Name: Richard

QUIZ 17 ♠

MATH 201
April 4, 2024

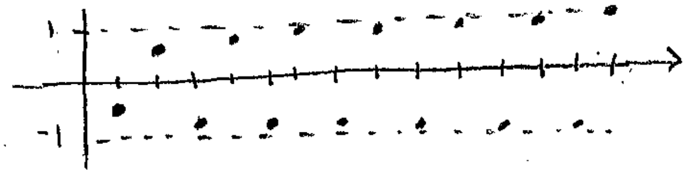
1. (4 pts.) Determine whether the sequence converges or diverges. If it converges, give the value if possible.

(a) $\left\{\dfrac{4\tan^{-1}(n)}{\pi}\right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty} \frac{4\tan^{-1}(n)}{\pi} = \frac{4}{\pi}\lim_{n\to\infty}\tan^{-1}(n) = \frac{4}{\pi}\cdot\frac{\pi}{2} = \boxed{2}$$

$\boxed{\text{Converges to 2}}$

(b) $\left\{\dfrac{(-1)^n(n+1)}{n+2}\right\}_{n=1}^{\infty}$ $\boxed{\text{Diverges}}$



Note that $\displaystyle\lim_{n\to\infty}\frac{n+1}{n+2} = 1$, but the $(-1)^n$ makes the sequence terms alternate positive and negative getting closer to 1 and −1. Limit DNE.

2. (6 pts.) Determine whether the series converges or diverges. If it converges, give the value if possible.

(a) $$\sum_{k=0}^{\infty}\frac{3}{4^k} = \frac{a}{1-r} = \frac{3}{1-\frac{1}{4}} = \frac{3}{3/4} = \boxed{4} \leftarrow \boxed{\text{converges}}$$

(Geometric series $a = 3$, $r = 1/4$)

(b) $$\sum_{k=1}^{\infty}\left(\frac{5}{k}-\frac{5}{k+1}\right) = \lim_{n\to\infty} S_n$$

$$= \lim_{n\to\infty}\left(\left(\frac{5}{1}-\frac{5}{2}\right)+\left(\frac{5}{2}-\frac{5}{3}\right)+\left(\frac{5}{3}-\frac{5}{4}\right)+\left(\frac{5}{4}-\frac{5}{5}\right)+\cdots+\left(\frac{5}{n}-\frac{5}{n+1}\right)\right)$$

$$= \lim_{n\to\infty}\left(5-\frac{5}{n+1}\right) = 5-0 = \boxed{5} \leftarrow \boxed{\text{converges}}$$

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Quiz 17 ♢

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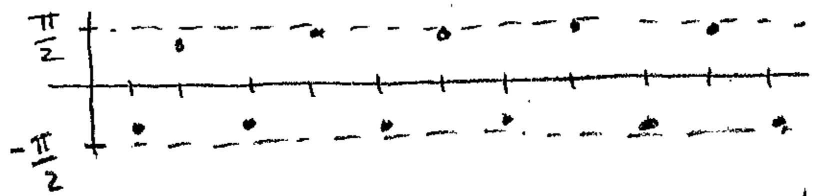
April 4, 2024

1. (4 pts.) Determine whether the sequence converges or diverges. If it converges, give the value if possible.

(a) $\left\{(-1)^n \tan^{-1}(n)\right\}_{n=1}^{\infty}$



$\boxed{\text{Diverges}}$ Sequence terms alternate signs, getting progressively closer to $\pi/2$ and $-\pi/2$

(b) $\frac{\ln(2)}{2}, \frac{\ln(3)}{3}, \frac{\ln(4)}{4}, \frac{\ln(5)}{5}, \ldots$ $= \left\{ \frac{\ln(n)}{n} \right\}_{n=2}^{\infty}$

$$\lim_{n\to\infty} \frac{\ln(n)}{n} = \lim_{n\to\infty} \frac{1/n}{1} = \frac{0}{1} = \boxed{0} \leftarrow \boxed{\text{converges!}}$$

form $\frac{\infty}{\infty}$ L'Hôpital

2. (6 pts.) Determine whether the series converges or diverges. If it converges, give the value if possible.

