BCT 2202 – Lecture 6 - DC circuit theory II

1 Thévenin's theorem

Thévenin's theorem states: *The current in any branch of a network is that which would result if an e.m.f. equal to the p.d. across a break made in the branch, were introduced into the branch, all other e.m.f.'s being removed and represented by the internal resistances of the sources.* The procedure adopted when using Thévenin's theorem to determine the current in any branch of an active network (i.e., one containing a source of e.m.f.) is,

i. remove the resistance R from that branch,
ii. determine the open-circuit voltage, E, across the break,
iii. remove each source of e.m.f. and replace them by their internal resistances and then determine the resistance, r, 'looking-in' at the break,
iv. determine the value of the current from the equivalent circuit shown in Figure 1, i.e.

$$I = \frac{E}{R + r}$$

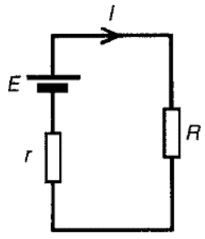

Figure 1

Example

Use Thévenin's theorem to find current flowing in the 10 Ω resistor in Figure 2

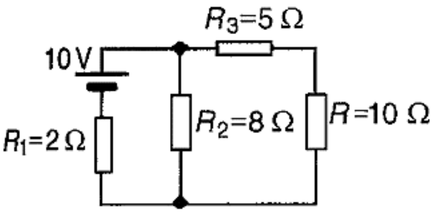

Figure 2

Solution

1. The 10 Ω resistance is removed from the circuit as shown in Figure 2(a)

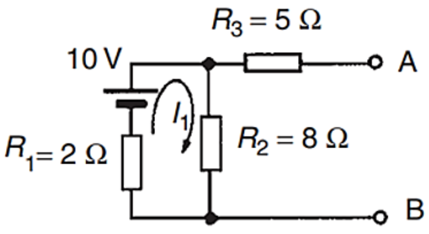

Figure 2(a)

There is no current flowing in the 5 Ω resistor and current I_1 is given by,

$$I_1 = \frac{10}{R_1 + R_2} = \frac{10}{2 + 8} = 1\ A$$

P.d. across $R_2 = I_1 R_2 = 1 \times 8 = 8V$. Hence p.d. across AB, i.e., the open-circuit voltage across the break, $E = 8V$

2. Removing the source of e.m.f. gives Figure 2(b).

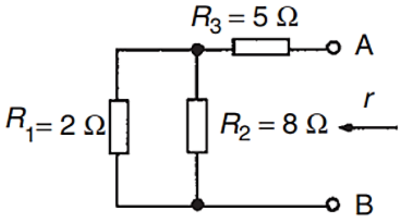

Figure 2(b)

Resistance,

$$r = R_3 + \frac{R_1R_2}{R_1 + R_2} = 5 + \frac{2x8}{2+8} = 5 + 1.6 = 6.6\Omega$$

3. The equivalent Thévenin's circuit is shown in Figure 2(c)

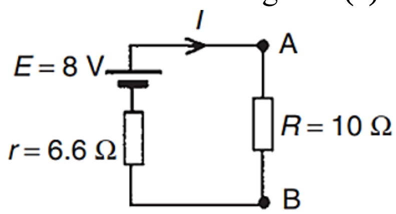

Figure 2(c)

Current

$$I = \frac{E}{R + r} = \frac{8}{10 + 6.6} = \frac{8}{16.6} = 0.482\ A$$

Hence the current flowing in the 10Ω resistor is 0.482A.

Example

Use Thévenin's theorem to determine the current flowing in the 4 Ω resistor in Figure 3. Find also the power dissipated in the 4 Ω resistor.

