FIRST-ORDER LINEAR LOGIC FOR NATURAL LANGUAGE ANALYSIS

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LACompLing 2017, 17-8-2017, Stockholm

https://github.com/RichardMoot/LinearOne

Outline

- Type-logical grammars and formal semantics
- Why first-order linear logic?
- Proof theory
- Parsing
- Conclusion & the future

THE LAMBEK CALCULUS AND THE SYNTAX-SEMANTICS INTERFACE

Background: Lambek calculus and the syntax-semantic interface

- Lambek (1958) extended earlier work by Ajdukiewicz and Bar-Hillel, casting it as a *logic*.

- With hindsight, this is the non-commutative, multiplicative, intuitionistic fragment of linear logic.

- “Leftward looking” implication B\A and “rightward looking” implication A/B

The Lambek calculus

$$\frac{A/B \quad B}{A}\ /E \qquad \frac{B \quad B\backslash A}{A}\ \backslash E$$

$$\frac{\begin{array}{c}[B]^i \\ \vdots \\ A\end{array}}{A/B}\ /I \qquad \frac{\begin{array}{c}[B]^i \\ \vdots \\ A\end{array}}{B\backslash A}\ \backslash I$$

The Lambek calculus

$$\frac{A/B \quad B}{A}\ /E \qquad \frac{B \quad B\backslash A}{A}\ \backslash E$$

$$\dfrac{\dfrac{\dfrac{\text{the}}{np/n}\ Lex \quad \dfrac{\text{student}}{n}\ Lex}{np}\ /E \qquad \dfrac{\text{slept}}{np\backslash s}\ Lex}{s}\ \backslash E$$

Every gambler visited a casino

$$\frac{\text{every}}{(s/(np\backslash s))/n} \quad \frac{\text{gambler}}{n} \quad \frac{\text{visited}}{(np\backslash s)/np} \quad \frac{\text{a}}{((s/np)\backslash s)/n} \quad \frac{\text{casino}}{n}$$

Every gambler visited a casino

$$\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n}\quad\dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\,/E\quad\dfrac{\text{visited}}{(np\backslash s)/np}\quad\dfrac{\text{a}}{((s/np)\backslash s)/n}\quad\dfrac{\text{casino}}{n}$$

Every gambler visited a casino

$$\dfrac{\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n} \quad \dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\,/E \quad \dfrac{\text{visited}}{(np\backslash s)/np} \quad \dfrac{\dfrac{\dfrac{\text{a}}{((s/np)\backslash s)/n} \quad \dfrac{\text{casino}}{n}}{(s/np)\backslash s}\,/E}{}$$

Every gambler visited a casino

$$\dfrac{\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n}\quad\dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\,/E\quad\dfrac{\text{visited}}{(np\backslash s)/np}\quad np\quad\dfrac{\dfrac{\dfrac{\text{a}}{((s/np)\backslash s)/n}\quad\dfrac{\text{casino}}{n}}{(s/np)\backslash s}\,/E}{s}$$

Every gambler visited a casino

$$\dfrac{\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n} \quad \dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\,/E \quad \dfrac{\dfrac{\text{visited}}{(np\backslash s)/np} \quad np}{np\backslash s}\,/E \quad \dfrac{\dfrac{\dfrac{\text{a}}{((s/np)\backslash s)/n} \quad \dfrac{\text{casino}}{n}}{(s/np)\backslash s}\,/E}{}$$

Every gambler visited a casino

$$\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n}\quad\dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\,/E\quad np\qquad\dfrac{\dfrac{\text{visited}}{(np\backslash s)/np}\quad np}{np\backslash s}\,/E\qquad\dfrac{\dfrac{\text{a}}{((s/np)\backslash s)/n}\quad\dfrac{\text{casino}}{n}}{(s/np)\backslash s}\,/E$$

Every gambler visited a casino

$$\dfrac{\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n}\quad\dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\,/E\quad\dfrac{np\qquad\dfrac{\dfrac{\text{visited}}{(np\backslash s)/np}\quad np}{np\backslash s}\,\backslash E}{s}\,/E\quad\dfrac{\dfrac{\dfrac{\text{a}}{((s/np)\backslash s)/n}\quad\dfrac{\text{casino}}{n}}{(s/np)\backslash s}\,/E}{\qquad}}{\qquad}$$

Every gambler visited a casino

$$\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n}\quad\dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\,/E\qquad\dfrac{\dfrac{np\qquad\dfrac{\dfrac{\text{visited}}{(np\backslash s)/np}\quad[np]^1}{np\backslash s}\,\backslash E}{s}}{s/np}\,/I_1\qquad\dfrac{\dfrac{\text{a}}{((s/np)\backslash s)/n}\quad\dfrac{\text{casino}}{n}}{(s/np)\backslash s}\,/E$$

Every gambler visited a casino

$$\dfrac{\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n}\quad\dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\,/E\qquad\dfrac{\dfrac{\dfrac{\dfrac{\text{visited}}{(np\backslash s)/np}\quad[np]^1}{np\backslash s}\,\backslash E}{s}\,/I_1}{s/np}\qquad\dfrac{\dfrac{\dfrac{\text{a}}{((s/np)\backslash s)/n}\quad\dfrac{\text{casino}}{n}}{(s/np)\backslash s}\,/E}{s}\,\backslash E$$

Every gambler visited a casino

$$\dfrac{\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n}\quad\dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\,/E\qquad\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{visited}}{(np\backslash s)/np}\quad\dfrac{}{[np]^1}}{np\backslash s}\,/E}{[np]^2}\backslash E}{s/np}\,/I_1\qquad\dfrac{\dfrac{\dfrac{\text{a}}{((s/np)\backslash s)/n}\quad\dfrac{\text{casino}}{n}}{(s/np)\backslash s}\,/E}{s/np\backslash s}\,\backslash I_2}{s}\,\backslash E$$

Every gambler visited a casino

$$\dfrac{\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n}\quad\dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\,/E\qquad\dfrac{\dfrac{\dfrac{\dfrac{\text{visited}}{(np\backslash s)/np}\quad\dfrac{}{[np]^1}}{np\backslash s}\,\backslash E\quad\dfrac{\dfrac{\dfrac{\text{a}}{((s/np)\backslash s)/n}\quad\dfrac{\text{casino}}{n}}{(s/np)\backslash s}\,/E}{s}\,\backslash I_2}{np\backslash s}\,/E}{s/np}\,/I_1}{[np]^2}\,\backslash E}{s}\,/E$$

Every gambler visited a casino “deep structure”

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{n \multimap ((np \multimap s) \multimap s) \quad n}{(np \multimap s) \multimap s} \multimap E \quad \dfrac{\dfrac{\dfrac{np \multimap (np \multimap s) \quad [np]^1}{np \multimap s} \multimap E \quad [np]^2}{s} \multimap E}{np \multimap s} \multimap I_1}{s} \multimap E \quad \dfrac{n \multimap ((np \multimap s) \multimap s) \quad n}{(np \multimap s) \multimap s} \multimap E}{np \multimap s} \multimap I_2}{s} \multimap E$$

The Lambek calculus is the intuitionistic, multiplicative, non-commutative fragment of linear logic. If we replace “/“ and “\” by “⊸” we obtain a linear logic proof.

Syntactic types to semantic types

$$np^* = e$$

$$n^* = e \to t$$

$$s^* = t$$

$$(A \multimap B)^* = A^* \to B^*$$

Semantic derivation and lambda term

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{z_0^{(e\to t)\to(e\to t)\to t} \quad z_1^{e\to t}}{(z_0\ z_1)^{(e\to t)\to t}}\to E \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{z_2^{e\to(e\to t)} \quad [y^e]^1}{(z_2\ y)^{e\to t}}\to E \quad [x^e]^2}{((z_2\ y)\ x)^t}\to E}{\lambda y.((z_2\ y)\ x)^{e\to t}}\to I_1 \qquad \dfrac{z_3^{(e\to t)\to(e\to t)\to t} \quad z_4^{e\to t}}{(z_3\ z_4)^{(e\to t)\to t}}\to E}{((z_3\ z_4)\ \lambda y.((z_2\ y)\ x))^t}\to E}{\lambda x.((z_3\ z_4)\ \lambda y.((z_2\ y)\ x))^{e\to t}}\to I_2}{((z_0\ z_1)\ (\lambda x.((z_3\ z_4)\ \lambda y.((z_2\ y)\ x))))^t}\to E$$

multiplicative intuitionistic linear logic proofs are isomorphic with linear lambda terms

Lexical substitution

$$((z_0\ z_1)\ (\lambda x.((z_3\ z_4)\ \lambda y.((z_2\ y)\ x))))$$

$$z_0 := \lambda P^{e\to t}.\lambda Q^{e\to t}.(\forall(\lambda x^{e}.((\Rightarrow\ (P\ x))(Q\ x))))$$

$$z_1 := gambler^{e\to t}$$

$$z_2 := visit^{e\to(e\to t)}$$

$$z_3 := \lambda P^{e\to t}.\lambda Q^{e\to t}.(\exists(\lambda x^{e}.((\wedge(P\ x))(Q\ x))))$$

$$z_4 := casino^{e\to t}$$

Normalisation

$$((\lambda P^{e\to t}.\lambda Q^{e\to t}.(\forall(\lambda v^{e}.((\Rightarrow (P\,v))(Q\,v))))\; gambler^{e\to t})$$
$$(\lambda x.((\lambda P'^{e\to t}.\lambda Q'^{e\to t}.(\exists(\lambda z^{e}.((\wedge(P'\,z))(Q'\,z)))))\; casino^{e\to t})$$
$$\lambda y.((visit^{e\to(e\to t)}\; y)\; x))))$$

Normalisation

$$((\lambda P^{e\to t}.\lambda Q^{e\to t}.(\forall(\lambda v^{e}.((\Rightarrow (P\, v))(Q\, v))))\; gambler^{e\to t})$$
$$(\lambda x.((\lambda P'^{e\to t}.\lambda Q'^{e\to t}.(\exists(\lambda z^{e}.((\wedge(P'\, z))(Q'\, z)))))\; casino^{e\to t})$$
$$\lambda y.((visit^{e\to(e\to t)}\; y)\, x))))$$

$$\rightsquigarrow_{\beta}\; (\forall(\lambda x^{e}.((\Rightarrow (gambler^{e\to t}\; x))(\exists(\lambda y^{e}.((\wedge(casino^{e\to t}\; y))((visit^{e\to(e\to t)}\; y)\, x))))))$$

Normalisation

$$((\lambda P^{e\to t}.\lambda Q^{e\to t}.(\forall(\lambda v^{e}.((\Rightarrow (P\, v))(Q\, v))))\; gambler^{e\to t})$$
$$(\lambda x.((\lambda P'^{e\to t}.\lambda Q'^{e\to t}.(\exists(\lambda z^{e}.((\wedge(P'\, z))(Q'\, z))))\; casino^{e\to t})$$
$$\lambda y.((visit^{e\to(e\to t)}\, y)\, x))))$$

$$\rightsquigarrow_{\beta} (\forall(\lambda x^{e}.((\Rightarrow (gambler^{e\to t}\, x))(\exists(\lambda y^{e}.((\wedge(casino^{e\to t}\, y))((visit^{e\to(e\to t)}\, y)\, x))))))$$

$$\equiv_{def} \forall x.[gambler(x) \Rightarrow \exists y.[casino(y) \wedge visit(x,y)]]$$

More complicated proofs: right-node raising

2. Howard loved but Geoffrey hated Syntactic Structures

$$\dfrac{\dfrac{\text{Howard}}{np}\,Lex \quad \dfrac{\dfrac{\text{loved}}{(np\backslash s)/np}\,Lex \quad [np]^1}{np\backslash s}\,/E}{\dfrac{s}{s/np}\,/I_1}\,\backslash E$$

More complicated proofs: right-node raising

2. Howard loved but Geoffrey hated Syntactic Structures

$$\dfrac{\dfrac{\dfrac{\dfrac{\text{Howard}}{np}\,Lex \quad \dfrac{\dfrac{\text{loved}}{(np\backslash s)/np}\,Lex \quad [np]^1}{np\backslash s}\,/E}{s}\,\backslash E}{s/np}\,/I_1 \quad \dfrac{\dfrac{\text{but}}{((s/np)\backslash(s/np))/(s/np)}\,Lex \quad \dfrac{\dfrac{\dfrac{\text{Geoffrey}}{np}\,Lex \quad \dfrac{\dfrac{\text{hated}}{(np\backslash s)/np}\,Lex \quad [np]^2}{np\backslash s}\,/E}{s}\,\backslash E}{s/np}\,/I_2}{(s/np)\backslash(s/np)}\,/E}{s/np}\,\backslash E \quad \dfrac{\dfrac{\text{Syntactic Structures}}{np}\,Lex}{s}\,/E$$

More complicated proofs: argument cluster coordination

3. Terry gave Robin flowers and Sue a book

$$\dfrac{\dfrac{[(s/np)/np]^3 \quad \dfrac{\text{Robin}}{np}\,Lex}{s/np}\,/E \quad \dfrac{\text{flowers}}{np}\,Lex}{\dfrac{s}{((s/np)/np)\backslash s}\,\backslash I_3}\,\backslash E$$

More complicated proofs: argument cluster coordination

3. Terry gave Robin flowers and Sue a book

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{Terry}}{np}\ Lex}{s/(np\backslash s)}\ /I_2 \quad \dfrac{\dfrac{\dfrac{\dfrac{\text{gave}}{((np\backslash s)/np)/np}\ Lex \quad [np]^1}{(np\backslash s)/np}\ /E \quad [np]^2}{np\backslash s}\ /E}{s}\ \backslash E}{(s/np)/np}\ /I_1 \quad \dfrac{\dfrac{\dfrac{\dfrac{\text{Robin}}{np}\ Lex}{[(s/np)/np]^3}\ /E \quad \dfrac{\text{flowers}}{np}\ Lex}{s/np}\ \backslash E}{s}\ \backslash I_3}{((s/np)/np)\backslash s}\ \backslash E \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{and}}{(X\backslash X)/X}\ Lex \quad \dfrac{\dfrac{\dfrac{\text{Sue}}{np}\ Lex}{[(s/np)/np]^4}\ /E \quad \dfrac{\text{a book}}{(s/np)\backslash s}\ Lex}{s/np}\ \backslash E}{s}\ \backslash I_4}{((s/np)/np)\backslash s}\ /E}{X\backslash X}\ \backslash E}{s}\ \backslash E$$

(back)

More complicated proofs: argument cluster coordination

4. Captain Jack served lobster yesterday and bananafish today

$$\dfrac{\dfrac{[np]^1 \qquad \dfrac{[(np\backslash s)/np]^2 \qquad \dfrac{\text{lobster}}{np}\,Lex}{np\backslash s}\,/E}{s}\,\backslash E \qquad \dfrac{\text{yesterday}}{s\backslash s}\,Lex}{\dfrac{s}{np\backslash s}\,\backslash I_1}\,\backslash E$$
$$\dfrac{}{((np\backslash s)/np)\backslash(np\backslash s)}\,\backslash I_2$$

Trouble in paradise: problems for Lambek grammars

- Though the Lambek calculus handles the basics of Montague grammar, it has problems with non-peripheral wide scope (and many other phenomena).

- Lambek grammars generate only context-free languages; some natural languages demonstrate non-context-free phenomena like copying and multiple/crossed dependencies.

