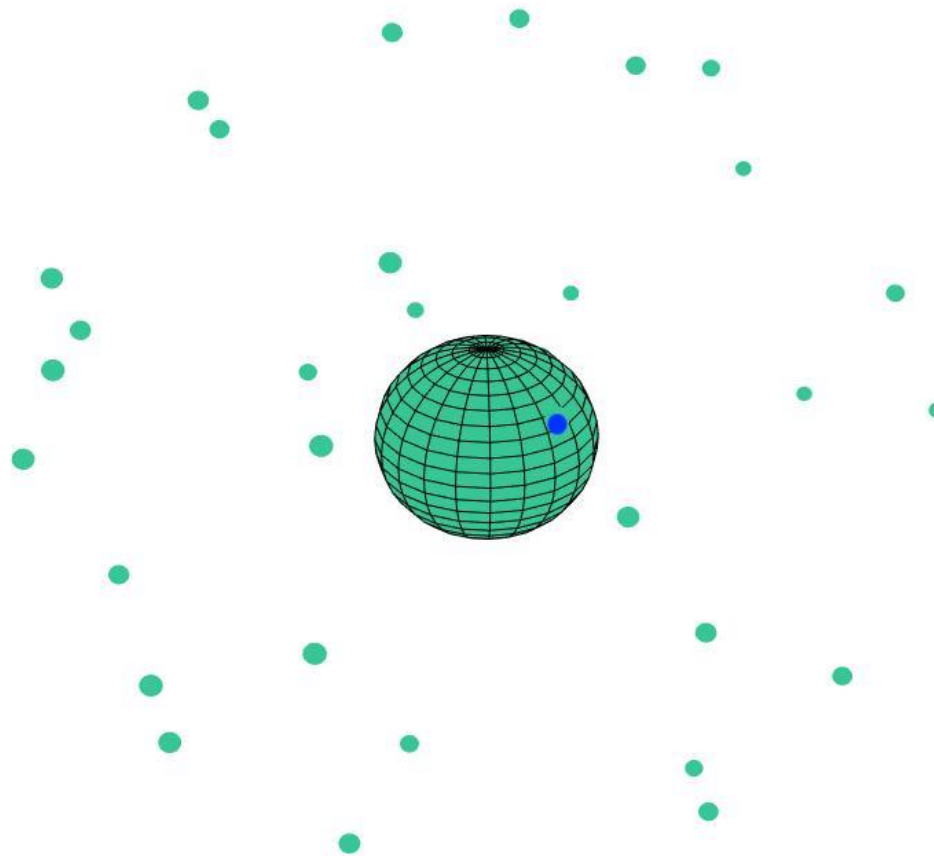
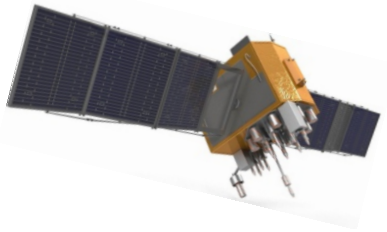


# Start with a simplified GPS model spherical Earth, receiver sync'd with satellite clocks



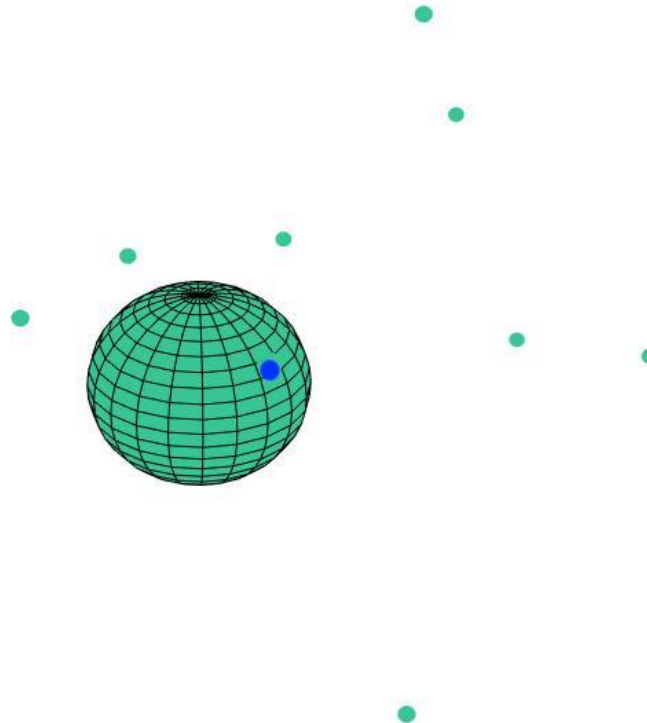
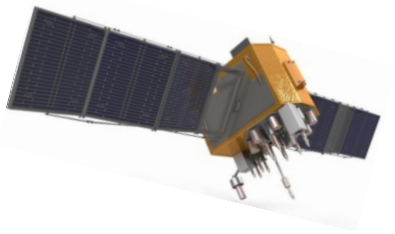
**31 U.S. GPS satellites active at 1:30 pm, June 12, 2019**

