



$$1. \int e^{2x^2} 4x \, dx = \int e^u \, du = e^u + C = \boxed{e^{2x^2} + C}$$

$$\begin{cases} u = 2x^2 \\ \frac{du}{dx} = 4x \\ du = 4x \, dx \end{cases}$$

$$2. \int \frac{1}{x^2} \sqrt{2 - \frac{1}{x}} \, dx = \int \sqrt{2 - \frac{1}{x}} \frac{1}{x^2} \, dx = \int \sqrt{u} \, du = \int u^{\frac{1}{2}} \, du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$\begin{cases} u = 2 - \frac{1}{x} \\ \frac{du}{dx} = 0 - \frac{-1}{x^2} = \frac{1}{x^2} \\ du = \frac{1}{x^2} \, dx \end{cases}$$

$$= \frac{2}{3} \sqrt{u}^3 + C = \boxed{\frac{2}{3} \sqrt{2 - \frac{1}{x}}^3 + C}$$

$$3. \int \frac{6x^3 + 6}{x^4 + 4x} \, dx = \int \frac{1}{x^4 + 4x} (6x^3 + 6) \, dx = \int \frac{1}{u} \frac{3}{2} \, du = \frac{3}{2} \int \frac{1}{u} \, du$$

$$\begin{cases} u = x^4 + 4x \\ \frac{du}{dx} = 4x^3 + 4 \\ du = 4(x^3 + 1) \, dx \end{cases} \rightarrow \begin{cases} \frac{6}{4} \, du = \frac{6}{4} 4(x^3 + 1) \, dx \\ \frac{3}{2} \, du = (6x^3 + 6) \, dx \end{cases} = \frac{3}{2} \ln|u| + C = \boxed{\frac{3}{2} \ln|x^4 + 4x| + C}$$

4. Find the area under the graph of $x \sin(x^2)$ between 0 and $\sqrt{\pi/6}$.

$$\int_0^{\sqrt{\pi/6}} x \sin(x^2) \, dx = \frac{1}{2} \int_0^{\sqrt{\pi/6}} \sin(u) \, du$$

$$\begin{cases} u = x^2 \\ \frac{du}{dx} = 2x \\ du = 2x \, dx \end{cases}$$

$$= \frac{1}{2} \left[-\cos(u) \right]_0^{\sqrt{\pi/6}} = \frac{1}{2} \left(-\cos\left(\frac{\pi}{6}\right) - (-\cos(0)) \right) \\ = \frac{1}{2} \left(-\frac{\sqrt{3}}{2} + 1 \right) = \boxed{\frac{1}{2} - \frac{\sqrt{3}}{4} \text{ square units}}$$