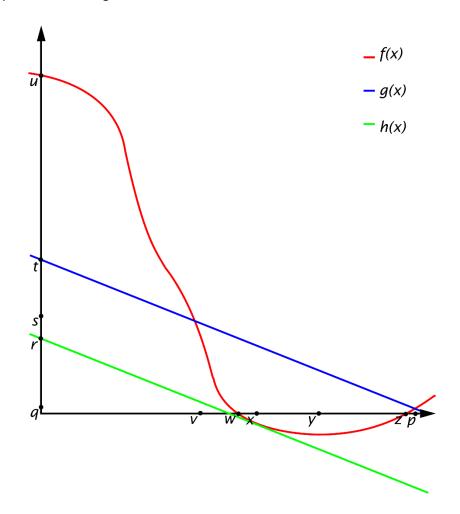
Data Science

Prerequisite Mathematics Exam

Section 1

For Section 1, please refer to Figure 1, below.



- 1. Which point corresponds to f(0)?
- 2. Which points, if any, would correspond to roots of the function f(x)?
- 3. Say g(x) and h(x) have exactly the same slope. Express the value of f'(x), that is, the derivative of f at the point labeled x, in terms of points and functions given in the figure above.
- 4. Give the coordinates, in the form (x, f(x)), of any optima found in the figure.
- 5. What can we say about the value of f''(y)?
- 6. What can we say about the value of f''(v)?
- 7. What can we say about the value of g''(p)?

Section 2

- 8. Let $u, v \in \mathbb{R}^N$. If we are told that the set $\{u, v\}$ spans \mathbb{R}^N , what can we say about N?
- 9. Let

$$A = \begin{bmatrix} 5 & 1 \\ 2 & 7 \\ 3 & 8 \end{bmatrix}$$

Are the columns of A^T linearly independent?

- 10. Let $u, v \in \mathbb{R}^N$. Show that $u^T v = v^T u$.
- 11. Let

$$W = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$$

Calculate $\sum_{i=1}^{2} \sum_{j=1}^{2} W_{ij}$.

12. A matrix A is said to be symmetric if, and only if, $A = A^T$. Let A be symmetric and invertible. Show that A^{-1} must also be symmetric.

Section 3

- 13. A friend of yours was just tested for the presence of a disease. Your friend tells you that the test is 99% accurate at detecting the disease when it's present, but that 10% of the time, the test comes back positive no matter what. Your friend's test came back positive, and the doctor has told him that he has a 9% of chance of having the disease, given the positive result. How common is this disease within the general population?
- 14. Say we have four possible events A, B, C, and E. Let A, B, and C be disjoint from each other, but not from E. How do we calculate P(E)?