Dutch verb clusters

5. (dat) Jan Henk Marie de nijlpaarden zag helpen voeren

Jan Henk Marie the hippopotami saw help feed

(that) Jan saw Henk help Marie feed the hippopotami

$$\dfrac{\dfrac{\dfrac{\text{zag}}{np \multimap (inf \multimap (np \multimap s))}\,Lex \quad \dfrac{\text{Henk}}{np}\,Lex}{inf \multimap (np \multimap s)}\multimap E \quad \dfrac{\dfrac{\text{Marie}}{np}\,Lex \quad \dfrac{\text{helpen}}{np \multimap (inf \multimap inf)}\,Lex}{inf \multimap inf}\multimap E \quad \dfrac{\dfrac{\text{de nijlpaarden}}{np}\,Lex \quad \dfrac{\text{voeren}}{np \multimap inf}\,Lex}{inf}\multimap E}{inf}\multimap E$$

$$np \multimap s$$

Dutch verb clusters

5. (dat) Jan Henk Marie de nijlpaarden zag helpen voeren

Jan Henk Marie the hippopotami saw help feed

(that) Jan saw Henk help Marie feed the hippopotami

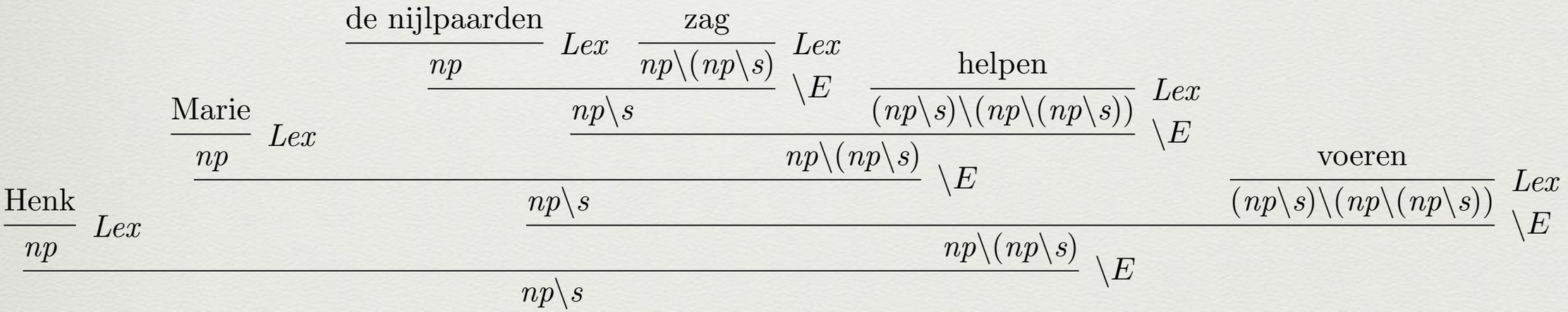
$$\dfrac{\dfrac{\dfrac{\text{zag}}{np \multimap (inf \multimap (np \multimap s))}\ Lex \quad \dfrac{\text{Henk}}{np}\ Lex}{inf \multimap (np \multimap s)} \multimap E \quad \dfrac{\dfrac{\text{Marie}}{np}\ Lex \quad \dfrac{\text{helpen}}{np \multimap (inf \multimap inf)}\ Lex}{inf \multimap inf} \multimap E \quad \dfrac{\dfrac{\text{de nijlpaarden}}{np}\ Lex \quad \dfrac{\text{voeren}}{np \multimap inf}\ Lex}{inf} \multimap E}{np \multimap s} \multimap E$$

Dutch verb clusters

5. (dat) Jan Henk Marie de nijlpaarden zag helpen voeren

Jan Henk Marie the hippopotami saw help feed

(that) Jan saw Henk help Marie feed the hippopotami



Medial extraction

6. report that John read yesterday

$$\dfrac{\dfrac{\dfrac{\text{John}}{np}\,Lex \quad \dfrac{\dfrac{\dfrac{\text{read}}{np \multimap (np \multimap s)}\,Lex \quad [np]^1}{np \multimap s} \multimap E}{s} \multimap E \quad \dfrac{\text{yesterday}}{s \multimap s}\,Lex}{s} \multimap E}{np \multimap s} \multimap I_1$$

Medial extraction

6. report that John read yesterday

$$\dfrac{\dfrac{\dfrac{\dfrac{\text{John}}{np}\,Lex \qquad \dfrac{\dfrac{\text{read}}{(np\backslash s)/np}\,Lex \qquad [np]^1}{np\backslash s}\,/E}{s}\,\backslash E \qquad \dfrac{\text{yesterday}}{s\backslash s}\,Lex}{s}\,\backslash E}{np \multimap s}\,\multimap I_1$$

De dicto/de re

7. John believes someone left

$$\dfrac{\dfrac{\text{John}}{np}\,Lex \quad \dfrac{\dfrac{\dfrac{\text{believes}}{s \multimap (np \multimap s)}\,Lex \quad \dfrac{\dfrac{\text{someone}}{(np \multimap s) \multimap s}\,Lex \quad \dfrac{\text{left}}{np \multimap s}\,Lex}{s}\multimap E}{np \multimap s}\multimap E}{s}\multimap E$$

De dicto/de re

7. John believes someone left

$$\dfrac{\dfrac{\text{John}}{np}\,Lex \qquad \dfrac{\dfrac{\text{believes}}{s \multimap (np \multimap s)}\,Lex \qquad \dfrac{[np]^1 \qquad \dfrac{\text{left}}{np \multimap s}\,Lex}{s}\,\multimap E}{np \multimap s}\,\multimap E}{s}\,\multimap E$$

$$\dfrac{\dfrac{\text{someone}}{(np \multimap s) \multimap s}\,Lex \qquad \dfrac{s}{np \multimap s}\,\multimap I_1}{s}\,\multimap E$$

This is not the forgetful mapping of any Lambek calculus proof!

(at least not given np\s for “left” and (np\s)/s for “believes”)

(back)

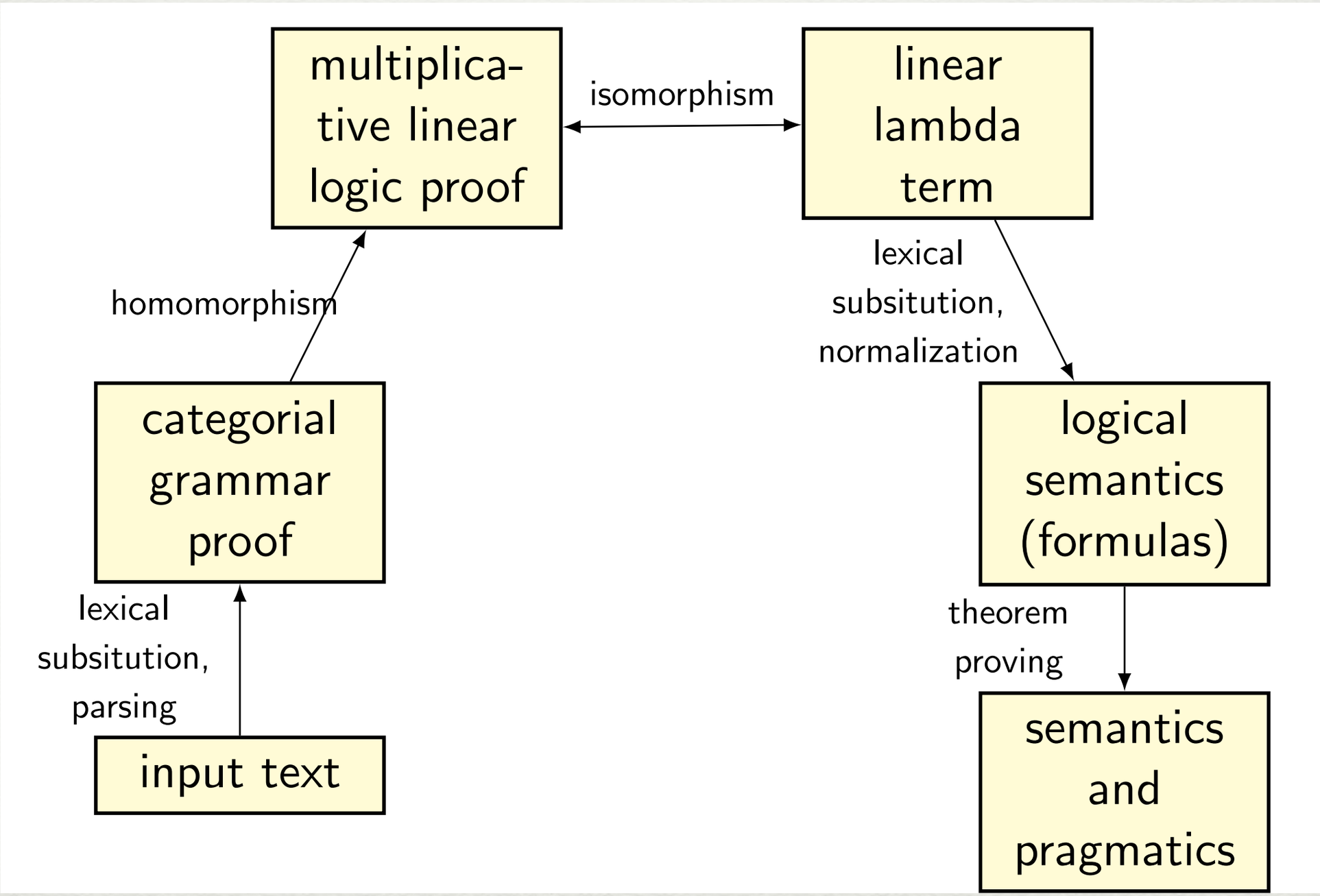
EXTENDING THE LAMBEK CALCULUS: FIRST-ORDER LINEAR LOGIC

Extending the Lambek calculus

- The Lambek calculus is a simple, elegant logic which gets a number of the basic facts about the syntax-semantics interface right (or, at least, *almost* right)

- How can we keep everything which worked correctly for the Lambek calculus without overly complicating the logic?

Core architecture of type-logical grammars



Why first-order linear logic?

- simple logic, nice proof theory
- can be seen as a sort of "machine language" underlying several modern type-logical grammars
- has many natural, interesting fragments (Lambek calculus but also several of item modern extensions: the "core" Displacement calculus, lambda grammars)
- a parser for first-order linear logic provides a single parse engine for the above frameworks; proof transformations can hide the internals (if desired)

Why first-order linear logic?

Things we can incorporate directly

1. John slept before Mary did.

2. mountain the painting of which by Cezanne John sold for $10.000.000.

3. John ate more donuts than Mary bought bagels.

4. John studies logic and Charles, phonetics.

5. No dog eats Whiskas or cat Alpo.

(1-4) from Morrill e.a. (2011),
(3-5) from Kubota & Levine (2012,2013)

Why first-order linear logic?

Other things with easy solutions

- Grammatical case/inflection or "lightweight features" (Morrill 1990)

- Constraints on derivability, notably so-called "island constraints" (de Groote)

  eg. avoid overgeneration for sentences like "book which Harry read Moby Dick and"

Displacement calculus (Morrill e.a.)

$$\dfrac{\textrm{john} \vdash n \qquad \dfrac{[r_0 \vdash (n\backslash s)/n]^2 \quad \textrm{logic} \vdash n}{r_0\ \textrm{logic} \vdash n\backslash s}\,/E}{\dfrac{\textrm{john}\ r_0\ \textrm{logic} \vdash s}{\textrm{john}, \textrm{logic} \vdash s \uparrow_{>} (n\backslash s)/n}\,\uparrow_{>} I_2}\,\backslash E$$

$$\dfrac{\textrm{charles} \vdash n \qquad \dfrac{[s_0 \vdash (n\backslash s)/n]^1 \quad \textrm{phonetics} \vdash n}{s_0\ \textrm{phonetics} \vdash n\backslash s}\,/E}{\dfrac{\dfrac{\textrm{charles}\ s_0\ \textrm{phonetics} \vdash s}{\textrm{charles}, \textrm{phonetics} \vdash s \uparrow_{>} (n\backslash s)/n}\,\uparrow_{>} I_1}{\textrm{charles phonetics} \vdash \hat{}(s \uparrow_{>} (n\backslash s)/n)}\,\hat{}I}\,\backslash E$$

$$\dfrac{\textrm{and} \vdash ((s \uparrow_{>} ((n\backslash s)/n))\backslash(s \uparrow_{>} ((n\backslash s)/n)))/\hat{}(s \uparrow_{>} (n\backslash s)/n) \qquad \textrm{charles phonetics} \vdash \hat{}(s \uparrow_{>} (n\backslash s)/n)}{\textrm{and charles phonetics} \vdash (s \uparrow_{>} ((n\backslash s)/n))\backslash(s \uparrow_{>} ((n\backslash s)/n))}\,/E$$

$$\dfrac{\textrm{john}, \textrm{logic} \vdash s \uparrow_{>} (n\backslash s)/n \qquad \textrm{and charles phonetics} \vdash (s \uparrow_{>} ((n\backslash s)/n))\backslash(s \uparrow_{>} ((n\backslash s)/n))}{\textrm{john}, \textrm{logic and charles phonetics} \vdash s \uparrow_{>} (n\backslash s)/n}\,\backslash E$$

$$\dfrac{\textrm{john}, \textrm{logic and charles phonetics} \vdash s \uparrow_{>} (n\backslash s)/n \qquad \textrm{studies} \vdash (n\backslash s)/n}{\textrm{john studies logic and charles phonetics} \vdash s}\,\uparrow_{>} E$$

$$\textrm{study}(\textrm{john}, \textrm{logic}) \wedge \textrm{study}(\textrm{charles}, \textrm{phonetics})$$

Hybrid type-logical grammars (Kubota and Levine)

$$\dfrac{\dfrac{\text{john}}{\text{np}} \quad \dfrac{\dfrac{\dfrac{\dfrac{\left[\begin{array}{l} r_0 \\ (\text{np}\backslash\text{s})/\text{np} \end{array}\right]^2}{\text{logic}}}{\text{np}}}{r_0 \circ \text{logic}}/E}{(\text{np}\backslash\text{s})}\backslash E}{\text{john} \circ r_0 \circ \text{logic}}|I_2 \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\left[\begin{array}{l} q_0 \\ (\text{np}\backslash\text{s})/\text{np} \end{array}\right]^1}{\text{phonetics}}}{\text{np}}}{q_0 \circ \text{phonetics}}/E}{(\text{np}\backslash\text{s})}\backslash E}{\text{charles} \circ q_0 \circ \text{phonetics}}|I_1 \quad \dfrac{\lambda s_4.\lambda t_4.\lambda p_5.(t_4\ p_5) \circ \text{and} \circ (s_4\ \epsilon)}{((\text{s}|((\text{np}\backslash\text{s})/\text{np}))|(\text{s}|((\text{np}\backslash\text{s})/\text{np})))|(\text{s}|((\text{np}\backslash\text{s})/\text{np}))}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\text{studies}}{(\text{np}\backslash\text{s})/\text{np}} \quad \dfrac{\lambda r_0.\text{john} \circ r_0 \circ \text{logic}}{\text{s}|((\text{np}\backslash\text{s})/\text{np})} \quad \dfrac{\lambda t_4.\lambda p_5.(t_4\ p_5) \circ \text{and} \circ \text{charles} \circ \text{phonetics}}{(\text{s}|((\text{np}\backslash\text{s})/\text{np}))|(\text{s}|((\text{np}\backslash\text{s})/\text{np}))}|E}{\lambda p_5.\text{john} \circ p_5 \circ \text{logic} \circ \text{and} \circ \text{charles} \circ \text{phonetics}}|E}{\text{john} \circ \text{studies} \circ \text{logic} \circ \text{and} \circ \text{charles} \circ \text{phonetics}}|E}{\text{s}}$$

$$\text{study}(\text{john}, \text{logic}) \wedge \text{study}(\text{charles}, \text{phonetics})$$

First-order linear logic

$$\dfrac{\dfrac{}{\mathrm{np}(4,5) \vdash \mathrm{np}(4,5)} \qquad \dfrac{}{\mathrm{s}(4,6) \vdash \mathrm{s}(4,6)}}{\mathrm{np}(4,5), \mathrm{np}(4,5) \multimap \mathrm{s}(4,6) \vdash \mathrm{s}(4,6)}\ L\multimap$$

$$\dfrac{\mathrm{np}(4,5), \mathrm{np}(4,5) \multimap \mathrm{s}(4,6) \vdash \mathrm{s}(4,6)}{\mathrm{np}(4,5), \forall B.[\mathrm{np}(B,5) \multimap \mathrm{s}(B,6)] \vdash \mathrm{s}(4,6)}\ L\forall$$

$$\dfrac{\dfrac{}{\mathrm{np}(5,6) \vdash \mathrm{np}(5,6)} \qquad \mathrm{np}(4,5), \forall B.[\mathrm{np}(B,5) \multimap \mathrm{s}(B,6)] \vdash \mathrm{s}(4,6)}{\mathrm{np}(5,6), \mathrm{np}(5,6) \multimap \forall B.[\mathrm{np}(B,5) \multimap \mathrm{s}(B,6)], \mathrm{np}(4,5) \vdash \mathrm{s}(4,6)}\ L\multimap$$

$$\dfrac{\mathrm{np}(5,6), \mathrm{np}(5,6) \multimap \forall B.[\mathrm{np}(B,5) \multimap \mathrm{s}(B,6)], \mathrm{np}(4,5) \vdash \mathrm{s}(4,6)}{\mathrm{np}(5,6), \forall A.[\mathrm{np}(5,A) \multimap \forall B.[\mathrm{np}(B,5) \multimap \mathrm{s}(B,A)]], \mathrm{np}(4,5) \vdash \mathrm{s}(4,6)}\ L\forall$$

$$\dfrac{\mathrm{np}(5,6), \forall A.[\mathrm{np}(5,A) \multimap \forall B.[\mathrm{np}(B,5) \multimap \mathrm{s}(B,A)]], \mathrm{np}(4,5) \vdash \mathrm{s}(4,6)}{\mathrm{np}(5,6), \mathrm{np}(4,5) \vdash \forall A.[\mathrm{np}(5,A) \multimap \forall B.[\mathrm{np}(B,5) \multimap \mathrm{s}(B,A)]] \multimap \mathrm{s}(4,6)}\ R\multimap$$

$$\dfrac{\dfrac{}{\mathrm{np}(0,1) \vdash \mathrm{np}(0,1)} \qquad \dfrac{}{\mathrm{s}(0,3) \vdash \mathrm{s}(0,3)}}{\mathrm{np}(0,1), \mathrm{np}(0,1) \multimap \mathrm{s}(0,3) \vdash \mathrm{s}(0,3)}\ L\multimap$$

$$\dfrac{\mathrm{np}(0,1), \mathrm{np}(0,1) \multimap \mathrm{s}(0,3) \vdash \mathrm{s}(0,3)}{\mathrm{np}(0,1), \forall D.[\mathrm{np}(D,1) \multimap \mathrm{s}(D,3)] \vdash \mathrm{s}(0,3)}\ L\forall$$

$$\dfrac{\dfrac{}{\mathrm{np}(2,3) \vdash \mathrm{np}(2,3)} \qquad \mathrm{np}(0,1), \forall D.[\mathrm{np}(D,1) \multimap \mathrm{s}(D,3)] \vdash \mathrm{s}(0,3)}{\mathrm{np}(2,3), \mathrm{np}(2,3) \multimap \forall D.[\mathrm{np}(D,1) \multimap \mathrm{s}(D,3)], \mathrm{np}(0,1) \vdash \mathrm{s}(0,3)}\ L\multimap$$

$$\dfrac{\mathrm{np}(2,3), \mathrm{np}(2,3) \multimap \forall D.[\mathrm{np}(D,1) \multimap \mathrm{s}(D,3)], \mathrm{np}(0,1) \vdash \mathrm{s}(0,3)}{\mathrm{np}(2,3), \forall C.[\mathrm{np}(2,C) \multimap \forall D.[\mathrm{np}(D,1) \multimap \mathrm{s}(D,C)]], \mathrm{np}(0,1) \vdash \mathrm{s}(0,3)}\ L\forall$$

