Giant Diffusion in Systems with bistable rate dynamics

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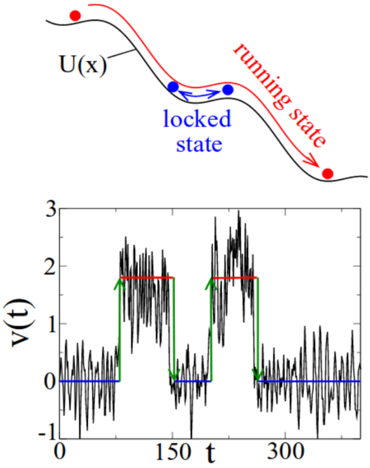
16 June 2020

Giant diffusion of Brownian particles[1]

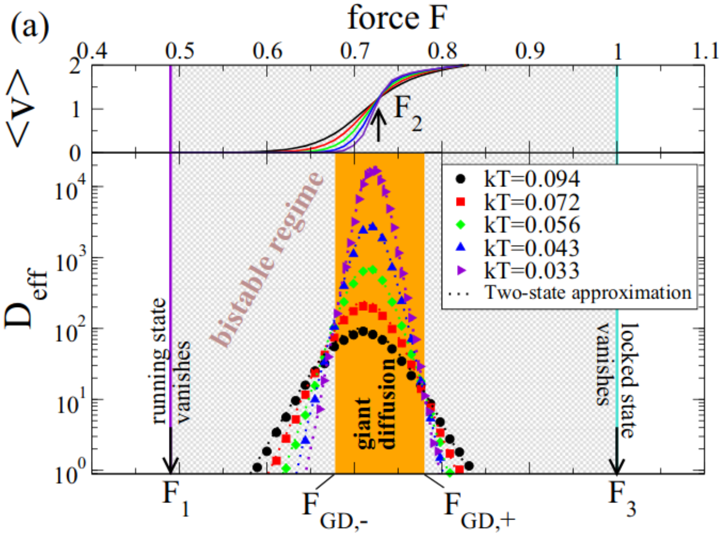


- Motion of particles in a tilted cosine potential
- **bistability** of velocity
- effective diffusion coefficient:

$$D_{eff} = lim_{t\to\infty} \frac{\langle x^2(t)\rangle - \langle x(t)\rangle^2}{2t}$$

[1] B. Lindner and I. M. Sokolov, "Giant diffusion of underdamped particles in a biased periodic potential", *PRE 93*, 2016.

Giant diffusion of Brownian particles



Brownian particle vs Neuron model

mechanical interpretation	neuroscience interpretation
position, phase	spike count
mean velocity	firing rate
diffusion coefficient	Fano factor x rate
Velocity power spectrum	Spike train power spectrum
temperature	inverse number of channels

Quantities of interest in neuron models

- effective diffusion coefficient

$$D_{eff} = lim_{t\to\infty} \frac{\left\langle x^2(t) \right\rangle - \langle x(t) \rangle^2}{2t} = \frac{\left\langle N^2(t) \right\rangle - \langle N(t) \rangle^2}{2t}$$

- mean velocity (firing rate):

$$r = lim_{t\to\infty} \frac{\langle x(t) - x(0) \rangle}{t} = \frac{\langle N(t) \rangle}{t}$$

- Fano factor:

$$F = \frac{\left\langle \Delta N^2(t) \right\rangle}{\langle N(t) \rangle}$$

Goals of this thesis

- Primary goal: induce stochastic **bursting** in **simple** neuron models and investigate spike count statistics
- find **critical points** where firing pattern changes drastically
- Secondary goal: simulate neurons under influence of periodic stimulus to explore **signal transmission**

Model

- $I_{Na,p} + I_K$ model:

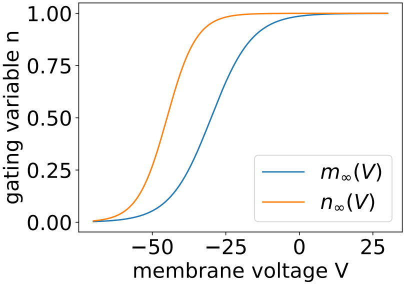
$$C\dot{V} = I - g_L(V - E_L) - g_{Na} m_\infty(V)(V - E_{Na}) - g_K n(V - E_K) + \sqrt{2D}\xi(t)$$
$$\dot{n} = (n_\infty(V) - n)/\tau(V)$$

