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Control Engineering Simulation: First and Second Order Systems

Note: This version of this report only contains the sections done by Richard Jimenez

Matriculation Number	Student Name	Workload in pre-report
2210013	Bhavesh	First Order System section
2210027	**Richard Jimenez**	Second Order System section

1 First Order System by Bhavesh

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Note: This version of this report only contains the sections done by Richard Jimenez

2 Second Order System by Richard Jimenez

2.1 Finding the Transfer Function and Defining the Parameters

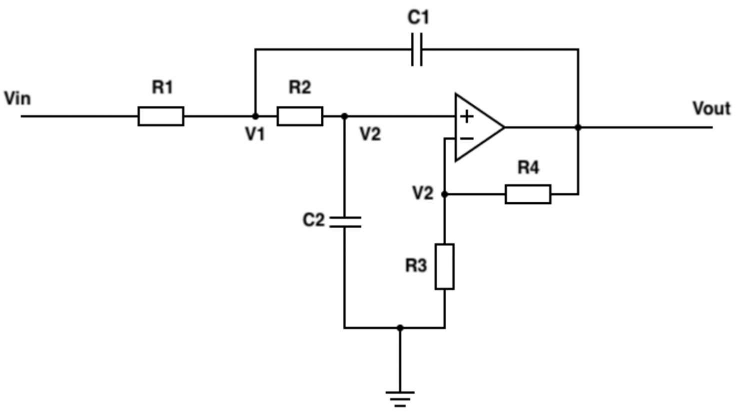

Figure 1: Circuit to be analyzed No.2. Source: Second Order Circuit, composed of one operational amplifier, two capacitors, and four resistors. Adapted from Instructions Control Engineering by Prof. Dr.-Ing. da Costa and Mrs. Faezeh.

The Simulink model that recreates the the second-order circuit shown in Figure 1 is as follows:

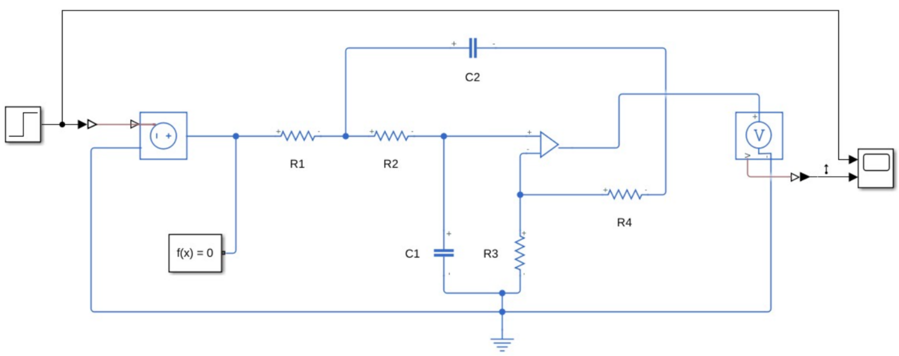

Figure 2: Circuit Second Order Circuit, composed of one operational amplifier, two capacitors, and four resistors in Simulink. Source: Own creation.

To find the transfer function the first step is to do a nodal analysis in nodes V1 and V2. That way it is possible to get the three functions bellow:

$$\frac{V_1 - V_{\text{in}}}{R_1} + \frac{V_1 - V_2}{R_2} + \frac{V_1 - V_{\text{out}}}{\frac{1}{SC_2}} = 0 \tag{1}$$

$$\frac{V_2 - V_{\text{out}}}{R_4} + \frac{V_2}{R_3} = 0 \tag{2}$$

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{\frac{1}{SC_1}} = 0 \tag{3}$$

These three equations can be mathematically manipulated or reorganized in such a way that the variables V_1 and V_2 are isolated as shown in the following steps:

$$R_2 V_1 - R_2 V_{\text{in}} + R_1 V_1 - R_1 V_2 + SR_1R_2C_2V_1 - SR_1R_2C_2V_{\text{out}} = 0 \tag{4}$$

$$V_2 = V_{\text{out}} \frac{R_3}{R_3 + R_4} \tag{5}$$

$$V_1 = V_{\text{out}} \frac{R_3}{R_3 + R_4} (1 + SR_2C_1) \tag{6}$$

Through the substitution of equations 5 and 6 into equation 4, the intermediate nodes can be eliminated, leading to a simplified configuration that includes only the input and output nodes.

