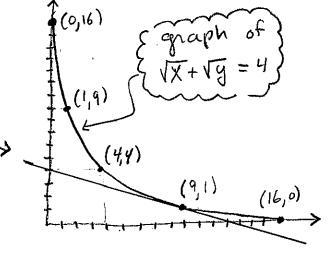1. Find the equation of the tangent line to the graph of $\sqrt{x} + \sqrt{y} = 4$ at the point $(9,1)$.

Here's a sketch of the graph, using some easy points ⟶

We need to find the equation of this line ⟶

We already have a <u>point</u> $(x_0 \, y_0) = (9,1)$ on the line.

(0,16)
(1,9)
(4,4)
(9,1)
(16,0)

graph of $\sqrt{x} + \sqrt{y} = 4$

To find the <u>equation</u> of the line, we need its slope. For this we'll use implicit differentiation to find $\dfrac{dy}{dx}$

$$D_x[\sqrt{x} + \sqrt{y}] = D_x[4] \qquad \{y = f(x)\}$$

$$D_x[x^{\frac{1}{2}} + y^{\frac{1}{2}}] = D_x[4]$$

$$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0$$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0 \quad \longleftarrow \{\text{Now solve for } \tfrac{dy}{dx}\}$$

$$\frac{1}{2\sqrt{y}}\frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}}$$

So tangent line has slope
$$m = \frac{dy}{dx}\Big|_{(x_0,y_0)=(9,1)} = -\frac{\sqrt{1}}{\sqrt{9}} = -\frac{1}{3}$$

Now use the point-slope formula for the line:
$$y - y_0 = m(x - x_0) \Rightarrow y - 1 = -\frac{1}{3}(x-9) \Rightarrow \underline{\text{Ans}} \boxed{y = -\frac{1}{3}x + 4}$$