Steady-State activation function:

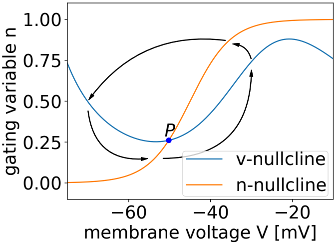
$$f_\infty(V) = \frac{1}{1 + \exp\{(V_{1/2} - V)/k\}}$$

where k is the slope factor and

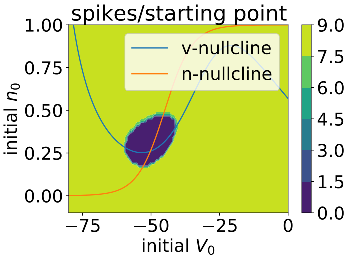
$$f_\infty(V_{1/2}) = 1/2$$



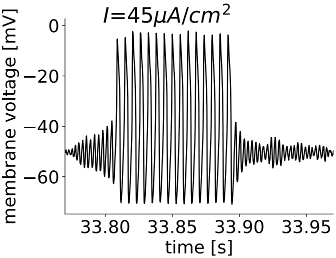
Phase space



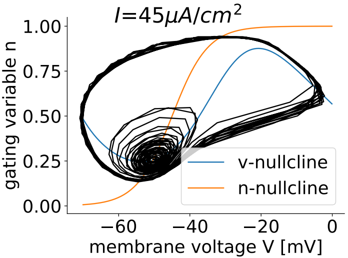
gating variable n
1.00
0.75
0.50
0.25
0.00
P
v-nullcline
n-nullcline
−60
−40
−20
membrane voltage V [mV]



spikes/starting point
1.00
0.75
0.50
0.25
0.00
initial n_0
v-nullcline
n-nullcline
−75
−50
−25
0
initial V_0
9.0
7.5
6.0
4.5
3.0
1.5
0.0

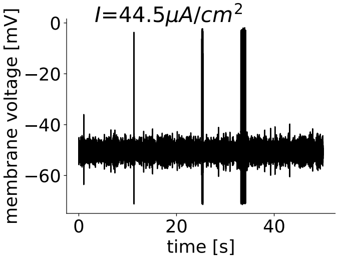


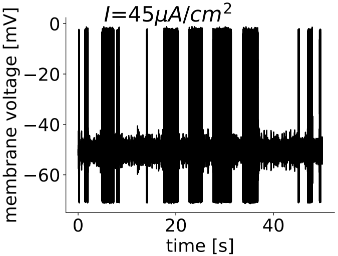
I=45$\mu A/cm^2$
membrane voltage [mV]
0
−20
−40
−60
33.80
33.85
33.90
33.95
time [s]

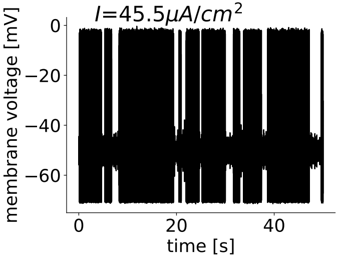


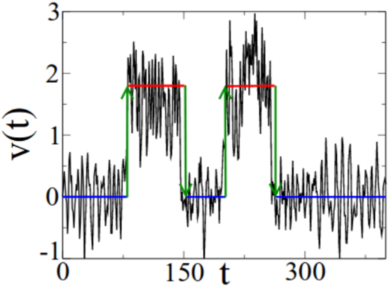
I=45$\mu A/cm^2$
gating variable n
1.00
0.75
0.50
0.25
0.00
v-nullcline
n-nullcline
−60
−40
−20
0
membrane voltage V [mV]

