CSCI 119 Lab 11

0. prove $\{w \mid w \in \Sigma_1^*, rev(w) = w\}$ is non-regular

consider the strings

ε, a, b, ab, ba, aab, baa ...

each string is its own equivalence class

$a \not\equiv_L b$ $ab \not\equiv_L ba$

this leads to an infinite # of equivalence classes, making it non-regular

1. $G_1 = S \to ABS \mid AB$

$A \to aA \mid a$

$B \to bA$

1) aabaab $\in L(G_1)$?

$S \mapsto^* ABS \mapsto^* A \mapsto^* aA \mapsto a$

$ABS \mapsto B \mapsto bA \mapsto aA \mapsto aA \mapsto a$

$ABS \mapsto S \mapsto ABS \mapsto A \mapsto aA \mapsto a$

$ABS \mapsto B \mapsto bA \mapsto a$

string won't be accepted if the last char is a "b"

2) aaaaba $\in L(G_1)$?

$S \mapsto^* AB \mapsto^* A \mapsto^* aA \mapsto a$

$aA \mapsto^* aA \mapsto a$

$aA \mapsto^* aA \mapsto a$

$aA \mapsto^* aA \mapsto a$

$AB \mapsto^* B \mapsto^* bA$

$bA \mapsto^* a$

aaaaba $\in L(G_1)$ ✓

$G_1 = S \to ABS \mid AB$
$A \to aA \mid a$
$B \to bA$

3) aabbaa $\in L(G_1)$?

$S \mapsto ABS \mapsto A \mapsto aA \mapsto A \mapsto a$
$ABS \mapsto B \mapsto bA \mapsto A \mapsto a$

aabbaa $\notin L(G_1)$

can't get to another "b" after immediately reading a "b"

4) abaaba $\in L(G_1)$?

$S \mapsto ABS \mapsto A \mapsto a$
$ABS \mapsto B \mapsto bA \mapsto a$
$ABS \mapsto S \mapsto ABS \mapsto A \mapsto a$
$ABS \mapsto B \mapsto bA \mapsto a$

abaaba $\in L(G_1)$ ✓

$r = ab^*(ba+b+bb)^*+aba+1$

2. G_2 = <exp> ↦ <fact> | <fact> + <exp> | <exp><exp> | <atom> | <star>

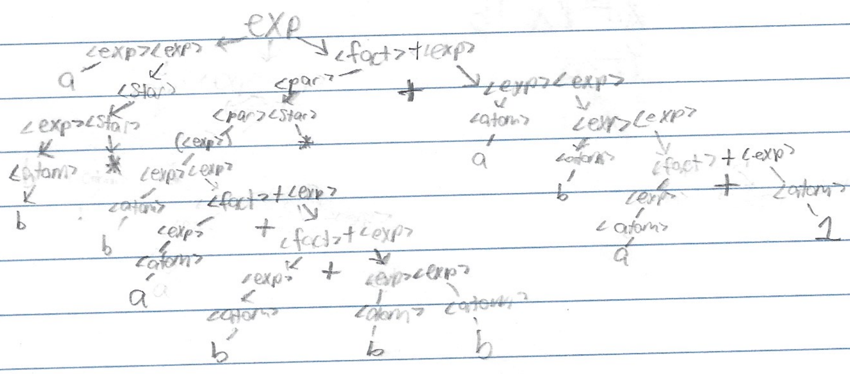
<fact> ↦ <par> | <exp>

<atom> ↦ Σ | 0 | 1

<par> ↦ (<exp>) | <par><star>

<star> ↦ * | <exp><star>

$r = ab^*(ba+b+bb)^*+aba+1$



produces

$ab^*(ba+b+bb)^*+aba+1$

3. $ab^*(ba+b+bb)^*+aba+1$

G_3 = S ↦ aBP | aba | 1

B ↦ bB | ε

P ↦ baP | bP | bbP | ε

4. $\{a^i b^j a^i \mid i \geq 0, j \geq 1\}$

$S \mapsto aXa$

$X \mapsto bX \mid b$

convert to CNF

$S \mapsto aXa$ $\quad$ $S \mapsto aW$

$X \mapsto bX \mid b$ $\quad$ $W \mapsto XY$

$X \mapsto bX \mid b$

$Y \mapsto a$

5. $\Sigma = \{a, b\}$

$S \to aSb \mid bSa \mid SS \mid \epsilon$

$a, b ; \Sigma^* \to \mathbb{N}$

Theorem for all $w \in \{S\} \cup \Sigma^*$ $S \mapsto^* w$ is balanced if w is balanced

Def a string $w \in \{S\} \cup \Sigma^*$ is balanced if

① $a(Sw) = b(Sw)$

Derivation

given a string aaabbb

$S \mapsto aSb \mapsto aSSb \mapsto aaSbb \mapsto aaabbb$

resulting in aaabbb

Proof

① $a(S) = 0 = b(S)$ ✓

case 1: $\alpha S_\beta \mapsto \alpha aSb_\beta$

assume αS_β is balanced

5 continued

case 1: $a(\alpha S_\beta) = b(\alpha S_\beta)$

$\downarrow \qquad \downarrow$

$a(\alpha) + a(\beta) = b(\alpha) + b(\beta)$ ✓

$a(\varepsilon) = 0$ $\qquad$ $b(\varepsilon) = 0$

$a(aw) = 1 + a(w)$ $\qquad$ $b(aw) = b(w)$

$a(bw) = a(w)$ $\qquad$ $b(bw) = 1 + b(w)$

$a(Sw) = a(w)$ $\qquad$ $b(Sw) = b(w)$

Case 2: $\alpha S_\beta \mapsto \alpha b S a \beta$

assume αS_β is balanced

$b(\alpha S_\beta) = a(\alpha S_\beta)$

$\downarrow \qquad \downarrow$

$b(\alpha) + b(\beta) = a(\alpha) + a(\beta)$ ✓

Case 3: $\alpha S_\beta \mapsto \alpha S S_\beta$ ✓

Case 4: $\alpha S_\beta \mapsto \alpha\beta$ ✓

6. $S \to bS \mid Sa \mid aSb \mid \varepsilon$ What set is generated by this grammar?

By Strong Induction prove any $w \in \{a,b\}^*$ has a derivation in G_6

$a(w) = $ # a's in w $\qquad$ $w \mapsto w'$

$b(w) = $ # b's in w

$a(\varepsilon) = 0$ $\qquad$ $b(\varepsilon) = 0$

$a(aw) = 1 + a(w)$ $\qquad$ $b(aw) = b(w)$

$a(bw) = a(w)$ $\qquad$ $b(bw) = 1 + b(w)$

$a(Sw) = a(w)$ $\qquad$ $b(Sw) = b(w)$

Case 1: $w = \varepsilon$ ✓

Case 2: $|w| > 0$ ✓

Claim