



Lookback Option

Zijun Zhou, Fangzheng Lu, Jialei Cai, Xiangyu Fan

Introduction to the Lookback Option

- **Floating Lookback Option**

The strike price is set automatically at maturity to the most favorable underlying price reached during the contract's life.

The option will then settle against the market price.

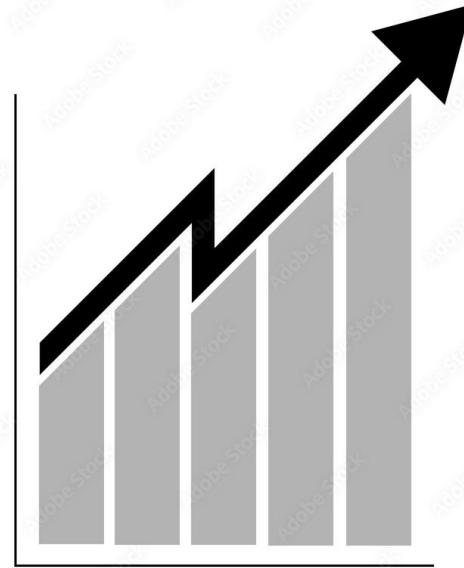
- **Fixed Lookback Option**

The strike price is set or fixed at purchase.

At the time of exercise, the most beneficial price of the underlying asset over the life of the contract is used instead of the current market price.

Why We Choose the Lookback Option

- **Increased flexibility**
- **Reduced market risk**
- **Potential for enhanced returns**



Data Selection And Simulation

- Stock: Coca-Cola(COKE) 06/28/2022—06/27/2023 Closing Price (Yahoo Finance)
- 12-day simulation of the stock price using the Monte Carlo method and Black-Scholes Formula (for expected stock price instead of the European call option price as this is an exotic option)

Simulations

Fixed Lookback option	COKE	
We assume that this option has 12 days		
r	5.18%	
sigma	0.36162363	
spot price	644.77	
delta	3.01%	
strike	660	
T	0.04761905	

sim	day1	day2	day3	day4	day5	day6	day7	day8	day9	day10	day11	day12	lookback payoff	standard payoff
1	662.2424701	645.588162	657.050349	638.062743	642.989826	647.684907	666.923883	683.336645	696.486442	703.471778	669.128659	635.66354	43.47177789	
2	638.583704	638.618508	609.737445	615.129393	692.921541	591.276191	557.295012	543.942274	544.558341	574.481877	615.952992	617.380043		
3	650.7535048	639.795178	635.105559	655.10332	687.92124	658.494906	666.490051	675.981772	686.223519	686.63534	699.004381	675.942405	39.00438072	15.94205059
4	651.1433078	651.485126	649.657789	658.891254	646.327102	632.287735	621.448641	621.943178	620.006785	627.881439	642.417832	678.225684	18.22568391	18.22568391
5	652.386411	634.195019	640.755745	648.079081	645.690477	648.684743	638.896813	636.729777	644.351583	630.410441	617.949946	603.818613	0	0
6	665.0613408	657.220967	655.338856	667.194038	690.970929	685.304969	700.73506	677.195927	680.701265	675.162008	690.25843	704.311249	44.31124939	44.31124939
7	651.3096613	651.518696	647.695814	641.920299	636.622486	639.625945	635.459174	610.136562	602.640643	591.205591	575.707936	557.297077		
8	646.0409531	616.83068	607.884051	638.480951	625.565205	621.390907	629.406458	648.158046	630.623351	634.931971	627.639643	637.693643	0	0
9	644.7354443	648.635384	636.667049	652.292053	630.135126	628.994358	624.231308	616.057572	630.324340	635.837054	651.638771	662.491321	2.491320692	2.491320692
10	648.9126584	629.005126	625.395044	636.875158	625.904176	634.613044	649.831962	646.174036	658.978202	683.887288	687.1798	686.15943	27.14980141	26.15496261
11	626.734549	575.920439	555.254049	593.00352	575.351279	527.820744	540.023884	547.789881	542.785819	527.585359	525.127964	528.291834	0	0
12	669.1925832	649.075197	668.985235	677.050442	647.271145	648.673753	646.988324	656.988324	669.642267	674.016741	674.016741	653.735254	17.05044212	17.05044212
13	625.692061	624.306641	617.193732	609.605896	619.823423	626.649084	654.485154	647.17262	647.199147	665.595596	654.529407	632.107509	5.595959605	5.595959605
14	652.4965533	631.57458	655.187829	644.085048	623.855017	634.623102	641.156167	644.433499	663.009826	658.10785	655.94734	639.385669	3.00982599	
15	612.966997	614.945985	611.743679	598.498753	578.527908	565.796448	570.744195	562.237427	564.060109	556.041797	553.222009	571.371764		
16	618.231359	619.598952	604.658681	598.977508	596.401591	601.291726	595.664191	593.894163	608.89154	605.301944	591.997727	572.082642		
17	644.1431361	627.668665	617.259201	614.639173	619.913809	643.515626	642.097143	642.38493	633.392486	645.239407	626.327772	635.892996		
18	641.3260193	608.346875	588.723147	607.319218	606.949257	605.353191	599.166439	599.667909	605.853041	607.613445	603.341783	588.224481		
19	661.9595004	676.194573	665.179855	655.458232	627.133865	628.569983	627.807415	607.052667	639.482321	628.044596	625.539715	611.057147	16.19457266	
20	647.8122074	606.85035	642.387344	650.175654	655.180916	655.977719	654.158954	656.173346	670.297582	649.905752	606.868366	593.53739	33.53373862	33.53373862