Variation of I

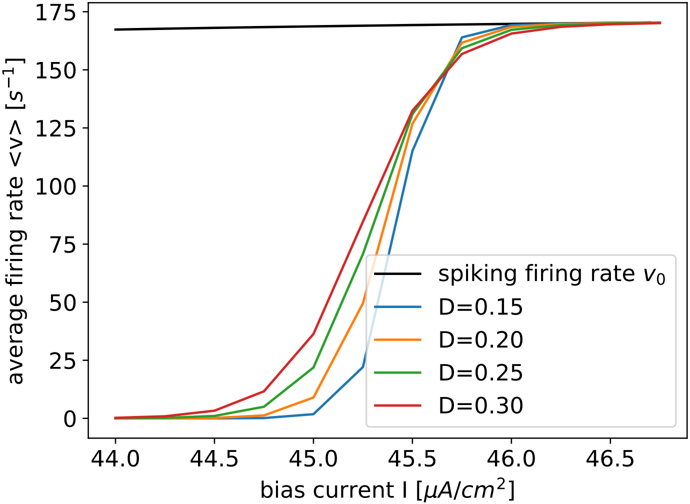




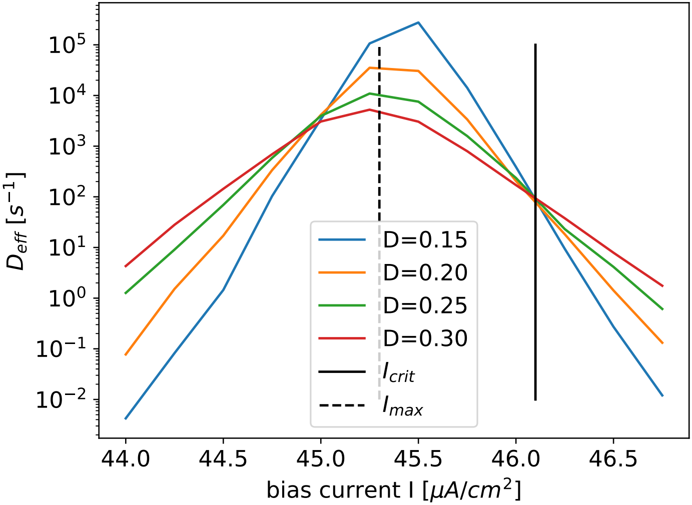




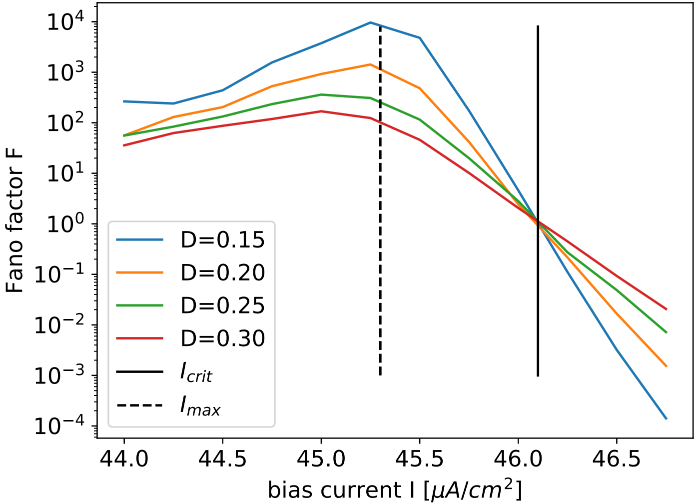
Simulation: firing rate



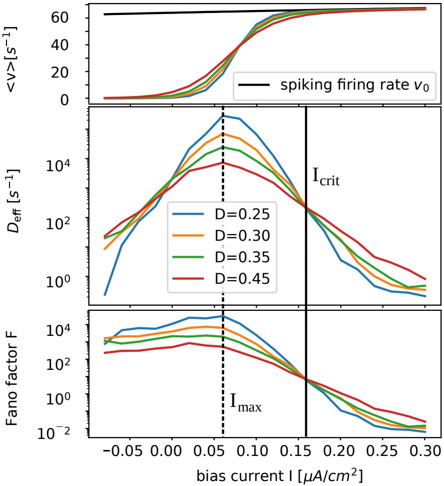
Simulation: effective diffusion coefficient