(a) $\sum_{k=0}^{\infty} \frac{5}{2^k} = \frac{a}{1-r} = \frac{5}{1-1/2} = \frac{5}{1/2} = \boxed{10} \leftarrow \boxed{\text{converges}}$

(Geometric series $a = 5$ $r = 1/2$)

(b) $\sum_{k=1}^{\infty} \left(\sqrt{\frac{2}{k}} - \sqrt{\frac{2}{k+1}}\right) = \lim_{n\to\infty} S_n$

$$= \lim_{n\to\infty} \left(\left(\sqrt{\tfrac{2}{1}} - \sqrt{\tfrac{2}{2}}\right) + \left(\sqrt{\tfrac{2}{2}} - \sqrt{\tfrac{2}{3}}\right) + \left(\sqrt{\tfrac{2}{3}} - \sqrt{\tfrac{2}{4}}\right) + \left(\sqrt{\tfrac{2}{4}} - \sqrt{\tfrac{2}{5}}\right) + \cdots + \left(\sqrt{\tfrac{2}{n}} - \sqrt{\tfrac{2}{n+1}}\right) \right)$$

$$= \lim_{n\to\infty} \left(\sqrt{2} - \sqrt{\frac{2}{n+1}}\right) = \sqrt{2} - \sqrt{0} = \boxed{\sqrt{2}}$$

$\boxed{\text{Converges}}$

Name: Richard

QUIZ 17 ♣

MATH 201
April 4, 2024

1. (4 pts.) Determine whether the sequence converges or diverges. If it converges, give the value if possible.

(a) $\left\{\cos\left(\frac{\pi n}{3n+1}\right)\right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty} \cos\left(\frac{\pi n}{3n+1}\right) = \cos\left(\lim_{n\to\infty}\frac{\pi n}{3n+1}\right) = \cos\left(\frac{\pi}{3}\right) = \boxed{\frac{1}{2}}$$

Converges!

(b) $\frac{\ln(2)+1}{\ln(2)+2}, \frac{\ln(3)+1}{\ln(3)+2}, \frac{\ln(4)+1}{\ln(4)+2}, \frac{\ln(5)+1}{\ln(5)+2}, \ldots$ $= \left\{\frac{\ln(n)+1}{\ln(n)+2}\right\}_{n=2}^{\infty}$

$$\lim_{n\to\infty}\frac{\ln(n)+1}{\ln(n)+2} = \lim_{n\to\infty}\frac{\frac{1}{n}+0}{\frac{1}{n}+0} = \lim_{n\to\infty} 1 = \boxed{1}$$

form $\frac{\infty}{\infty}$

L'Hôpital

Converges!

2. (6 pts.) Determine whether the series converges or diverges. If it converges, give the value if possible.

(a) $\sum_{k=0}^{\infty}\frac{7}{5^k} = \frac{a}{1-r} = \frac{7}{1-1/5} = \frac{7}{4/5} = \boxed{\frac{35}{4}}$

Converges!

(Geometric series, $a=7$, $r=1/5$)

(b) $\sum_{k=1}^{\infty}\left(e^{1-k}-e^{-k}\right) = \lim_{n\to\infty} S_n$

$$= \lim_{n\to\infty}\left((e^0-e^{-1})+(e^{-1}-e^{-2})+(e^{-2}-e^{-3})+(e^{-3}-e^{-4})+\cdots+(e^{1-n}-e^{-n})\right)$$

$$= \lim_{n\to\infty}\left(e^0-e^{-n}\right) = \lim_{n\to\infty}\left(1-\frac{1}{e^n}\right) = 1-0 = \boxed{1}$$

Converges!

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QUIZ 17 ♡

MATH 201
April 4, 2024

1. (4 pts.) Determine whether the sequence converges or diverges. If it converges, give the value if possible.

(a) $\left\{\frac{\sin(n)}{n}\right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty} \frac{\sin(n)}{n} = \boxed{0} \leftarrow \boxed{\text{converges!}}$$

(b) $\left\{\frac{n}{e^n+n}\right\}_{n=1}^{\infty}$

$$\underbrace{\lim_{n\to\infty} \frac{n}{e^n+1}}_{\text{form } \frac{\infty}{\infty}} = \underbrace{\lim_{n\to\infty} \frac{1}{e^n+0}}_{\text{L'Hôpital}} = \lim_{n\to\infty} \frac{1}{e^n} = \boxed{0}\ \boxed{\text{converges!}}$$

2. (6 pts.) Determine whether the series converges or diverges. If it converges, give the value if possible.

(a) $\sum_{k=0}^{\infty} \frac{4}{3^k} = \frac{a}{1-r} = \frac{4}{1-1/3} = \frac{4}{2/3} = \boxed{6}\ \boxed{\text{Converges!}}$

(b) $\sum_{k=1}^{\infty} \left(\ln(k) - \ln(k+1)\right) = \lim_{n\to\infty} S_n$

$$= \lim_{n\to\infty}\Big(\big(\ln(1)-\ln(2)\big)+\big(\ln(2)-\ln(3)\big)+\big(\ln(3)-\ln(4)\big)+\cdots+\big(\ln(n)-\ln(n-1)\big)\Big)$$

$$= \lim_{n\to\infty}\big(0-\ln(n-1)\big) = \boxed{-\infty}\ \boxed{\text{Diverges!}}$$