

# Lecture 4

## DC Resistivity: Basic Principles

GEOL 4397: Electromagnetic Methods for Exploration

GEOL 6398: Special Problems

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UNIVERSITY of  
**HOUSTON**

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EARTH AND ATMOSPHERIC SCIENCES

# Take attendance on CourseKey

# Agenda

- Recap
- Two current electrodes
- Apparent resistivity
- Understanding charges

# Ohm's law

- In 1827, Georg Ohm discovered an empirical relationship between the current flowing through a wire and the voltage required to drive that current.

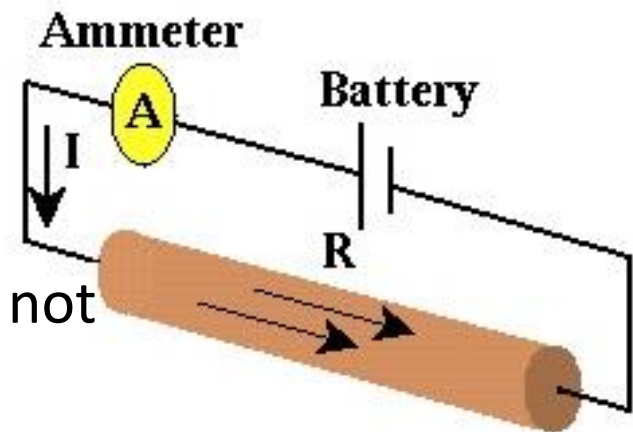
$$V = IR$$

$$R = \frac{V}{I}$$

- Note that here  $R$  represents resistance, not the resistivity. In fact, resistivity  $\rho = \frac{RA}{L}$



[https://en.wikipedia.org/wiki/Georg\\_Ohm](https://en.wikipedia.org/wiki/Georg_Ohm)



[https://pburnley.faculty.unlv.edu/GEOL442\\_642/RES/NOTES/ResistivityNotes04Ohm.html](https://pburnley.faculty.unlv.edu/GEOL442_642/RES/NOTES/ResistivityNotes04Ohm.html)

# What does Ohm's law tell us?

- Given current and voltage, we can estimate the resistance (which is related to resistivity).

# Another version of Ohm's law

$$\mathbf{J} = \sigma \mathbf{E}$$

# Question

- Is it possible to apply Ohm's law to determine the **electrical resistivities** of the **Earth** materials in the subsurface?

# Simple answer

- Yes!
- That is exactly what DC resistivity does.



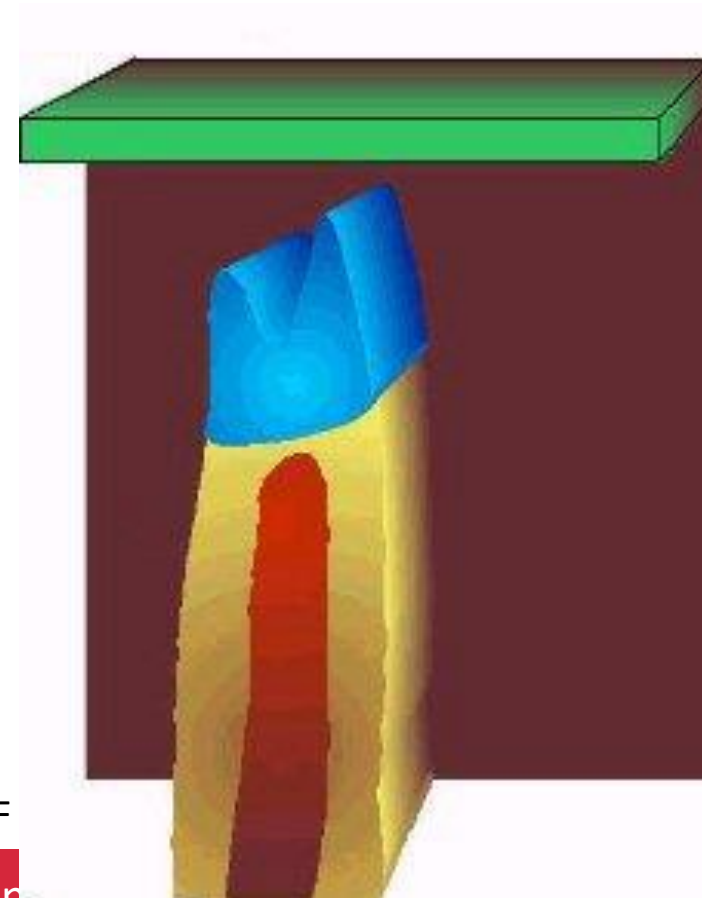
# Basic Experiment

- **Target:**

Ore body. Mineralized regions less resistive than host

Elura Orebody Electrical resistivities

<i>Rock Type</i>	<i>Ohm-m</i>
Overburden	12
Host rocks	200
Gossan	420
Mineralization (pyritic)	0.6
Mineralization (pyrrhotite)	0.6



Credit: Doug Oldenburg, Seogi Kang and Linsey Heagy from UBC-GIF

# Basic Experiment

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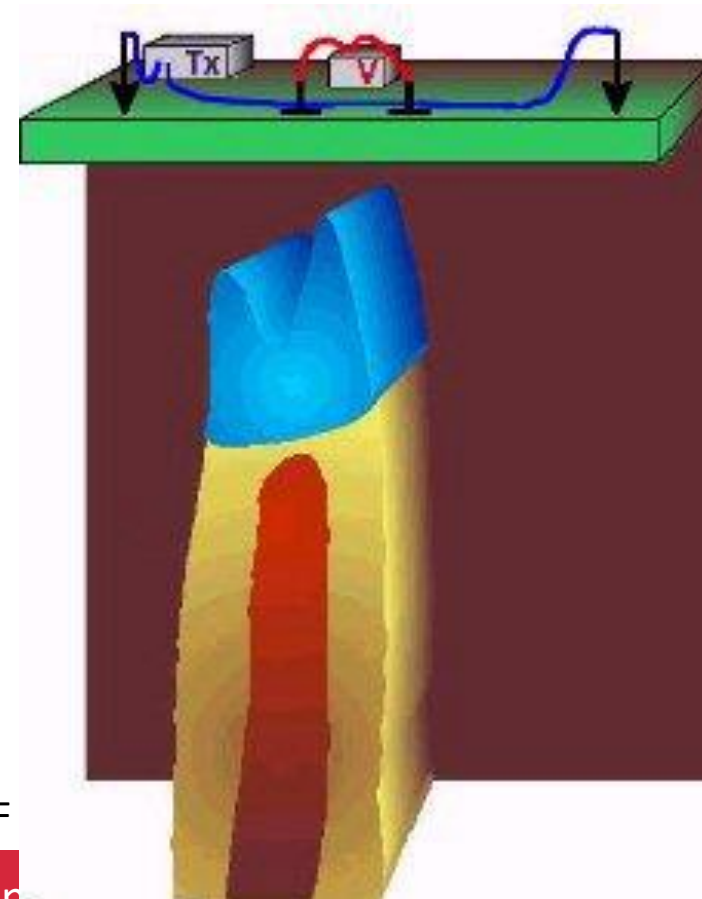
- **Setup:**

Tx: Current electrodes

Rx: Potential electrodes

Elura Orebody Electrical resistivities

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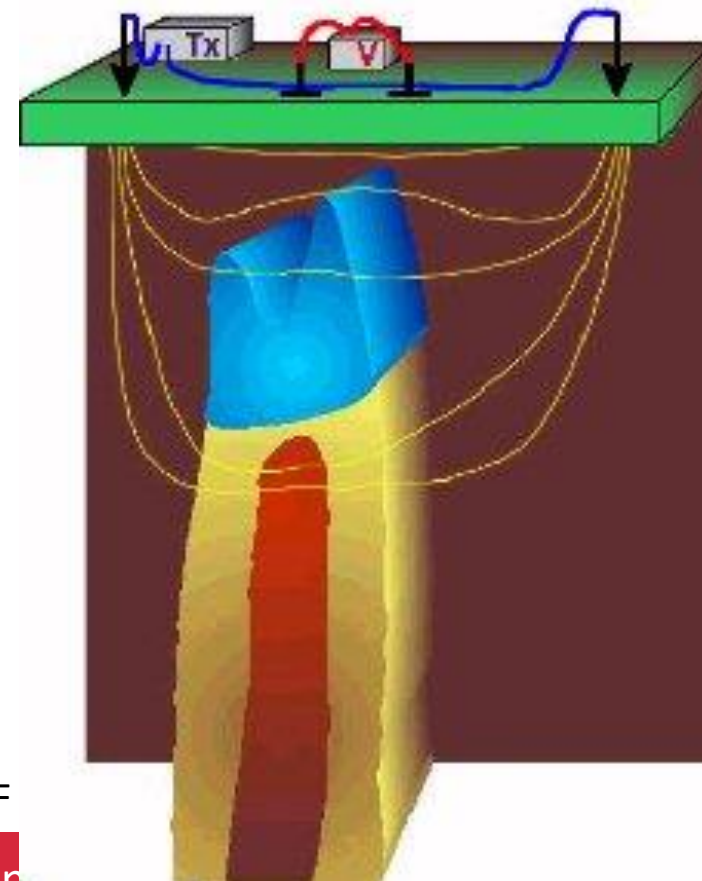
Rx: Potential electrodes

