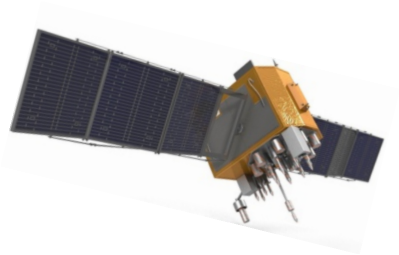
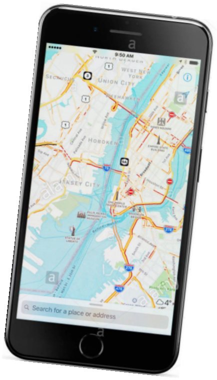
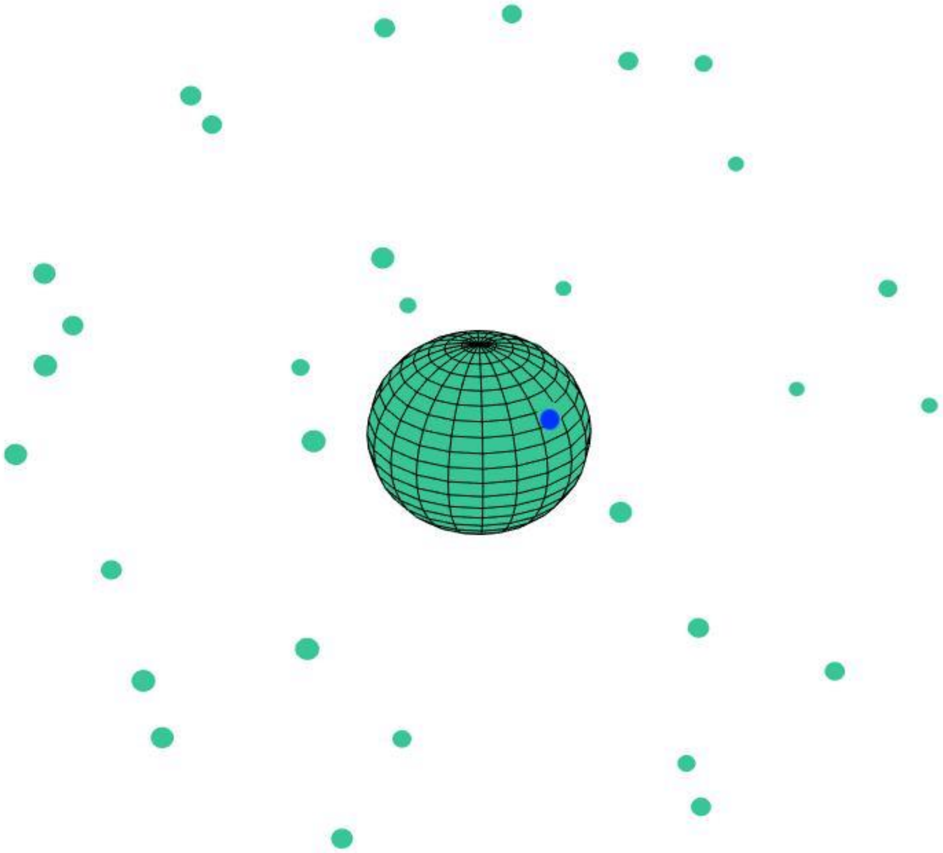
Start with a simplified GPS model
spherical Earth, receiver sync'd with satellite clocks







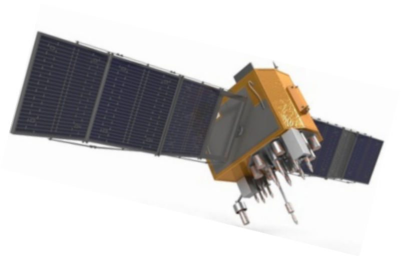
31 U.S. GPS satellites active at 1:30 pm, June 12, 2019

