Name: Richard | Quiz 23 ♠ | MATH 201
April 25, 2024

1. Find the interval of convergence for the power series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}(x-1)^k}{k}$. Test endpoints (if any). Show all work.

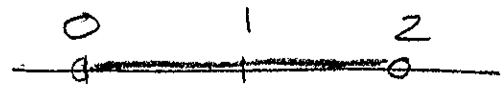
Ratio Test : $\lim_{k\to\infty} \left| \frac{\frac{(-1)^{k+2}(x-1)^{k+1}}{k+1}}{\frac{(-1)^{k+1}(x-1)^k}{k}} \right| = \lim_{k\to\infty} \left| \frac{k}{k+1} \frac{(x-1)^{k+1}}{(x-1)^k} \right|$

$= \lim_{k\to\infty} \frac{k}{k+1} |x-1| = 1 \cdot |x-1| = |x-1|.$

For convergence, we must have $|x-1| < 1$

$-1 < x-1 < 1$

$0 < x < 2$



So the radius of convergence is $R = 1$.

Test 0 $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}(0-1)^k}{k} = \sum_{k=1}^{\infty} \frac{(-1)^{2k+1}}{k} = -1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \frac{1}{5} \cdots$

and this diverges (harmonic)

Test 2 $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}(2-1)}{k} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \cdots$

and this converges (alternating harmonic)

Thus the $\boxed{\text{interval of convergence is } (0, 2]}$

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QUIZ 23

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1. Find the interval of convergence of the power series $\sum_{k=1}^{\infty} \frac{(4x-1)^k}{k^2+4}$. Test endpoints (if any). Show work.

Ratio Test: $\lim_{k\to\infty} \left| \dfrac{\frac{(4x-1)^{k+1}}{(k+1)^2+4}}{\frac{(4x-1)^k}{k^2+4}} \right| = \lim_{k\to\infty} \left| \frac{(4x-1)^{k+1}}{(4x-1)^k} \frac{k^2+4}{(k+1)^2+4} \right|$

$= \lim_{k\to\infty} |4x-1| \frac{k^2+4}{k^2+2k+5} = |4x-1|.$

For convergence we must have $|4x-1| < 1$

$-1 < 4x-1 < 1$

$0 < 4x < 2$

$0 < x < \frac{1}{2}$

Test $x=0$: $\sum_{k=1}^{\infty} \frac{(4\cdot 0-1)^k}{k^2+4} = \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2+4} = -\frac{1}{5} + \frac{1}{8} - \frac{1}{13} + \frac{1}{20} - \cdots$

Converges by alternating series test.

Test $x = \frac{1}{2}$ $\sum_{k=1}^{\infty} \frac{(4\cdot\frac{1}{2}-1)^k}{k^2+4} = \sum_{k=1}^{\infty} \frac{1}{k^2+4} = \frac{1}{5} + \frac{1}{8} + \frac{1}{13} + \frac{1}{20} + \cdots$

Converges by comparison with p-series

$\sum_{k=1}^{\infty} \frac{1}{k^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$

Thus the $\boxed{\text{interval of convergence is } [0, \tfrac{1}{2}]}$

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QUIZ 23 ♣

MATH 201

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1. Find the interval of convergence for the power series $\sum_{k=1}^{\infty} \frac{k^2 x^{2k}}{k!}$. Test endpoints (if any). Show all work.

<u>Ratio Test</u> $\lim_{k\to\infty} \left| \frac{\frac{(k+1)^2 x^{2(k+1)}}{(k+1)!}}{\frac{k^2 x^{2k}}{k!}} \right| = \lim_{k\to\infty} \left| \frac{(k+1)^2 x^{2k+2}\, k!}{k^2 x^{2k}\, (k+1)!} \right|$

$= \lim_{k\to\infty} \frac{(k+1)^2}{k^2} \cdot \frac{x^{2k+2}}{x^{2k}} \cdot \frac{k!}{(k+1)\, k!}$

$= \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^2 x^2 \frac{1}{k+1} = 1^2 \cdot x^2 \cdot 0 = 0$

Because $0 < 1$, we get convergence for any x, so the interval of convergence is

$\boxed{(-\infty, \infty)}$

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QUIZ 23 ♡

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1. Find the interval of convergence for the power series $\sum_{k=1}^{\infty} \frac{x^{2k+1}}{3^{k-1}}$. Test endpoints (if any). Show all work.

Ratio Test $\lim_{k\to\infty} \left| \dfrac{\frac{x^{2(k+1)+1}}{3^k}}{\frac{x^{2k+1}}{3^{k-1}}} \right|$

$$= \lim_{k\to\infty} \left| \frac{x^{2k+3}}{x^{2k+1}} \cdot \frac{3^{k-1}}{3^k} \right| = \lim_{k\to\infty} |x^2| \frac{1}{3} = \frac{x^2}{3}$$

For convergence we need $\frac{x^2}{3} < 1$

$$x^2 < 3$$

$$-\sqrt{3} < x < \sqrt{3}$$

Test $\sqrt{3}$: $\sum_{k=1}^{\infty} \frac{\sqrt{3}^{2k+1}}{3^{k-1}} = \sum_{k=1}^{\infty} \frac{\sqrt{3}^{2k}\sqrt{3}}{3^{k-1}} = \sum_{k=1}^{\infty} \frac{3^k\sqrt{3}}{3^{k-1}}$

$= \sum_{k=1}^{\infty} 3\sqrt{3} \leftarrow$ Diverges

Test $-\sqrt{3}$ $\sum_{k=1}^{\infty} \frac{(-\sqrt{3}^{2k+1})}{3^{k-1}} = \sum_{k=1}^{\infty} \frac{-\sqrt{3}^{2k}\sqrt{3}}{3^{k-1}} = \sum_{k=1}^{\infty} \frac{-3^k\sqrt{3}}{3^{k-1}}$

$= \sum_{k=1}^{\infty} -3\sqrt{3} \leftarrow$ Diverges

$\therefore$ $\boxed{\text{Interval of Convergence is } (-\sqrt{3}, \sqrt{3})}$