (2000); Scotchmer (1991). In general, firms can only appropriate a share of the overall welfare increases that result from their inventions. Since this share is not constant across inventions, firms over-invest in inventions where they can appropriate a high share of the returns (Bryan and Lemus, 2017). In the model presented here, producing firms can only appropriate the returns from incremental innovation, which drives aggregate behavior.

Beyond the theoretical literature, there is substantial empirical support for the monopolization of research fields, which is conceptually adjacent to the proposed model: Thompson and Kuhn (2017) use patent races between firms to compare the first and second research team and thus patent holders and followers. They find that patents preclude competitors from follow-up innovation and make the winner of patent races more dominant in the associated technology field. In the semiconductor industry, increased patent protection seems to have led to defensive patenting instead of innovation (Hall and Ziedonis, 2001). Across industries, the correlation between patent protection and innovation is negative, which (Bessen and Maskin, 2009) explain by the negative effect of patents on sequential inventions. This study extends the principal insights of this literature to a context of inventor-firm labor market matching in an endogenous growth model.

This paper also links into the literature around the documented rise of firm profits and markups (Barkai, 2017; De Loecker and Eeckhout, 2017). The model predicts that firms with high market power engage in qualitatively different R&D. Only small, competitive firms invest in disruptive technology to – if successful – themselves become large firms linked to a technology. After that, their research portfolio will become much more incremental.

In a larger context, the paper relates to literature on the efficacy of the current system to reward innovative firms. The theoretical and experimental literature suggests that patents are not able to optimally steer the direction of innovation in general: If only a finite number of research direction is available, firms race each other to the most lucrative patents and incur wasteful parallel investment (Zizzo, 2002; Silipo, 2005; Breitmoser et al., 2010). Both in the US (Jaffe, 2000) and Japan (Sakakibara and Branstetter, 2001), firms do not react conclusively to substantial changes in patenting protection. Nevertheless, in my model, the market failure can be corrected by policy interventions. Since technology monopolists are misdirecting innovation, policy should break up existing monopolies and prevent mergers and buy-outs of start-ups. Likewise, any policy that increases the transferability of inventor skills makes technology markets larger and thus harder to monopolize.

The remainder of the paper is structured as follows: Section 2 documents stylized facts about disruptive vs. incremental innovation. Section 3 lays out the assumptions and mechanisms of the model. The section also discusses various possibilities for extensions of the model and their implications. Section 4 discusses the equilibrium behavior of the economy and response to various policy interventions. Section 5 concludes the analysis.

2 Stylized Facts

2.1 Literature on disruptive vs. incremental innovation

There is an active literature using firm level data to discuss the growth slowdown in developed economies. This research has generally concluded that there is a real slowdown in productivity growth, not just a measurement issue (Syverson, 2017; Reinsdorf et al., 2016; Antolin-Diaz et al., 2017). Gordon (2016) proposed that new (impactful) ideas are getting harder and harder to find as more and more discoveries are made. They demonstrate this by estimating the worldwide researcher productivity in a series of tasks, e.g. doubling the number of transistors on a chip (Moore's Law) or crop yields per acre. Andrews et al. (2016) and Akcigit and Ates (2019) show that firm productivity dispersion has increased at the same time.

Park et al. (2023) have documented a trend towards more incremental, less disruptive research both in publications and patents. Poege et al. (2019) show that increased incrementalism decreases the economic value of patents: Patents connected to high quality research through citations are roughly twice as valuable as other patents.

There is also substantial evidence that large firms – in contrast to small firms – lean more towards incremental improvements of existing technology (Acemoglu et al., 2016; Kerr et al., 2014; Kueng et al., 2014). The incentives that cause this behavior are also well understood theoretically (Akcigit and Kerr, 2018).

From these results, I draw four different stylized facts that the model has to replicate:

1. Aggregate productivity growth is slowing down.
2. Researcher productivity measured for specific targets is declining.
3. Firms' research is becoming more incremental.
4. Large firms' research is more incremental than small firms' research.

2.2 The PATSTAT Data

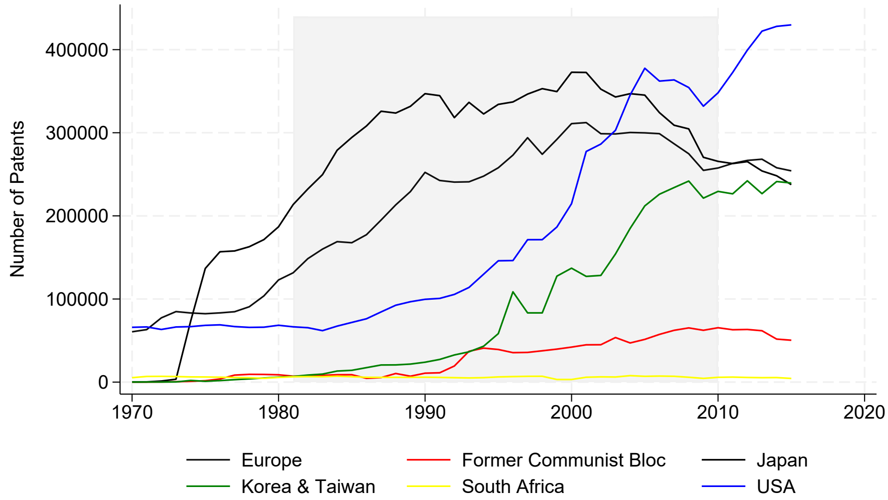
Patent data from across the world gathered in the PATSTAT database forms the basis of my empirical strategy. This data contains the filing date of any patent application, a description of the technology and the names of firms and inventors involved. For some participating countries, the data starts in 1850, however, coverage pre-WW2 is generally low. Patents from some countries are only available from a later date onwards: E.g., Japan enters the database in the mid-seventies. Around the same time, coverage rates improve in general and the data can give a reliable picture of worldwide patent activity.

The following graph shows the number of patents over time for selected countries. Note that the stable or shrinking number of national patents for EU countries is offset by a large increase in EU-wide EPO applications.

Importantly, PATSTAT does not contain unique firm or person identifiers. Instead, it contains a character string written into the fields "inventor" and "applicant" on the patent. In my empirical application, I avoid using individual or firm level indicators because of this problem (see Magerman et al. (2006); Toole et al. (2021); Li et al. (2019) for a discussion of this issue).

PATSTAT contains detailed descriptions of patents' content: Beside titles, abstracts and patent texts, the EPO assigns one or more harmonized 8 digit IPC classes to every patent. I use these IPC classes as technology fields for the theoretical section of the paper. The EPO also groups patents for the same inventions together as families and provides citations between patent families. I use these patent family citations to determine how incremental or disruptive any given patent is and aggregate these measures to the IPC-class level.

Figure 1: *Overview over PATSTAT*



Notes: Number of patents in PATSTAT per region. The gray region marks the time period of data used in the event study in section 2.5.
Sources: PATSTAT (European Patent Office).

2.3 Measuring patent "disruptiveness"

To better understand the effect of disruptive inventions, I use the PATSTAT data base provided by the European Patent Office to conduct my own analysis around disruption events. PATSTAT has world-wide coverage and covers most information on patents. However, the EPO mostly relies on partner patent offices for digitization, so both coverage and the available variables vary by country, especially before 1975. I start my analysis in 1980, when the data from the major patent offices contains citations and coverage is satisfactory.

To determine how disruptive specific technologies are, I follow the general strategy of Park et al. (2023) and Funk and Owen-Smith (2017) and look at the citation patterns around patent p. Both papers use two indices derived from the other citations of patents citing patent p: If patents that cite p also cite the older literature that p is referencing, p did not disrupt the technology. If however p starts a new literature that does not cite pre-existing patents anymore, p is classified as disruptive. I simplify their exact specification by not counting the citations between cited patents, p and citing patents, but instead just by observing the average filing year of the other citations from patents citing p.

$$CYG_p = \frac{\sum^C \bar{t}_{o,c}}{C_p} - t_p \tag{1}$$

where p is the patent of interest, c indexes patents filed up to five years after patent p and citing patent p and o indexes the other patents cited by c. $\bar{t}_{o,c}$ is the average year of the patents o cited by citing patent c and t_p is the filing year of the original patent. Intuitively, the citing year gap CYG is the difference between the average year referenced by the patents cited by the patent citing p. A positive number means that patents citing p on average reference patents filed after the patent of interest p, i.e. they are referencing new research instead of the old patents that p is based on. Thus, this measure intuitively is very close to the original by Funk and Owen-Smith (2017). However, because it is essentially continuous, it outperforms the original in small samples, where the fact that most patents only have one or two citations really matters: Indices based on referenced patent count only have a couple of values in practice, making their movements quite jumpy. I prefer CYG_p as a measure since I follow "disruptiveness" within (sometimes) small technology classes, not the entire patent sample, unlike Funk and Owen-Smith (2017).

2.4 Aggregate Trends

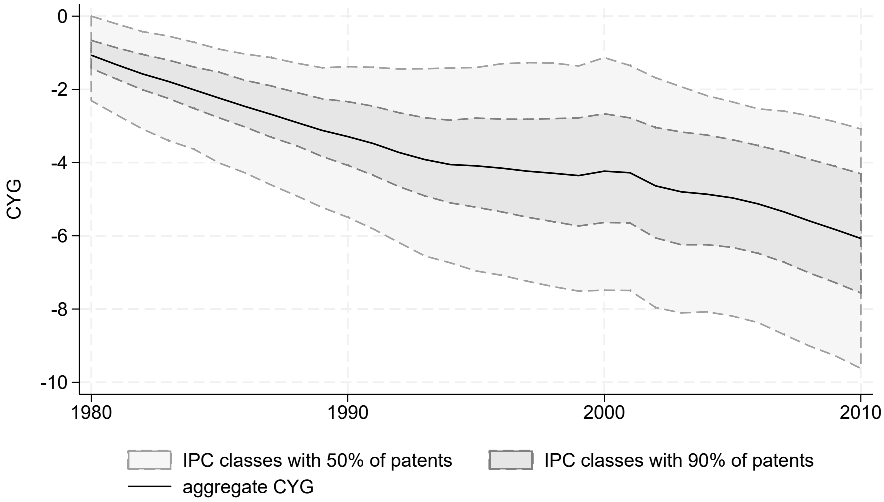
To understand what drives the aggregate decline in disruptive research, I split the data into different time consistent technology classes provided by PATSTAT. I create balanced panel of 75613 IPC classes from 1970 to 2010, though I only use data from 1980 onward due to coverage issues. Figure 2 reports the trend of CYG for all technology classes. Most IPC classes mirror the aggregate downward trend: Patents filed in 1985 get cited by patents that reference work on average ˜1 years older than the original patent. In 2010, the number has increased to ˜6. Though most IPC classes experience a decline in the CYG, the difference between the most disrupted and the most incremental technologies is rising, with some IPC classes even exhibiting rising disruptiveness as measured by the CYG during the 90s. A major factor in this is the revolution in ICT technology: Of the 25 IPC classes with the highest CYG in 2010, 19 are categorized as telecommunications and 9 among those as "transmission of digital information", 3 more are in "computing and image processing" and another 2 are in "games (including video)". The measure thus produces sensible results. Table 1 reports summary statistics for the IPC class panel.