[https://in-the-sky.org/satmap\\_worldmap.php](https://in-the-sky.org/satmap_worldmap.php)

**Russia, China and the EU also have systems**

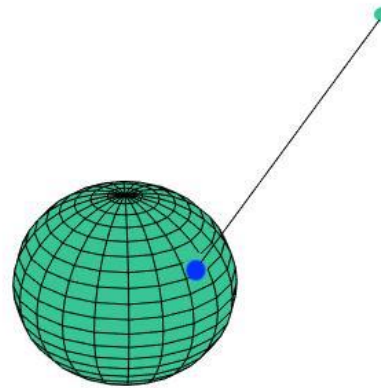
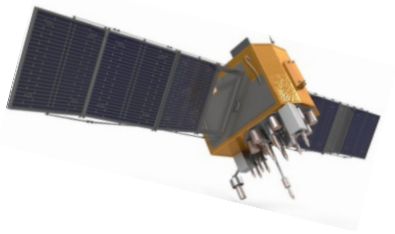
satellite sizes are exaggerated, blue dot is arbitrary Earth receiver location

# Simplified GPS model



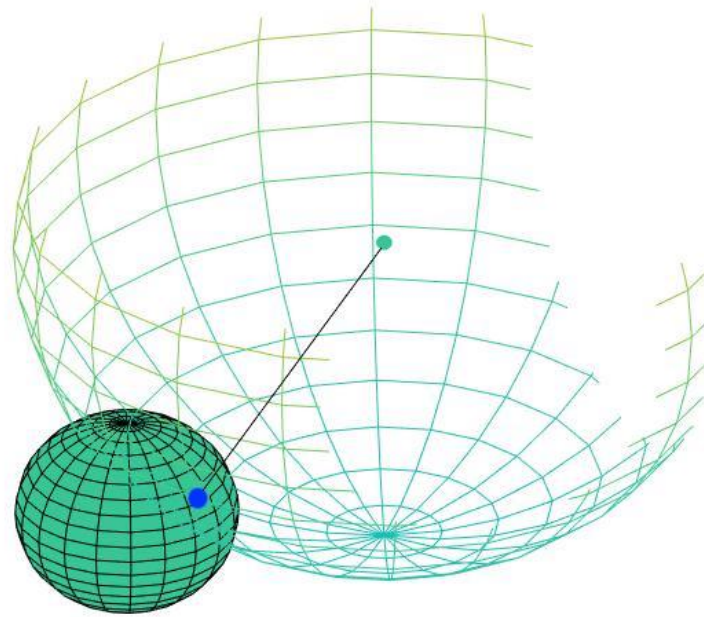
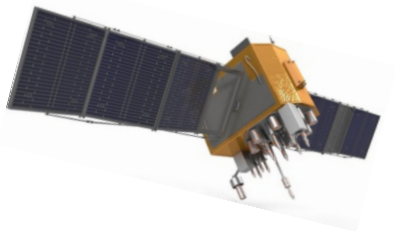
**8 satellites are  $10^\circ$  or more above horizon at San Diego, CA  
 $32.7^\circ$  latitude,  $-117^\circ$  longitude  
GPS receiver at blue dot does NOT know it is there yet,  
only that it's somewhere on Earth**

