

Neural Networks in Artificial Intelligence

Artificial intelligence is the process of using computers to analyze complex information in order to assist with making decisions.

"Is that really you looking at me?"



There are many approaches to artificial intelligence.

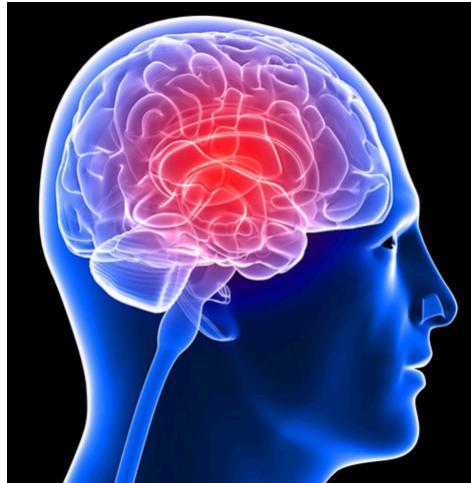
The approach we introduce here is the use of [artificial neural networks](#).

There are software tools available which allow people to apply neural networks to solve problems without having to understand what is going on inside the tools.

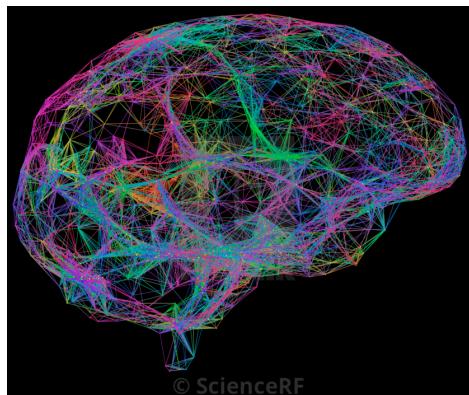
Our purpose is to explain the computational aspects behind simple artificial neural networks.

The goal is an understanding of the basics which underly the software tools.

MATLAB code used here is available at <https://github.com/RichardHerz/neural-networks>

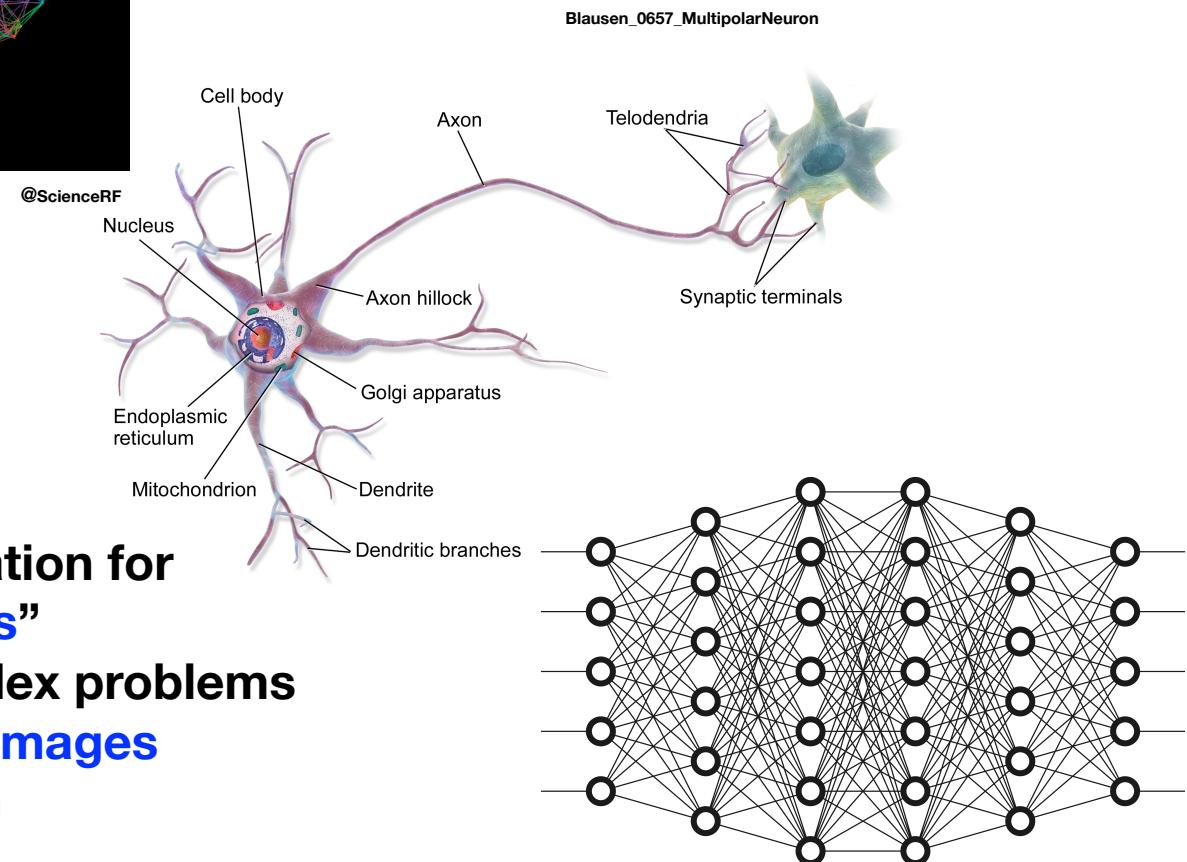


www.braininjuryaustralia.org.au



**These networks are the inspiration for
“artificial neural networks”
which can be trained to solve complex problems
such as object recognition in images
and speech recognition**

**Our brains sense and think using connected networks
of cells called neurons**



neural_network_shutterstock_all_is_magic.jpg

github.com/RichardHerz

Neural Network

Simulates XOR logic - exclusive or
Output is TRUE when one input is TRUE but not both

**2 inputs, 1 output,
1 hidden layer
with 3 neurons &
9 synapses**

We ignore biases here
for simplicity but include
them in the next model.
An XOR network with
biases only needs 2
hidden nodes.

EXAMPLE for input of
1
0

node value = sigmaFunc(sum of (connection weight * node activation))

where $\text{sigmaFunc}(x) = \exp(x) / (1 + \exp(x))$ >> converts all input x values into range 0 to 1

INPUT > HIDDEN LAYER

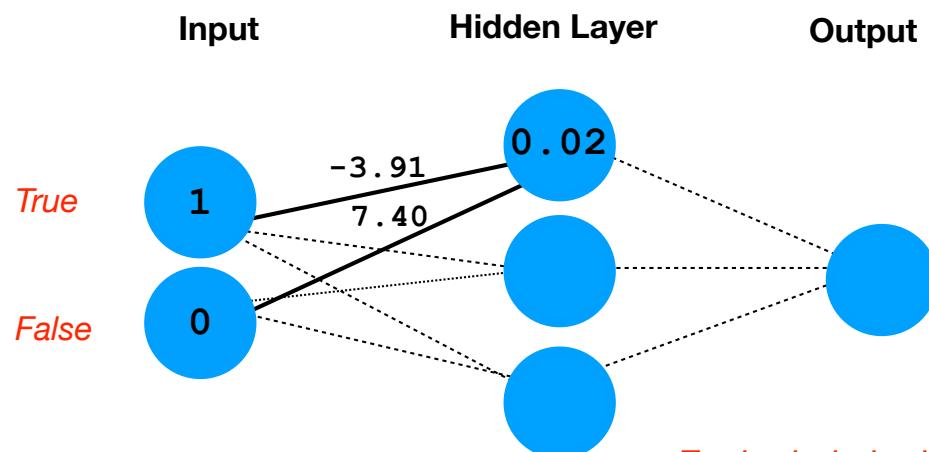
$\text{sigmaFunc}((-3.91 * 1) + (7.40 * 0)) = 0.02 = \text{hidden node 1 activation}$

