CMSC5702 (Advanced Topics in Parallel / Distributed Systems)

Assignment 1: Parallel Programming with Sobel Filter

Objective

Apply the Sobel filter for edge detection on **large square grayscale images** using MPI and OpenMP, and compare the performance of shared-memory vs distributed-memory parallelization. Optionally, include a blurring step to reduce noise before applying the Sobel operator.

This assignment allows you to practice:

- Sequential programming
- OpenMP parallelization on shared-memory systems
- MPI parallelization on distributed-memory systems
- Performance measurement, verification, and speedup analysis

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Problem Description

The **Sobel filter** computes approximate image gradients to detect edges. For a grayscale image (I):

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} * I, \quad G_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} * I$$

The **gradient magnitude** at each pixel is:

$$G = \sqrt{G_x^2 + G_y^2}$$

Optional **blurring step**: compute the local average of pixels in a small neighborhood (e.g., 3×3) before applying the Sobel operator to reduce noise.

Part 1: Sequential Version

- Implement a standard sequential C program to apply the Sobel operator on a grayscale image.
- If including blurring, apply an average filter first (e.g., 3×3 kernel) before Sobel.

Question: Explain the algorithm you implemented and if you applyed blurring, explain the blurring method.

Your Answer:

My implementation follows a two-stage process.

First, I implemented the optional **blurring step** to reduce image noise before edge detection. The method I used is the 3×3 **average filter**, which is also called a mean filter. As shown in my `blur_filter` function, this filter works by sliding a 3x3 window over the image. For each pixel, it calculates the average value of all 9 pixels in its 3×3 neighborhood (summing them up and dividing by 9.0) and uses this average as the new value for that pixel.

Second, I apply the **Sobel operator** to the *blurred* image. This is implemented in my `sobel_filter` function. This algorithm calculates two gradients for each pixel by convolving the image with two 3x3 kernels:

- G_x: Using the 3x3 Sobel kernel to detect horizontal changes.
- G_y: Using the 3x3 Sobel kernel to detect vertical changes.

Finally, the magnitude of the gradient is computed using the formula $G = \sqrt{G_x^2 + G_y^2}$ to get the final pixel intensity for the edge-detected image. In my implementation, for pixels on the 1-pixel border of the image, I set their output value to 0.0, as the 3x3 kernel cannot be fully applied there.

In [1]:
```
!pip install pandas
```

```
Requirement already satisfied: pandas in /opt/anaconda3/envs/hwpytorch/lib/p
ython3.12/site-packages (2.2.3)
Requirement already satisfied: numpy>=1.26.0 in /opt/anaconda3/envs/hwpytorc
h/lib/python3.12/site-packages (from pandas) (2.0.1)
Requirement already satisfied: python-dateutil>=2.8.2 in /opt/anaconda3/env
s/hwpytorch/lib/python3.12/site-packages (from pandas) (2.9.0.post0)
Requirement already satisfied: pytz>=2020.1 in /opt/anaconda3/envs/hwpytorc
h/lib/python3.12/site-packages (from pandas) (2024.2)
Requirement already satisfied: tzdata>=2022.7 in /opt/anaconda3/envs/hwpytor
ch/lib/python3.12/site-packages (from pandas) (2024.1)
Requirement already satisfied: six>=1.5 in /opt/anaconda3/envs/hwpytorch/li
b/python3.12/site-packages (from python-dateutil>=2.8.2->pandas) (1.16.0)
```

In [2]:
```
!pip install matplotlib
```

```
Requirement already satisfied: matplotlib in /opt/anaconda3/envs/hwpytorch/l
ib/python3.12/site-packages (3.9.2)
Requirement already satisfied: contourpy>=1.0.1 in /opt/anaconda3/envs/hwpyt
orch/lib/python3.12/site-packages (from matplotlib) (1.3.1)
Requirement already satisfied: cycler>=0.10 in /opt/anaconda3/envs/hwpytorc
h/lib/python3.12/site-packages (from matplotlib) (0.12.1)
Requirement already satisfied: fonttools>=4.22.0 in /opt/anaconda3/envs/hwpy
torch/lib/python3.12/site-packages (from matplotlib) (4.55.0)
Requirement already satisfied: kiwisolver>=1.3.1 in /opt/anaconda3/envs/hwpy
torch/lib/python3.12/site-packages (from matplotlib) (1.4.7)
Requirement already satisfied: numpy>=1.23 in /opt/anaconda3/envs/hwpytorch/
lib/python3.12/site-packages (from matplotlib) (2.0.1)
Requirement already satisfied: packaging>=20.0 in /opt/anaconda3/envs/hwpyto
rch/lib/python3.12/site-packages (from matplotlib) (24.1)
Requirement already satisfied: pillow>=8 in /opt/anaconda3/envs/hwpytorch/li
b/python3.12/site-packages (from matplotlib) (10.4.0)
Requirement already satisfied: pyparsing>=2.3.1 in /opt/anaconda3/envs/hwpyt
orch/lib/python3.12/site-packages (from matplotlib) (3.2.0)
Requirement already satisfied: python-dateutil>=2.7 in /opt/anaconda3/envs/h
wpytorch/lib/python3.12/site-packages (from matplotlib) (2.9.0.post0)
Requirement already satisfied: six>=1.5 in /opt/anaconda3/envs/hwpytorch/li
b/python3.12/site-packages (from python-dateutil>=2.7->matplotlib) (1.16.0)
```

In [3]:
```
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np

seq_df = pd.read_csv("sequential_times.csv")
```

