Data Science Statistics and Probability Homework

1. For her entire life, Ruth has always been known to be extremely compassionate and kind. As a high school and college student, Ruth was extremely active in lobbying for the rights of animals, and for equal representation and compensation for women in the workplace. Now Ruth is a working professional and a mother of two daughters. She has two dogs and two cats, all rescue animals. Rank the following statements about Ruth from most to least likely and explain your reasoning.
    a. Ruth is an executive at her company.
    b. Ruth is an executive at her company and a chapter leader in her local chapter of National Organization for Women (NOW).
    c. Ruth is an executive at her company, a chapter leader in her local chapter of NOW, and she regularly volunteers at the local animal shelter.
2. You are a data scientist at Duracell. It has been discovered by quality assurance that some of the raw materials used in the manufacture of a large batch of AA batteries was impure, causing about 1 battery in every 500 to be defective. Duracell has a method of testing for defective batteries, which is 99% accurate. If a battery is identified as defective by this test, what is the probability that the battery is actually defective?
3. You're a data scientist at Amazon.com. A consulting firm was hired to improve the load time of the website. Prior to hiring them, the home page had an average load time of 3.125 seconds with a standard deviation of 0.702 seconds. Management wants you to evaluate the result of the consulting engagement, and they've asked for a 99% confidence level. You loaded the webpage 40 times and found that the home page had an average load time of 2.875 seconds. Can we say with statistical significance that the consulting firm was able to improve the load time of the website?
4. Recall the definition of variance is $Var[X] = E[(X - E[X])^2]$. Show that $Var[X] = E[X^2] - E^2[X]$.