

1. Use either the first or second derivative test to find the local extrema of  $f(x) = e^x - x$ .

$$f'(x) = e^x - 1 \leftarrow \begin{array}{l} \text{Defined for all } x, \text{ so solve } f'(x) = 0 \\ \text{to find } \underline{\text{all}} \text{ critical points} \end{array}$$
$$e^x - 1 = 0$$
$$e^x = 1$$

$$\ln(e^x) = \ln(1)$$

$$x = 0$$

critical point

Second derivative:  $f''(x) = e^x$

Since  $f''(0) = e^0 = 1 > 0$ , second derivative test says:

$f$  has a local minimum at  $x=0$   
 $f$  has no local maximum

2. Find the global extrema (i.e. absolute extrema) of  $f(x) = x^3 - 3x$  on  $[0, 2]$ .

$$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x-1)(x+1) = 0$$

Critical points:  $x=1$     $x=-1$

But only  $x=1$  is in  $[0, 2]$ .

$$f(0) = 0^3 - 3 \cdot 0 = 0$$

$$f(2) = 2^3 - 3 \cdot 2 = 2 \leftarrow \text{MAX}$$

$$f(1) = 1^3 - 3 \cdot 1 = -2 \leftarrow \text{MIN}$$

$f$  has a global maximum of  $f(2)=2$  at  $x=2$   
 $f$  has a global minimum of  $f(1)=-2$  at  $x=1$