Directions: Show all steps (within reason). Simplify your answer.

1. Use a limit definition of the derivative to find the derivative of the function $f(x) = \dfrac{1}{2x+1}$.

$$
\begin{aligned}
f'(x) &= \lim_{z \to x} \frac{f(z) - f(x)}{z - x} \\[2em]
&= \lim_{z \to x} \frac{\dfrac{1}{2z+1} - \dfrac{1}{2x+1}}{z - x} \\[2em]
&= \lim_{z \to x} \frac{\dfrac{1}{2z+1} - \dfrac{1}{2x+1}}{z - x} \cdot \frac{(2z+1)(2x+1)}{(2z+1)(2x+1)} \\[2em]
&= \lim_{z \to x} \frac{(2x+1) - (2z+1)}{(z-x)(2z+1)(2x+1)} \\[2em]
&= \lim_{z \to x} \frac{2x+1 - 2z - 1}{(z-x)(2z+1)(2x+1)} \\[2em]
&= \lim_{z \to x} \frac{2x - 2z}{(z-x)(2z+1)(2x+1)} \\[2em]
&= \lim_{z \to x} \frac{-2(z-x)}{(z-x)(2z+1)(2x+1)} \\[2em]
&= \lim_{z \to x} \frac{-2}{(2z+1)(2x+1)} \\[2em]
&= \frac{-2}{(2x+1)(2x+1)} \\[2em]
&= \frac{-2}{(2x+1)^2}
\end{aligned}
$$

Answer: $\boxed{f'(x) = \dfrac{-2}{(2x+1)^2}}$

Directions: Show all steps (within reason). Simplify your answer.

1. Use a limit definition of the derivative to find the derivative of the function $f(x) = \sqrt{3x + 1}$.

$$
\begin{aligned}
f'(x) \;&=\; \lim_{z \to x} \frac{f(z) - f(x)}{z - x} \\[2mm]
&=\; \lim_{z \to x} \frac{\sqrt{3z + 1} - \sqrt{3x + 1}}{z - x} \\[2mm]
&=\; \lim_{z \to x} \frac{\sqrt{3z + 1} - \sqrt{3x + 1}}{z - x} \cdot \frac{\sqrt{3z + 1} + \sqrt{3x + 1}}{\sqrt{3z + 1} + \sqrt{3x + 1}} \\[2mm]
&=\; \lim_{z \to x} \frac{\sqrt{3z + 1}^2 + \sqrt{3z + 1}\sqrt{3x + 1} - \sqrt{3x + 1}\sqrt{3z + 1} - \sqrt{3x + 1}^2}{(z - x)\left(\sqrt{3z + 1} + \sqrt{3x + 1}\right)} \\[2mm]
&=\; \lim_{z \to x} \frac{\sqrt{3z + 1}^2 - \sqrt{3x + 1}^2}{(z - x)\left(\sqrt{3z + 1} + \sqrt{3x + 1}\right)} \\[2mm]
&=\; \lim_{z \to x} \frac{(3z + 1) - (3x + 1)}{(z - x)\left(\sqrt{3z + 1} + \sqrt{3x + 1}\right)} \\[2mm]
&=\; \lim_{z \to x} \frac{3z + 1 - 3x - 1}{(z - x)\left(\sqrt{3z + 1} + \sqrt{3x + 1}\right)} \\[2mm]
&=\; \lim_{z \to x} \frac{3z - 3x}{(z - x)\left(\sqrt{3z + 1} + \sqrt{3x + 1}\right)} \\[2mm]
&=\; \lim_{z \to x} \frac{3(z - x)}{(z - x)\left(\sqrt{3z + 1} + \sqrt{3x + 1}\right)} \\[2mm]
&=\; \lim_{z \to x} \frac{3(z - x)}{(z - x)\left(\sqrt{3z + 1} + \sqrt{3x + 1}\right)} \\[2mm]
&=\; \lim_{z \to x} \frac{3}{\sqrt{3z + 1} + \sqrt{3x + 1}} \\[2mm]
&=\; \frac{3}{\sqrt{3x + 1} + \sqrt{3x + 1}} \\[2mm]
&=\; \frac{3}{2\sqrt{3x + 1}}
\end{aligned}
$$

Answer: $\boxed{\,f'(x) = \dfrac{3}{2\sqrt{3x + 1}}\,}$