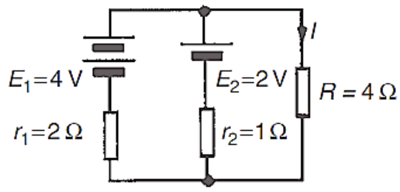

Figure 3

Solution

1. The 4 Ω resistor is removed from the circuit as shown in Figure 3(a)

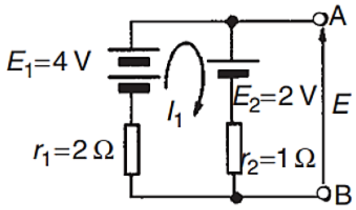

Figure 3(a)

Current

$$I_1 = \frac{E_1 - E_2}{r_1 + r_2} = \frac{4 - 2}{2 + 1} = \frac{2}{3} A$$

P.d across AB,

$$E = E_2 + I_1r_2 = 2 + \frac{2}{3}(1) = 2\left(\frac{2}{3}\right) V$$

2. Removing the sources of e.m.f. gives Figure 3(b).

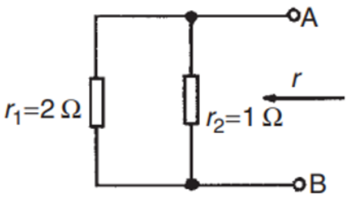

Figure 3(b)

From which, resistance

$$r = \frac{2x1}{2+1} = \frac{2}{3}\Omega$$

3. The equivalent Thévenin's circuit is shown in Figure 3(c).

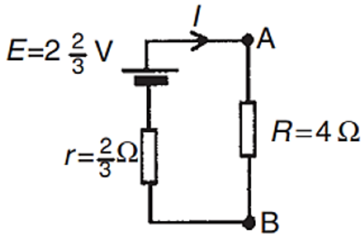

Figure 3(c)

From which, current in the 4Ω resistor

$$I = \frac{E}{r+R} = \frac{2\left(\frac{2}{3}\right)}{\frac{2}{3}+4} = \frac{8/3}{14/3} = \frac{8}{14} = 0.571 \text{ A}$$

Power dissipated in the 4Ω resistor,

$$P = I^2R = (0.571^2)4 = 1.304 \text{ W}$$

Example

Determine the current in the 5 Ω resistance of the network shown in Figure 4 using Thévenin's theorem. Hence find the currents flowing in the other two branches.

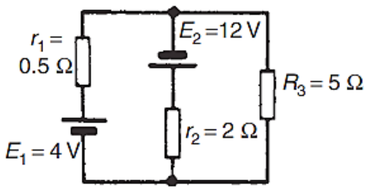

Figure 4

Solution

1. The 5 Ω resistance is removed from the circuit as shown in Figure 4(a).

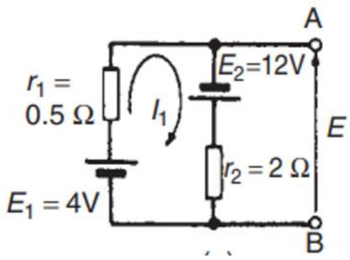

Figure 4(a)

Current

$$I_1 = \frac{12+4}{0.5+2} = \frac{16}{2.5} = 6.4\text{A}$$

P.d across AB

$$E = -E_2 + I_1 r_1 = -12 + (6.4)(2) = 0.8\ \mathrm{V}$$

2. Removing the sources of e.m.f. gives the circuit shown in Figure 4(b).

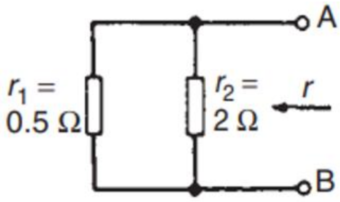

Figure 4(b)

From which resistance

$$r = \frac{0.5 \times 2}{0.5 + 2} = 0.4\Omega$$

3. The equivalent Thévenin's circuit is shown in Figure 4(c).

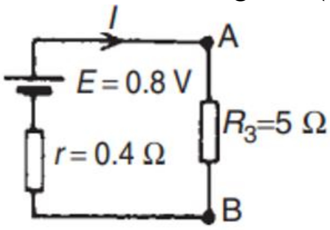

Figure 4(c)