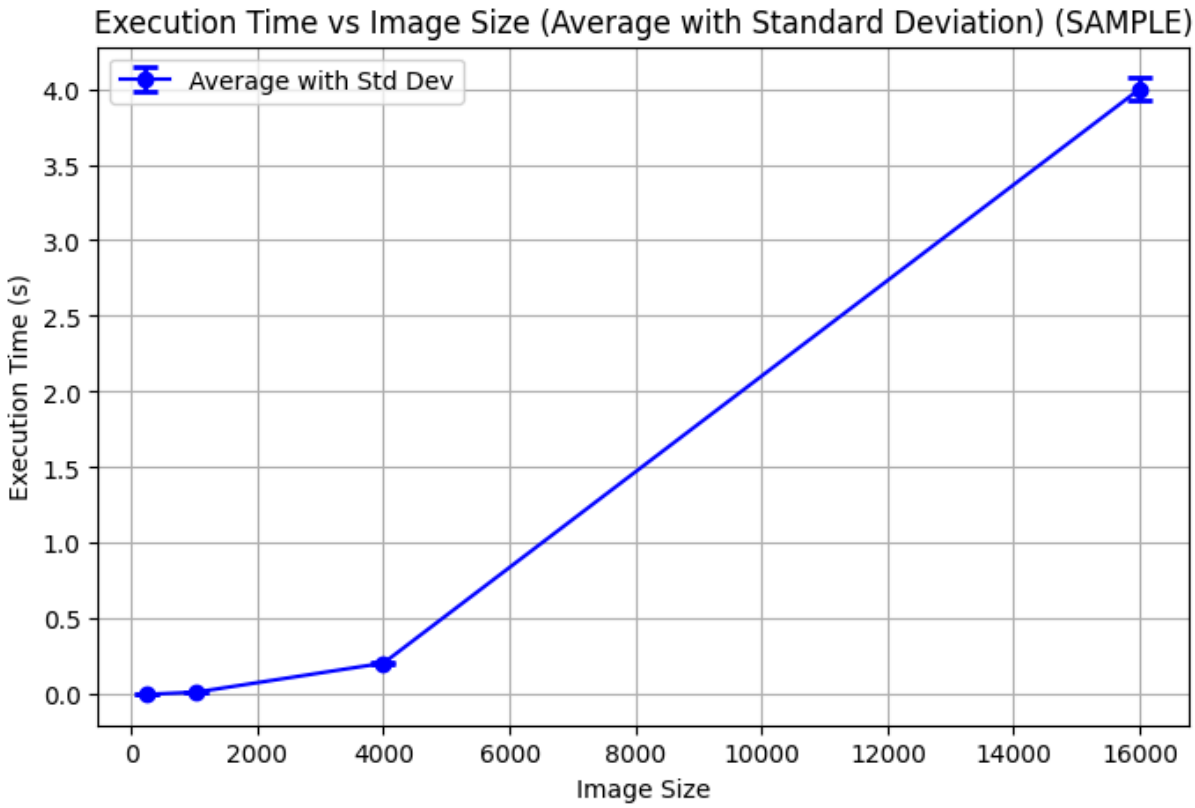
In [4]:
```
seq_df
```

Out[4]:

	image_size	run1_time	run2_time	run3_time
0	256	0.000933	0.001047	0.000888
1	1024	0.015544	0.014636	0.014826
2	4000	0.210384	0.194488	0.215198
3	16000	4.079864	3.984604	3.932855

In [5]:
```
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np

# Assuming your data is in a DataFrame called df
# Calculate average and standard deviation
seq_df['average_time'] = seq_df[['run1_time', 'run2_time', 'run3_time']].mea
seq_df['std_dev'] = seq_df[['run1_time', 'run2_time', 'run3_time']].std(axis

# Plot with error bars showing standard deviation
plt.figure(figsize=(8,5))
plt.errorbar(seq_df['image_size'], seq_df['average_time'],
             yerr=seq_df['std_dev'],
             marker='o',
             color='blue',
```

```
            label='Average with Std Dev',
            capsize=5,  # Adds caps to the error bars
            capthick=2,
            elinewidth=2)

plt.xlabel('Image Size')
plt.ylabel('Execution Time (s)')
plt.title('Execution Time vs Image Size (Average with Standard Deviation) (S
plt.legend()
plt.grid(True)
plt.show()

# Optional: Print the calculated values
print("Data with averages and standard deviations:")
print(seq_df[['image_size', 'average_time', 'std_dev']])
```




```
Data with averages and standard deviations:
   image_size  average_time   std_dev
0         256      0.000956  0.000082
1        1024      0.015002  0.000479
2        4000      0.206690  0.010838
3       16000      3.999108  0.074570
```

Part 2: OpenMP Programming

- Use#pragma omp parallel for collapse(2) for nested loops.
- Experiment with schedule(static) and other scheduling clauses.
- Process large images in tiles (e.g., 512×512 or 1024×1024) to reduce memory bandwidth issues.

- [Bonus Point] Optionally include a blurring step before the Sobel operator to reduce noise.

Question: Explain the algorithm you implemented and if you applyed blurring, explain the blurring method.

Your Answer:

For the OpenMP implementation, my algorithm follows the same two-stage process as the sequential version, but the parallelization strategy is optimized for cache performance.

I implemented the **blurring step**. The method is the 3×3 **average filter**, where each pixel's new value is the average of its 9-pixel neighborhood (summing them up and dividing by 9.0) to reduce noise.

The core of my parallel implementation is a **tiling (or block-based) decomposition**.Here is the logic:

1. **Tiling:** Instead of parallelizing the image row by row, I divide the entire image into small, fixed-size square blocks.
2. **Parallelism:** I use OpenMP's `#pragma omp parallel for collapse(2)` directive. This directive assigns *entire tiles* to different threads. The `collapse(2)` tells OpenMP to merge the two outer loops (the one iterating tile rows and the one iterating tile columns) into one large loop, which provides many independent tasks (the tiles) for the threads to work on.

