Introduction to Mathematical Reasoning	MATH 300 R. Hammack	April 11, 2007 Test #3

Name: ____________________ Score: ______

Directions: This is a take-home test. It is due at the beginning of class on Monday, April 16. Please answer all questions in the space provided. Consider working the problems on scratch paper, then rewriting them neatly on the test. Additional copies of this test can be downloaded from my web page if needed.

- Do not discuss this test with anyone other than the instructor. Ask me if you have any questions.
- You may consult your text and notes, but **no** other source.
- To get full credit you must show and explain all of your work.
- Each problem is worth 10 points.

1. Suppose that R is a symmetric and transitive relation on a set A, and there is an element $a \in A$ with the property that aRx for every $x \in A$. Prove that R is reflexive.

2. Let $B = \{1, 2, 3, 4\}$. Define a relation R on $\mathcal{P}(B)$ as $X \, R \, Y$ if $|X| = |Y|$.

   (a) Show that R is an equivalence relation.

   (b) List the equivalence classes of R.

3. Let $\mathbb{Q}^+$ denote the set of positive rational numbers. That is, $\mathbb{Q}^+ = \{x \in \mathbb{Q} \mid x > 0\}$. Consider the function $f : \mathbb{N} \times \mathbb{N} \to \mathbb{Q}^+$ defined as $f((x, y)) = \dfrac{x}{y}$.

   (a) Is f is injective? Explain.

   (b) Is f is surjective? Explain.

4. Consider the function $g : \mathbb{N} \to \mathbb{Q}^+$ defined as $g(x) = \dfrac{1}{x}$.

   (a) Is g is injective? Explain.

   (b) Is g is surjective? Explain.

5. Write the addition table for $\mathbb{Z}_7$.

6. Consider the function $f : \mathbb{R} - \{1\} \to \mathbb{R} - \{2\}$ defined as $f(x) = \sqrt[3]{\dfrac{8x}{x-1}}$.

(a) Show that f is injective.

(b) Show that f is surjective.

(c) Find a formula for f^{-1}.

7. Use mathematical induction to prove that $1 + 3 + 5 + 7 + \cdots + (2n + 1) = (n + 1)^2$ for every integer $n \geq 0$.

8. Use mathematical induction to prove that $n! > 2^n$ for every integer $n > 4$.

The problems on this page concern the Fibonacci Sequence $F_1, F_2, F_3, \ldots$.
Recall that this sequence is defined as $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for any $n > 2$.

9. Use mathematical induction to prove that $\sum_{i=1}^{n} F_i = F_{n+2} - 1$ for every $n \in \mathbb{N}$.

10. Prove that $3 | F_{4n}$ for each $n \in \mathbb{N}$.