From which, current in the 5Ω resistor

$$I = \frac{E}{r + R} = \frac{0.8}{0.4 + 5} = 0.148\Omega$$

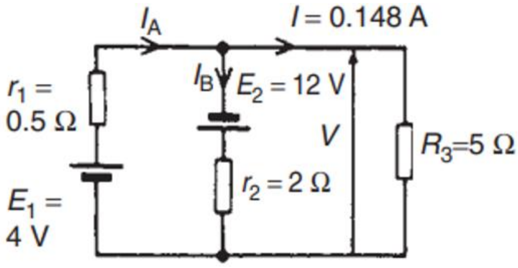

Figure 4(d)

From Figure 4(d), voltage

$$V = IR_3 = (0.148)(5) = 0.74\mathrm{V}$$
$$V = E_1 - I_A r_1$$
$$0.74 = 4 - (I_A)(0.5)$$

Hence

$$I_A = \frac{4 - 0.74}{0.5} = 6.52\ \mathrm{A}$$

Also, from Figure 4(d)

$$V = -E_2 + I_B r_2$$
$$0.74 = -12 + I_B(2)$$
$$I_B = \frac{12 + 0.74}{2} = 6.37\ \mathrm{A}$$

2 Constant-current source

The Thévenin constant-voltage source consisted of a constant e.m.f. E in series with an internal resistance r. However, this is not the only form of representation. A source of electrical energy can also be represented by a constant-current source in parallel with a resistance.

i. An ideal constant-voltage generator is one with zero internal resistance so that it supplies the same voltage to all loads.
ii. An ideal constant-current generator is one with infinite internal resistance so that it supplies the same current to all loads.

The symbol for an ideal current source is shown in Figure 5.

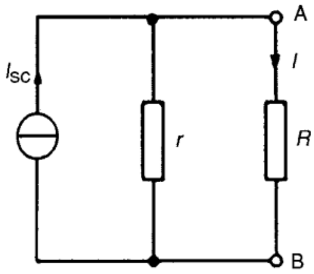

Figure 5

3 Norton's theorem

Norton's theorem states: *The current that flows in any branch of a network is the same as that which would flow in the branch if it were connected across a source of electrical energy, the short-circuit current of which is equal to the current that would flow in a short-circuit across the branch, and the internal resistance of which is equal to the resistance which appears across the open-circuited branch terminals.* The procedure adopted when using Norton's theorem to determine the current flowing in a resistance R of a branch AB of an active network is

i. short-circuit branch AB,
ii. determine the short-circuit current I_{SC} flowing in the branch,
iii. remove all sources of e.m.f. and replace them by their internal resistance (or, if a current source exists, replace with an open-circuit), then determine the resistance r, 'looking-in' at a break made between A and B
iv. determine the current flowing in resistance R from the Norton equivalent network shown in Figure 6, i.e.,

$$I = \left(\frac{r}{r + R}\right) I_{sc}$$

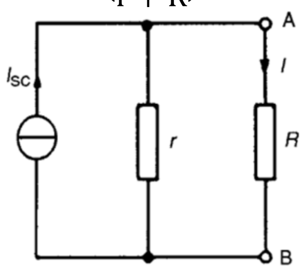

Figure 6

Example

Use Norton's theorem to determine the current flowing in the 10 Ω resistance for the circuit shown in Figure 7.

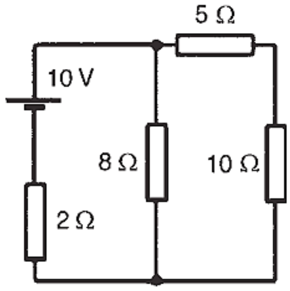

Figure 7

Solution

1. The branch containing the 10 Ω resistance is short-circuited as shown in Figure 7(a).

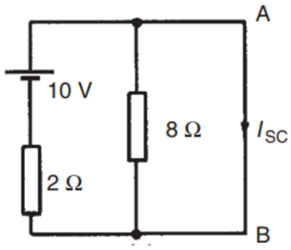

Figure 7(a)

