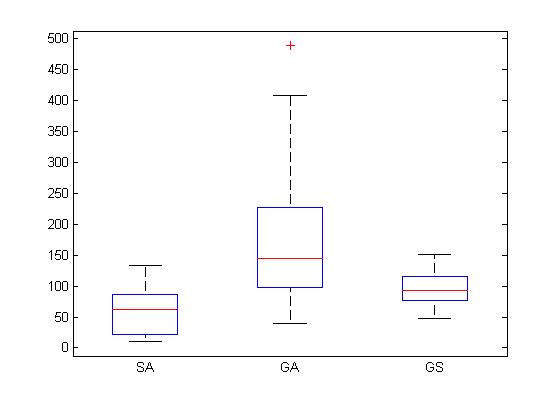
## Problem 1

### Part i)

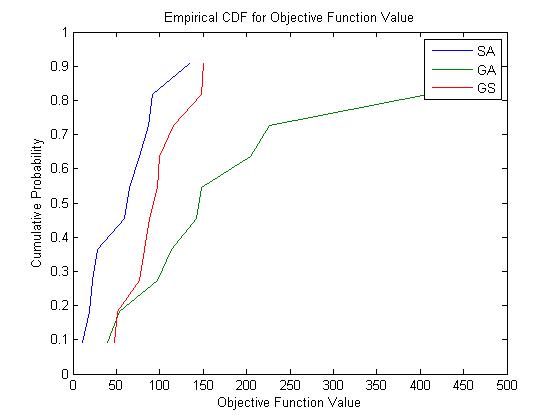
The box plot is at follows:



From the boxplot for the given data, we can see that GA has the worst performance with highest mean and variance. SA has lower mean but with larger variance comparing with GS. There is an obvious outlier for the data of GA with value 488.83. We think that SA has the best performance as 1. It has the lowest mean, 2. It has a fairly stable performance and 3. Among all 3 algorithms, SA is more likely to reach the lowest objective value.

### Part ii)

The plot is as follows:



From the Empirical CDF above, we can see that SA gives the most vertical plot, which means it has the highest probability to produce the lowest objective value. GS gives the second best plot, with a relatively high probability of finding the lowest objective value. And GA has the worst performance over the 3, as its empirical CDF is more flat, which means it often produces very large objective value.

The empirical CDF also indicates the spread of the solutions. SA and GS have more than 0.8 probability of performing well, while GA only performs 7 out of 10 trials. Therefore from both perspectives, SA and GS are better than GA in performance, and SA is relatively better than GS.

There is evidence of stochastic dominance from looking at the graph, because if you look at any objective function value, the current data suggests that SA always achieves the same or better results than the other algorithms with a higher probability.

### Part iii)

We do pairwise comparison of all 3 algorithms

1. **SA vs. GA**

Let the mean of the lowest objective function value generated by SA be , standard deviation be

Let the mean of the lowest objective function value generated by GA be , standard deviation be

If we calculate this, we see that

And so

***Hypothesis Test 1***

Let (2 algorithms are the same)

(2 algorithms are not same)

Since we chose α = 0.05 > p-Value, so we reject null hypothesis. These 2 algorithms are not same.

***Hypothesis Test 2***

Let (2 algorithms are the same)

(SA is better)

So as we are doing one-sided test.

Since we chose α = 0.05 > p-Value, so we reject null hypothesis. Therefore SA is better than GA.

1. **GA vs. GS**

Let the mean of the lowest objective function value generated by GA be , standard deviation be

Let the mean of the lowest objective function value generated by GS be , standard deviation be

If we calculate this, we see that

And so

***Hypothesis Test 1***

Let (2 algorithms are the same)

(2 algorithms are not same)

So

Since we chose α = 0.05<p-Value, so we accept the null hypothesis. These 2 algorithms give the same performance.

***Hypothesis Test 2***

Let (2 algorithms are the same)

(GS is better)

Since we chose α = 0.05 > p-Value, so we reject null hypothesis. Therefore, GS is better than GA.

1. **SA vs. GS**

Let the mean of the lowest objective function value generated by SA be , standard deviation be

Let the mean of the lowest objective function value generated by GS be , standard deviation be

If we calculate this, we see that

And so

***Hypothesis Test 1***

Let (2 algorithms are the same)

(2 algorithms are not same)

So

Since we chose α = 0.05>p-Value, so we reject the null hypothesis. These 2 algorithms do not give the same performance.

***Hypothesis Test 2***

Let (2 algorithms are the same)

(GS is better)

Since we chose α = 0.05 > p-Value, so we reject null hypothesis. Therefore, SA is better than GS.

### Part iv)

Given that SA and GS had the same starting solution in each trial, I would use the paired t-test as it’s more accurate and appropriate for the task.

is as follows:

Since , we can reject the null hypothesis and conclude that the two algorithms have different performance.

### Part v)

We do pairwise non-parametric comparison of all 3 algorithms

Let SA be algorithm 1, GA be algorithm 2, GS be algorithm 3.

