# **GSLSP**

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#### 1 Introduction

These routines provide support for constructing and manipulating sparse matrices in GSL, using an API similar to the  $\mathtt{gsl\_matrix}$  machinery. The basic structure is called  $\mathtt{gsl\_spmatrix}$ . There are two supported storage formats for sparse matrices: the triplet and compressed column storage (CCS) formats. The triplet format stores triplets (i,j,x) for each non-zero element of the matrix. This notation means that the (i,j) element of the matrix A is  $A_{ij} = x$ . Compressed column storage stores each column of non-zero values in the sparse matrix in a continuous memory block, keeping pointers to the beginning of each column in that memory block, and storing the row indices of each non-zero element. The triplet format is ideal for adding elements to the sparse matrix structure while it is being constructed, while the compressed column storage is better suited for matrix-matrix multiplication or linear solvers.

GSL does not provide a linear sparse matrix solver for Ax = b, since this is a highly complex problem and many advanced software packages already exist to efficiently solve it. The routines in this section provide a simple interface to construct a sparse matrix and convert it to a compressed format, where it can easily be passed to external linear sparse matrix solvers.

The gsl\_spmatrix structure is defined as

```
typedef struct
{
    size_t size1;
    size_t size2;
    size_t *i;
    double *data;
    size_t *p;
    size_t nzmax;
    size_t nz;
    size_t *work;
    size_t flags;
} gsl_spmatrix;
```

This defines a size1-by-size2 sparse matrix. The number of non-zero elements currently in the matrix is given by nz. For the triplet representation, i, p, and data are arrays of size nz which contain the row indices, column indices, and element value, respectively. So if data[k] = x, then x = A(i[k], p[k]). For compressed column storage, i and data are arrays of size nz containing the row indices and element values, similar to the triplet case. p is an array of size size2+1 where p[j] points to the index in data of the start of column j. Thus, if data[k] = A(i,j), then i = i[k] and p[j] <= k < p[j+1].

work is additional workspace needed for various operations like converting from triplet to compressed column storage. flags indicates the type of storage format being used (triplet or compressed column).

The routines in this extension are defined in the header file 'gsl\_spmatrix.h'.

### 2 Sparse matrix allocation

The functions for allocating memory for a sparse matrix follow the style of malloc and free. They also perform their own error checking. If there is insufficient memory available to allocate a matrix then the functions call the GSL error handler with an error code of GSL\_ENOMEM in addition to returning a null pointer.

This function allocates a sparse matrix of size n1-by-n2 and initializes it to all zeros. If the size of the matrix is not known at allocation time, both n1 and n2 may be set to 1, and they will automatically grow as elements are added to the matrix. This function sets the matrix to the triplet representation, which is the easiest for adding and accessing matrix elements. This function tries to make a reasonable guess for the number of non-zero elements (nzmax) which will be added to the matrix by assuming a sparse density of 10%. The function  $gsl_spmatrix_alloc_nzmax$  can be used if this number is known more accurately. The workspace is of size O(nzmax).

gsl\_spmatrix \* gsl\_spmatrix\_alloc\_nzmax (const size\_t n1, const size\_t n2, const size\_t nzmax, const size\_t flags) [Function]

This function allocates a sparse matrix of size n1-by-n2 and initializes it to all zeros. If the size of the matrix is not known at allocation time, both n1 and n2 may be set to 1, and they will automatically grow as elements are added to the matrix. The parameter nzmax specifies the maximum number of non-zero elements which will be added to the matrix. It does not need to be precisely known in advance, since storage space will automatically grow using  $gsl_spmatrix_realloc$  if nzmax is not large enough. Accurate knowledge of this parameter reduces the number of reallocation calls required. The parameter flags specifies the storage format of the sparse matrix. Possible values are

GSL\_SPMATRIX\_TRIPLET

This flag specifies triplet storage.

GSL\_SPMATRIX\_COMPCOL

This flag specifies compressed column storage.

The allocated  $gsl\_spmatrix$  structure is of size O(nzmax).

int gsl\_spmatrix\_realloc (const size\_t nzmax, gsl\_spmatrix \* m) [Function]

This function reallocates the storage space for m to accommodate nzmax non-zero elements. It is typically called internally by gsl\_spmatrix\_set if the user wants to add more elements to the sparse matrix than the previously specified nzmax.

void gsl\_spmatrix\_free (gsl\_spmatrix \* m) [Function]

This function frees the memory associated with the sparse matrix m.

