Introduction to Mathematical Reasoning — MATH 300 — February 7, 2007

R. Hammack — Test #1

Name: ________________________ Score: _______

Directions: This is a take-home test. It is due at the beginning of class on Monday, February 12. Please answer all questions in the space provided. Consider working the problems on scratch paper, then rewriting them neatly on the test. Additional copies of this test can be downloaded from my web page if needed.

- Do not discuss this test with anyone other than the instructor. Ask me if you have any questions
- You may consult your text and notes, but **no** other source.
- To get full credit on problems 4–10, you must show all of your work.
- Each problem is worth 10 points.

1. Write each of the following sets by listing its elements or describing it with a familiar symbol.

   (a) $\{x \in \mathbb{Z} : |x| \leq 3\} =$

   (b) $\{X \in \mathcal{P}(\mathbb{N}) : |X \cup \{1,2,3\}| \leq 3\} =$

   (c) $\{(x,y) \in \mathbb{N} \times \mathbb{R} : x^2 = 4, y^2 = 2\} =$

   (d) $\{(x,y) \in \mathbb{R} \times \mathbb{R} : y = x^2\} \cap \{(x,y) \in \mathbb{R} \times \mathbb{R} : y = x\} =$

   (e) $\mathbb{R} - \mathcal{P}(\mathbb{R}) =$

2. Write each of the following sets by listing its elements or describing it with a familiar symbol.

   (a) $\mathcal{P}(\mathcal{P}(\{\emptyset\})) =$

   (b) $\{\emptyset\} \times \{\emptyset\} =$

   (c) $\emptyset \times \mathbb{N} =$

   (d) $(\mathbb{R} - \mathbb{Z}) \cap \mathbb{N} =$

   (e) $\displaystyle\bigcup_{X \in \mathcal{P}(\mathbb{N})} \overline{X} =$

3. For each $n \in \mathbb{N}$, let $A_n = \{(x,y) \in \mathbb{R} \times \mathbb{R} : 1 - \frac{1}{n} < x^2 + y^2 \leq 1 + \frac{1}{n}\}$.

   (a) Describe the set A_2. A carefully drawn picture will suffice.

   (b) $\displaystyle\bigcap_{n=1}^{\infty} A_n =$

   (c) $\displaystyle\bigcup_{n=1}^{\infty} A_n =$

4. Write an expression that is logically equivalent to $\sim (\ \forall x, P(x) \vee \sim Q(x)\ )$ and contains only one $\sim$.

5. Use a truth table to show $P \wedge (Q \vee R)$ and $(P \wedge Q) \vee (P \wedge R)$ are logically equivalent.

6. Write out truth tables for the statements $P \Rightarrow Q$ and $(P \wedge \sim Q) \Rightarrow (Q \wedge \sim Q)$.
How do these two statements compare?

7. Let $x \in \mathbb{Z}$. Prove that $5x - 11$ is even if and only if x odd.

8. Prove that if $n \in \mathbb{Z}$, then $n^2 - 3n + 9$ is odd.

9. Suppose $a, b \in \mathbb{Z}$. Prove that if ab is odd, then a and b are both odd.

10. Recall that if a and b are integers, we say that a *divides* b, written $a|b$, if there is an integer n for which $b = an$. Prove that if $a|b$ and $a|c$, then $a|(b + c)$.