MATH 501, Section 11 Solutions

(2) Consider the group $\mathbb{Z}_3 \times \mathbb{Z}_4 = \{(0,0),(1,0),(2,0),(0,1),(0,1),(2,1),(0,2),(1,2),(2,2),(0,4),(1,4),(2,4)\}$.
Let's look at the cyclic groups generated by these elements in order to find their orders.
$\langle(0,0)\rangle = \{(0,0)\}$, so $(0,0)$ has order 1.
$\langle(1,0)\rangle = \{(1,0),(2,0),(0,0)\}$, so $(1,0)$ has order 3.
$\langle(2,0)\rangle = \{(2,0),(1,0),(0,0)\}$, so $(2,0)$ has order 3.
$\langle(0,1)\rangle = \{(0,1),(0,2),(0,2),(0,0)\}$, so $(0,1)$ has order 4.
$\langle(1,1)\rangle = \{(1,1),(2,2),(0,3),(1,0),(2,1),(0,2),(1,3),(2,0),(0,1),(1,2),(2,3),(0,0)\}$, so $(1,1)$ has order 12.
$\langle(2,1)\rangle = \{(2,1),(1,2),(0,3),(2,0),(1,1),(0,2),(2,3),(1,0),(0,1),(2,2),(1,3),(0,0)\}$, so $(2,1)$ has order 12.
$\langle(0,2)\rangle = \{(0,2),(0,0)\}$, so $(0,2)$ has order 2.
$\langle(1,2)\rangle = \{(1,2),(2,0),(0,2),(1,0),(2,2),(0,0)\}$, so $(1,2)$ has order 6.
$\langle(2,2)\rangle = \{(2,2),(1,0),(0,2),(2,0),(1,2),(0,0)\}$, so $(2,2)$ has order 6.
$\langle(0,3)\rangle = \{(0,3),(0,2),(0,1),(0,0)\}$, so $(0,3)$ has order 4.
$\langle(1,3)\rangle = \{(1,3),(2,2),(0,1),(1,0),(2,3),(0,2),(1,1),(2,0),(0,3),(1,2),(2,1),(0,0)\}$, so $(1,3)$ has order 12.
$\langle(2,3)\rangle = \{(2,3),(1,2),(0,1),(2,0),(1,3),(0,2),(2,1),(1,0),(0,3),(2,2),(1,1),(0,0)\}$, so $(2,3)$ has order 12.

From this, you can see that the group $\mathbb{Z}_3 \times \mathbb{Z}_4$ is cyclic because it can be generated by a single element.

(12) Find all subgroups of $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_4$ that are isomorphic to the Klein 4-group.
Here are the ones I found:
$H = \{(0,0,0),(1,0,0),(0,1,0),(1,1,0)\}$
$H = \{(0,0,0),(1,0,0),(0,0,2),(1,0,2)\}$
$H = \{(0,0,0),(0,1,0),(0,0,2),(0,1,2)\}$
$H = \{(0,0,0),(1,1,0),(0,0,2),(1,1,2)\}$
$H = \{(0,0,0),(1,0,2),(0,1,0),(1,1,2)\}$
$H = \{(0,0,0),(0,1,2),(1,0,0),(1,1,2)\}$
$H = \{(0,0,0),(1,0,2),(1,1,0),(0,1,2)\}$

(16) Are $\mathbb{Z}_2 \times \mathbb{Z}_{12}$ and $\mathbb{Z}_4 \times \mathbb{Z}_6$ isomorphic?
Notice that:
$\mathbb{Z}_2 \times \mathbb{Z}_{12} \cong \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_4$ and
$\mathbb{Z}_4 \times \mathbb{Z}_6 \cong \mathbb{Z}_4 \times \mathbb{Z}_3 \times \mathbb{Z}_2$
so from this you can see that $\mathbb{Z}_2 \times \mathbb{Z}_{12} \cong \mathbb{Z}_4 \times \mathbb{Z}_6$.

(24) List all finite abelian groups of order 720, up to isomorphism. (That is, no two groups on you list should be isomorphic, but if G is a given abelian group of order 720, your list must contain G or something isomorphic to G.)

Since $720 = 2^4 \cdot 5 \cdot 3^2$, the groups are as follows.

$\mathbb{Z}_{16} \times \mathbb{Z}_5 \times \mathbb{Z}_3 \times \mathbb{Z}_3$
$\mathbb{Z}_{16} \times \mathbb{Z}_5 \times \mathbb{Z}_9$
$\mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_5 \times \mathbb{Z}_3 \times \mathbb{Z}_3$
$\mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_5 \times \mathbb{Z}_9$
$\mathbb{Z}_2 \times \mathbb{Z}_8 \times \mathbb{Z}_5 \times \mathbb{Z}_3 \times \mathbb{Z}_3$
$\mathbb{Z}_2 \times \mathbb{Z}_8 \times \mathbb{Z}_5 \times \mathbb{Z}_9$
$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_5 \times \mathbb{Z}_3 \times \mathbb{Z}_3$
$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_5 \times \mathbb{Z}_9$
$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_5 \times \mathbb{Z}_3 \times \mathbb{Z}_3$
$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_5 \times \mathbb{Z}_9$