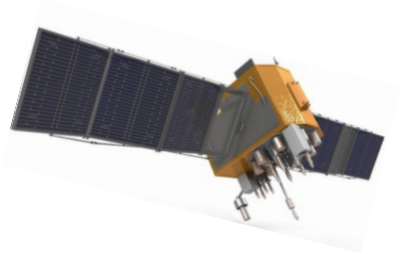
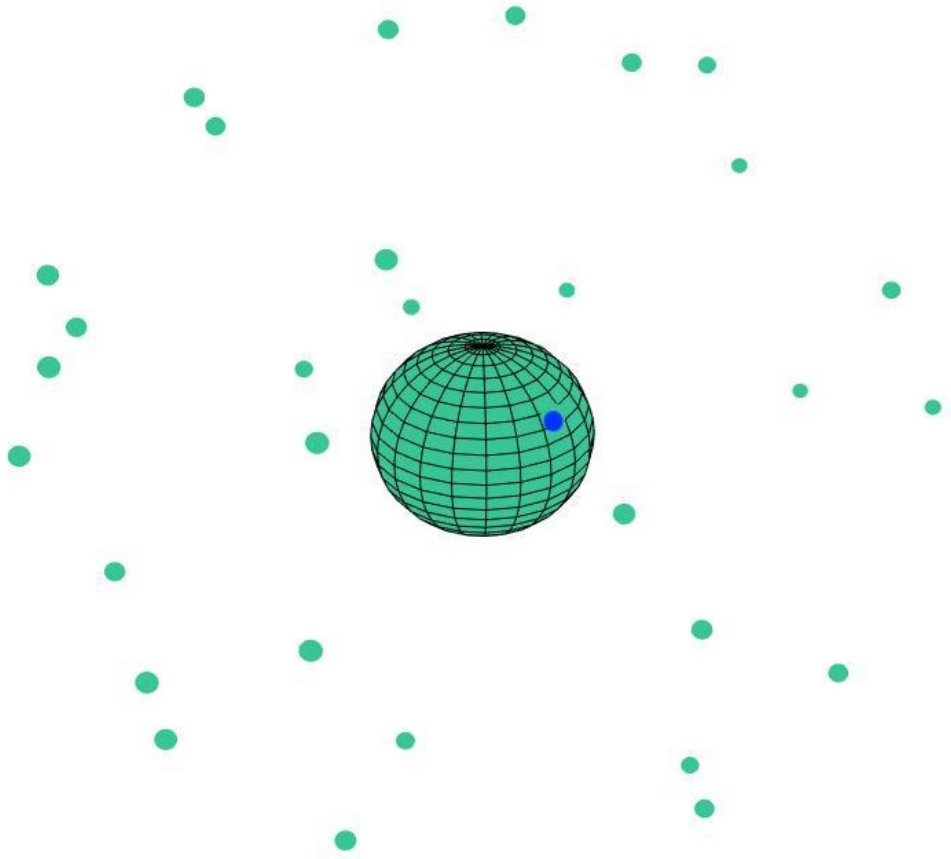
Simplified GPS
spherical Earth, receiver sync'd with satellite clocks





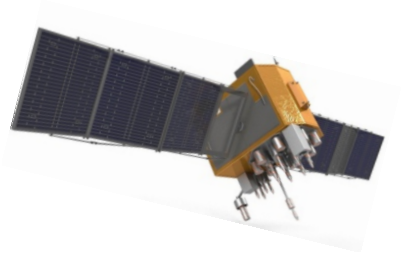
31 U.S. GPS satellites active at 1:30 pm, June 12, 2019

