

Determining the optimal zone fares for urban transit

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1 Problem statement

Given a number of stations in an urban area, find the optimal zone fares that maximize the revenue.

1.1 Some background information on fare prices

- The fare from point i to point j is generally composed of two components: a fixed charge c_{fix} per ride and a variable, mileage charge c_{mile} which scales linearly with the distance.
- The ridership of the urban transit network depends on the fare price f_{ij} . To represent this dependency, we use the following linear relationship:

$$P_{ij} = P_{ij}^0 \left(1 + e_{ij} \frac{f_{ij} - f_{ij}^0}{f_{ij}^0} \right),$$

where P_{ij} is the ridership from i to j , P_{ij}^0 is the ridership at the base fare price f_{ij}^0 and e_{ij} is an elasticity parameter¹ which determines how large the impact of a fare change is.

1.2 Other considerations to make

- There is a minimum and maximum fare charged between each i and j , i.e. f_{ij}^{\min} and f_{ij}^{\max} respectively.
- There is a minimum and maximum difference between fixed and mileage charge c_{diff}^{\min} and c_{diff}^{\max} .

¹The elasticity parameter is negative and bound between 0 and 1.

Table 1: Distances between the stations in km.

-	Nørreport	Kastrup	Glostrup	Klampenborg	Herlev	Christianshavn
Nørreport	-	7.9	12.1	11.5	9.6	1.8
Kastrup	-	-	18.3	19.1	17.5	6.2
Glostrup	-	-	-	20.2	7.5	13.1
Klampenborg	-	-	-	-	13.7	13.2
Herlev	-	-	-	-	-	11.6

Table 2: The elasticity to fare changes.

-	Nørreport	Kastrup	Glostrup	Klampenborg	Herlev	Christianshavn
Nørreport	-	-0.6	-0.7	-0.6	-0.8	-0.9
Kastrup	-	-	-0.7	-0.5	-0.9	-0.9
Glostrup	-	-	-	-0.6	-0.8	-0.9
Klampenborg	-	-	-	-	-0.4	-0.9
Herlev	-	-	-	-	-	-0.9

2 Tasks

1. Formulate a quadratic programming program which maximizes the overall revenue.
2. Solve the problem with the data provided in Tables 2, 2 and 2 with a base fare price of 15 DKK and $f_{ij}^{\min} = 10$, $f_{ij}^{\min} = 40$, $c_{diff}^{\min} = 0$ and $c_{diff}^{\max} = 30$.
3. Imagine there was also a price for each change (bus to train etc.) levied. How could you incorporate this in this model?
4. Analyze the solution: (a) what are the key bottlenecks in the system? (b) Where should the main focus of improvement be for this setup?

Table 3: The base ridership in 1000 people/day.

-	Nørreport	Kastrup	Glostrup	Klampenborg	Herlev	Christianshavn
Nørreport	-	50	3	9	15	80
Kastrup	-	-	8	9	5	60
Glostrup	-	-	-	16	26	34
Klampenborg	-	-	-	-	35	12
Herlev	-	-	-	-	-	19