- **Currents:**

Preferentially flow through conductors

Elura Orebody Electrical resistivities

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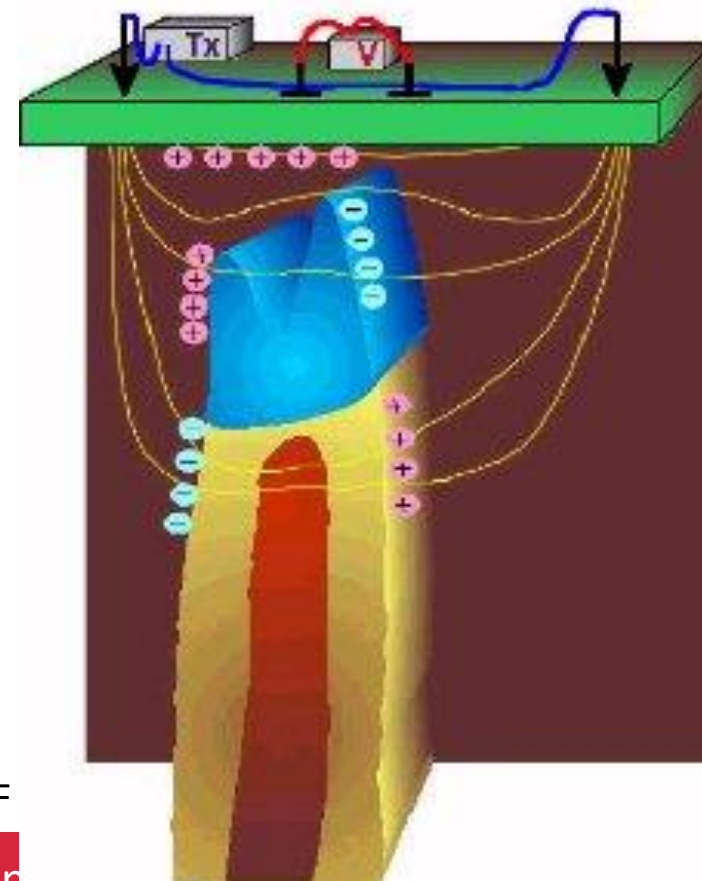
Preferentially flow through conductors

- **Charges:**

Build up at interfaces

Elura Orebody Electrical resistivities

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# Basic Experiment

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Tx: Current electrodes

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Preferentially flow through conductors

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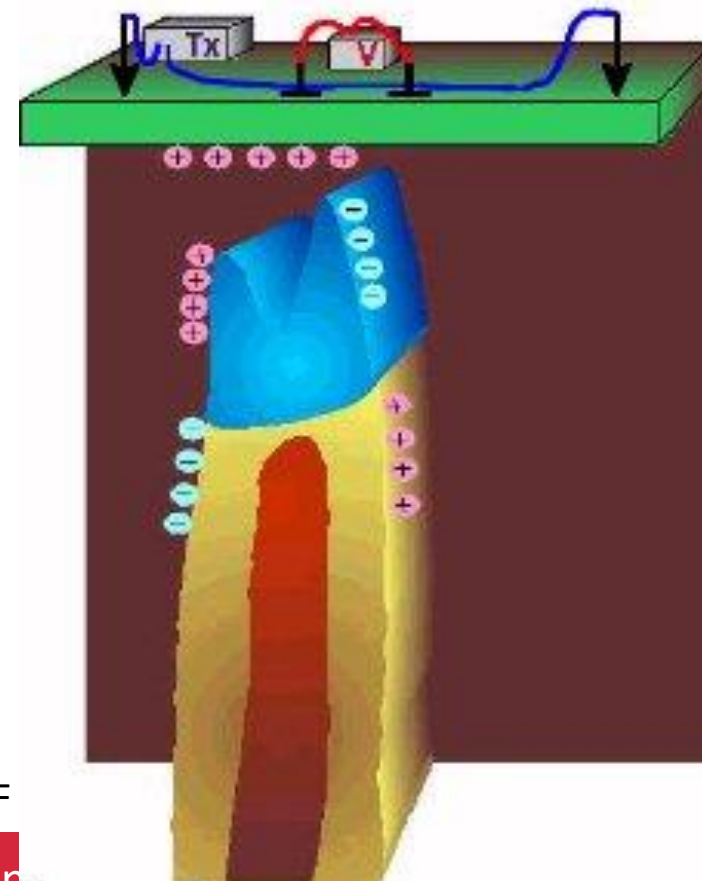
Build up at interfaces

- **Potentials:**

Associated with the charges are measured at the surface

Elura Orebody Electrical resistivities

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How do we obtain resistivity?

# Simple answer for uniform halfspace

$$\rho = \frac{2\pi rV}{I}$$

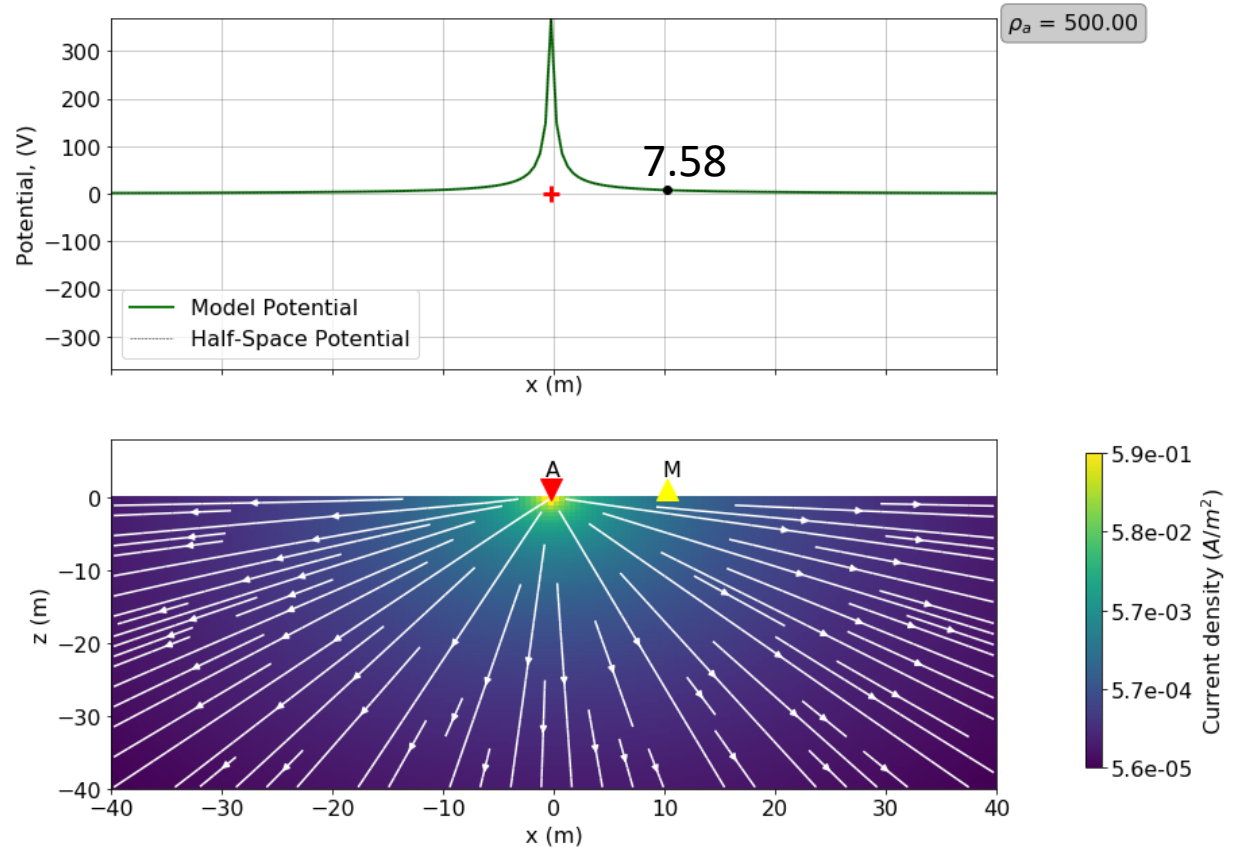


Image generated using DC\_Plate2\_5D. Pole-Pole. A -0.25 m, M 10.25 m.  
DC\_Layer\_Cylinder\_2\_5D would also do.

