MATH 501, Section 14 Solutions

2. Find the order of $G = (\mathbb{Z}_4 \times \mathbb{Z}_{12})/(\langle 2\rangle \times \langle 2\rangle)$.

   Notice that $|\langle 2\rangle \times \langle 2\rangle| = 2 \cdot 6 = 12$. Since this subgroup has 12 elements and $G = \mathbb{Z}_4 \times \mathbb{Z}_{12}$ has 48 elements, the group G must have 48/12=4 elements. $\boxed{\text{Thus } G \text{ has order 4.}}$

6. Find the order of $G = (\mathbb{Z}_{12} \times \mathbb{Z}_{18})/\langle (4,3)\rangle$.

   Notice that $\langle(4,3)\rangle = \{(0,0),(4,3),(8,6),(0,9),(4,12),(8,15)\}$has 6 elements, and $\mathbb{Z}_{12} \times \mathbb{Z}_{18}$ has 216 elements. $\boxed{\text{Thus } |G| = 216/6 = 36}$

10. Find the order of $26 + \langle 12\rangle$ in the group $\mathbb{Z}_{60}/\langle 12\rangle$.

    Doing calculations in the group $\mathbb{Z}_{60}/\langle 12\rangle$, we have:
    $1(26 + \langle 12\rangle) = 26 + \langle 12\rangle$
    $2(26 + \langle 12\rangle) = 52 + \langle 12\rangle$
    $3(26 + \langle 12\rangle) = 18 + \langle 12\rangle$
    $4(26 + \langle 12\rangle) = 44 + \langle 12\rangle$
    $5(26 + \langle 12\rangle) = 10 + \langle 12\rangle$
    $6(26 + \langle 12\rangle) = 36 + \langle 12\rangle = 0 + \langle 12\rangle$

    $\boxed{\text{Thus } 26 + \langle 12\rangle \text{ has order 6.}}$

30. Suppose that H is a normal subgroup of G and $m = (G : H)$.Show $a^m \in H$ for all $a \in G$.

    Proof. Let a be an element of G. Since $|G/K| = (G : H) = m$, and aH is an element of G/K, we know that the order k of aH in G/K must divide m (Theorem 10.12). Thus there is an integer n for which $m = kn$. Now we have $a^m H = (aH)^m = (aH)^{kn} = ((aH)^k)^n = (eH)^n = e^n H = eH = H$. Thus$a^m H = H$, and this means $a^m \in H$.