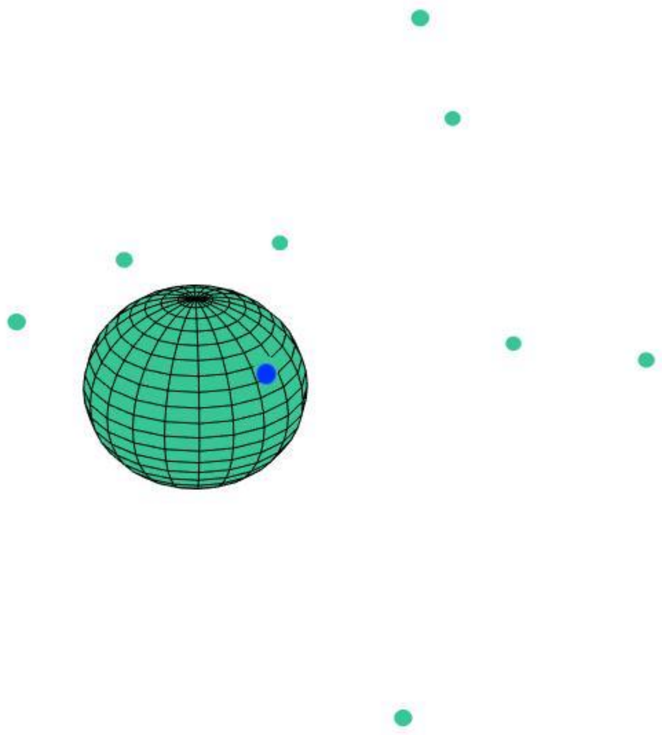
`https://in-the-sky.org/satmap_worldmap.php`

Russia, China and the EU also have systems

satellite sizes are exaggerated, blue dot is arbitrary Earth receiver location

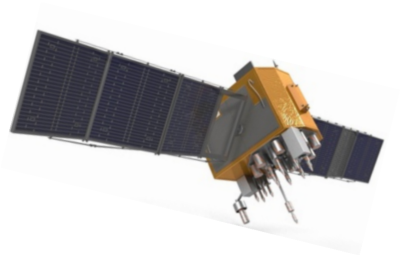
Simplified GPS model

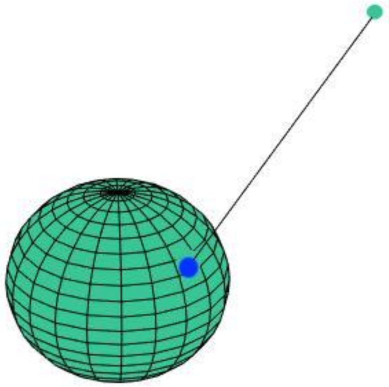




8 satellites are 10° or more above horizon at San Diego, CA
32.7° latitude, -117° longitude
GPS receiver at blue dot does NOT know it is there yet,
only that it's somewhere on Earth

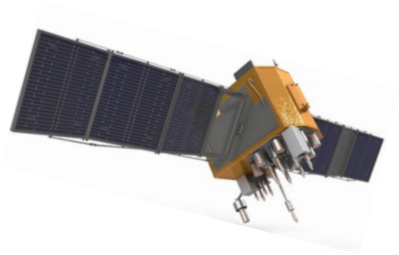
Simplified GPS model

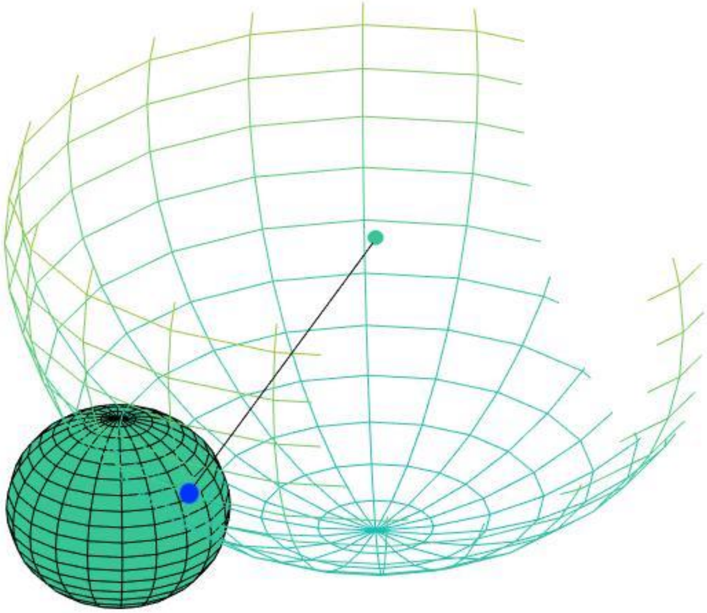


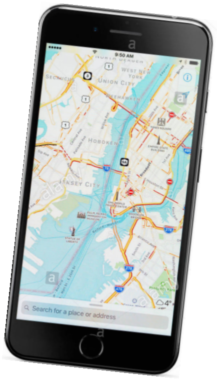


Each satellite transmits signals with time sent and satellite location, the time-sync’d GPS receiver computes distance to satellite from the time difference between broadcast and reception, using the speed of light