Special thanks to Thibaut Astic from UBC-GIF

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# Think about Ohm's law

$$V = IR$$

Note that  $R$  is resistance

But we want to relate potential to resistivity

$$\rho = \frac{RA}{L}$$

$$R = \frac{\rho L}{A}$$

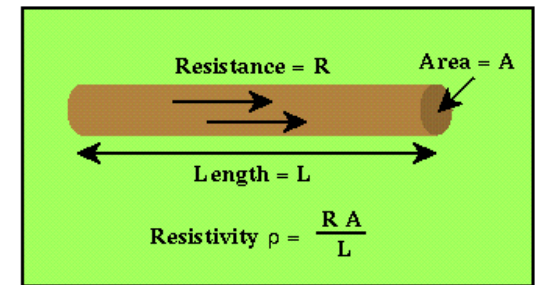
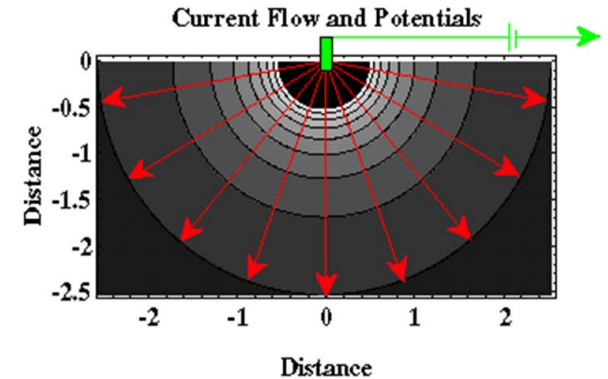
What is the  $L$  and  $A$  for our DC survey?

$$L = r$$

$$A = 2\pi r^2$$

$$\text{Therefore, } R = \frac{\rho L}{A} = \frac{\rho}{2\pi r}$$

$$\text{Therefore, } V = IR = \frac{\rho I}{2\pi r}$$



[https://pburnley.faculty.unlv.edu/GEOL442\\_642/RES/NOTES/ResistivityNotes05Resistivity.htm](https://pburnley.faculty.unlv.edu/GEOL442_642/RES/NOTES/ResistivityNotes05Resistivity.htm)



# A summary thus far

$$V = \frac{\rho I}{2\pi r}$$

$$\rho = \frac{2\pi r V}{I}$$

If we know current  $I$   
then, measure the potential  
value at any location  
We can derive the resistivity  
of the Earth!!!

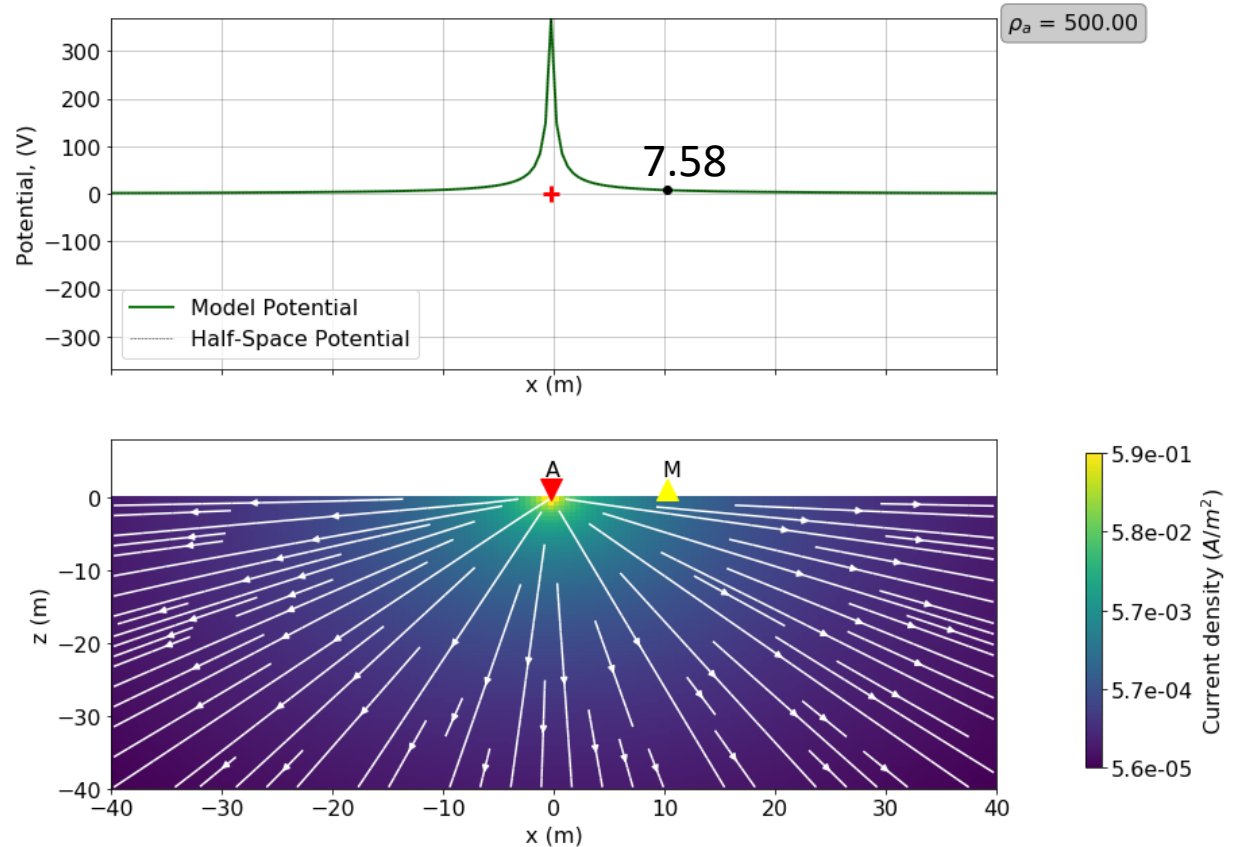


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# Exercise

- Calculate the potential at 10.5 m.
- Calculate the resistivity of the Earth, given a measured potential value.

# Electric potential from single current electrode

$$V = \frac{\rho I}{2\pi r}$$

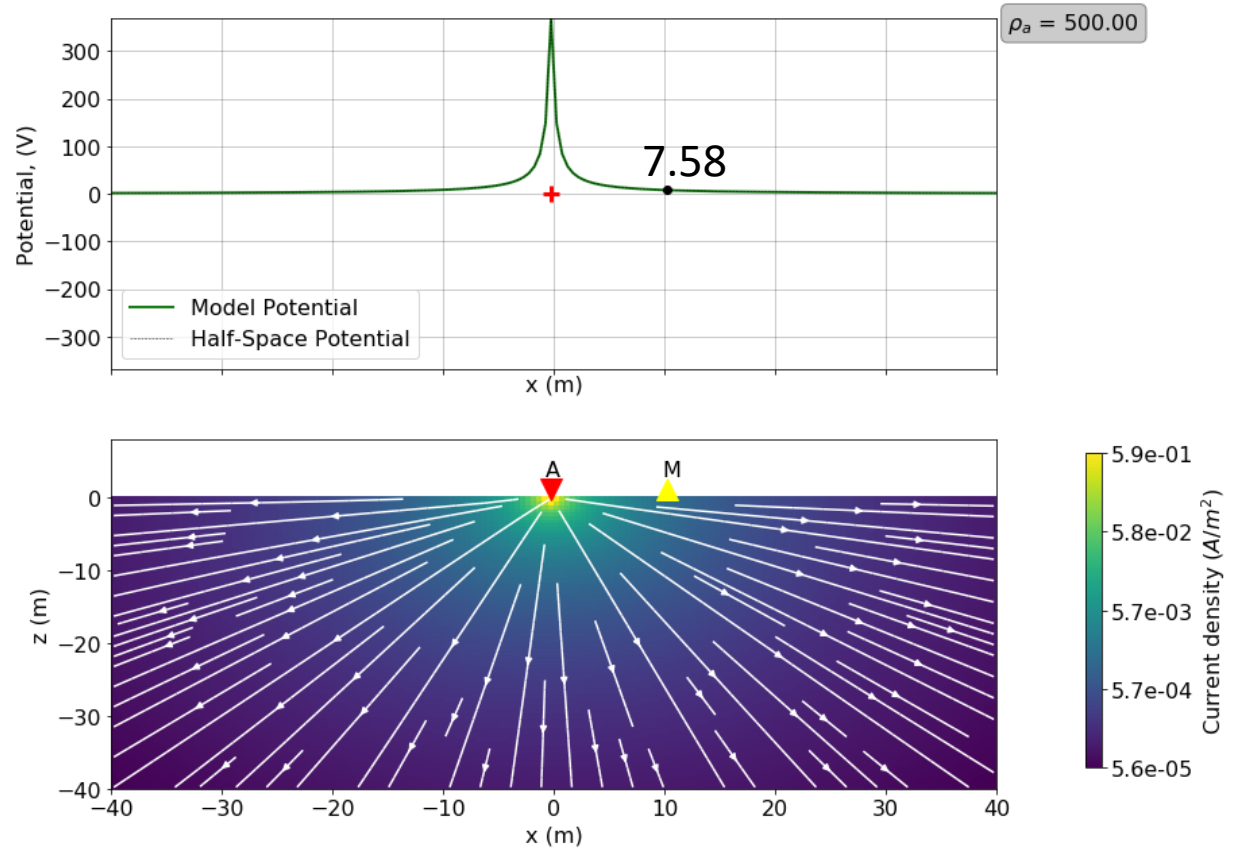


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# Two electrode current sources