## 3 Accessing sparse matrix elements

This function returns element (i,j) of the matrix m. The matrix may be in triplet or compressed format.

This function sets element (i,j) of the matrix m to the value x. The matrix must be in triplet representation.

## 4 Initializing sparse matrix elements

Since the sparse matrix format only stores the non-zero elements, it is automatically initialized to zero upon allocation. The function <code>gsl\_spmatrix\_set\_zero</code> may be used to re-initialize a matrix to zero after elements have been added to it.

int gsl\_spmatrix\_set\_zero (gsl\_spmatrix \* m) [Function]
This function sets (or resets) all the elements of the matrix m to zero.

### 5 Copying sparse matrices

- gsl\_spmatrix \* gsl\_spmatrix\_memcpy (const gsl\_spmatrix \* src) [Function]
  This function makes a copy of the sparse matrix src and returns a pointer to a newly allocated copy, which must be freed by the caller when no longer needed. The matrix src may be in either triplet or compressed format.

This function computes the transpose of *src* and stores it in a newly allocated matrix which is returned by the function. This matrix should be freed by the caller using <code>gsl\_spmatrix\_free</code> when no longer needed. The matrix *src* may be in either triplet or compressed format.

### 6 Sparse matrix operations

This function adds the two matrices a+b and stores the result in a newly allocated matrix which is returned. The result should be freed with  ${\tt gsl\_spmatrix\_free}$  when no longer needed. The two matrices must have the same dimensions.

int gsl\_spmatrix\_scale (gsl\_spmatrix \* m, const double x) [Function] This function scales all elements of the matrix m by the constant factor x. The result  $m(i,j) \leftarrow xm(i,j)$  is stored in m.

## 7 Sparse matrix properties

size\_t gsl\_spmatrix\_nnz (const gsl\_spmatrix \* m) [Function]
This function returns the number of non-zero elements in m.

This function returns 1 if the matrices a and b are equal (by comparison of element values) and 0 otherwise. The matrices a and b must be either both triplet format or both compressed format for comparison.

# 8 Finding maximum and minimum elements of sparse matrices

This function returns the minimum and maximum elements of the matrix m, storing them in  $min\_out$  and  $max\_out$ .

# 9 Sparse matrix compressed format

GSL supports the compressed column format, in which the non-zero elements in each column are stored contiguously in memory.

 $gsl\_spmatrix * gsl\_spmatrix\_compress (const gsl\_spmatrix * T)$  [Function] This function creates a sparse matrix in compressed column format from the input sparse matrix T which must be in triplet format. A pointer to a newly allocated matrix is returned. The calling function should free the newly allocated matrix when it is no longer needed.

### 10 Conversion between sparse and dense matrices

The gsl\_spmatrix structure can be converted into the dense gsl\_matrix format and vice versa with the following routines.

- int gsl\_spmatrix\_d2sp (gsl\_spmatrix \* S, const gsl\_matrix \* A) [Function]

  This function converts the dense matrix A into sparse triplet format and stores the result in S.
- int gsl\_spmatrix\_sp2d (gsl\_matrix \* A, const gsl\_spmatrix \* S) [Function]
  This function converts the sparse matrix S into a dense matrix and stores the result in A. S must be in triplet format.

### 11 Sparse BLAS operations

GSL supports a limited number of BLAS operations for sparse matrices.

int gsl\_spblas\_dgemv (const double alpha, const gsl\_spmatrix \* A, const gsl\_vector \* x, const double beta, gsl\_vector \* y) [Function]

This function computes the matrix-vector product and sum  $y \leftarrow \alpha Ax + \beta y$ , where A is sparse and the vectors x and y are dense. The matrix A may be in triplet or compressed format.

This function computes the sparse matrix-matrix product  $C = \alpha AB$ . A pointer to the newly allocated matrix C is returned and should be freed using gsl\_spmatrix\_free when no longer needed. The matrices A and B must be in compressed format.

# 12 Examples

# 13 References and Further Reading

The algorithms used by these functions are described in the following sources:

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