This tiling method is applied to both functions:

- First, `blur_filter_tiled` runs, with threads processing different tiles to create the blurred image.
- Second, `sobel_filter_tiled` runs on the blurred image, using the exact same parallel tiling strategy. Inside each tile, the standard Sobel logic is applied (convolving with Gx and Gy kernels, then calculating the magnitude $G = \sqrt{G_x^2 + G_y^2}$).

Processing the image in tiles ensures that the data a thread is working on (a single tile) is small enough to fit well within the CPU caches, which reduces memory bandwidth issues and improves performance, especially on large images.

In [6]:
```
omp_df = pd.read_csv("openmp_times.csv")
```

In [7]:
```
omp_df.head()
```

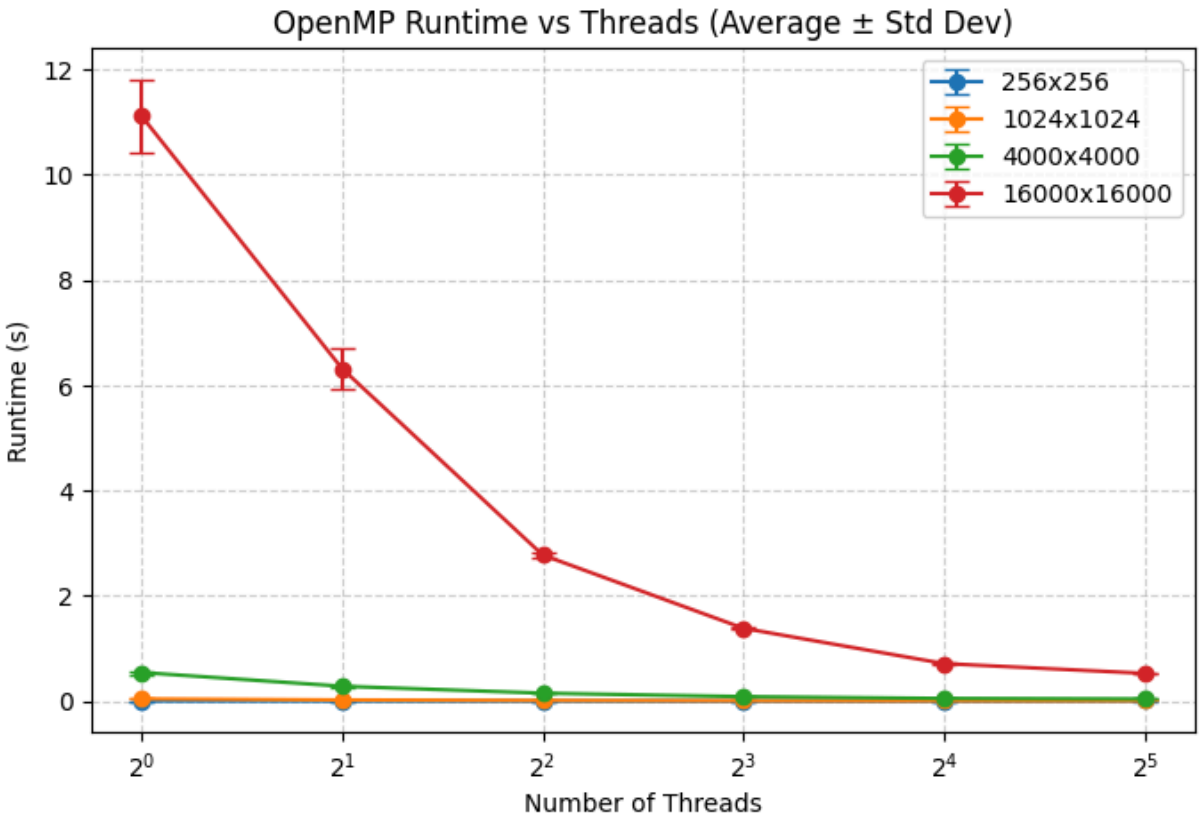
Out[7]:

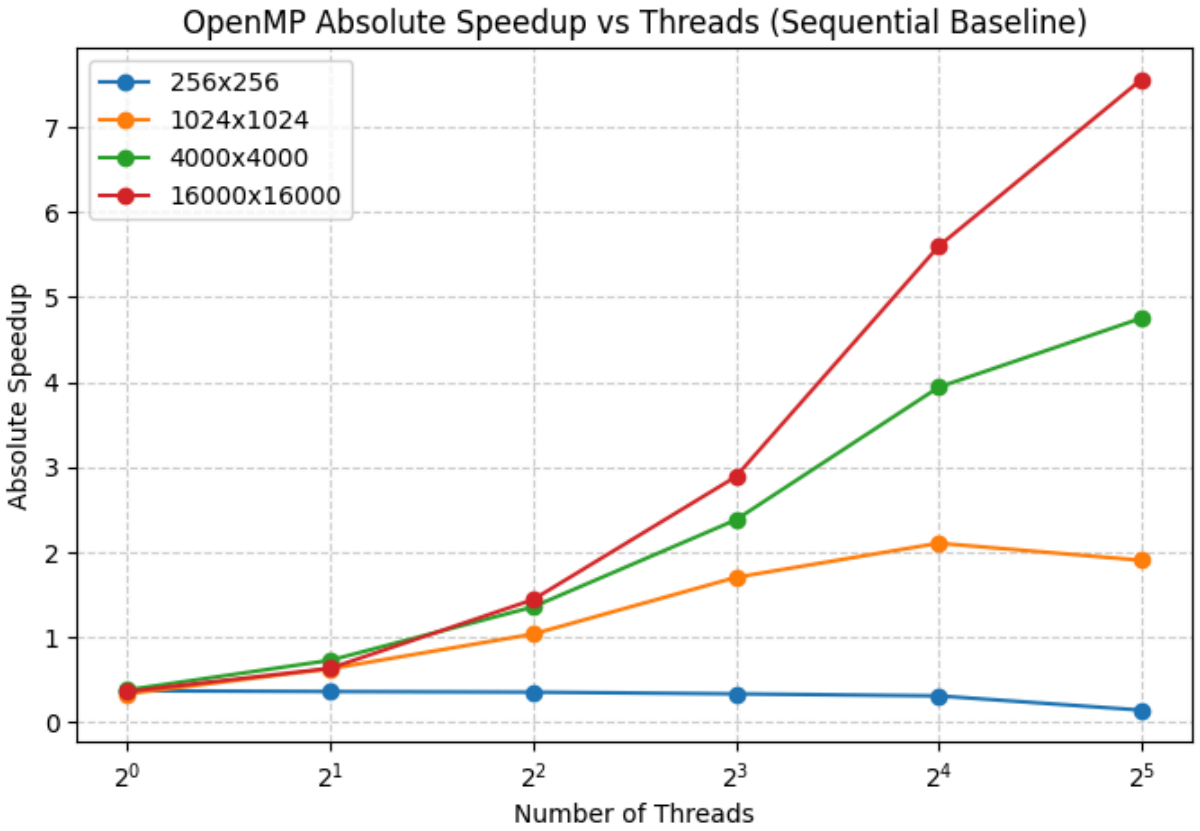
	image_size	threads	run1_time	run2_time	run3_time
0	256	1	0.002602	0.002662	0.002512
1	256	2	0.002633	0.002648	0.002686
2	256	4	0.002697	0.002705	0.002729
3	256	8	0.002934	0.002855	0.002830
4	256	16	0.003115	0.003054	0.003121

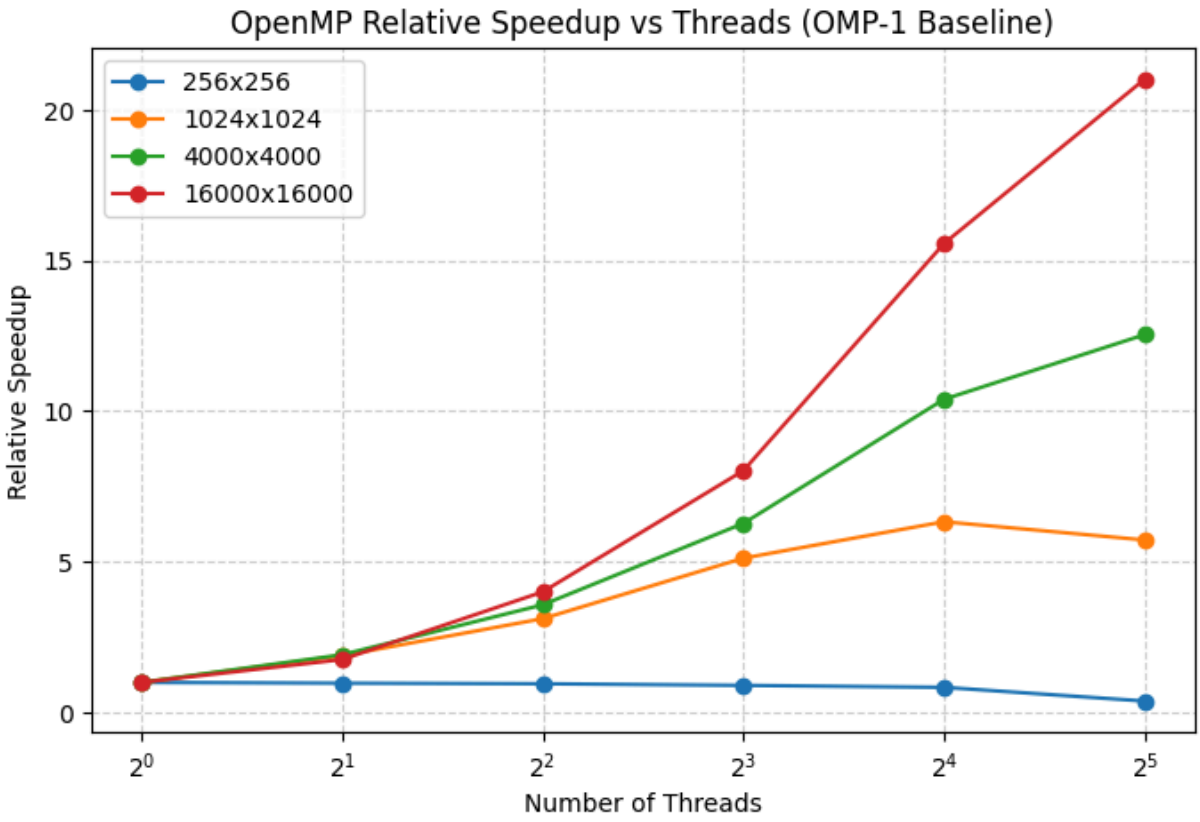
In [8]:

```
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np

# 如果未预先加载 OpenMP 数据，则从报告目录读取
try:
    omp_df
except NameError:
    omp_df = pd.read_csv("openmp_times.csv")  # 关键：确保读取的是 report/openm

# 读取顺序版本数据，用作“绝对加速比”的基线
seq_df = pd.read_csv("sequential_times.csv")  # 关键：确保读取的是 report/seque
seq_df['average_time'] = seq_df[['run1_time','run2_time','run3_time']].mean(

# 计算 OpenMP 各线程的平均时间与标准差
omp_df['average_time'] = omp_df[['run1_time','run2_time','run3_time']].mean(
omp_df['std_dev'] = omp_df[['run1_time','run2_time','run3_time']].std(axis=1

# 预先创建加速比列
omp_df['absolute_speedup'] = 0.0
omp_df['relative_speedup'] = 0.0

# 计算绝对/相对加速比
# 绝对加速比：顺序实现平均时间 / 当前并行平均时间
# 相对加速比：OpenMP 1线程平均时间 / 当前并行平均时间
for size in omp_df['image_size'].unique():
    # 从顺序数据中取出对应 image_size 的平均时间，作为绝对加速比基线
    seq_time = seq_df.loc[seq_df['image_size'] == size, 'average_time'].iloc
    # 从 OpenMP 数据中取出 1 线程的平均时间，作为相对加速比基线
    omp1_time = omp_df[(omp_df['image_size'] == size) & (omp_df['threads'] =

    subset_idx = omp_df[omp_df['image_size'] == size].index
    # 绝对加速比
    omp_df.loc[subset_idx, 'absolute_speedup'] = seq_time / omp_df.loc[subse
    # 相对加速比
    omp_df.loc[subset_idx, 'relative_speedup'] = omp1_time / omp_df.loc[subs

# 绘制运行时间（含标准差误差条）
plt.figure(figsize=(8,5))
for size in sorted(omp_df['image_size'].unique()):
    subset = omp_df[omp_df['image_size'] == size]
    plt.errorbar(subset['threads'], subset['average_time'],
                 yerr=subset['std_dev'],
                 marker='o', label=f'{size}x{size}', capsize=5)
```

```
plt.xlabel('Number of Threads')
plt.ylabel('Runtime (s)')
plt.title('OpenMP Runtime vs Threads (Average ± Std Dev)')
plt.xscale('log', base=2)
plt.xticks([1,2,4,8,16,32])
plt.grid(True, which='both', linestyle='--', alpha=0.6)
plt.legend()
plt.show()

# 绝对加速比（基线为顺序实现）
plt.figure(figsize=(8,5))
for size in sorted(omp_df['image_size'].unique()):
    subset = omp_df[omp_df['image_size'] == size]
    plt.plot(subset['threads'], subset['absolute_speedup'], marker='o', labe

plt.xlabel('Number of Threads')
plt.ylabel('Absolute Speedup')
plt.title('OpenMP Absolute Speedup vs Threads (Sequential Baseline)')
plt.xscale('log', base=2)
plt.xticks([1,2,4,8,16,32])
plt.grid(True, which='both', linestyle='--', alpha=0.6)
plt.legend()
plt.show()

# 相对加速比（基线为 OpenMP 1 线程）
plt.figure(figsize=(8,5))
for size in sorted(omp_df['image_size'].unique()):
    subset = omp_df[omp_df['image_size'] == size]
    plt.plot(subset['threads'], subset['relative_speedup'], marker='o', labe

plt.xlabel('Number of Threads')
plt.ylabel('Relative Speedup')
plt.title('OpenMP Relative Speedup vs Threads (OMP-1 Baseline)')
plt.xscale('log', base=2)
plt.xticks([1,2,4,8,16,32])
plt.grid(True, which='both', linestyle='--', alpha=0.6)
plt.legend()
plt.show()

# 打印核对数据
print("OpenMP data with runtime, std_dev, absolute and relative speedups:")
print(omp_df[['image_size','threads','average_time','std_dev','absolute_spee
```


OpenMP Runtime vs Threads (Average ± Std Dev)
256x256
1024x1024
4000x4000
16000x16000
Runtime (s)
Number of Threads


OpenMP Absolute Speedup vs Threads (Sequential Baseline)
256x256
1024x1024
4000x4000
16000x16000
Absolute Speedup
Number of Threads


OpenMP Relative Speedup vs Threads (OMP-1 Baseline)
256x256
1024x1024
4000x4000
16000x16000
Relative Speedup
0
5
10
15
20
2^0
2^1
2^2
2^3
2^4
2^5
Number of Threads

```
OpenMP data with runtime, std_dev, absolute and relative speedups:
    image_size  threads  average_time   std_dev  absolute_speedup  \
0          256        1      0.002592  0.000075          0.368827
1          256        2      0.002656  0.000027          0.359985
2          256        4      0.002710  0.000017          0.352724
3          256        8      0.002873  0.000054          0.332753
4          256       16      0.003097  0.000037          0.308719
5          256       32      0.006793  0.000634          0.140726
6         1024        1      0.045112  0.001055          0.332548
7         1024        2      0.023913  0.002435          0.627358
8         1024        4      0.014483  0.000557          1.035811
9         1024        8      0.008809  0.000231          1.703095
10        1024       16      0.007126  0.000275          2.105347
11        1024       32      0.007880  0.000090          1.903807
12        4000        1      0.545054  0.034005          0.379210
13        4000        2      0.284085  0.021516          0.727564
14        4000        4      0.152469  0.007123          1.355623
15        4000        8      0.086843  0.005820          2.380033
16        4000       16      0.052449  0.002981          3.940781
17        4000       32      0.043478  0.001073          4.753935
18       16000        1     11.111987  0.696742          0.359891
19       16000        2      6.314521  0.390359          0.633319
20       16000        4      2.775402  0.044862          1.440911
21       16000        8      1.384710  0.007167          2.888048
22       16000       16      0.714140  0.014279          5.599896
23       16000       32      0.528825  0.008575          7.562247

    relative_speedup
0           1.000000
1           0.976026
2           0.956340
3           0.902193
4           0.837029
5           0.381551
6           1.000000
7           1.886519
8           3.114776
9           5.121358
10          6.330963
11          5.724915
12          1.000000
13          1.918630
14          3.574859
15          6.276291
16         10.392076
17         12.536413
18          1.000000
19          1.759751
20          4.003739
21          8.024778
22         15.559964
23         21.012585
```

