Introduction to Mathematical Reason	Midterm MATH 300	March 17, 2008
Name: ____________________	R. Hammack	Score: ______

Directions: Please answer the questions in the space provided. To get full credit you must show all of your work. Use of calculators and other computing or communication devices is not allowed on this test.

1. Write each of the following sets by listing its elements or describing it with a familiar symbol or symbols.

   (a) $\{n \in \mathbb{Z} : |n| \leq 2\} \;=$

   (b) $\{x \in \mathbb{R} \;:\; \cos(\pi x) = -1\} \;=$

   (c) $\{X \in \mathscr{P}(\mathbb{N}) \;:\; X \cap \{1,2\} = X\} \;=$

   (d) $\displaystyle\bigcap_{n \in \mathbb{N}} \left[1, 2 + \tfrac{1}{n}\right] \;=$

   (e) $\mathscr{P}(\{1\}) \times \mathscr{P}(\{2\}) \;=$

2. Write a truth table to decide if $(\sim P) \Rightarrow Q$ and $(P \wedge Q) \Rightarrow P$ are logically equivalent.

3. This problem concerns the following statement.
   $P:$ For every subset X of $\mathbb{N}$, there is an integer m for which $|X| = m$.

   (a) Is the statement P true or false? Explain.

   (b) Form the negation $\sim P$. Write your answer as an English sentence.

4. Suppose that $(R \Rightarrow S) \vee \sim (P \wedge Q)$ is **false**.
   Is there enough information to determine the truth values of P, Q, R and S? If so, what are they?
   (This is most easily done without a truth table.)

5. Suppose $a, b, c, d \in \mathbb{Z}$ and $n \in \mathbb{N}$. Prove that if $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $ac \equiv bd \pmod{n}$. (Suggestion: Try direct proof.)

6. Suppose $a, b \in \mathbb{Z}$. If $a^2(b^2 - 2b)$ is odd, then both a and b are odd. (Suggestion: Try contrapositive proof.)

7. Prove: If $a, b \in \mathbb{Z}$, then $a^2 - 4b - 2 \neq 0$.
   (Suggestion: Contradiction may be easiest.)

8. Suppose $x \in \mathbb{Z}$. Prove that x is even if and only if $3x + 5$ is odd.

9. Prove that $\{9^n : n \in \mathbb{Q}\} = \{3^n : n \in \mathbb{Q}\}$.

10. Suppose A and B are sets. Prove that if $A \times A \subseteq A \times B$, then $A \subseteq B$.