Introduction to Mathematical Reason	MATH 300 Test #1	Febraury 23, 2010

Name:____________________ R. Hammack Score:_______

Directions This is just a quick quiz to test your knowledge of various definitions. Most questions are short-answer. You need not explain your work unless asked.

1. **Short answer.** Write each of the following sets by listing its elements between braces or denoting it with a familiar symbol or symbols.

   (a) $\{x \in \mathbb{Z} \,:\, |3x| \leq 10\} = \{-3, -2, -1, 0, 1, 2, 3\}$

   (b) $[5, 7] \cap [7, 10] = \{7\}$

   (c) $\{x \in \mathbb{R} \,:\, \sin(\pi x) = 0\} - \mathbb{Z} = \mathbb{Z} - \mathbb{Z} = \emptyset$

   (d) $\mathscr{P}(\{1, 2\} \times \{\emptyset\}) = \mathscr{P}(\{1, \emptyset), (2, \emptyset)) = \{\emptyset, \{(1, \emptyset)\}, \{(2, \emptyset)\}, \{(1, \emptyset), (2, \emptyset)\}\}.$

   (e) $\bigcap_{n \in \mathbb{N}} [3,\ 5 + 1/n] = [3, 5]$

   (f) $(\{0, 3\} \times \mathbb{N}) \cap (\mathbb{N} \times \{5, 6\}) = \{(3, 5), (3, 6)\}$

   (g) $(\mathbb{R} - \mathbb{N}) \cap \mathbb{Z} = \{0, -1, -2, -3, -4, -5, \ldots\}$

   (h) $\{X : X \subseteq \{3, 4\} \cap X\} = \{\{\}, \{3\}, \{4\}, \{3, 4\}\}$

2. **Short answer.** Write the following sets in set-builder notation.

   (a) $\{\ldots, -3, 2, 7, 12, 17, 22, 27, \ldots\} = \{2 + 5n : n \in \mathbb{Z}\}$

   (b) $\left\{\frac{1}{3}, \frac{2}{9}, \frac{3}{27}, \frac{4}{81}, \ldots\right\} = \left\{\frac{n}{3^n} : n \in \mathbb{Z}\right\}$

3. Write a truth table for the expression: $(P \Leftrightarrow Q) \Rightarrow \sim (P \vee Q)$

P	Q	$P \Leftrightarrow Q$	$P \vee Q$	$\sim (P \vee Q)$	$(P \Leftrightarrow Q) \Rightarrow \sim (P \vee Q)$
T	T	T	T	F	F
T	F	F	T	F	T
F	T	F	T	F	T
F	F	T	F	T	T

4. Consider the following statement:
For every subset $X \subseteq \mathbb{N}$, *there is a subset* $Y \subseteq \mathbb{N}$ *for which* $|X - Y| = 1$.

   (a) Is this statement true or false? Explain.
   This statement is FALSE. Consider that X could be the empty set. In that case $|X - Y| = 0 \neq 1$.

   (b) Write the statement in symbolic form.
   $\forall X \subseteq \mathbb{N}, \exists Y \subseteq \mathbb{N}, |X - Y| = 1$

   (c) Write the negation of the statement as an English sentence.
   The negation is:
   $\sim (\forall X \subseteq \mathbb{N}, \exists Y \subseteq \mathbb{N}, |X - Y|) =$
   $\exists X \subseteq \mathbb{N}, \sim (\exists Y \subseteq \mathbb{N}, |X - Y|) =$
   $\exists X \subseteq \mathbb{N}, \forall Y \subseteq \mathbb{N}, \sim (|X - Y|) =$
   $\exists X \subseteq \mathbb{N}, \forall Y \subseteq \mathbb{N}, |X - Y| \neq 1$

   Final Answer: $\boxed{\textit{There is a subset } X \subseteq \mathbb{N} \textit{ for which } |X - Y| \neq 1 \textit{ for every subset } Y \subseteq \mathbb{N}.}$

5. This question involves lists made from the symbols A, B, C, D, E, F. How many length-6 lists can be made from these symbols if repetition is allowed and the first **or** last entry must be an A? (Show your work. It is OK to leave your final answer in unsimplified form.)

METHOD 1:
(All Lists) – (Those Lists where first and last entry is not A) $= 6^6 - 5 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 5 = \boxed{6^6 - 5^2 6^4}$
$= 6^4(6^2 - 5^2) = \boxed{11 \cdot 6^4.}$

METHOD 2:
Lists will be of three types:
Type 1: (A, $\boxed{\text{anything}}$, $\boxed{\text{anything}}$, $\boxed{\text{anything}}$, $\boxed{\text{anything}}$, A) .... 6^4 of these
Type 2: (A, $\boxed{\text{anything}}$, $\boxed{\text{anything}}$, $\boxed{\text{anything}}$, $\boxed{\text{anything}}$, $\boxed{\text{not A}}$ .... $6^4 \cdot 5$ of these
Type 3: ($\boxed{\text{not A}}$, $\boxed{\text{anything}}$, $\boxed{\text{anything}}$, $\boxed{\text{anything}}$, $\boxed{\text{anything}}$, A) .... $5 \cdot 6^4$ of these

The total number of lists is thus $6^4 + 10 \cdot 6^4 = \boxed{11 \cdot 6^4}$