Introduction to Mathematical Reason | Test #2 MATH 300 | March 7, 2007

Name: _____ R. Hammack Score: _____

Directions: Please answer the questions in the space provided. To get full credit you must show all of your work. Use of calculators and other computing or communication devices is not allowed on this test.

1. Complete the following definitions.

   (a) Suppose $a, b \in \mathbb{Z}$. Then $a|b$ if _____

   (b) Suppose $a, b, n \in \mathbb{Z}$. Then $a \equiv b \pmod{n}$ if _____

   (c) A number r is rational if _____

   (d) A number r is irrational if _____

   (e) If X and Y are sets, then $X - Y =$ _____

2. Suppose $a, b, c \in \mathbb{Z}$, and $a \neq 0$. Prove the following statement: If $a \nmid bc$, then $a \nmid b$ and $a \nmid c$.
   [Suggestion: Contrapositive may be easiest.]

3. Suppose $a, b, c, d, n \in \mathbb{Z}$, and $n \geq 2$. Prove the following statement.
   If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $a + c \equiv b + d \pmod{n}$.

4. Let $x \in \mathbb{R}$. Prove the following statement: If $3x^4 + 1 \leq x^7 + x^3$, then $x \geq 0$.

5. Prove that $\sqrt{2}$ is irrational. [Suggestion: proof by contradiction is probably easiest.]

6. Suppose A, B, C and D are sets. Prove the following statement.
If $A \subseteq C$ and $B \subseteq D$, then $A \times B \subseteq C \times D$.

FOR THE PROBLEMS ON THIS PAGE:
Decide if the statement is true or false. If it is true, prove it; if it is false, give a counterexample.

7. Let A and B be sets. If $A - B = B - A$, then $A - B = \emptyset$.

8. If $x, y \in \mathbb{R}$ and $x^2 < y^2$, then $x < y$.

9. For every two sets A and B, $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.

10. Suppose A, B, C and D are sets. If $A \times B \subseteq C \times D$, then $A \subseteq C$ and $B \subseteq D$.