Name: _____ MIDTERM EXAM ♣ MATH 200 October 26, 2022

1. (35 pts.) Evaluate the following limits. Show steps, as appropriate.

(a) $\lim_{x\to 0} \dfrac{1-\cos(x)}{x} =$

(b) $\lim_{x\to\infty} \sin^{-1}\left(\dfrac{1}{2}+\dfrac{1}{x}\right) =$

(c) $\lim_{x\to-\infty} e^x =$

(d) $\lim_{x\to\infty} \dfrac{x^2-4x+3}{x^2+4x-5} =$

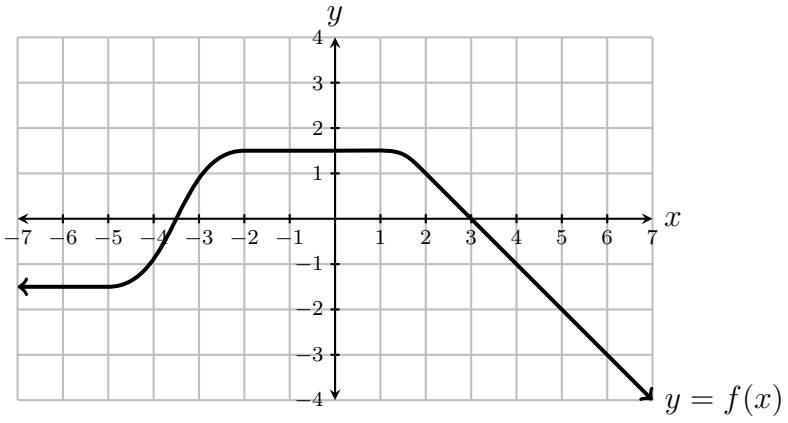
(e) $\lim_{x\to 1} \dfrac{x^2-4x+3}{x^2+4x-5} =$

(f) $\lim_{x\to -5^+} \dfrac{x^2-4x+3}{x^2+4x-5} =$

(g) $\lim_{x\to 16} \dfrac{\sqrt{x}-4}{x-16} =$

2. (5 pts.) Use a limit definition of a derivative to find the derivative of $f(x) = 2x^2 - 3$.

3. (5 pts.) The graph of a function $f(x)$ is shown. Using the same grid, sketch the graph of $f'(x)$.



4. (5 pts.) Find all points (x, y) on the graph of $y = x + \dfrac{1}{x-3}$ where the tangent line is horizontal.

5. (30 pts.) Find the indicated derivatives.

(a) $f(\theta) = \sqrt{\theta^3} + \ln(\theta)$

$f'(\theta) =$

$f''(\theta) =$

(b) $D_x \left[\dfrac{x^3 + x^2 + 1}{x} \right] =$

(c) $D_x \left[4xe^{\sqrt{3x+1}} \right] =$

(d) $D_x \left[\left(\sec\left(\ln(x) \right) \right)^3 \right] =$

(e) $D_x \left[\sin^{-1}\left(\pi x \right) \right] =$

6. (5 pts.) Consider the equation $y\sin(x) = y^3$. Use implicit differentiation to find $\dfrac{dy}{dx}$.

7. (5 pts.) Use logarithmic differentiation to find the derivative of $f(x) = x^{1+2x}$.

8. (10 pts.) A rock is thrown from a tower at time $t = 0$. At time t (in seconds) it has a height of $s(t) = 48 + 32t - 16t^2$ feet. Please show your work in answering the following questions.

   (a) When does the rock hit the ground?

   (b) What is its velocity when it hits the ground?

9. (**Bonus:** 5 pts.) A spherical balloon is inflated and its volume increases at a rate of 15 cubic inches per minute. What is the rate of change of its radius when the radius is 10 inches?

Sphere formulas: Volume $= \frac{4}{3}\pi r^3$ Area $= \frac{1}{3}\pi r^2$