$$\begin{gathered} \frac{R_2R_3}{R_3 + R_4} (1 + SR_2C_1) V_{\text{out}} - R_2V_{\text{in}} + \frac{R_1R_3}{R_3 + R_4} (1 + SR_2C_1) V_{\text{out}} - \\ \frac{R_1R_3}{R_3 + R_4} V_{\text{out}} + \frac{SR_1R_2R_3C_2}{R_3 + R_4} (1 + SR_2C_1) V_{\text{out}} - SR_1R_2C_2V_{\text{out}} = 0 \end{gathered} \tag{7}$$

After rearranging equation 7 to isolate V_{in} and V_{out}, it is possible to establish the relationship between the two nodes. This relationship shows the transfer function of the second order system from Figure ??, and it is shown in the final equation 8.

$$\frac{V_{out}}{V_{in}} = \frac{\frac{R_3 + R_4}{R_3}}{S^2(R_1R_2C_1C_2) + S(C_1(R_1 + R_2) - R_1C_2(\frac{R_3 + R_4}{R_3} - 1)) + 1} \tag{8}$$

The transfer function from equation 8 of the standard second-order system can be compared to the general form of a second-order system transfer function shown as follows in equation 9. This form of the second-order transfer function is composed of the standard coefficient, unity s^2 coefficient, unity constant coefficient:

$$G(s) = \frac{K}{\left(\frac{1}{\omega_n^2}\right) s^2 + \left(\frac{2\zeta}{\omega_n}\right) s + 1} \tag{9}$$

Now after mathematically manipulating both equations 8 and 9, it is possible to define specific expressions of these parameters: the system gain K, the damping ratio ζ, and the natural frequency ω_n as described in the following paragraphs.

Starting with the system gain K, it can be derived its expression from equations 8 and 9:

$$K = \frac{R_3 + R_4}{R_3} \tag{10}$$

Similarly, it can be found the natural frequency, ω_n:

$$\frac{1}{\omega_n^2} = R_1R_2C_1C_2 \tag{11}$$

$$\iff \frac{1}{R_1R_2C_1C_2} = \omega_n^2 \tag{12}$$

$$\iff \quad \omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}} \tag{13}$$

Lastly, in similar manner the damping ratio, ζ, can be found from equations 8 and 9, as shown in the following calculations:

$$\frac{2\zeta}{\omega_n} = (C_1(R_1 + R_2) - R_1C_2(\frac{R_3 + R_4}{R_3} - 1)) \tag{14}$$

$$\iff \zeta = \frac{\omega_n(C_1(R_1 + R_2) - R_1C_2(\frac{R_3 + R_4}{R_3} - 1))}{2} \tag{15}$$

By simplifying further with previous findings of the system gain K in equation 10, the expression of the damping ratio ζ yields as follows:

$$\zeta = \frac{\omega_n(C_1(R_1 + R_2) - R_1C_2(K - 1))}{2} \tag{16}$$

2.2 Projecting Components Values

The first requirement, valid for all the specifications, is that K must be equal to 2. For that purpose, K only equals to 2 if $R_3 = R_4$. Another hint given in the report is to use $C_1 = C_2$. With these two requirements it is possible now to simplify the parameters equations.

To simplify the expression of the system gain K that was found in equation 10, and having the new condition results to following result:

$$K = \frac{R_3 + R_3}{R_3} \tag{17}$$

$$\iff K = \frac{2R_3}{R_3} \tag{18}$$

$$\iff K = 2 \tag{19}$$

Similarly, the expression of the natural frequency ω_n from equation 13 may be simplified with the new conditions:

$$\omega_n = \frac{1}{\sqrt{R_1R_2C_1^2}} \tag{20}$$

$$\iff \quad \omega_n = \frac{1}{C_1}\sqrt{\frac{1}{R_1R_2}} \tag{21}$$

Lastly, simplifying the damping ratio ζ from equation 16 under the new conditions yields to the following expression:

$$\zeta = \frac{C_1(R_1 + R_2) - R_1C_1(2 - 1)}{2C_1\sqrt{R_1R_2}} \tag{22}$$

$$\iff \zeta = \frac{C_1(R_1 + R_2 - R_1)}{2C_1\sqrt{R_1R_2}} \tag{23}$$

$$\iff \zeta = \frac{R_2}{2\sqrt{R_1R_2}} \tag{24}$$

The next step is to find the relation between R_1 and R_2 so that the system has the desired output waveform.