We set this option to expire in 12 days.

We first generate 12000 random numbers from the normal distribution $N(0,1)$ using the function 'normsinv(rand())' and then use the Black-Scholes formula to simulate 1000 times, each containing the closing prices of 12 days. We finally take the average of all 1000 cases.

The Black-Scholes stock pricing formula: $S_T = S_0 e^{(\alpha - \frac{1}{2}\sigma^2)T + z\sigma\sqrt{T}}$

In our case, S_0 is 644.77, real interest rate alpha is 5.18%-3.01% (nominal interest rate r minus inflation δ), the standard deviation of stock annual return sigma is 0.36162363, maturity T is $\frac{12}{252}$ years (252 trading days per year), and z is the random number generated from the normal distribution with mean 0 and standard deviation 1.

Comparisons

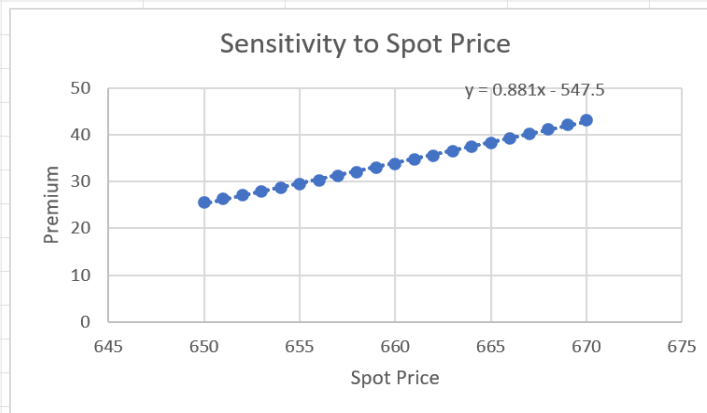
	Fixed Lookback	Standard
Average	21.75382339	13.35882171
Discounted	21.70023009	13.32591055

Since the fixed lookback option calculates the difference between the highest stock price and the strike price, the lookback payoff is determined by the maximum between (1) zero and (2) the difference between the highest stock price in the 12-day period and the strike price of 660. For the convenience of comparing with the standard option, we also calculate the standard European option payoff, which is the maximum between (1) zero and (2) the difference between the price of day 12 (last day price) and the strike price of 660.

Then we take the average of both lookback and standard payoffs and discount them by the risk-free rate 5.18%. The payoff of the fixed lookback option is 21.70 dollars and that of the standard option is only 13.32 dollars. It is clear that the fixed lookback option has a much greater payoff than the standard option.