Simplified GPS model

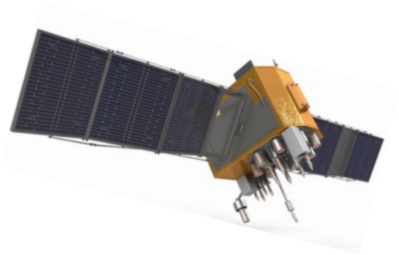


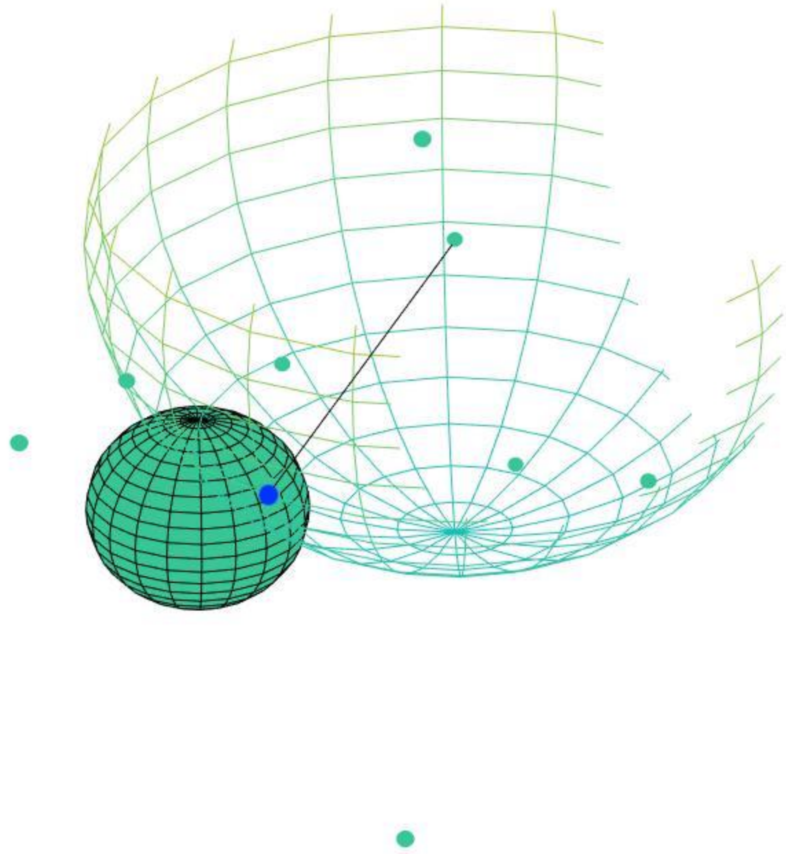


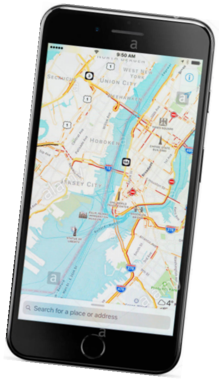


Distance to satellite gives equation for sphere of that radius around known satellite location - receiver only knows it is located somewhere on that sphere and on Earth's surface

Simplified GPS model







The GPS receiver then computes its location from the intersection of 3 or more satellite spheres with Earth's spherical surface (a 4th sphere), where the intersection of 2 spheres is a circle, 3 is 2 points, 4 is one point, a problem in "linear algebra," solving multiple, coupled algebraic equations

