MATH 211 Test #2 ♠ November 16, 2017

Name:_____ R. Hammack Score:_____

Directions No calculators. Please put all phones, etc., away.

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1. (4 points) Complete the following truth tables.

P	Q	$P \Leftrightarrow Q$
T	T	
T	F	
F	T	
F	F	

P	Q	$P \Rightarrow Q$
T	T	
T	F	
F	T	
F	F	

2. (12 points) Complete the truth table to decide if $P \vee (Q \wedge R)$ and $(\sim Q \vee \sim R) \Rightarrow P$ are logically equivalent.

P	Q	R	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Are they logically equivalent? Why or why not?

3. (6 points) Suppose the statement $\sim\Big(S \Rightarrow (P \vee Q \vee \sim R) \Big)$ is **true**.
Find the truth values of P, Q, R and S. (This can be done without a truth table.)

4. (12 points) This problem concerns the following statement.
P : There is a number $n \in \mathbb{Z}$ for which $m \mid n$ for every $m \in \mathbb{Z}$.

(a) Is the statement P true or false? **Explain**.

(b) Write the statement P in symbolic form.

(c) Form the negation $\sim P$ of your answer from (b), and simplify.

(d) Write the negation $\sim P$ as an English sentence.
(The sentence may use mathematical symbols.)

5. (6 points) Complete the first and last lines of each of the following proof outlines.

Proposition: If P, then Q.
Proof: (Direct)
Suppose _____
⋮
Therefore _____ . ■

Proposition: If P, then Q.
Proof: (Contradiction)
Suppose _____
⋮
Therefore _____ . ■

Proposition: If P, then Q.
Proof: (Contrapositive)
Suppose _____
⋮
Therefore _____ . ■

6. (15 points) Let $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$.
Prove: If $a \equiv b \pmod{n}$, then $ab \equiv b^2 \pmod{n}$. [Use direct proof.]

7. (15 points) Suppose $a, b, c \in \mathbb{Z}$. **Prove:** If $a \nmid bc$, then $a \nmid b$ and $a \nmid c$. [Use contrapositive.]

8. (15 points) **Prove:** If $4 | (a^2 + b^2)$, then a and b are not both odd. [Use contradiction.]

9. (15 points) Suppose $a, b, c \in \mathbb{Z}$. **Prove:** If $a | b$ and $a | (b + c)$, then $a | c$.