2.2.1 Underdamped

For an underdamped system ζ must be smaller than 1, that is, $1 > \frac{R_2}{2\sqrt{R_1 R_2}}$. For a underdamped response the components must meet the requirements in equation 25.

$$4R_1 > R_2 \tag{25}$$

2.2.2 Overdamped

For an overdamped system ζ must be greater than 1, that is, $1 < \frac{R_2}{2\sqrt{R_1 R_2}}$. For a overdamped response the components must meet the requirements in equation 26.

$$4R_1 < R_2 \tag{26}$$

2.2.3 Critically Damped

For an critically damped system ζ must be equal to 1, that is, $1 = \frac{R_2}{2\sqrt{R_1 R_2}}$. For a critically damped response the components must meet the requirements in equation 27.

$$4R_1 = R_2 \tag{27}$$

2.2.4 Components Values

With the specifications in equations 25, 26, and 27 and the values available in the laboratory, it is possible to define the table below with the components values and parameter values.

Output Shape	R1	R2	R3	R4	C1	C2	K	ζ	ω_n
Overdamped	220Ω	10kΩ	680Ω	680Ω	100μF	100μF	2	3.7009	67419.986
Critically Damped	253Ω	1.01kΩ	4.7kΩ	4.7kΩ	10μF	10μF	2	0.9990	197.8240
Underdamped	680Ω	10Ω	330Ω	330Ω	10μF	10μF	2	0.0606	1212.6781

Table 1: Components values and parameter values - Chosen from laboratory availability

To choose the values of R_1 and R_2, the following three sets of MATLAB code was used to finder the suitable values for the available resistors. It is worth to mention that there multiple combinations of resistor values to achieve either an underdamped system or an overdamped system.

According to the output from the MATLAB code below for the critically damped system, there is no possible combination that can achieve the requirements for this type of system. The only solution would be to use a series of two resistors for R_1 in order to achieve an approximate value of the necessary requirements of a critically damped system. In similar approach, it is necessary to combine two resistors values for R_2. These values are filled in the table above. In this sense it was calculated the values in these combinations for both R_1 and R_2:

$$R_{1CD} = 220\Omega + 33\Omega \tag{28}$$

$$R_{2CD} = 680\Omega + 330\Omega \tag{29}$$

where these values for R_1 and R_2 approximately achieved both requirements for a critically damped system, that are stated in subsection 2.2.3:

$$1 \approx \frac{R_{2CD}}{2\sqrt{R_{1CD} R_{2CD}}} \tag{30}$$

$$4R_{1CD} \approx R_{2CD} \tag{31}$$

In the case of choosing the values of the capacitors, any of the nine capacitors available in the laboratory can be used since all of them satisfy the initial condition mentioned in the hint, which states that C_1 must be equal to C_2.

```
%Here is included the values for all resistors available in the laboratory
resistor_available = [10, 22, 33, 100, 220, 330, 470, 680, 1000, 2200, 4700, 10000,
    22000, 47000, 100000, 1000000];
valid_combinations = [];

% For loop to test the values
for i = 1:length(resistor_available)
    for j = 1:length(resistor_available)
        R1 = resistor_values(i);
        R2 = resistor_values(j);

        %It checks the conditions for Underdamped system
        if (R2/(2*sqrt(R1/R2)) < 1) && (4*R1 > R2)
            valid_combinations = [valid_combinations; R1, R2];
        end
    end
end
% Prints the results
disp('Valid combinations (R1, R2):');
disp(valid_combinations);
```

Listing 1: MATLAB code - Underdamped system

```
%Here is included the values for all resistors available in the laboratory
resistor_available = [10, 22, 33, 100, 220, 330, 470, 680, 1000, 2200, 4700, 10000,
    22000, 47000, 100000, 1000000];
valid_combinations = [];

% For loop to test the values
for i = 1:length(resistor_available)
    for j = 1:length(resistor_available)
        R1 = resistor_values(i);
        R2 = resistor_values(j);

        %It checks the conditions for Critically Damped system
        if (R2/(2*sqrt(R1/R2)) == 1) && (4*R1 == R2)
            valid_combinations = [valid_combinations; R1, R2];
        end
    end
end

% Prints the results
disp('Valid combinations (R1, R2):');
disp(valid_combinations);
```

