Introduction to Mathematical Reason	Test #3 MATH 300	November 28, 2007

Name: ____________________ R. Hammack Score: ________

Directions: Please answer the questions in the space provided. To get full credit you must show all of your work. Use of calculators and other computing or communication devices is not allowed on this test.

1. (10 points) Let $A = \{a, b, c, d\}$. Suppose R is an equivalence relation on A for which $(a, b) \in R$, $(c, b) \in R$, but $(a, d) \notin R$. List out the set R as a subset of $A \times A$.

   $R = \{$ $\}$

2. (10 points) The relation $\{(x, y) : x, y \in \mathbb{Z},\ 2|(x + y)\}$ is an equivalence relation on $\mathbb{Z}$.
   Describe its equivalence classes.

3. (10 points) Do the following operations in $\mathbb{Z}_5$. In each case your answer should be one of [0], [1], [2], [3], or [4].

   (a) $[4] \cdot [4] =$

   (b) $[3] + [4] =$

   (c) $([2] + [4]) \cdot [3] =$

4. (15 points) Suppose R and R' are two symmetric relations on a set A.
   Show that the relation $S = R \cap R'$ is also symmetric.

5. (10 points) Consider the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ defined as $f((x, y)) = xy + 5$.

   (a) Is f injective? Explain.

   (b) Is f surjective? Explain.

6. (15 points) Consider the function $f : \mathbb{R} \to \mathbb{R}^+$ defined as $f(x) = e^{3x+4}$.

   (a) Show that f is injective.

   (b) Show that f is surjective.

   (c) Find a formula for f^{-1}.

7. (15 points) Prove that $3|(4^n - 1)$ for every $n \in \mathbb{N}$.

8. (15 points) Prove that $1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \dfrac{n^2(n+1)^2}{4}$ for every natural number n.