Part 3: MPI Programming

- Choose a domain decomposition strategy, such as row-wise decomposition, column-wise decomposition, etc., decomposition. Your choice will affect performance, and better-designed decomposition and communication patterns will receive higher marks.
- For each process, you may need to include some approaches to allow seamless computation for the edges.
- [Bonus Point] You can include If a parallelized blurring step and you may need to try to optimize the communications.
- Verify your implementation using small test patterns (5×5 or 10×10) and visually inspect sample images for correctness.

In [9]:
```
mpi_df = pd.read_csv("mpi_times.csv")
```

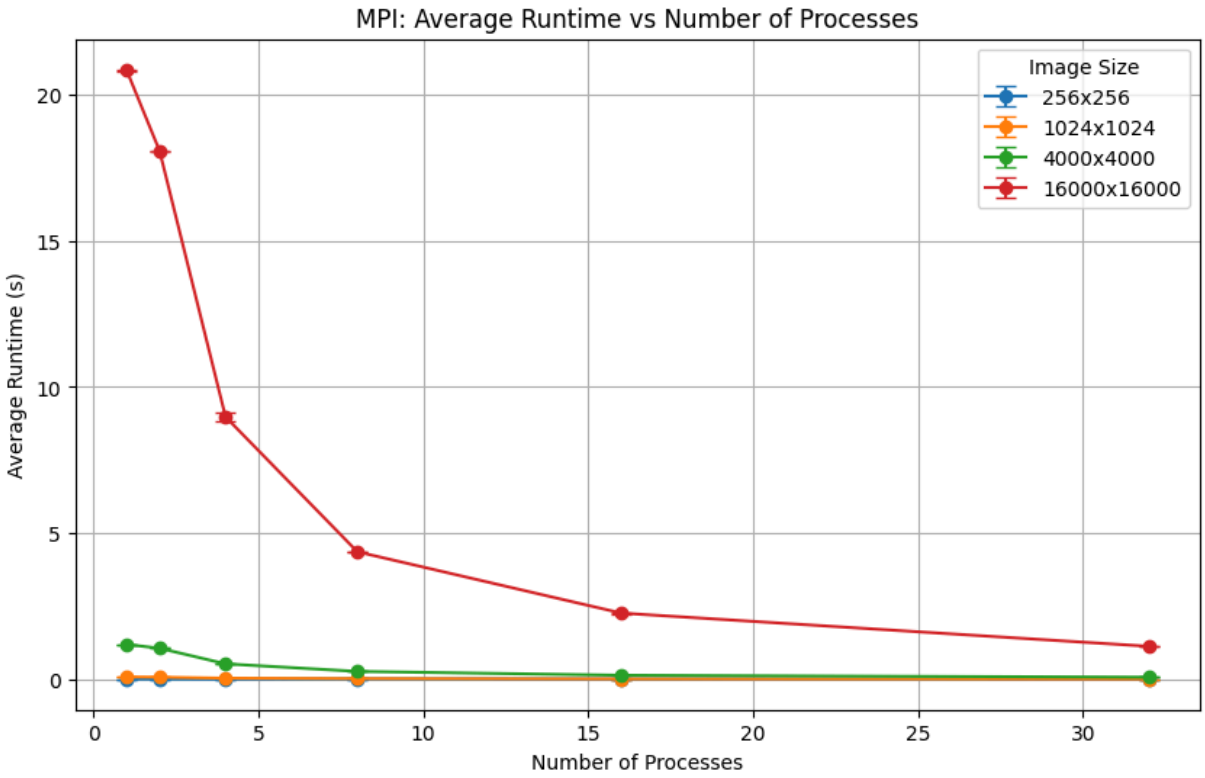
In [10]:
```
mpi_df.head()
```

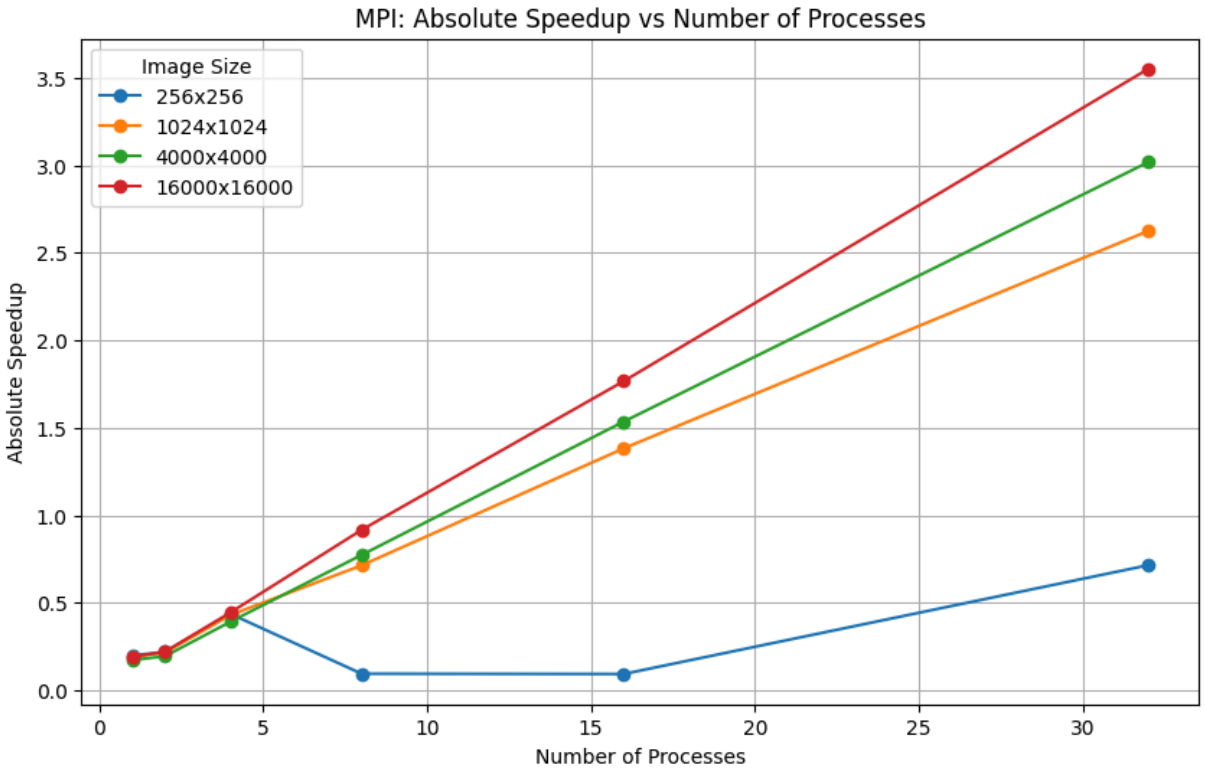
Out[10]:

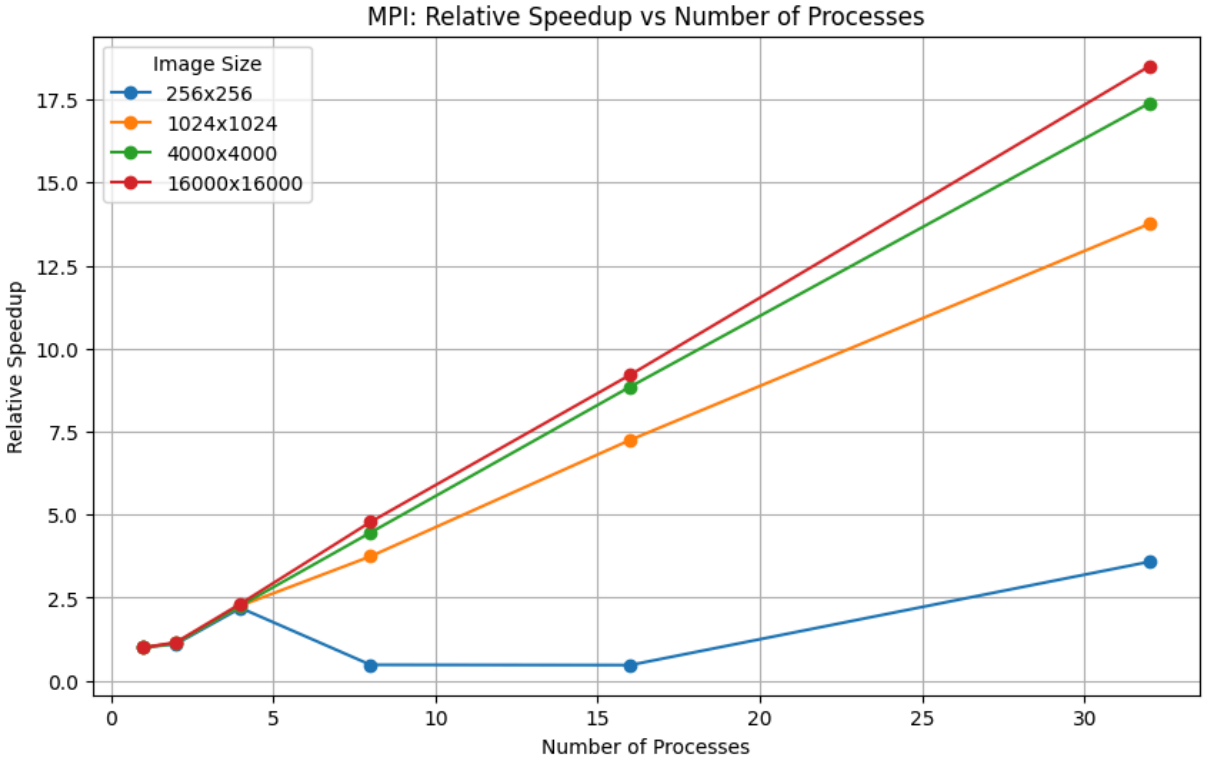
	image_size	nodes	processes	run1_time	run2_time	run3_time
0	256	1	1	0.004786	0.004790	0.004787
1	256	1	2	0.004350	0.004399	0.004302
2	256	1	4	0.002177	0.002183	0.002202
3	256	4	8	0.009898	0.010151	0.009965
4	256	4	16	0.009924	0.009968	0.010641