2.5 Effects of radical innovations

I define a disruption event as a patent with more than 100 citations and a CYG of larger than 0, meaning that the citing patents refer to on average newer work than the original patent, i.e. the patent has made older work obsolete and is very impactful. Such patents are very rare (3347 in the sample): There is a double digit number of such patents per year in the 80s and 00s and between 100 and 500 per year in the 90s, in concordance with the overall time trends in disruption frequency discussed above.

To understand what happens after such a disruption event with the IPC class, I perform an event study: I match IPC classes to never-disrupted IPC classes in the same year and compare their evolution around the disruption event. Apart from exact matching on the year, I perform Mahanalobis distance matching on the CYG of the four years prior to the disruption as well as the number of patents in the IPC class. Table 1 reports summary

Figure 2: *Aggregate Evolution of Disruptive Innovation*



Notes: Average CYG per technology class over time. CYG measures how disrupted a technology is and is defined in 1 and discussed subsequently. In the aggregate over all IPC classes, the measure declines from -1 to -6. The CYG of individual IPC classes containing 50% of patents are contained in the dark gray area, the light gray area contains the CYG of 90% of IPC classes. The aggregate behavior is not driven by outlier IPC classes, declining CYG is widespread. However, some IPC classes maintain their average disruptiveness, especially during the 90ies. These are almost exclusively ICT-related.
Sources: PATSTAT (European Patent Office).

statistics for the two groups prior to matching as well as a comparison of the control and treatment group together with significance tests.

After the matching procedure, I obtain a sample of 1747 disrupted IPC classes and their nearest neighbor as a control. This is a substantial reduction from the previous 7316 disrupted IPC classes and is mainly due to the difficulty of finding matches for smaller, less cited and already less incremental IP classes: The Difference in CYG between unmatched Disrupted IPC classes and their potential control is large and does not allow to find matches for all disrupted IPC classes despite the large number of potential candidates.

The event study itself is estimated using OLS

$$y_{r,i} = \sum_{r=-5}^{r=15} \beta^r t_i^r + \Theta_i + u \tag{2}$$

where Θ_i is a matched pair fixed effect for pair i, t_i^r is relative time since the disruption and y stands in for different outcome variables of interest. Specifically, I look at the share of inventions that are categorized as ”disruptive” as a measure of the likelihood of a consecutive disruption and CYG as a continuous measure of the ”disruptiveness” of an IPC class. Figure 3 reports the results from 2. The matched pairs of IPC classes behave nearly identical prior to the disruption. After the disruption, the probability for

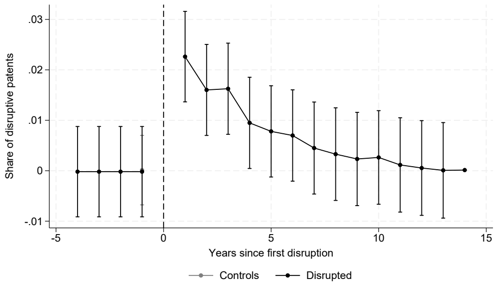
Table 1: *Summary Statistics on IPC classes before and after Matching*

	Panel 1: Before Matching			Panel 2: After Matching		
	Controls	Disrupted	Difference	Controls	Disrupted	Difference
CYG_{T-1}	-4.346	-0.477	3.869***	-2.445	-2.358	0.087*
	(3.119)	(1.850)	(0.042)	(1.355)	(1.344)	(0.046)
CYG_{T-2}	-4.226	-0.567	3.659***	-2.256	-2.167	0.089*
	(3.064)	(1.890)	(0.045)	(1.377)	(1.390)	(0.047)
CYG_{T-3}	-4.106	-0.592	3.514***	-2.132	-2.063	0.069
	(3.014)	(2.013)	(0.048)	(1.434)	(1.453)	(0.049)
CYG_{T-4}	-3.984	-0.771	3.213***	-1.993	-1.947	0.046
	(2.961)	(2.025)	(0.051)	(1.458)	(1.469)	(0.050)
$nr_{patents}$	4.025	8.735	4.710***	17.966	18.332	0.366
	(12.920)	(22.984)	(0.152)	(34.060)	(35.113)	(1.170)
$nr_{citations}$	20.004	67.807	47.803***	138.213	143.128	4.916
	(64.665)	(142.815)	(0.761)	(202.185)	(205.071)	(6.890)
$cohort_{T-1}$	20.179	41.625	21.446***	132.501	136.995	4.493
	(62.349)	(110.791)	(0.732)	(168.863)	(172.684)	(5.779)
Observations	2,797,758	7,316	2,805,074	1,747	1,747	3,494

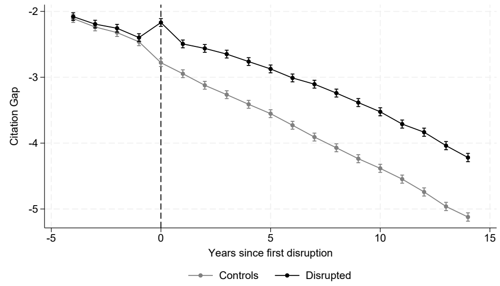
Notes: Unit of observation: harmonized IPC class first disrupted in year T. Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. CYG measures how disrupted a technology is and is defined in 1 and discussed subsequently. $cohort_{T-1}$ refers to the number of inventors that entered the IPC class in the year prior to the disruption. In the population, disruptions happen in already much less incremental IPC classes, measured by past CYG. Soon to be disrupted IPC classes are also larger, more cited and have more inventors enter. After the matching procedure, the differences are controlled for. It is worth noting that matching mainly works for larger, well cited IPC classes and the matched sample reduces substantially.

Source: PATSTAT (European Patent Office).

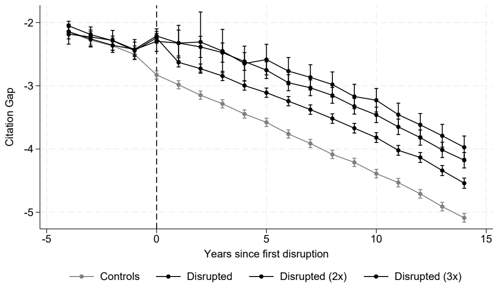
Figure 3: *Effect of Disruption on IPC class*



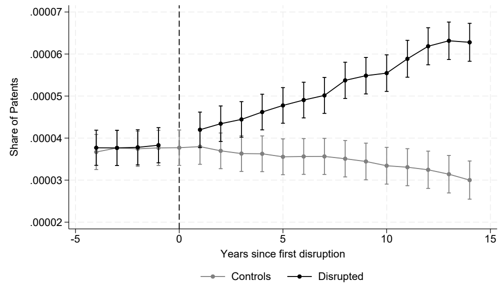
(a) Probability of new disruption



(b) Average CYG in the technology



(c) Multiple disruptions



(d) Share of Patents

Notes: Result of an event study in the nearest neighbor matched sample of firms. Panel a depicts the evolution of the probability of a new disruption after the first disruption. Panel 3b depicts the evolution of the citation year gap as defined in 2.3 as a measure of the overall "disruptiveness" of average patents in the technology class. Panel 3c is similar to 3b, but splits the group of disrupted IPC classes into those with one, two, three or four consecutive disruptions. Panel 3d depict the evolution of registered patents in the disrupted and undisrupted IPC class, expressed as a share of all patent of that year. *Sources*: PATSTAT (European Patent Office).

a consecutive disruption increases markedly. The effect size of a 0.6% chance one year after a disruption is substantial compared to the average probability of 0.2% per year. A disruption also increases the citation year gap by 0.75 (Panel B), i.e. patents referencing patents in the technology class now reference literature that is roughly 0.75 years younger. After a disruption, the IPC class again begins to trend back towards incrementalism, in accordance with Gordon (2016). The effect of consecutive disruptions (Panel C) is smaller than that of the first disruption, but the general patterns remain. Panels D depicts the evolution of the number of patent applications: Disrupted IPC classes grow in terms of pure quantity, while undisrupted IPC classes shrink.

The results from this analysis present additional facts that the model should be able to explain:

5. Technologies' trend towards incrementalism is reversed by discrete, high impact, disruptive inventions.

6. Disruptive inventions increase the likelihood for consecutive disruption.

7. The first disruption has the highest impact on the technology class.

3 Model

This section develops a tractable endogenous growth model that captures the stylized facts discussed in section 2. In this model, firms have to choose between disruptive and incremental research, taking into account other firms' decisions. The strategic interaction occurs on the labor market for inventors, where firms poach each others' inventors to pursue research. The model builds on Akcigit and Kerr (2018), but both the inventor labor market and the strategic interactions it causes are new additions.

3.1 Technology and product structure

The inventors who drive technological progress are at the heart of this endogenous growth model. Inventors choose the firm they work with and the type of innovation they pursue. Producing firms (=incumbents in the terminology of Akcigit and Kerr, 2018) are threatened by disruption and can poach inventors from disruptive firms (=entrants) to protect their technologies.

Technology is differentiated into broad fields or disciplines like "telecommunications" or "electricity generation". Each field corresponds to one product. Within each of these fields, technology clusters denote distinct areas of knowledge like the clusters "telegraphy" or "satellite communications" in the field "telecommunications". These clusters are areas of expertise for individual inventors, who cannot be experts in whole fields or even all sciences. Within each field, there are old obsolete clusters ("telegraphy"), a currently active cluster ("satellite communication") and as of yet still unknown future clusters.

Firms cannot conduct research on their own and have to hire inventors. The majority of inventors are specialists who studied one specific technology cluster and are dedicated to improving it further. Every invention these incremental inventors make increases the product quality of their firm, but does not change the general technology structure. An example of incremental inventors are the engineers who improve the internal combustion engine. Incremental inventors no longer contribute to the economy if this technology becomes obsolete. Because of this restriction, technology clusters play a large role in inventors' and firms' calculations. Throughout the rest of the paper, I will use the words cluster and technology cluster interchangeably.

Occasionally, major breakthroughs in a technology field create an entirely new, better technology cluster within the same field. An example are current efforts to use hydrogen or electric energy to power cars. Hydrogen or electric cars will then form another technology cluster within the broader field of "vehicle construction". Disruptive inventions are proofs of concepts for better technologies: The first telegraph, the first power line or the first electrical train were not viable consumer products, but demonstrated the feasibility of the technology. Subsequent incremental innovations then create actual products that can enter the market. Each cluster is better than the last one in the sense that it enables more impactful incremental follow-up innovation: The quality improvement from one incremental invention in a technology cluster is

$$\Delta q(c) = \omega^c \tag{3}$$

where c denotes the number of the cluster. In the above example, an incremental invention that improves the telegraph would generate ω^1 additional quality for the inventing firm. An incremental improvement of the telephone would create ω^2. Thus, parameter $\omega > 1$ determines how substantial the gains from disruptive inventions are: If $\omega = 1.20$, a telephone improvement would generate 20% more quality than a telegraph improvement.

Clusters are indexed by their field and a running number c. Taking the field of telecommunication as an example, the telegraph might be $c = 1$, the telephone might be $c = 2$ and so forth. Slightly abusing notation, I will drop the index $field$ for now, since all fields in the model are symmetrical. Thus, the index is only relevant when aggregating over the whole economy. In the following, capitalized variables denote aggregate variables (like the probability for disruption in a technology field Λ^{dis}) and lower case letters describe microeconomic variables (like the number of patents for firm f $nr_f^{patents}$). Parameter notations follow precedents in the literature whenever possible.

Within each technology field, there exists a continuum of disruptive inventors who aim to create breakthroughs. These generate prototypes of future production technologies. Whenever these firms are successful, a new technology cluster is born and the old cluster becomes obsolete. Old incremental inventors can no longer contribute to products based on the new technology, but disruptive inventors and firms can immediately work on disrupting the new technology again. The disrupting firm also founds a new producing firm which will use the newly created technology.

3.2 Research actors

The innovations described in section 3.1 are pursued by four different actors: entrepreneurs, incremental inventors, disruptive inventors and producing firms. Entrepreneurs enter each technology field economy at constant rate eta_e. Entering entrepreneurs, incremental inventors and disruptive inventors match with each other and entrepreneurs each found new incremental firms with randomly drawn firm quality y_f. Entrepreneurs exit the economy at rate δ. When they do, the inventors that are matched with them exit with them. This assumption simplifies the dynamics of firm size, but it will still ensure that inventors and firms exit with probability δ.

Incremental firms match with incremental inventors and bargain about wages. They also must decide whether they want to enter the inventor labor market for disruptive inventors (to poach them and keep them from making disruptive inventions). There are no costs associated with entering, since I abstract from vacancy creation in the baseline specification. The patents that firms and incremental inventors jointly produce increase the quality of the product in the associated research field. Demand and supply are such that no matter which firm holds patents improving the quality of the product by Δq, they yield a revenue stream of $\Delta q * \pi$. The demand structure is a derivative of Akcigit and Kerr (2018) and is discussed in detail in section 3.6. In the baseline model, a firm's incremental inventors will become obsolete after a disruptive invention, but its patents will remain functional for simplification. Therefore, patents earn their profit stream indefinitely and are being valued at

$$V^p(\Delta q) = \frac{\pi}{r} * \Delta q \tag{4}$$

Incremental inventors enter each technology field at rate eta^{inc} and draw a random skill level x_i from the uniform distribution between 0 and 1. Since the baseline model has no vacancies, they then match with a random newly entering entrepreneur and will receive no further matches after that. The match remains existant whether or not the two potential partners work together and both sides can freely choose to stop or resume working with each other. When not working for their matched firm, incremental inventors do not produce any output. Incremental inventors exit the economy at rate δ due to their matched firms exiting.

Disruptive inventors enter the field at rate eta^{dis}, assumed to be much lower than eta^{inc}. Like incremental inventors, they match with a random firm which is searching for disruptive inventors. In the same way as with incremental inventors, the match with "their" firm is stable and continues to exist no matter their current employment. However, unlike incremental inventors, disruptive inventors do not produce anything when working for an incremental firm and produce their actual output when not connected to a firm: In that case, they will work on disrupting the current technology, succeeding at rate x_i. If the producing firm they matched with offers them a good enough wage, they no longer try to disrupt the field. Without vacancies, incremental firms pay no costs to connect to disruptive inventors, so the key question determining whether disruptive inventors sell out is whether incremental firms want to pay enough to suppress disruptive inventions. Disruptive inventors also exit the economy at rate δ due to their matched firms exiting.

With the above assumptions, the amount of (incremental) R&D is essentially fixed, since the number of firms and incremental inventors is exogenous. This is because the novel facts the model aims to explain concern the type of innovation performed, not how much. Since the number of researchers is increasing globally and within countries, the amount of research is not a likely explanation for the productivity slowdown.

Note that right after a disruption, each technology field looks the same, except for a factor ω^c: All disruptive inventors are working and no incremental inventors and no producing firms exist (yet). Incremental inventors then enter at rate eta^{inc}.

3.3 Inventor Firm matching

After an incremental inventor has matched with a producing firm, the two engage in Nash bargaining. Since both can freely terminate and renegotiate the contract, they bargain over the wage of the current period only. However, neither side has a credible outside option. The inventor will not get additional matches and will thus be stuck negotiating with the firm again. The firm might have additional matches with inventors, but rejecting the match does not yield any additional match opportunities. The inventor produces $\pi\omega^c y_f x_i$, where π is the constant profits that can be expected from increasing productivity, ω^c is the productivity improvement of one incremental invention which depends on the index of the current technology cluster c and $y_f x_i$ is the rate at which new inventions are generated, a function of firm quality and inventor skill. Since both outside options are 0, the entire output of the match is the surplus, which will be divided between firms and inventors according to the bargaining parameter α:

$$w(x_i) = (1-\alpha)\pi\omega^c y_f x_i \tag{5}$$

In equilibrium, incremental firms and inventors will always work together to produce patents, with the inventor i earning share $(1-\alpha)$ of the expected profits as wage and firm f earning the remaining share of α.

Disruptive inventors entering the economy and meeting with a producing firm also Nash bargain over the price. In contrast to incremental inventors, they have an outside option, namely to disrupt the economy with probability x_i. With this outside option, the disruptive inventor will earn

$$f(x_i) = x_i * E(V^f(\Delta q = 0, y_f, \Lambda^{dis} = \frac{\eta^{dis}}{\delta}, c = c+1)) \tag{6}$$

where the value of the outside option depends on the skill of the inventor x_i because it determines the rate of his inventions. If the disruptive invention is successful, the inventor will gain a new firm in the newly created cluster. This firm will have a patent portfolio of $\Delta q = 0$, a randomly drawn firm quality y_f, with all disruptive inventors active $\Lambda^{dis} = \frac{\eta^{dis}}{\delta}$ and in the technology cluster one above the current one $c = c+1$. In Nash bargaining, the match will always be formed if surplus is positive, i.e. if the producing firm gains more from stopping disruptive innovation than $V_i^{dis}(x_i, \Lambda^{dis})$, no matter the bargaining parameter (Rogerson et al., 2005). The firm's optimization determines whether it will poach disruptive inventors.

3.4 Firm optimization

The incremental firm takes inventor behavior as given when maximizing its own value. Each technology field is described by t, the time since the last disruptive invention. Since the model is inhabited by masses of incremental firms and inventors, incremental invention will follow the expected path due to the law of large numbers. At each point in t, any field is described by

- $\Lambda^{inc}(t)$, the rate of incremental innovations and also the output weighted number of incremental inventors at time t
- $\Lambda^{dis}(t)$, the rate of disruptive innovations and also the output weighted number of disruptive inventors at time t
- $M^f(t)$, the mass of firms in the field at time t
- c, the index of the current technology cluster, which will stay constant until a disruptive invention is actually made
- eta^{inc}, eta^{dis} & eta^e, the number of incremental inventors, disruptive inventors and entrepreneurs entering the field and matching with each other

New firms have obtained their matches exogenously. All firms have to decide whether or not to pay the wages of incremental and disruptive inventors in order to actually work with them. The value of a firm is derived from its stock of patents ($V_f^p(q)$), its quality (y_f), the stock of matches with which it can potentially work (x_f^{inc} & x_f^{dis}), the technology cluster index c and the rate of disruptive inventions ($\Lambda^{dis}(t)$), which will determine for how long it can expect to use these assets.

This yields the firm value function:

$$V_f^f(q, y_f, x_f^{inc}, x_f^{dis}, \Lambda^{dis}) = V_f^p(q) + V_f^{inc}(y_f, x_f^{inc}, x_f^{dis}, X^{dis})$$

i.e. the value of a firm is derived from the sum of its patents and the value of its stock of inventors. Note that the stock of patents (and its value) is not affected by any decision the firm makes now: These patents will continue to return $\pi\Delta q$ forever no matter what. Hence the additive separability of firm value $V_f^f(q, y_f, x_f^{inc}, x_f^{dis}, \Lambda^{dis})$.

I conjecture that the value of a firm's stock of inventors is a linear function of ω^c and y_f, since all patent quality gains are multiplied by these factors and firms pay wages as a fraction of their expected profits from these quality gains. I will verify later that this is actually the case.

The firm maximizes the value of its inventor matches by maximizing

$$rV_f^{inc}(y_f, x_f^{inc}, x_f^{dis}, X^{dis}) = \alpha\frac{\pi}{r} * \omega^c \lambda_f^{inc} - w(\lambda_f^{dis})$$

$$-(\Lambda^{dis}+\delta)V_f^{inc}(y_f,x_f^{inc},x_f^{dis},X^{dis})+\dot{V}_f^f(q,y_f,x_f^{inc},x_f^{dis},\Lambda^{dis}) \tag{7}$$

The firm can decide on λ_f^{inc} and λ_f^{dis}. The first term describes the firm's gains from incremental inventors, already net of the wages paid to these inventors as per equation 5. The second term in the first lines describes wages paid to disruptive inventors, which depend on how many of the matched inventors the firm actually wants to hire, paying at least their outside option as described in equation 6. The second line contains the risks of operations: the rate of agent exit δ, which will force the termination of the firm, and the rate of disruptive inventions Λ^{dis}. The last term represents the change in value over time (due to changes of the macro parameters). In the baseline model, the firm will trivially always hire matched incremental inventors, so $\lambda_f^{inc}=y_f*x_i$. In order to decide whether to hire disruptive inventors, the firm has to maximize equation 7, taking into account that $\Lambda^{dis}=\int_f \lambda_f^{dis}$. Taking the derivative with respect to λ_f^{dis} and rearranging, one gets the first order condition in terms of y_f, the research quality of the firm

$$y^{sklerosis}=\frac{\gamma*\omega*V_f^{inc}(y_f=1,x_f^{inc},x_f^{dis},X^{dis}=\frac{\eta^{dis}}{\delta})}{V_f^{inc}(y_f=1,x_f^{inc},x_f^{dis},X^{dis})} \tag{8}$$

where the nominator represents the value of a new firm in the next technology cluster (i.e. the return to a disruptive innovation) and the denominator represents the value of a firm in the current technology cluster.

Equation 8 yields the paramters under which the model will adhere to the observed data: Recall from section 2 that the rate of disruptive inventions jumps up after a disruptive invention but begins to fall again immediately. After a disruptive invention, $X^{dis}=\frac{\eta^{dis}}{\delta}$, which will simplify equation 8 to

$$y^{sklerosis}=\gamma*\omega \tag{9}$$

i.e. firms that have a research quality higher than $\gamma*\omega$ will immediately start to poach inventors. If the disrupting inventor gains a large advantage in the newly created technology cluster γ or if the research productivity gain from disruption ω is large, only higher quality firms of the current cluster will find it profitable to poach disruptive inventors. If the product of both parameters is high enough, no firm will find it profitable to poach and the rate of disruption will stay at its initial level indefinitely. In this case, no endogenous productivity growth slowdown will occur.

In the baseline labor market, only newly entering disruptive inventors match with poaching firms. They match with exactly one firm (if such a firm exists) and a match will be formed. The exact wage the disruptive inventor earns depends on the research quality of his match and the time since the last disruption (since incremental inventor stocks become more valuable over time as more disruptive inventors get poached and the risk declines). Since all newly entering inventors get poached if $y^{sklerosis}<1$, the equilibrium stock of actually disrupting inventors declines with rate δ, with which active disruptive inventors leave the economy but do not get replaced.

This yields

$$\Lambda^{dis}(t) = \frac{\eta^{dis}}{\delta} e^{-\delta t}$$
$$\Lambda^{inc}(t) = \frac{1}{2}\frac{\eta^{inc}}{\delta}(1 - e^{-\delta t}) \tag{10}$$

as the equilibrium arrival rates of disruptive and incremental patents as a function of the time since the last disruption: The stock of disruptive inventors decays at rate δ because new inventors get poached and current inventors exit the economy. In contrast, incremental inventors were all forced to exit with the disruptive invention and are now entering again at rate η^{inc}. The stock of incremental inventors accumulated this way then also decays at rate δ.

3.5 Extension: Vacancies and Unemployment

New inventors search for firms' open vacancies. In contrast to the standard search and matching labor market model, I assume that new inventors enter the labor market, immediately find a match among the available vacancies and that unmatched inventors have to leave the economy because they lose their connection to recent developments. The research avenues that are represented by vacancies also become superseded by new approaches if they do not match. This reduces the complexity of the labor market, because the mass of unemployed inventors does not matter for the equilibrium anymore, since they cannot contribute to the economy. This simplifying assumption eliminates two state variables from each field's inventor labor market: the number of unmatched vacancies and the number of unmatched inventors. This assumption leads to the same steady state outcome, but the path towards that steady state is much more tractable. Figure (4) describes the path towards labor market equilibrium after a disruptive innovation for both specifications.

How many vacancies firms will create in this setting depends on the value of obtaining an additional inventor. This value is determined by the number of patents the new inventor will produce and by how much the firm has to pay to the inventor.

3.6 Consumer Demand, Patent Value and Static Profits

Throughout the previous discussion, I assumed that patents yield a steady stream of profits equal to a constant π times the quality increase that each patent represents.

This assumption can be microfounded in a number of ways, most notably as in Akcigit and Kerr (2018). In their model, firms sell their products to a love-of-variety final goods sector and profits only depend on product quality and exogenous demand and cost parameters. As is standard in these settings, firms compete against the (appropriately weighted) average product in the market and not any specific firms. For the baseline specification of this paper, I present a close derivative of this model where I increase the role of technology

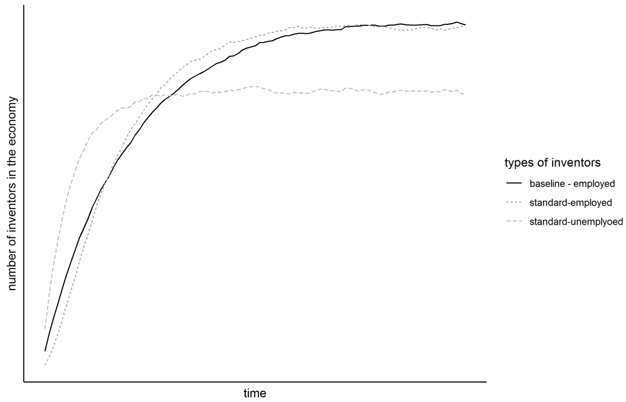


Figure 4: The graph shows the evolution of the number of incremental inventors in a technology cluster after its foundation. Over time, more and more inventors enter the cluster, until the steady state level is reached. The baseline specification of the model is presented in black. The grey lines depict the stock of employed and unemployed inventors in a more standard model for comparison. Such a model has slightly less employed inventors early on, because inventors enter into unemployment and leave it over time. However, not only do both models give the same kind of path qualitatively, the two paths are also quantitatively close. Assuming that inventors cannot be unemployed increases tractability without greatly changing even the quantitative results.

clusters and introduce technology fields.

I also present an alternative specification of consumer demand that also leads to profits linear in quality in appendix A. This is to emphasize that the specifics of demand do not drive my conclusions and labor markets are tractable enough so that they can be inserted in different GE-models. This alternative justification of linear profits is based on a Salop circle demand framework. In this framework, every firm competes against specific firms (its neighbors on the circle), which opens up the possibility to extend the model for strategic interactions between individual firms, not only all firms in a submarket.

Consumers are part of a representative household and derive logarithmic utility from consuming a final good (Y) in continuous time. This final good is the numeraire good.

$$U = \int_0^\infty e^{-rt} \ln(Y(t))dt$$

Consumers are impatient and discount with factor r. They neither face a tradeoff between leisure and consumption, nor do they experience inequality. Households evenly share income from all sources between their members.

A final goods industry produces the consumption good from labor and a variety of intermediate inputs and sells it to consumers. The industry produces according to

$$Y(t) = \frac{1}{1-\beta} L_c^\beta(t) \int_0^1 q_j^\beta z_j^{1-\beta} dj \tag{11}$$

where q_j is the quality of good j, z_j is its quantity and $L_c(t)$ is the labor expended in final goods production. If all product qualities are fixed, the production function exhibits constant returns to scale in labor and intermediate inputs. With increasing product qualities q_j, the production function exhibits increasing returns to scale.

Each product j corresponds to a technology field. To become a producing firm for product j, firms enter the current frontier technology cluster in that field and hire inventors.

The economy contains a mass 1 of production workers which I will call technicians. Technicians have undergone vocational training and cannot become inventors. However, they can contribute to the production of any good, regardless of the specific technology. Since all technicians are perfect substitutes and firms' research quality does not matter for production, no matching is necessary. There is a perfectly competitive spot market for technicians' labor without search costs.

The final goods industry is a price taker, consisting of a multitude of small competing firms. Hence, its inverse demand for any one intermediate good is

$$p_j = L_c^\beta(t) * q_j^\beta * z_j^{-\beta}$$

The price that the final goods industry is willing to accept for variety j of the intermediate good increases as more technicians work in the final goods industry $L_c(t)$ and the quality of the variety becomes higher. If the final goods industry buys a higher quantity z_j, the acceptable price declines.

In each of these intermediate goods sectors j, producing firms compete to satisfy this demand. They produce intermediate goods using labor:

$$z_j^f = l_j^f * \bar{q}$$

Firms use one unit of labor to produce one unit of an intermediate good of average quality.

As is standard in the literature, these firms compete in a two-stage Bertrand game: In stage one, every firm decides whether it wants to incur an arbitrarily small set-up cost ϵ to be able to produce. In stage two, all remaining firms engage in Bertrand competition. Since the result of Bertrand competition will be that only the firm with the highest quality produces, only this firm will incur the cost ϵ and it will be the monopolist in the second stage of the game.

A single monopolist with a given product quality will set the profit maximizing price and produce quantity

$$z_j^* = q_j * L_c(t)(\frac{(1-\beta)\bar{q}}{w})^{\frac{1}{\beta}} \tag{12}$$

Importantly, demand for the monopolist's products depends on the amount of labor employed in the final goods industry since production workers process the intermediate inputs.