- Recall, for single current electrode,

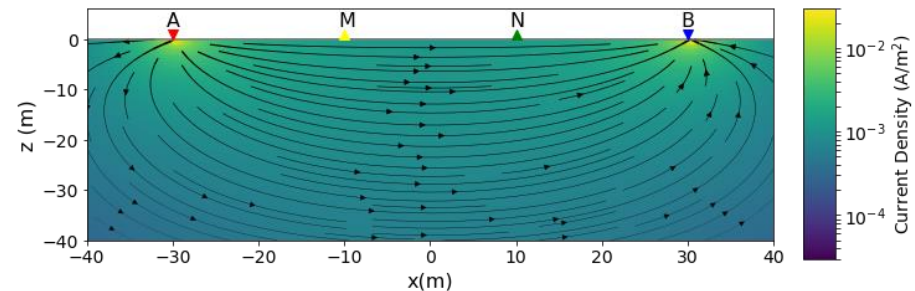
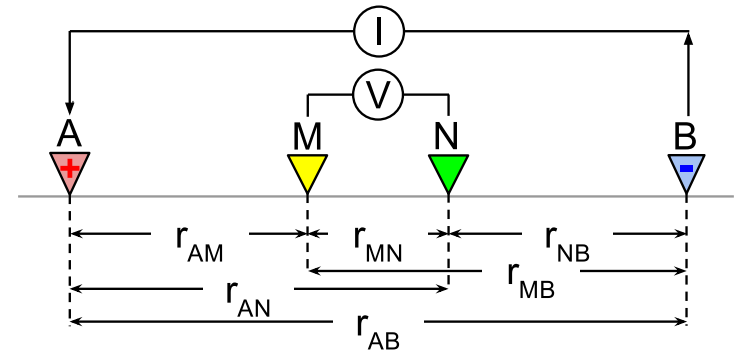
$$V = IR = \frac{\rho I}{2\pi r}$$

# Two electrode current sources

- Recall, for single current electrode,

$$V = IR = \frac{\rho I}{2\pi r}$$

- What if we have two current electrodes (a positive and a negative electrode)?



Images generated using DC\_Layered Earth.ipynb.

A: -30 m, B: 30 m, M: -10 m, N = 10 m.  $\rho = 500\Omega\cdot\text{m}$

Observation:

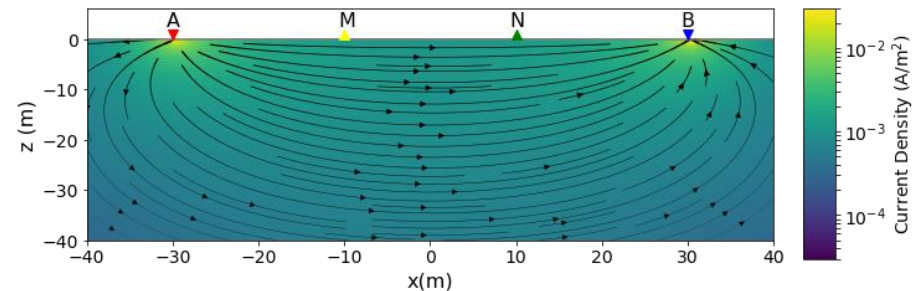
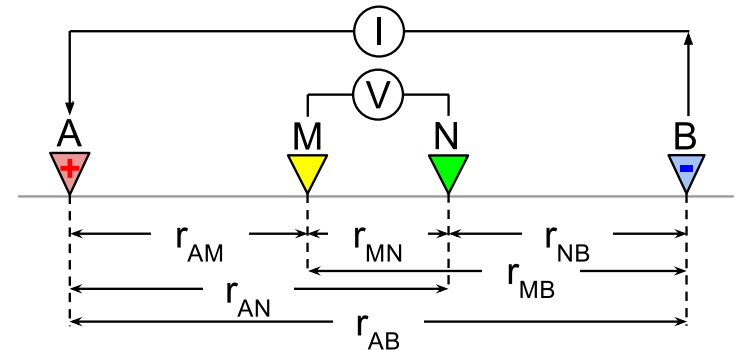
- Current flows along the curved paths connecting the two electrodes

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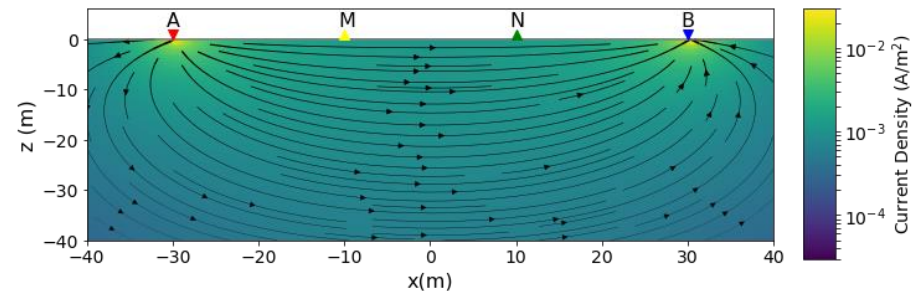
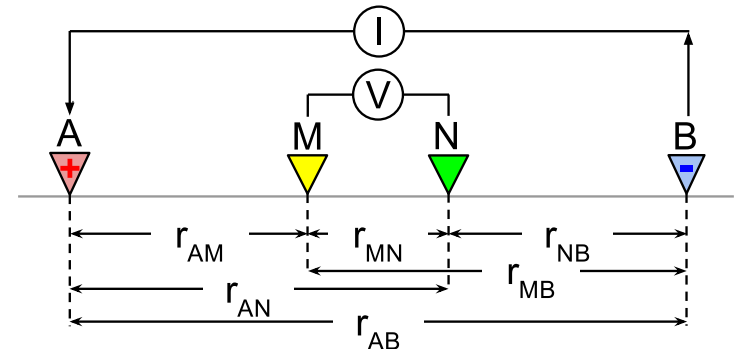
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$$V_M = \frac{\rho I}{2\pi} \left( \frac{1}{r_{AM}} - \frac{1}{r_{BM}} \right)$$

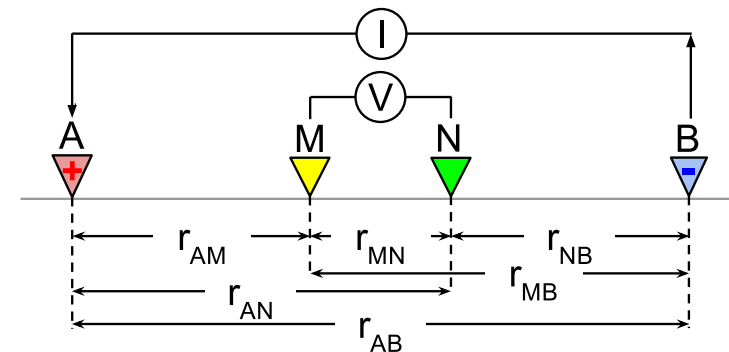
Observation:

- Current flows along the curved paths connecting the two electrodes

# Voltage

- What we measure in practice is voltage (using voltmeter)
- That is, potential difference

What is the potential difference between potential electrodes M and N?

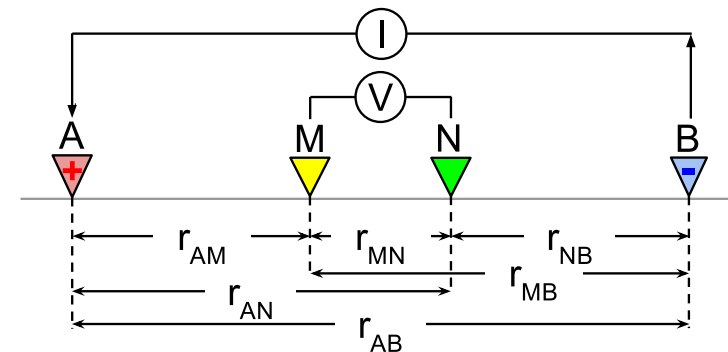




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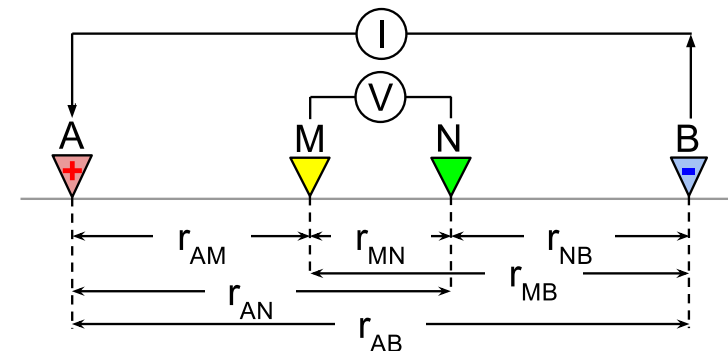