Listing 2: MATLAB code - Critically Damped system

```
%Here is included the values for all resistors available in the laboratory
resistor_available = [10, 22, 33, 100, 220, 330, 470, 680, 1000, 2200, 4700, 10000,
    22000, 47000, 100000, 1000000];
valid_combinations = [];

% For loop to test the values
for i = 1:length(resistor_available)
    for j = 1:length(resistor_available)
        R1 = resistor_values(i);
        R2 = resistor_values(j);

        %It checks the conditions for Overdamped system
```

```
        if (R2/(2*sqrt(R1/R2)) > 1) && (4*R1 < R2)
            valid_combinations = [valid_combinations; R1, R2];
        end
    end
end

% Prints the results
disp('Valid combinations (R1, R2):');
disp(valid_combinations);
```

Listing 3: MATLAB code - Overdamped system

2.3 Simulation

2.3.1 Overdamped

Replacing the values for the overdamped system response from table 1 in equation (8) we get the transfer function 32.

$$G(s) = \frac{2}{(2.2 \times 10^{-10})s^2 + (0.0001)s + 1} \tag{32}$$

With the transfer function in 32 the block diagram in figure 3 was assembled and fed with a square wave of 2Vpp and 10Hz frequency. The system was simulated for 5 seconds.

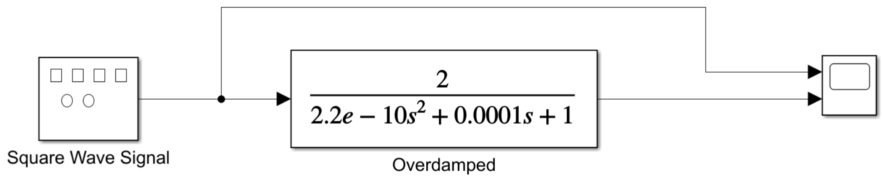

Figure 3: Block Diagram for the overdamped second order system with the projected parameters

Using the Simulink horizontal scope cursor, in figure 4, is possible to see that the gain is 2 and it is possible to observe that the output waveform is overdamped, as expected.

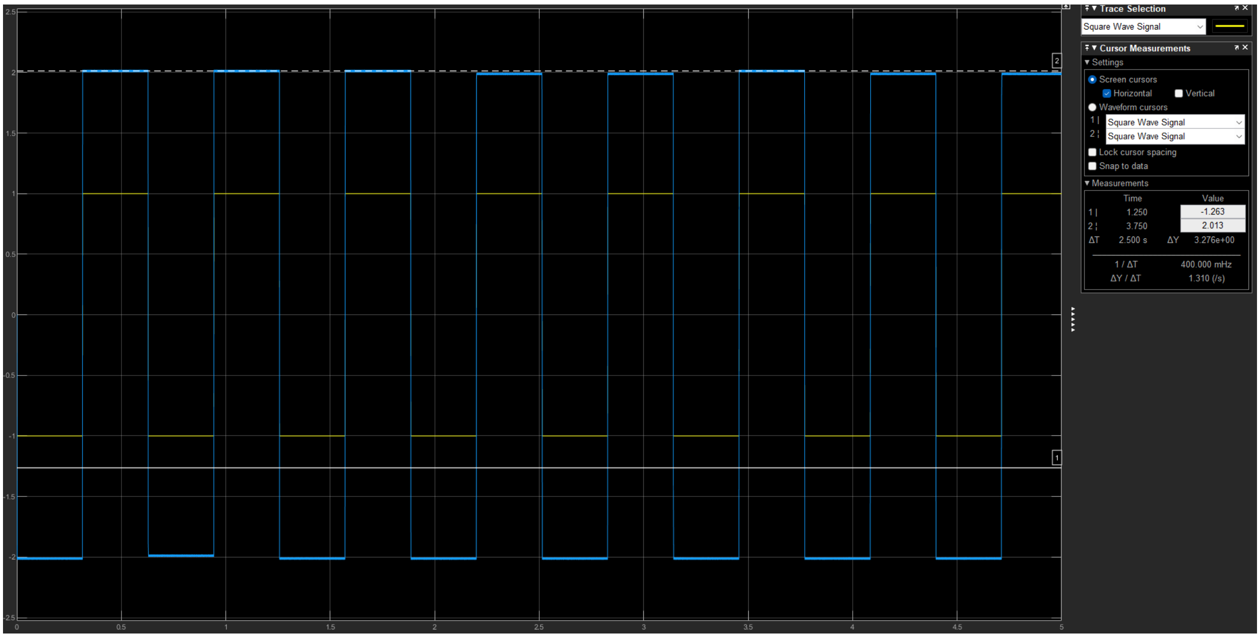

Figure 4: Waveform of input and output for the overdamped second order system with horizontal cursor to verify the gain value