$$\dfrac{\mathrm{np}(2,3), \forall C.[\mathrm{np}(2,C) \multimap \forall D.[\mathrm{np}(D,1) \multimap \mathrm{s}(D,C)]], \mathrm{np}(0,1) \vdash \mathrm{s}(0,3)}{\mathrm{np}(2,3), \mathrm{np}(0,1) \vdash \forall C.[\mathrm{np}(2,C) \multimap \forall D.[\mathrm{np}(D,1) \multimap \mathrm{s}(D,C)]] \multimap \mathrm{s}(0,3)}\ R\multimap$$

$$\dfrac{\dfrac{}{\mathrm{np}(x_1,1) \vdash \mathrm{np}(x_1,1)} \qquad \dfrac{}{\mathrm{s}(x_1,x_0) \vdash \mathrm{s}(x_1,x_0)}}{\mathrm{np}(x_1,1), \mathrm{np}(x_1,1) \multimap \mathrm{s}(x_1,x_0) \vdash \mathrm{s}(x_1,x_0)}\ L\multimap$$

$$\dfrac{\mathrm{np}(x_1,1), \mathrm{np}(x_1,1) \multimap \mathrm{s}(x_1,x_0) \vdash \mathrm{s}(x_1,x_0)}{\mathrm{np}(x_1,1), \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,x_0)] \vdash \mathrm{s}(x_1,x_0)}\ L\forall$$

$$\dfrac{\dfrac{}{\mathrm{np}(2,x_0) \vdash \mathrm{np}(2,x_0)} \qquad \mathrm{np}(x_1,1), \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,x_0)] \vdash \mathrm{s}(x_1,x_0)}{\mathrm{np}(2,x_0), \mathrm{np}(2,x_0) \multimap \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,x_0)], \mathrm{np}(x_1,1) \vdash \mathrm{s}(x_1,x_0)}\ L\multimap$$

$$\dfrac{\mathrm{np}(2,x_0), \mathrm{np}(2,x_0) \multimap \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,x_0)], \mathrm{np}(x_1,1) \vdash \mathrm{s}(x_1,x_0)}{\mathrm{np}(2,x_0), \forall E.[\mathrm{np}(2,E) \multimap \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,E)]], \mathrm{np}(x_1,1) \vdash \mathrm{s}(x_1,x_0)}\ L\forall$$

$$\dfrac{\mathrm{np}(2,x_0), \forall E.[\mathrm{np}(2,E) \multimap \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,E)]], \mathrm{np}(x_1,1) \vdash \mathrm{s}(x_1,x_0)}{\mathrm{np}(2,x_0), \forall E.[\mathrm{np}(2,E) \multimap \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,E)]] \vdash \mathrm{np}(x_1,1) \multimap \mathrm{s}(x_1,x_0)}\ R\multimap$$

$$\dfrac{\mathrm{np}(2,x_0), \forall E.[\mathrm{np}(2,E) \multimap \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,E)]] \vdash \mathrm{np}(x_1,1) \multimap \mathrm{s}(x_1,x_0)}{\mathrm{np}(2,x_0), \forall E.[\mathrm{np}(2,E) \multimap \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,E)]] \vdash \forall x_1.[\mathrm{np}(x_1,1) \multimap \mathrm{s}(x_1,x_0)]}\ R\forall$$

$$\dfrac{\mathrm{np}(2,x_0), \forall E.[\mathrm{np}(2,E) \multimap \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,E)]] \vdash \forall x_1.[\mathrm{np}(x_1,1) \multimap \mathrm{s}(x_1,x_0)]}{\forall E.[\mathrm{np}(2,E) \multimap \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,E)]] \vdash \mathrm{np}(2,x_0) \multimap \forall x_1.[\mathrm{np}(x_1,1) \multimap \mathrm{s}(x_1,x_0)]}\ R\multimap$$

$$\dfrac{\forall E.[\mathrm{np}(2,E) \multimap \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,E)]] \vdash \mathrm{np}(2,x_0) \multimap \forall x_1.[\mathrm{np}(x_1,1) \multimap \mathrm{s}(x_1,x_0)]}{\forall E.[\mathrm{np}(2,E) \multimap \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,E)]] \vdash \forall x_0.[\mathrm{np}(2,x_0) \multimap \forall x_1.[\mathrm{np}(x_1,1) \multimap \mathrm{s}(x_1,x_0)]]}\ R\forall$$

$$\dfrac{\forall E.[\mathrm{np}(2,E) \multimap \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,E)]] \vdash \forall x_0.[\mathrm{np}(2,x_0) \multimap \forall x_1.[\mathrm{np}(x_1,1) \multimap \mathrm{s}(x_1,x_0)]] \qquad \dfrac{}{\mathrm{s}(0,6) \vdash \mathrm{s}(0,6)}}{\forall E.[\mathrm{np}(2,E) \multimap \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,E)]], \forall x_0.[\mathrm{np}(2,x_0) \multimap \forall x_1.[\mathrm{np}(x_1,1) \multimap \mathrm{s}(x_1,x_0)]] \multimap \mathrm{s}(0,6) \vdash \mathrm{s}(0,6)}\ L\multimap$$

$$\dfrac{\mathrm{np}(2,3), \mathrm{np}(0,1) \vdash \forall C.[\mathrm{np}(2,C) \multimap \forall D.[\mathrm{np}(D,1) \multimap \mathrm{s}(D,C)]] \multimap \mathrm{s}(0,3) \qquad \forall E.[\mathrm{np}(2,E) \multimap \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,E)]], \forall x_0.[\mathrm{np}(2,x_0) \multimap \forall x_1.[\mathrm{np}(x_1,1) \multimap \mathrm{s}(x_1,x_0)]] \multimap \mathrm{s}(0,6) \vdash \mathrm{s}(0,6)}{\mathrm{np}(2,3), \mathrm{np}(0,1), (\forall C.[\mathrm{np}(2,C) \multimap \forall D.[\mathrm{np}(D,1) \multimap \mathrm{s}(D,C)]] \multimap \mathrm{s}(0,3)) \multimap \forall x_0.[\mathrm{np}(2,x_0) \multimap \forall x_1.[\mathrm{np}(x_1,1) \multimap \mathrm{s}(x_1,x_0)]] \multimap \mathrm{s}(0,6), \forall E.[\mathrm{np}(2,E) \multimap \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,E)]] \vdash \mathrm{s}(0,6)}\ L\multimap$$

$$\dfrac{\mathrm{np}(5,6), \mathrm{np}(4,5) \vdash \forall A.[\mathrm{np}(5,A) \multimap \forall B.[\mathrm{np}(B,5) \multimap \mathrm{s}(B,A)]] \multimap \mathrm{s}(4,6) \qquad \mathrm{np}(2,3), \mathrm{np}(0,1), (\forall C.[\mathrm{np}(2,C) \multimap \forall D.[\mathrm{np}(D,1) \multimap \mathrm{s}(D,C)]] \multimap \mathrm{s}(0,3)) \multimap \forall x_0.[\mathrm{np}(2,x_0) \multimap \forall x_1.[\mathrm{np}(x_1,1) \multimap \mathrm{s}(x_1,x_0)]] \multimap \mathrm{s}(0,6), \forall E.[\mathrm{np}(2,E) \multimap \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,E)]] \vdash \mathrm{s}(0,6)}{\mathrm{np}(5,6), \mathrm{np}(4,5), (\forall A.[\mathrm{np}(5,A) \multimap \forall B.[\mathrm{np}(B,5) \multimap \mathrm{s}(B,A)]] \multimap \mathrm{s}(4,6)) \multimap (\forall C.[\mathrm{np}(2,C) \multimap \forall D.[\mathrm{np}(D,1) \multimap \mathrm{s}(D,C)]] \multimap \mathrm{s}(0,3)) \multimap \forall x_0.[\mathrm{np}(2,x_0) \multimap \forall x_1.[\mathrm{np}(x_1,1) \multimap \mathrm{s}(x_1,x_0)]] \multimap \mathrm{s}(0,6), \mathrm{np}(2,3), \mathrm{np}(0,1), \forall E.[\mathrm{np}(2,E) \multimap \forall F.[\mathrm{np}(F,1) \multimap \mathrm{s}(F,E)]] \vdash \mathrm{s}(0,6)}\ L\multimap$$

$$\mathrm{study}(\mathrm{john}, \mathrm{logic}) \wedge \mathrm{study}(\mathrm{charles}, \mathrm{phonetics})$$

PROOF THEORY

Parsing using string position pairs

0 1 2 3 4

Jim	proved	the	theorem
np	(np\s)/np	np/n	n

Parsing using string position pairs

0 1 2 3 4

Jim	proved	the	theorem
np	(np\s)/np	np/n	n

∀x.n(3,x)–∘np(2,x) n(3,4)

Parsing using string position pairs

0 1 2 3 4

Jim	proved	the	theorem
np	(np\s)/np	np/n	n

$$\frac{\forall x.n(3,x) \multimap np(2,x) \qquad n(3,4)}{np(2,4)}$$

Parsing using string position pairs

0 1 2 3 4

Jim	proved	the	theorem
np	(np\s)/np	np/n	n

$$\frac{\forall z.np(2,z) \multimap \forall y.np(y,1) \multimap s(y,z) \qquad \frac{\forall x.n(3,x) \multimap np(2,x) \qquad n(3,4)}{np(2,4)}}{}$$

Parsing using string position pairs

0 1 2 3 4

Jim	proved	the	theorem
np	(np\s)/np	np/n	n

$$\dfrac{\dfrac{\forall z.np(2,z) \multimap \forall y.np(y,1) \multimap s(y,z) \qquad \dfrac{\forall x.n(3,x) \multimap np(2,x) \qquad n(3,4)}{np(2,4)}}{\forall y.np(y,1) \multimap s(y,4)} \qquad np(0,1)}$$

Parsing using string position pairs

0 1 2 3 4

Jim	proved	the	theorem
np	(np\s)/np	np/n	n

$$\dfrac{\dfrac{np(0,1) \qquad \forall z.np(2,z) \multimap \forall y.np(y,1) \multimap s(y,z)}{\forall y.np(y,1) \multimap s(y,4)} \qquad \dfrac{\forall x.n(3,x) \multimap np(2,x) \qquad n(3,4)}{np(2,4)}}{s(0,4)}$$

Natural deduction

$$\frac{A \quad A \multimap B}{B} \multimap E \qquad \frac{\begin{array}{c}[A]^i \\ \vdots \\ B\end{array}}{A \multimap B} \multimap I_i$$

$$\frac{\forall x.A}{A[x := t]} \forall E \qquad \frac{A}{\forall x.A} \forall I^*$$

John believes someone left (revisited)

$$\frac{\text{John}}{np(0,1)}\ Lex \quad \frac{\text{believes}}{\forall D.[s(2,D) \multimap \forall E.[np(E,1) \multimap s(E,D)]]}\ Lex \quad \frac{\text{someone}}{\forall A.\forall B.[(np(2,3) \multimap s(A,B)) \multimap s(A,B)]}\ Lex \quad \frac{\text{left}}{\forall C.[np(C,3) \multimap np(C,4)]}\ Lex$$

John believes someone left (revisited)

left

$$\frac{\forall C.[\mathrm{np}(C,3) \multimap \mathrm{s}(C,4)]}{\mathrm{np}(2,3) \multimap \mathrm{s}(2,4)}\ \forall E$$

$$\frac{[\mathrm{np}(2,3)]^1 \qquad \mathrm{np}(2,3) \multimap \mathrm{s}(2,4)}{\mathrm{s}(2,4)}\ \multimap E$$

believes

$$\frac{\forall D.[\mathrm{s}(2,D) \multimap \forall E.[\mathrm{np}(E,1) \multimap \mathrm{s}(E,D)]]}{\mathrm{s}(2,4) \multimap \forall E.[\mathrm{np}(E,1) \multimap \mathrm{s}(E,4)]}\ \forall E$$

$$\frac{\mathrm{s}(2,4) \qquad \mathrm{s}(2,4) \multimap \forall E.[\mathrm{np}(E,1) \multimap \mathrm{s}(E,4)]}{\forall E.[\mathrm{np}(E,1) \multimap \mathrm{s}(E,4)]}\ \multimap E$$

$$\frac{\forall E.[\mathrm{np}(E,1) \multimap \mathrm{s}(E,4)]}{\mathrm{np}(0,1) \multimap \mathrm{s}(0,4)}\ \forall E$$

John

$$\frac{\mathrm{np}(0,1) \qquad \mathrm{np}(0,1) \multimap \mathrm{s}(0,4)}{\mathrm{s}(0,4)}\ \multimap E$$

$$\frac{\mathrm{s}(0,4)}{\mathrm{np}(2,3) \multimap \mathrm{s}(0,4)}\ \multimap I_1$$

someone

$$\frac{\dfrac{\forall A.\forall B.[(\mathrm{np}(2,3) \multimap \mathrm{s}(A,B)) \multimap \mathrm{s}(A,B)]}{\forall B.[(\mathrm{np}(2,3) \multimap \mathrm{s}(0,B)) \multimap \mathrm{s}(0,B)]}\ \forall E}{(\mathrm{n}(2,3) \multimap \mathrm{s}(0,4)) \multimap \mathrm{s}(0,4)}\ \forall E$$

$$\frac{\mathrm{np}(2,3) \multimap \mathrm{s}(0,4) \qquad (\mathrm{n}(2,3) \multimap \mathrm{s}(0,4)) \multimap \mathrm{s}(0,4)}{\mathrm{s}(0,4)}\ \multimap E$$

When we keep only the propositional part,
we are left with <u>the "missing proof" from before</u>

John believes someone left (revisited)

$$\dfrac{\dfrac{\text{john} \vdash n \qquad \dfrac{\text{believes} \vdash (n\backslash s)/s \qquad \dfrac{[p_0 \vdash n]^1 \quad \text{left} \vdash n\backslash s}{p_0\ \text{left} \vdash s}\,/E}{\text{believes}\ p_0\ \text{left} \vdash n\backslash s}\,\backslash E}{\text{john believes}\ p_0\ \text{left} \vdash s}\,\backslash E}{\dfrac{\text{john believes}, \text{left} \vdash s \uparrow_> n}{\text{john believes someone left} \vdash s}\,\uparrow_> I_1 \qquad \text{someone} \vdash (s \uparrow_> n) \downarrow_> s}\,\downarrow_> E$$

John believes someone left (revisited)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\left[\begin{array}{l}q_0\\ \text{np}\end{array}\right]^1 \qquad \begin{array}{l}\text{left}\\ (\text{np}\backslash\text{s})\end{array}}{q_0 \circ \text{left}}\backslash E \qquad \begin{array}{l}\text{believes}\\ (\text{np}\backslash\text{s})/\text{s}\end{array}}{\begin{array}{l}\text{s}\end{array}}/E \qquad \begin{array}{l}\text{john}\\ \text{np}\end{array}}{\begin{array}{l}\text{believes} \circ q_0 \circ \text{left}\\ (\text{np}\backslash\text{s})\end{array}}\backslash E}{\begin{array}{l}\text{john} \circ \text{believes} \circ q_0 \circ \text{left}\\ \text{s}\end{array}}|I_1 \qquad \begin{array}{l}\lambda q_2.(q_2\ \text{someone})\\ \text{s}|(\text{s}|\text{np})\end{array}}{\begin{array}{l}\text{john} \circ \text{believes} \circ \text{someone} \circ \text{left}\\ \text{s}\end{array}}|E$$

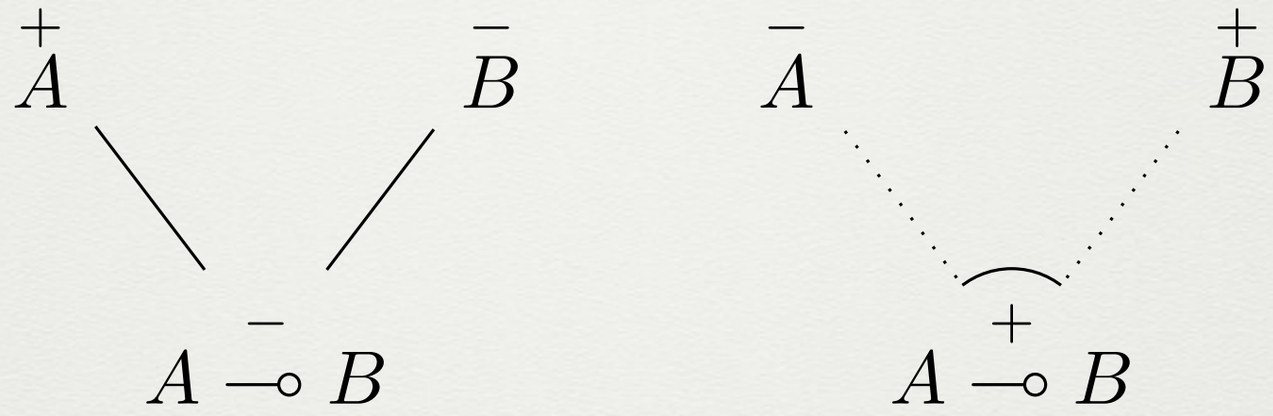
Proof nets

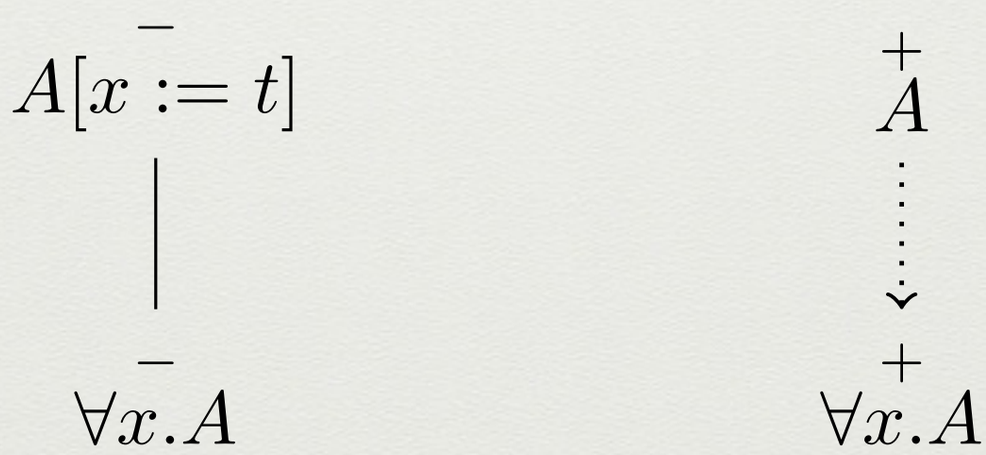
- Natural deduction is a system which works well enough for proof search (though less so when we include the existential quantifier and the product)

- There is a nice alternative, a graph-based presentation of proofs called *proof nets*

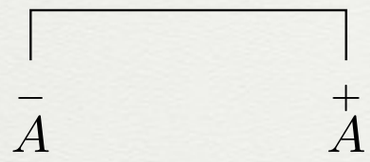
- Proof search for proof nets is very simple: we write down a formula decomposition tree (for each formula), connect leaves, then verify a condition on the resulting graph.

