Production scheduling of a fermentation process

June 8, 2019

1 Problem statement

Industrial fermentation processes are generally batch operations. The process consists of a number of phases or stages, each of which requires a considerable amount of time. Because of the time factor, the system cannot react quickly to variations in demand, so that the scheduling of production in each stage is of considerable importance.

Consider a fermentation process which has the stages as shown in Table 1. m_i (f_i, p_i, etc.) is the quantity that goes into stage m (from the previous stage) at every day i. d_i is the demand (by customers) from the warehouse in period i. The demand d_i is known (but not precisely). Given d_i, a set of values m_i, f_i, p_i, b_i, w_i constitutes a production schedule. The capital letters indicate the total quantity in the stage, i.e. M_j is the total amount of material in the mixing stage in period j.

Table 1: The different stages of the fermentation process.

(1)	Mixing and cooking	M, m
(2)	Fermentation	F, f
(3)	Purification	P, p
(4)	Blending and packaging	B, b
(5)	Warehousing	W, w

Remark 1. A planning horizon is considered to be a large number of periods i; for our production schedule, the planning horizon is a 3-months' duration, $1 \leq i \leq N$, where $N = 105$.

1. **Formulate and implement the variables and the mathematical relations between them.**

There are also the following considerations to make:

- For each schedule, the mixing and cooking stage is of fixed duration, $t_m = 3$ periods.
- There is a minimum time required for fermentation and a maximum time allowed by the process. The minimum time is 10 days, and the maximum time is 30 days.

  Remark 2. We assume that you always take out of the fermenter first what has been added first (i.e. first-in first-out).
- The purification process requires a minimum of 2 days and a maximum of 5 days.
- Blending and packaging takes one day to perform; the total amount passing through the blending stage is restricted only by capacity $Z_b = 5000L$.
- Warehousing is limited by the age of the product, where $A_{\max} = 8$ is the maximum number of time periods allowed for storage.
- The output expressed in barrels or tons can be converted to man hours by introducing the factors:

$$\mu = \text{number of man hours/unit of production in mixing} = 2\frac{\text{hr}}{1000L}$$
$$\phi = \text{number of man hours/unit of production in fermentation} = 5\frac{\text{hr}}{1000L}$$
$$\pi = \text{number of man hours/unit of production in purification} = 7\frac{\text{hr}}{1000L}$$
$$\beta = \text{number of man hours/unit of production in blending} = 2\frac{\text{hr}}{1000L}$$
$$\chi = \text{number of man hours/unit of production in warehousing} = 1\frac{\text{hr}}{1000L}$$

- The basic cost factor in every stage is the labor cost, as this is the only part of the variable costs which depends on the production rate of a stage is considered (raw material costs are ignored).

2. **Formulate and implement these considerations in the model.**

The entire process is driven by the demand d_i. Suppose we start with empty tanks, and get a 20-day heads-up (i.e. the first 20 days no demand).

3. **Solve the problem with the data provided in Table 1, and the following demand patterns:**

   - **Constant demand:** $d_i = 3000$**,** $d_i = 4000$**,** $d_i = 4500$**.**

Table 2: The data for the fermentation problem.

Process	Cost/man hour $\left[\frac{DKK}{\mathrm{hr}}\right]$
Mixing - straight time	350
Fermentation	410
Purification	380
Blending	250
Warehousing	200

- **Time-varying demand:** $d_i = 5000 + 1000 \sin(i/10)$**.**

4. **Analyze the solution: (a) what are the key bottlenecks in the system? (b) Where should the main focus of improvement be for this setup?**

5. **[Extra exercise] Rewrite the model using a class-based formulation with the different processing stages and times as properties.**