

1. Find the derivative:  $y = \sin^{-1}(x^5 - 3x^2)$ 

$$y' = \frac{1}{\sqrt{1 - (x^5 - 3x^2)^2}} (5x^4 - 6x) = \frac{5x^4 - 6x}{\sqrt{1 - (x^5 - 3x^2)^2}}$$

$$= \frac{5x^4 - 6x}{\sqrt{1 - x^{10} + 6x^7 - 9x^4}}$$

2. Find the derivative:  $y = (\tan^{-1}(x))^5$ 

$$y' = 5(\tan^{-1}(x))^4 \frac{1}{1+x^2} = \frac{5(\tan^{-1}(x))^4}{1+x^2}$$

3. Find the derivative:  $y = \frac{\sec^{-1}(x)}{e^x}$ 

$$y' = \frac{\frac{1}{|x|\sqrt{x^2-1}} e^x - \sec^{-1}(x) e^x}{(e^x)^2}$$

$$= \frac{e^x \left( \frac{1}{|x|\sqrt{x^2-1}} - \sec^{-1}(x) \right)}{e^x e^x} = \frac{\frac{1}{|x|\sqrt{x^2-1}} - \sec^{-1}(x)}{e^x}$$

4. Suppose  $f(x)$  is the number of liters of fuel in a rocket when it is  $x$  miles above the Earth's surface. Explain in simple terms the meaning of the statement  $f'(20) = -8$ .

When the rocket is 20 miles high, it is using fuel at a rate of -8 liters per mile. At that rate it would burn 8 liters to go an additional mile high.

1. Find the derivative:  $y = \tan^{-1}(x^5 - 3x^2)$ 

$$y' = \frac{1}{1 + (x^5 - 3x^2)^2} (5x^4 - 6x) = \boxed{\frac{5x^4 - 6x}{1 + (x^5 - 3x^2)^2}}$$

$$= \boxed{\frac{5x^4 - 6x}{1 + x^{10} - 6x^7 + 9x^4}}$$

2. Find the derivative:  $y = (\sin^{-1}(x))^5$ 

$$y' = 5(\sin^{-1}(x))^4 \frac{1}{\sqrt{1-x^2}} = \boxed{\frac{5(\sin^{-1}(x))^4}{\sqrt{1-x^2}}}$$

3. Find the derivative:  $y = \ln(x) \sec^{-1}(x)$ 

$$y' = \frac{1}{x} \sec^{-1}(x) + \ln(x) \frac{1}{|x|\sqrt{x^2-1}}$$

$$= \boxed{\frac{\sec^{-1}(x)}{x} + \frac{\ln(x)}{|x|\sqrt{x^2-1}}}$$

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 At that rate it would burn 8 liters to go an additional mile high.

1. Find the derivative:  $y = \sec^{-1}(x^5 - 3x^2)$

$$y' = \frac{1}{|x^5 - 3x^2| \sqrt{(x^5 - 3x^2)^2 - 1}} D_x [x^5 - 3x^2]$$

$$= \boxed{\frac{5x^4 - 6x}{|x^5 - 3x^2| \sqrt{(x^5 - 3x^2)^2 - 1}}}$$

2. Find the derivative:  $y = (\sin^{-1}(x))^5$

$$y' = 5 (\sin^{-1}(x))^4 D_x [\sin^{-1}(x)]$$

$$= \boxed{\frac{5 (\sin^{-1}(x))^4}{\sqrt{1 - x^2}}}$$

3. Find the derivative:  $y = e^{5x} \tan^{-1}(x)$

$$y' = D_x [e^{5x}] \tan^{-1}(x) + e^{5x} D_x [\tan^{-1}(x)]$$

$$= \boxed{5e^{5x} \tan^{-1}(x) + \frac{e^{5x}}{1+x^2}}$$

4. Consider the function  $h(x)$ , where  $h(x)$  equals the elevation (in feet above sea level)  $x$  miles due west of your present location. Suppose  $h'(75) = 5$ . Explain what this means.

At the point 75 miles due west from your present location, elevation is increasing at a rate of 5 feet per mile (so that point is on a slight incline, increasing 5 feet in one mile)  
 [i.e. go one mile further west, and expect to go up 5 feet.]

1. Find the derivative:  $y = \sin^{-1}(x^5 - 3x^2)$ 

$$\begin{aligned} y' &= \frac{1}{\sqrt{1-(x^5-3x^2)^2}} D_x [x^5 - 3x^2] = \boxed{\frac{5x^4 - 6x}{\sqrt{1-(x^5-3x^2)^2}}} \\ &= \boxed{\frac{5x^4 - 6x}{\sqrt{1-x^{10}+6x^7-9x^4}}} \end{aligned}$$

2. Find the derivative:  $y = 3(\tan^{-1}(x))^4$ 

$$y' = 12(\tan^{-1}(x))^3 D_x [\tan^{-1}(x)] = \boxed{\frac{12(\tan^{-1}(x))^3}{1+x^2}}$$

3. Find the derivative:  $y = \sec(x) \sec^{-1}(x)$ 

$$\begin{aligned} y' &= D_x[\sec(x)] \sec^{-1}(x) + \sec(x) D_x[\sec^{-1}(x)] \\ &= \boxed{\sec(x) \tan(x) \sec^{-1}(x) + \sec(x) \frac{1}{1+\sqrt{x^2-1}}} \end{aligned}$$

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At the point 75 miles due west of your present location, elevation is increasing at a rate of 5 feet per mile (i.e. that point is on a slight incline, climbing 5 feet in one mile)  
 [Go one mile further west, expect to go up 5 feet]