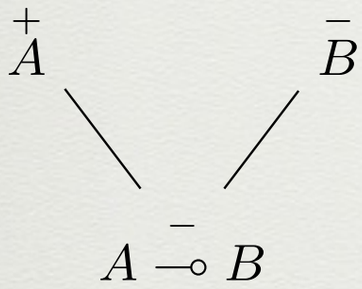
Proof nets: logical links

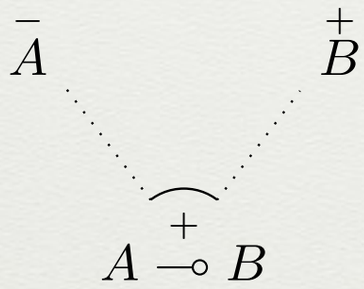


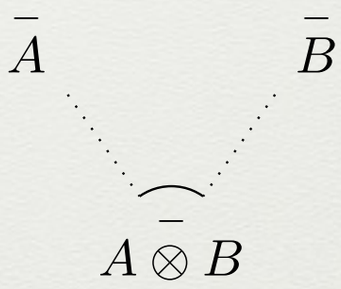


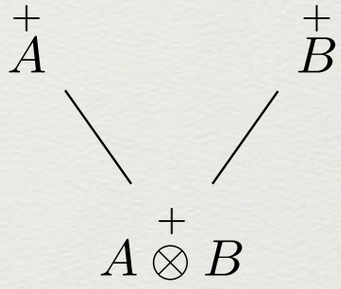
Proof nets: all links

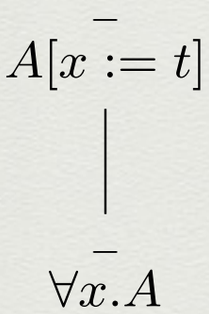


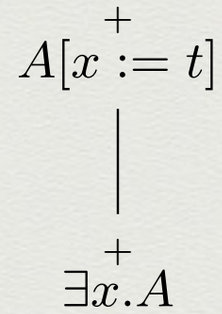








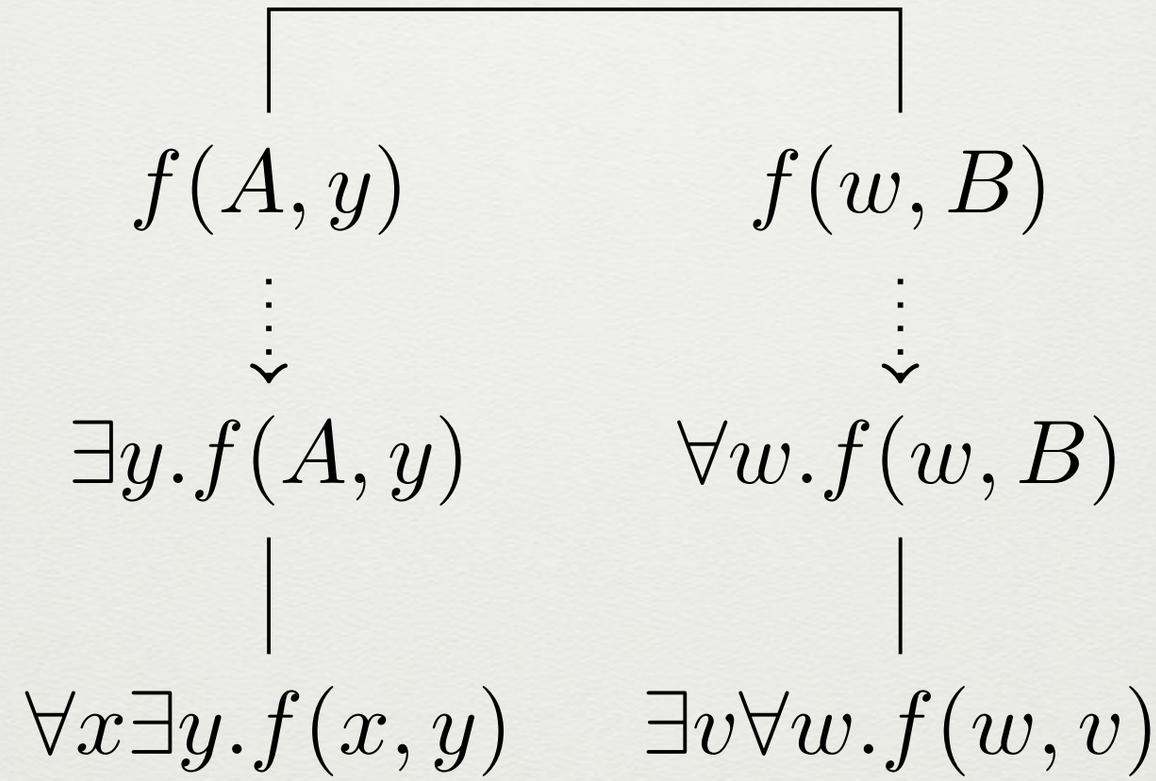




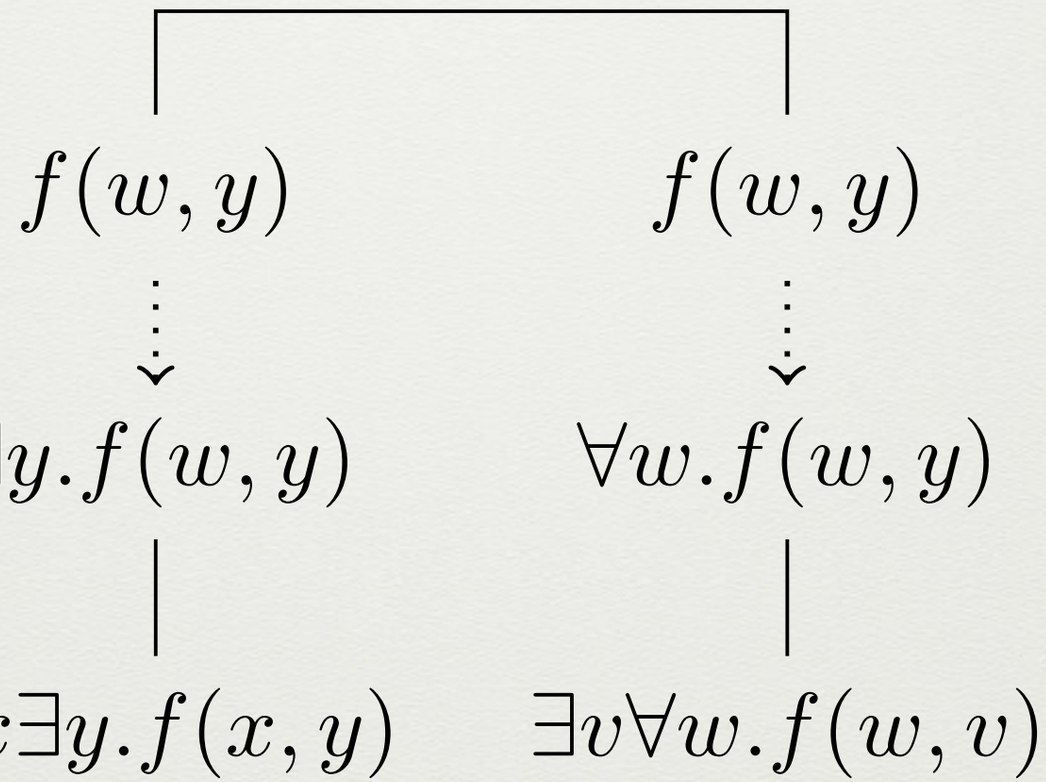
Example: an underivable statement

$$\begin{array}{cc} f(A,y) & f(w,B) \\ \vdots & \vdots \\ \exists y.f(A,y) & \forall w.f(w,B) \\ | & | \\ \forall x \exists y.f(x,y) & \exists v \forall w.f(w,v) \end{array}$$

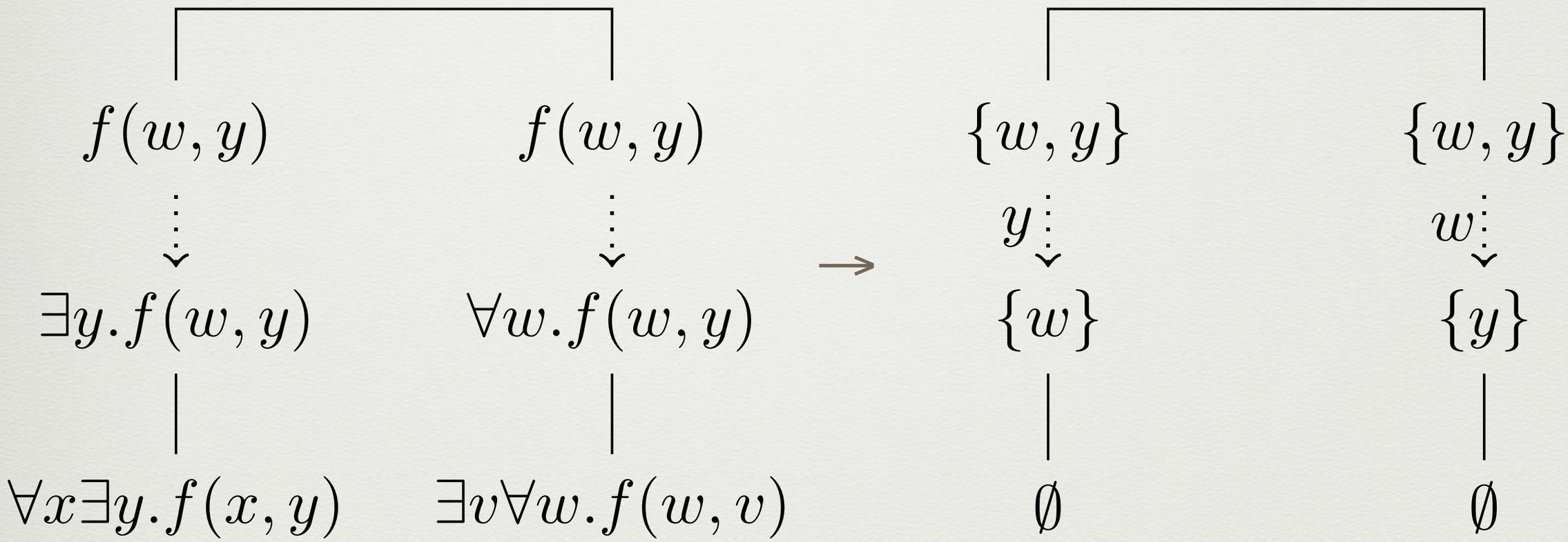
Example


$f(A, y)$
$f(w, B)$
$\exists y.f(A, y)$
$\forall w.f(w, B)$
$\forall x \exists y.f(x, y)$
$\exists v \forall w.f(w, v)$

Example

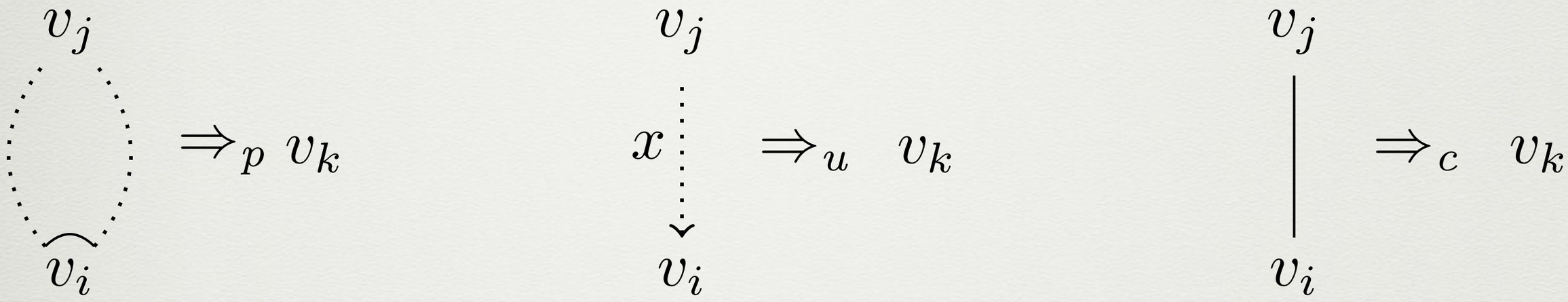

$f(w,y)$
$f(w,y)$
$\exists y.f(w,y)$
$\forall w.f(w,y)$
$\forall x \exists y.f(x,y)$
$\exists v \forall w.f(w,v)$

