1. Suppose $a, b, c, d \in \mathbb{Z}$. Use direct proof to prove the following statement.

Proposition: If $a \mid b$ and $c \mid d$, then $ac \mid bd$.

Proof: (Direct)

Suppose $a \mid b$ and $c \mid d$.

This means $b = ae$ and $d = cf$ for some $e, f \in \mathbb{Z}$.

Therefore $bd = ae \cdot cf = ac \cdot ef$.

Consequently $bd = ac \cdot g$ for $g = ef \in \mathbb{Z}$.

Thus $ac \mid bd$.

1. Suppose $a, b, c \in \mathbb{Z}$. Use direct proof to prove the following statement. Use completely formed sentences. Use definitions when appropriate.

Proposition: If $a \mid c$ and $c \mid b$, then $a \mid b$.

Proof (Direct):

Suppose $a \mid c$ and $c \mid b$.

This means $c = ad$ and $b = ce$ for $d, e \in \mathbb{Z}$.

Thus $b = ce = (ad)e = a(de)$, that is,

$b = af$ for $f = de \in \mathbb{Z}$.

Therefore $a \mid b$