The mass of small firms in the final goods sector will optimize their labor and intermediate goods intake and through this set the wage rate. Optimizing equation (11) with respect to labor and inserting the equilibrium on the intermediate goods market (equation 12) gives the optimal wage as

$$w = \beta^{\beta}(1-\beta)^{1-2\beta} * \bar{q} \tag{13}$$

i.e. the final goods industry will adjust its labor demand to achieve a wage rate as a multiple of the average quality $\bar{q}$ in the economy. The precise multiple is dictated by labor's output elasticity β. This behavior is optimal because the supply of intermediate varieties is itself a function of $L_c(t)$ (equation 12).

Producing firms make the important decisions in the model, since their decisions about hiring inventors will determine technological progress and dynamic equilibrium. However, their downstream decisions have no dynamic component: Labor input, quantity sold and price can be adjusted at any point in time. Taking into account that the final goods industry will always fix the wage rate (equation 13), the optimal quantity decision for a producing firm gives equilibrium profits as

$$\pi_{mon}^* = L_c(t) * (1-\beta) * \beta^{\beta}(1-\beta)^{1-2\beta} \tag{14}$$

Thus, a monopolist's profits are a linear function of quality and (from the viewpoint of the firm) an exogenous factor called π throughout the rest of the paper.

So far, this framework is deviating from the setup in Akcigit and Kerr (2018) in two ways: First, I introduce technology fields, equate them with products and prohibit producing firms from creating disruptive inventions. Together, these changes mean that producing firms no longer face a general threat of disruption from firms throughout the whole economy. Instead, only a distinct set of disruptive inventors within their own technology field pose a threat to producing firms.

Second, there are now multiple producing firms within one technology cluster. As in Akcigit and Kerr (2018), incremental inventions increase product quality, but I now have to make some assumptions about how producing firms split the revenues and how this is

affected by new inventions. To keep the model tractable, I will assume that incremental inventions are unique, non-substitutable and additive. Hence, a producing firm that makes an incremental invention will not necessarily displace the current best product as in most ladder models, but just gain ω^c product quality. I assume that all producing firms within a technology cluster then pool all their patents to create the best possible product and split the revenues from selling that product according to the quality contribution that each firm was able to make with its patents. Since all inventions are unique and non-substitutable, market power lies with whoever holds each individual patent, who can make a take-it-or-leave-it offer to the pool of the other firms. Thus, each producing firm can extract the value of its patents, no matter which firm will actually produce.
Using the HJB, the value of the firm's patent portfolio is:

$$V(\Delta q_f) = \frac{\Delta q_f \pi}{r} \tag{15}$$

The value of the patent portfolio of a firm thus only depends on the impatience to consume r and Δq_f, the quality improvement that this patent portfolio makes possible. For simplicity, patents do not expire. Importantly, the value of a patent portfolio is independent of the number of researchers in any firm.

This assumption is unusual insofar as the typical quality ladder model would assume that a successful incremental invention creates a product one step above the currently existing one. Instead, in my model, an invention represents a quality improvement that any firm in the cluster could in principle use to improve their product. If the firm is not currently producing, it can license the invention to the currently producing firm to increase the quality of their product further.

3.7 Technological Progress

Technological progress in the economy is twofold: Incremental inventions improve the average quality of the products in the economy and ultimatively increase the utility of consumers. Disruptive progress increases the value of future incremental progress and ensures long term growth: Without disruptive innovation, the economy still grows as new incremental inventions increase quality, but growth as a percentage of GDP declines because incremental inventions can only create linear growth.

Each technology field faces a chance of disruption Λ^{dis}, upon which a new, more valuable technology cluster emerges. The rate of disruptive inventions is declining as more and more disruptive inventors get poached by producing firms (**??**). For any specific technology field, this creates a saw blade like graph of the rate of disruption (Figure 5a).

In the steady state growth path, the number of technology fields with any specific rate of disruption $N_field(\Lambda^{dis})$ is stable, which keeps both the aggregate rate of disruptions and the rate of incremental inventions constant. Note that Λ^{dis}_{field} fully characterizes a field, since it is monotonely dependent on t (time since last disruption), as is Λ^{inc}_{field}. Λ^{dis}_{field} or t both define the type of a field.

In the steady state, the inflows into any type must equal the outflows. Since fields that are disrupted move to $\Lambda^{dis}_{field}(t=0)$ and t_{field} increases linearly for all other fields. This translates to the differential equation $-N_{field}(t_{field}) * \Lambda^{dis}_{field}(t_{field}) = \dot{N}_{field}(t_{field})$, which is solved by

$$S(\Lambda^{dis}_{field}(t_{field})) \stackrel{!}{=} e^{\Lambda^{dis}_{field}(t_{field})\frac{1}{\delta}} \tag{16}$$

where $S(\Lambda^{dis}_{field}(t_{field}))$ denotes the share of fields that are in state $\Lambda^{dis}_{field}(t_{field})$: The stability condition enforces a certain distribution. The number of fields in each state depends on the total number of fields. The steady state distribution of types requires that the number of fields per $\Lambda^{dis}_{field}(t_{field})$ increases the higher $\Lambda^{dis}_{field}(t_{field})$ is: fields' rate of disruption declines over time, but this requires no disruption for a longer time period, so the share of fields that get to this point has to become smaller if the economy is supposed to be in equilibrium. In contrast, the rate of exit decreases the difference between incremental and disruptive fields: The stock of disruptive inventors and thus the rate of disruptions declines with rate δ as time goes on, so a high δ means that fields change their state more slowly.

At each point of this distribution of fields, fields get disrupted with $\Lambda^{dis}_{field}(t_{field})$. The number of these disrupted fields has to add up to $N(\frac{\eta^{dis}}{\delta})$ to ensure that the distribution remains unchanged. Integrating over equation 16 yields

$$e^{\frac{\eta^{dis}}{\delta} * \frac{1}{\delta}} \stackrel{!}{=} (\frac{\eta^{dis}}{\delta} - \delta)\delta * e^{\frac{\eta^{dis}}{\delta}\frac{1}{\delta}} \tag{17}$$

The left side represents the number of newly disrupted fields to maintain the stable distribution, while the right side represents the inflows from all states (including $t=0$, i.e. fields that get disrupted again immediately). Clearly, 17 is only fulfilled for $\eta^{dis} \stackrel{!}{=} \delta^2 + 1$. Since both are unrelated continuous parameters, this is never the case exactly. Thus, the economy has no steady growth path except a corner solution: In the steady state equilibrium, disruptive innovation is 0, while all incremental inventors work.

The whole economy consists of many such sectors and is thus not subject to the randomness of any one sector. The expected change in Λ^{dis} for any one technology field or sector is

$$E(\Delta\Lambda^{dis}) = \Lambda^{dis}(\Lambda^{dis}(0) - \Lambda^{dis}) - \Lambda^{dis}(r + \delta + \frac{1}{2}\Lambda^{dis}) \tag{18}$$

The first term describes that with probability Λ^{dis}, a disruptive invention will occur and set Λ^{dis} to $\Lambda^{dis}(0)$. The second term encapsulates the poaching efforts of producing firms, which will decrease the rate of disruptive inventions by $\Lambda^{dis}(r + \delta + \frac{1}{2}\Lambda^{dis})$. Note that the rate of disruptive inventions is clearly declining for every value of Λ^{dis} as long as

$\Lambda^{dis}(0) < r + \delta$. This restriction will hold in most real world applications: **?** estimate a separation rate of around 1.5% for the average OECD country, which together with a real interest rate of a conservative 2% would imply that the average sector of the economy stays undisrupted for 18 years or more. Even if this parameter restriction is violated, the qualitative result remains, but it becomes much harder to show analytically. Appendix B details the results of the empirical simulation.

The exact parameters notwithstanding, it is clear from equation 18 that the only possible equilibrium for any one sector is that the rate of disruptive inventions is zero: It is the only point where both the expected change and the change in case of no disruptive inventions are zero, so the technology field will never leave this point. Any technology field will thus eventually experience the decline of disruptive inventions to zero, even if $\Lambda^{dis}(0)$ is very high. More and more technology fields will be stuck in this equilibrium, until eventually all of them are.

Nevertheless, incremental inventors will add to the product quality of producing firms at each point in time. How much quality they add to the economy depends on the technology cluster they are in: Technology fields with large inventor portfolios and more disruptive inventions in the past will contribute more quality increases (equation **??**). Technological progress through incremental invention in one technology field then is

$$\Delta q_c(t_c^e) = \frac{\eta}{\delta}(1 - e^{-t_c^e})\frac{1}{6} * \omega^c \tag{19}$$

i.e., progress is a function of the number of inventors in that cluster and the quality increase that an incremental invention in the frontier cluster of the field creates.

Technological progress in a field changes as additional inventors enter the field, old inventors leave and disruptive inventions make the whole stock of inventors obsolete:

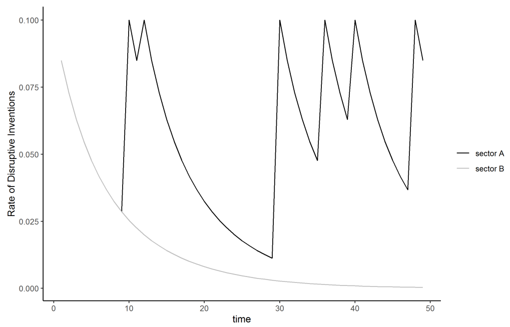
$$E(\dot{\Delta q_c}(t_c^e)) = \omega^c * \eta\frac{1}{6} - \delta\Delta q_c(t_c^e) - \Lambda^{dis}\Delta q_c(t_c^e) \tag{20}$$

If no disruptive invention happens, the frontier cluster will eventually absorb all inventors in the field. At this point, technological progress will be linear, as each inventor produces a set amount of inventions, each of which adds a fixed amount of quality ω^c. This is the steady state outcome: The rate of disruption will eventually decline to 0 and after that, all inventors will eventually work in that cluster as $(t_c^e) \to \infty$.

