



# Algorithm

To approach this problem, I used a method similar to binary search, which is to divide the array into sections and perform comparison.

Here is a high-level algorithm description for the problem:

1. The problem is to find the index  $P$  where the peak value occurs in an array  $A$  of length  $n$ .
2. Divide the array  $A$  from indices low to high into two halves.
3. Compare the middle element  $A[\text{mid}]$  to the element after it  $A[\text{mid}+1]$ .
4. If  $A[\text{mid}] < A[\text{mid}+1]$ , the peak is in the right half. Recursively search the right half by setting  $\text{low} = \text{mid}+1$ .
5. Else,  $A[\text{mid}] \geq A[\text{mid}+1]$ , the peak is in the left half. Recursively search the left half by setting  $\text{high} = \text{mid}$ .
6. Base case is when  $\text{low} == \text{high}$ , return this index as the peak  $P$ .
7. Return the index  $P$  and value  $A[P]$  from the recursive calls.

A drawn diagram of the algorithm is shown below:

[ 1 3 5 9 7 2 ]

$$\text{mid} = \frac{0+5}{2} = (\text{int})2.5 = 2$$

$$A[2] = 5 < A[3] \text{ and} \\ > A[2]$$



$\therefore$  recurse on the right side

$$\text{mid} = \frac{3+5}{2} = 4$$

$$A[4] = 5 > A[5] = 2 \\ < A[3] = 9$$



$\therefore$  recurse on the left side

$$\text{mid} = \frac{3+3}{2} = 3$$

$$A[3] = 9 > A[4] = 5 \\ > A[2] = 5$$

$\therefore$  return, this is the peak and its value is 9

This algorithm leverages divide and conquer to reduce the search space in half each recursion, resulting in  $O(\log n)$  time complexity. The peak index and value are returned once the base case of a single element is reached.

## Time Complexity Analysis

Let  $n$  be the size of the input array  $A$ .

Let  $T(n)$  denote the time complexity to find the peak index and value in an array of size  $n$ .

The algorithm works as follows:

Divide the array from indices low to high into two halves. Let's assume this takes  $O(1)$  time.

Compare  $A[\text{mid}]$  and  $A[\text{mid}+1]$  to determine which half has the peak. This takes  $O(1)$  time.  
Recursively call the algorithm on the selected half. Let the size of the half be  $n/2$ .  
Return the index and value once  $\text{low} == \text{high}$  (base case). This takes  $O(1)$  time.  
Therefore, the recurrence relation is:

$$\begin{aligned}T(n) &= T(n/2) + O(1) + O(1) \\&= T(n/2) + O(1)\end{aligned}$$

$$a = 1(\text{constant dividing factor})$$

$$b = 2(n/2)$$

$$f(n) = O(1)(\text{work done outside recursion})$$

$$\text{Therefore, } T(n) = \Theta(n \log_b(a)) = \Theta(\log(n))$$

Hence, the time complexity of this divide and conquer algorithm is  $O(\log(n))$ .

This makes sense since we are reducing the problem size by half at each recursive call, resulting in a  $O(\log n)$  binary search style algorithm.