PROBABILITY RULES

- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
  - Note: $A|B$ means "A given B" or "A conditional on the fact that B happens."

PROBABILITY RULES

- $P(A \cap B) = P(A|B)P(B)$
  - We took the last rule, multiplied both sides of $P(B)$, and voila!
  - We can rearrange these, as well! $P(B \cap A) = P(B|A)P(A)$

BAYES' THEOREM

- Bayes' Theorem (Bayes' Rule) relates $P(A|B)$ to $P(B|A)$.

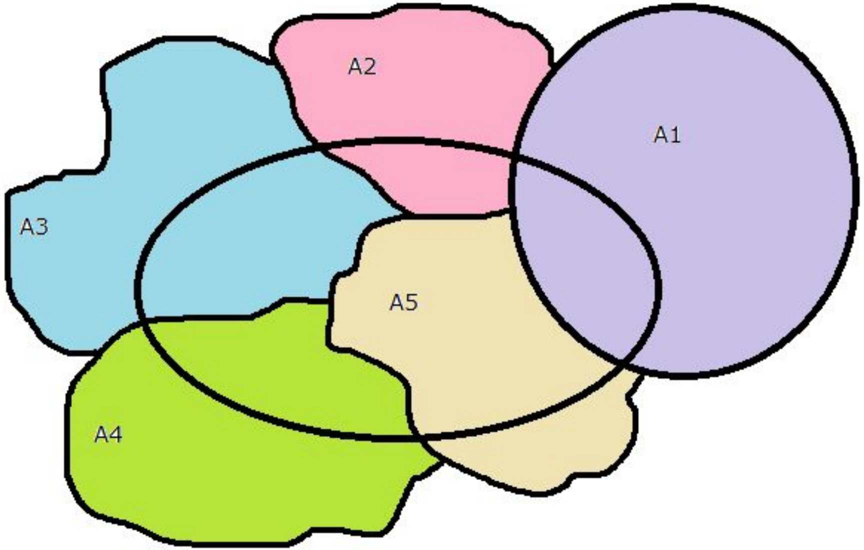
BREAKING DOWN BAYES' THEOREM

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- $P(A)$ is the probability that A occurs given no supplemental information.
- $P(B|A)$ is the likelihood of seeing evidence (data) B assuming that A is true.
- $P(B)$ is the probability that B occurs given no supplemental information.
  - $P(B)$ what we scale $P(B|A)P(A)$ by to ensure we are only looking at A within the context of B occurring.

PROBABILITY RULES

- $P(B) = \sum_{i=1}^{n} P(B \cap A_i)$
  - "Law of Total Probability"



Cookies!

- We have two bowls of cookies:

	Bowl 1	Bowl 2
Vanilla	30	20
Chocolate	10	20

- Without knowing which is which, we randomly select one and pull out a vanilla cookie.
- What is the probability that the cookie came from Bowl 1?

Source: http://www.greenteapress.com/thinkbayes/html/thinkbayes002.html

Cookies!

$$P(B1 \mid Vanilla) = \frac{P(Vanilla \mid B1) * P(B1)}{P(Vanilla)}$$

Using the law of total probability:

$$P(B1 \mid Vanilla) = \frac{P(Vanilla \mid B1) * P(B1)}{P(Vanilla \mid B1) * P(B1) + P(Vanilla \mid B2) * P(B2)}$$

Cookies!

Filling in our priors is pretty straightforward:

$P(B1) = 1/2$
$P(B2) = 1/2$

$$P(B1 \mid Vanilla) = \frac{P(Vanilla \mid B1) * P(B1)}{P(Vanilla \mid B1) * P(B1) + P(Vanilla \mid B2) * P(B2)}$$

$$P(B1 \mid Vanilla) = \frac{P(Vanilla \mid B1) * 1/2}{P(Vanilla \mid B1) * 1/2 + P(Vanilla \mid B2) * 1/2}$$

Cookies!

We can use our original table to fill out the rest:

	Bowl 1	Bowl 2
Vanilla	30	20
Chocolate	10	20

$$P(B1 \mid Vanilla) = \frac{P(Vanilla \mid B1) * P(B1)}{P(Vanilla \mid B1) * P(B1) + P(Vanilla \mid B2) * P(B2)}$$

$$P(B1 \mid Vanilla) = \frac{30/40 * 1/2}{30/40 * 1/2 + 20/40 * 1/2}$$

Cookies!

$$P(B1 \mid Vanilla) = \frac{P(Vanilla \mid B1) * P(B1)}{P(Vanilla \mid B1) * P(B1) + P(Vanilla \mid B2) * P(B2)}$$

$$P(B1 \mid Vanilla) = \frac{30/40 * 1/2}{30/40 * 1/2 + 20/40 * ½}$$

$$P(B1 \mid Vanilla) = \frac{15/40}{15/40 + 10/40}$$

$$P(B1 \mid Vanilla) = \frac{15/40}{25/40}$$

$$P(B1 \mid Vanilla) = 3/5$$

Monty Hall

"Let's Make a Deal" features three doors labeled "A," "B," and "C."