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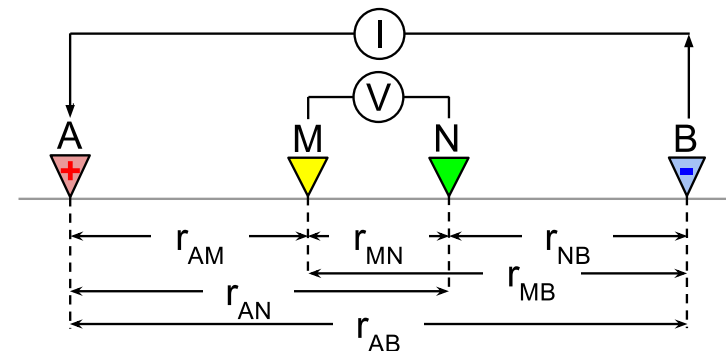
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# Voltage

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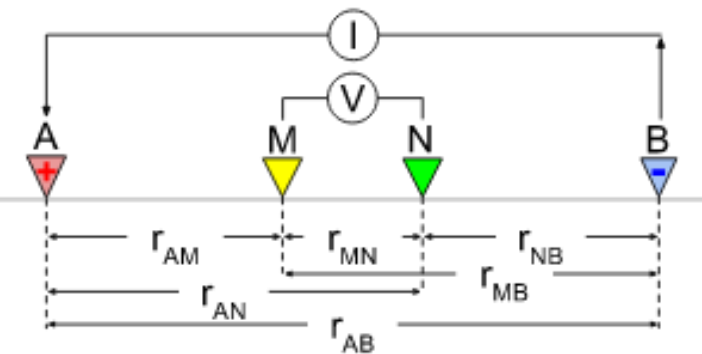
$$V_N = \frac{\rho I}{2\pi} \left( \frac{1}{r_{AN}} - \frac{1}{r_{BN}} \right)$$

$$\Delta V_{MN} = V_M - V_N = \rho I \frac{1}{2\pi} \left( \frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}} \right)$$

# From voltage to resistivity

- Remember that, we want to estimate resistivity from the measured voltage

$$\Delta V_{MN} = V_M - V_N = \rho I \frac{1}{2\pi} \left( \frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}} \right)$$

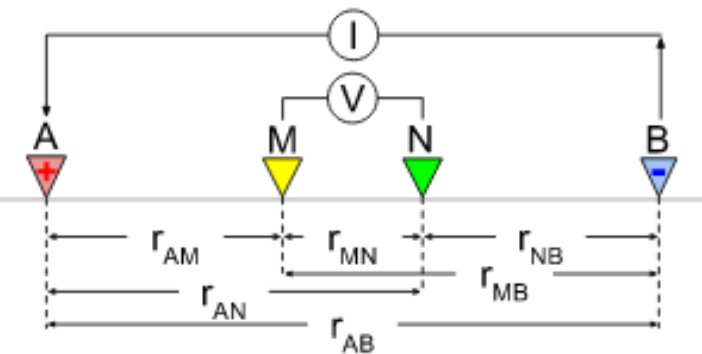


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Geometric constant  $G = \frac{1}{2\pi} \left( \frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}} \right)$

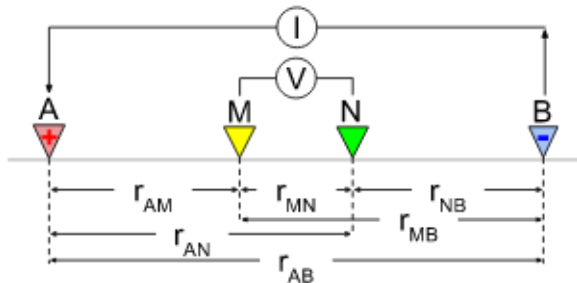


$$\rho = \frac{\Delta V_{MN}}{IG}$$

$G$  is purely determined by **survey geometry** (i.e., locations of the electrodes)

# Currents and potentials: 4-electrode array

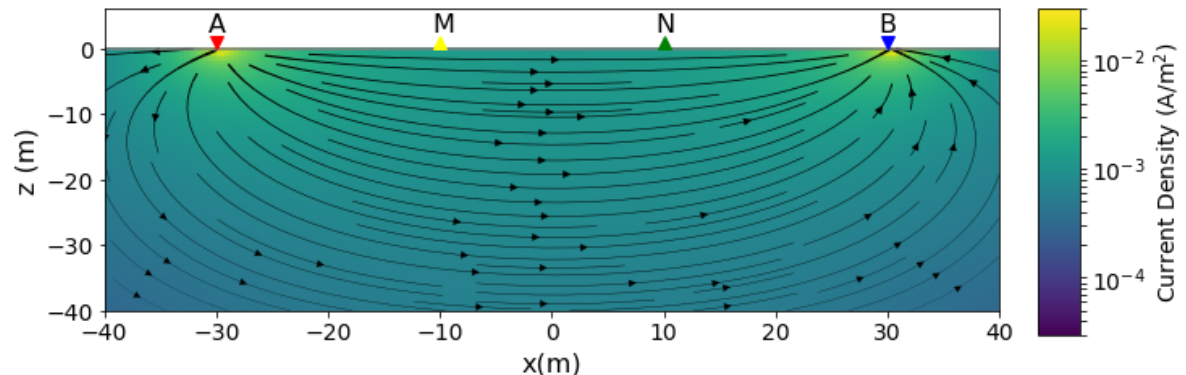
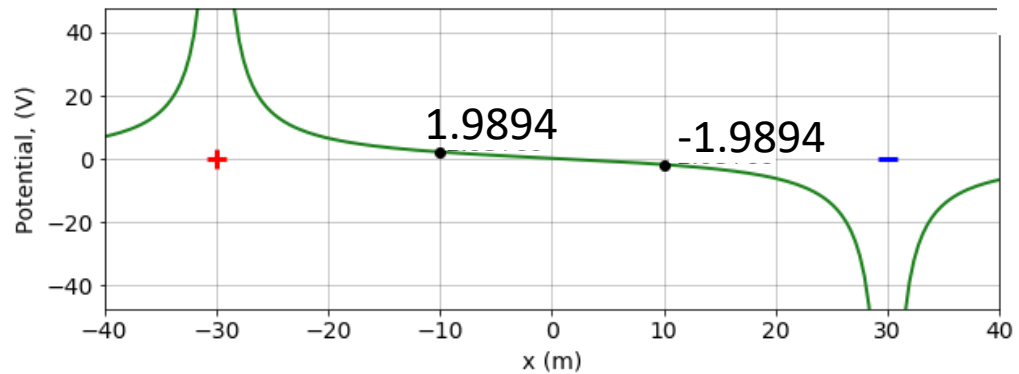
Halfspace ( $500 \Omega m$ )



$$\Delta V_{MN} = \rho I \underbrace{\frac{1}{2\pi} \left[ \frac{1}{AM} - \frac{1}{MB} - \frac{1}{AN} + \frac{1}{NB} \right]}_G$$

Resistivity

$$\rho = \frac{\Delta V_{MN}}{IG}$$



Images generated using DC\_Layered Earth.ipynb.

A: -30 m, B: 30 m, M: -10 m, N = 10 m.  $\rho = 500 \Omega \cdot m$

Credit: Doug Oldenburg, Seogi Kang and Linsey Heagy from UBC-GIF

# Exercise

- Calculate the Earth's resistivity based on the following information:

$$V_M = 1.9894 \text{ V}$$

$$V_N = -1.9894 \text{ V}$$

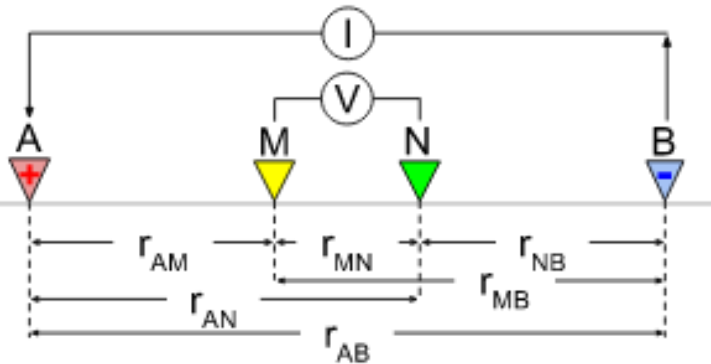
$$I = 1 \text{ A}$$

$$A: -30 \text{ m}$$

$$B: 30 \text{ m}$$

$$M: -10 \text{ m}$$

$$N: 10 \text{ m}$$

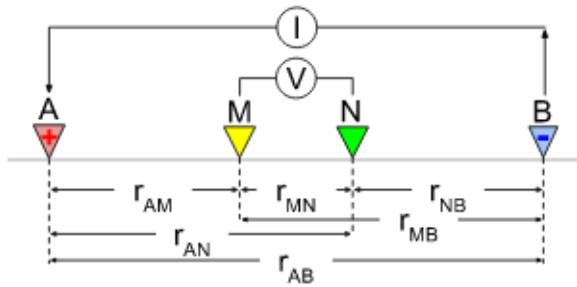


$$\rho = \frac{\Delta V_{MN}}{IG}$$

$$G = \frac{1}{2\pi} \left( \frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}} \right)$$

