1. Use a limit definition of the derivative to find the derivative of the function $f(x) = \dfrac{1}{x+1}$.

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x}$$

$$= \lim_{z \to x} \frac{\frac{1}{z+1} - \frac{1}{x+1}}{z - x}$$

$$= \lim_{z \to x} \frac{\frac{1}{z+1} - \frac{1}{x+1}}{z - x} \cdot \frac{(z+1)(x+1)}{(z+1)(x+1)}$$

$$= \lim_{z \to x} \frac{(x+1) - (z+1)}{(z-x)(z+1)(x+1)}$$

$$= \lim_{z \to x} \frac{x - z}{(z-x)(z+1)(x+1)}$$

$$= \lim_{z \to x} \frac{-(z - x)}{(z-x)(z+1)(x+1)}$$

$$= \lim_{z \to x} \frac{-1}{(z+1)(x+1)} = \frac{-1}{(x+1)(x+1)} = \frac{-1}{(x+1)^2}$$

$$\underline{\text{Answer :}} \quad \boxed{f'(x) = \frac{-1}{(x+1)^2}}$$

1. Use a limit definition of the derivative to find the derivative of the function $f(x) = 3x^2 - 2$.

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x}$$

$$= \lim_{z \to x} \frac{3z^2 - 2 - (3x^2 - 2)}{z - x}$$

$$= \lim_{z \to x} \frac{3z^2 - 3x^2}{z - x}$$

$$= \lim_{z \to x} \frac{3(z^2 - x^2)}{z - x}$$

$$= \lim_{z \to x} \frac{3(z - x)(z + x)}{z - x}$$

$$= \lim_{z \to x} 3(z + x) = 3(x + x) = 3 \cdot 2x = 6x$$

Answer $f'(x) = 6x$

1. Use a limit definition of the derivative to find the derivative of the function $f(x) = \sqrt{x+5}$.

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x}$$

$$= \lim_{z \to x} \frac{\sqrt{z+5} - \sqrt{x-5}}{z - x}$$

$$= \lim_{z \to x} \frac{\sqrt{z+5} - \sqrt{x+5}}{z - x} , \frac{\sqrt{z+5} + \sqrt{x+5}}{\sqrt{z+5} + \sqrt{x+5}}$$

$$= \lim_{z \to x} \frac{\sqrt{z+5}^2 + \sqrt{z+5}\sqrt{x+5} - \sqrt{x+5}\sqrt{z+5} - \sqrt{x+5}^2}{(z-x)(\sqrt{z+5} + \sqrt{x+5})}$$

$$= \lim_{z \to x} \frac{(z+5) - (x+5)}{(z-x)(\sqrt{z+5} + \sqrt{x+5})}$$

$$= \lim_{z \to x} \frac{z-x}{(z-x)(\sqrt{z+5} + \sqrt{x+5})}$$

$$= \lim_{z \to x} \frac{1}{\sqrt{z+5} + \sqrt{x+5}}$$

$$= \frac{1}{\sqrt{x+5} + \sqrt{x+5}} = \frac{1}{2\sqrt{x+5}}$$

Answer:

$$f'(x) = \frac{1}{2\sqrt{x+5}}$$

1. Use a limit definition of the derivative to find the derivative of the function $f(x) = \frac{2}{3x}$.

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x}$$

$$= \lim_{z \to x} \frac{\frac{2}{3z} - \frac{2}{3x}}{z - x}$$

$$= \lim_{z \to x} \frac{\frac{2}{3z} - \frac{2}{3x}}{z - x} \cdot \frac{3zx}{3zx}$$

$$= \lim_{z \to x} \frac{2x - 2z}{(z - x) \, 3zx}$$

$$= \lim_{z \to x} \frac{-2(z - x)}{(z - x) \, 3zx}$$

$$= \lim_{z \to x} \frac{-2}{3zx}$$

$$= \frac{-2}{3 \cdot x \cdot x} = \frac{-2}{3x^2}$$

$$\boxed{\text{Answer: } f'(x) = \frac{-2}{3x^2}}$$