As the contestant, you are told that, behind exactly one door, there’s a new car. Behind the other two doors are goats. Your goal is to select the door with the car behind it.

After you choose a door (Door A), Monty Hall will show you a goat in one of the other two doors (Door B). He then gives you the option to either keep your original choice (Door A) or switch to the last remaining door (Door C).

Is it better to switch to Door C, or stay with Door A?

Monty Hall: Strategy 1 (Stick with Door A)

$$P(A|mB) = \frac{P(mB|A) * P(A)}{P(mB)}$$

Using Law of Total Probability:

$$P(A|mB) = \frac{P(mB|A) * P(A)}{P(mB|A) * P(A) + P(mB|B) * P(B) + P(mB|C) * P(C)}$$

Monty Hall: Strategy 1 (Stick with Door A)

First, let's knock out the priors:

$P(A) = 1/3$

$P(B) = 1/3$

$P(C) = 1/3$

$$P(A|mB) = \frac{P(mB|A) * P(A)}{P(mB|A) * P(A) + P(mB|B) * P(B) + P(mB|C) * P(C)}$$

$$P(A|mB) = \frac{P(mB|A) * 1/3}{P(mB|A) * 1/3 + P(mB|B) * 1/3 + P(mB|C) * 1/3}$$

Monty Hall: Strategy 1 (Stick with Door A)

Now let's look at this term: $P(mB|B)$

Essentially we're saying "Assuming the car is behind Door B, what is the probability that Monty Hall shows us Door B?"

$P(mB|B) = 0$

$$P(A|mB) = \frac{P(mB|A) * P(A)}{P(mB|A) * P(A) + P(mB|B) * P(B) + P(mB|C) * P(C)}$$

$$P(A|mB) = \frac{P(mB|A) * 1/3}{P(mB|A) * 1/3 + 0 * 1/3 + P(mB|C) * 1/3}$$

Monty Hall: Strategy 1 (Stick with Door A)

Now let's look at this term: $P(mB|C)$

Essentially we're saying "Assuming the car is behind Door C, what is the probability that Monty Hall shows us Door B?" Remember, we chose A, so Monty will ***never show us Door A.***

$P(mB|C) = 1$

$$P(A|mB) = \frac{P(mB|A) * P(A)}{P(mB|A) * P(A) + P(mB|B) * P(B) + P(mB|C) * P(C)}$$

$$P(A|mB) = \frac{P(mB|A) * 1/3}{P(mB|A) * 1/3 + 0 * 1/3 + 1 * 1/3}$$

Monty Hall: Strategy 1 (Stick with Door A)

Now let's look at this term: P(mB|A)

Essentially we're saying "Assuming the car is behind Door A (the door we chose), What is the probability that Monty shows us Door B?" In this case, Monty could show us either B or C, since they both have goats.

$P(mB|A) = 1/2$

$$P(A|mB) = \frac{P(mB|A) * P(A)}{P(mB|A) * P(A) + P(mB|B) * P(B) + P(mB|C) * P(C)}$$

$$P(A|mB) = \frac{1/2 * 1/3}{1/2 * 1/3 + 0 * 1/3 + 1 * 1/3}$$

Monty Hall: Strategy 1 (Stick with Door A)

Now we can finally solve the probability that the car is behind Door A, given that Monty showed us a goat behind Door B.

$$P(A|mB) = \frac{1/2 * 1/3}{1/2 * 1/3 + 0 * 1/3 + 1 * 1/3}$$

$$P(A|mB) = \frac{1/6}{1/6 + 0 + 2/6}$$

$$P(A|mB) = \frac{1/6}{3/6}$$

$$P(A|mB) = 1/3$$

Monty Hall: Strategy 2 (Switch to Door C)

Now we're going to consider the alternative strategy. We're going to ***initially choose Door A***. Monty Hall will still show us Door B. Only this time we're going to switch to Door C. So we're interested in the following:

$$P(C|mB) = \frac{P(mB|C) * P(C)}{P(mB)}$$

Using Law of Total Probability:

$$P(C|mB) = \frac{P(mB|C) * P(C)}{P(mB|A) * P(A) + P(mB|B) * P(B) + P(mB|C) * P(C)}$$

Monty Hall: Strategy 2 (Switch to Door C)

We already solved the terms below in the first strategy, so all we have to do is plug in those numbers.

$$P(C|mB) = \frac{1 * 1/3}{1/2 * 1/3 + 0 * 1/3 + 1 * 1/3}$$

$$P(C|mB) = \frac{2/6}{1/6 + 0 + 2/6}$$

$$P(C|mB) = \frac{2/6}{3/6}$$

$$P(C|mB) = 2/3$$