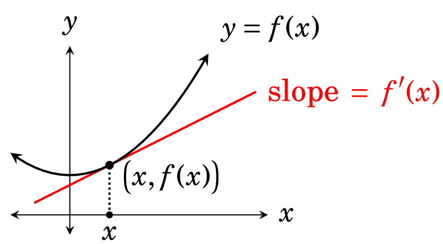
CHAPTER 17

Derivative Rules

Chapter 16 presented the main idea of calculus. Definition 16.1 states that the **derivative** of a function $f(x)$ is another function $f'(x)$, where

$$f'(x) \;=\; \lim_{z \to x} \frac{f(z)-f(x)}{z-x} \;=\; \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}.$$

The meaning of $f'(x)$ is that it gives the slope of the tangent line to the graph of $y = f(x)$ at the point $(x, f(x))$, as the following picture reminds us.



In Chapter 16 we computed the derivatives of several functions by working out one of the limits from Definition 16.1.

In Example 16.2 we saw that if $f(x) = \sqrt{x}$, then $f'(x) = \frac{1}{2\sqrt{x}}$.

In Example 16.3 we saw that if $f(x) = x^2 - x + 2$, then $f'(x) = 2x+1$.

In Example 16.4 we saw that if $f(x) = x^3$, then $f'(x) = 3x^2$.

Derivatives are so significant that there are many different notations for them. Two common notations for the derivative of $f(x)$ are

$$D_x\Big[f(x)\Big] \;=\; f'(x) \qquad \text{and} \qquad \frac{d}{dx}\Big[f(x)\Big] \;=\; f'(x).$$

Think of D_x and $\frac{d}{dx}$ as *verbs* (pronounced "*dee ex*" and "*dee dee ex*") meaning "*take the derivative of*". From the examples above,

$$D_x\Big[\sqrt{x}\Big] = \frac{1}{2\sqrt{x}}, \qquad D_x\Big[x^2-x+2\Big] = 2x-1, \qquad D_x\Big[x^3\Big] = 3x^2.$$

$$\frac{d}{dx}\Big[\sqrt{x}\Big] = \frac{1}{2\sqrt{x}}, \qquad \frac{d}{dx}\Big[x^2-x+2\Big] = 2x-1, \qquad \frac{d}{dx}\Big[x^3\Big] = 3x^2.$$

One of the primary goals of Calculus I is to find the derivative $D_x[f(x)]$ of a function $f(x)$. (Of course this is not the *only* goal. Once *found*, a derivative can be put to *use*. But our first goal is to learn how to find them.)

Definition 16.1 says that a derivative can be got from working out either one of two limits:

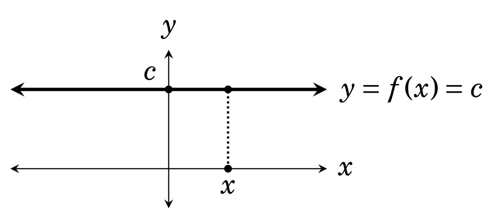
$$f'(x) \;=\; D_x\Big[f(x)\Big] \;=\; \lim_{z\to x}\frac{f(z)-f(x)}{z-x} \;=\; \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}.$$

These limits (which come from the formulas for tangent line slope) give the derivative its meaning. (And in Chapter 26 we will see that they give it other important meaning as well.) But it turns out that derivatives of most functions can be computed quickly—and without a limit—through a set of simple formulas, or *rules*. Beginning now, and continuing through Chapter 25, we will develop a list of 25 rules for finding derivatives. Once developed, these rules can be used in combination to quickly find derivatives. For example, you will soon be able to find a derivative like

$$D_x\left[3\cos\left(x^2\right)+\frac{\ln(x)+e^x}{x^2+3x+1}\right]$$

in one step, without resorting to a limit.

But we will get there gradually. We will start with rules for the simplest functions imaginable. First, let c be a constant, and consider the function $f(x)=c$. Such a function is called a **constant** function. Whatever the input x, the output is always the same, c. Think of this as a $f(x)=0\cdot x+c$, so its graph is a straight line with slope 0 and y-intercept c.



The graph is a horizontal line. The slope of a tangent at any point $(x,f(x))=(x,c)$ is zero. Therefore we would expect that $f'(x)=0$. Indeed, this follows immediately from Definition 16.1. If $f(x)=c$, then

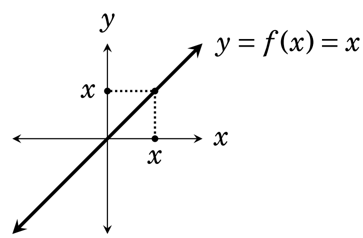
$$f'(x) \;=\; \lim_{z\to x}\frac{f(z)-f(x)}{z-x} \;=\; \lim_{z\to x}\frac{c-c}{z-x} \;=\; \lim_{z\to x}0 \;=\; 0.$$

This is our first rule. The derivative of a constant function is zero.

Rule 1. (Constant Rule) If c is a constant, then $\quad D_x[c] \;=\; 0$

The rule is so simple that the examples are not very interesting. But here are a few: $D_x[2] = 0$, $D_x\left[\frac{11}{3}\right] = 0$, $D_x[\pi] = 0$, and $D_x[0] = 0$.

For the next-simplest function after constant functions, consider the function $f(x) = x$. Whatever its input is, that is also its output, so, for example, $f(1) = 1$, $f(2) = 2$, $f(\pi) = \pi$. In general, $f(z) = z$. Because it doesn't alter its input, this function is sometimes called the **identity function**. Think of it as $f(x) = 1 \cdot x + 0$, a linear function with slope 1 and y-intercept 0.



Because its tangents all have slope 1, you'd probably guess (correctly) that the derivative of $f(x) = x$ is the constant function $f'(x) = 1$. And that is exactly confirmed by Definition 16.1: If $f(x) = x$, then

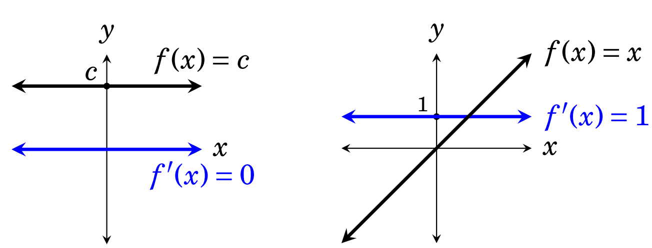
$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x} = \lim_{z \to x} \frac{z - x}{z - x} = \lim_{z \to x} 1 = 1.$$

This is our second rule.

Rule 2. (Identity Rule) $D_x[x] = 1$

There are even fewer examples of the identity rule than there are for the constant rule; in fact there is only one example: $D_x[x] = 1$.

When we say $D_x[x] = 1$, we do not mean that the derivative equals the *number 1*, but rather that it equals the *constant function* 1. (For, remember, the derivative of a function is a *function*, not a number.) Below (right) we have graphed the function $y = f(x) = x$ and its derivative $y = f'(x) = D_x[x] = 1$.



Likewise, in saying $D_x[c] = 0$, we mean the the derivative of a constant function is the *zero function*. See the graph of $f(x) = c$ and its derivative $f'(x) = 0$ above (left).

Next we turn to power functions. We ask: If $f(x) = x^n$, what is $f'(x)$? We will have an answer by the end of this page. It turns out that the second limit in Definition 16.1 is more convenient for the computation. It says

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h}.$$

To work this limit we hope to somehow factor an h from the top to cancel the h on the bottom. To do this, we need to expand out the $(x+h)^n$. You may recall how to do this. The first few powers of $(x+h)$ look like this:

$$\begin{array}{rcl}
(x+h)^2 & = & x^2 + 2xh + h^2 \\
(x+h)^3 & = & x^3 + 3x^2h + 3xh^2 + h^3 \\
(x+h)^4 & = & x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 \\
(x+h)^5 & = & x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5 \\
\vdots & \vdots & \vdots \qquad\qquad\qquad\qquad\qquad\qquad \ddots \\
(x+h)^n & = & x^n + nx^{n-1}h + \cdots \qquad \cdots \qquad \cdots + nxh^{n-1} + h^n
\end{array} \tag{*}$$

The first two terms in the expansion of $(x+h)^n$ are x^n followed by $nx^{n-1}h$. In each subsequent term the power of x decreases by 1 and the power of h increases by 1, until reaching the last two terms, which are $nxh^{n-1} + h^n$. The coefficients of the intermediate terms are called *binomial coefficients*; their exact values will not concern us here. The important thing is that we have enough to work out our limit: If $f(x) = x^n$, then

$$\begin{aligned}
f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0} \frac{(x+h)^n - x^n}{h} \qquad \text{(now use (*) above)} \\
&= \lim_{h\to 0} \frac{\left(x^n + nx^{n-1}h + \cdots + nxh^{n-1} + h^n\right) - x^n}{h} \\
&= \lim_{h\to 0} \frac{nx^{n-1}h + \cdots + nxh^{n-1} + h^n}{h} \\
&= \lim_{h\to 0} \frac{h\left(nx^{n-1} + \cdots + nxh^{n-2} + h^{n-1}\right)}{h} \\
&= \lim_{h\to 0} \left(nx^{n-1} + \cdots + nxh^{n-2} + h^{n-1}\right) \\
&= \left(nx^{n-1} + \cdots + nx\cdot 0^{n-2} + 0^{n-1}\right) = nx^{n-1}
\end{aligned}$$

We have our answer: If $f(x) = x^n$, then $f'(x) = nx^{n-1}$. This is our new rule.

Rule 3. (Power Rule) $D_x\left[x^n\right] = nx^{n-1}$

Let's look at some examples of our new rule, the power rule.

$$\begin{aligned} D_x\left[x^2\right] &= 2x^{2-1} = 2x^1 = 2x \\ D_x\left[x^3\right] &= 3x^{3-1} = 3x^2 \end{aligned}$$

Notice that this agrees with Example 16.4, which used the limit definition of the derivative to show that if $f(x)=x^3$, then $f'(x)=3x^2$. Continuing,

$$\begin{aligned} D_x\left[x^4\right] &= 4x^{4-1} = 4x^3 \\ D_x\left[x^{100}\right] &= 100x^{100-1} = 100x^{99} \end{aligned}$$

The power rule is very convenient. You would **not** want to find the derivative of x^{100} using a limit! The power rule allows us to bypass the limit when finding the derivative of a power function.

On the previous page we derived the power rule $D_x[x^n]=nx^{n-1}$ under the assumption that n is a *positive integer*. However, it is a fact (which we will prove later) that the power rule holds for any real value of n, positive or negative. For example,

$$\begin{aligned} D_x\left[x^{-3}\right] &= -3x^{-3-1} = -3x^{-4} \\ D_x\left[x^{\frac{3}{2}}\right] &= \frac{3}{2}x^{\frac{3}{2}-1} = \frac{3}{2}x^{\frac{1}{2}} \end{aligned}$$

Using this you can find derivatives of radical functions and reciprocals of powers of x by converting to a power and applying the power rule. Here are some examples.

$$\begin{aligned} D_x\left[\frac{1}{x}\right] &= \frac{d}{dx}\left[x^{-1}\right] = -1\cdot x^{-1-1} = -\frac{1}{x^2} \\ D_x\left[\frac{1}{x^2}\right] &= \frac{d}{dx}\left[x^{-2}\right] = -2\cdot x^{-2-1} = -\frac{2}{x^3} \\ D_x\left[\sqrt{x}\right] &= \frac{d}{dx}\left[x^{\frac{1}{2}}\right] = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{-\frac{1}{2}}} = \frac{1}{2\sqrt{x}} \end{aligned}$$

Notice that these agree with our previous examples and exercises. In the previous chapter you may have used a limit to show that $D_x\left[\frac{1}{x}\right]=-\frac{1}{x^2}$. In Example 16.2 we used a limit to show that $D_x\left[\sqrt{x}\right]=\frac{1}{2\sqrt{x}}$.