Simplified GPS model in MATLAB

github.com/RichardHerz/GPS >> gps3D_spheres

```
% simplified GPS in MATLAB - receiver clock sync'd with satellites
re = 6370; % (km), spherical earth radius

% specify GPS receiver latitude, longitude and altitude (altitude == 0)
rec = [32.7,-117,0]; % San Diego, CA, USA is [32.7,-117,0]
[x,y,z] = fLatLongToXYZ(rec, re);
xyzRec = [x,y,z]; % xyz coordinates of receiver, earth center is origin

% specify >= 3 satellite latitude (deg), longitude (deg), altitude (km)
% 31 listed in file sat.txt taken 1:30 pm, June 12, 2019 from data at
% https://in-the-sky.org/satmap_worldmap.php
load sat.txt

% get xyz coordinates of satellites
[x, y, z] = fLatLongToXYZ(sat,re);
xyz = [x, y, z];

% get satellites above horizon and in view of receiver
degdel = 10; % min degree above horizon for sat in view
rView = fReturnSatViewRows(sat,xyz,xyzRec,re,degdel);
xyz = xyz(rView,:);
r = fDistance(xyz,xyzRec); % sats to receiver

% END SETUP

% GIVEN:
% radius of spherical earth, re
% lat, long and altitude of >= 3 satellites
% distance of each satellite from receiver

% FIND:
% lat and long of receiver on earth's surface

% matrix eqn for sphere intersects is A * xyzCalc = c
A = xyz; % xyz of satellites
c = fCcoef(xyz,r,re);

% xyzCalc = inv(A) * c; % only for A and c rows == 3
xyzCalc = A \ c; % for A and c rows >= 3

% compute receiver lat and long
[latCalc, longCalc, altCalc] = fXYZtoLatLong(xyzCalc', re);

fprintf('location:   lat, long, alt, %6.3f, %6.3f, %4.3e \n', rec)
fprintf('calculated: lat, long, alt, %6.3f, %6.3f, %4.3e \n', ...
    latCalc, longCalc, altCalc)
```

2 key functions

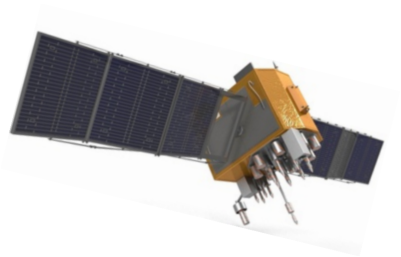
```
function rowView = fReturnSatViewRows(sat,xyz,xyzRec,re,degdel)
    % returns row numbers of satellites >= degdel above horizon

    dRec = fDistance(xyz,xyzRec); % distances from sats to receiver
    dOrig = re + sat(:,3); % distances from sats to earth center

    % we know 3 sides of triangle between sat, rec, earth center
    % use law of cosines to find the angle we want
    num = re^2 + dRec.^2 - dOrig.^2;
    denom = 2 * re * dRec;
    gamma = -90 + acosd(num ./ denom);

    % find and return satellite row numbers where gamma >= degdel
    rowView = find(gamma >= degdel);
```

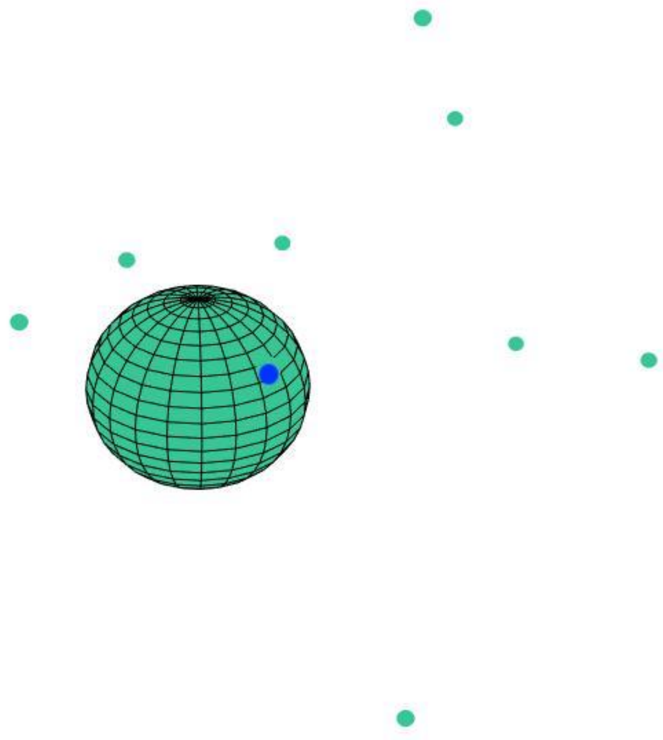
```
function c = fCcoef(xyz,r,re)
    % input xyz are locations of satellites (each row is satellite)
    % input r are distances from satellites to receiver
    % input re is radius of spherical earth
    % returns vector of coefficients for matrix solution
    % option 2 for sum( ,2) sums each row

    c = ( re^2 + sum(xyz.^2, 2) - r.^2 ) / 2 ;
```

```
>> gps3
location:   lat, long, alt, 32.700, -117.000, 0.000e+00
calculated: lat, long, alt, 32.700, -117.000, -9.095e-13
```

