Name: Richard

QUIZ 5

MATH 200
September 5, 2024

1. Use a limit definition of the derivative to find the derivative of the function $f(x) = \dfrac{1}{x+1}$.

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x} = \lim_{z \to x} \frac{\frac{1}{z+1} - \frac{1}{x+1}}{z - x}$$

$$= \lim_{z \to x} \frac{\frac{1}{z+1} - \frac{1}{x+1}}{z - x} \cdot \frac{(z+1)(x+1)}{(z+1)(x+1)}$$

$$= \lim_{z \to x} \frac{(x+1) - (z+1)}{(z-x)(z+1)(x+1)} = \lim_{z \to x} \frac{-\cancel{(z-x)}}{\cancel{(z-x)}(z+1)(x+1)}$$

$$= \lim_{z \to x} \frac{-1}{(z+1)(x+1)} = \frac{-1}{(x+1)(x+1)} = \frac{-1}{(x+1)^2}$$

Therefore: $\boxed{f'(x) = \dfrac{-1}{(x+1)^2}}$

Alternatively:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} \cdot \frac{(x+h+1)(x+1)}{(x+h+1)(x+1)}$$

$$= \lim_{h \to 0} \frac{(x+1) - (x+h+1)}{h(x+h+1)(x+1)} = \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}(x+h+1)(x+1)}$$

$$= \lim_{h \to 0} \frac{-1}{(x+h+1)(x+1)} = \frac{-1}{(x+0+1)(x+1)} = \frac{-1}{(x+1)^2}$$

Therefore: $\boxed{f'(x) = \dfrac{-1}{(x+1)^2}}$

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1. Use a limit definition of the derivative to find the derivative of the function $f(x) = \sqrt{x+1}$.

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x} = \lim_{z \to x} \frac{\sqrt{z+1} - \sqrt{x+1}}{z - x}$$

$$= \lim_{z \to x} \frac{\sqrt{z+1} - \sqrt{x+1}}{z - x} \cdot \frac{\sqrt{z+1} + \sqrt{x+1}}{\sqrt{z+1} + \sqrt{x+1}}$$

$$= \lim_{z \to x} \frac{\sqrt{z+1}^2 + \sqrt{z+1}\sqrt{x+1} - \sqrt{x+1}\sqrt{z+1} - \sqrt{x+1}^2}{(z-x)(\sqrt{z+1} + \sqrt{x+1})}$$

$$= \lim_{z \to x} \frac{z+1 - (x+1)}{(z-x)(\sqrt{z+1} + \sqrt{x+1})} = \lim_{z \to x} \frac{\cancel{(z-x)}}{\cancel{(z-x)}(\sqrt{z+1} + \sqrt{x+1})}$$

$$= \lim_{z \to x} \frac{1}{\sqrt{z+1} + \sqrt{x+1}} = \frac{1}{\sqrt{x+1} + \sqrt{x+1}} = \frac{1}{2\sqrt{x+1}}$$

Therefore: $\boxed{f'(x) = \frac{1}{2\sqrt{x+1}}}$

Alternatively:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}}$$

$$= \lim_{h \to 0} \frac{(x+h+1) - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h+1} + \sqrt{x+1})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} = \frac{1}{\sqrt{x+0+1} + \sqrt{x+1}} = \boxed{\frac{1}{2\sqrt{x+1}}}$$