Problem 2

1. We claim that the new dataset is linearly separable by an unbiased plane with normal vector.

To prove the claim, we look at. By definition of dot product, we have:

For, we know, and for. We also know that by our construction, is same as for the first coordinates. Thus, we know:

We also know that by the indicator construction,  is 0 except when and. Effectively, this means that the second part of our summation is merely the coordinate of multiplied with, which is. That is,

Thus, we have:

We notice that is simply in our original case. We know from question statement that. Thus, we have.

For, we notice that since is 0 except when. Thus, the second summation is 0. Also, since for, we have:

Again, this is also simply in our original case. Thus, we have.

As we have covered all cases for, we can conclude that. In other words, our constructed indeed separates

1. We firstly notice that the margin we calculated for part a is actually a functional margin, as . Thus, the geometric margin can be calculated as:

With that, we can express the mistake bound for as:

Firstly, we could bound with, since we merely swapped one coordinate out with the value k. That is, the maximum of can be no more than the norm of added another coordinate with value, which is square root of plus. That is,

As for the length of, we could bound it by and , where . Since in the problem statement we were given that . We can further simplify to:

Thus, we know that the mistake bound for is simply:

If we notice that , since . The mistake bound could be further loosening to: