

Name: \_\_\_\_\_

R. Hammack

Score: \_\_\_\_\_

**Directions:** Please answer the questions in the space provided. To get full credit you must show all of your work. Use of calculators and other computing or communication devices is not allowed on this test.

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1. SHORT ANSWER. Write each of the following sets by listing its elements or describing it with a familiar symbol.

(a)  $\{x \in \mathbb{Z} : |x| \leq 3\} =$

(b)  $\{(x, y) \in \mathbb{N} \times \mathbb{R} : x^2 = 4, y^2 = 2\} =$

(c)  $\{(x, y) \in \mathbb{R} \times \mathbb{R} : y = x^2\} \cap \{(x, y) \in \mathbb{R} \times \mathbb{R} : y = x\} =$

(d)  $\mathbb{R} - \mathcal{P}(\mathbb{R}) =$

(e)  $\{x \in \mathbb{R} : \cos(\pi x) = -1\} =$

(f)  $\{X \in \mathcal{P}(\mathbb{N}) : X \cap \{1, 2\} = X\} =$

(g)  $\mathcal{P}(\{1\}) \times \mathcal{P}(\{2\}) =$

(h)  $\mathcal{P}(\mathcal{P}(\{\emptyset\})) =$

(i)  $\{\emptyset\} \times \{\emptyset\} =$

(j)  $\emptyset \times \mathbb{N} =$

(k)  $(\mathbb{R} - \mathbb{Z}) \cap \mathbb{N} =$

(l)  $\bigcup_{X \in \mathcal{P}(\mathbb{N})} \overline{X} =$

2. This problem concerns the following statement.

$P$ : For every subset  $X$  of  $\mathbb{N}$ , there is an integer  $m$  for which  $|X| = m$ .

(a) Is the statement  $P$  true or false? Explain.

(b) Form the negation  $\sim P$ . Write your answer as an English sentence.

3. Suppose that  $(R \Rightarrow S) \vee \sim (P \wedge Q)$  is **false**.

Is there enough information to determine the truth values of  $P$ ,  $Q$ ,  $R$  and  $S$ ? If so, what are they?  
(This is most easily done without a truth table.)

4. Write out a truth table to decide if  $(\sim P) \wedge (P \Rightarrow Q)$  and  $\sim (Q \Rightarrow P)$  are logically equivalent.

5. How many 10-digit integers contain no 0's and exactly three 6's?