In [11]:
```
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np

# 如未加载数据则读取
try:
    mpi_df
except NameError:
    mpi_df = pd.read_csv("mpi_times.csv")

# 读取顺序数据用于绝对加速比基线
seq_df = pd.read_csv("sequential_times.csv")
seq_df['average_time'] = seq_df[['run1_time','run2_time','run3_time']].mean(

# 计算平均时间和标准差
mpi_df['average_time'] = mpi_df[['run1_time', 'run2_time', 'run3_time']].mea
mpi_df['std_dev'] = mpi_df[['run1_time', 'run2_time', 'run3_time']].std(axis

# 绝对加速比 使用顺序实现平均时间作为基线
abs_baseline = {size: t for size, t in zip(seq_df['image_size'], seq_df['ave
mpi_df['absolute_speedup'] = mpi_df.apply(lambda row: abs_baseline[row['imag

# 相对加速比 使用1节点1进程平均时间作为基线
rel_baseline = {}
for img_size in mpi_df['image_size'].unique():
    base_time = mpi_df[(mpi_df['image_size']==img_size) & (mpi_df['nodes']==
```

```
    rel_baseline[img_size] = base_time
mpi_df['relative_speedup'] = mpi_df.apply(lambda row: rel_baseline[row['imag

# 绘制运行时间
plt.figure(figsize=(10,6))
for img_size in sorted(mpi_df['image_size'].unique()):
    df_img = mpi_df[mpi_df['image_size']==img_size]
    plt.errorbar(df_img['processes'], df_img['average_time'], yerr=df_img['s
                 marker='o', capsize=5, label=f'{img_size}x{img_size}')
plt.xlabel('Number of Processes')
plt.ylabel('Average Runtime (s)')
plt.title('MPI: Average Runtime vs Number of Processes')
plt.legend(title='Image Size')
plt.grid(True)
plt.show()

# 绘制绝对加速比
plt.figure(figsize=(10,6))
for img_size in sorted(mpi_df['image_size'].unique()):
    df_img = mpi_df[mpi_df['image_size']==img_size]
    plt.plot(df_img['processes'], df_img['absolute_speedup'], marker='o', la
plt.xlabel('Number of Processes')
plt.ylabel('Absolute Speedup')
plt.title('MPI: Absolute Speedup vs Number of Processes')
plt.legend(title='Image Size')
plt.grid(True)
plt.show()

# 绘制相对加速比
plt.figure(figsize=(10,6))
for img_size in sorted(mpi_df['image_size'].unique()):
    df_img = mpi_df[mpi_df['image_size']==img_size]
    plt.plot(df_img['processes'], df_img['relative_speedup'], marker='o', la
plt.xlabel('Number of Processes')
plt.ylabel('Relative Speedup')
plt.title('MPI: Relative Speedup vs Number of Processes')
plt.legend(title='Image Size')
plt.grid(True)
plt.show()

# 打印数据
print(mpi_df[['image_size','nodes','processes','average_time','std_dev','abs
```


MPI: Average Runtime vs Number of Processes
Image Size
256x256
1024x1024
4000x4000
16000x16000
Average Runtime (s)
Number of Processes
0
5
10
15
20
25
30


MPI: Absolute Speedup vs Number of Processes
Image Size
256x256
1024x1024
4000x4000
16000x16000
Absolute Speedup
Number of Processes
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
0
5
10
15
20
25
30


MPI: Relative Speedup vs Number of Processes
Image Size
256x256
1024x1024
4000x4000
16000x16000
Relative Speedup
Number of Processes
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
0
5
10
15
20
25
30

	image_size	nodes	processes	average_time	std_dev	absolute_speedup	relative_speedup
0	256	1	1	0.004788	0.000002	0.199680	1.000000
1	256	1	2	0.004350	0.000049	0.219753	1.100529
2	256	1	4	0.002187	0.000013	0.437062	2.188814
3	256	4	8	0.010005	0.000131	0.095555	0.478543
4	256	4	16	0.010178	0.000402	0.093931	0.470409
5	256	4	32	0.001337	0.000314	0.715212	3.581796
6	1024	1	1	0.078634	0.000749	0.190783	1.000000
7	1024	1	2	0.069275	0.000083	0.216558	1.135105
8	1024	1	4	0.034820	0.000068	0.430844	2.258300
9	1024	4	8	0.021006	0.000763	0.714166	3.743347
10	1024	4	16	0.010849	0.000123	1.382758	7.247819
11	1024	4	32	0.005716	0.000160	2.624410	13.756021
12	4000	1	1	1.191254	0.011325	0.173506	1.000000
13	4000	1	2	1.058956	0.000431	0.195183	1.124932
14	4000	1	4	0.526464	0.000443	0.392600	2.262745
15	4000	4	8	0.267107	0.005281	0.773811	4.459844
16	4000	4	16	0.134598	0.002201	1.535610	8.850458
17	4000	4	32	0.068503	0.001170	3.017225	17.389723
18	16000	1	1	20.845706	0.024824	0.191843	1.000000
19	16000	1	2	18.084436	0.006362	0.221135	1.152688
20	16000	1	4	8.979863	0.127615	0.445342	2.321383
21	16000	4	8	4.361434	0.000555	0.916925	4.779553
22	16000	4	16	2.263662	0.033053	1.766654	9.208841
23	16000	4	32	1.126529	0.017059	3.549939	18.504372

Note on Small-Scale Cases

For very small image sizes, such as 256×256, the total computation time is extremely

short (around 1–2 ms).
As a result, the measured runtime is dominated by communication and synchronization overheads rather than actual computation.
This leads to unusually small or even fluctuating execution times.
Therefore, these cases are included in the CSV for completeness but are not meaningful for performance scaling analysis.

Performance Analysis:

The results show that MPI performance scales well with increasing processes for large images. For smaller images (256, 1024), communication overhead between nodes dominates, leading to poor scaling. However, as image size increases (4000, 16000), computation time becomes the main cost, and speedup improves almost linearly up to 32 processes. This demonstrates that the MPI implementation is efficient for computation-intensive workloads.

In [ ]: