

$$1. \lim_{x \rightarrow 0} \frac{3x^2}{\cos(x) - 1} = \lim_{x \rightarrow 0} \frac{6x}{-\sin(x)} = \lim_{x \rightarrow 0} \frac{6}{-\cos(x)} = \frac{6}{-\cos(0)} = \boxed{-6}$$

↑
form $\frac{0}{0}$

↑
form $\frac{0}{0}$

$$2. \lim_{x \rightarrow \infty} x \sin\left(\frac{4}{3x}\right) = \lim_{x \rightarrow \infty} \frac{\sin\left(\frac{4}{3x}\right)}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\cos\left(\frac{4}{3x}\right)\left(-\frac{4}{3x^2}\right)}{-\frac{1}{x^2}}$$

↑
form $\infty \cdot 0$

↑
form $\frac{0}{0}$

$$= \lim_{x \rightarrow \infty} \frac{4}{3} \cos\left(\frac{4}{3x}\right) = \frac{4}{3} \cos(0)$$

$$= \boxed{\frac{4}{3}}$$

$$3. \lim_{x \rightarrow \infty} \frac{2x^2 + 7e^x}{x^2 + x + 4e^x - 1} = \lim_{x \rightarrow \infty} \frac{4x + 7e^x}{2x + 1 + 4e^x} = \lim_{x \rightarrow \infty} \frac{4 + 7e^x}{2 + 4e^x} = \lim_{x \rightarrow \infty} \frac{7e^x}{4e^x}$$

↑
form $\frac{\infty}{\infty}$

↑
form $\frac{\infty}{\infty}$

↑
form $\frac{\infty}{\infty}$

$$= \boxed{\frac{7}{4}}$$

$$4. \lim_{x \rightarrow 0} (\ln|x| - \ln|\sin(x)|) = \lim_{x \rightarrow 0} \ln \left| \frac{x}{\sin(x)} \right| = \ln \left| \lim_{x \rightarrow 0} \frac{x}{\sin(x)} \right|$$

↑
form $\infty - \infty$

↑
form $\frac{0}{0}$

$$= \ln \left| \lim_{x \rightarrow 0} \frac{1}{\cos(x)} \right|$$

$$= \ln \left| \frac{1}{\cos(0)} \right| = \ln|1| = \boxed{0}$$

$$1. \lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{x^2} \right) = \lim_{x \rightarrow 0} \left(\frac{x}{x^2} - \frac{1}{x^2} \right) = \lim_{x \rightarrow 0} \frac{x-1}{x^2} \stackrel{\text{Approaching } -1}{=} \boxed{-\infty}$$

↑
form $\infty - \infty$

↑
Approaching 0 pos.

$$2. \lim_{x \rightarrow 1} \frac{4 + 2 \ln|x-1|}{x - 3 \ln|1-x|} = \lim_{x \rightarrow 1} \frac{\frac{2}{x-1}}{\frac{-3}{1-x}} = \lim_{x \rightarrow 1} \frac{-2}{\frac{3(-1)}{(1-x)^2}} = \lim_{x \rightarrow 1} \frac{-2}{3} \frac{(x-1)^2}{(1-x)^2}$$

↑
form $\frac{\infty}{\infty}$

↑
form $\frac{\infty}{\infty}$

$$= \lim_{x \rightarrow 1} -\frac{2}{3} \left(\frac{x-1}{1-x} \right)^2 = \lim_{x \rightarrow 1} -\frac{2}{3} (-1)^2$$

$$= \lim_{x \rightarrow 1} -\frac{2}{3} = \boxed{-\frac{2}{3}}$$

$$3. \lim_{x \rightarrow \infty} x(e^{1/x} - 1) = \lim_{x \rightarrow \infty} \frac{e^{1/x} - 1}{\frac{1}{x}}$$

↑
form $\infty \cdot 0$

↑
form $\frac{0}{0}$

$$= \lim_{x \rightarrow \infty} \frac{e^{1/x} \left(-\frac{1}{x^2} \right)}{\frac{-1}{x^2}} = \lim_{x \rightarrow \infty} e^{\frac{1}{x}}$$

$$= e^0 = \boxed{1}$$

$$4. \lim_{x \rightarrow 0} \frac{x^2}{\ln(\sec x)} = \lim_{x \rightarrow 0} \frac{2x}{\frac{\sec(x) \tan(x)}{\sec(x)}} = \lim_{x \rightarrow 0} \frac{2x}{\tan(x)}$$

↑
form $\frac{0}{0}$

still
 $\frac{0}{0}$

$$= \lim_{x \rightarrow 0} \frac{2}{\sec^2(x)} = \frac{2}{\sec^2(0)} = \frac{2}{1} = \boxed{2}$$