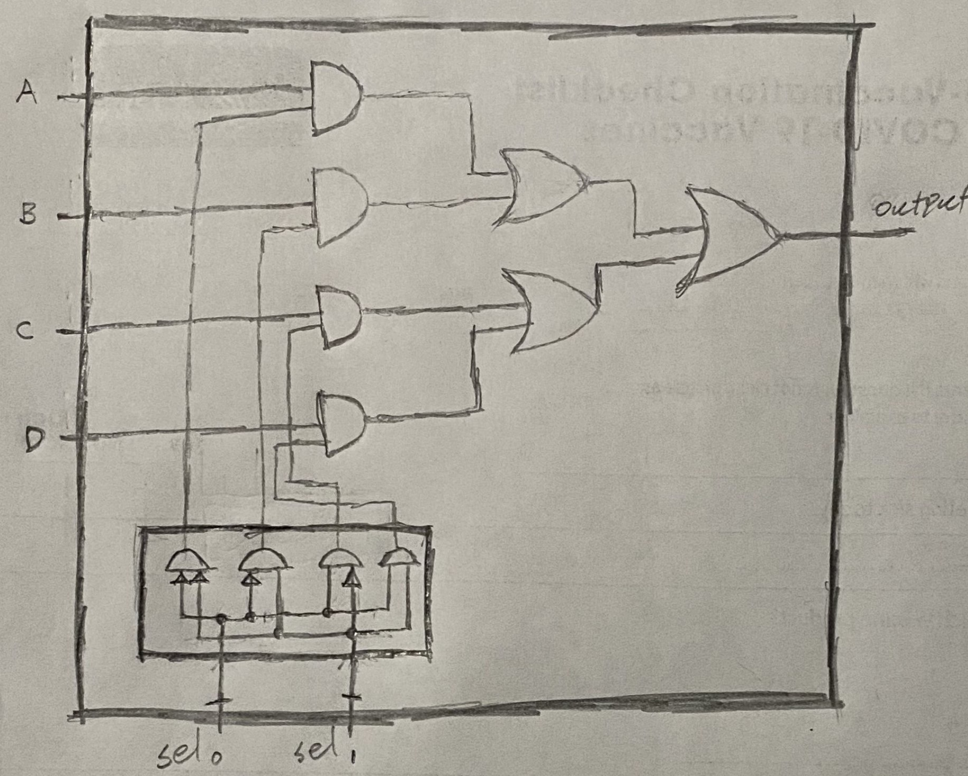
Assignment . 1

Mingzan Liu

#1



#2

$$F1 = (\bar{X}_0 \cdot X_1 \cdot X_2) + (X_0 \cdot \bar{X}_1 \cdot X_2) + (X_0 \cdot X_1 \cdot \bar{X}_2)$$

$$F2 = (\bar{X}_0 \cdot \bar{X}_1 \cdot X_2) + (\bar{X}_0 \cdot X_1 \cdot \bar{X}_2) + (X_0 \cdot \bar{X}_1 \cdot \bar{X}_2)$$

$$F3 = (\bar{X}_0 \cdot \bar{X}_1 \cdot \bar{X}_2) + (\bar{X}_0 \cdot \bar{X}_1 \cdot X_2) + (\bar{X}_0 \cdot X_1 \cdot \bar{X}_2) + (\bar{X}_0 \cdot X_1 \cdot X_2)$$

$$F4 = (X_0 \cdot \bar{X}_1 \cdot \bar{X}_2) + (X_0 \cdot \bar{X}_1 \cdot X_2) + (X_0 \cdot X_1 \cdot \bar{X}_2) + (X_0 \cdot X_1 \cdot X_2)$$

X_0	X_1	X_2	F_1	F_2	F_3	F_4	XOR
0	0	0	0	0	1	0	0
0	0	1	0	1	1	0	1
0	1	0	0	1	1	0	1
0	1	1	1	0	1	0	0
1	0	0	0	1	0	1	1
1	0	1	1	0	0	1	0
1	1	0	1	0	0	1	0
1	1	1	0	0	0	1	1

$$F_2XOR = X_0 \oplus X_1 \oplus X_2$$

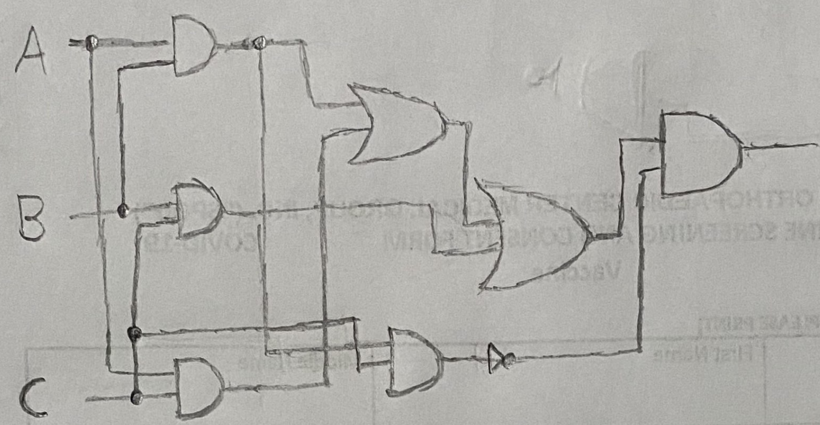
$$F_2XOR = (\bar{X}_0 \cdot \bar{X}_1 \cdot X_2) + (\bar{X}_0 \cdot X_1 \cdot \bar{X}_2) + (X_0 \cdot \bar{X}_1 \cdot \bar{X}_2) + (X_0 \cdot X_1 \cdot X_2)$$

