Directions: Differentiate the following functions.

1. $y = 1 + x^5 + e^{x^2 + \sin(x)}$

$$y' = 0 + 5x^4 + e^{x^2 + \sin(x)}\left(2x + \cos(x)\right)$$

$$= \boxed{5x^4 + (2x + \cos(x)) e^{x^2 + \sin(x)}}$$

2. $y = \left(\dfrac{x + x^2}{1 + x^3}\right)^{10}$

$$y' = 10\left(\frac{x + x^2}{1 + x^3}\right)^9 \frac{(1 + 2x)(1 + x^3) - (x + x^2)(0 + 3x^2)}{(1 + x^3)^2}$$

$$= 10\left(\frac{x + x^2}{1 + x^3}\right)^9 \frac{1 + x^3 + 2x + 2x^4 - 3x^3 - 3x^4}{(1 + x^3)^2} = \boxed{10\left(\frac{x + x^2}{1 + x^3}\right)^9 \frac{1 + 2x - 2x^3 - x^4}{(1 + x^3)^2}}$$

3. $y = \sin^3(4x) = \left(\sin(4x)\right)^3$

$$y' = 3\left(\sin(4x)\right)^2 D_x\left[\sin(4x)\right] = 3\sin^2(4x)\cos(4x)\cdot 4$$

$$= \boxed{12\sin^2(4x)\cos(4x)}$$

4. $y = \sqrt{x^2 + e^{5x}} = \left(x^2 + e^{5x}\right)^{\frac{1}{2}}$

$$y' = \frac{1}{2}\left(x^2 + e^{5x}\right)^{-\frac{1}{2}} D_x\left[x^2 + e^{5x}\right] = \frac{1}{2(x^2 + e^{5x})^{\frac{1}{2}}}\left(2x + e^{5x}\cdot 5\right)$$

$$= \boxed{\frac{2x + 5e^{5x}}{2\sqrt{x^2 + e^{5x}}}}$$

5. $y = xe^{-x}$

$$y' = D_x\left[xe^{-x}\right] = D_x[x]e^{-x} + x D_x\left[e^{-x}\right]$$

$$= 1\cdot e^{-x} + x e^{-x}(-1)$$

$$= \boxed{e^{-x} - xe^{-x}}$$

Directions: Differentiate the following functions.

1. $y = \frac{1}{3}e^{x^3+3x}$ $y' = \frac{1}{3}e^{x^3+3x}(3x^2+3) = \boxed{e^{x^3+3x}(x^2+1)}$

2. $y = (2+3e^x)^5$ $y' = 5(2+3e^x)^4 D_x\left[2+3e^x\right]$

$= 5(2+3e^x)^4(0+3e^x) = \boxed{15e^x(2+3e^x)^4}$

3. $y = \sqrt{x+\sin(x)}$ $= \left(x+\sin(x)\right)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}\left(x+\sin(x)\right)^{-\frac{1}{2}}D_x\left[x+\sin(x)\right]$

$= \frac{1}{2(x+\sin(x))^{1/2}}(1+\cos(x)) = \boxed{\dfrac{1+\cos(x)}{2\sqrt{x+\sin(x)}}}$

4. $y = \frac{1}{\tan(3x+1)}$ $= \left(\tan(3x+1)\right)^{-1}$

$y' = -\left(\tan(3x+1)\right)^{-2}D_x\left[\tan(3x+1)\right]$

$= -\frac{1}{\tan^2(3x+1)}\sec^2(3x+1)\cdot 3 = \boxed{-\dfrac{3\sec^2(3x+1)}{\tan^2(3x+1)}}$

$= \boxed{-3\sin^2(3x+1)}$

5. $y = \sec(e^x) + e^{\sec(x)}$

$y' = \boxed{\sec(e^x)\tan(e^x)e^x + e^{\sec(x)}\sec(x)\tan(x)}$