Problem 2

1. We claim that the new dataset is linearly separable by an unbiased plane with normal vector.

To prove the claim, we look at. By definition of dot product, we have:

For, we know, and for. We also know that by our construction, is same as for the first coordinates. Thus, given that is separable by, we have the following inequality:

We also know that by the indicator construction,  is 0 except when and. Effectively, this means that the second part of our summation is merely the coordinate of multiplied with. That is,

Thus, we have:

We notice that is simply in our original case. Thus, we have.

For, we notice that since is 0 except when. Thus, the second summation is 0. Also, since for, we have:

Again, this is also simply in our original case. Thus, we have.

As we have covered all cases for, we can conclude that. In other words, our indeed separates