*Every node - neuron - has a connection - synapse - to every neuron in nearest-neighbor layers
of neurons in this basic type of neural network.*

*The values are held in memory locations and the CPU executes the math - there are no
physical, hardware neurons and synapses.*

A simple example

*This logic can be computed in a single IF statement in
a procedural program but is useful here to start
learning about neural networks*



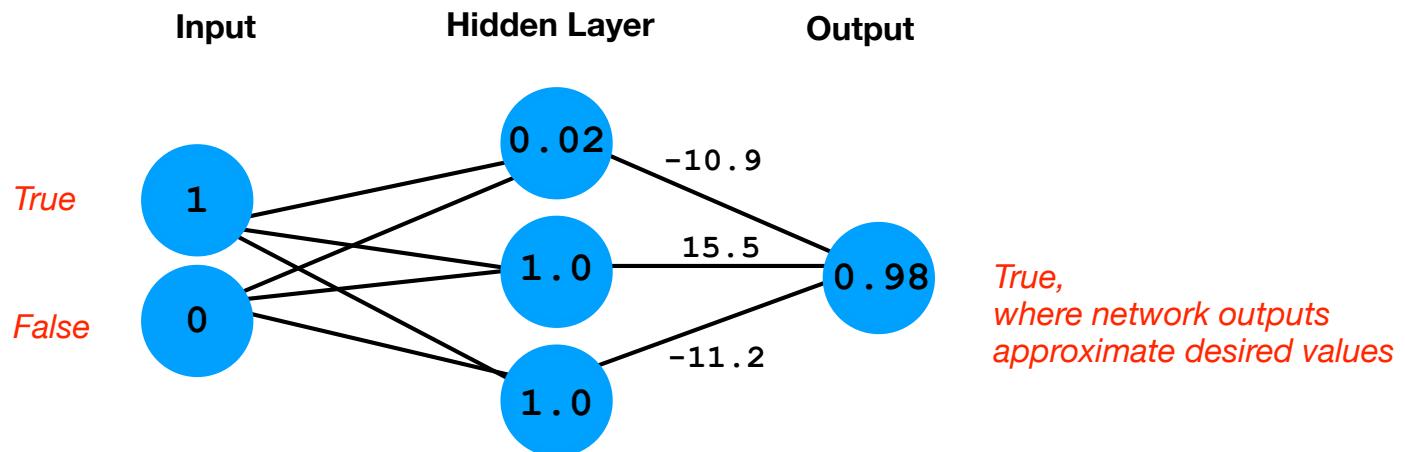
Each circle in the diagram represents a node or “neuron.” Each line represents a connection or “synapse.” The value in a neuron is its “activation.” Each synapse has a connection “weight.”

Neural Network

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HIDDEN LAYER > OUTPUT

$$\text{sigmaFunc}((-10.9 * 0.02) + (15.5 * 1.0) + (-11.2 * 1.0)) = 0.98 = \text{output node}$$

The MATLAB code to solve for the output remains the same as that below, regardless of the size of the network:

```
for i = 2 : numHiddenLayers + 2
    a{i} = sigmaFunc( W{i-1} * a{i-1} );
end
```

W is a MATLAB cell array whose elements are the matrices of synapse weights for each layer; **a** is a cell array whose elements are the vectors of neuron activation values. Each set of **W** and **a** are matrix-multiplied to obtain the neuron activation values for the next layer in the series of neuron layers.

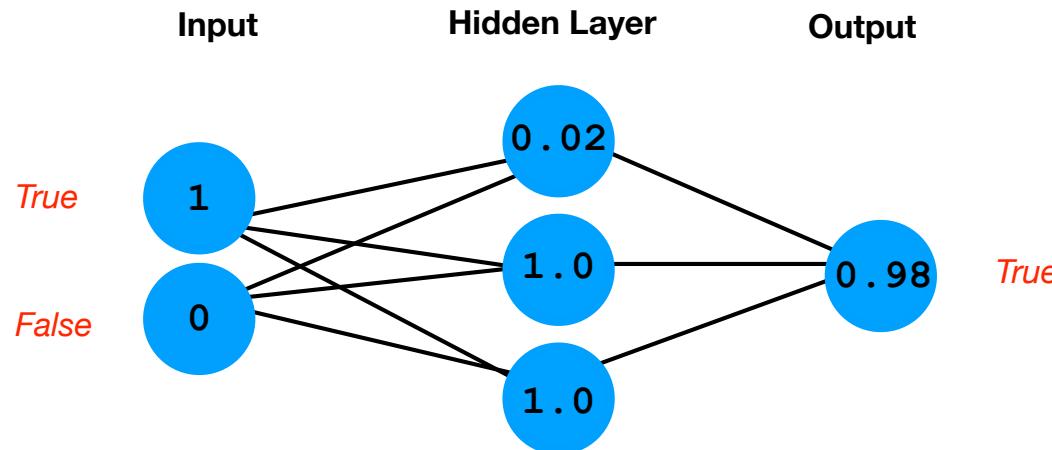
Matrix multiplication is well suited to being accelerated in hardware Graphical Processing Units, since graphic transformations also involve matrix multiplication.