There are two more rules to come in this chapter. But rather than giving derivatives of specific functions, they deal with derivatives of combinations of functions. For example, given two functions f and g, we now obtain a rule for the derivative of their sum $F(x) = f(x)+g(x)$, or difference $F(x) = f(x)-g(x)$.

Before beginning, quickly note how Definition 16.1 works if the function is F or g (or anything else) instead of f:

Given a function $F(x)$, its derivative is $F'(x) = \lim_{z\to x} \dfrac{F(z)-F(x)}{z-x}$.

Given a function $g(x)$ its derivative is $g'(x) = \lim_{z\to x} \dfrac{g(z)-g(x)}{z-x}$, etc.

Now consider a function $F(x)$ that is the sum $F(x) = f(x)+g(x)$ of two other functions. What is its derivative?

$$
\begin{aligned}
F'(x) &= \lim_{z\to x} \frac{F(z)-F(x)}{z-x} \\
&= \lim_{z\to x} \frac{\big(f(z)+g(z)\big)-\big(f(x)+g(x)\big)}{z-x} \\
&= \lim_{z\to x} \frac{f(z)-f(x) \ + \ g(z)-g(x)}{z-x} \\
&= \lim_{z\to x} \left(\frac{f(z)-f(x)}{z-x} + \frac{g(z)-g(x)}{z-x}\right) \\
&= \lim_{z\to x} \frac{f(z)-f(x)}{z-x} + \lim_{z\to x} \frac{g(z)-g(x)}{z-x} \quad = \quad f'(x)+g'(x).
\end{aligned}
$$

We have just shown that if $F(x) = f(x)+g(x)$, then $F'(x) = f'(x)+g'(x)$, which we can also express as

$$D_x\Big[f(x)+g(x)\Big] = D_x\big[f(x)\big] + D_x\big[g(x)\big].$$

The derivative of the sum of two functions is the sum of their derivatives.

Doing the same computation for $F(x) = f(x)-g(x)$, you will get

$$D_x\Big[f(x)-g(x)\Big] = D_x\big[f(x)\big] - D_x\big[g(x)\big].$$

These combine into our fourth rule for derivatives.

Rule 4. (Sum-Difference Rule) Given two functions $f(x)$ and $g(x)$,

$$
\begin{aligned}
D_x\Big[f(x)+g(x)\Big] &= f'(x)+g'(x) \\
D_x\Big[f(x)-g(x)\Big] &= f'(x)-g'(x)
\end{aligned}
$$

Example 17.1 Actually this is *four* examples with a common theme. In each one we're asked for the derivative of a sum or difference of two functions. In each case we first apply the sum-difference rule to break the problem into a sum or difference of two derivatives. Then we complete the problem by using whichever of the first three rules applies.

$$D_x\Big[x^5+x\Big] = D_x\Big[x^5\Big]+D_x\Big[x\Big] = \boxed{5x^4+1}$$
$$D_x\Big[x^2-8\Big] = D_x\Big[x^2\Big]-D_x\Big[8\Big] = 2x-0 = \boxed{2x}$$
$$D_x\left[x^2-\frac{1}{x^2}\right] = D_x\Big[x^2\Big]-D_x\Big[x^{-2}\Big] = 2x-\left(-2x^{-2-1}\right) = 2x+2x^{-3} = \boxed{2x+\frac{2}{x^3}}$$
$$D_x\left[\sqrt{x}+\frac{1}{x}\right] = D_x\Big[x^{\frac{1}{2}}\Big]+D_x\Big[x^{-1}\Big] = \frac{1}{2}x^{-\frac{1}{2}}+\left(-1\cdot x^{-2}\right) = \boxed{\frac{1}{2\sqrt{x}}-\frac{1}{x^2}}$$