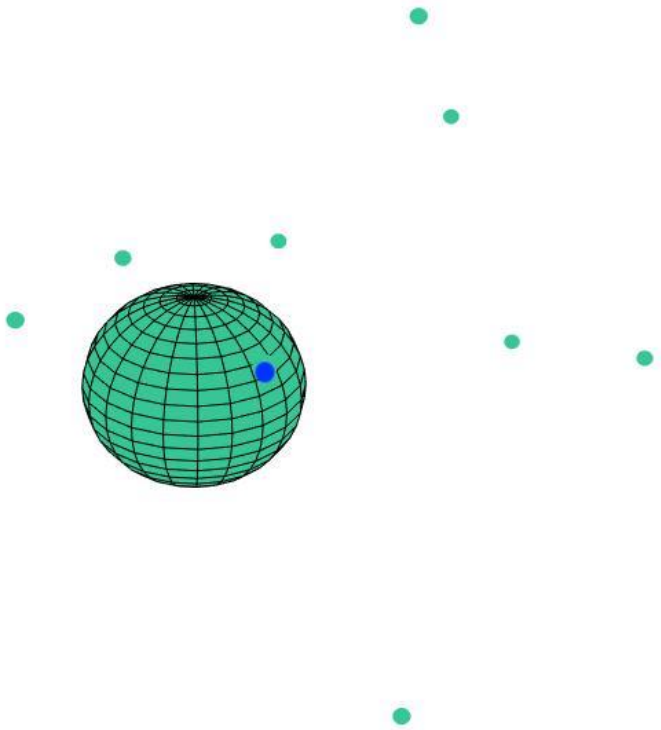
https://in-the-sky.org/satmap_worldmap.php

Russia, China and the EU also have GPS systems

satellite sizes are exaggerated

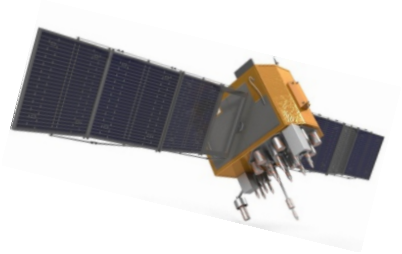
Simplified GPS

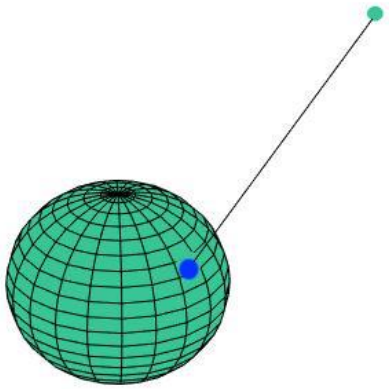




8 satellites are 10° or more above horizon at San Diego, CA
32.7° latitude, -117° longitude
GPS receiver at blue dot does NOT know it is there yet,
only that it's somewhere on Earth

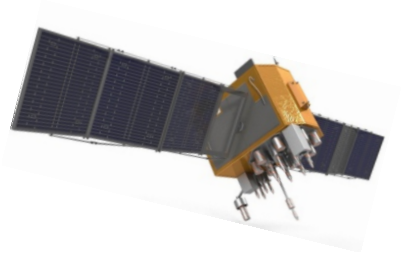
Simplified GPS

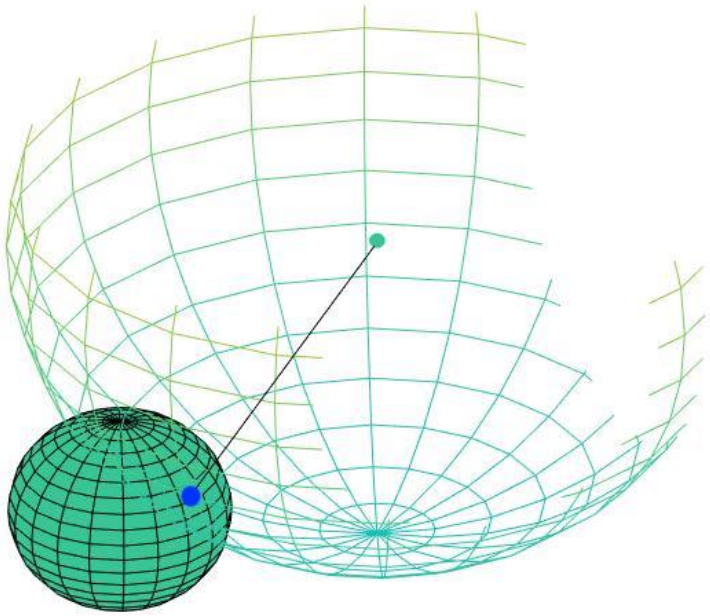




**Each satellite transmits signals with time sent and satellite location,
the time-sync'd GPS receiver computes distance to satellite
from the time difference between broadcast and reception,
using the speed of light**