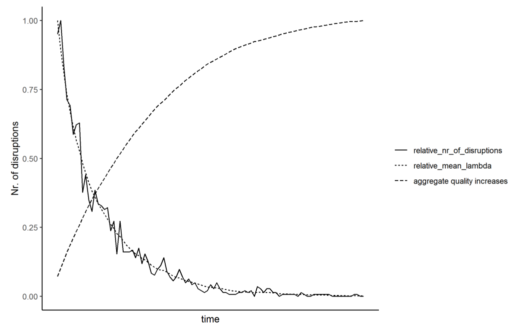
4 Description of Equilibrium and Policy Implications

The economy presented in the baseline specification has several major decision points, only some of which the market economy handles efficiently.

Figure 5: *Simulation result for example economy*



(a) Example evolution of one sector



(b) Example evolution of simulated economy with 2000 sectors

Notes: Example of a simulated economy in Equilibrium. Panel 5a depicts the evolution of the expected rate of disruption in two technology fields. Sector A experienced several disruptive inventions. Sector B did not make a disruptive invention and thus experiences continuously declining chances of future disruption. Panel 5b depicts the simulation result for the entire economy: The solid black line denotes the number of disruptive inventions at each point in time for a simulated economy with 2000 sectors. The dotted line denotes the theoretically expected number. As disruptions become less frequent, the aggregate rate of quality growth in the economy slows down until linear growth is reached in the steady state. *Sources*: Own simulations.

First, there is the demand of the final goods sector for intermediate products to turn into the final consumer product. The economy has a fixed number of products defined by how many technology fields there are and all of them are produced in equilibrium. However, the quantity produced is smaller than in the optimum because of the monopoly power of intermediate goods producers. This inefficiency depresses output by a fixed share, but has no impact on equilibrium growth rates.

Second, intermediate goods producers have to hire incremental inventors to improve their product. Producers hire all incremental inventors by assumption (because new inventors are guaranteed to draw a job). So, there is no inefficiency in this dimension.

Third, disruptive inventors work on disrupting the economy and get poached by producing firms to prevent this. The market economy weighs the costs of disruptive inventions against the entry costs for producing firms: A successful disruptive inventor is not able to appropriate all the benefits from his invention as profits because other entrepreneurs can enter the new technology cluster that he has created. Producing firms bear all costs from disruption and receive none of the benefits, thus they have a strong incentive to prevent disruption. A social planner that maximizes the utility of representative households makes a very different calculation: He weighs the value of getting inventors empowered by the disruptive invention in the future against the costs of losing all current inventors:

$$\int \frac{1}{4}\eta V_i^{inc}(t)e^{-rt}\,\mathrm{d}t \geq \frac{V^{patent}\Lambda^{inc}(t)}{(r+\delta+\frac{1}{2}\Lambda^{dis})} \tag{21}$$

Firms do not make this calculation since the value of the future inventors that other firms will get does not factor into their profits. It is apparent that a social planner might even arrive at the same conclusion as the market economy if the discount rate is sufficiently high: Empowering future inventors takes longer to pay off than current incremental inventions.

This highlights an important point about the tradeoffs involved in the decision about which type of research to pursue: Increasing long run economic growth in this model requires unambiguously hurting the current generation. The currently living incremental inventors and firms have a vested interest in slowing economic growth. Fast productivity growth through disruption does not benefit them, but the inventors and firms who will enter the newly created cluster. A social planner might want to solve this via transfers, but even that might not work: The current stock of incremental inventors is made obsolete, temporarily decreasing GDP. While it will eventually be rebuilt and growth will increase, many incremental inventors and firm owners that were hurt by the disruption will already have left the economy. Effectively, the current generations prefer to increase the level of economic activity through incremental inventions at the cost of economic growth. Of course, the linear technological progress of incremental improvements is still progress, but it means that the growth rate of the economy will continuously decline.

If the social planner wants to preserve the arrival rate of disruptive inventions in the economy, he has to slow down the rate at which producing firms poach disruptive inventors. There are in principle two ways to achieve this: One is to decrease the value of the stock of incremental inventors, which decreases the incentive to poach. Increasing the separation rate of inventors and firms or decreasing the market power that firms enjoy on the goods market would both work in this direction. However, these are large interventions into the markets. The second route is to decrease the ability of large producing firms to poach disruptive inventors. An easy step in this direction would be to restrict startup acquisitions significantly. There is an active literature on the questions of whether startup acquisitions are welfare enhancing (Cabral, 2018; Piazza and Zheng, 2019). My paper offers an additional argument for prohibiting such acquisitions.

Income in the economy is derived from the wages of technicians, the profits of firms and the wages of incremental and disruptive inventors. As in the base model of (Akcigit and Kerr, 2018), the revenue of producing firms within each technology field/product is constant. Of that revenue, a fixed share goes to technicians pay for labor input into production. The remainder pays the rents of firms and their investments into inventors.

Firms pay out a fixed share α of the quality increases that incremental inventions produce to their incremental inventors. If firms hire disruptive inventors, they pay them out of their share $1 - \alpha$. This reduces their stream of profits, but increases the expected duration of this stream. Poaching disruptive inventors is only profitable if the expected profits of disruptive inventors ($\Lambda^{dis}\omega^c f_e$) are smaller than what firms can earn from incremental inventions ($(1-\alpha)\Lambda^{inc}V^{Patent}$). Otherwise, equation (**??**) will yield a sclerosis threshold larger than 1 and no firms will actually poach inventors.

5 Conclusion

As productivity growth is declining across frontier economies, it is urgent to understand firm innovation as a determinant of productivity development. The main contribution of the paper is to build an endogenous growth model around the difference between radical/disruptive or incremental innovation and the strategic decision that this poses on firms which produces declining aggregate growth. This model fits empirical data on innovation, both from other studies and estimated in the empirical part of the paper using global patent data between 1980 and 2010 (PATSTAT). In the model, firms have to build a portfolio of specialized inventors to do research on a search and matching labor market. With these inventors, firms improve technology and their product incrementally. However, such firms are invested in existing technologies and are threatened by disruptive inventions that might make "their" technology obsolete. However, firms can use the inventor labor market to poach disruptive inventors and mitigate this threat. Over time, incremental firms prevent an increasing share of technology disruption and become even more valuable, increasing the costs of disruption further. In aggregate, the economy stagnates due to a lack of technology disruptions.

The model describes the situation found in empirical work well: Section **??** documents stylized facts about incremental vs. disruptive inventions. These are partly collected from previous work: Funk and Owen-Smith (2017) and Park et al. (2023) have documented that research is becoming more incremental and less disruptive. Poege et al. (2019) and Akcigit and Kerr (2018) have grouped patents into incremental improvements and more radical innovation using the quality of scientific literature linked to the patent and the citations from other firms. Both have found that more ambitious patents are more valuable to the applicant. Despite that, firms' research has become more incremental (Arora et al., 2019). Firms produce more and more patents with an increasing number of researchers, whose productivity is falling, yet whose wages do not decrease (Cowen and Southwood, 2019; Bloom et al., 2017). To supplement these results from the literature, I estimate an event study to understand how technology fields respond to disruptive inventions. Technology fields after a disruptive invention have a higher chance for successive disruptions while more patents are filed and patents garner more citations. However, the effects is decaying over time. Technology fields without disruptive inventions continuously decline in both relevance and the chance for future disruptions.

My model offers a reinterpretation of this finding: Inventors are technology experts and cannot work in any field. Firms that pursue incremental innovation are thus linked to certain technologies through their inventor portfolio. Disruptive inventions that make technologies obsolete or rearrange the technology structure of the economy are thus threats these firms have to manage. The inventor labor market gives these firms the possibility to poach disruptive inventors and prevent disruptive inventions. This is optimal for the firms, but causes declining growth: Creating exponential growth with incremental improvements is getting harder and harder. As a result, there is a troublesome misallocation of inventive talent to incremental innovation with declining returns.

To achieve these results, the paper also makes a technical contribution by demonstrating how an elementary search and matching labor market can be inserted into a general equilibrium model without greatly increasing its complexity. Given that the equilibria in search and matching models are most often found through numerical simulation (Rogerson et al., 2005; Hagedorn et al., 2017), this represents a significant step in its own right.

The model implies several levers for policy intervention: zzzzzzzzzzzzzzz

One example is banning the acquisition of startups. Such regulation, while difficult to codify and enforce, would prohibit large firms from monopolizing research talent and inhibiting technology disruption. This is equivalent to making it more difficult to poach disruptive inventors in my model. Extending merger controls to labor markets has been suggested e.g. by Naidu et al. (2018), but for different reasons than the ones presented here. However, there have also been arguments in favor of startup buyouts. Among others, Cabral (2018) argues that allowing incumbents to buy startups can increase the incentive to innovate: Potential entrepreneurs should start more innovative firms because they expect to sell to big corporations. This argument need not even hold in traditional endogenous growth models, where the amount of research alone determines growth (Piazza and Zheng, 2019). If different types of innovation exist, the argument breaks down: Large firms can also acquire startups to suppress disruptive innovation and redirect R&D into incremental research.

Appendices

A Alternative Demand Specification

This section describes a completely different demand system that nevertheless gives the same result of profits being a linear function of quality. The purpose of this section is to demonstrate the flexibility of the inventor labor market setup and to showcase a setup in which firms have direct competitors which could be used for model extensions with strategic interactions between firms. To avoid confusion with the baseline specification, I will index firms f and consumers con in this section.