www.ReactorLab.net

Improved GPS models





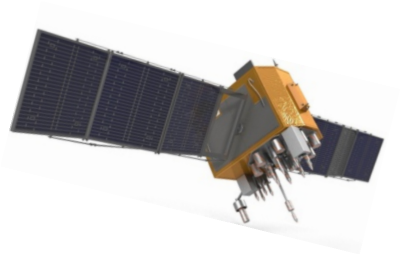
GPS receivers are not usually sync'd with the satellites such that there is an unknown offset in the distance determinations. In a model with the receiver on the surface of a spherical Earth, a unique linear algebra solution can be obtained with 4 satellites

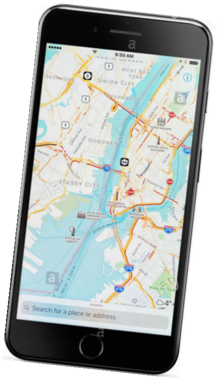
(assuming offset << distance to satellite)

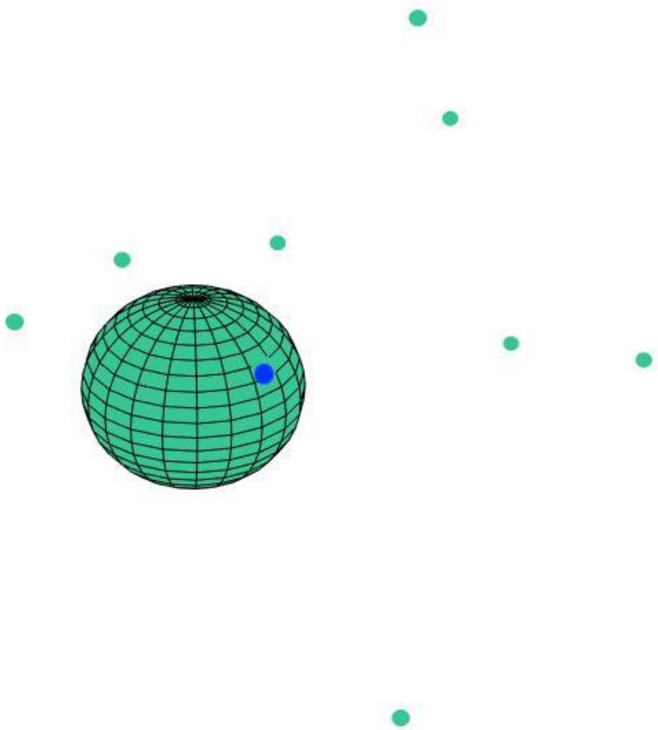
`github.com/RichardHerz/GPS  >>  gps4_spheres_offset`

