MATH 211 FINAL EXAM May 7, 2006

Name: ____________________ R. Hammack Score: ________

Directions: Prove the following statements in the space provided. To get full credit you must show all of your work. Use of calculators is **not** allowed on this test.

1. Prove that if A, B and C are nonempty sets and $A \times B = A \times C$, then $B = C$.

2. Prove that if x and y are real numbers that are both greater than zero, then $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$. (Suggestion: consider proof by contradiction or contrapositive.)

3. Suppose $x \in \mathbb{Z}$. Prove $7x - 3$ is even if and only if x is odd.

4. Prove or disprove: If A and B are nonempty sets, then $\mathscr{P}(A) \cap \mathscr{P}(B) = \mathscr{P}(A \cap B)$.

5. Prove or disprove: For all real numbers x and y, if $|x+y| = |x-y|$, then $y = 0$.

6. Prove or disprove: If an equivalence relation on a set A has finitely many equivalence classes, then A is finite.

7. Suppose a, b and c are integers. Prove that if $a|b$ and $a|(b+c)$, then $a|c$.

8. Suppose A and B are sets. Use the technique of contrapositive proof to prove the following:
If $A \times B = \emptyset$, then $A = \emptyset$ or $B = \emptyset$.

9. Prove that if $a \equiv 1 \pmod 5$, then $a^2 \equiv 1 \pmod 5$.

10. Prove that $\sqrt{2}$ is irrational.

11. Suppose R is a transitive relation on a set A, and $x \not R\, x$ for all $x \in A$. Show that if xRy, then $y \not R\, x$.

The questions on this page involve the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined as $f((x, y)) = (x + y, x)$

12. Prove that f is injective.

13. Prove that f is surjective.

14. Find a formula for f^{-1}

15. Use mathematical induction to prove $1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \dfrac{n^2(n+1)^2}{4}$ for all $n \in \mathbb{N}$.

16. Use mathematical induction to prove $4|(5^n - 1)$ for every $n \in \mathbb{N}$.

17. Use mathematical induction to prove $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + n \cdot n! = (n+1)! - 1$ for every $n \in \mathbb{N}$.

18. A bag contains 20 identical red balls, 20 identical blue balls, 20 identical green balls, and one white ball. You reach in and grab 15 balls. How many different outcomes are possible?

19. This problem concerns 4-letter codes that can be made from the letters A,B,C,D,E, …, Z of the English Alphabet.

(a) How many such codes can be made?

(b) How many such codes are there that have no two consecutive letters the same?

20. A 4-letter list is made from the letters L,I,S,T,E,D according to the following rule: Repetition is allowed, and the first two letters on the list are vowels or the list ends in D. How many such lists are possible?