Production scheduling of a fermentation process

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1 Problem statement

Industrial fermentation processes are generally batch operations. The process consists of a number of phases or stages, each of which requires a considerable amount of time. Because of the time factor, the system cannot react quickly to variations in demand, so that the scheduling of production in each stage is of considerable importance.

Consider a fermentation process which has the stages as shown in Table 1. m_i (f_i, p_i, etc.) is the quantity that goes into stage m (from the previous stage) in period i; a period may be a week, a day, or even a shift. In our case, we will look at daily periods, i.e. each i is one day. d_i is the demand (by customers) from the warehouse in period i. The demand d_i is known (but not precisely). Given d_i, a set of values m_i, f_i, p_i, b_i, w_i constitutes a production schedule. The capital letters indicate the total quantity in the stage, i.e. M_j is the total amount of material in the mixing stage in period j.

Table 1: The different stages of the fermentation process.

(1)	Mixing and cooking	M, m
(2)	Fermentation	F, f
(3)	Purification	P, p
(4)	Blending and packaging	B, b
(5)	Warehousing	W, w

The basic cost factor in every stage is the labor cost, if only that portion of variable costs which depends on the production rate of a stage is considered. Extra production may require overtime operations by the crews, which include both production and ”clean-up” workers. Overtime production in a stage will be indicated by a prime (m_i'), and the costs are indicated as follows:

$$\begin{aligned}
C_m &= \text{cost/man-hour for straight time labor in stage } m \\
C'_m &= \text{cost/man-hour for overtime labor in stage } m \\
C_f &= \text{cost/man-hour for straight time labor in stage } f \\
&\text{etc.}
\end{aligned}$$

A planning horizon is considered to be a large number of periods (i); for our production schedule, the planning horizon is a 3-months' duration, $1 \le i \le N$, where $N = 105$.

There are also the following considerations to make:

- For each schedule, the mixing and cooking stage is of fixed duration, $t_m = 3$ periods.
- There is a minimum time required for fermentation and a maximum time allowed by the process. The minimum time is 10 days, and the maximum time is 30 days.
- The purification process requires a minimum of 2 days and a maximum of 5 days.
- Blending and packaging takes one day to perform; the total amount passing through the blending stage is restricted only by capacity $Z_b = 5000L$.
- Warehousing is limited by the age of the product, where $A_{\max} = 8$ is the maximum number of time periods allowed for storage.
- There are restrictions on the available man hours in any stage for both straight time and overtime:

$$\begin{aligned}
H_{m_i} &= \text{max. available hours in mixing in period } i\text{, straight time} \\
H'_{m_i} &= \text{max. available hours in mixing in period } i\text{, overtime,} \\
H_{f_i} &= \text{max. available hours in fermentation in period } i\text{, straight time} \\
&\text{etc.}
\end{aligned}$$

- The output expressed in barrels or tons can be converted to man hours by introducing

Table 2: The data for the fermentation problem.

Process	Cost/man hour $\left[\frac{DKK}{\text{hr}}\right]$	Max. available hours per period [hr]
Mixing - straight time	350	200
Mixing - overtime	700	40
Fermentation - straight time	410	250
Fermentation - overtime	800	250
Purification - straight time	380	200
Purification - overtime	1000	150
Blending - straight time	250	600
Blending - overtime	500	100
Warehousing - straight time	200	400
Warehousing - overtime	300	400

the factors:

$$\mu = \text{number of man hours/unit of production in mixing} = 2\frac{\text{hr}}{1000L}$$
$$\phi = \text{number of man hours/unit of production in fermentation} = 5\frac{\text{hr}}{1000L}$$
$$\pi = \text{number of man hours/unit of production in purification} = 7\frac{\text{hr}}{1000L}$$
$$\beta = \text{number of man hours/unit of production in blending} = 2\frac{\text{hr}}{1000L}$$
$$\chi = \text{number of man hours/unit of production in warehousing} = 1\frac{\text{hr}}{1000L}$$

2 Tasks

1. Formulate a linear program which describes the fermentation process

2. Solve the problem with the data provided in Table 2, and the following demand patterns [the first 20 days no demand]:

   - Constant demand: $d_i = 3000$, $d_i = 4000$, $d_i = 4500$.
   - Time-varying demand: $d_i = 5000 + 1000 \sin(i/10)$.

3. Analyze the solution: (a) what are the key bottlenecks in the system? (b) Where should the main focus of improvement be for this setup?