Consumers derive utility from a generic numeraire good a that represents consumer goods with low research content. In addition, they derive utility from satisfying a continuum of their needs located on a Salop circle of circumference 1. The needs on this circle are more advanced and can only be fulfilled with research intensive products. Needs that are located closer to each other are more substitutable. E.g., a section of the circle might represent different modes of transportation, while another section might signify entertainment. In the transportation section, one point might represent short distance trips for one person, another point might represent longer commutes and a more distant point might be intercontinental travel. Crucially, these are general needs and not existing products.
The utility function of consumer con is

$$U_{con} = \prod_{n} x_n^{\beta} * \frac{q_f}{d_{n \to f(n)}} * a_{con}^{1-\beta} \tag{22}$$

Utility comes from the amount of goods purchased (x_n) for each need, from the quality of the products (q_f) for each need and from the distance between this need and the product that the consumer actually bought ($d_{n \to f(n)}$). Since each point on the circle represents a need and not a product, consumers have to search for the best product to meet any specific need. The Cobb-Douglas utility function implies that consumers spend a fixed share of their income on research intensive goods, spread equally over their continuum of needs. In effect, consumers assign a constant budget to any of their needs n on the circle.
There is only a finite number of firms, each of which produces exactly one product. Firms and thus also products will be indexed with f. Firms have to position themselves on the circle and will attract customers intent on satisfying their needs in the vicinity. E.g. a firm might decide to rent out bicycles suited for short distance trips. However, this firm's product might also be the best option for longer commutes if the bike has a high quality (e.g. an electric engine), if there are no competing products in the vicinity (e.g. because the only other transportation firm is an airline), or if the firm is charging a comparatively low price.
Consumers will buy a firm's product multiple times: They will search for the best offer for any one of their needs. E.g., consumers will search for the best firm for short trips and then again search for the best firm for commutes. The success of a firm f depends on for how many of these different needs it can make the best offer. Consumers are indifferent to a product of double the quality which is twice as far away from the desired variety. The quality and quantity of variety v are complements and the consumer derives utility from their joint consumption. Thus, the lower the price of the research intensive good, the more the consumer can buy, which again makes the quality of the research intensive good more useful to him.

From the viewpoint of firms, each need n is a separate winner-takes-all market of equal size $I * \beta$. How many of these markets a firm wins determines its revenue and size. Firms will always be able to control the markets closest to them because quality is divided by the distance of the firm to the market $\frac{q_f}{dn \to n}$. Thus, any firm can offer infinite utility in the market at its location. Demand for the product of firm f is determined by the marginal market $n_{m;f;f+1}$, i.e. the market where consumers are just indifferent between product f and $f+1$ of the neighboring firm.

$$\frac{q_f}{d_{n_{m;f;f+1} \to f}} * (\frac{1}{p_f})^{\beta} = \frac{q_{f+1}}{d_{n_{m;f;f+1} \to f+1}} * (\frac{1}{p_{f+1}})^{\beta} \tag{23}$$

Additionally, product f competes with product $f-1$ on the other side of f. The number of markets that firm f can capture depends on the quality of its product (q_f) and the pricing and location decisions of its competitors.

In static equilibrium, firm f has to take the quality of its product as given. It first sets prices and then positions itself on the circle, considering the fixed quality of its competitors. Firm f will have to take the quality of all firms into account when setting prices, anticipating that the prices it sets will affect where its competitors position themselves.

E.g., consider a bicycle, a car and a train company all competing for markets in the transportation sector. The car company has to do research to increase the quality of the cars it can produce, set a price and then decide whether it would like to compete for short-distance inner-city trips with bicycles or for long-distance traveling with trains. Setting a low, competitive price will induce both the bicycle and the train competitor to move more into their specific niches, as will having a high quality product.

Thus, every firm owner has to position the firm taking all other variables as given. Solving equation (23) for the number of markets firm f captures to its left (against firm $f-1$) and to its right (against firm $f+1$) yields the profits the owner of firm f can reap:

$$\pi_f = I * \beta \frac{q_f}{p_f^{\beta}} [\frac{p_{f-1}^{\beta} d_{n_{m;f-1;f} \to f-1}}{q_{f-1}} + \frac{p_{f-1}^{\beta} d_{n_{m;f-1;f} \to f-1}}{q_{f-1}}](1 - \frac{mc}{p_f}) \tag{24}$$

Given that the firm owner has already set prices, maximizing profits now comes down to maximizing the number of markets that the firm can capture: The markup $\frac{p_f}{mc}$ is given. Note that the firm owner conceptually could influence $d_{n_{m;f-1;f}}$ by moving firm f closer to firm $f-1$ and taking its markets.

The markets that f and $f+1$ capture between them have to sum up to the distance between the two firms, so the profits of firm f can be expressed only in exogenous variables and the strategy choices of its competitors:

$$\pi_f = I * \beta [d_{f \to f+1} \frac{\frac{q_f}{p_f^{\beta}}}{\frac{q_f}{p_f^{\beta}} + \frac{q_{f+1}}{p_{f+1}^{\beta}}} + d_{f \to f-1} \frac{\frac{q_f}{p_f^{\beta}}}{\frac{q_f}{p_f^{\beta}} + \frac{q_{f-1}}{p_{f-1}^{\beta}}}](1 - \frac{mc}{p_f}) \tag{25}$$

where the term in brackets denotes the markets won by firm f: $d_{f\to f+1}$ is the distance between firm f and its competitor $f+1$. The two firms split the markets between them according to the ratio of the attractiveness of their products $\frac{\frac{q_f}{p_f^\beta}}{\frac{q_f}{p_f^\beta}+\frac{q_{f+1}}{p_{f+1}^\beta}}$. In the same way, firm f and firm $f-1$ share the markets between them. From equation (25), it is clear that there is no Nash equilibrium if firm $f-1$ and firm $f+1$ have different qualities and prices: Firm f will always move to the firm that offers the stronger product. However, Salop circles do not have Nash equilibria in general. An equilibrium is only possible if firms take the location reaction of their competitors into account.

Consider the reactions of firm $f+1$ to the actions of firm f. Because firm $f+1$ can freely move on the circle, its profits must be independent of f. Otherwise, the firm will costlessly move to a different part of the circle. Firm $f+1$ will react to any price and quality changes of f to restore this indifference.

$$\frac{\partial \pi_{f+1}}{\partial l_f} = 0 = -\frac{\frac{q_{f+1}}{p_{f+1}^\beta}}{\frac{q_f}{p_f^\beta}+\frac{q_{f+1}}{p_{f+1}^\beta}} + \frac{\partial l_f}{\partial l_{f+1}}\frac{\frac{q_{f+1}}{p_{f+1}^\beta}}{\frac{q_f}{p_f^\beta}+\frac{q_{f+1}}{p_{f+1}^\beta}} - \frac{\partial l_{f+1}}{\partial l_f}\frac{\frac{q_{f+1}}{p_{f+1}^\beta}}{\frac{q_{f+1}}{p_{f+1}^\beta}+\frac{q_{f+2}}{p_{f+2}^\beta}} + \frac{\partial l_{f+2}}{\partial l_{f+1}}\frac{\frac{q_{f+1}}{p_{f+1}^\beta}}{\frac{q_{f+1}}{p_{f+1}^\beta}+\frac{q_{f+2}}{p_{f+2}^\beta}} \quad (26)$$

which yields $\frac{\partial l_{f+1}}{\partial l_f} = \frac{\partial l_{f+2}}{\partial l_{f+1}} = 1$ as the solution: If firm f moves 0.1 units closer to firm $f+1$, $f+1$ will also move 0.1 units towards $f+2$. Firm $f+1$ can do this because it expects firm $f+2$ (and $f+3$, $f+4$,...) to do the same, restoring the original positioning.

Now consider the case where firm f has set a higher price. Again, firm $f+1$ cannot profit from that, since otherwise firms from other parts of the circle would move to the spot of firm $f+1$: $\frac{\partial \pi_{f+1}}{\partial p_f} = 0$. Thus,

$$\frac{\partial d_{f\to f+1}}{\partial p_f} = -\beta p_f^{-\beta} * \frac{\frac{q_f}{p_f^\beta}}{\frac{q_f}{p_f^\beta}+\frac{q_{f+1}}{p_{f+1}^\beta}} \quad (27)$$

I.e., by increasing its price, firm f captures a slightly smaller share of the markets between f and $f+1$. $f+1$ then moves closer so as to exactly maintain the number of markets it captures itself. Two firms share the markets between them according to the ratio of the attractiveness of their products. So when f becomes less attractive because of price increases, firm $f+1$ has to move closer to shorten the distance between the two firms. If the price of product f was already high, f only has a tiny share of the contested markets and additional price increases only require small changes in location. Even though this movement lowers its profits, firm $f+1$ has to do this to protect itself against other firms moving into the resulting gap.

Equations (26) and (27) imply that the utility of the marginal consumer between two firms is constant across the economy. Intuitively, this follows more or less directly from the free movement condition: Since the profits of firm f depend directly on its own quality, its price and the utility of the marginal consumers it can still capture (24), it stands to reason that one spot on the circle cannot have marginal consumers with higher utility, since firms would otherwise move there. Thus, I denote the sum of the utility of the two marginal consumers of each firm as **C**. **C** is a competition parameter describing how low firms have to set prices to stave off competing firms. It rises with how many firms of a given quality are in the economy.

Mathematically, inserting **C** into equation (24), firm profits are

$$\pi_f = I\beta D_f - I\beta D_f^{(1+\frac{1}{\beta})}[\mathbf{C}]^{\frac{1}{\beta}} q_f^{\frac{-1}{\beta}} \frac{mc}{\beta} \tag{28}$$

where D_f denotes the number of captured markets, i.e. the number of markets for which the product of f is the best product. Firms earn $I\beta$ per captured market, but the costs of servicing these markets increase non-linearly, because lowering prices forces a firm to serve its already captured markets with more produce or leave revenue on the table. Equation (28) takes into account that firm f expects its neighboring firms to keep their profits and thus the fractions in equation (24) constant. Thus, if f increases its price, f expects the other firms to move closer, tightening competition compared to equation (25). Likewise, if firm f decreases its price, it expects to cater to additional markets partly because its direct competitors move away and partly because its products become more attractive.

Maximizing equation (28) yields

$$p_f = (1 + \frac{1}{\beta})mc \tag{29}$$

for the optimal price: Firms charge a fixed markup over marginal costs depending on the demand parameter β, which denotes how long additional quantity still generates value for the customers for any given variety. If additional quantity does not lead to much additional utility, firms cannot gain many customers by lowering prices and charge a high markup.