2.3.2 Critically Damped

Replacing the values for the critically damped system response from table 1 in equation (8) we get the transfer function 33.

$$G(s) = \frac{2}{(2.5553 \times 10^{-5})s^2 + (10.0998 \times 10^{-3})s + 1} \tag{33}$$

With the transfer function in 33 the block diagram in figure 5 was assembled and fed with a square wave of 2 Vpp, and 10 Hz frequency. The system was simulated for seconds.

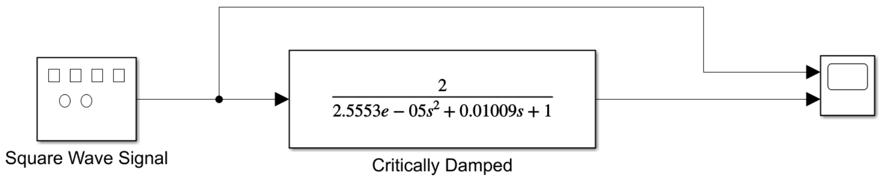

Figure 5: Block Diagram for the critically damped second order system with the projected parameters

Using the Simulink horizontal scope cursor, in figure 6, is possible to see that the gain is 2 and it is possible to observe that the output waveform is critically damped, as expected.

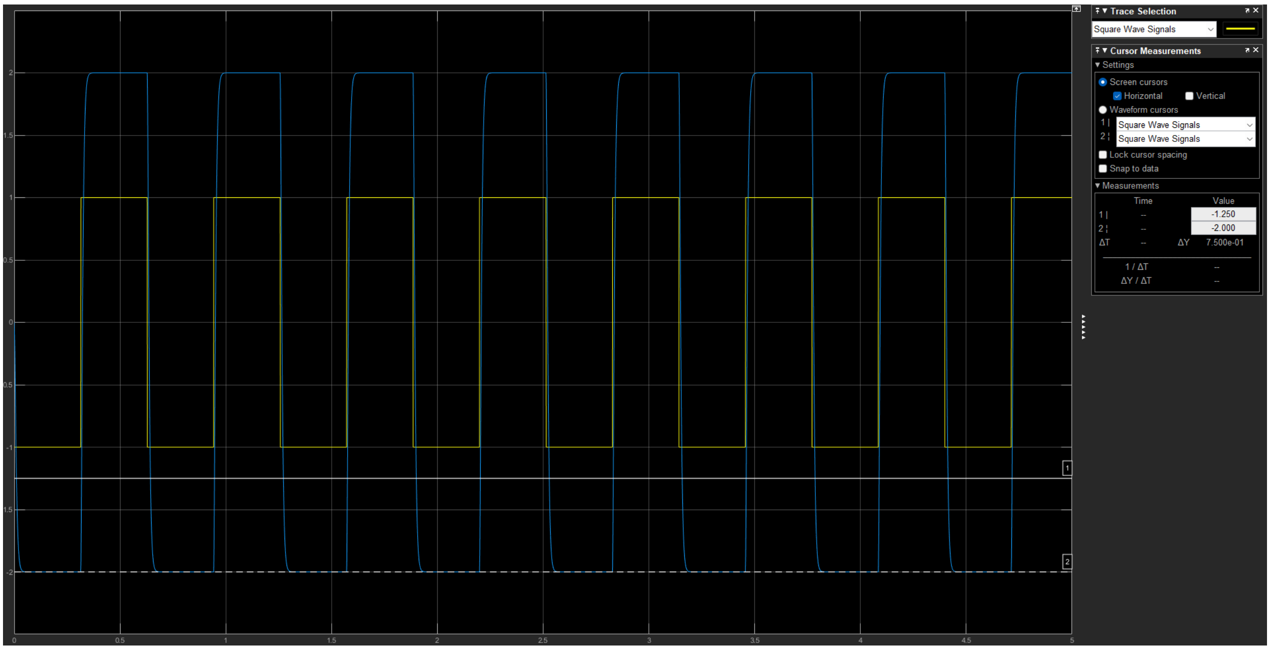

Figure 6: Waveform of input and output for the critically damped second order system with horizontal cursor to verify the gain value