Sensitivity Analysis Part A -- Spot price

spot price	option premium
	21.70023009
650	25.49618917
651	26.26072898
652	27.04262194
653	27.8404536
654	28.65171312
655	29.47409445
656	30.30711007
657	31.14918983
658	32.0066457
659	32.87816245
660	33.76122444
661	34.65355958
662	35.55520206
663	36.46320387
664	37.37996602
665	38.30437214
666	39.24082884
667	40.18806268
668	41.14360657
669	42.10396906
670	43.07004249

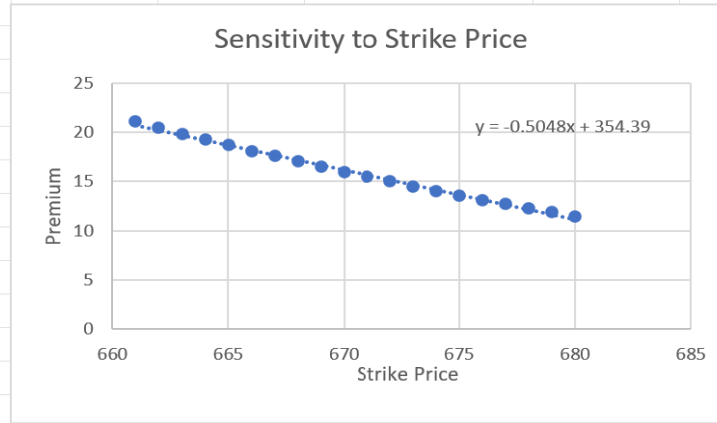


slope=0.88

1 dollar increase in spot price causes premium to increase by 0.88 dollar.

Sensitivity Analysis Part B -- Strike Price

strike price	option premium
	21.70023009
661	21.06236899
662	20.44224329
663	19.84053731
664	19.2512072
665	18.67176801
666	18.10519144
667	17.55520898
668	17.02226745
669	16.50203428
670	15.99071094
671	15.48277855
672	14.98523902
673	14.49862826
674	14.02624142
675	13.56856854
676	13.12470558
677	12.68965935
678	12.26946452
679	11.86063713
680	11.46189686

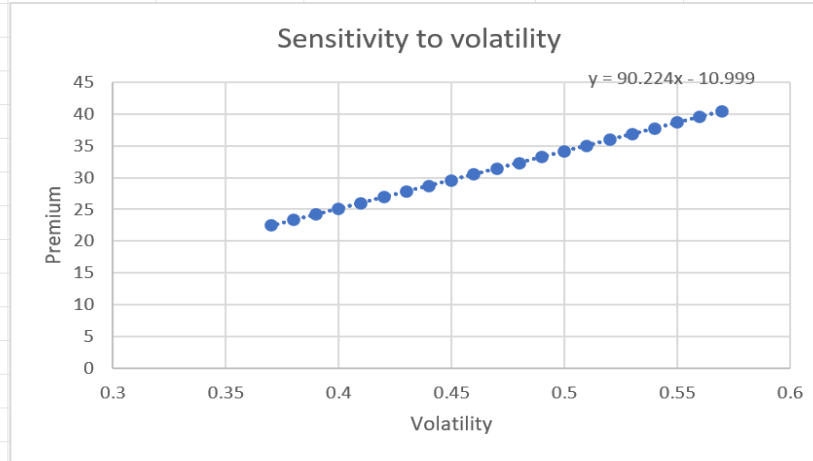


slope=-0.50

1 dollar increase in strike price causes premium to decrease by 0.5 dollar.

Sensitivity Analysis Part C -- Volatility

volatility(sigma)	option premium
	21.70023009
0.37	22.43900312
0.38	23.32283173
0.39	24.20925939
0.4	25.09797481
0.41	25.98940395
0.42	26.88342078
0.43	27.77944763
0.44	28.67684655
0.45	29.57566181
0.46	30.47583747
0.47	31.37767076
0.48	32.28090233
0.49	33.18545156
0.5	34.09200453
0.51	34.99991427
0.52	35.90906445
0.53	36.81989623
0.54	37.73218729
0.55	38.64597731
0.56	39.56093204
0.57	40.47730157



slope=90.22

1 percent increase in volatility causes premium to increase by 0.9022 dollar.

Conclusion (1/2)

- Traditional Options:

European Option

American Option

- Exotic Options



Conclusion (2/2)

- The Dream of Investors:

Buy Low / Sell High

- Lookback Option

Helps investors maximize returns