1. **SA vs. GA**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Rank | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Value | 10.93 | 18.6 | 22.23 | 28.63 | 39.76 | 53.76 | 59.75 | 64.58 | 77.13 | 86.52 |
| SA/GA | 1 | 1 | 1 | 1 | 2 | 2 | 1 | 1 | 1 | 1 |
| Rank | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Value | 91.94 | 97.88 | 113 | 134.11 | 141.97 | 147.9 | 204.48 | 226.95 | 408.2 | 488.83 |
| SA/GA | 1 | 2 | 2 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |

In this case m = n = 10.

***Hypothesis Test 1***

Let

So we reject null hypothesis. These two algorithms do not have similar performance.

***Hypothesis Test 2***

Let

(SA is better)

< α = 0.05 so Therefore we reject the null hypothesis and accept . SA is better than GA.

1. **GA vs. GS**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Rank | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Value | 39.76 | 47.66 | 51.73 | 53.76 | 76.52 | 82.62 | 87.84 | 97.04 | 97.88 | 99.89 |
| GA/GS | 2 | 3 | 3 | 2 | 3 | 3 | 3 | 3 | 2 | 3 |
| Rank | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Value | 113 | 115.98 | 141.97 | 147.51 | 147.9 | 150.53 | 204.48 | 226.95 | 408.2 | 488.83 |
| GA/GS | 2 | 3 | 2 | 3 | 2 | 3 | 2 | 2 | 2 | 2 |

In this case m = n = 10.

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***Hypothesis Test 1***

Let

So we failed to reject null hypothesis. These two algorithms have the same statistical performance.

***Hypothesis Test 2***

Let

(GS is better)

< α = 0.05 so Therefore we reject the null hypothesis and accept . GS is better than GA.

1. **SA vs. GS**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Rank | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Value | 10.93 | 18.6 | 22.23 | 28.63 | 47.66 | 51.73 | 59.75 | 64.58 | 76.52 | 77.13 |
| SA/GS | 1 | 1 | 1 | 1 | 3 | 3 | 1 | 1 | 3 | 1 |
| Rank | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Value | 82.62 | 86.52 | 87.84 | 91.94 | 97.04 | 99.89 | 115.98 | 134.11 | 147.51 | 150.53 |
| SA/GS | 3 | 1 | 3 | 1 | 3 | 3 | 3 | 1 | 3 | 3 |

In this case m = n = 10.

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***Hypothesis Test 1***

Let

We reject null hypothesis, and conclude that these 2 algorithms are not the same.

***Hypothesis Test 2***

Let

(SA is better)

α = 0.05. Again we reject the null hypothesis and conclude that SA is better than GS.

From the six hypotheses from iii), we can conclude that:

1. SA and GA do not give the same results, and SA is better than GA.
2. Cannot prove GA and GS give different results, and GS is better than GA.
3. SA and GS do not give the same results, and SA is better than GS

From the six hypotheses from above, we can conclude that:

1. SA and GA do not give the same results, and SA is better than GA.
2. Cannot prove GA and GS give different results, and GS is better than GA.
3. SA and GS do not give the same results, and SA is better than GS.

### Part vi)

The graphs in part i and ii both suggests that SA is better than GS, which is better than GA.

Then we performed two-sample t-test on all pairs of algorithms. From these calculations, we found that SA is better than GA, GS is better than GA, and SA is better than GS. This result is consistent with our observations from the graphs.

Next, after noting that SA and GS had the same starting solution, we performed paired t-test on the pair of algorithms and, again, found that SA is better than GS.

Lastly, we performed non-parametric comparisons for all pairwise algorithms. Once again, we obtained the result that in order of performance is SA > GS > GA.

Based on the tests above, I would choose SA as the preferred algorithm to use.

## Problem 2

### Part a)

Channels

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 |

The minimal channel number we need is 5 to satisfy the interference matrix. We know this must be the optimal number of channels because of the following:

We assume there is a max of 4 channels. Without loss of generality, we put channel x in cell 1.

Given , cells 2 and 7 must each have either a channel or . Since we said there is a max of 4 channels, cell 2 and 7 must have the same channel number (both or both ).

Given , cells 2 and 7 must have channel numbers two apart.

But we just showed that cells 2 and 7 must have the same channel. This is a contradiction, so there must be at least 5 channel numbers to satisfy the interference matrix.

Since we showed 5 channels is possible, that must be a optimal solution.

### Part b)

We define a conflict (a, b) as cell a has a channel that conflicts with a channel in cell b.

Here are the conflicts:

which is 11. (note that there are two pairs of channels that conflict in cell 7 with itself.)

However, if we count (x,y) and (y,x) as two conflicts, since the interference matrix is not necessarily symmetrical (in this case, it is), then we have the following conflicts:

Where the bolded pairs are the newly introduced conflicts. In this case, we would have 17 conflicts.

### Part c)

In each (cell, channel) pair, you can either have a 1 (meaning that channel exists in that cell), or a 0 (meaning there isn’t that channel in that cell). Since there are (cell, channel) pairs, there needs to be 91 decision variables.