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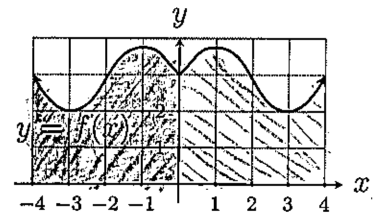
QUIZ 15

MATH 200

June 20, 2022

1. A function $f(x)$ is graphed below. If $\int_{-4}^{4} f(x)\,dx = 22.6$, what is $\int_0^4 f(x)\,dx$?



By symmetry $\int_{-4}^{4} f(x)\,dx = 2\int_0^4 f(x)\,dx$

$22.6 = 2\int_0^4 f(x)\,dx$

$\Rightarrow \int_0^4 f(x)\,dx = \frac{22.6}{2} = \boxed{11.3}$

2. Suppose f and g are functions for which $\int_0^5 f(x)\,dx = 3$, $\int_0^2 3g(x)\,dx = 12$, and $\int_2^5 g(x)\,dx = -1$. Find $\int_0^5 3f(x) - g(x)\,dx$.

$\int_0^2 3g(x)\,dx = 12 \Rightarrow 3\int_0^2 g(x)\,dx = 12 \Rightarrow \boxed{\int_0^2 g(x)\,dx = 4}$

$\int_0^5 3f(x) - g(x)\,dx = 3\int_0^5 f(x)\,dx - \int_0^5 g(x)\,dx = 3\cdot 3 - \int_0^5 g(x)\,dx$

$= 9 - \left(\int_0^2 g(x)\,dx + \int_2^5 g(x)\,dx\right) = 9 - (4 + (-1)) = \boxed{6}$

3. Write $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{1+(2+7k/n)^2}\,\frac{7}{n}$ as a definite integral.

As k goes from 0 to n, $2 + \frac{7k}{n}$ goes from 2 to 9.
This suggests $a = 2$, $b = 9$ and $\Delta x = \frac{b-a}{n} = \frac{9-2}{n} = \frac{7}{n}$.
Then: $x_k = a + k\Delta x = 2 + k\frac{7}{n} = 2 + \frac{7k}{n}$

and $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{1+\left(2+\frac{7k}{n}\right)^2}\frac{7}{n} = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{1+x_k^2}\Delta x = \boxed{\int_2^9 \frac{1}{1+x^2}\,dx}$

4. Write $\int_3^4 \sin(x)\,dx$ as a limit of Riemann sums (such as in problem 3 above).

$a = 3$

$b = 4$

$\Delta x = \frac{4-3}{n} = \frac{1}{n}$

$x_k = a + k\Delta x = 3 + \frac{k}{n}$

$\displaystyle\int_3^4 \sin(x)\,dx = \lim_{n\to\infty}\sum_{k=1}^{n}\sin(x_k)\,\Delta x$

$\displaystyle = \boxed{\lim_{n\to\infty}\sum_{k=1}^{n}\sin\left(3+\frac{k}{n}\right)\frac{1}{n}}$