Checking correctness



We erase all formulas, keeping track only of the free variables

Graph contractions

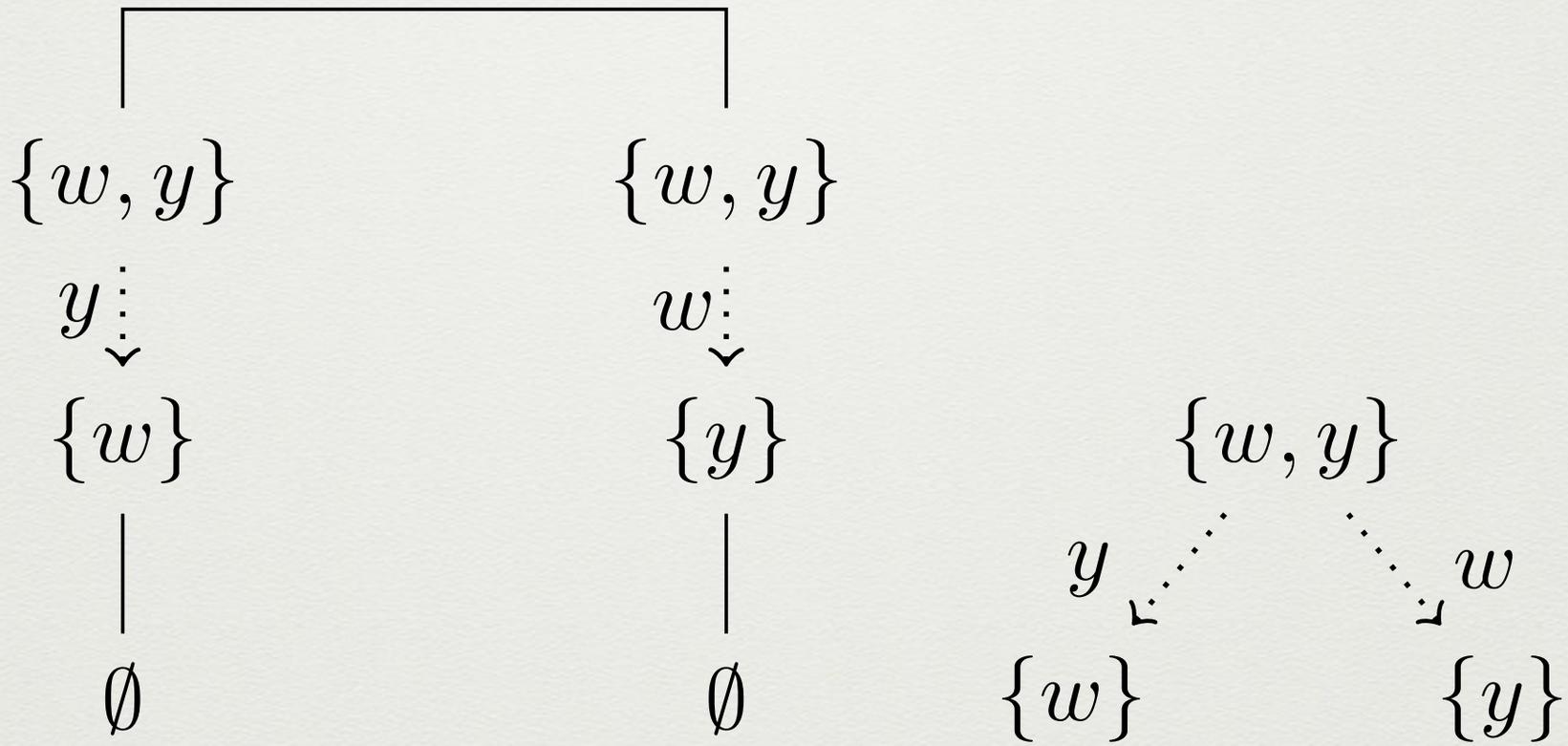


Conditions

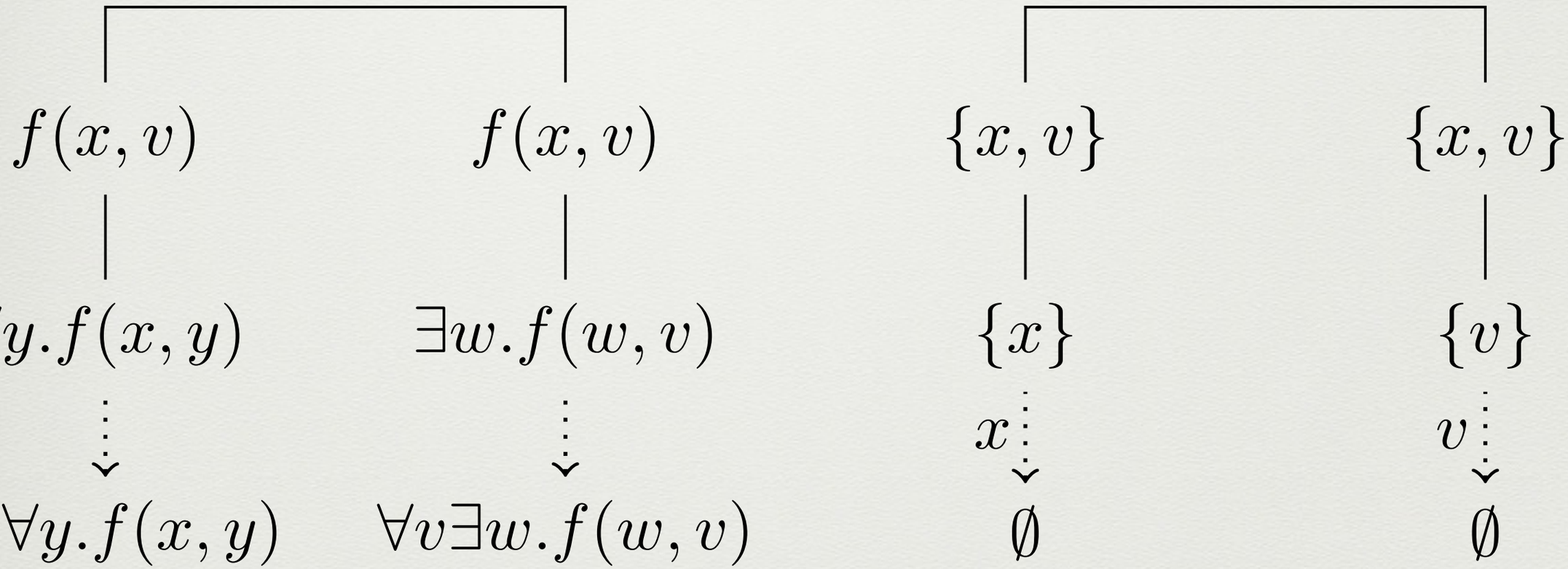
- all links: $v_i \neq v_j$; $\mathrm{VAR}(v_k) = \mathrm{VAR}(v_i) \cup \mathrm{VAR}(v_j)$
- p: two "connected" branches only
- u: all occurrences of x must be at v_j

Lemma a structure is *correct* iff it contacts to a single vertex using the contractions above

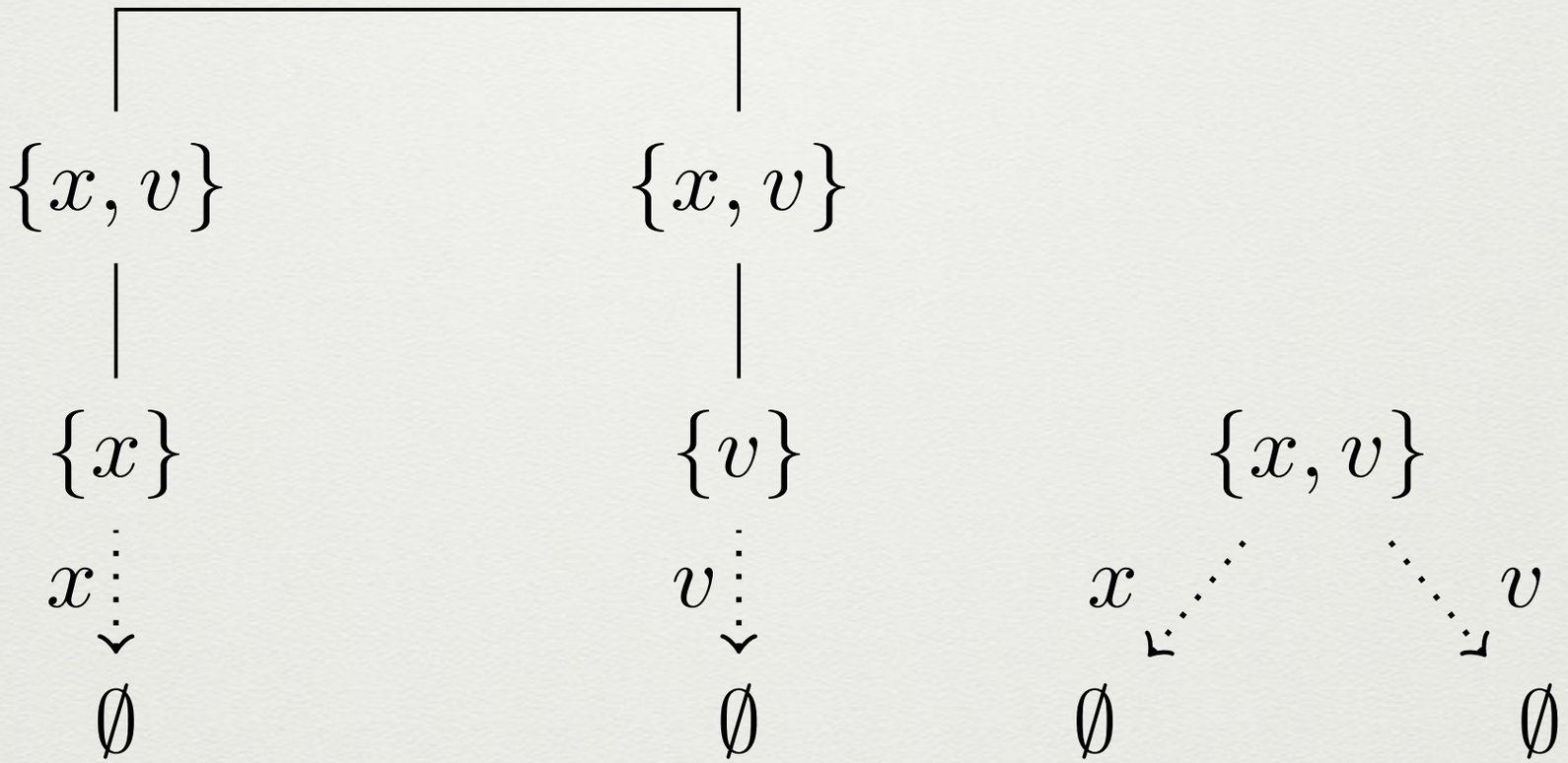
Example: contractions



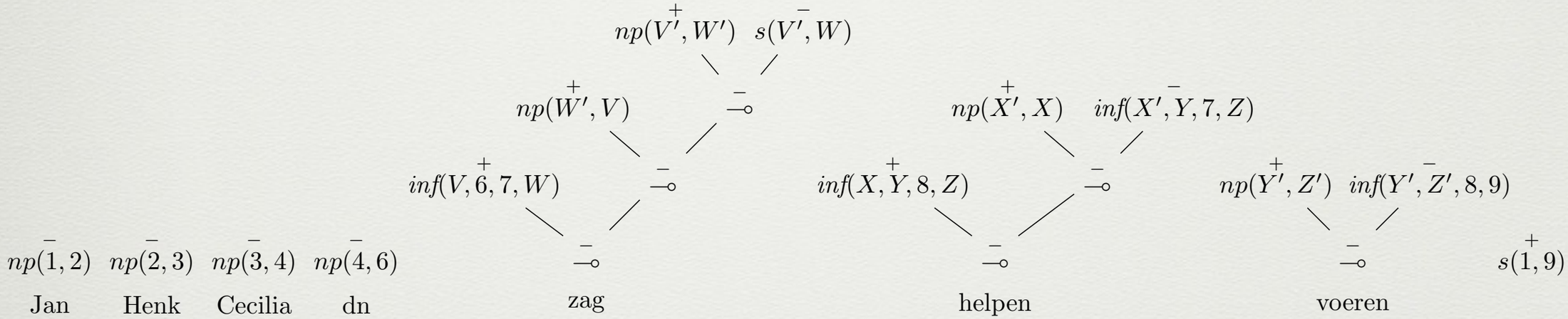
Another example



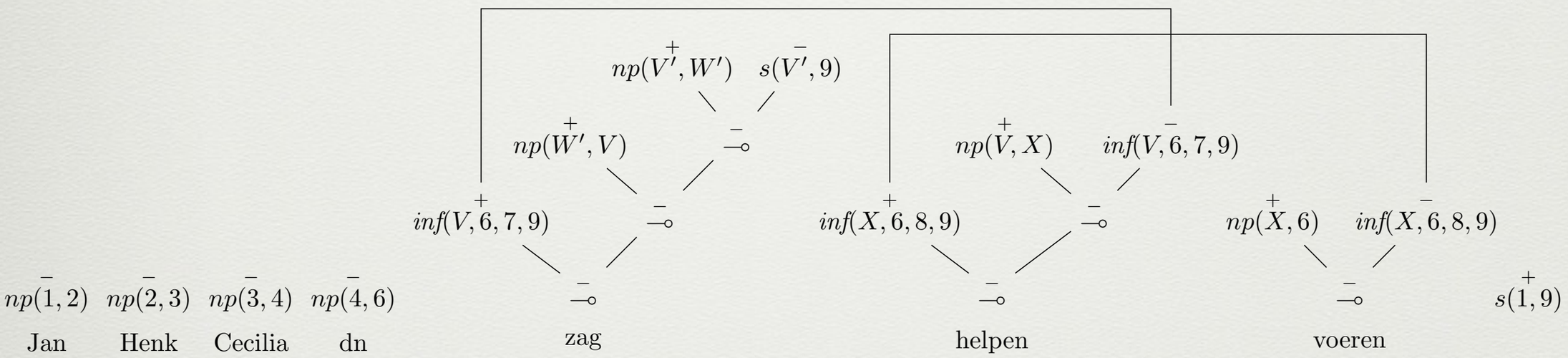
Example: contractions



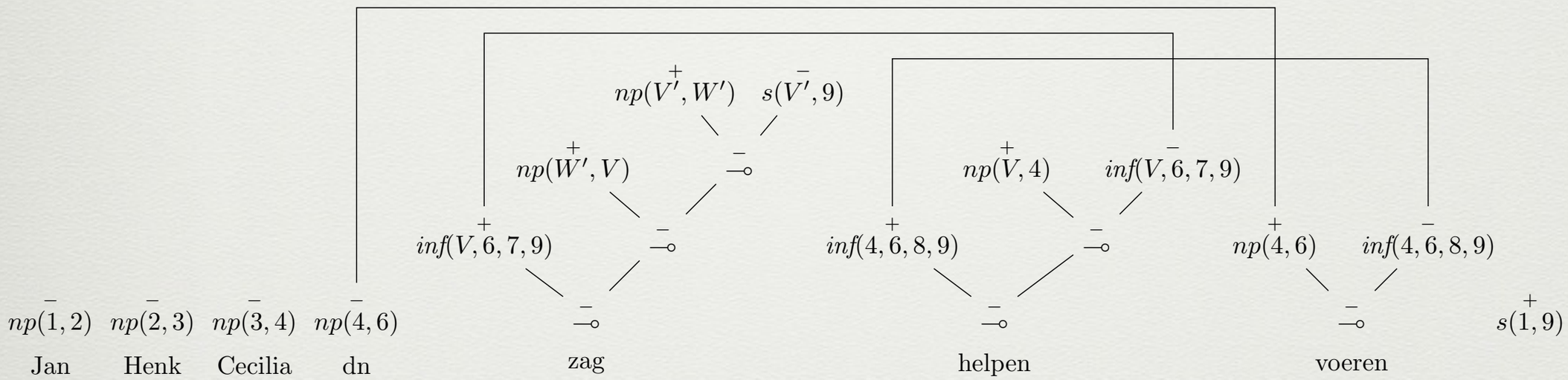
Nijlpaarden (revisited)



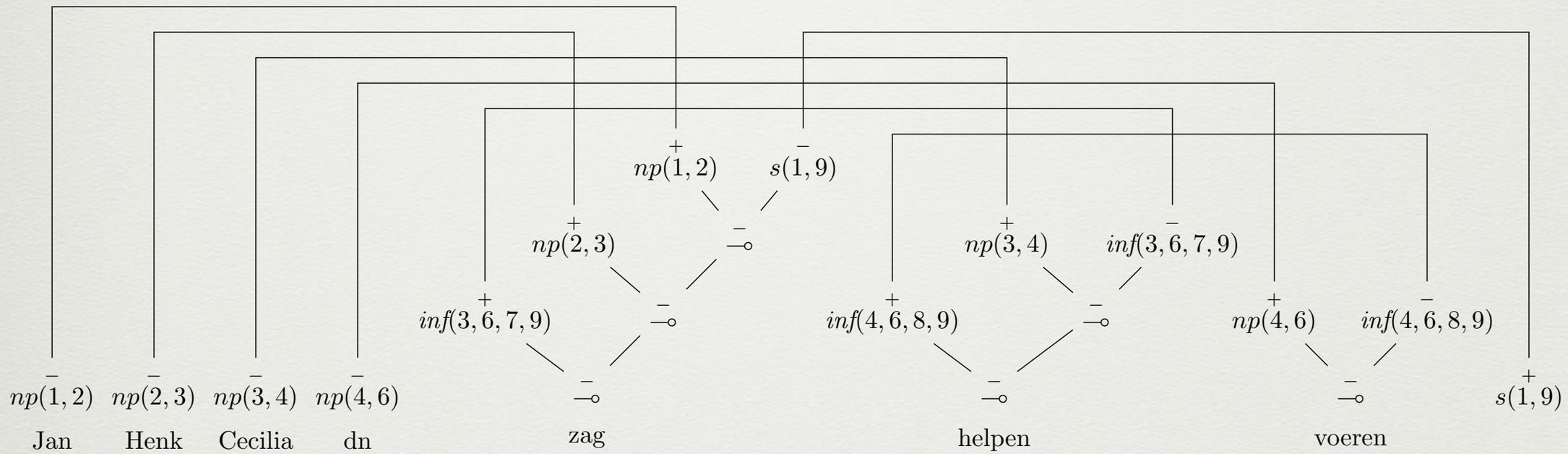
Nijlpaarden (revisited)



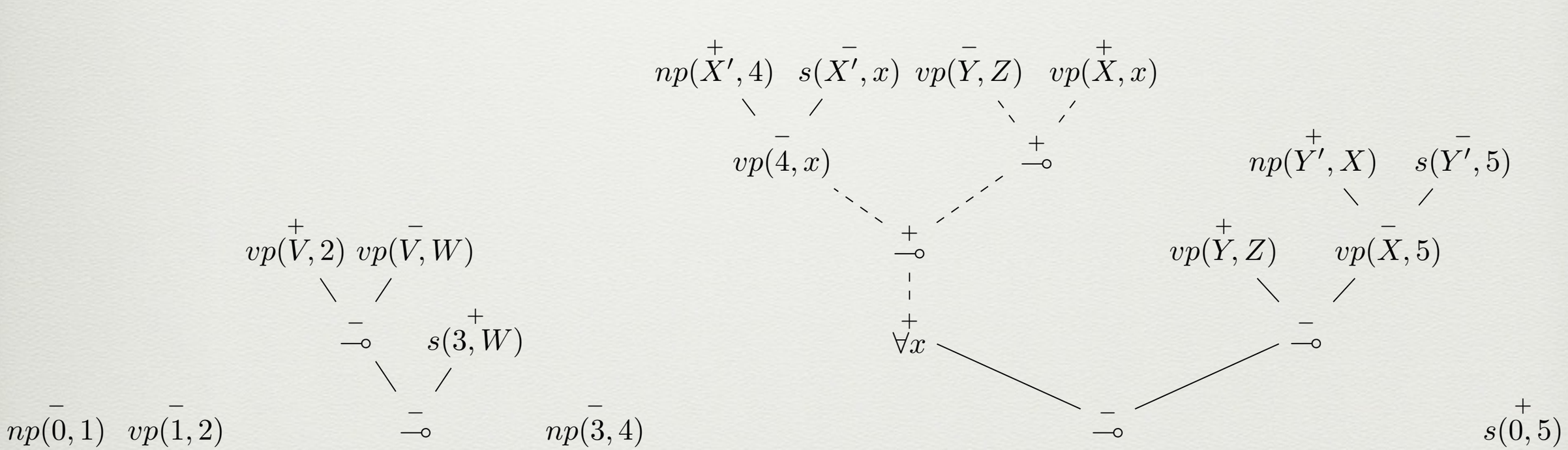
Nijlpaarden (revisited)



