Intelligent Learning and Analysis Systems SS19, Exercise Sheet 1

Andreas Hene, Niklas Mertens, Richard Palme

April 13, 2019

1. Finding Two Missing Items

2. Identifying the Majority

Let $\sigma = \langle a_1, \ldots, a_m \rangle$ and $a_i \in [n]$.

Algorithm 1 Algorithm for finding the majority of σ, if it exists

$c \leftarrow 0$
for all $i \in [m]$ **do**
  if $c = 0$ **then**
    $s \leftarrow a_i$
    $c \leftarrow 1$
  end if
  if $s = a_i$ **then**
    $c \mathrel{+}= 1$
  else
    $c \mathrel{-}= 1$
  end if
end for
return s

Let $A(\sigma) = s$ be the output of the algorithm.

Claim: $A(\sigma)$ is the majority of σ, if it exists.
Proof: Let k be the number of times c is set to zero in the for loop. We prove the claim by induction over k.

Base case:
Let $k = 0$. Since c is never set to 0 during the loop, we know that s never changes after iteration 1, i.e. $A(\sigma) = a_1$. Also, c increases at least one time more often than it decreases, so $A(\sigma) = a_1$ has to be the majority of σ.

Step case:
Let $k > 0$. Let d be the majority of σ. Let j be the first iteration where c is zero. j must be even. d can occur at most $\frac{j}{2}$ times in $\tau := \langle a_1, \ldots, a_j \rangle$, otherwise d would be the majority of τ and c couldn't be zero in iteration j. Consequently d will occur at least

$$\left\lfloor \frac{m}{2} \right\rfloor + 1 - \frac{j}{2} = \left\lfloor \frac{m-j}{2} \right\rfloor + 1$$

times in $\sigma' = \langle a_{j+1}, \ldots, a_m \rangle$. Because σ' is a stream of length $m - j$, d is the majority of σ'. Since the number of zeros in σ' is smaller than k, we can apply the inductive hypothesis to see that $A(\sigma') = d$. Since in iteration j we have $c = 0$, applying the algorithm to σ is the same as applying it to τ and then to σ'. Hence, $A(\sigma) = d$.

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3. Reservoir Sampling I.

4. Reservoir Sampling II.

5. The ϕ-HH Problem: Lower Bounds