You will quickly reach the point of doing such problems in one step. ✍

We can write the sum-difference rule by combining its two statements as

$$D_x\Big[f(x)\pm g(x)\Big] = f'(x)\pm g'(x).$$

Actually (as is easy to check) this works for more than just two functions:

$$D_x\Big[f(x)\pm g(x)\pm h(x)\pm\cdots\Big] = f'(x)\pm g'(x)\pm h'(x)\pm\cdots.$$

For example, $D_x\Big[x^2-x+2\Big] = 2x-1+0 = \boxed{2x-1}$. To appreciate how far we've come, compare this to Example 16.3 in Chapter 16, where we solved the same problem with a limit. It took a page. Here we did it in one step!

Now for this chapter's final rule. Given a constant c and a function $f(x)$, make a new function $F(x)=cf(x)$ by multiplying f by c. Its derivative is

$$\begin{aligned} F'(x) &= \lim_{z\to x}\frac{F(z)-F(x)}{z-x} = \lim_{z\to x}\frac{cf(z)-cf(x)}{z-x} \\ &= \lim_{z\to x}\frac{c\big(f(z)-f(x)\big)}{z-x} = c\cdot\lim_{z\to x}\frac{f(z)-f(x)}{z-x} = cf'(x). \end{aligned}$$

Thus the derivative of $cf(x)$ is $cf'(x)=c\cdot D_x\big[f(x)\big]$. That is our new rule.

Rule 5. (Constant Multiple Rule) $D_x\Big[cf(x)\Big] = cD_x\Big[f(x)\Big]$.

The derivative of c times a function is c times the derivative of the function.

Example 17.2 This is actually *six* examples with a common theme. In each case we apply the constant multiple rule, and follow up with the power rule (Rule 3) or Rule 2.

$$D_x\left[3x^5\right] = 3\cdot D_x\left[x^5\right] = 3\cdot 5x^4 = \boxed{15x^4}$$

$$D_x\left[3x\right] = 3\cdot D_x\left[x\right] = 3\cdot 1 = \boxed{3}$$

Sometimes a little algebra is needed to change the function into something to which the constant multiple rule applies. In the next example we factor a constant of $\frac{1}{2}$ out of the $\frac{x}{2}$ before applying the constant multiple rule.

$$D_x\left[\frac{x}{2}\right] = D_x\left[\frac{1}{2}x\right] = \frac{1}{2}\cdot D_x\left[x\right] = \frac{1}{2}\cdot 1 = \boxed{\frac{1}{2}}$$

In the next example we convert a reciprocal to a power, then follow up with the constant multiple rule followed by the power rule.

$$D_x\left[\frac{3}{7x^5}\right] = D_x\left[\frac{3}{7}x^{-5}\right] = \frac{3}{7}\cdot D_x\left[x^{-5}\right] = \frac{3}{7}\cdot\left(-5x^{-6}\right) = -\frac{15}{7}x^{-6} = \boxed{-\frac{15}{7x^6}}$$

(Here the calculus was finished at the third step—with the power rule—and the last two steps were just algebraic simplifications.) In the next example we have to convert a radical to a power so that we can apply the constant multiple rule, followed by the power rule.

$$D_x\left[\pi\sqrt{x}\right] = D_x\left[\pi x^{1/2}\right] = \pi\cdot D_x\left[x^{1/2}\right] = \pi\cdot\frac{1}{2}x^{1/2-1} = \frac{\pi}{2}x^{-1/2} = \boxed{\frac{\pi}{2\sqrt{x}}}$$

Here's one more example involving a radical.

$$D_x\left[\frac{3}{7\sqrt{x}^3}\right] = D_x\left[\frac{3}{7x^{3/2}}\right] = D_x\left[\frac{3}{7}x^{-3/2}\right] = \frac{3}{7}\cdot D_x\left[x^{-3/2}\right] = \frac{3}{7}\cdot\frac{-3}{2}x^{-3/2-1}$$

