

$$1. \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = \lim_{x \rightarrow \infty} e^{\ln((1 + \frac{1}{x})^x)} = \lim_{x \rightarrow \infty} e^{x \ln(1 + \frac{1}{x})}$$

↑  
form  $1^\infty$

$$= e^{\lim_{x \rightarrow \infty} \frac{\ln(1 + \frac{1}{x})}{\frac{1}{x}}}$$

form  $\frac{0}{0}$

$$= e^{\lim_{x \rightarrow \infty} \frac{\frac{-1/x^2}{1+1/x}}{-1/x^2}} = e^{\lim_{x \rightarrow \infty} \frac{-1/x^2}{1+1/x} \cdot \frac{1}{-1/x^2}}$$

$$= e^{\lim_{x \rightarrow \infty} \frac{1}{1+1/x}} = e^{\frac{1}{1+0}} = e^1 = \boxed{e}$$

$$2. \int \frac{1}{\sqrt{x}} dx = \int x^{-1/2} dx = \frac{x^{-1/2+1}}{-1/2+1} + C = \frac{x^{1/2}}{1/2} + C$$

$$= \boxed{2\sqrt{x} + C}$$

$$3. \int \left(\frac{1}{x} - \sec^2(x) + \pi\right) dx = \boxed{\ln|x| - \tan(x) + \pi x + C}$$

$$4. \int \left(x^2 + \frac{1}{x^2}\right) dx = \int (x^2 + x^{-2}) dx = \frac{x^{2+1}}{2+1} + \frac{x^{-2+1}}{-2+1} + C$$

$$= \frac{x^3}{3} + \frac{x^{-1}}{-1} + C = \boxed{\frac{x^3}{3} - \frac{1}{x} + C}$$



$$\begin{aligned}
 1. \quad \lim_{x \rightarrow 0^+} (1+x)^{1/x} &= \lim_{x \rightarrow 0^+} e^{\ln((1+x)^{1/x})} = \lim_{x \rightarrow 0^+} e^{\frac{1}{x} \ln(1+x)} \\
 &\text{↑} \\
 &\text{form } 0^\infty \quad = \lim_{x \rightarrow 0^+} e^{\frac{\ln(1+x)}{x}} = e^{\lim_{x \rightarrow 0^+} \frac{\ln(1+x)}{x}} \quad \text{← form } \frac{0}{0} \\
 &= e^{\lim_{x \rightarrow 0^+} \frac{\frac{1}{1+x}}{1}} = e^{\frac{1}{1+0}} = e^1 = \boxed{e}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \int \left( 14x^6 - \frac{1}{x} + e^x \right) dx &= \boxed{14 \frac{x^7}{7} - \ln|x| + e^x + C} \\
 &= \boxed{2x^7 - \ln|x| + e^x + C}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \int x\sqrt{x} dx &= \int x^{\frac{1}{2}} x^{\frac{1}{2}} dx = \int x^{1+\frac{1}{2}} dx = \int x^{\frac{3}{2}} dx
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C = \boxed{\frac{2}{5} \sqrt{x^5} + C}
 \end{aligned}$$