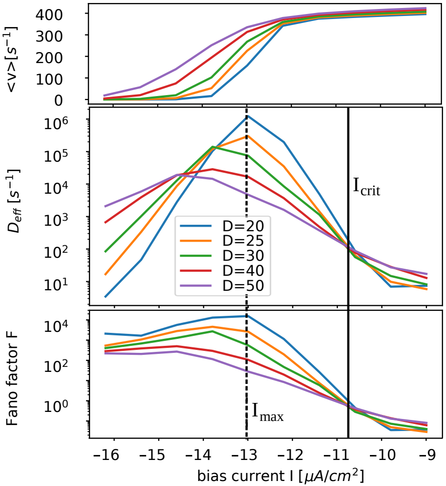


Simulation: Fano factor



Simulation: other models





Two-state theory

- **low noise**: system mostly determined by **transitions** between states
- alternative calculation of D_{eff} using transition rates $r_\pm$[1]

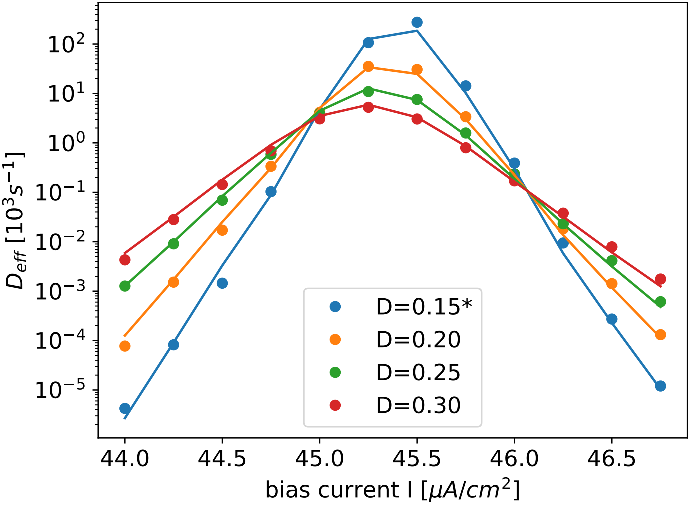
$$D_{\mathrm{eff}} = \frac{(\Delta \langle v \rangle)^2 r_+ r_-}{(r_+ + r_-)^3}$$

- fit with **Arrhenius equation**: effective potential barriers $\Delta U_\pm$ and prediction

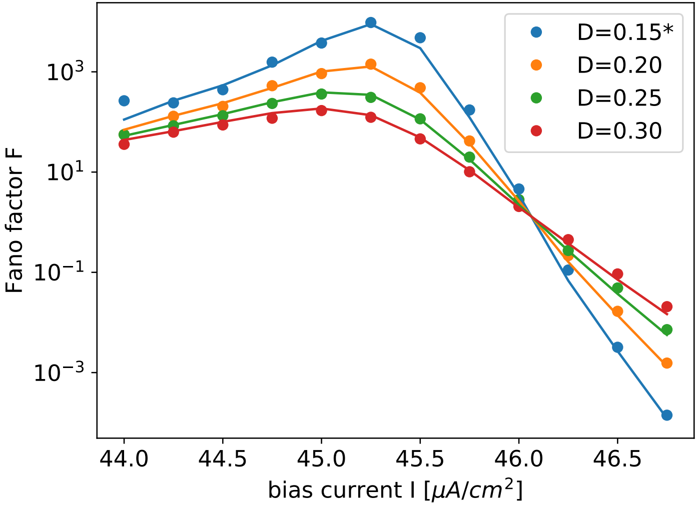
$$r_\pm = r_{0,\pm} \mathrm{e}^{-\frac{\Delta U_\pm}{D}}$$

[1] C. Van den Broeck, "Taylor dispersion revisited", *Physica A 168*, 1990.

