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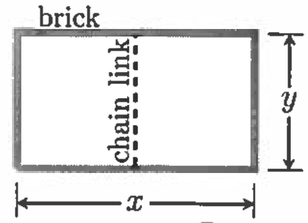
Quiz 10 — MATH 200, Section 9, April 16, 2021

1. (12 points) A total area of 2000 square feet is to be enclosed by two pens, as shown. The outside walls will be made of brick, and the inner dividing wall is chain link. The brick wall costs \$10 per foot. Chain link costs \$5 per foot. Find the dimensions x and y that minimize the cost of materials.



Cost = (cost of brick wall) + (cost of chain link)

$= 10 \cdot (\text{length of brick}) + 5(\text{length of chain})$

$= 10(2x + 2y) + 5y$

$= 20x + 25y$

$= 20x + 25\dfrac{2000}{x}$

$= 20x + \dfrac{50000}{x}$

Constraint: Area $= 2000 = xy$. Therefore: $y = \dfrac{2000}{x}$

Thus Cost $= C(x) = 20x + \dfrac{50000}{x}$

Find x that gives global maximum of this on $(0, \infty)$

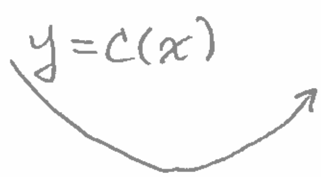
$$C'(x) = 20 - \frac{50000}{x^2} = 0$$

$$20 = \frac{50000}{x^2}$$

$$20x^2 = 50000$$

$$x^2 = 2500$$

$$x = \sqrt{2500} = 50$$ ← critical point



0 50

$- - - \mid + + +$ $C'(x) = 20 - \dfrac{50000}{x^2}$

So we get a global minimum cost when $x = 50$ and $y = \dfrac{2000}{50} = 40$

Answer $\boxed{x = 50 \quad y = 40}$

2. (8 points) Find $\displaystyle\lim_{x \to 0} \frac{e^{-x} - 1}{\sin(x)}$ ← form $\frac{0}{0}$

$$= \lim_{x \to 0} \frac{e^{-x}(-1)}{\cos(x)} = \frac{-e^0}{\cos(0)} = \frac{-1}{1} = \boxed{-1}$$