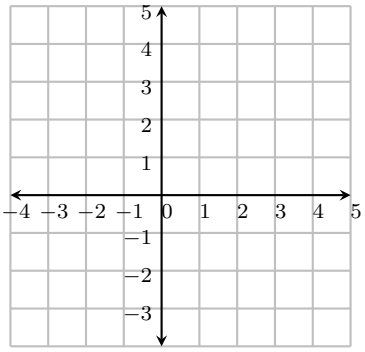
Exponents and Logarithms Diagnostic Quiz

Take this quiz to see if you need Lectures 5A, 5B, 5C (Exponents and Logarithms). Answers on page 2.

Important: Pencil or pen only. **No calculators**.

1. State the inverse of the function $f(x) = e^x$.

2. Sketch the graphs of $y = e^x$ and $y = \ln(x)$.



3. Write as a single logarithm: $\ln\left(x^3\right) - \dfrac{1}{2}\ln(x+2)$

4. $\log_2(\sqrt{2}) =$

5. $\ln(1/e) =$

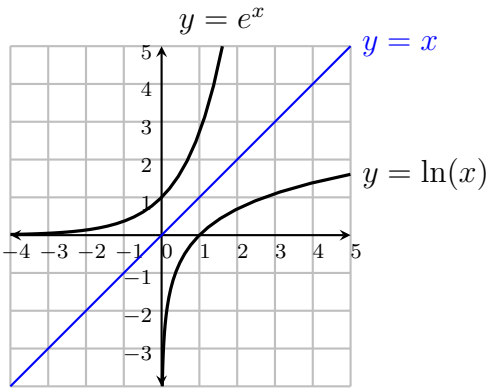
6. $\ln(1) =$

7. Find the inverse of $f(x) = 2 + \ln(x-3)$.

Here are the solutions. If your answers are not all correct, then you probably need Lectures 5ABC.

1. State the inverse of the function $f(x) = e^x$. Inverse is $\boxed{\ln(x)}$.

2. Sketch the graphs of $y = e^x$ and $y = \ln(x)$.



3. Write as a single logarithm: $\ln\left(x^3\right) - \frac{1}{2}\ln(x+2)$

$$\begin{aligned} \ln\left(x^3\right) - \frac{1}{2}\ln(x+2) &= \ln\left(x^3\right) - \ln\left((x+2)^{1/2}\right) \\ &= \ln\left(x^3\right) - \ln\left(\sqrt{x+2}\right) \\ &= \boxed{\ln\left(\frac{x^3}{\sqrt{x+2}}\right)} \end{aligned}$$

4. $\log_2(\sqrt{2}) = \boxed{\frac{1}{2}}$

5. $\ln(1/e) = \boxed{-1}$

6. $\ln(1) = \boxed{0}$

7. Find the inverse of $f(x) = 2 + \ln(x-3)$.

$$\begin{aligned} y &= 2 + \ln(x-3) \\ x &= 2 + \ln(y-3) && \text{(interchange variables)} \\ x - 2 &= \ln(y-3) && \text{(solve for } y\text{)} \\ e^{x-2} &= e^{\ln(y-3)} \\ e^{x-2} &= y - 3 \\ e^{x-2} &= e^{\ln(y-3)} \\ 3 + e^{x-2} &= y \end{aligned}$$

Therefore $\boxed{f^{-1}(x) = 3 + e^{x-2}}$.