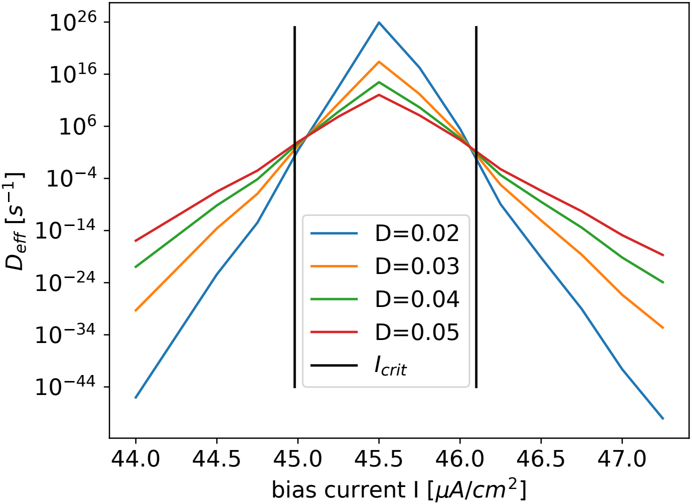
Comparison with Two-state model: D_{eff}



Comparison with Two-state model: F



Prediction with Two-state model



Consequences

for

Signal transmission

Measurement of SNR

- System with **weak+slow** cosine Signal:

$$C\dot{V} = f(V,n,t) \rightarrow C\dot{V} = f(V,n,t) + \varepsilon\cos(\omega t + \phi)$$

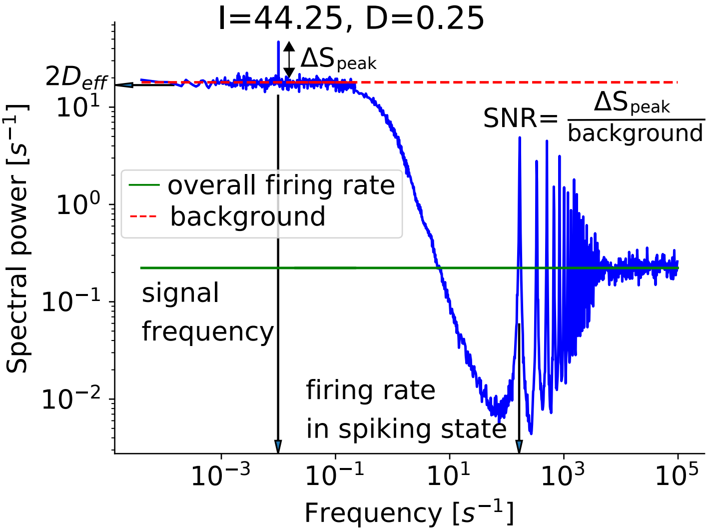
- Spectrum from **Fourier-Trafo** of the δ-Spike-train
- Signal-to-noise ratio SNR depends on firing rate $\langle v \rangle$ and D_{eff}[1]:

$$SNR = \frac{\Delta S_{peak}}{S_{bg}} = \frac{\varepsilon^2 T |d\langle v \rangle / dI|^2}{8 \cdot D_{eff}}$$

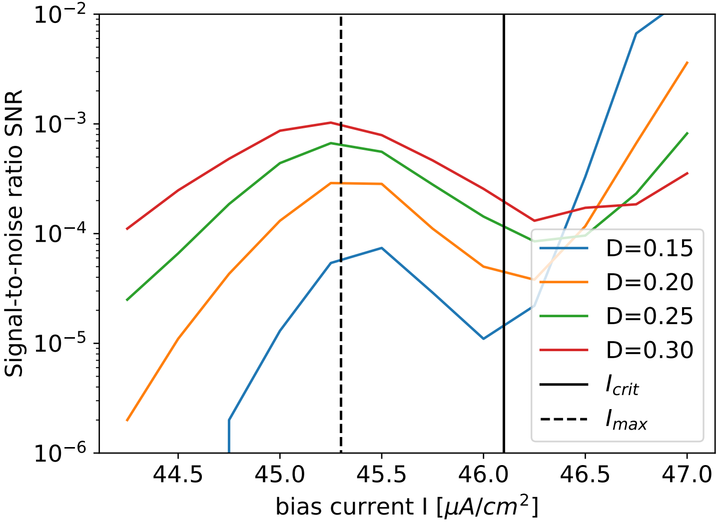
ε...signal amplitude
T..simulation time
I....bias current

[1] L. Gammaitoni et al., " Stochastic resonance", *Rev. Mod. Phys. 70*, 1998.