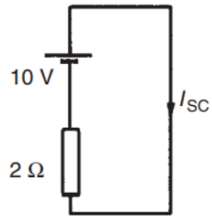

Figure 7(b

Figure 7(b) is equivalent to Figure 7(a). Hence

$$I_{SC} = \frac{10}{2} = 5\ \mathrm{A}$$

2. If the 10 V source of e.m.f. is removed from Figure 7(a) the resistance 'looking-in'at a break made between A and B is given by

$$r = \frac{2x8}{2+8} = 1.6\Omega$$

3. From the Norton equivalent network shown in Figure 7(c).

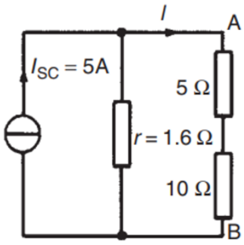

Figure 7(c)

The current in the 10 Ω resistance, by current division, is given by:

$$I = \left(\frac{1.6}{1.6+5+10}\right)(5) = 0.482\ \mathrm{A}$$

Example

Use Norton's theorem to determine current flowing in the 4 Ω resistance shown in Figure 8.

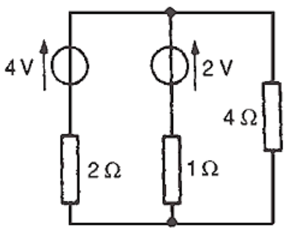

Figure 8

Solution

1. The 4 Ω branch is short-circuited as shown in Figure 8(a)

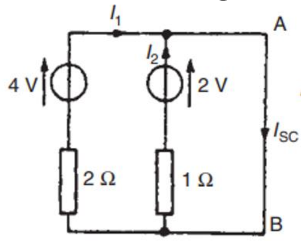

Figure 8(a)

From Figure 8(a),

$$I_{SC} = I_1 + I_2 = \frac{4}{2} + \frac{2}{1} = 4\ A$$

2. If the sources of e.m.f. are removed the resistance 'looking-in' at a break made between A and B is given by:

$$r = \frac{2x1}{2+1} = \frac{2}{3}\Omega$$

3. From the Norton equivalent network shown in Figure 8(b)

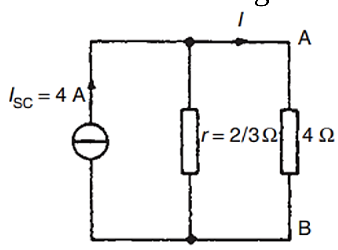

Figure 8(b)

The current in the 4 Ω resistance is given by:

$$I = \left(\frac{\frac{2}{3}}{\frac{2}{3}+4}\right)(4) = 0.571\ A$$

Example

Determine the current in the 5 Ω resistance of the network shown in Figure 9 using Norton's theorem. Hence find the currents flowing in the other two branches.

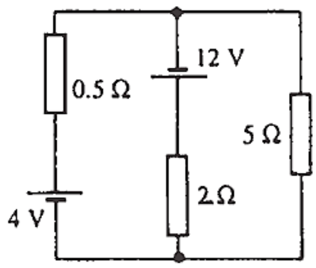

Figure 9

Solution

1. The 5 Ω branch is short-circuited as shown in Figure 9(a).

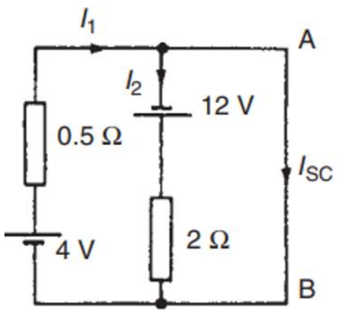

Figure 9(a)

From Figure (a)

$$I_{SC} = I_1 - I_2 = \frac{4}{0.5} - \frac{12}{2} = 2 \text{ A}$$

2. If each source of e.m.f. is removed the resistance 'looking-in' at a break made between A and B is given by:

$$r = \frac{0.5 \text{ x } 2}{0.5 + 2} = 0.4 \ \Omega$$

3. From the Norton equivalent network shown in Figure 9(b)

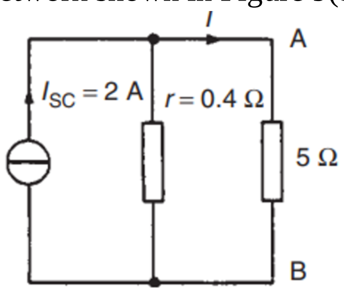

Figure 9(b)

Current in the 5 Ω resistance is given by:

$$I = \left(\frac{0.4}{0.4 + 5}\right)(2) = 0.148 \text{ A}$$

The currents flowing in the other two branches are obtained in the same way. Hence the current flowing from the 4V source is 6.52A and the current flowing from the 12V source is 6.37A.

