Name: Richard

TEST 2 — MATH 200, SECTION 9

April 2, 2021

Directions: Closed book, closed notes, no calculators. Put all phones, etc., away. You will need only a pencil or pen.

1. (36 points) Find the derivatives of these functions. You do **not** need to simplify your answers.

(a) $\frac{d}{dx}\left[e^x \ln(x)\right] = e^x \ln(x) + e^x \frac{1}{x} = \boxed{e^x \ln(x) + \frac{e^x}{x}}$

(b) $\frac{d}{dx}\left[\sin^{-1}(x)\right] = \boxed{\frac{1}{\sqrt{1-x^2}}}$

(c) $\frac{d}{dx}\left[\left(2+\ln(x+e^x)\right)^4\right] = \boxed{4\left(2+\ln(x+e^x)\right)^3 \cdot \frac{1+e^x}{x+e^x}}$

(d) $\frac{d}{dx}\left[\frac{\ln(x)}{x}\right] = \frac{\frac{1}{x}x - \ln(x)\cdot 1}{x^2} = \boxed{\frac{1-\ln(x)}{x^2}}$

(e) $\frac{d}{dx}\left[\frac{1}{\sqrt{\ln(x)}}\right] = \frac{d}{dx}\left[\left(\ln(x)\right)^{-\frac{1}{2}}\right] = -\frac{1}{2}\left(\ln(x)\right)^{-\frac{3}{2}}\frac{1}{x} = \boxed{\frac{-1}{2x\sqrt{\ln(x)}^3}}$

(f) $\frac{d}{dx}\left[\tan^{-1}\left(x^3+3x\right)\right] = \frac{1}{1+(x^3+3x)^2}\left(3x^2+3\right)$

2. (4 points) Find: $\lim_{h\to 0}\frac{\ln(4+h)-\ln(4)}{h} = \boxed{\frac{1}{4}}$

Reason: Let $f(x)=\ln(x)$. Then $f'(x)=\frac{1}{x}$ and this limit gives $f'(4)=\frac{1}{4}$

3. (12 points) Given the equation $\ln|x+y| = xy+1$, find y'.

$$\frac{d}{dx}\left[\ln|x+y|\right] = \frac{d}{dx}\left[xy+1\right]$$

$$\frac{1+y'}{x+y} = 1\cdot y + xy' + 0$$

$$1+y' = (x+y)(y+xy')$$

$$1+y' = xy + x^2y' + y^2 + xyy'$$

$$1-xy-y^2 = x^2y' + xyy' - y'$$

$$1-xy-y^2 = (x^2+xy-1)y'$$

$$\boxed{y' = \frac{1-xy-y^2}{x^2+xy-1}}$$

4. (12 points) A spherical balloon is deflating in such a way that its volume is decreasing at a rate of 18 cubic feet per hour. At what rate is the radius changing when the radius is 3 feet?

Know $\frac{dV}{dt} = -18\ ft^3/hr$

V = volume

r = radius

Want $\frac{dr}{dt}$ (when $r=3$)

$$V = \frac{4}{3}\pi r^3$$

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$$

$$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2\frac{dr}{dt}$$

$$-18 = 4\pi r^2\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{-18}{4\pi r^2} = \frac{-9}{2\pi r^2}$$

Now insert $r=3$:

$$\left.\frac{dr}{dt}\right|_{r=3} = \frac{-9}{2\pi 3^2} = \boxed{\frac{-1}{2\pi}\ ft/hr}$$

Sphere formulas Volume: $V = \frac{4}{3}\pi r^3$ Surface area: $S = 4\pi r^2$

5. (12 points) A rocket has a height of $t+t^2$ meters t seconds after it is launched. How high is the rocket when its velocity is 101 meters per second?

position (height) $= S(t) = t + t^2$ meters at time t.

Velocity $= V(t) = S'(t) = 1 + 2t$ m/sec

To find when velocity is 101 m/sec, solve

$$V(t) = 101$$
$$1 + 2t = 101$$
$$2t = 100$$
$$t = 50 \text{ sec}.$$

Height at this time is $S(50) = 50 + 50^2 = \boxed{2550 \text{ m.}}$

6. (12 points) Find the locations (x-coordinates) of any local extrema of $f(x) = x^2e^x$.

$f'(x) = 2xe^x + x^2e^x = e^x x(2+x)$.

This is defined for all x and equals 0 for $x=0$ & $x=-2$.

Thus the $\boxed{\text{critical points are } 0 \text{ and } -2}$

	-2	0	
+ + +	+ + + +	+ + + +	e^x
− − −	− − − −	+ + + +	x
− − −	+ + + +	+ + + +	$(2+x)$
+ + +	− − − −	+ + + +	$f'(x) = e^x x(2+x)$

f has a local max at $x = -2$

f has a local min at $x = 0$

(by 1st derivative test)

7. (12 points) The graph of the **derivative** $f'(x)$ of a function f is shown below.

(a) State the critical points of f.

$\boxed{x = 5}$ (because $f'(5)=0$)

(b) State the interval(s) on which f increases.

$\boxed{(-\infty, 5)}$ (that's where $f'(x)>0$)

(c) State the interval(s) on which f decreases.

$\boxed{(5, \infty)}$ (that's where $f'(x)<0$)