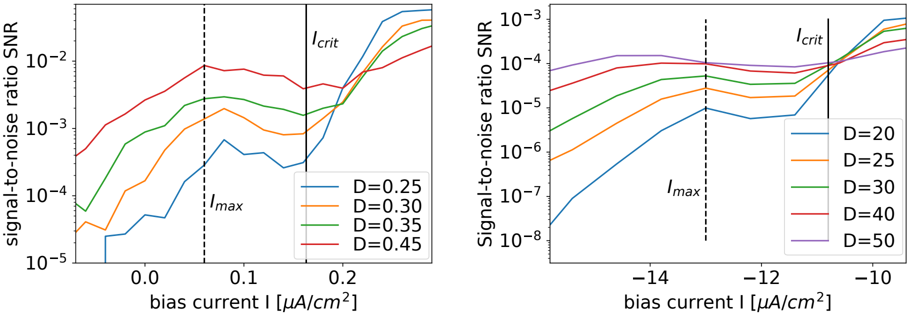
Frequency Power Spectrum



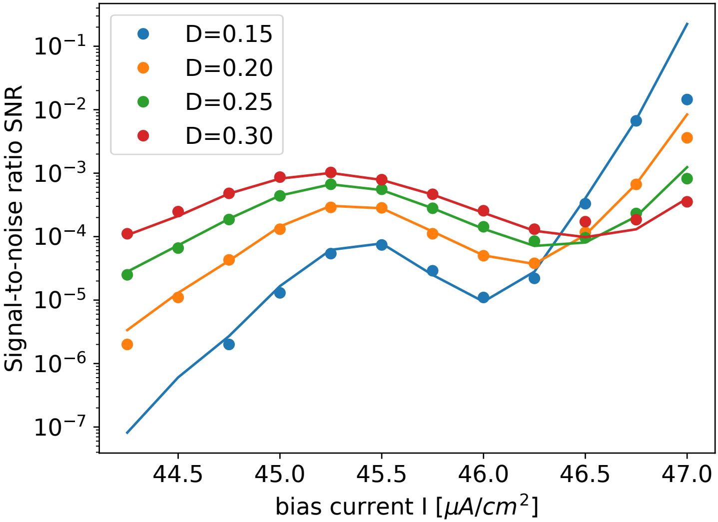
SNR


Signal-to-noise ratio SNR
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
44.5
45.0
45.5
46.0
46.5
47.0
bias current I [$\mu A/cm^2$]
D=0.15
D=0.20
D=0.25
D=0.30
I_{crit}
I_{max}

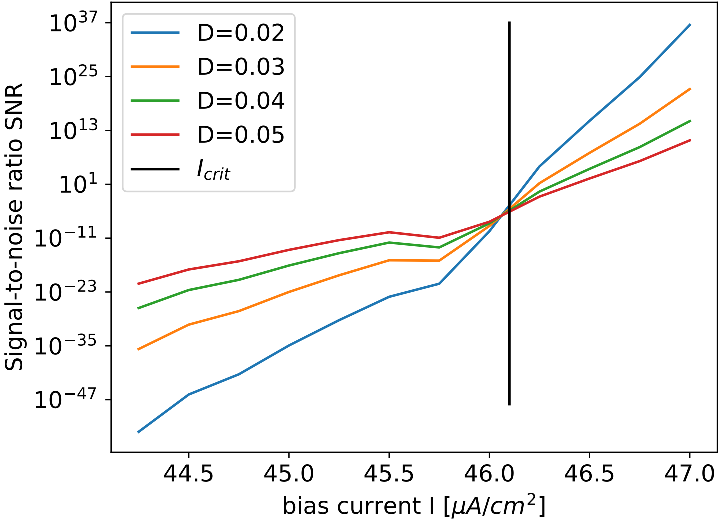
SNR: other models



Comparison SNR - Two-state theory



Prediction with Two-state theory



Conclusion and outlook

- giant diffusion in multiple models $\rightarrow$ **bistability only requirement**?
- **critical points** where D_{eff} noise-independent and **steep changes** of D_{eff} and SNR observed
- **Two-state theory** gives **good** description
- future: experimental investigation of bistable neurons