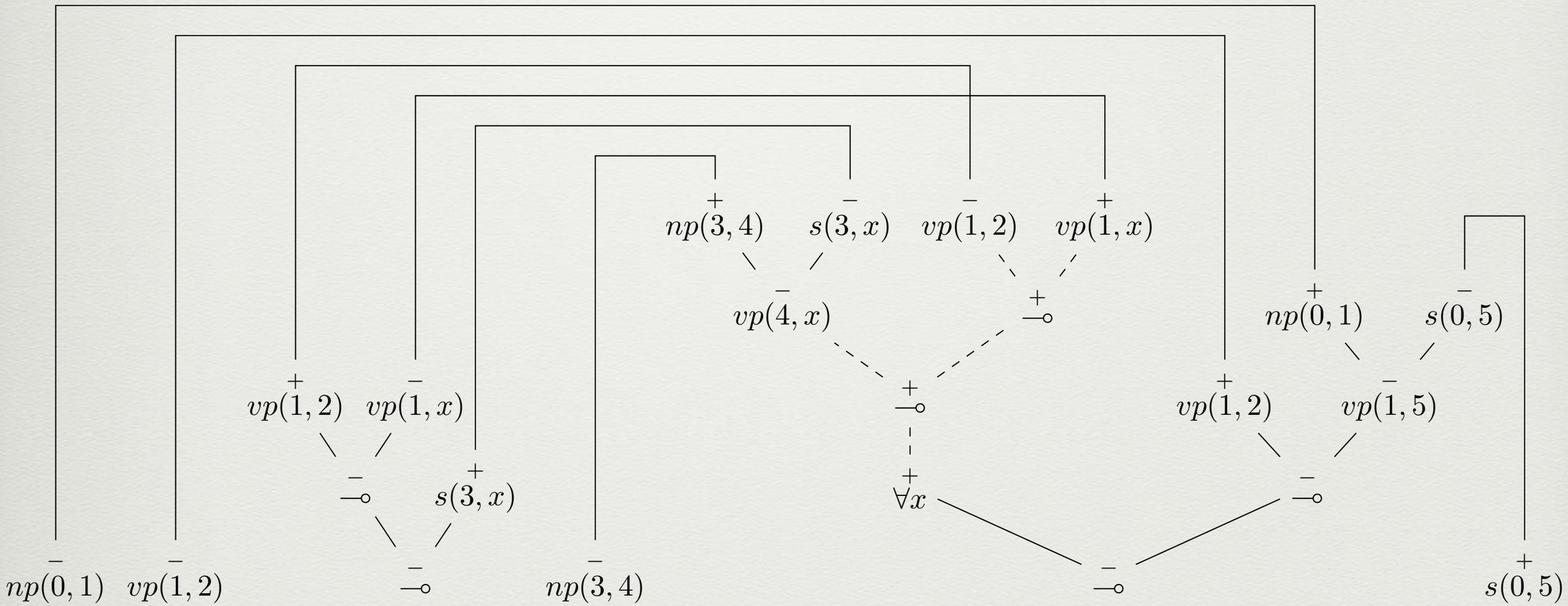
Nijlpaarden (revisited)



John left before Mary did



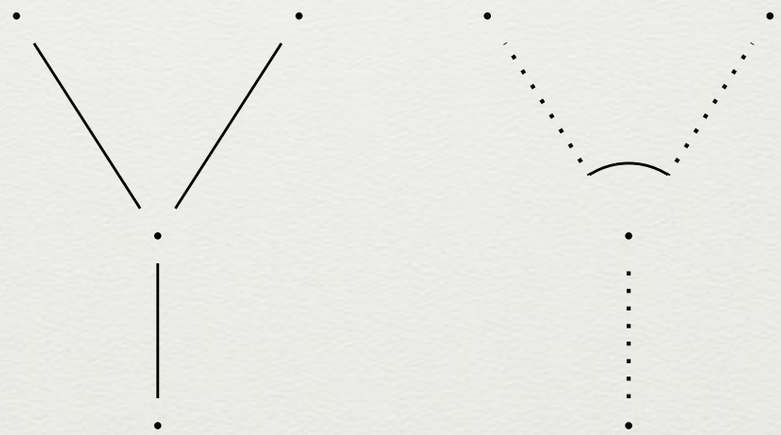
John left before Mary did



Lambek calculus vs first-order linear logic

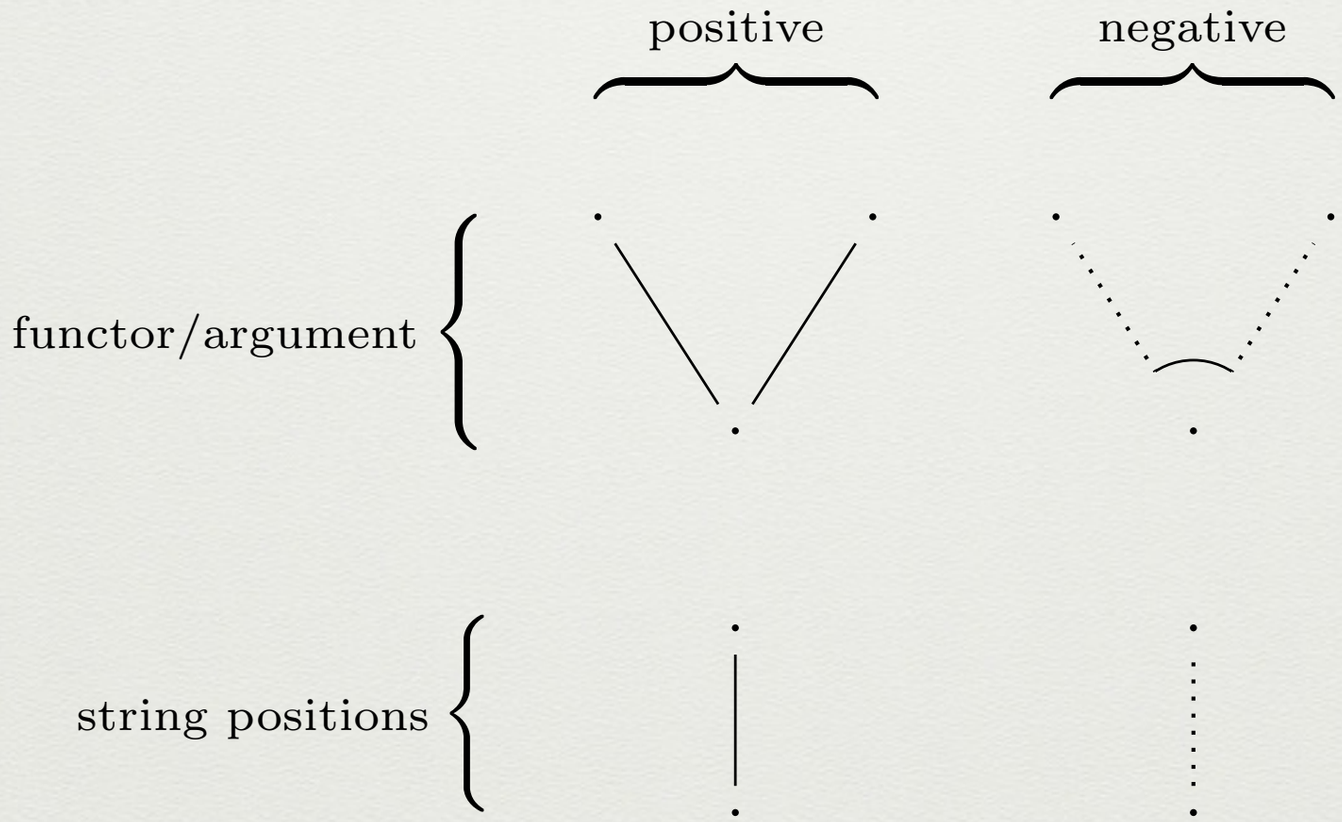
Lambek calculus

AB-grammar



Lambek calculus vs first-order linear logic

First-order linear logic



Measures of complexity: order

$$\text{order}(p) = 0$$

$$\text{order}(A \multimap B) = \max(\text{order}(A) + 1, \text{order}(B))$$

- roughly speaking, the order of the formulas used in type-logical grammars is an indication of the complexity of the *semantic* operations

- in treebanks, order 3 or 4 seems to suffice (order 4 occurs for gapping of auxiliaries and the copula)

Measures of complexity: width

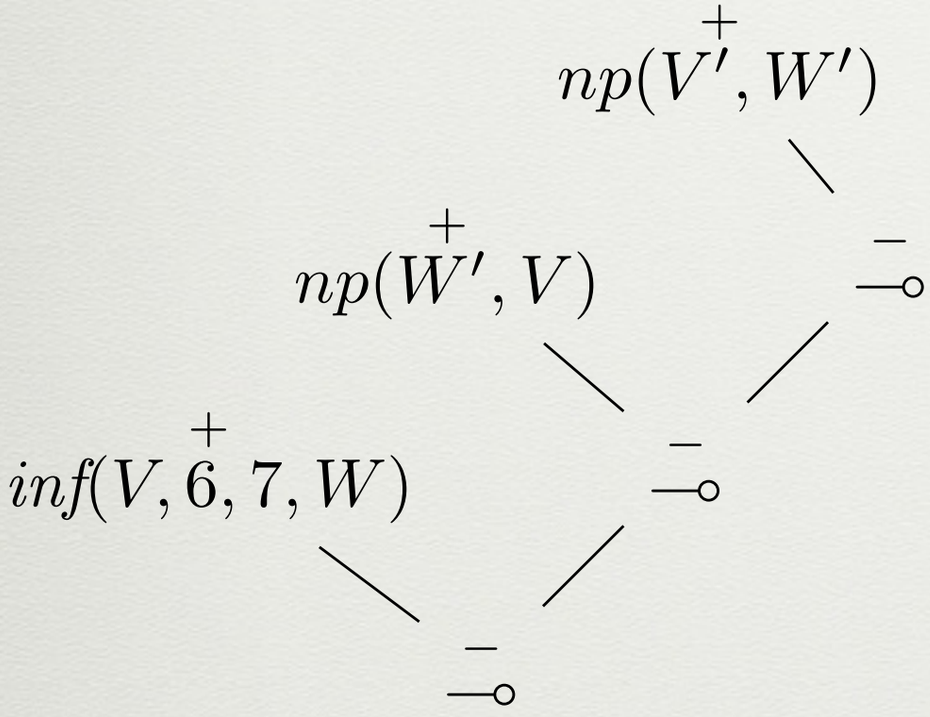
the width of a formula is the maximum number of free variables occurring in its subformulas

constants are treated like free variables,
connectives are treated as "synthetic" as much as possible

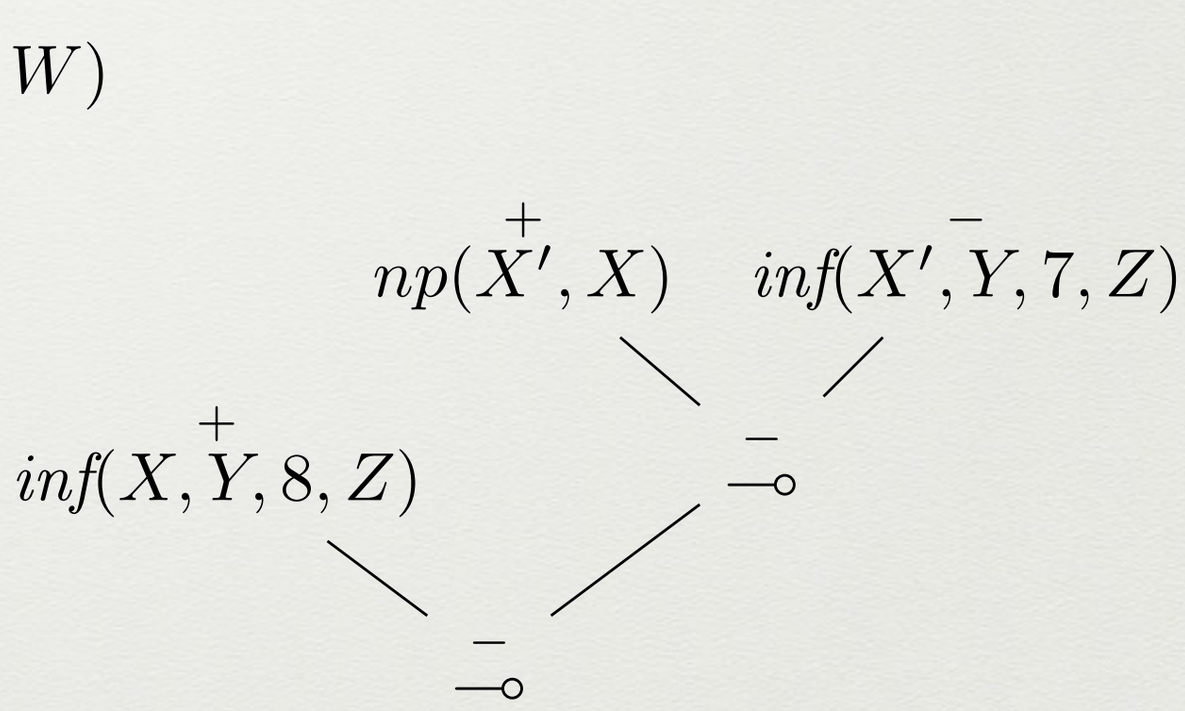
- roughly speaking, formula width corresponds to the complexity of the *string* operations: width 2 corresponds to operations on strings, width 4 pairs of strings etc.

