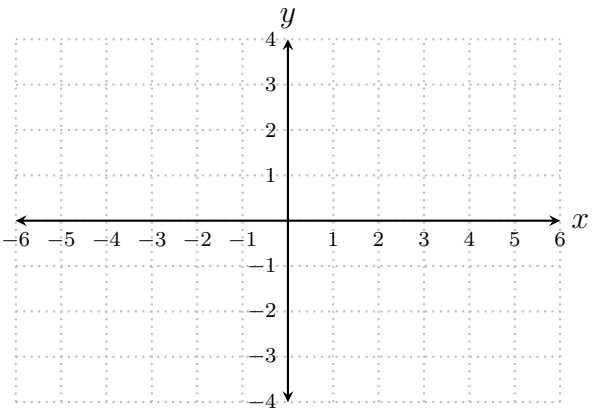
Name: _____ MATH 200 MIDTERM EXAM MARCH 17, 2022

Directions: Closed book, closed notes, no calculators. Put all phones, etc., away. You will need only a pencil or pen.

1. (10 points) Draw the graph of one function $f(x)$ meeting **all** of the following conditions.
   (a) $\lim_{z\to -3} f(x) = \infty$
   (b) $\lim_{z\to -\infty} f(x) = \infty$
   (c) $\lim_{z\to \infty} f(x) = 2$
   (d) f is continuous on $(-\infty, -3)\cup(-3, 2)\cup(2, \infty)$.
   (e) $f(-1) = 1$
   (f) $f'(-1) = 0$
   (g) $f'(1)$ does not exist
   (h) $\lim_{z\to 2^-} f(x) = 1$
   (i) $\lim_{z\to 2^+} f(x) = 3$



2. (24 points) Find the limits.
   (a) $\lim_{x\to \sqrt{2}/2} \sin^{-1}(x) =$
   (b) $\lim_{x\to -\infty} \tan^{-1}(x) =$
   (c) $\lim_{z\to 3} \dfrac{\ln(z) - \ln(3)}{z-3} =$
   (d) $\lim_{x\to 3} \dfrac{1-\dfrac{3}{x}}{x-3} =$
   (e) $\lim_{x\to 1} \dfrac{1-\dfrac{3}{x}}{x-3} =$
   (f) $\lim_{x\to \infty} \dfrac{1-\dfrac{3}{x}}{x-3} =$

3. (6 points) Use a **limit definition** of the derivative to find the derivative of $f(x) = \dfrac{1}{3x}$.

4. (6 points) Find all x for which the tangent to the graph of $y = \dfrac{x^3}{3} - \dfrac{3x^2}{2} + 2x + 1$ has slope 20.

5. (6 points) Suppose it costs $C(x)$ dollars to build a transmitting tower that is x meters high. Suppose it happens that $C'(100) = 1000$. Explain in simple terms what this means.

6. (35 points) Find the derivatives of these functions. You do **not** need to simplify your answers.

(a) $f(x) = 3x^2 + e^3$

(b) $f(x) = \dfrac{4}{\sqrt{x}}$

(c) $f(x) = \tan\left(\dfrac{1}{x^2+1}\right)$

(d) $f(x) = 3x^4 \cos(x)$

(e) $f(x) = \left(\tan^{-1}(x)\right)^4$

(f) $f(x) = \dfrac{6x+1}{x^3+4x+9}$

(g) $y = \sec\left(\ln\left(x^3+x\right)\right)$

7. (7 points) Given the equation $x\ln(y) + x^2 = 5y$, find y'.

8. (6 points) A spherical balloon is inflated at a rate of 100π cubic feet per minute. How fast is the radius increasing at the instant that the radius is 5 feet?

(A sphere of radius r has volume $V = \frac{4}{3}\pi r^3$ cubic units, and surface area $S = 4\pi r^2$ square units.)