CEE 5290/COM S 5722/ORIE 5340 Heuristic Methods for Optimization

Homework 11: No Free Lunch Theorem

(Homework only for students taking the "Theory" part of the course.)

Assigned:, November 19, 2011

Due:Nov. 28, 2011

TA Office Hours: Tue & Thu 2:00-3:30in Hollister 203;

Prof. Shomaker office hours Nov.21,23,28,30 are 2:30-3:30

(Note this homework might not be graded before the final so you are advised to have a copy of your answers so you can compare them with the solutions to be posted on Blackboard.)

No Free Lunch Theorem: (This question uses notation given in lecture slides)
Assume you want to compare a genetic algorithm with an evolutionary search (ES) algorithm by how well they can maximize a function. The decision vector is a binary string of length 5. The value of the objective function is an integer between 1 and 10. So the domain is the set of binary strings of length 5 and the range is {1,2,…,10}. The initial population for the optimization is picked randomly (uniform distribution) from the domain. All comparisons below assume algorithms are run so that 1000 objective function evaluations of J are made at distinctly different binary strings for each J.

a) How many possible problems (i.e. **how many different J_K 's**) are there for this situation?

b) How many of the possible J_K *'s* have an objective function value of strictly less than 6 for all possible binary strings of length 5? To show you understand this concept, give one example of one objective function $J_K(x)$ such that $J_K(x)$ is less than 6 for all x in domain S (*This is very easy—the question is just to make you think concretely about what a J in a subset of Z is*). Let us call this set $G=\{ J_K , K=1,\dots,H \mid J_K(x)\leq 5\}$ *(corrected)* . So the question is how big is $H=|G|$? What is the **ratio of |Z| to |G|** (expressed as a power of 2)? Approximate that as function of 10 (e.g. ratio $|Z|/|G|= 10^b$ where b= is an integer). This base 10 question is to make sure you appreciate the difference in size of these function sets.

c) Assume that you divide the J_K *'s* in Z into two disjoint sets Z1 and Z2 so that |Z1| + |Z2|= |Z|, and the intersection of Z1 and Z2 is empty. Assume also that ¾ of the J's are in set Z1 and the remainder are in set Z2. You want to compare GA with Evolutionary Search (ES). Assume you can estimate that the average (over all J_K in *Z1*) probability that GA will find the x that maximizes $J_K(x)$ in 1000 evaluations of each $J_K(x)$ is .5 and that the probability ES will find the maximizing x in 1000 evaluations of each of the $J_{K\ in}$ in *Z1* is .4. Assume also that the average value of Jbest for GA's applied to J's in *Z1* for GA is 5.2 and for ES is 4.9 in *Z1*. Then what is the average probability for GA and ES of finding the maximum value for each of all the $J_K(x)$ that are in Z2? **Give an equation and explain your answer**. If you use an equation summing over the J's be sure to specify from which set the the sum is over.

Reference for Problem 1 (which you are not required to read): Wolpert, D.H., W.G. Macready, "No Free Lunch Theorems for Optimization," *IEEE Transactions on Evolutionary Computation*, 1(1) 67-82,1997