3

$$E = ((A \cdot B) + (A \cdot C) + (B \cdot C)) \cdot (\overline{A \cdot B \cdot C})$$

↓

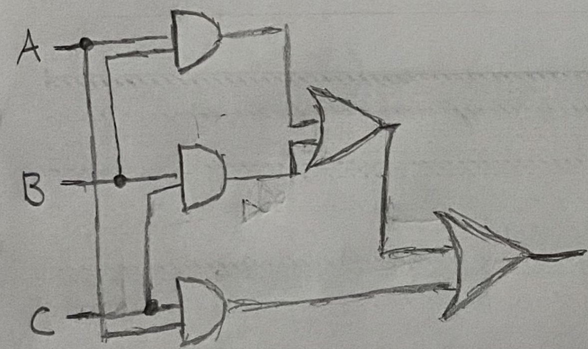
A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0



$$E = (A \cdot B \cdot \bar{C}) + (A \cdot \bar{B} \cdot C) + (\bar{A} \cdot B \cdot C)$$

↓

A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0



- Equation 2 is more efficient in terms of 2-input gates as it only require 5 gates. While Equation 1 require 7 gates.

#4

$$E = ((A \cdot B) + (A \cdot C) + (B \cdot C)) \cdot (\overline{A \cdot B \cdot C})$$

$= ((A \cdot B) + (A \cdot C) + (B \cdot C)) \cdot (\bar{A} + \bar{B} + \bar{C})$ — DeMorgan's laws

$= [(A \cdot B) \cdot (\bar{A} + \bar{B} + \bar{C})] + [(A \cdot C) \cdot (\bar{A} + \bar{B} + \bar{C})]$ $+ [(B \cdot C) \cdot (\bar{A} + \bar{B} + \bar{C})]$ — Distributive laws

$= [(A \cdot B \cdot \bar{A}) + (A \cdot B \cdot \bar{B}) + (A \cdot B \cdot \bar{C})] +$
$[(A \cdot C \cdot \bar{A}) + (A \cdot C \cdot \bar{B}) + (A \cdot C \cdot \bar{C})] +$
$[(B \cdot C \cdot \bar{A}) + (B \cdot C \cdot \bar{B}) + (B \cdot C \cdot \bar{C})]$ — Distributive laws

$= [(B \cdot \emptyset) + (A \cdot \emptyset) + (A \cdot B \cdot \bar{C})] +$
$[(C \cdot \emptyset) + (A \cdot C \bar{B}) + (A \cdot \emptyset)] +$
$[(B \cdot C \cdot \bar{A}) + (C \cdot \emptyset) + (B \cdot \emptyset)]$ — Basic laws $A \cdot \bar{A} = \emptyset$

$= (A \cdot B \cdot \bar{C}) + (A \cdot C \cdot \bar{B}) + (B \cdot C \cdot \bar{A})$ — Basic laws $A \cdot \emptyset = \emptyset$

$= (A \cdot B \cdot \bar{C}) + (A \cdot \bar{B} \cdot C) + (\bar{A} \cdot B \cdot C)$ — Associative laws

Therefore:

$$((A \cdot B) + (A \cdot C) + (B \cdot C)) \cdot (\overline{A \cdot B \cdot C}) \equiv (A \cdot B \cdot \bar{C}) + (A \cdot \bar{B} \cdot C) + (\bar{A} \cdot B \cdot C)$$

#5

$XOR = (A + B) \cdot (\overline{A \cdot B})$ — DeMorgan's laws

$= (A + B) \cdot (\bar{A} + \bar{B})$

$= (A \cdot (\bar{A} + \bar{B})) + (B \cdot (\bar{A} + \bar{B}))$ — Distributive laws

$= [(A \cdot \bar{A}) + (A \cdot \bar{B})] + [(B \cdot \bar{A}) + (B \cdot \bar{B})]$ — Distributive laws

$= [\emptyset + (A \cdot \bar{B})] + [(B \cdot \bar{A}) + \emptyset]$ — basic laws $\bar{A} \cdot A = \emptyset$

$= (A \cdot \bar{B}) + (B \cdot \bar{A})$ — basic laws $A + 0 = A$

$= (A \cdot \bar{B}) + (\bar{A} \cdot B)$ — commutative laws

Therefore:

$$(A + B) \cdot (\overline{A \cdot B}) \equiv (A \cdot \bar{B}) + (\bar{A} \cdot B)$$

#6

3-input XOR

A	B	C	Output
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$3XOR = (\bar{A} \cdot \bar{B} \cdot C) + (\bar{A} \cdot B \cdot \bar{C}) + (A \cdot \bar{B} \cdot \bar{C}) + (A \cdot B \cdot C)$