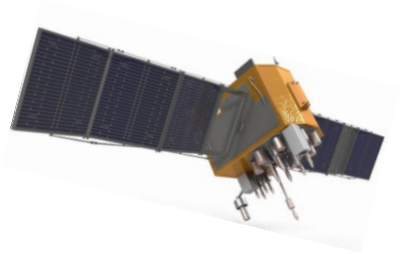
Simplified GPS

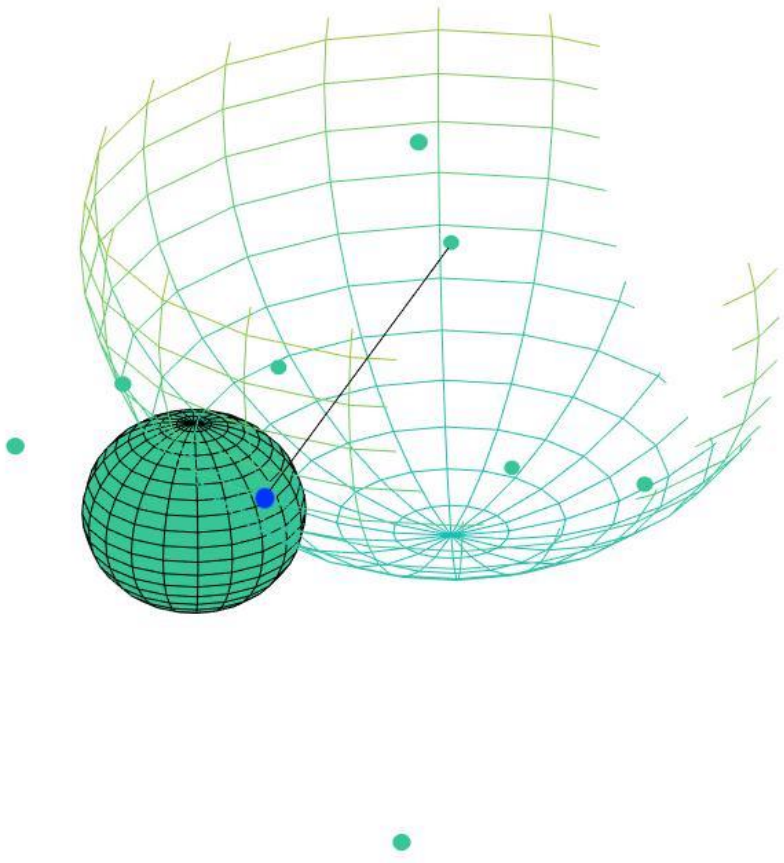




Distance to satellite gives equation for sphere of that radius around known satellite location - receiver only knows it is located somewhere on that sphere and on Earth's surface

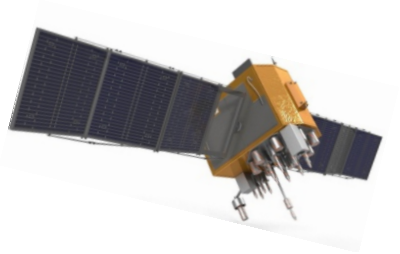
Simplified GPS

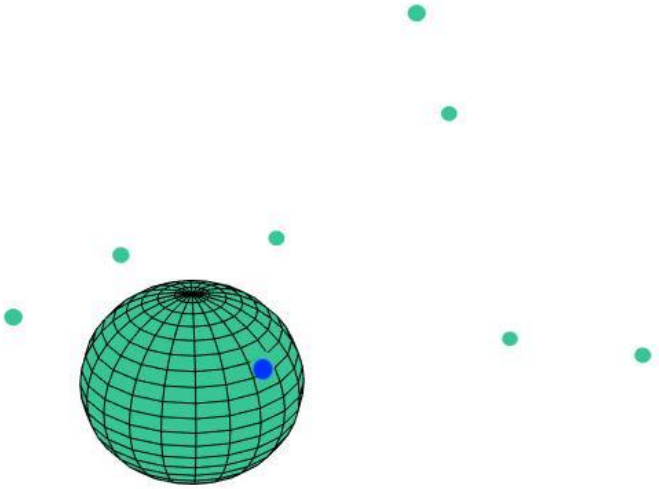




The GPS receiver then computes its location from the intersection of 3 or more satellite spheres with Earth's surface, where intersection of 2 spheres is a circle, 3 is 2 points, 4 is one point, a problem in Linear Algebra, solving multiple, coupled algebraic equations

Improved GPS





GPS receivers are not usually sync'd with the satellites, so receivers can only determine the time and distance difference between satellite pairs, which defines 2-sheet hyperboloids of revolution, then solve for intersection of hyperboloids & oblate spheroid Earth.

B. Fang, "Simple Solutions for Hyperbolic and Related Position Fixes," IEEE Transactions on Aerospace and Electronic Systems, vol. 26, no. 5, pp. 748–753, 1990
https://ieeexplore.ieee.org/abstract/document/102710

Cell phones use additional info: last know location, cell & wifi tower locations…

Simplified GPS in MATLAB

github.com/RichardHerz/GPS >> gps3D_spheres

```
% simplified GPS in MATLAB - receiver clock sync'd with satellites
re = 6370; % (km), spherical earth radius

% specify GPS receiver latitude, longitude and altitude (altitude == 0)
rec = [32.7,-117,0]; % San Diego, CA, USA is [32.7,-117,0]
[x,y,z] = fLatLongToXYZ(rec, re);
xyzRec = [x,y,z]; % xyz coordinates of receiver, earth center is origin

% specify >= 3 satellite latitude (deg), longitude (deg), altitude (km)
% 31 listed in file sat.txt taken 1:30 pm, June 12, 2019 from data at
% https://in-the-sky.org/satmap_worldmap.php
load sat.txt

% get xyz coordinates of satellites
[x, y, z] = fLatLongToXYZ(sat,re);
xyz = [x, y, z];

% get satellites above horizon and in view of receiver
degdel = 10; % min degree above horizon for sat in view
rView = fReturnSatViewRows(sat,xyz,xyzRec,re,degdel);
xyz = xyz(rView,:);
r = fDistance(xyz,xyzRec); % sats to receiver

% END SETUP

% GIVEN:
% radius of spherical earth, re
% lat, long and altitude of >= 3 satellites
% distance of each satellite from receiver

% FIND:
% lat and long of receiver on earth's surface

% matrix eqn for sphere intersects is A * xyzCalc = c
A = xyz; % xyz of satellites
c = fCcoef(xyz,r,re);

% xyzCalc = inv(A) * c; % only for A and c rows == 3
xyzCalc = A \ c; % for A and c rows >= 3

% compute receiver lat and long
[latCalc, longCalc, altCalc] = fXYZtoLatLong(xyzCalc', re);

fprintf('location:   lat, long, alt, %6.3f, %6.3f, %4.3e \n', rec)
fprintf('calculated: lat, long, alt, %6.3f, %6.3f, %4.3e \n', ...
    latCalc, longCalc, altCalc)
```

2 key functions

```
function rowView = fReturnSatViewRows(sat,xyz,xyzRec,re,degdel)
    % returns row numbers of satellites >= degdel above horizon

    dRec = fDistance(xyz,xyzRec); % distances from sats to receiver
    dOrig = re + sat(:,3); % distances from sats to earth center

    % we know 3 sides of triangle between sat, rec, earth center
    % use law of cosines to find the angle we want
    num = re^2 + dRec.^2 - dOrig.^2;
    denom = 2 * re * dRec;
    gamma = -90 + acosd(num ./ denom);

    % find and return satellite row numbers where gamma >= degdel
    rowView = find(gamma >= degdel);
```

```
function c = fCcoef(xyz,r,re)
    % input xyz are locations of satellites (each row is satellite)
    % input r are distances from satellites to receiver
    % input re is radius of spherical earth
    % returns vector of coefficients for matrix solution
    % option 2 for sum( ,2) sums each row

    c = ( (re^2 + sum(xyz.^2, 2) - r.^2) / 2 );
```

```
>> gps3
location:   lat, long, alt, 32.700, -117.000, 0.000e+00
calculated: lat, long, alt, 32.700, -117.000, -9.095e-13
```