- it is a more robust notion than predicate arity

Measures of complexity: width

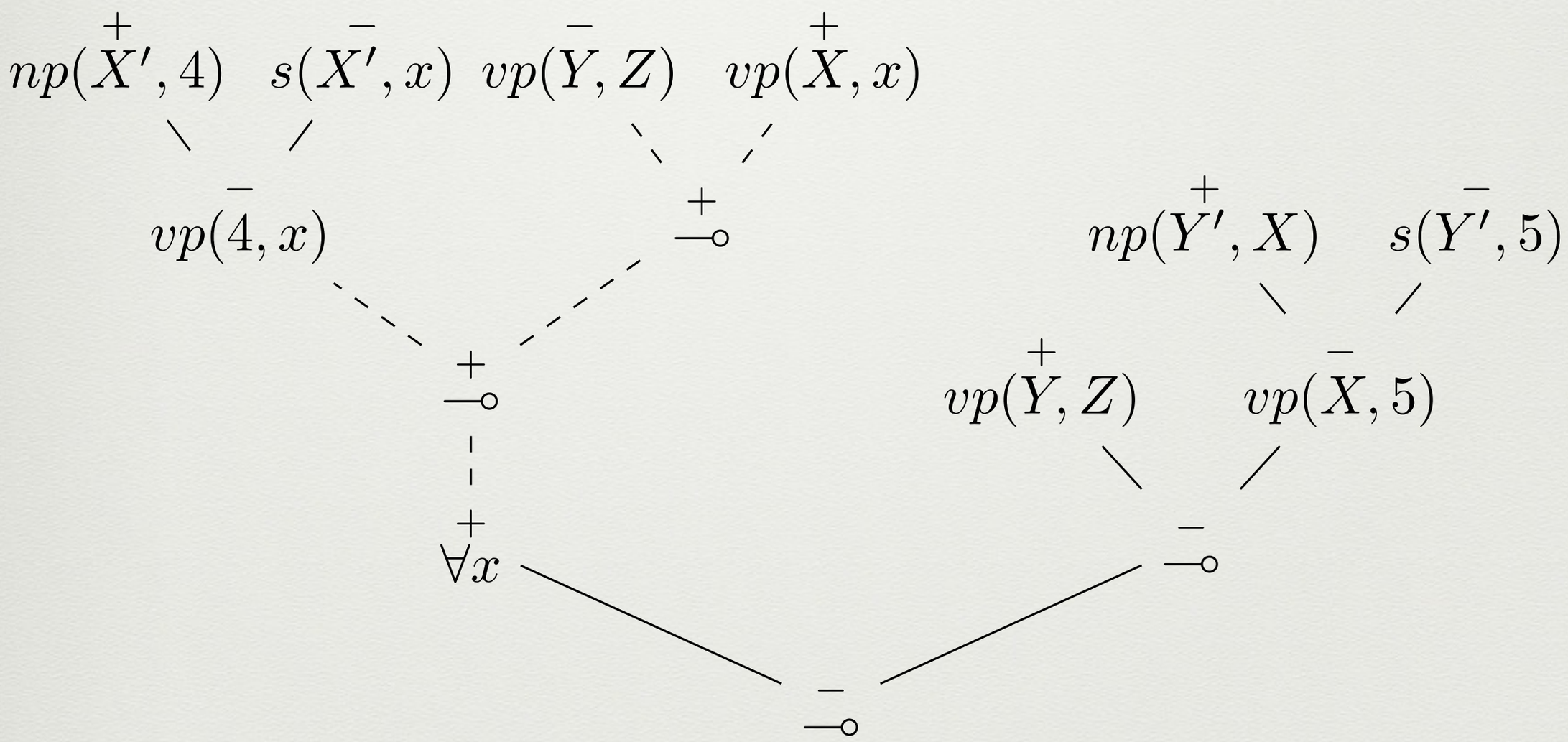


zag

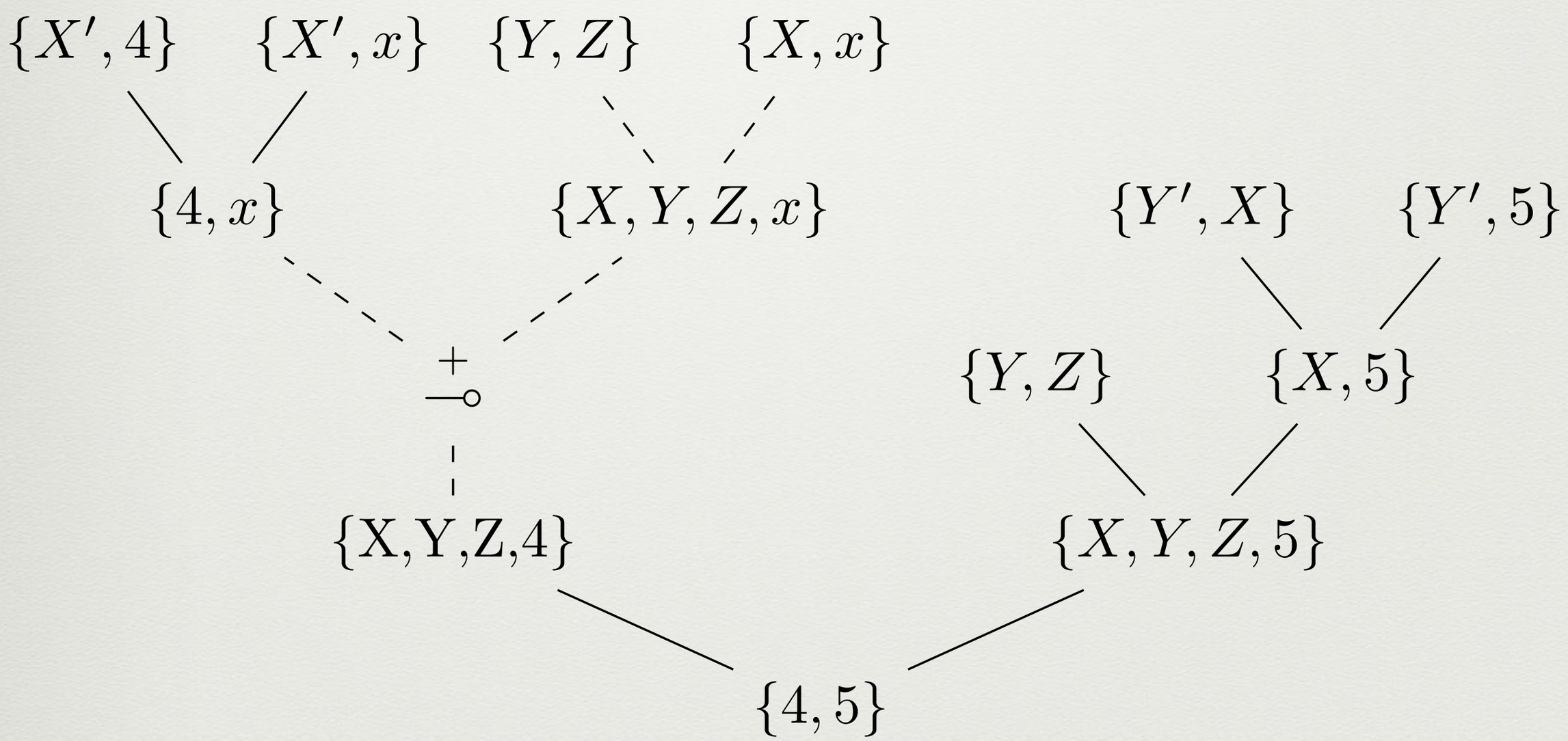


helpen

Measures of complexity: width



Measures of complexity: width



PARSING

Parsing

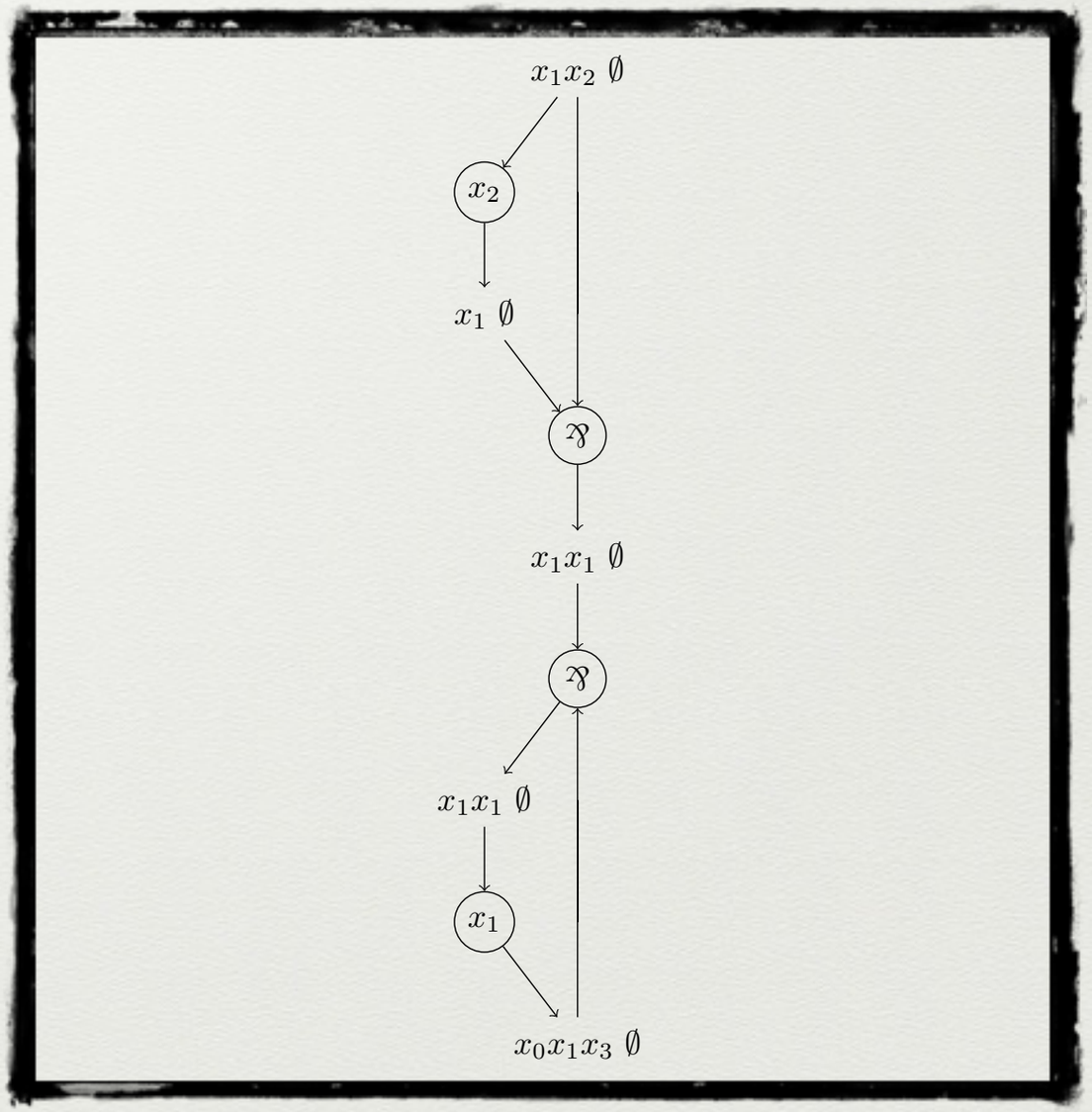
- The proof net calculus of the previous section suggests a simple, direct implementation of first-order linear logic (and by translation ACGs, hybrid-type-logical grammars, the Displacement calculus, etc.)

Parsing

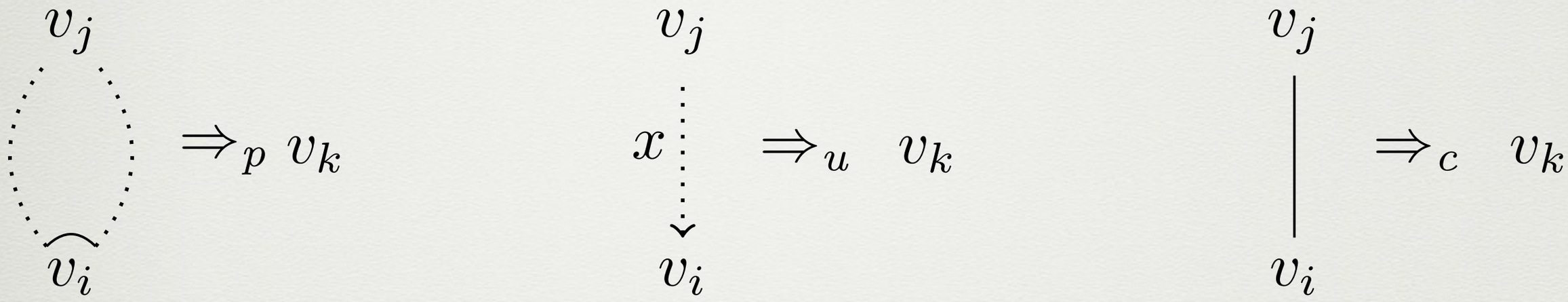
What makes theorem proving/parsing complicated?

1. Lexical ambiguity (for more realistic, wide-coverage grammars)

2. Combinatorics for the graph connections.

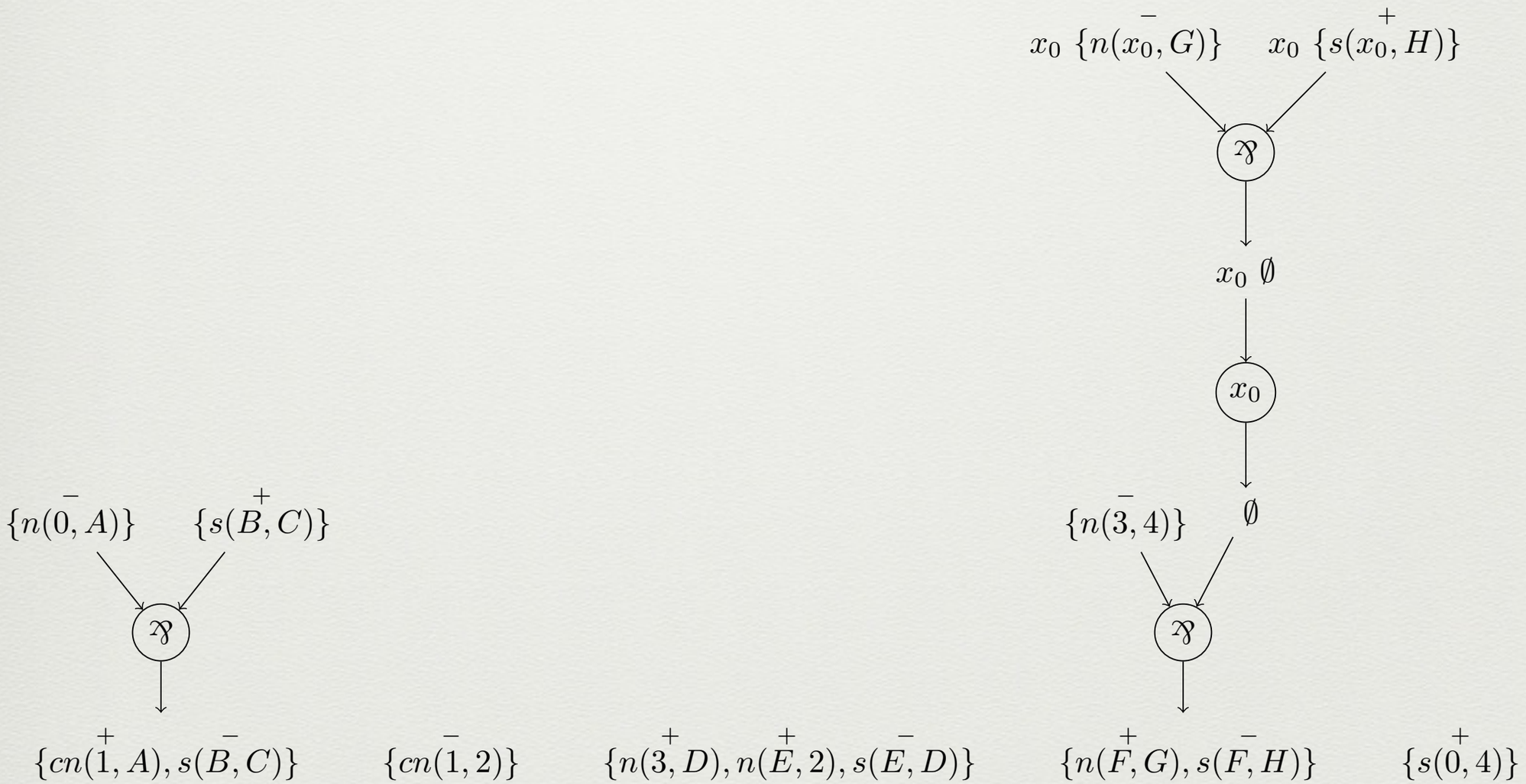


Parsing

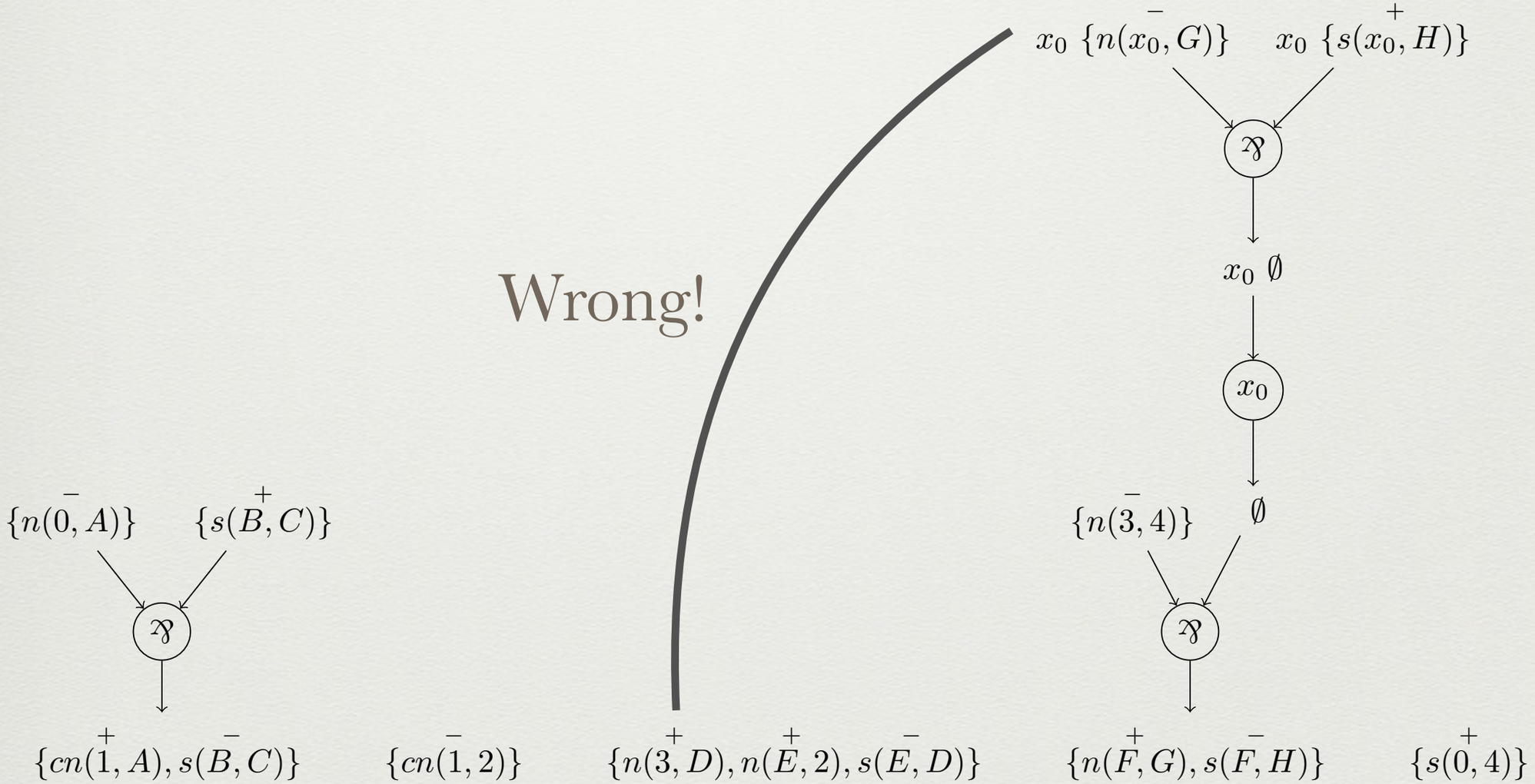


- After lexical lookup (and translation to first-order linear logic) we unfold the links and contract the structure as much as possible.
- We need to keep track both of the free variables and of the unlinked atoms in the graph.