Given this pricing behavior, customers search for the best product for each different variety. This yields the number of varieties serviced by each firm as

$$D_f = q_f\beta[\frac{1}{(1+\beta)mc}]^\beta[\mathbf{C}] \tag{30}$$

Serviced markets are a linear function of a firm's quality, given that every firm charges the same price, regardless of its quantity. The number of markets served reacts more strongly to quality if the marginal costs are small, so that the costs of serving additional markets do not matter so much. The effect of the demand parameter β is more ambiguous, because a high β raises the costs of servicing a new market (because consumers demand more goods), but also means that consumers spend more in each market. Firms leverage their quality to service more markets, not to raise their prices. Since the circumference of the Salop circle is finite, this is a predatory strategy: High quality firms push out their competitors.

Since only the number of served markets rises with quality, profits are also linear in quality:

$$\pi_f = D_f * I\beta * \frac{1}{1+\beta} = q_f[\frac{\beta}{(1+\beta)mc}]^{\beta}[\mathbf{C}]I\beta\frac{1}{(1+\beta)} \tag{31}$$

Every firm faces the same marginal costs and charges the same price, thus profits in every market are the same ($I\beta * \frac{1}{1+\beta}$). Profits per market are higher if β increases, because consumers allot a higher budget to each need. This is partly counteracted because a higher β also means that consumers draw more utility from the quantity that firms produce, which harms firms: Since consumers value lower prices, firms try to steal each others' markets by lowering prices. Yet, the higher overall spending for the research intensive good prevails.

Equilibrium requires that the whole circle is serviced, i.e. $\sum D_f = 1$. This allows to solve for the equilibrium value of competition strength $[\mathbf{C}] = \frac{1}{\sum q_{f_f}}(1+\beta)^{\beta}mc^{\beta}$. The strength of equilibrium competition is rising in the sum of all qualities of active firms: Since every firm captures markets on the Salop circle in relation to their quality, if there are more high quality firms, every firm has to receive a smaller number of markets. Marginal costs lower the competition each firm feels, because higher marginal costs decrease the incentive for each firm to spread out over multiple markets. Given the level of competition, firms can cater to $q_f * \sum {q_f}_f^{-1}$ markets. Consequently, every firm makes profits of $q_f * \sum {q_f}_f^{-1} I\beta(1+\beta)^{-1}$.

The economy is closed by the labor market and the market for the numeraire good. The economy produces the numeraire good with the fixed amount of labor L with the technology of all firms $\sum q_f$. Thus, the research intensive sector increases the productivity of the numeraire sector. Since the numeraire sector is competitive, its whole revenue is earned by its laborers. Thus, the equilibrium income in the economy is

$$I = \sum q_f L(1+\beta) \tag{32}$$

i.e., the labor income from the numeraire sector plus the profits from the firms in the research intensive sector. Labor income increases the higher the productivity of the economy and the more labor L households supply. The higher β, the higher the profit share of the economy, as well as nominal income for a given productivity level. However, a higher β would also lead to higher prices for the research intensive good, so real income is not rising.

Given this nominal income level, the profit of any given firm is

$$\pi_f = q_f * \beta \tag{33}$$

and thus is only a function of a firm's product quality q_f and the constant parameter β. However, potential entrants do not only care about current conditions, but are motivated by potential future profits. Thus, the number of firms in equilibrium is determined by future prospects for quality improvements through research.

Within a cluster currently at the technology frontier, i.e. a cluster that is the best in its field, patents improve the product quality of firms and thus represent a steady stream of profits for the firm that holds them. The value of a patent

$$V(p_c) = \omega^c * \frac{\beta}{r} \tag{34}$$

is a function of parameters of the model and thus fixed. It rises with c, as patents in more advanced clusters create more quality (parameter ω determines the strength of this effect). The value of a patent also rises in β, which governs the markups of firms and the amount of money consumers spend on the research intensive good.

Given this value of patents, the value of an inventor is the stream of patents he represents. The value of the inventor portfolio of all firms with a given quality y_f is thus

$$V_f(N_i^c(y_f, x_i, t_c^e)) = \int_0^1 (V(p_c) y_f x_i (1 - e^{-t_c^e}) \frac{2\eta}{\delta r} \,\mathrm{d}x_i = y_f V_f(N_i^c(y_f = 1, t_c^e)) \tag{35}$$

I.e. the value of the patent portfolio of firms is increasing in y_f because high quality firms produce more patents with the inventors they have. The value of a firm's patent portfolio increases as long as the current technology cluster is still on the edge and additional inventors are still entering the cluster.

A potential entrant does not have an inventor portfolio, but expects to hire inventors in the future. The value of this stream is dependent on the research quality y_f the entrant will draw.

$$V_f(y_f) = \frac{1}{N_f(y)} \int_0^1 (\eta x_i * V_f(y_f; x_i) \,\mathrm{d}x_i \frac{1}{r + \Lambda_{dis}} = y_f V_f(y_f = 1) \tag{36}$$

The stream of inventors matched with firms of quality y is shared between all firms of that quality $\frac{1}{N_f(y)}$. Again, the value of the stream of hires is increasing in y_f because a higher quality firm gets more patents out of each hire. If the value of a patent in the technology cluster $V(p_c)$ is higher, firms value the stream of inventors they will hire more. The share of profits flowing to the firm makes future inventors more valuable, too. The likelihood of disruptive inventions decreases the value of future inventors: If a disruptive invention occurs, new inventors will not enter the now obsolete cluster of the firm. The stream of hires will dry up.

Whenever a new technology cluster is created (and only then), new firms can enter.

New firms entering the economy do not yet have inventors or patents. However, entrants gain access to the inventor labor market and will hire inventors and produce patents in the future. Firms pay an entry fee f_e to become experts in a technology cluster proportional to ω^c: The more disruptive inventions were necessary to form the cluster, the more sophisticated the technology is and the more setup is necessary. In equilibrium, the ex ante expected value of future hired inventors must equal this setup cost. Thus,

$$N_f = \eta \frac{2}{9} f_e \frac{\beta}{r} \frac{\alpha}{r+\delta} \frac{1}{r+\Lambda_{dis}} \tag{37}$$

Since entrants draw a quality y_f randomly from a uniform distribution between 0 and 1, there is an equal mass of entrants (and firms) at every quality $N_f(y)$. The expected value of entry declines as more firms enter, because a higher number of entrants compete for a fixed number of graduates. However, the value of entry is independent of patents or the inventors already in the cluster. Hence, firms will enter as soon as the disruptive invention creates the cluster and drive the expected returns from entering down to the entry fee f_e.

Some firms will ex post regret entering: They draw a bad research quality and do not make enough profits to recoup their entry costs. Using equation (36), ex post profits are $3y_f f_e$. Thus, all firms that draw a quality of $\frac{1}{\sqrt{3}} = 0.58$ or worse will not recoup f_e. These firms will not exit, since there is no continuous cost of operation apart from inventor wages. Thus, such firms will participate in the search for inventors and hire those with which they can recover at least some part of their entry fee.

B Numerical Simulation with High Λ

This appendix details the result of a simulation of the model economy where Λ^{dis} is much higher than the sum of the interest rate r and the separation rate δ. For this purpose, $\Lambda^{dis}(0)$ is set to 50%, the interest rate to 5% and the separation rate to 5%. 50% is clearly too high for the rate of disruptive technology change, as it would imply that every second sector of the economy is disrupted every year, making incremental inventors obsolete. Nonetheless, even under these extreme conditions, the qualitative results of the model hold:

While the economy converges to the non-disruptive equilibrium much slower (100 is the simulated time in the main paper), the qualitative path is very similar to that of the economy where the parameter restriction holds.

A closer look at the expected change in the disruptive rate makes clear why this is the case (figure 7): Even with these extreme assumptions, the expected change in the rate of disruption is only positive when the risk of a disruptive invention in the technology field is already very low, only to converge to 0 from above.

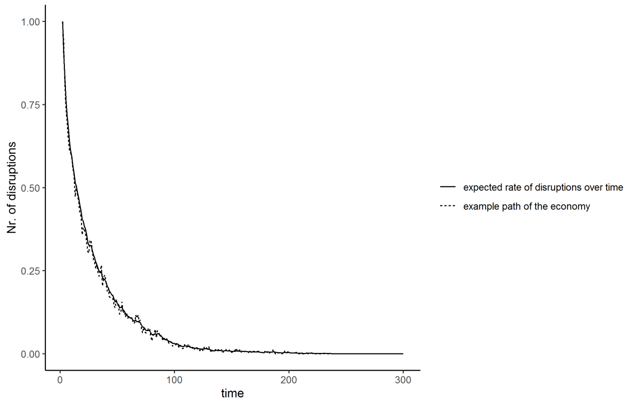


Figure 6: The graph shows the average rate of disruptive inventions throughout the whole economy for $\delta = r = 0.05; \Lambda^{dis}(0) = 0.5$.

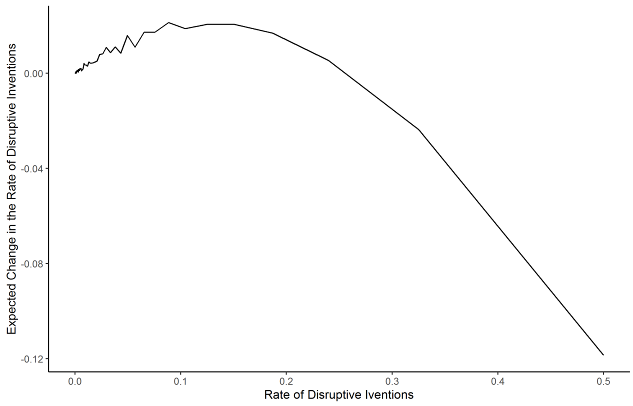


Figure 7: The expected change in the arrival rate of disruptive inventions for a sector. The rate is expected to go down when it is high and to slightly increase when it is already very low. However, if no disruption happens, the expected change approaches 0 as the rate of disruption becomes zero itself.

Clearly, an equilibrium where technology fields with increasing and decreasing rates of disruption cancel each other out in the aggregate is not achievable for all plausible values of $\Lambda^{dis}(0)$.

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