Name: _____

Test 3

MATH 200, Section 9

April 23, 2021

Directions: Closed book, closed notes, no calculators. Put all phones, etc., away. You will need only a pencil or pen.

1. (7 points each) Find the indefinite integrals.

   (a) $\int \left(x^3 + 2x + e^x\right)\, dx$

   (b) $\int 5x^{-1}\, dx$

   (c) $\int \left(\sec^2(x) + 3\sin(x)\right)\, dx$

   (d) $\int \frac{1}{\sqrt{x}}\, dx$

   (e) $\int \frac{\pi}{3 + 3x^2}\, dx$

   (f) $\int \frac{5x + 1}{x}\, dx$

2. (8 points) Suppose $f(x)$ and $g(x)$ are differentiable functions. Find $\int \left(f'(x)g(x) + f(x)g'(x) \right) dx$.

3. (8 points) Suppose $f(x)$ is a function for which $f'(x) = \dfrac{1}{x} + \dfrac{1}{x^2} - 1$ and $f(1) = 3$. Find $f(x)$.

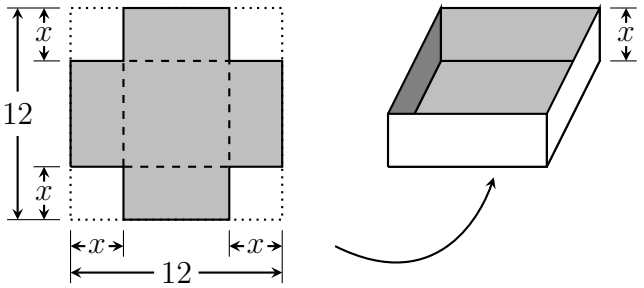
4. (8 points each) Find the limits.

   (a) $\lim\limits_{x\to\infty} x(e^{1/x} - 1)$

   (b) $\lim\limits_{x\to 0} \dfrac{e^x - 1 - x}{x^2}$

   (c) $\lim\limits_{x\to\infty} \Big(\ln(2x) - \ln(x+1)\Big)$

5. (10 points) An open-top box is made from a 12 by 12 inch piece of cardboard by cutting a square from each corner, and folding up. What should x be to maximize the volume of the box?



6. (8 points) Below is the graph of the **derivative** $f'(x)$ of a function $f(x)$.
Answer the following question about the function $f(x)$.

(a) On what intervals is $f(x)$ is concave up?

(b) On what intervals is $f(x)$ is concave down?