Neural Network

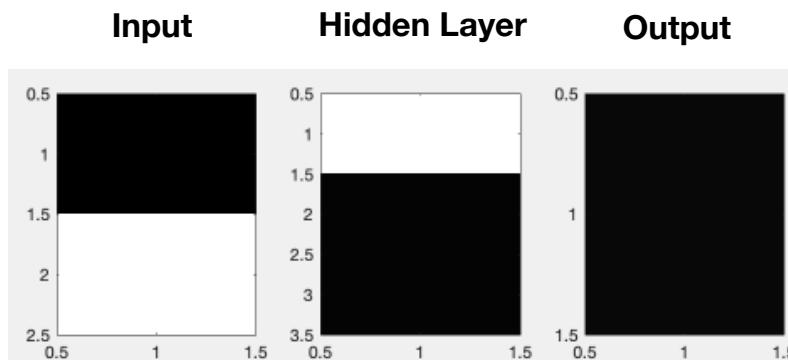
Simulates XOR logic - exclusive or

Output is TRUE when one input is TRUE but not both

**2 inputs, 1 output,
1 hidden layer
with 3 neurons &
9 synapses**



Visualization of neuron values - “activations” - for this input



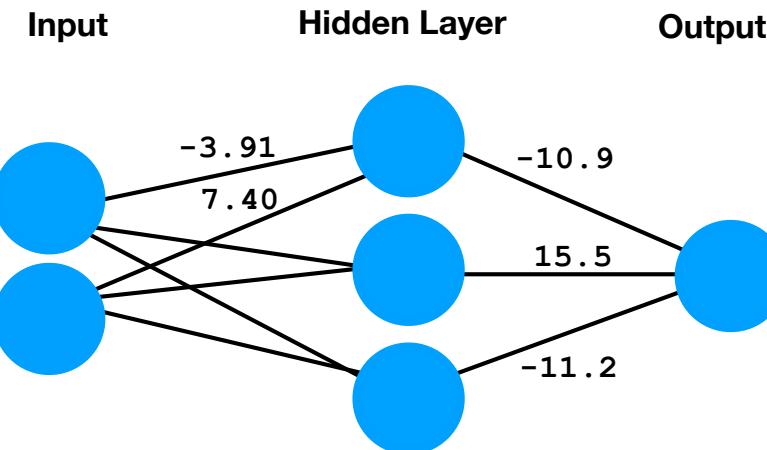
Input	Hidden Layer	Output
1	0.02	0.98
0	1.00	True
	1.00	

Neural Network

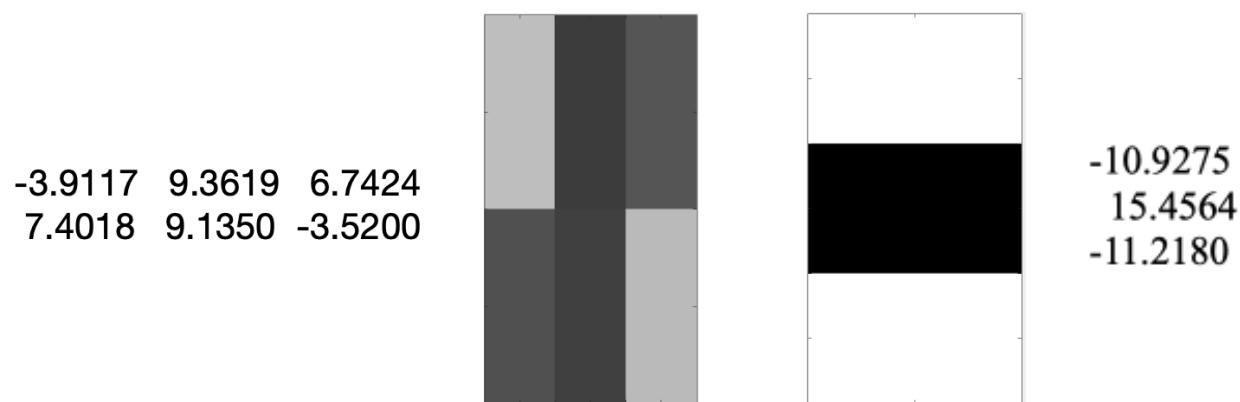
Simulates XOR logic - exclusive or

Output is TRUE when one input is TRUE but not both

**2 inputs, 1 output,
1 hidden layer
with 3 neurons &
9 synapses**

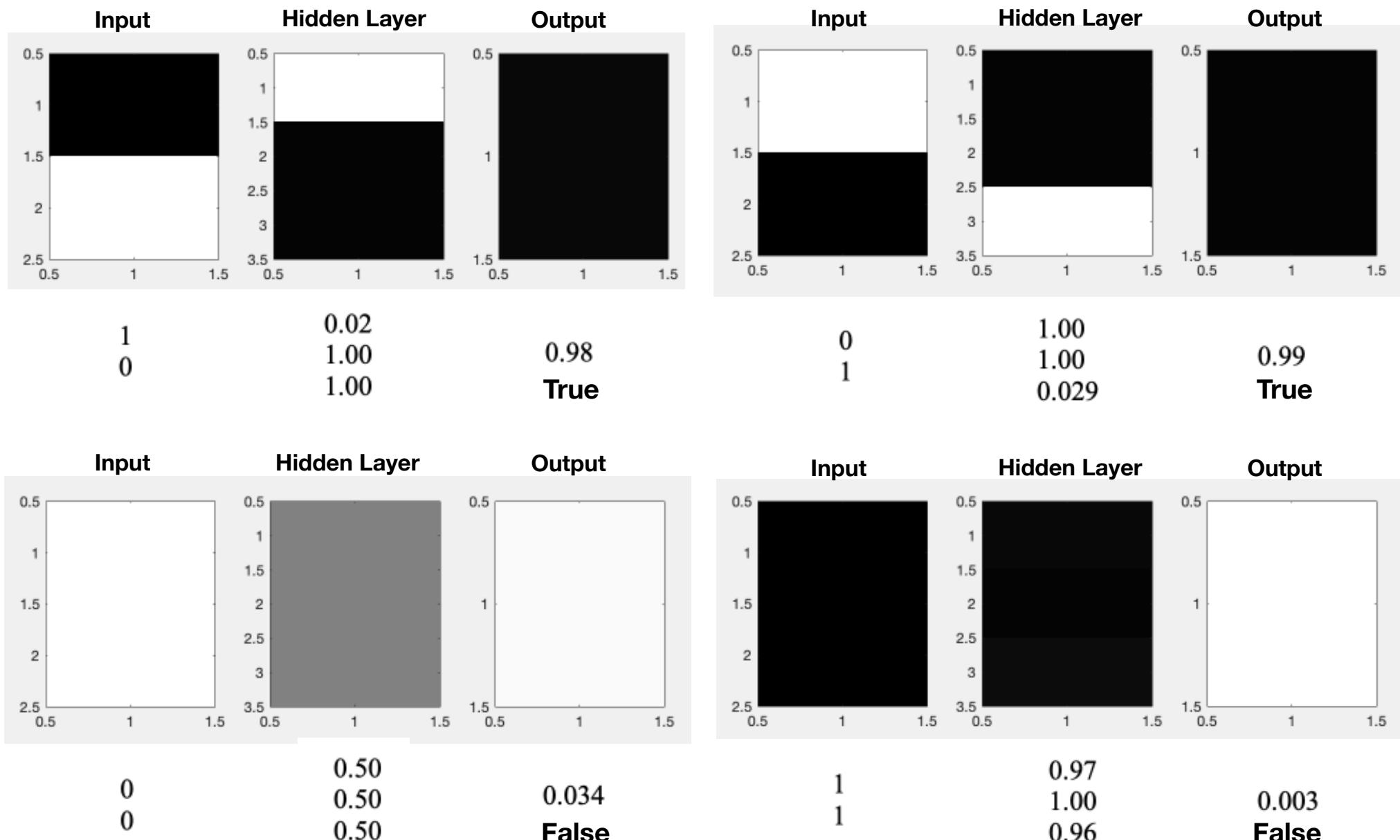


**Visualization of synapse connection “weights” to hidden layer and to output
min = -11.2 (white), max = +15.5 (black)**



*The synapse connection weights were determined when the network was “trained”
using combinations of known inputs and outputs.
Training is discussed later in these notes.*

Visualizations of node activations: input > hidden layer > output



Output is TRUE when one input is TRUE but not both

The first paper on back-propagation network training also had illustrations of network weights and activations

Rumelhart, D., Hinton, G. & Williams, R. Learning representations by back-propagating errors.