This is the derivative, but a little algebra gets it into a nicer form:

$$= -\frac{9}{14}x^{-5/2} = -\frac{9}{14x^{5/2}} = \boxed{-\frac{9}{14\sqrt{x}^5}}$$

With practice, you will soon find yourself doing problems like these in one or two steps, doing most of the work in your head. ✍

Example 17.3 Typically in finding a derivative you will use multiple rules in combination. This example uses all five of this chapter's rules:

$$\begin{aligned} D_x\left[7x^3-2x^2+6x+1\right] &= D_x\left[7x^3\right] - D_x\left[2x^2\right] + D_x\left[6x\right] + D_x\left[1\right] \\ &= 7\cdot D_x\left[x^3\right]-2\cdot D_x\left[x^2\right]+6\cdot D_x\left[x\right]+D_x\left[1\right] \\ &= 7\cdot 3x^2 - 2\cdot 2x + 6\cdot 1 + 0 \\ &= 21x^2-4x+6. \end{aligned}$$

Work some exercises like this one! After doing one or two problems this way, you will start to skip steps. You'll do it in one step:

$$D_x\left[7x^3-2x^2+6x+1\right] = 21x^2-4x+6.$$

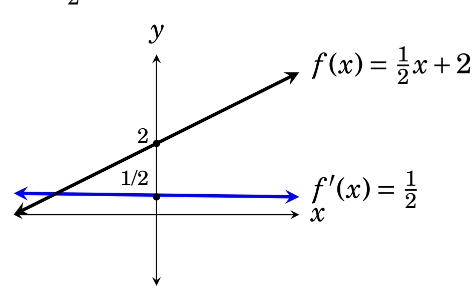
Don't forget that the notation $\frac{d}{dx}$ is sometimes used instead of D_x. The above derivative can be expressed as

$$\frac{d}{dx}\left[7x^3-2x^2+6x+1\right] = 21x^2-4x+6.$$ ✍

The rules make finding derivatives almost automatic. Things become so easy that we can lose sight of the derivative's *meaning*. Our final example reminds us of that meaning, which is that $f'(x)$ is the slope of the tangent line to $y=f(x)$ at the point $(x,f(x))$.

Example 17.4 Find the derivative of $f(x)=\frac{1}{2}x+2$.

Answer: By this chapter's rules, we immediately get $f'(x)=\frac{1}{2}$, that is, the derivative is the constant function $y=\frac{1}{2}$. This makes sense because the graph of $y=f(x)$ is a linear function line with slope $\frac{1}{2}$. The derivative $f'(x)$ gives the slope of the tangent to the graph of $f(x)$ at $(x,f(x))$. At any such point this slope is $\frac{1}{2}$. ✍



Now test your skill by working some exercises!

Exercises for Chapter 17

Find the derivatives of the following functions.

1. $f(x) = x^4 + x^3 + x^2 + x + 1$
2. $f(x) = \frac{1}{2}x^6 + 3x^2$
3. $\frac{1}{x^5} + \frac{1}{5}$
4. $f(x) = \frac{1}{x} + x$
5. $y = 3x^5 - 2x^3 + x^2 - 4$
6. $f(x) = \frac{1}{2x^4} + x^4$
7. $f(x) = \sqrt[3]{x}^2$
8. $f(x) = \sqrt{x} + x^7$
9. $f(x) = \frac{1}{\sqrt{x}}$
10. $f(x) = \sqrt{7x}$
11. $f(x) = \frac{1}{x^5} + x^5$
12. $f(x) = \sqrt[3]{x^2}$
13. $f(x) = (7x)^2$
14. $f(x) = \frac{5}{x^5} + \sqrt{x}$
15. $f(x) = \frac{7}{3x^2} + \sqrt{2}$
16. $f(x) = \sqrt[3]{8x}$
17. $y = \sqrt{\frac{5x^3}{3}}$
18. $f(x) = \frac{5x^2 + x^4}{2}$
19. $f(x) = \frac{5x^3 - 3x^2 + 2x - 4}{3}$
20. $f(x) = \frac{2\sqrt{x} + 2}{3}$
21. $f(x) = \frac{x^2}{\sqrt{3}}$
22. $f(x) = \sqrt{\frac{5}{2x}}$
23. $f(x) = \frac{x}{1+\pi}$
24. $f(x) = \frac{\pi^3}{x}$
25. $f(x) = \frac{1+\pi}{x}$
26. $f(x) = \frac{x}{1+\sqrt{2}}$