# Currents and potentials: 4-electrode array

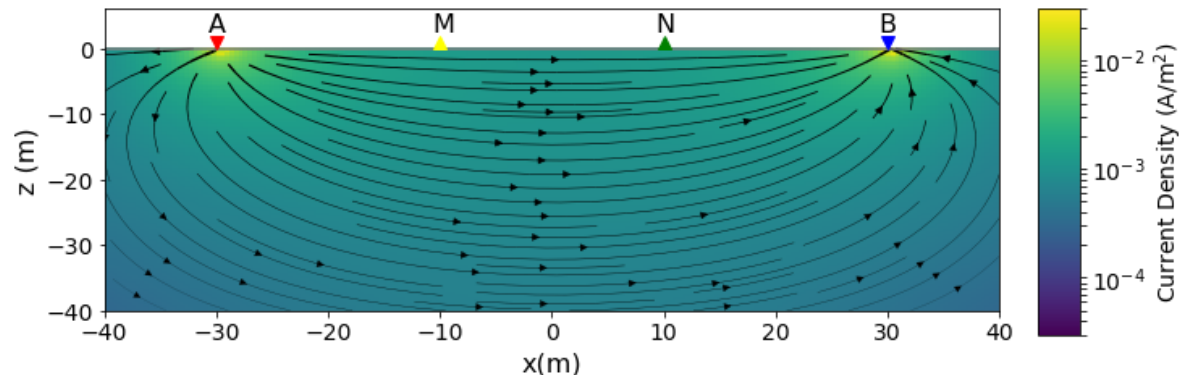
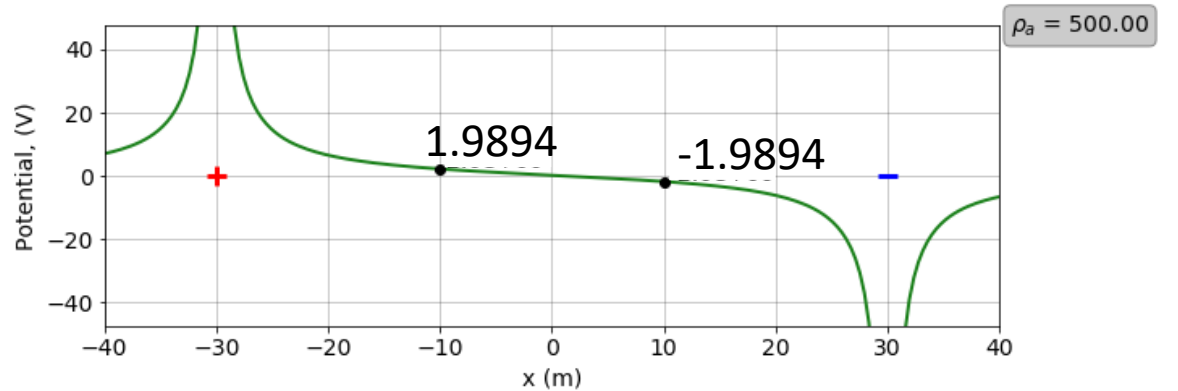
Halfspace ( $500 \Omega m$ )



$$\Delta V_{MN} = \rho I \underbrace{\frac{1}{2\pi} \left[ \frac{1}{AM} - \frac{1}{MB} - \frac{1}{AN} + \frac{1}{NB} \right]}_G$$

Resistivity

$$\rho = \frac{\Delta V_{MN}}{IG}$$



Images generated using DC\_Layered Earth.ipynb.

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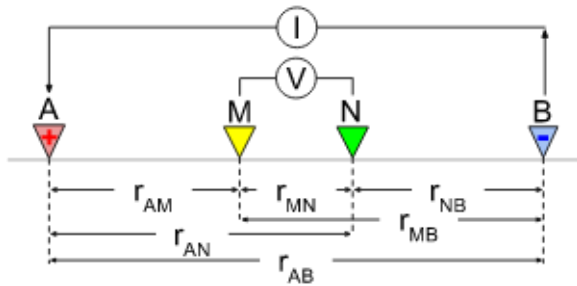


# Inhomogeneous Earth

- So far so good!
- But remember that we have assumed a homogeneous Earth
- The real Earth is, unfortunately, not homogeneous!

# Currents and Apparent Resistivity

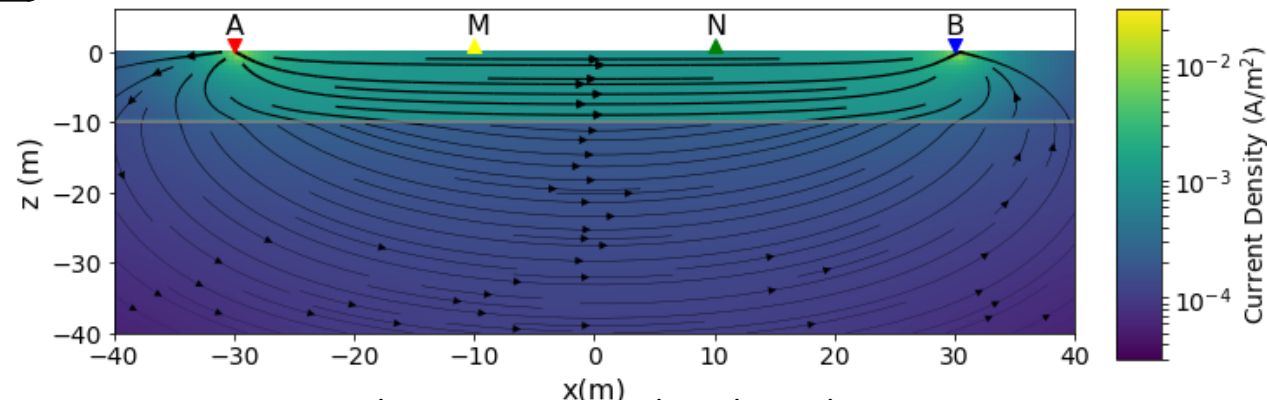
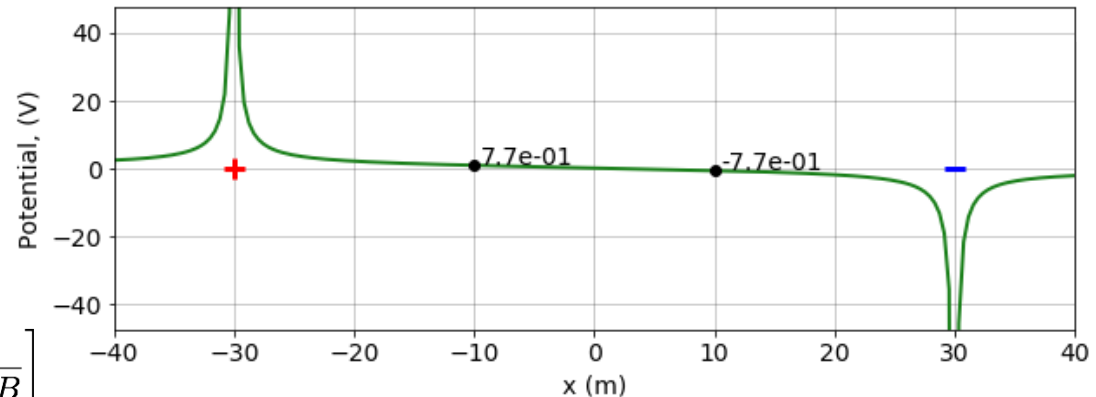
Conductive overburden ( $100 \Omega m$ )



$$\Delta V_{MN} = \rho I \underbrace{\frac{1}{2\pi} \left[ \frac{1}{AM} - \frac{1}{MB} - \frac{1}{AN} + \frac{1}{NB} \right]}_G$$

Apparent resistivity

$$\rho_a = \frac{\Delta V_{MN}}{IG}$$



Images generated using DC\_Layered Earth.ipynb. A: -30 m, B: 30 m, M: -10 m, N = 10 m.  $\rho_1 = 100 \Omega \cdot m$ ,  $\rho_2 = 500 \Omega \cdot m$ .  $h = 10$  m.

