Name: Richard

TEST 3

MATH 200, SECTION 9

April 23, 2021

Directions: Closed book, closed notes, no calculators. Put all phones, etc., away. You will need only a pencil or pen.

1. (7 points each) Find the indefinite integrals.

(a) $\int (x^3 + 2x + e^x)\, dx$ $= \frac{x^4}{4} + 2\frac{x^2}{2} + e^x + C = \boxed{\frac{x^4}{4} + x^2 + e^x + C}$

(b) $\int 5x^{-1}\, dx$ $= 5\int \frac{1}{x}\, dx = \boxed{5 \ln|x| + C}$

(c) $\int (\sec^2(x) + 3\sin(x))\, dx$ $= \boxed{\tan(x) - 3\cos(x) + C}$

(d) $\int \frac{1}{\sqrt{x}}\, dx$ $= \int x^{-\frac{1}{2}}\, dx = \frac{1}{-\frac{1}{2}+1} x^{-\frac{1}{2}+1} + C = \frac{1}{\frac{1}{2}} x^{\frac{1}{2}} + C$

$= \boxed{2\sqrt{x} + C}$

(e) $\int \frac{\pi}{3+3x^2}\, dx$ $= \frac{\pi}{3}\int \frac{1}{1+x^2}\, dx = \boxed{\frac{\pi}{3}\tan^{-1}(x) + C}$

(f) $\int \frac{5x+1}{x}\, dx$ $= \int \left(\frac{5x}{x} + \frac{1}{x}\right) dx = \int \left(5 + \frac{1}{x}\right) dx$

$= \boxed{5x + \ln|x| + C}$

2. (8 points) Suppose $f(x)$ and $g(x)$ are differentiable functions. Find $\int \left(f'(x)g(x) + f(x)g'(x) \right) dx$.

$\int (f'(x)g(x) + f(x)g'(x))\, dx = \boxed{f(x)g(x) + C}$

because $\frac{d}{dx}\left[f(x)g(x) + C\right] = f'(x)g(x) + f(x)g'(x)$

3. (8 points) Suppose $f(x)$ is a function for which $f'(x) = \frac{1}{x} + \frac{1}{x^2} - 1$ and $f(1) = 3$. Find $f(x)$.

$$f(x) = \int\left(\frac{1}{x} + x^{-2} - 1\right)dx = \ln|x| + \frac{1}{-2+1}x^{-2+1} - x + C$$

$$= \ln|x| - x^{-1} - x + C$$

So $f(x) = \ln|x| - \frac{1}{x} - x + C$

To find C, $3 = f(1) = \ln|1| - \frac{1}{1} - 1 + C$

$$3 = 0 - 1 - 1 + C$$
$$5 = C$$

Therefore $\boxed{f(x) = \ln|x| - \frac{1}{x} - x + 5}$

4. (8 points each) Find the limits.

(a) $\lim_{x\to\infty} x(e^{1/x} - 1) = \lim_{x\to\infty} \frac{e^{\frac{1}{x}} - 1}{\frac{1}{x}} = \lim_{x\to\infty} \frac{e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}}$

form $\infty \cdot 0$ form $\frac{0}{0}$

$$= \lim_{x\to\infty} e^{\frac{1}{x}} = e^0 = \boxed{1}$$

(b) $\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} = \lim_{x\to 0} \frac{e^x - 0 - 1}{2x} = \lim_{x\to 0} \frac{e^x}{2} = \frac{e^0}{2} = \boxed{\frac{1}{2}}$

form $\frac{0}{0}$ form $\frac{0}{0}$ again

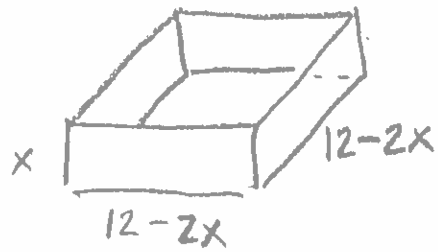
(c) $\lim_{x\to\infty} \left(\ln(2x) - \ln(x+1)\right) = \lim_{x\to\infty} \ln\left(\frac{2x}{x+1}\right) = \ln\left(\lim_{x\to\infty}\frac{2x}{x+1}\right)$

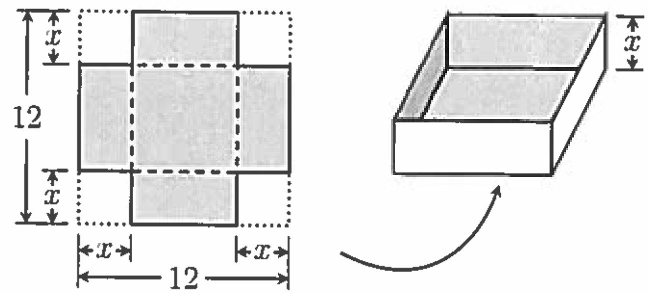
form $\infty - \infty$

$$= \ln\left(\frac{2}{1}\right)$$

$$= \boxed{\ln(2)}$$

5. (10 points) An open-top box is made from a 12 by 12 inch piece of cardboard by cutting a square from each corner, and folding up. What should x be to maximize the volume of the box?





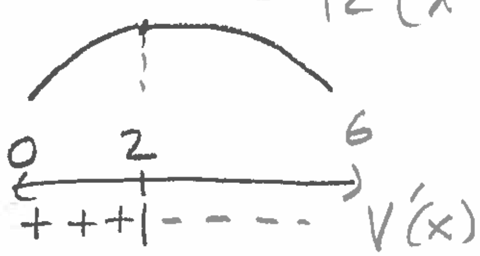
Box has dimensions x by $12-2x$ by $12-2x$, so

$$\text{Volume} = V(x) = x(12-2x)(12-2x)$$
$$V(x) = x(114 - 48x + 4x^2)$$
$$V(x) = 114x - 48x^2 + 4x^3$$

We need to find x giving global maximum of this on $(0, 6)$ ← (Note: x can't exceed $\frac{12}{2} = 6$)

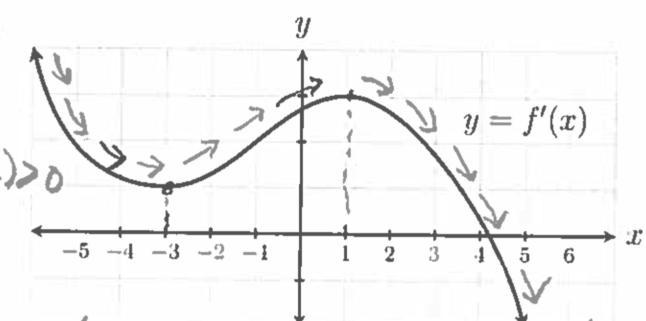
$$V'(x) = 114 - 96x + 12x^2$$
$$= 12(12 - 8x + x^2)$$
$$= 12(x^2 - 8x + 12)$$
$$= 12(x-6)(x-2) = 0$$

Critical points are $x = 2$ and $x = 6$, but only $x = 2$ is in $(0, 6)$



Answer Volume maximized if $x = 2$

6. (8 points) Below is the graph of the **derivative** $f'(x)$ of a function $f(x)$. Answer the following question about the function $f(x)$.



(a) On what intervals is $f(x)$ is concave up?

$(-3, 1)$ because that's where f' increases, so $f''(x) > 0$

(b) On what intervals is $f(x)$ is concave down?

$(-5, -3)$ and $(1, \infty)$ because that's where f' decreases, so $f''(x) < 0$