

Name:\_\_\_\_\_

R. Hammack

Score:\_\_\_\_\_

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**PART I.** (14 points each) Prove the following statements.

1. Prove that an integer  $a$  is even if and only if  $a^2 + 2a + 9$  is odd.

2. Suppose  $A, B$  and  $C$  are nonempty sets. Prove that if  $A \times B \subseteq B \times C$ , then  $A \subseteq C$ .

3. Use induction to prove that  $1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$ .

4. There exists a set  $X$  for which  $\mathbb{Z} \in X$ ,  $\mathbb{N} \in \mathcal{P}(X)$  and  $\mathbb{R} \in \mathcal{P}(X)$ .

5. Use induction to prove that  $24|(5^{2n} - 1)$  for every integer  $n \geq 0$ .

**PART II.** (10 points each)

Decide if the following statements are true or false. Prove the true statements; disprove the false ones.

6. If  $A, B$  and  $C$  are sets, then  $A \cup (B - C) = (A \cup B) - (A \cup C)$ .

7. Suppose  $a$  and  $b$  are integers. If  $a|b$  and  $b|a$ , then  $a = b$ .

8. If  $A, B, C$  are finite sets and  $|A \cap B \cap C| = 0$ , then  $|A \cup B \cup C| = |A| + |B| + |C|$ .