24-703/12-703 Numerical Methods in Engineering S25 Homework 1
Due 22 January 2025, 10:00AM

The program start.cpp is available on Canvas in the HW folder. The program shows some examples of input/output and data structures. If you are not familiar with programming, spend some time figuring out what this program does and how it works. You may find it useful to use start.cpp as the template for the code development in this homework.

Problem 1 (5 points) Use the log tables on Canvas to evaluate the following. For each, compute the relative error compared to the exact answer.

(a) 458×29846
(b) 56789/345
(c) $92984^{4/5}$

Problem 2 (10 points) Download the program hw1_problem2.cpp from Canvas. Note that the "fabs" function returns the absolute value of a float or double variable.

(a) Run the program and provide the output in your submission.
(b) What is the purpose of this program?
(c) Carefully explain why methods 1, 2, and 3 give different results for the larger b values. Which method is best? Explain your choice.
(d) Run the code using "double" variables instead of "float" variables (Line 19). Provide the output in your submission and comment on the results compared to (a). What is the difference between "float" and "double" variables?

Problem 3 (10 points) Consider the summation

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

(a) Using "int" variables, write a program to evaluate this expression. Test the program for $n = 30$, 300, 3000, and 30000. Put all your results and the exact answers in a table. Identify where/why your code goes wrong when it gives the wrong answer. What do your results tell you about the "int" variable? Submit a copy of your code and output.
(b) How can you modify your code so that it will give a reasonable answer for large n values?

Problem 4 (15 points) In class, we defined the following operators:

- Forward Difference: $\Delta f_i = f_{i+1} - f_i$
- Backward Difference: $\nabla f_i = f_i - f_{i-1}$
- Stepping: $Ef_i = f_{i+1}$
- Derivative: $Df_i = \left.\frac{df}{dx}\right|_i$

(a) Show that:
(i) $\nabla = 1 - E^{-1}$
(ii) $E\nabla = \Delta$
(iii) $D = \frac{1}{h}\ln\left(\frac{1}{1-\nabla}\right)$

(b) Expand (a)(iii) as a Taylor series to show that

$$D = \frac{1}{h}\left(\nabla + \frac{1}{2}\nabla^2 + \frac{1}{3}\nabla^3 + \cdots\right)$$

(c) Use the result of (b) to develop the backward difference formulas for Df_i to the first and second order. Note that here the first (or second) order corresponds to including the first term (or the first two terms).

(d) Use the result of (b) to find an expression for D^2 that depends on ∇ and its higher powers. Use the result to find the first and second order backwards difference formulas for $D^2 f_i$.