Name: ———————————  R. Hammack  Score: ————

**Directions:** Please answer the questions in the space provided. To get full credit you must show all of your work. Use of calculators and other computing or communication devices is not allowed on this test.

1. Consider the sets $A = \{n \in \mathbb{Z} : 24|n\}$, $B = \{n \in \mathbb{Z} : 3|n\}$ and $C = \{n \in \mathbb{Z} : 4|n\}$.
   Prove that $A \subseteq B \cap C$.

2. Suppose $x$ and $y$ are real numbers. Prove the following statement.
   If $x^2 - 4x = y^2 - 4y$ and $x \neq y$, then $x + y = 4$.

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3. Suppose $A$, $B$, and $C$ are sets, and $C \neq \emptyset$. Prove the following statement.
   If $B \times C = A \times C$, then $A = B$.

4. Suppose $x$ and $y$ are nonzero real numbers. Use proof by contradiction to prove the following result.
   If $x$ is rational and $y$ is irrational, then $xy$ is irrational.

5. If $a$ and $b$ are integers, then $(a+b)^3 \equiv a^3 + b^3 \pmod 3$.

6. Suppose $a, b$ and $c$ are integers. If $ab$, $ac$ and $bc$ are all even, then $a, b$ and $c$ are all even

7. For all integers $a$ and $b$, if $a \equiv b \pmod{56}$, then $a \equiv b \pmod 8$.