

1. This problem concerns the equation $xy^3 = xy + 6$

(a) Find $\frac{dy}{dx}$.

$$\begin{aligned} D_x [xy^3] &= D_x [xy + 6] \\ &= 1 \cdot y^3 + x \cdot 3y^2 y' = 1y + xy' + 0 \\ 3xy^2 y' - xy' &= y - y^3 \\ y'(3xy^2 - x) &= y - y^3 \\ y' &= \frac{y - y^3}{3xy^2 - x} \end{aligned}$$

(b) Use your answer from part (a) to find the slope of the tangent to the graph of $xy^3 = xy + 6$ at the point $(1, 2)$.

$$y' \Big|_{(x,y)=(1,2)} = \frac{2 - 2^3}{3 \cdot 1 \cdot 2^2 - 1} = \frac{2 - 8}{12 - 1} = \boxed{-\frac{6}{11}}$$

1. This problem concerns the equation
- $x^2 + xy + y^2 = 7$

$$y = f(x)$$

- (a) Find
- $\frac{dy}{dx}$
- .

$$D_x [x^2 + xy + y^2] = D_x [7]$$

$$2x + 1y + xy' + 2yy' = 0$$

$$xy' + 2yy' = -2x - y$$

$$y'(x + 2y) = -2x - y$$

$$y' = \frac{-2x - y}{x + 2y}$$

- (b) Use your answer from part (a) to find the slope of the tangent to the graph of
- $x^2 + xy + y^2 = 7$
- at the point
- $(2, -3)$
- .

$$y' \Big|_{(x,y)=(2,-3)} = \frac{-2(2) - (-3)}{2 + 2(-3)} = \frac{-4 + 3}{2 - 6} = \boxed{\frac{1}{4}}$$