Nature 323, 533–536 (1986).

<https://doi.org/10.1038/323533a0>

They showed how the network was trained to determine the name of a relative in a family tree given the name of the first person and the relationship of the relative to the first person

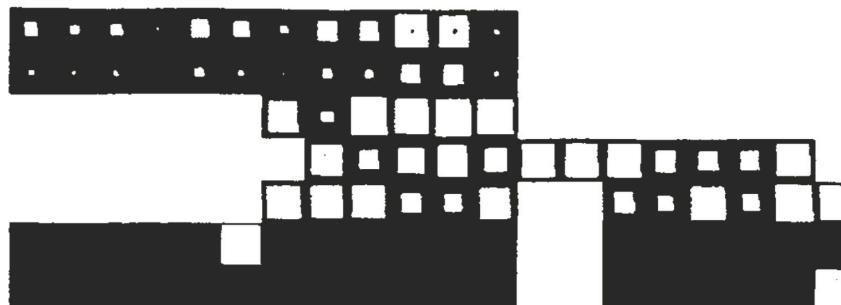


Fig. 3 Activity levels in a five-layer network after it has learned.

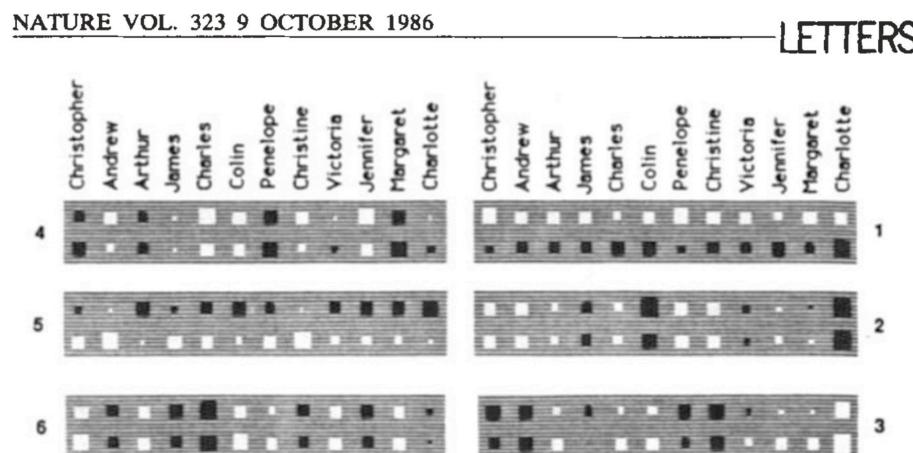
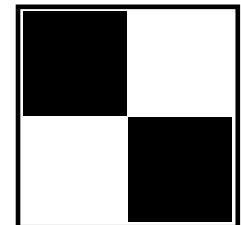


Fig. 4 The weights from the 24 input units that represent people to the 6 units in the second layer that learn distributed representations of people. White rectangles, excitatory weights; black rectangles, inhibitory weights; area of the rectangle encodes the magnitude of the weight.

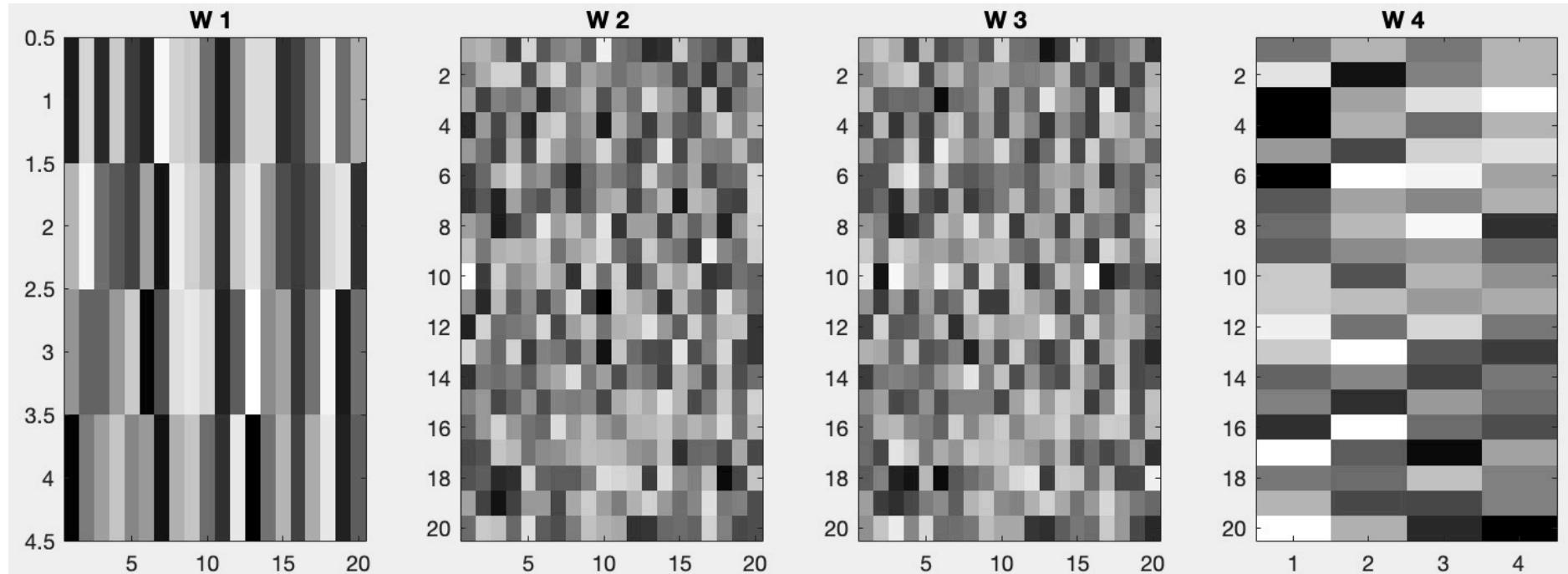
Neural Network

**4 inputs, 4 outputs
3 hidden layers, each
with 20 neurons =
60 neurons &
960 synapses,
plus 64 bias values**

**A more complex network which
detects diagonal, horizontal and vertical
inputs to a 2×2 “touch screen”**



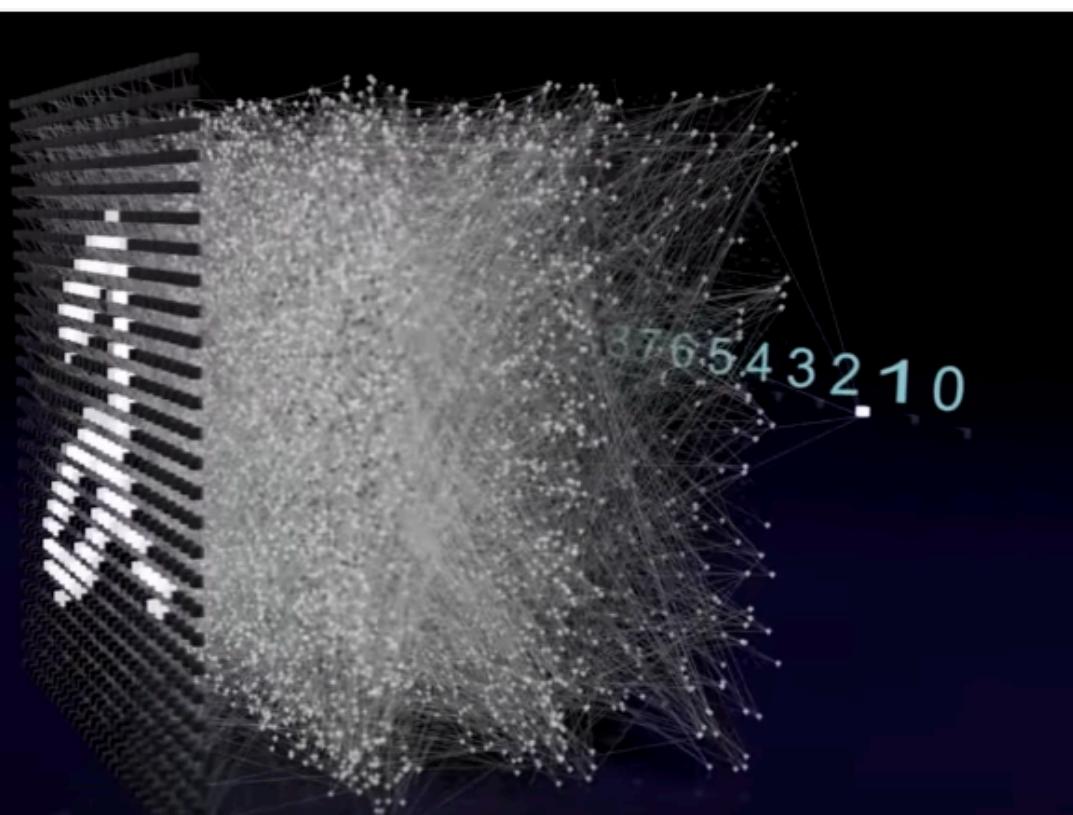
**Visualization of synapse weights to hidden
layers 1-3 and to output,
min = -2.27 (white), max = +2.42 (black)**



A bias value is added at each of the hidden and output nodes before the sigma function is applied at each node in this simulation. Prior to training the network with known input and output cases, the connection weights and biases were assigned random values in the range -1 to +1. Then the values were adjusted during training in order to match input cases with their corresponding outputs. The resulting values are not random. Different sets of values will be obtained with different random initializations

A neural network for a 28 x 28 “touch screen” that identifies handwritten numerals 0-9

Type: ML Perceptron
Data Set: MNIST
Hidden Layers: 3
Hidden Neurons: 10000
Synapses: 24864180
Synapses shown: 2%
Learning: BP



Denis Dmitriev <https://youtu.be/3JQ3hYko51Y>

This network was trained with a set of known inputs and outputs that is much smaller than all the possible ways in which numbers can be drawn. The power of a trained network is to be able to give correct outputs for inputs for which the network wasn't trained. That's the main purpose of neural networks!

A neural network represents a large number of coupled equations which, when given a set of input values, can produce a set of desired output values.

The more neurons and synapses - the more equations - and the greater complexity of inputs and outputs which can be "fit" by the system of equations. Note the significant increase in complexity going from the XOR example (9 synapse weights) to the 2 x 2 "touch screen" (960) to the 28 x 28 touch screen in the figure above (24.9 million) to Open AI's GPT-3 (175 billion)!

A neural network might be thought of as a general function which can fit anything given enough terms...

In a sense, neural networks are math functions which can “fit” any desired input and output data given enough adjustable parameters, which are the “synapse” connection weights and, thus, enough neurons.

Using a neural network is somewhat similar to using a polynomial function to fit a series of data points (empirical fit) vs. using a functional form that represents the underlying physics (theoretical fit).

In a neural network, the functional form is fixed by the network structure. The values of the constants in the function are the connection weights, whose values are determined during training.

For the XOR network above, this is the Matlab code which computes the output $a\{3\}$ given the input $a\{1\}$

```
for i = 2:3
    a{i} = sigmaFunc( W{i-1}*a{i-1} );
end
```

Matrix $W\{i-1\}$ and vector $a\{i-1\}$ are elements of the cell arrays W and a . They are matrix multiplied. The Matlab code is very compact. We can see the form of this network’s function by looking at the expanded equation, which shows the individual terms. The output $a\{3\}$ is a function of the inputs $a\{1\}$:

$$\begin{aligned} a^{\{3\}} = f(a^{\{1\}}) &= \sigma \left(W_1^{\{2\}} a_1^{\{2\}} + W_2^{\{2\}} a_2^{\{2\}} + W_3^{\{2\}} a_3^{\{2\}} \right) \\ &= \sigma \left(W_1^{\{2\}} \sigma \left(W_{1,1}^{\{1\}} a_1^{\{1\}} + W_{1,2}^{\{1\}} a_2^{\{1\}} \right) + W_2^{\{2\}} \sigma \left(W_{2,1}^{\{1\}} a_1^{\{1\}} + W_{2,2}^{\{1\}} a_2^{\{1\}} \right) + W_3^{\{2\}} \sigma \left(W_{3,1}^{\{1\}} a_1^{\{1\}} + W_{3,2}^{\{1\}} a_2^{\{1\}} \right) \right) \end{aligned}$$

where, for more compact notation, the superscript $\{n\}$ of cell arrays a and W denotes a matrix in cell array element n , and the subscripts are the indices within that matrix. The hidden layer activations are $a\{2\}$. The nonlinear activation function for this network, which constrains activation values between 0 and 1, is

$$\sigma(x) = \frac{e^x}{1 + e^x}$$

For a larger neural network of this type, there are more terms but the functional form remains unchanged. With the continued development of computers, larger networks can be computed more rapidly.

Training Artificial Neural Networks

The networks shown earlier in these notes had already been "trained" to produce correct outputs given various inputs. The connection weights are fixed in the initial training stage. This is the hard part of neural networks!

First, a collection of paired inputs and their correct outputs is obtained to use in training the network.

Initial weights are set to random values. Then an input is fed to the network and an output is obtained. This output is compared to the correct output and an error value is computed. The initial error value will be large.

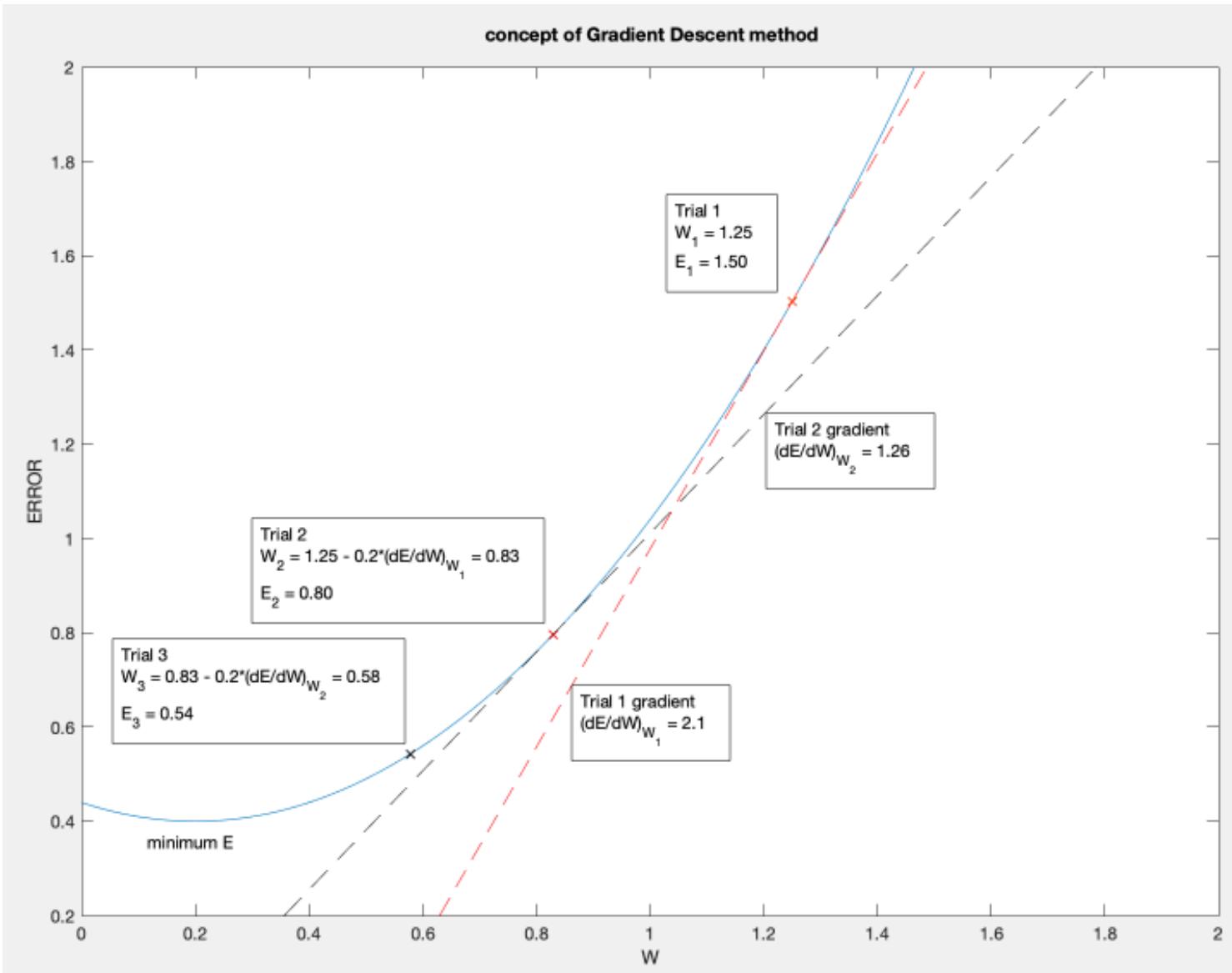
Next, for each connection weight in the network, the rate of change of the error with respect to a change in the value of that weight is computed. After the rates of change for all the weights are obtained, the weights are changed by multiplying a small constant times each of the rates of change, and then subtracting the result from corresponding weights. This small adjustment to the weights, when the input is again fed to the network, will result in a smaller error in the output.

This procedure is called the "gradient descent method." The rates of change are also called gradients. You wish to "descend" to smaller errors.

This gradient descent procedure is repeated until the error reaches a minimum value. The concept is illustrated in the figure on the next slide.

Gradient Descent Method

The blue line represents how the output error E changes with this connection weight W. In Trial 1, the value of W is randomly specified. Then E and the gradient (dE/dW) is computed. The value of W in Trial 2 is computed from the value in Trial 1 minus a factor (0.2 here) times the gradient from Trial 1. This results in a smaller error. The process repeats until E approaches the minimum value.



Training Artificial Neural Networks

After training of a network that is properly structured for the problem, the network will give approximately correct results, even for inputs that are not in the training set.

In fact, that is the whole point of neural networks: giving correct outputs for inputs that are not in the training set!

Note that the simple XOR and 2x2 touch screen networks had all possible inputs used in training, whereas the 28x28 touch screen was trained with a finite set of the extraordinarily large possible combinations of pixel activations that are possible.

The 28x28 touch screen network shown in a previous slide was trained with the MNIST data set <http://yann.lecun.com/exdb/mnist/>. Other sets of data are available on the web for other types of problems such as image and speech analysis.

The gradient descent method is applied to all pairs of inputs and outputs in the training set, either individually or in batches.

Next, we will examine how the gradient values in the gradient descent method are obtained.

Training Artificial Neural Networks

In the simple network structure we are considering, information signals move in one direction: from the input to the output. A change in one connection weight near the input causes a change in the signal that propagates through the rest of the network and eventually results in a change in the final output error.

The way that this change in signal propagates through the network is determined by the connection weights and nodes through which it passes. Those weights and nodes are known to us.

One way to determine the gradients would be to make a change in each connection weight separately, then recompute the outputs and obtain the change in error. Then repeat for each connection weight in the network. This would be a lot of computational work!

A more efficient way to compute the same gradient values is to work backwards from the output to each preceding connection weight in a procedure called "back propagation." This is possible because of the straightforward structure of the network.

This method proceeds from the output layer to the last hidden layer, then back to each preceding pair of hidden layers, computing the gradients of error with change in connection weight at each step by simple analytical differentiation. This process is more efficient because the entire network from input to output doesn't have to be computed for each connection weight.

After all the gradients are computed, then all the weights are updated. Then the input is fed to the modified network and a new output and error is computed.

The gradient descent process is repeated and continues until only small changes in output error are obtained.

Back Propagation and Gradient Descent in Network Training

Below, instead of standard math notation, we use MATLAB pseudo-code in order to make easier comparison of this derivation to the MATLAB code, which is available at <https://github.com/RichardHerz/neural-networks>.

A derivative similar to dx_dy below are the *partial* derivatives of array x with respect to array y . We do not use $\partial x / \partial y$ since that shows division in code and we can't use ∂ in MATLAB code.

Relationships in the XOR network:

layer	input	hidden	output
activation	$a\{1\}$	$a\{2\}$	$a\{3\}$
weight		$w\{1\}$	$w\{2\}$

The errors at the output nodes, given a training input $a\{1\}$, are

$$E = 0.5 * (y - a\{3\})^2$$

where y is the array of correct outputs in the training set, which correspond to the inputs $a\{1\}$.

See similar activation map images to those here in Figs. 3 & 4 of the original paper introducing back propagation:
Rumelhart, D., Hinton, G. & Williams, R. Learning representations by back-propagating errors. Nature 323, 533–536 (1986). <https://www.nature.com/articles/323533a0>

Back Propagation and Gradient Descent in Network Training

The rates of change of errors E with respect to the outputs are

$$dE_da\{3\} = -(y - a\{3\})$$

Define the inputs I from layer 2 going to the output layer 3 as

$$I\{2\} = w\{2\} * a\{2\}$$

where

$$a\{3\} = \text{sigmaFunc}(I\{2\})$$

$$\begin{aligned} da\{3\}_dI\{2\} &= d\text{sigmaFunc}(I\{2\})_dI\{2\} \\ &= a\{3\} .* (1 - a\{3\}) \end{aligned}$$

The rates of change of error E with respect to $I\{2\}$, the signals from layer 2 to layer 3, are

$$\begin{aligned} dE_dI\{2\} &= dE_da\{3\} * da\{3\}_dI\{2\} \\ &= -(y - a\{3\}) .* a\{3\} .* (1 - a\{3\}) \end{aligned}$$

Back Propagation and Gradient Descent in Network Training

Working backward toward the inputs, in "back propagation" of the error...

$$I\{1\} = W\{1\} * a\{1\}$$

$$a\{2\} = \text{sigmaFunc}(I\{1\})$$

$$da\{2\}_dI\{1\} = d\text{sigmaFunc}(I\{1\})_dI\{1\}$$

$$= a\{2\} .* (1 - a\{2\})$$

$$dE_da\{2\} = dI\{2\}_da\{2\}' * dE_dI\{2\}$$

$$dI\{2\}_da\{2\} = W\{2\}$$

$$dE_da\{2\} = W\{2\}' * dE_dI\{2\}$$

where $W\{2\}'$ is the transpose of $W\{2\}$ and $*$ is matrix multiplication in Matlab such that the results are the sums of the contributions of each node in layer 2.

Back Propagation and Gradient Descent in Network Training

The rates of change of error E with respect to $I\{1\}$, the signals from layer 1 to layer 2, are

$$dE_dI\{1\} = dE_da\{2\} * da\{2\}_I\{1\}$$

$$dE_dI\{1\} = w\{2\}' * dE_dI\{2\} .* a\{2\} .* (1 - a\{2\})$$

For the XOR network with only one hidden layer, we stop here in getting the d's.

For a network with more hidden layers, we would continue the process by which we got $dE_dI\{1\}$, with corresponding changes in array index.

Back Propagation and Gradient Descent in Network Training

Now that we have the rates of change of error E with respect to the inputs to each layer, we can compute the rates of change of error E with respect to the weights themselves.

$$dE_dW\{i\} = dE_dI\{i\} * dI\{i\}_dW\{i\}'$$

$$dI\{i\}_dW\{i\} = a\{i\}$$

$$dE_dW\{i\} = dE_dI\{i\} * a\{i\}'$$

where $a\{i\}'$ is the transpose of $a\{i\}$ and $*$ is matrix multiplication in Matlab such that the results are the sums of the contributions to each weight.

Back Propagation and Gradient Descent in Network Training

Now we can compute the new values of the connection weights $w\{i\}$, which will be used in the next iteration of the gradient descent method

$$w\{i\} = w\{i\} - \text{alpha} * dE_dW\{i\}$$

where alpha is a constant value in the range 0-1.

With these new values of weights, the training inputs are used to compute new outputs. The new errors are computed, and the process is repeated.

The process is stopped when some specified criterion is met with regard to the approach to the minimum error or a maximum number of iterations.

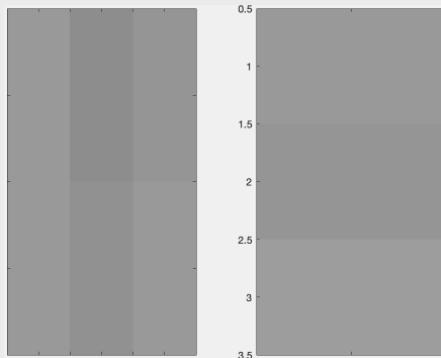
At that point, the artificial neural network has been trained. The performance of the network should then be tested by using additional inputs for which the outputs are known.

On the next slide is a visualization of how the XOR network weights change during training.

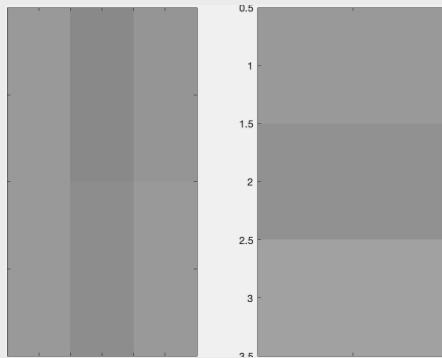
In a network with biases, $B\{i\} = B\{i\} - \text{alpha} * dE_dI\{i\}$

XOR network weight changes during training

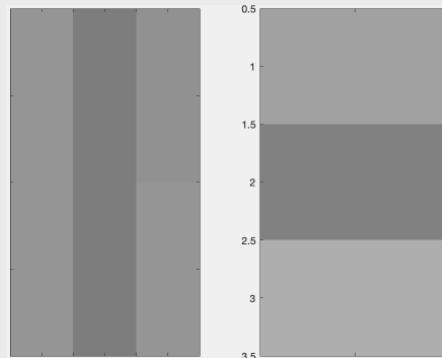
Visualization of the weights during training, starting from and ending at the weights shown above. These are for a particular random initialization. Other initializations will result in different weight patterns, but similar final error, E, which is shown for only the [1;0] input, although the training used all four possible inputs. Some elements of the initial pattern persist during training.