4 Maximum power transfer theorem

The maximum power transfer theorem states: *The power transferred from a supply source to a load is at its maximum when the resistance of the load is equal to the internal resistance of the source.*

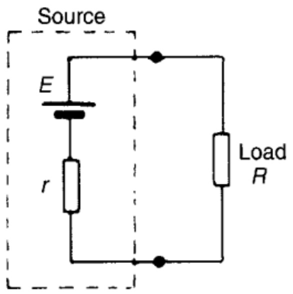

Figure 10

Hence, in Figure 10, when R = r the power transferred from the source to the load is a maximum. Typical practical applications of the maximum power transfer theorem are found in stereo amplifier design, seeking to maximise power delivered to speakers, and in electric vehicle design, seeking to maximise power delivered to drive a motor.

Example

A DC source has an open-circuit voltage of 30V and an internal resistance of 1.5 **Ω**. State the value of load resistance that gives maximum power dissipation and determine the value of this power.

Solution

The circuit diagram is shown in Figure 11. From the maximum power transfer theorem, for maximum power dissipation, $R_L = r = 1.5\ \Omega$.

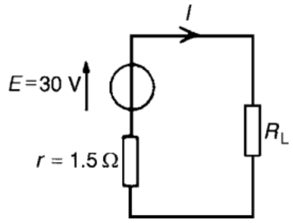

Figure 11

From Figure 11, current I,

$$I = \frac{E}{(r + R_L)} = \frac{30}{(1.5 + 1.5)} = 10\ \text{A}$$

Maximum power dissipated by load,

$$P = I^2 R_L = (10^2)(1.5) = 150\text{W}$$

Example

Find the value of the load resistor R_L in Figure 12 that gives maximum power dissipation and determine the value of this power.

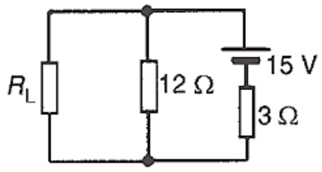

Figure 12

Using the procedure for Thévenin's theorem:

1. Resistance R_L is removed from the circuit as shown in Figure 12(a)

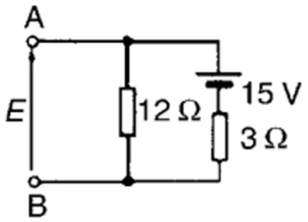

Figure 12(a)

The p.d. across AB is the same as the p.d. across the 12 **Ω** resistor. Hence

$$E = \left(\frac{12}{12 + 3}\right)(15) = 12\text{V}$$

2. Removing the source of e.m.f. gives Figure 12(b),

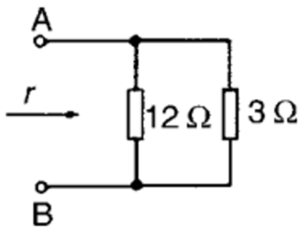

Figure 12(b)

from which, resistance,

$$r = \frac{12x3}{12 + 3} = 2.4\ \Omega$$

3. The equivalent Thévenin's circuit supplying terminals AB is shown in Figure 12(c),

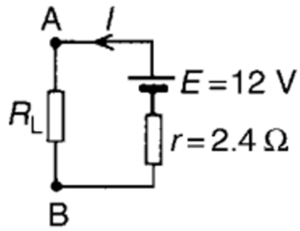

Figure 12 (c)

from which current

$$I = \frac{E}{r + R_L}$$

For maximum power, $R_L = r = 2.4\ \Omega$

Thus current

$$I = \frac{12}{2.4 + 2.4} = 2.5A$$

Power dissipated in load,

$$P = I^2R_L = 2.5^2(2.4) = 15\ W$$