Introduction to Mathematical Reasoning	MATH 300 R. Hammack	September 24, 2007 Test #1

Name: _____ Score: _____

Directions: Please answer each question in the space provided. Be sure to show your work when appropriate. Calculators may not be used on this test.

1. (8 points) Write each of the following sets in set builder notation (e.g. $\{x^2 : x \in \mathbb{N}\}$, etc.).

   (a) $\{\ldots, -4, -2, 0, 2, 4, 6, 8, 10, \ldots\} =$

   (b) $\{\ldots, -2, 3, 8, 13, 18, 23, 28, 33, \ldots\} =$

   (c) $\{-1, -4, -9, -16, -25, -36, \ldots\} =$

   (d) $[2, 7) =$

2. (8 points) Write each of the following sets by listing its elements between curly brackets.

   (a) $\{x^2 - 1 : x \in \mathbb{N}\} =$

   (b) $\{x \in \mathbb{R} : x^2 - x = 0\} =$

   (c) $\{(x, y) \in \mathbb{Z} \times \mathbb{N} : x^2 = 4 \text{ and } y^2 = 9\} =$

   (d) $\{X \in \mathcal{P}(\{a, b, c\}) : |X| = 2\} =$

3. (8 points) Answer the following questions, where $A = \{2, 3\}$, $B = \{a, b\}$, and $C = \{3, 4\}$.

   (a) $(A \cap C) \times B =$

   (b) $A \cap (C \times B) =$

4. (8 points) Consider the sets $A_1 = \{0, 1, 2, 3\}$, $A_2 = \{0, 2, 3, 4\}$, $A_3 = \{0, 3, 4, 5\}$, $A_4 = \{0, 3, 5, 6\}$, and $I = \{1, 2, 3, 4\}$.

   (a) $\bigcap_{n \in I} A_n =$

   (b) $\bigcup_{n \in I} A_n =$

5. (8 points) Write truth tables for the logical connectives $\Rightarrow$ and $\Leftrightarrow$.

6. (8 points) Suppose you know that P is false, and that the statement $(R \Rightarrow S) \Leftrightarrow (P \wedge Q)$ is true.
Can the true/false values of R and S be determined? Explain. (This can be done without a truth table.)

7. (8 points) Write an expression that is logically equivalent to $(\sim P) \vee (\sim Q)$ and contains only one $\sim$.

8. (8 points) Write out a truth table to decide if $(\sim P) \wedge (P \Rightarrow Q)$ and $\sim (Q \Rightarrow P)$ are logically equivalent.

9. (18 points) Let $x \in \mathbb{Z}$. Prove that if x is odd, then $x^2 + 1$ is even.

10. (18 points) Suppose $x, y \in \mathbb{Z}$. Prove that if xy is odd, then x and y are both odd.