Credit: Doug Oldenburg, Seogi Kang and Linsey Heagy from UBC-GIF

# Exercise

- Calculate the Earth's resistivity based on the following information:

$$V_M = 0.77 \text{ V}$$

$$V_N = -0.77 \text{ V}$$

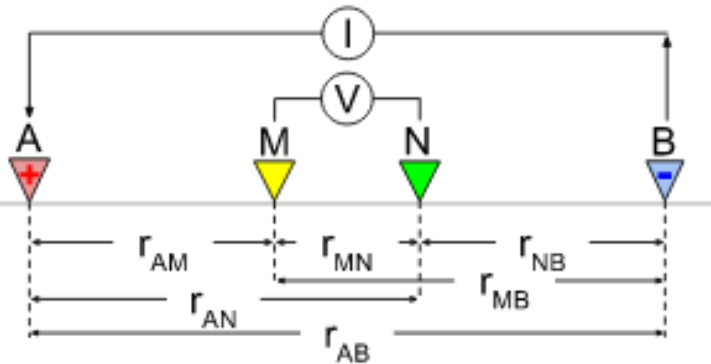
$$I = 1 \text{ A}$$

$$A: -30 \text{ m}$$

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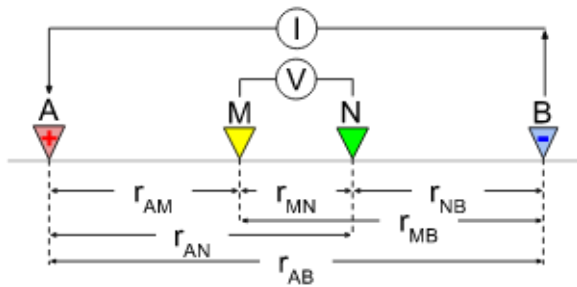
$$N: 10 \text{ m}$$



$$\rho = \frac{\Delta V_{MN}}{IG}$$

$$G = \frac{1}{2\pi} \left( \frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}} \right)$$

# Currents and Apparent Resistivity

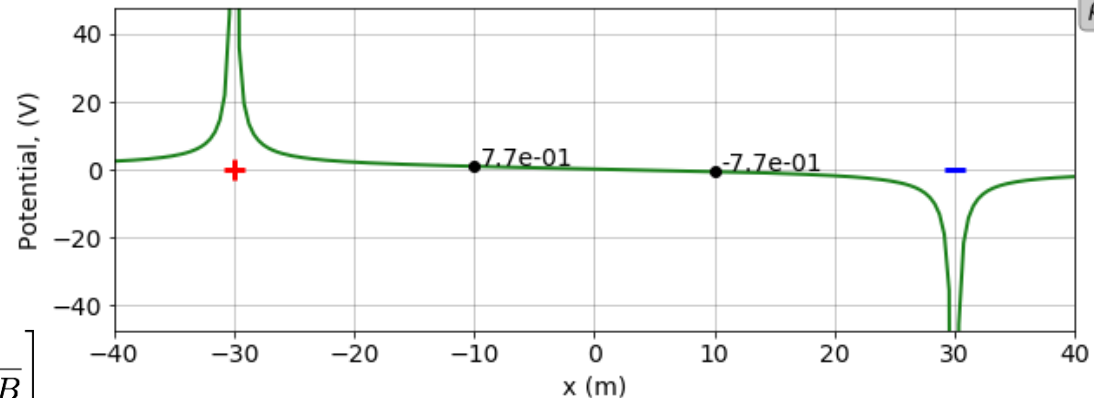


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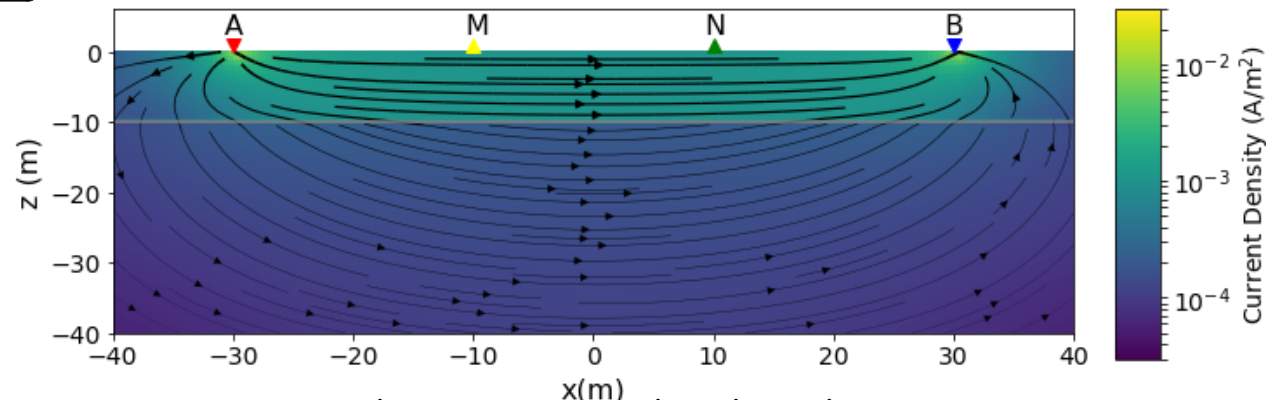
Apparent resistivity

$$\rho_a = \frac{\Delta V_{MN}}{IG}$$

Conductive overburden ( $100 \Omega m$ )



$\rho_a = 193.01$



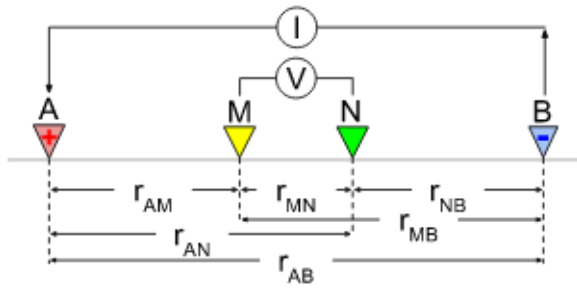
Images generated using DC\_Layered Earth.ipynb. A: -30 m, B: 30 m, M: -10 m, N = 10 m.  $\rho_1 = 100 \Omega \cdot m$ ,  $\rho_2 = 500 \Omega \cdot m$ .  $h = 10$  m.

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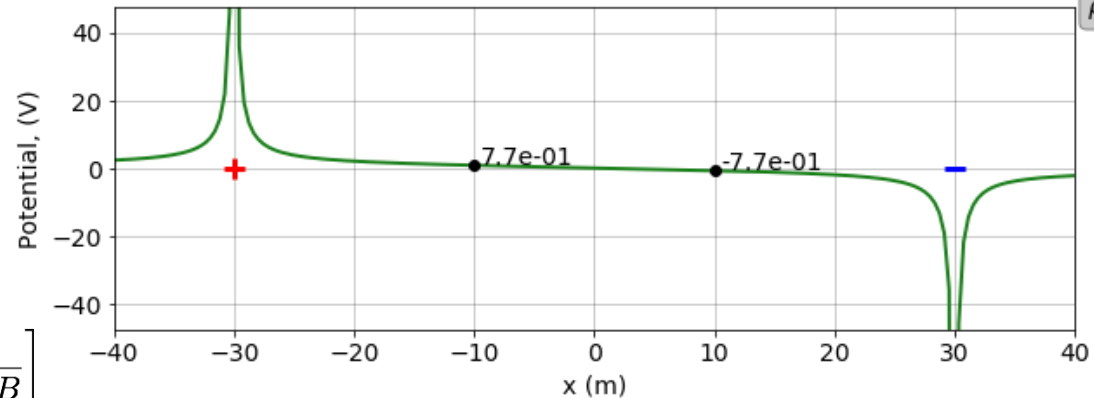
# Currents and Apparent Resistivity

Conductive overburden ( $100 \Omega m$ )

$\rho_a = 193.01$

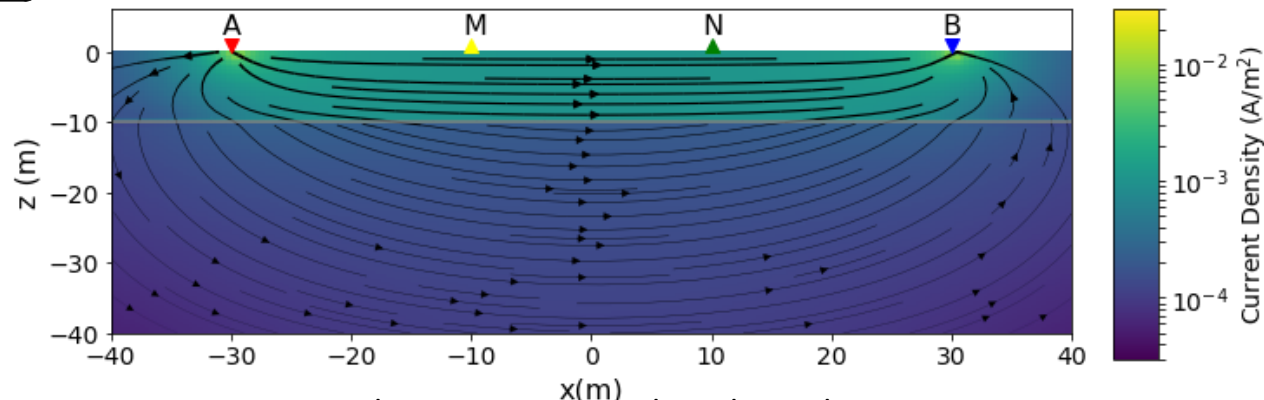


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Apparent resistivity

$$\rho_a = \frac{\Delta V_{MN}}{IG}$$



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# Apparent resistivity

$$\rho_a = \frac{\Delta V_{MN}}{IG}$$

- Compare it with **Ohm's law**:

# Apparent resistivity

$$\rho_a = \frac{\Delta V_{MN}}{IG} \qquad R = \frac{V}{I}$$

- Compare it with **Ohm's law**: apparent resistivity is equal to voltage divided by current (multiplied by G)

# Apparent resistivity

$$\rho_a = \frac{\Delta V_{MN}}{IG}$$

- If the Earth is **homogeneous**, the **apparent resistivity** is equal to the **true resistivity** of the Earth.