2.3.3 Underdamped

Replacing the values for the underdamped system response from table 1 in equation (8) we get the transfer function 34.

$$G(s) = \frac{2}{(6.8 \times 10^{-7})s^2 + (9.9944 \times 10^{-5})s + 1} \tag{34}$$

With the transfer function in equation (34), the block diagram in Figure 7 was assembled and fed with a square wave of 2 Vpp, this means that in Simulink, the amplitude was set to 1. And the frequency was set to 10 Hz. The system was simulated for 5 seconds.

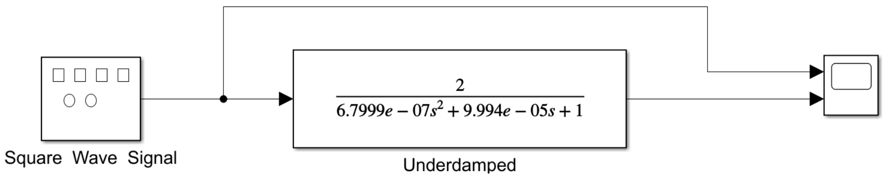

Figure 7: Block Diagram for the underdamped second order system with the projected parameters

Using the Simulink horizontal scope cursor, in figure 8, is possible to see that the gain is 2 and it is possible to observe that the output waveform is underdamped, as expected.

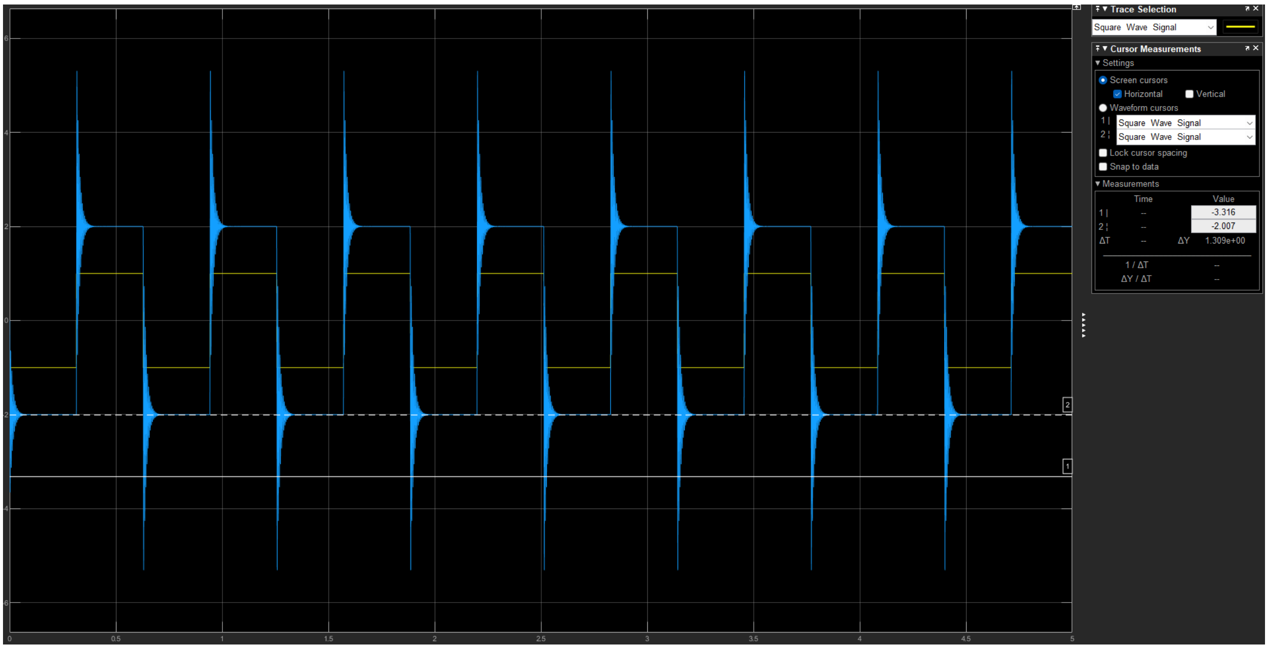

Figure 8: Waveform of input and output for the underdamped second order system with horizontal cursor to verify the gain value