## Simplified GPS model



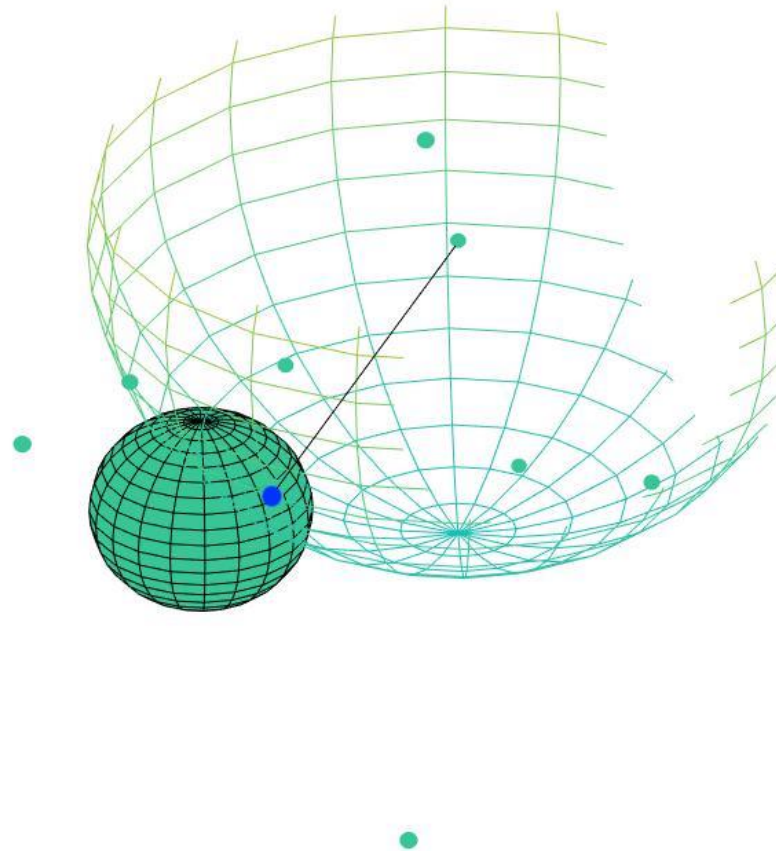
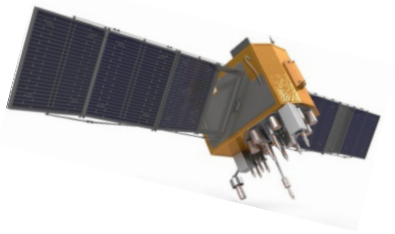
**Each satellite transmits signals with time sent and satellite location, the time-sync'd GPS receiver computes distance to satellite from the time difference between broadcast and reception, using the speed of light**

# Simplified GPS model



**Distance to satellite gives equation for sphere of that radius around known satellite location - receiver only knows it is located somewhere on that sphere and on Earth's surface**

## Simplified GPS model



**The GPS receiver then computes its location from the intersection of 3 or more satellite spheres with Earth's spherical surface (a 4th sphere), where the intersection of 2 spheres is a circle, 3 is 2 points, 4 is one point, a problem in "linear algebra," solving multiple, coupled algebraic equations**

# Simplified GPS model in MATLAB

[github.com/RichardHerz/GPS](https://github.com/RichardHerz/GPS) >> `gps3D_spheres`

```
% simplified GPS in MATLAB - receiver clock sync'd with satellites
re = 6370; % (km), spherical earth radius

% specify GPS receiver latitude, longitude and altitude (altitude == 0)
rec = [32.7,-117,0]; % San Diego, CA, USA is [32.7,-117,0]
[x,y,z] = fLatLongToXYZ(rec, re);
xyzRec = [x,y,z]; % xyz coordinates of receiver, earth center is origin

% specify >= 3 satellite latitude (deg), longitude (deg), altitude (km)
% 31 listed in file sat.txt taken 1:30 pm, June 12, 2019 from data at
% https://in-the-sky.org/satmap_worldmap.php
load sat.txt
```

```
% get xyz coordinates of satellites
[x, y, z] = fLatLongToXYZ(sat,re);
xyz = [x, y, z];

% get satellites above horizon and in view of receiver
degdel = 10; % min degree above horizon for sat in view
rView = fReturnSatViewRows(sat,xyz,xyzRec,re,degdel);
xyz = xyz(rView,:);
r = fDistance(xyz,xyzRec); % sats to receiver
```

```
% END SETUP
```

```
% GIVEN:
% radius of spherical earth, re
% lat, long and altitude of >= 3 satellites
% distance of each satellite from receiver
```

```
% FIND:
% lat and long of receiver on earth's surface
```

```
% matrix eqn for sphere intersects is  $A * xyzCalc = c$ 
A = xyz; % xyz of satellites
c = fCcoef(xyz,r,re);
```

```
%  $xyzCalc = inv(A) * c$ ; % only for A and c rows == 3
xyzCalc = A \ c; % for A and c rows >= 3
```

```
% compute receiver lat and long
[latCalc, longCalc, altCalc] = fXYZtoLatLong(xyzCalc', re);
```

```
fprintf('location:   lat, long, alt, %6.3f, %6.3f, %4.3e \n', rec)
fprintf('calculated: lat, long, alt, %6.3f, %6.3f, %4.3e \n', ...
    latCalc, longCalc, altCalc)
```

2 key functions

```
function rowView = fReturnSatViewRows(sat,xyz,xyzRec,re,degdel)
% returns row numbers of satellites >= degdel above horizon

dRec = fDistance(xyz,xyzRec); % distances from sats to receiver
dOrig = re + sat(:,3); % distances from sats to earth center

% we know 3 sides of triangle between sat, rec, earth center
% use law of cosines to find the angle we want
num = re^2 + dRec.^2 - dOrig.^2;
denom = 2 * re * dRec;
gamma = -90 + acosd(num ./ denom);

% find and return satellite row numbers where gamma >= degdel
rowView = find(gamma >= degdel);
```

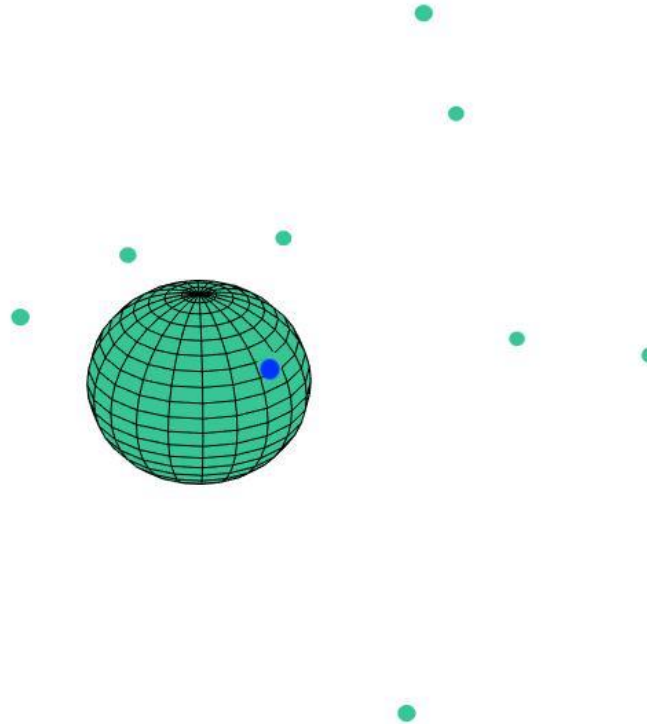
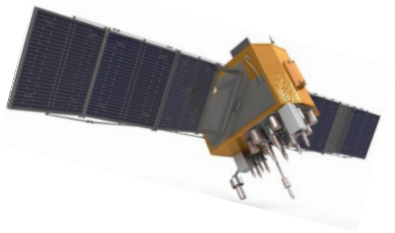
```
function c = fCcoef(xyz,r,re)
% input xyz are locations of satellites (each row is satellite)
% input r are distances from satellites to receiver
% input re is radius of spherical earth
% returns vector of coefficients for matrix solution
% option 2 for sum(,2) sums each row

c = ( (re^2 + sum(xyz.^2, 2) - r.^2) / 2 );
```

```
>> gps3
```

```
location:   lat, long, alt, 32.700, -117.000, 0.000e+00
calculated: lat, long, alt, 32.700, -117.000, -9.095e-13
```

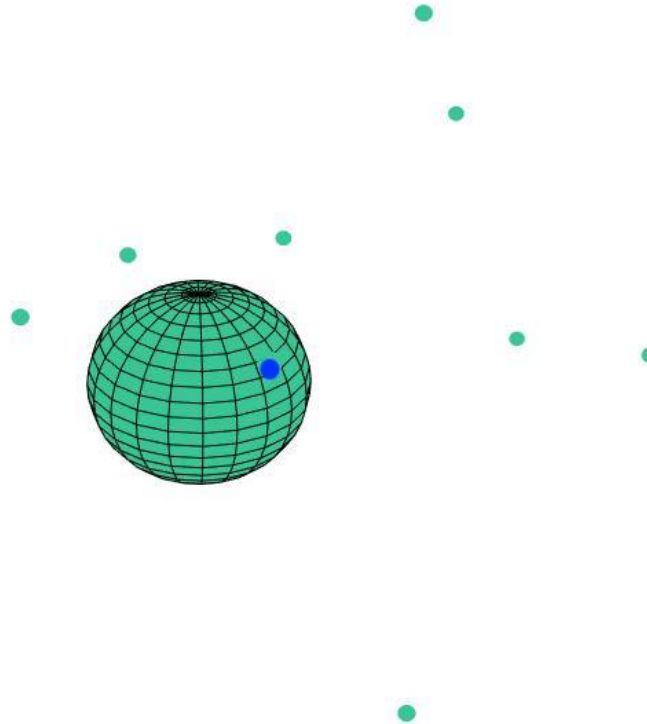
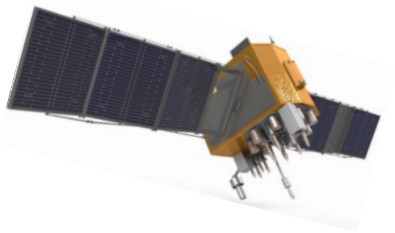
# Improved GPS models



**GPS receivers are not usually sync'd with the satellites  
such that there is an unknown offset in the distance determinations.  
In a model with the receiver on the surface of a spherical Earth, a unique  
linear algebra solution can be obtained with 4 satellites  
(assuming offset  $\ll$  distance to satellite)**

[github.com/RichardHerz/GPS](https://github.com/RichardHerz/GPS) >> [gps4\\_spheres\\_offset](#)

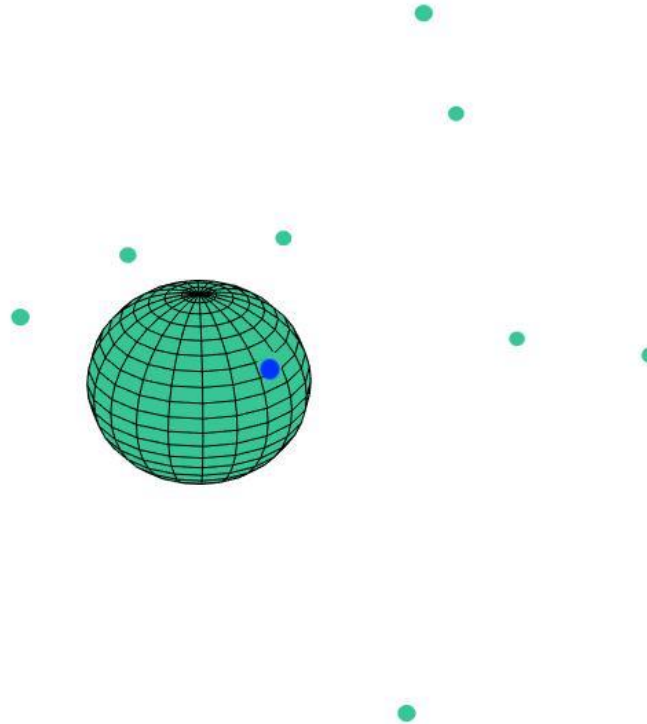
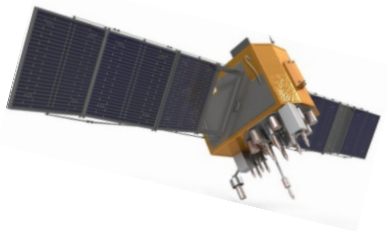
# Improved GPS models



**With an unknown clock offset and a receiver at an unknown altitude on or above a nonspherical Earth, a solution can be obtained with 4 or more satellite distance spheres using a nonlinear solution.**



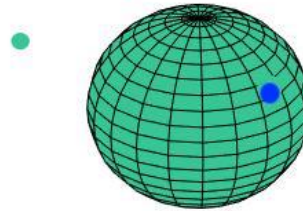
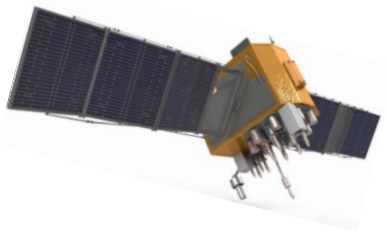
# Improved GPS models



**Another solution method with an unknown clock offset is to determine the time and distance differences between satellite pairs, which are independent of the offset and which define 2-sheet hyperboloids of revolution, then solve for the common intersection of the hyperboloids.**

B. Fang, "Simple Solutions for Hyperbolic and Related Position Fixes,"  
IEEE Transactions on Aerospace and Electronic Systems, vol. 26, no. 5, pp. 748–753, 1990  
<https://ieeexplore.ieee.org/abstract/document/102710>

# Improved GPS models



**GPS solutions also have to consider many complications such as effects of General and Special Relativity, signal transmission through the Ionosphere, and variations in Earth's spin axis and satellite orbits.**

**Cell phones use additional info: last known location, cell & wifi tower locations, etc.**

**For a video history of GPS, see “The Lonely Halls Meeting” documentary at**

<https://www.imdb.com/title/tt7093186/videoplayer/vi1679932185>

**Old men (now) but see current 20-something women and men operators at 1:20:20**

[www.ReactorLab.net](http://www.ReactorLab.net)