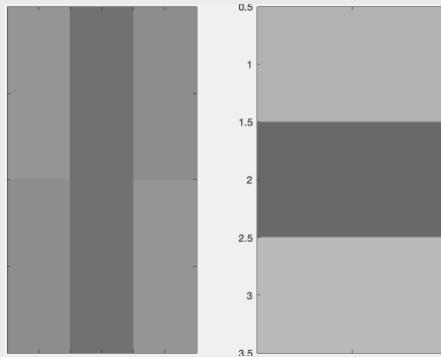
initial, $E = 0.1672$



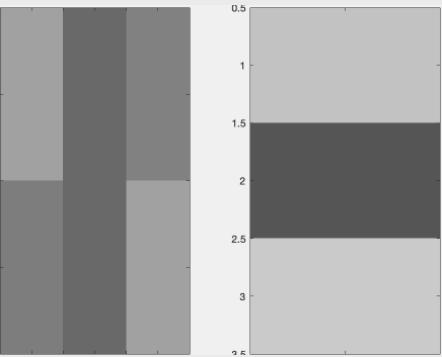
10%, $E = 0.1167$



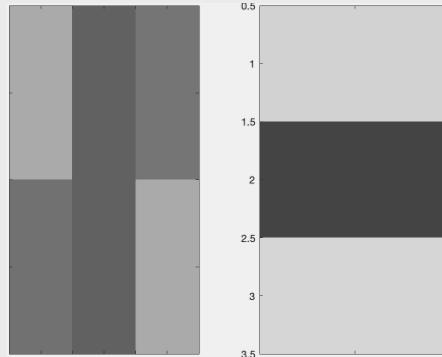
20%, $E = 0.0988$



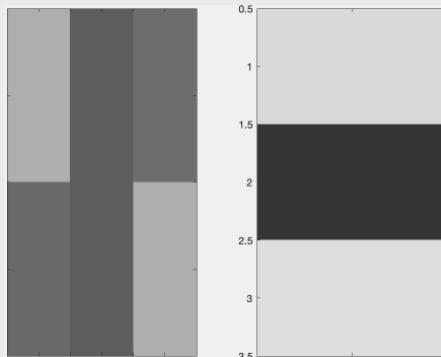
30%, $E = 0.0690$



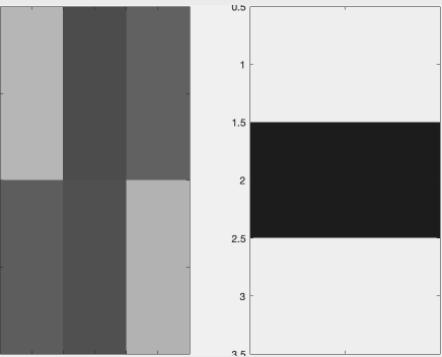
40%, $E = 0.0296$



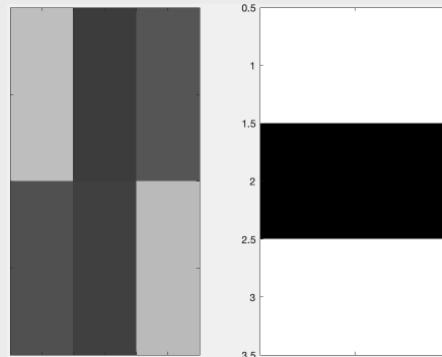
50%, $E = 0.0085$



60%, $E = 0.0035$



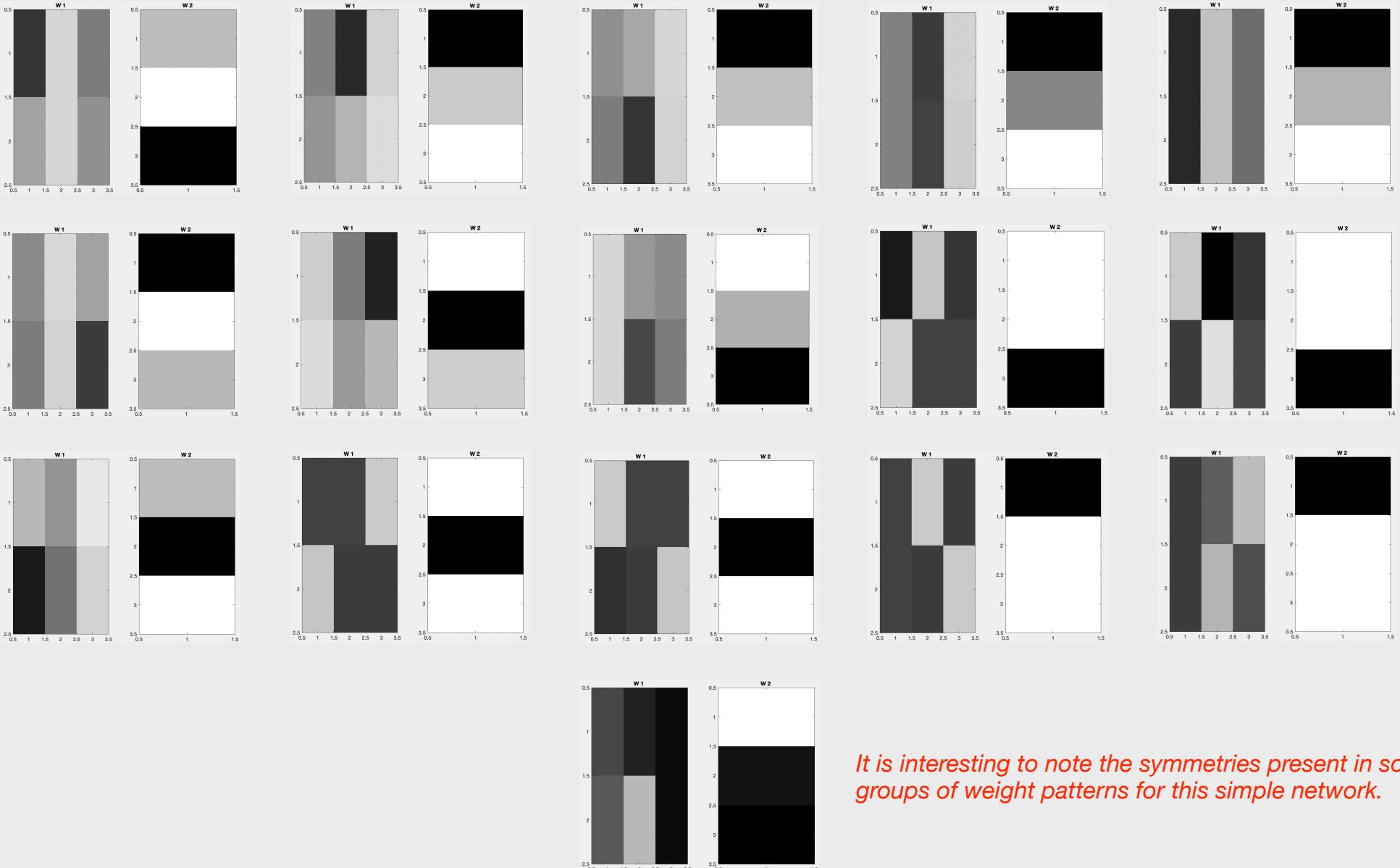
80%, $E = 0.00075$



100%, $E = 0.00015$

XOR network weight patterns vary with random initialization

Visualization of final connection weights produced from different random initializations of training. All of these final weight patterns after training produce the same valid outputs for the same inputs. Not all initializations result in valid networks.



It is interesting to note the symmetries present in some groups of weight patterns for this simple network.

More Complex Neural Networks

These notes have discussed the simplest structure used in neural networks.

There are many more network structures to learn about, including

- The addition of bias values to each hidden and output layer, and use of other activation functions.
- Deep networks with many hidden layers used in “deep learning” - essentially very complex functions to relate complex inputs and outputs.
- Convolutional networks, in which various matrices are mathematically convoluted with input submatrices and scanned across an input matrix in order to help identify specific features, such as faces in an image or words in a sound spectrum.
- Recurrent networks with feedforward and feedback of information, useful in analyzing sequential inputs such as speech.
- Different types of networks arranged in parallel and series.

Search the website of the MathWorks, makers of MATLAB, for their neural network tools
<https://mathworks.com>