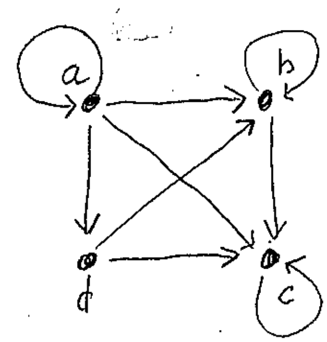
Name: Richard QUIZ 16 MATH 300
October 29, 2024

1. Let $A = \{a, b, c, d\}$ and consider the following relation on A:
$R = \{(a,a), (a,c), (b,c), (b,b), (d,c), (a,b), (c,c), (d,b), (a,d)\}$.

(a) Draw a diagram of this relation.



(b) Is this relation reflexive?

No, because $d \not R d$.

(c) Is this relation symmetric?

No, because, for instance aRb but $b \not R a$

(d) Is this relation transitive?

Yes by inspection, $xRy \wedge yRz \Longrightarrow xRz$ for all $x, y, z \in A$.

2. Consider the $\equiv \pmod 3$ relation on $\mathbb{Z}$. Prove that this relation is transitive.

We must show that if $x \equiv y \pmod 3$ and $y \equiv z \pmod 3$, then $x \equiv z \pmod 3$.

We will use direct proof.

Assume $x \equiv y \pmod 3$ and $y \equiv z \pmod 3$

This means $3 \mid (x-y)$ and $3 \mid (y-z)$.

Consequently $\boxed{x - y = 3k}$ and $\boxed{y - z = 3l}$ for $k, l \in \mathbb{Z}$

Adding the boxed equations results in

$y - z = 3k + 3l = 3(k+l)$. Hence $3 \mid (y-z)$,

and consequently $y \equiv z \pmod 3$

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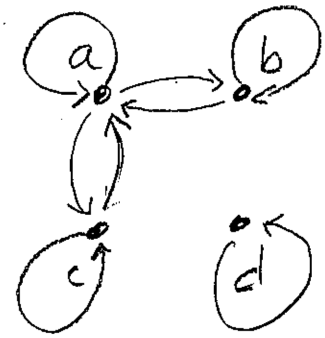

(b) Is this relation reflexive?

Yes! xRx for all $x \in A$.

(c) Is this relation symmetric?

Yes! $xRy \Rightarrow yRx \;\; \forall x, y \in A$.

(d) Is this relation transitive?

No. For instance, $cRa \land a \land b$ but $c \not R\, b$

2. Prove that the $\mid$ (divides) relation on $\mathbb{Z}$ is transitive.

We need to prove that if $x \mid y$ and $y \mid z$, then $x \mid z$.
We will use direct proof.

Suppose $x \mid y$ and $y \mid z$.

This means $y = xk$ and $z = y\ell$ for some $k, \ell \in \mathbb{Z}$.

Then $z = y\ell = xk\ell$, that is $\boxed{z = x(k\ell)}$ for $k\ell \in \mathbb{Z}$.

Therefore $x \mid z$ ∎