

1. This problem concerns the equation $2\sin(xy) = \sqrt{2}y^2$

(a) Find $\frac{dy}{dx}$.

$$D_x [2\sin(xy)] = D_x [\sqrt{2}y^2]$$

$$2\cos(xy)(1 \cdot y + xy') = \sqrt{2} \cdot 2 \cdot y y'$$

$$2\cos(xy)y + 2\cos(xy)xy' = 2\sqrt{2}yy'$$

$$2\cos(xy)xy' - 2\sqrt{2}yy' = -2\cos(xy)y$$

$$y' \cdot 2(\cos(xy)x - \sqrt{2}y) = -2\cos(xy)y$$

$$y' = \frac{-2y \cos(xy)}{2(x \cos(xy) - \sqrt{2}y)}$$

$$\boxed{\frac{dy}{dx} = \frac{-y \cos(xy)}{x \cos(xy) - \sqrt{2}y}}$$

(b) Use your answer from part (a) to find the slope of the tangent to the graph of $2\sin(xy) = \sqrt{2}y^2$ at the point $(\pi/4, 1)$.

$$\begin{aligned} \left. \frac{dy}{dx} \right|_{(\pi/4, 1)} &= \frac{-1 \cos(\frac{\pi}{4} \cdot 1)}{\frac{\pi}{4} \cos(\frac{\pi}{4} \cdot 1) - \sqrt{2} \cdot 1} = \frac{-\frac{\sqrt{2}}{2}}{\frac{\pi}{4} \frac{\sqrt{2}}{2} - \sqrt{2}} \\ &= \frac{-\sqrt{2}(\frac{1}{2})}{\sqrt{2}(\frac{\pi}{8} - 1)} = \frac{-\frac{1}{2}}{\frac{\pi}{8} - 1} = \frac{-\frac{1}{2}}{\frac{\pi}{8} - \frac{8}{8}} = \frac{-\frac{1}{2}}{\frac{\pi - 8}{8}} = \boxed{\frac{-4}{\pi - 8}} \end{aligned}$$

1. This problem concerns the equation $\ln(xy) = x - y$

(a) Find $\frac{dy}{dx}$.

$$D_x [\ln(xy)] = D_x [x - y]$$

$$\frac{1}{xy} (1 \cdot y + xy') = 1 - y'$$

$$1 \cdot y + xy' = (1 - y')xy$$

$$y + xy' = xy - xy y'$$

$$xy' + xy y' = xy - y$$

$$y'(x + xy) = xy - y$$

$$y' = \frac{xy - y}{x + xy}$$

$$\boxed{\frac{dy}{dx} = \frac{xy - y}{x + xy}}$$

(b) Use your answer from part (a) to find the slope of the tangent to the graph of $\ln(xy) = x - y$ at the point $(1, 1)$.

$$\left. \frac{dy}{dx} \right|_{(x,y)=(1,1)} = \frac{1 \cdot 1 - 1}{1 + 1 \cdot 1} = \frac{0}{2} = \boxed{0}$$

1. This problem concerns the equation $xy + \cos(xy) = 1$

(a) Find $\frac{dy}{dx}$.

$$D_x [xy + \cos(xy)] = D_x [1]$$

$$1 \cdot y + x y' - \sin(xy)(1 \cdot y + x y') = 0$$

$$y + x y' - \sin(xy)y - \sin(xy)x y' = 0$$

$$x y' - \sin(xy)x y' = \sin(xy)y - y$$

$$y'(x - \sin(xy)x) = \sin(xy)y - y$$

$$y' = \frac{\sin(xy)y - y}{x - \sin(xy)x}$$

$$\boxed{\frac{dy}{dx} = -\frac{y}{x}}$$

$$y' = \frac{y(\sin(xy) - 1)}{x(1 - \sin(xy))}$$

$$\boxed{y' = -\frac{y}{x}}$$

(b) Use your answer from part (a) to find the slope of the tangent to the graph of $xy + \cos(xy) = 0$ at the point $(1, 0)$.

$$m = \left. \frac{dy}{dx} \right|_{(x,y)=(1,0)} = -\frac{0}{1} = \boxed{0}$$

1. This problem concerns the equation $x^4 + 2xy + y^4 = \cos(x)$

(a) Find $\frac{dy}{dx}$.

$$\begin{aligned} D_x [x^4 + 2xy + y^4] &= D_x [\cos(x)] \\ 4x^3 + 2y + 2xy' + 4y^3y' &= -\sin(x) \\ 2xy' + 4y^3y' &= -\sin(x) - 4x^3 - 2y \\ y'(2x + 4y^3) &= -\sin(x) - 4x^3 - 2y \\ y' &= \frac{-\sin(x) - 4x^3 - 2y}{2x + 4y^3} \end{aligned}$$

$$\boxed{\frac{dy}{dx} = \frac{-\sin(x) - 4x^3 - 2y}{2x + 4y^3}}$$

(b) Use your answer from part (a) above to find the slope of the tangent to the graph of $x^4 + 2xy + y^4 = \cos(x)$ at the point $(0, 1)$.

$$\begin{aligned} \left. \frac{dy}{dx} \right|_{(x,y)=(0,1)} &= \frac{-\sin(0) - 4 \cdot 0^3 - 2 \cdot 1}{2 \cdot 0 + 4 \cdot 1^3} = \frac{-2}{4} \\ &= \boxed{-\frac{1}{2}} \end{aligned}$$