Name: Richard

QUIZ 24

MATH 200
November 21, 2024

1. Find the area under the graph of $y = 3\sqrt{x}$ between $x = 0$ and $x = 9$.

$$\int_0^9 3\sqrt{x}\,dx = \int_0^9 3x^{1/2}\,dx = \left[3\,\frac{x^{3/2}}{3/2}\right]_0^9 = \left[2\sqrt{x}^{\,3}\right]_0^9 = 2\sqrt{9}^{\,3} - 2\sqrt{0}^{\,3} = 2\cdot 3^3 = \boxed{54 \text{ sq units}}$$

2. $$\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx = \left[\sin^{-1}(x)\right]_0^1 = \sin^{-1}(1) - \sin^{-1}(0) = \frac{\pi}{4} - 0 = \boxed{\frac{\pi}{4}}$$

3. $$\int_0^2 \left(\frac{x^2}{3} + 2x + 1\right) dx = \left[\frac{x^3}{9} + x^2 + x\right]_0^2 = \left(\frac{2^3}{9} + 2^2 + 2\right) - \left(\frac{0^3}{9} + 0^2 + 0\right)$$

$$= \frac{8}{9} + 6 = \frac{8}{9} + \frac{54}{9} = \boxed{\frac{62}{9}}$$

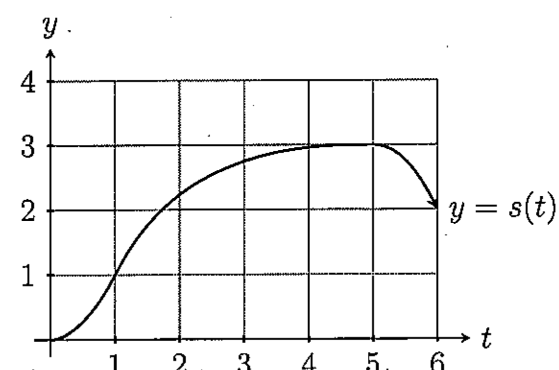
4. Find the derivative of the function $F(x) = \displaystyle\int_1^x \frac{t^5 + \sin(\pi t)}{e^t}\,dt$.

By FTC I, $\boxed{F'(x) = \dfrac{x^5 + \sin(\pi x)}{e^x}}$

5. An object moving on a line has position $s(t)$ and velocity $v(t)$ at time t.
The position function $s(t)$ is graphed below.

(a) $$\int_5^6 v(t)\,dt = \left[s(t)\right]_5^6 = s(6) - s(5)$$

$$= 2 - 3 = \boxed{-1}$$



(b) What does your answer to part (a) mean?

At time $t = 6$, object is one unit left of where it was at time 5.

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1. Find the derivative of the function $F(x) = \int_1^x \frac{\cos(t)\ln(t^2+7)}{t^5+e^t}\,dt$.

By FTC I, $\boxed{F'(x) = \frac{\cos(x)\ln(x^2+7)}{x^5+e^x}}$

2. $\int_1^4 \frac{1}{\sqrt{x}}\,dx = \int_1^4 x^{-1/2}\,dx = \left[\frac{x^{1/2}}{1/2}\right]_1^4 = \left[2\sqrt{x}\right]_1^4 = 2\sqrt{4} - 2\sqrt{1}$

$= 2 \cdot 2 - 2 = \boxed{2}$

3. $\int_0^1 \frac{1}{1+x^2}\,dx = \left[\tan^{-1}(x)\right]_0^1 = \tan^{-1}(1) - \tan^{-1}(0)$

$= \frac{\pi}{4} - 0 = \boxed{\frac{\pi}{4}}$

4. Find the area under the graph of $y = x^3 + 1$ between $x = 0$ and $x = 2$.

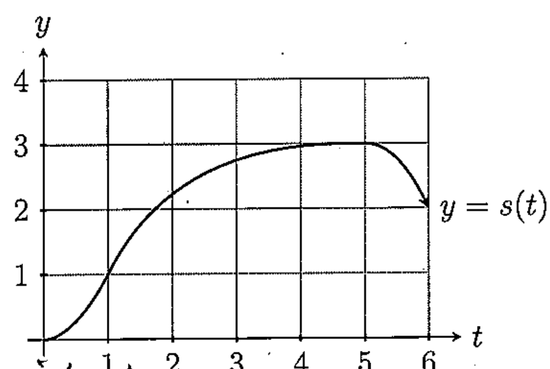
$A = \int_0^2 x^3 + 1\,dx = \left[\frac{x^4}{4} + x\right]_0^2 = \left(\frac{2^4}{4} + 2\right) - \left(\frac{0^4}{4} + 0\right)$

$= \frac{16}{4} + 2 = 4 + 2 = \boxed{6 \text{ sq. units}}$

5. An object moving on a line has position $s(t)$ and velocity $v(t)$ at time t.
The position function $s(t)$ is graphed below.

(a) $\int_1^6 v(t)\,dt = \left[s(t)\right]_1^6 = s(6) - s(1)$

$= 2 - 1 = \boxed{1}$



(b) What does your answer to part (a) mean?

At time $t=6$, object is one unit to right of where it was at time $t=1$.