www.ReactorLab.net

Improved GPS models



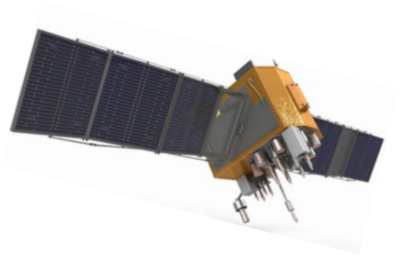


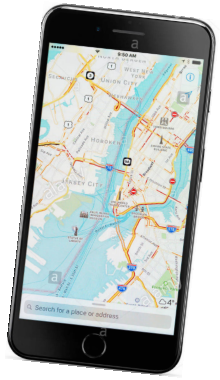


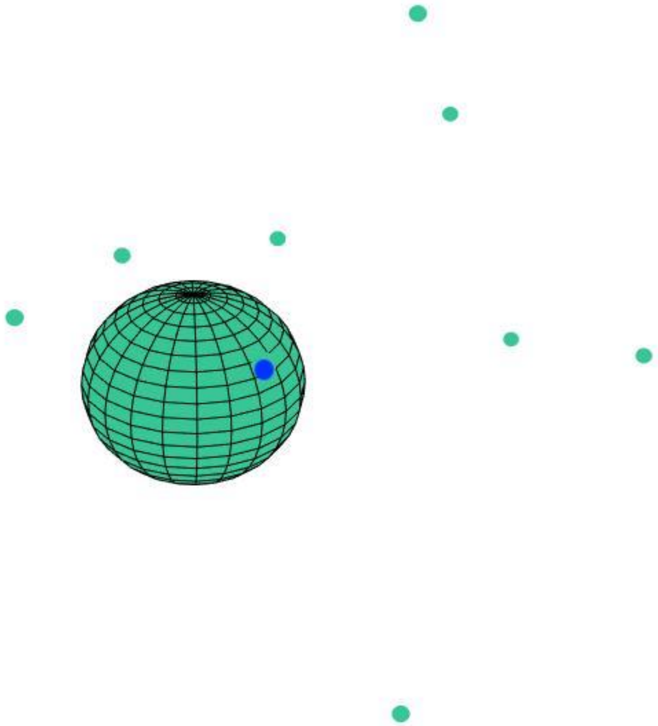
With an unknown clock offset and a receiver at an unknown altitude on or above a nonspherical Earth, a solution can be obtained with 4 or more satellite distance spheres using a nonlinear solution.

`github.com/RichardHerz/GPS >> gps5_spheres_offset`

Improved GPS models



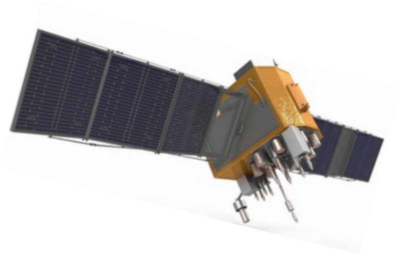


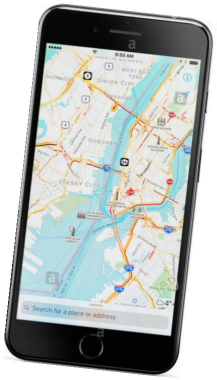


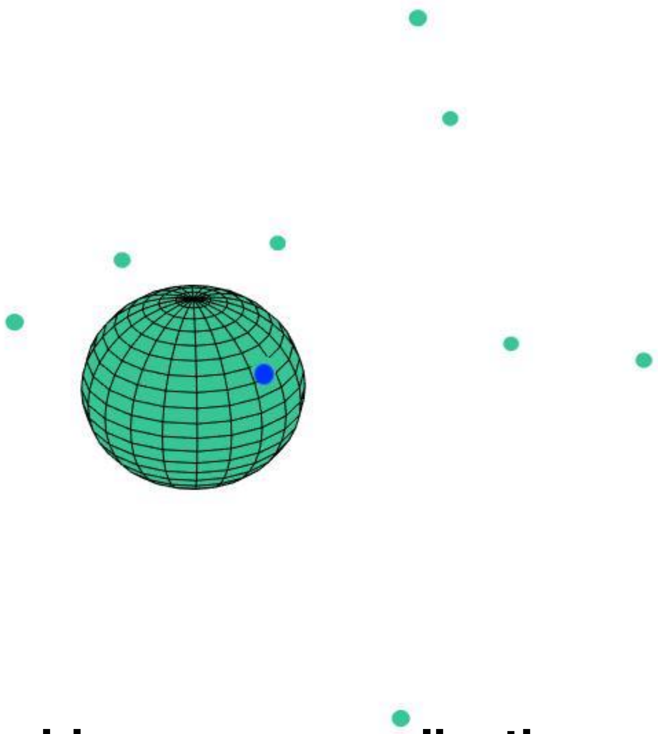
Another solution method with an unknown clock offset is to determine the time and distance differences between satellite pairs, which are independent of the offset and which define 2-sheet hyperboloids of revolution, then solve for the common intersection of the hyperboloids.

B. Fang, “Simple Solutions for Hyperbolic and Related Position Fixes,”
IEEE Transactions on Aerospace and Electronic Systems, vol. 26, no. 5, pp. 748–753, 1990
https://ieeexplore.ieee.org/abstract/document/102710

Improved GPS models







GPS solutions also have to consider many complications such as effects of General and Special Relativity, signal transmission through the Ionosphere, and variations in Earth's spin axis and satellite orbits.
Cell phones use additional info: last know location, cell & wifi tower locations, etc.

For a video history of GPS, see "The Lonely Halls Meeting" documentary at

https://www.imdb.com/title/tt7093186/videoplayer/vi1679932185

Old men (now) but see 20-something women and men system operators at 1:20:20

www.ReactorLab.net