Parsing

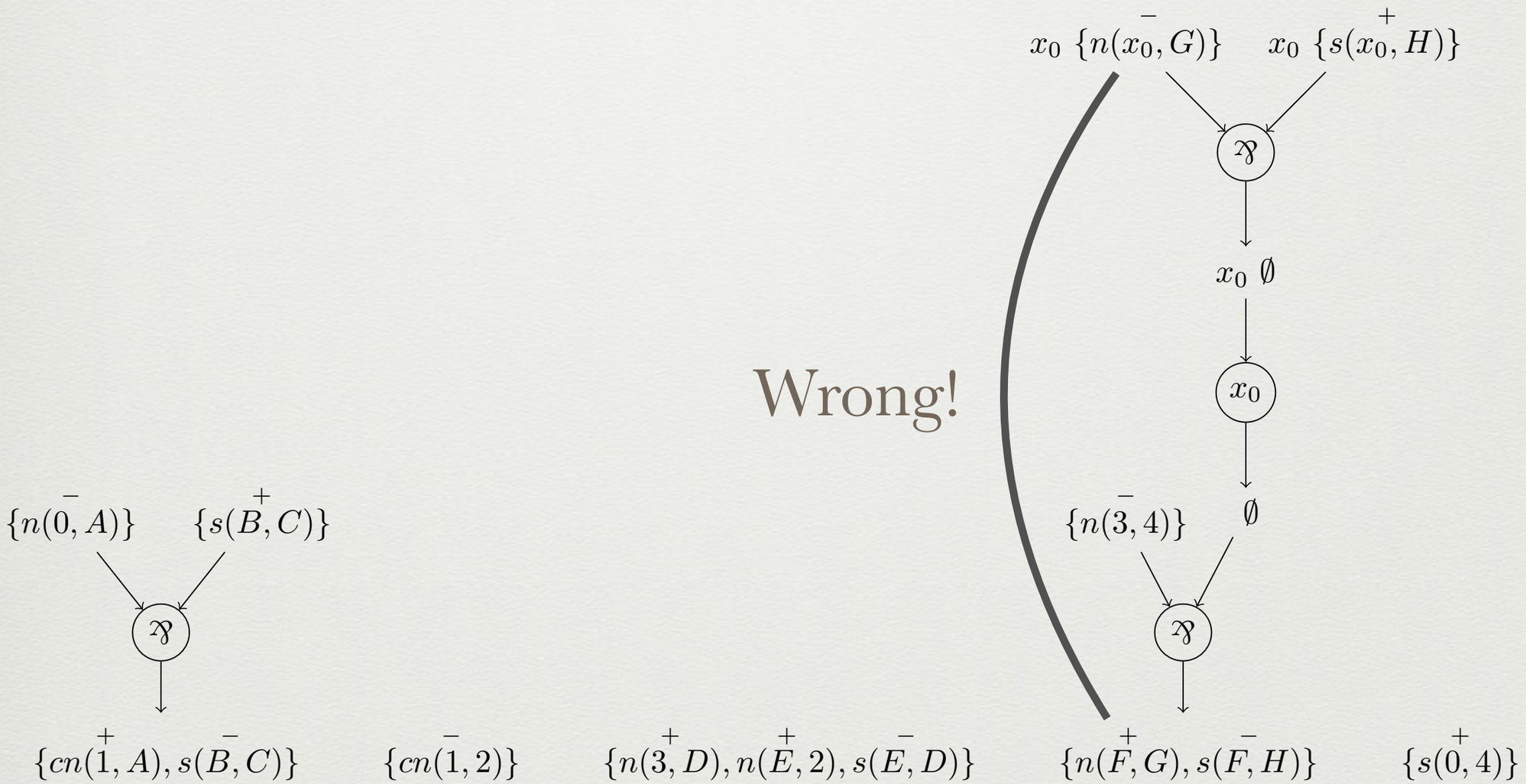

$x_0\ \{n(\bar{x_0}, G)\}$
$x_0\ \{s(\dot{x_0}, H)\}$
$\mathcal{Y}$
$x_0\ \emptyset$
x_0
$\{n(\bar{0}, A)\}$
$\{s(\dot{B}, C)\}$
$\mathcal{Y}$
$\{n(\bar{3}, 4)\}$
$\emptyset$
$\mathcal{Y}$
$\{cn(\dot{1}, A), s(\bar{B}, C)\}$
$\{cn(\bar{1}, 2)\}$
$\{n(\dot{3}, D), n(\dot{E}, 2), s(\bar{E}, D)\}$
$\{n(\dot{F}, G), s(\bar{F}, H)\}$
$\{s(\dot{0}, 4)\}$

Parsing



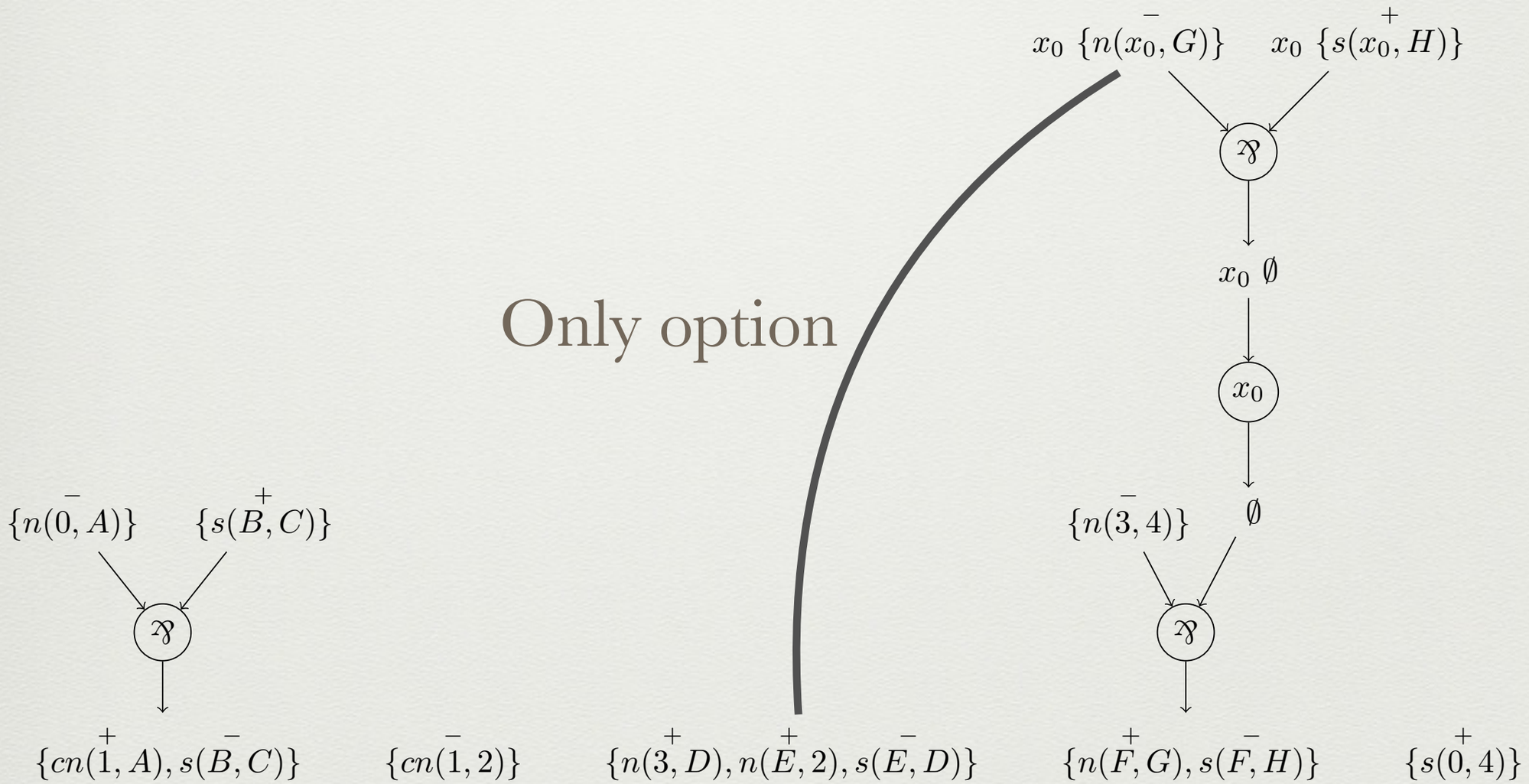
Wrong!

Parsing



Wrong!

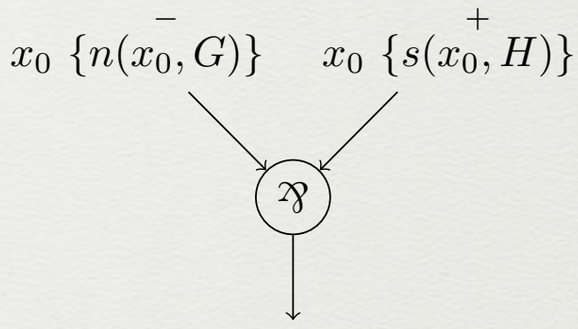
Parsing

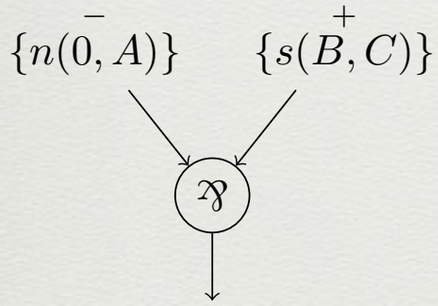


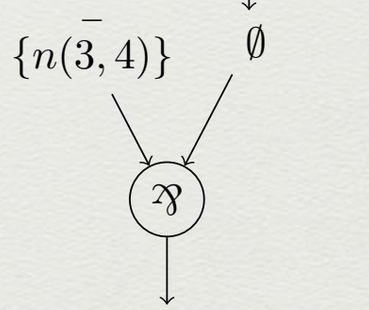
Only option

Parsing

	$n(3,D)$	$n(E,2)$	$n(F,G)$
$n(0,A)$		E=0,A=2	F=0,G=A
$n(3,4)$	D=4		F=3,G=4
$n(x_0,G)$		E=x_0,G=2	







$\{\bar{cn}(1, 2)\}$

$\{\bar{n}(3, D), \bar{n}(E, 2), \bar{s}(E, D)\}$

$\{\bar{s}(0, 4)\}$

Dancing links

- We use an implementation of Knuth's simple "dancing links" algorithm to traverse the search space in a smart way.
- In our case, it means connecting the atom which has the fewest possibilities (checking unifiability and absence of a directed path and "islands")

Proof generation/transformation

- When a proof is found, we can use the axiom connections and the contraction sequence to obtain a sequent proof (in first-order linear logic).

- There are optional proof transformations to natural deduction for first-order linear logic, ACGs, hybrid type-logical grammars and the Displacement calculus.

“book which every student loves”

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{n}(3,4)\vdash\mathrm{n}(3,4)}\quad\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{np}(5,5)\vdash\mathrm{np}(5,5)}\quad\dfrac{\dfrac{}{\mathrm{np}(2,4)\vdash\mathrm{np}(2,4)}\quad\dfrac{}{\mathrm{s}(2,5)\vdash\mathrm{s}(2,5)}}{\mathrm{np}(2,4),\mathrm{np}(2,4)\multimap\mathrm{s}(2,5)\vdash\mathrm{s}(2,5)}L\multimap}}{\mathrm{np}(5,5),\mathrm{np}(5,5)\multimap\mathrm{np}(2,4)\multimap\mathrm{s}(2,5),\mathrm{np}(2,4)\vdash\mathrm{s}(2,5)}L\multimap}}{\mathrm{np}(5,5),\mathrm{np}(5,5)\multimap\mathrm{np}(2,4)\multimap\mathrm{s}(2,5)\vdash\mathrm{np}(2,4)\multimap\mathrm{s}(2,5)}R\multimap\quad\dfrac{}{\mathrm{s}(2,5)\vdash\mathrm{s}(2,5)}}{\mathrm{np}(5,5),\mathrm{np}(5,5)\multimap\mathrm{np}(2,4)\multimap\mathrm{s}(2,5),(\mathrm{np}(2,4)\multimap\mathrm{s}(2,5))\multimap\mathrm{s}(2,5)\vdash\mathrm{s}(2,5)}L\multimap}}{\mathrm{n}(3,4),\mathrm{n}(3,4)\multimap(\mathrm{np}(2,4)\multimap\mathrm{s}(2,5))\multimap\mathrm{s}(2,5),\mathrm{np}(5,5),\mathrm{np}(5,5)\multimap\mathrm{np}(2,4)\multimap\mathrm{s}(2,5)\vdash\mathrm{s}(2,5)}L\multimap}}{\mathrm{n}(3,4),\mathrm{n}(3,4)\multimap(\mathrm{np}(2,4)\multimap\mathrm{s}(2,5))\multimap\mathrm{s}(2,5),\mathrm{np}(5,5)\multimap\mathrm{np}(2,4)\multimap\mathrm{s}(2,5)\vdash\mathrm{np}(5,5)\multimap\mathrm{s}(2,5)}R\multimap\quad\dfrac{\dfrac{}{\mathrm{n}(0,1)\vdash\mathrm{n}(0,1)}\quad\dfrac{}{\mathrm{n}(0,5)\vdash\mathrm{n}(0,5)}}{\mathrm{n}(0,1),\mathrm{n}(0,1)\multimap\mathrm{n}(0,5)\vdash\mathrm{n}(0,5)}L\multimap}}{\mathrm{n}(3,4),\mathrm{n}(3,4)\multimap(\mathrm{np}(2,4)\multimap\mathrm{s}(2,5))\multimap\mathrm{s}(2,5),\mathrm{np}(5,5)\multimap\mathrm{np}(2,4)\multimap\mathrm{s}(2,5),(\mathrm{np}(5,5)\multimap\mathrm{s}(2,5))\multimap\mathrm{n}(0,1)\multimap\mathrm{n}(0,5),\mathrm{n}(0,1)\vdash\mathrm{n}(0,5)}L\multimap$$

“book which every student loves”

$$\dfrac{\dfrac{\dfrac{[\mathrm{np}(2,4)]^1 \quad \dfrac{[\mathrm{np}(5,5)]^2 \quad \mathrm{np}(5,5) \multimap \mathrm{np}(2,4) \multimap \mathrm{s}(2,5)}{\mathrm{np}(2,4) \multimap \mathrm{s}(2,5)} \multimap E}{\mathrm{s}(2,5)} \multimap E}{\dfrac{\mathrm{s}(2,5)}{\mathrm{np}(2,4) \multimap \mathrm{s}(2,5)} \multimap I_1} \quad \dfrac{\mathrm{n}(3,4) \quad \mathrm{n}(3,4) \multimap (\mathrm{np}(2,4) \multimap \mathrm{s}(2,5)) \multimap \mathrm{s}(2,5)}{(\mathrm{np}(2,4) \multimap \mathrm{s}(2,5)) \multimap \mathrm{s}(2,5)} \multimap E}{\mathrm{s}(2,5)} \multimap E$$

$$\dfrac{\mathrm{n}(0,1) \quad \dfrac{\dfrac{\mathrm{s}(2,5)}{\mathrm{np}(5,5) \multimap \mathrm{s}(2,5)} \multimap I_2 \quad (\mathrm{np}(5,5) \multimap \mathrm{s}(2,5)) \multimap \mathrm{n}(0,1) \multimap \mathrm{n}(0,5)}{\mathrm{n}(0,1) \multimap \mathrm{n}(0,5)} \multimap E}{\mathrm{n}(0,5)} \multimap E$$

“book which every student loves”

(1) $\lambda x_0.((\text{book}\, x_0) \wedge \forall x_1.[(\text{student}\, x_1) \to ((\text{loves}\, x_0)\, x_1)])$

$$\dfrac{\begin{bmatrix} r_0 \\ \text{np} \end{bmatrix}^2 \qquad \begin{matrix} \lambda s_0.\lambda t_0.t_0 \circ \text{loves} \circ s_0 \\ \text{np} \multimap (\text{np} \multimap \text{s}) \end{matrix}}{\begin{matrix} \lambda t_0.t_0 \circ \text{loves} \circ r_0 \\ \text{np} \multimap \text{s} \end{matrix}} \multimap E$$

$$\dfrac{\begin{bmatrix} q_0 \\ \text{np} \end{bmatrix}^1 \qquad \begin{matrix} \lambda t_0.t_0 \circ \text{loves} \circ r_0 \\ \text{np} \multimap \text{s} \end{matrix}}{\begin{matrix} q_0 \circ \text{loves} \circ r_0 \\ \text{s} \end{matrix}} \multimap E$$

$$\dfrac{\begin{matrix} q_0 \circ \text{loves} \circ r_0 \\ \text{s} \end{matrix}}{\begin{matrix} \lambda q_0.q_0 \circ \text{loves} \circ r_0 \\ \text{np} \multimap \text{s} \end{matrix}} \multimap I_1$$

$$\dfrac{\begin{matrix} \text{student} \\ \text{n} \end{matrix} \qquad \begin{matrix} \lambda s_2.\lambda t_2.(t_2\, \text{every} \circ s_2) \\ \text{n} \multimap ((\text{np} \multimap \text{s}) \multimap \text{s}) \end{matrix}}{\begin{matrix} \lambda t_2.(t_2\, \text{every} \circ \text{student}) \\ (\text{np} \multimap \text{s}) \multimap \text{s} \end{matrix}} \multimap E$$

$$\dfrac{\begin{matrix} \lambda q_0.q_0 \circ \text{loves} \circ r_0 \\ \text{np} \multimap \text{s} \end{matrix} \qquad \begin{matrix} \lambda t_2.(t_2\, \text{every} \circ \text{student}) \\ (\text{np} \multimap \text{s}) \multimap \text{s} \end{matrix}}{\begin{matrix} \text{every} \circ \text{student} \circ \text{loves} \circ r_0 \\ \text{s} \end{matrix}} \multimap E$$

$$\dfrac{\begin{matrix} \text{every} \circ \text{student} \circ \text{loves} \circ r_0 \\ \text{s} \end{matrix}}{\begin{matrix} \lambda r_0.\text{every} \circ \text{student} \circ \text{loves} \circ r_0 \\ \text{np} \multimap \text{s} \end{matrix}} \multimap I_2$$

$$\dfrac{\begin{matrix} \lambda r_0.\text{every} \circ \text{student} \circ \text{loves} \circ r_0 \\ \text{np} \multimap \text{s} \end{matrix} \qquad \begin{matrix} \lambda s_4.\lambda t_4.t_4 \circ \text{which} \circ (s_4\, \epsilon) \\ (\text{np} \multimap \text{s}) \multimap (\text{n} \multimap \text{n}) \end{matrix}}{\begin{matrix} \lambda t_4.t_4 \circ \text{which} \circ \text{every} \circ \text{student} \circ \text{loves} \\ \text{n} \multimap \text{n} \end{matrix}} \multimap E$$

$$\dfrac{\begin{matrix} \text{book} \\ \text{n} \end{matrix} \qquad \begin{matrix} \lambda t_4.t_4 \circ \text{which} \circ \text{every} \circ \text{student} \circ \text{loves} \\ \text{n} \multimap \text{n} \end{matrix}}{\begin{matrix} \text{book} \circ \text{which} \circ \text{every} \circ \text{student} \circ \text{loves} \\ \text{n} \end{matrix}} \multimap E$$

The future

- Better treatment of lexical ambiguity, eg. à la Perrier e.a. but additives and chart-like strategies are other lines of attack.

- Can we cast first-order linear logic parsing as a constraint satisfaction problem?

- Would a connection with dependent types be useful for the semantics? (Pompigne&Pogodalla, Luo)

- More grammars

Conclusions

- First-order linear logic can be used for developing and testing grammars for many interesting phenomena on the syntax-semantics interface.

- This gives a uniform proof search strategy for different frameworks, though proof transformations give us back proofs in the formalism which interests us.

- Source code is freely available:

  https://github.com/RichardMoot/LinearOne

Partial orders