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QUIZ 24

MATH 200
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$$1. \int_1^2 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_1^2 = \left(\frac{2^3}{3} + 2 \right) - \left(\frac{1^3}{3} + 1 \right) \\ = \frac{8}{3} + 2 - \frac{1}{3} - 1 = \frac{7}{3} + 1 = \boxed{\frac{10}{3}}$$

$$2. \int_0^\pi \sin(x) dx = \left[-\cos(x) \right]_0^\pi = (-\cos(\pi) - (-\cos(0))) \\ = -(-1) - (-1) = \boxed{2}$$

3. Find the area under the graph of $y = x^2$ between $x = 0$ and $x = 2$.

$$\int_0^2 x^2 dx = \left[\frac{x^3}{3} \right]_0^2 = \frac{2^3}{3} - \frac{0^3}{3} = \boxed{\frac{8}{3} \text{ sq. units}}$$

4. Find the derivative of the function $F(x) = \int_1^x \frac{1 + \cos(t)}{\sqrt{t+4}} dt$.

$$F'(x) = \frac{1 + \cos(x)}{\sqrt{x+4}}$$

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5. Find the derivative of the function $y = \int_1^{x^2+x} \frac{1 + \cos(t)}{\sqrt{t+4}} dt$.

Use chain rule

$$y = \begin{cases} \int_1^u \frac{1 + \cos(t)}{\sqrt{t+4}} dt \\ u = x^2 + x \end{cases}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1 + \cos(u)}{\sqrt{u+4}} (2x+1) = \boxed{\frac{1 + \cos(x^2+x)}{\sqrt{x^2+x+4}} (2x+1)}$$



$$1. \int_{-1}^1 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_{-1}^1 = \left(\frac{1^3}{3} + 1 \right) - \left(\frac{(-1)^3}{3} + (-1) \right)$$

$$= \frac{1}{3} + 1 + \frac{1}{3} + 1 = \frac{2}{3} + 2 = \boxed{\frac{8}{3}}$$

$$2. \int_0^1 \sqrt{x} dx = \int_0^1 x^{\frac{1}{2}} dx = \left[\frac{\frac{1}{2} + 1}{\frac{1}{2} + 1} x^{\frac{1}{2} + 1} \right]_0^1 = \left[\frac{2}{3} x^{3/2} \right]_0^1$$

$$= \left[\frac{2}{3} \sqrt{x^3} \right]_0^1 = \frac{2}{3} \sqrt{1^3} - \frac{2}{3} \sqrt{0} = \boxed{\frac{2}{3}}$$

3. Find the area under the graph of $y = \sin(x)$ between $x = 0$ and $x = \pi$.

$$\int_0^{\pi} \sin(x) dx = \left[-\cos(x) \right]_0^{\pi} = -\cos(\pi) - (-\cos(0))$$

$$= -(-1) - (-1) = \boxed{2 \text{ sq. units}}$$

4. Find the derivative of the function $F(x) = \int_1^x \frac{\sqrt{t+4}}{1+\cos(t)} dt$.

$$F'(x) = \frac{\sqrt{x+4}}{1+\cos(x)}$$

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5. Find the derivative of the function $y = \int_1^{\sin(x)} \frac{\sqrt{t+4}}{1+\cos(t)} dt$.

Use chain rule

$$\begin{cases} y = \int_1^u \frac{\sqrt{t+4}}{1+\cos(t)} dt \\ u = \sin(x) \end{cases}$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = \frac{\sqrt{u+4}}{1+\cos(u)} \cos(x) = \boxed{\frac{\sqrt{\sin(x)+4} \cdot \cos(x)}{1+\cos(\sin(x))}}$$