Name: ———————————   R. Hammack   Score: ————

Directions: Please answer the questions in the space provided. To get full credit you must show all of your work. Use of calculators and other computing or communication devices is not allowed on this test.

1. Write each of the following sets by listing its elements or describing it with a familiar symbol.

   (a) $\{x \in \mathbb{Z} : |x| \leq 3\} =$

   (b) $\{X \in \mathscr{P}(\mathbb{N}) : |X \cup \{1, 2, 3\}| \leq 3\} =$

   (c) $\{(x, y) \in \mathbb{N} \times \mathbb{R} : x^2 = 4, y^2 = 2\} =$

   (d) $\{(x, y) \in \mathbb{R} \times \mathbb{R} : y = x^2\} \cap \{(x, y) \in \mathbb{R} \times \mathbb{R} : y = x\} =$

   (e) $\mathbb{R} - \mathscr{P}(\mathbb{R}) =$

2. Write each of the following sets by listing its elements or describing it with a familiar symbol.

   (a) $\mathscr{P}(\mathscr{P}(\{\emptyset\})) =$

   (b) $\{\emptyset\} \times \{\emptyset\} =$

   (c) $\emptyset \times \mathbb{N} =$

   (d) $(\mathbb{R} - \mathbb{Z}) \cap \mathbb{N} =$

   (e) $\displaystyle\bigcup_{X \in \mathscr{P}(\mathbb{N})} \overline{X} =$

3. (a) Suppose you know that P is false, and that the statement $(R \Rightarrow S) \Leftrightarrow (P \wedge Q)$ is true. Can the true/false values of R and S be determined? Explain. (This can be done without a truth table.)

   (b) Write an expression that is logically equivalent to $(\sim P) \vee (\sim Q)$ and contains only one $\sim$.

4. Write out a truth table to decide if $(\sim P) \wedge (P \Rightarrow Q)$ and $\sim (Q \Rightarrow P)$ are logically equivalent.

5. Suppose $a, b, c \in \mathbb{Z}$, and $a \neq 0$. Prove the following statement: If $a \nmid bc$, then $a \nmid b$ and $a \nmid c$. [Suggestion: Contrapositive may be easiest.]

6. Prove that $\sqrt{2}$ is irrational. [Suggestion: proof by contradiction is probably easiest.]

7. Suppose A and B are sets. Prove that $\mathscr{P}(A \cap B) = \mathscr{P}(A) \cap \mathscr{P}(B)$.

8. Prove that if $n \in \mathbb{Z}$, then $n^2 - 3n + 9$ is odd.

9. Suppose $a, b \in \mathbb{Z}$. Prove that $(a - 3)b^2$ is even if and only if a is odd or b is even.

10. Suppose A, B and C are sets. If $B \subseteq C$, then $A \times B \subseteq A \times C$.