Exercises Solutions for Chapter 17

1. The derivative of $f(x) = x^4 + x^3 + x^2 + x + 1$ is $f'(x) = 4x^3 + 3x^2 + 2x + 1$

3. $D_x\left[\frac{1}{x^5} + \frac{1}{5}\right] = \frac{d}{dx}\left[x^{-5} + \frac{1}{5}\right] = -5x^{-6} + 0 = \frac{-5}{x^6}$

5. $D_x\left[3x^5 - 2x^3 + x^2 - 4\right] = 15x^4 - 6x^2 + 2x$

7. $D_x\left[\sqrt[3]{x}^2\right] = D_x\left[x^{2/3}\right] = \frac{2}{3}x^{2/3-1} = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}} = \frac{2}{3\sqrt[3]{x}}$

9. $f(x)=\dfrac{1}{\sqrt{x}}=x^{-1/2}$ $\qquad f'(x)=-\dfrac{1}{2}x^{-1/2-1}=-\dfrac{1}{2}x^{-3/2}=-\dfrac{1}{2\sqrt{x^3}}$

11. $f(x)=\dfrac{1}{x^5}+x^5=x^{-5}+x^5$ $\qquad f'(x)=-5x^{-6}+5x^4=-\dfrac{5}{x^6}+5x^4$

13. $f(x)=(7x)^2=49x^2$ $\qquad f'(x)=49\cdot 2x=98x$

15. $f(x)=\dfrac{7}{3x^2}+\sqrt{2}=\dfrac{7}{3}x^{-2}+\sqrt{2}$ $\qquad f'(x)=\dfrac{7}{3}(-2x^{-3})+0=-\dfrac{14}{3x^3}$

17. $D_x\left[\sqrt{\dfrac{5x^3}{3}}\right]=D_x\left[\sqrt{\dfrac{5}{3}}\sqrt{x^3}\right]=\sqrt{\dfrac{5}{3}}\cdot D_x\left[\sqrt{x^3}\right]=\sqrt{\dfrac{5}{3}}\cdot D_x\left[x^{3/2}\right]=\sqrt{\dfrac{5}{3}}\cdot\dfrac{3}{2}x^{1/2}=\dfrac{3\sqrt{5}\sqrt{x}}{2\sqrt{3}}$

19. $D_x\left[\dfrac{5x^3-3x^2+2x-4}{3}\right]=\dfrac{1}{3}\cdot D_x\left[5x^3-3x^2+2x-4\right]=\dfrac{1}{3}\left(15x^2-6x+2\right)=5x^2-2x+\dfrac{2}{3}$

21. $f(x)=\dfrac{x^2}{\sqrt{3}}=\dfrac{1}{\sqrt{3}}x^2$ $\qquad f'(x)=\dfrac{2}{\sqrt{3}}x$

23. $f(x)=\dfrac{x}{1+\pi}=\dfrac{1}{1+\pi}x$ $\qquad f'(x)=\dfrac{1}{1+\pi}$

25. $f(x)=\dfrac{1+\pi}{x}=(1+\pi)x^{-1}$ $\qquad f'(x)=(1+\pi)(-1)x^{-1-1}=-\dfrac{1+\pi}{x^2}$