Linear Algebra Homework

Section 1

1. Let $u, v \in R^n$. Show that $u^T v = v^T u$.
2. Let A be an $n \times n$ matrix. Let B be a matrix such that $BA = I$, and let C be a matrix such that $AC = I$. Show that $B = C$.
3. Let V_F be a vector space over the field F. Recall the axioms for the inner product as follows. For $u, v, w \in V_F$ and $\alpha, \beta \in F$:
    a. $\langle \alpha u + \beta v, w\rangle = \alpha\langle u, w\rangle + \beta\langle v, w\rangle$
    b. $\langle u, v\rangle = \overline{\langle v, u\rangle}$
    c. $\langle u, u\rangle \geq 0$ where $\langle u, u\rangle = 0$ if and only if $u = 0$.

    Show that $\langle u, \alpha v + \beta w\rangle = \bar{\alpha}\langle u, v\rangle + \bar{\beta}\langle u, w\rangle$.

4. A matrix A is said to be symmetric if and only if $A = A^T$. Assuming A is both symmetric and invertible. Show that A^{-1} is also symmetric.

Section 2

For Section 2, consider the following matrices.

$X_{n\times d}$ $A_{d\times q}$ $B_{q\times r}$ $C_{r\times s}$ $D_{s\times t}$ $M_{t\times u}$ $N_{u\times k}$ $P_{n\times k}$ $Y_{n\times k}$ $V_{n\times 1}$ $S_{n\times q}$ $T_{n\times r}$ $U_{n\times s}$ $W_{n\times t}$ $Z_{n\times u}$

Determine the dimensionality of the result of each following expression. If the given expression is undefined, state that the dimensions are undefined.

1. $N(P - Y)(P \odot Y)^T$
2. $XAS^T NZ \odot ZM^T M$
3. $VV^T(P + Y)N^T \odot ZZ^T Z$
4. $(SBC \odot U)^T \odot U^T$
5. $A - X^T\left[\left(\left(\left(\left((P - Y)N^T \odot Z\right)M^T \odot W\right)D^T \odot U\right)C^T \odot T\right)B^T \odot S\right]$