# Apparent resistivity

$$\rho_a = \frac{\Delta V_{MN}}{IG}$$

- If the Earth is **homogeneous**, the **apparent resistivity** is equal to the **true resistivity** of the Earth.
- For **inhomogeneous** Earth, the apparent resistivity is some **complicated averaging** of the resistivities of all materials encountered by the currents.

# Apparent resistivity

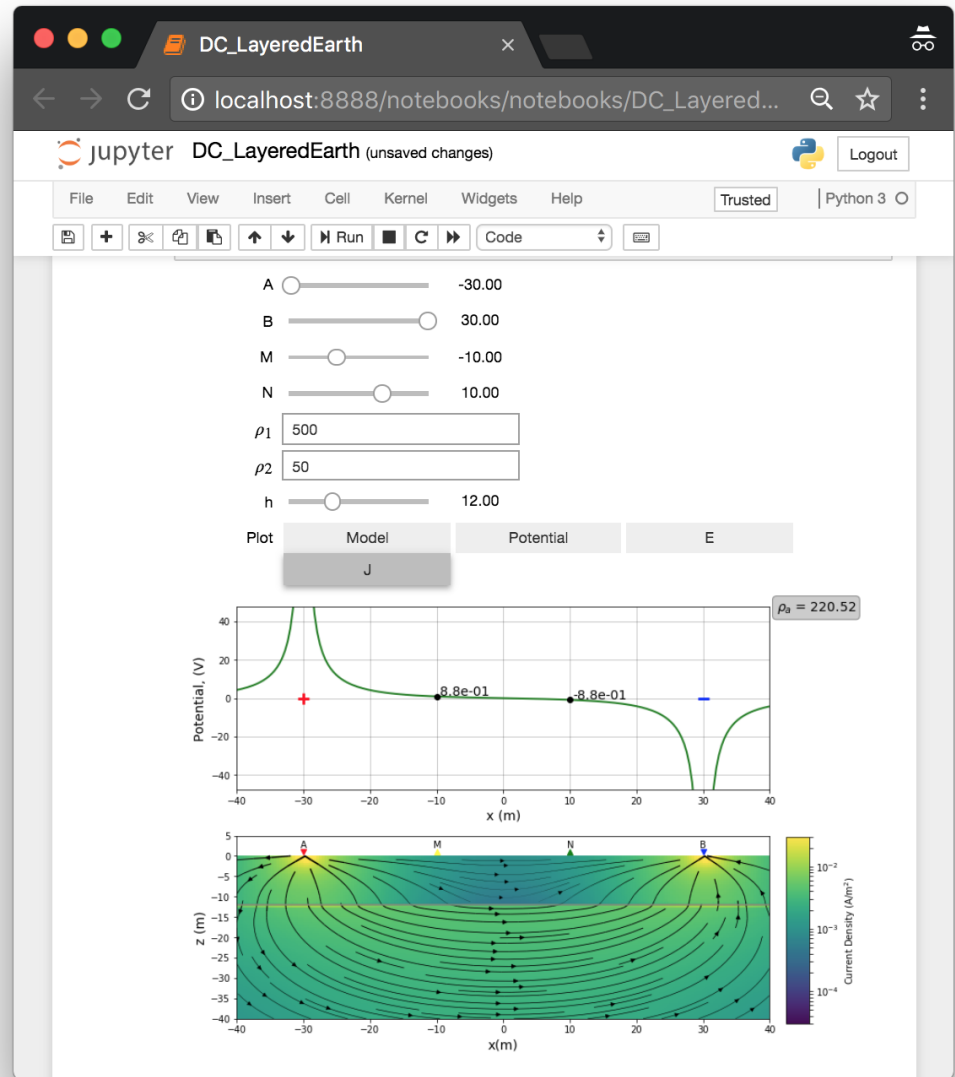
$$\rho_a = \frac{\Delta V_{MN}}{IG}$$

- If the Earth is **homogeneous**, the **apparent resistivity** is equal to the **true resistivity** of the Earth.
- For **inhomogeneous** Earth, the apparent resistivity is some **complicated averaging** of the resistivities of all materials encountered by the currents.
- It can be interpreted as the resistivity **that would have been measured if the Earth were homogeneous**.

# DC Simulation App

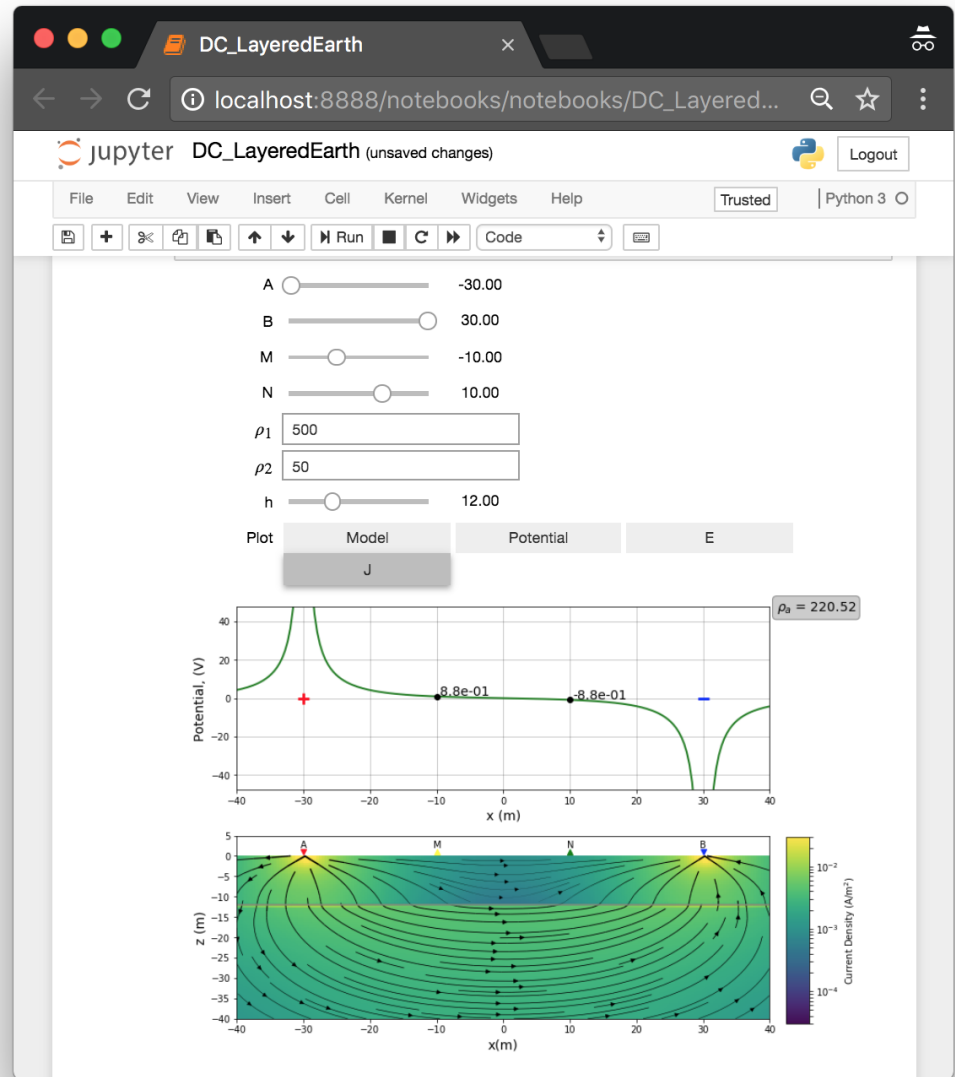
## Why interactive apps?

- Visualization aids understanding
- Learn through interaction
  - ask questions and investigate
- Open source:
  - Free to use
  - Welcome contributions!



# DC Simulation App

- DC\_LayeredEarth.ipynb
- Parameters:
  - Layer resistivities
  - Layer thickness
  - Electrode locations
- View:
  - Model
  - Electric potential
  - Electric field
  - Current density



# Current density (optional)

Remember that

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

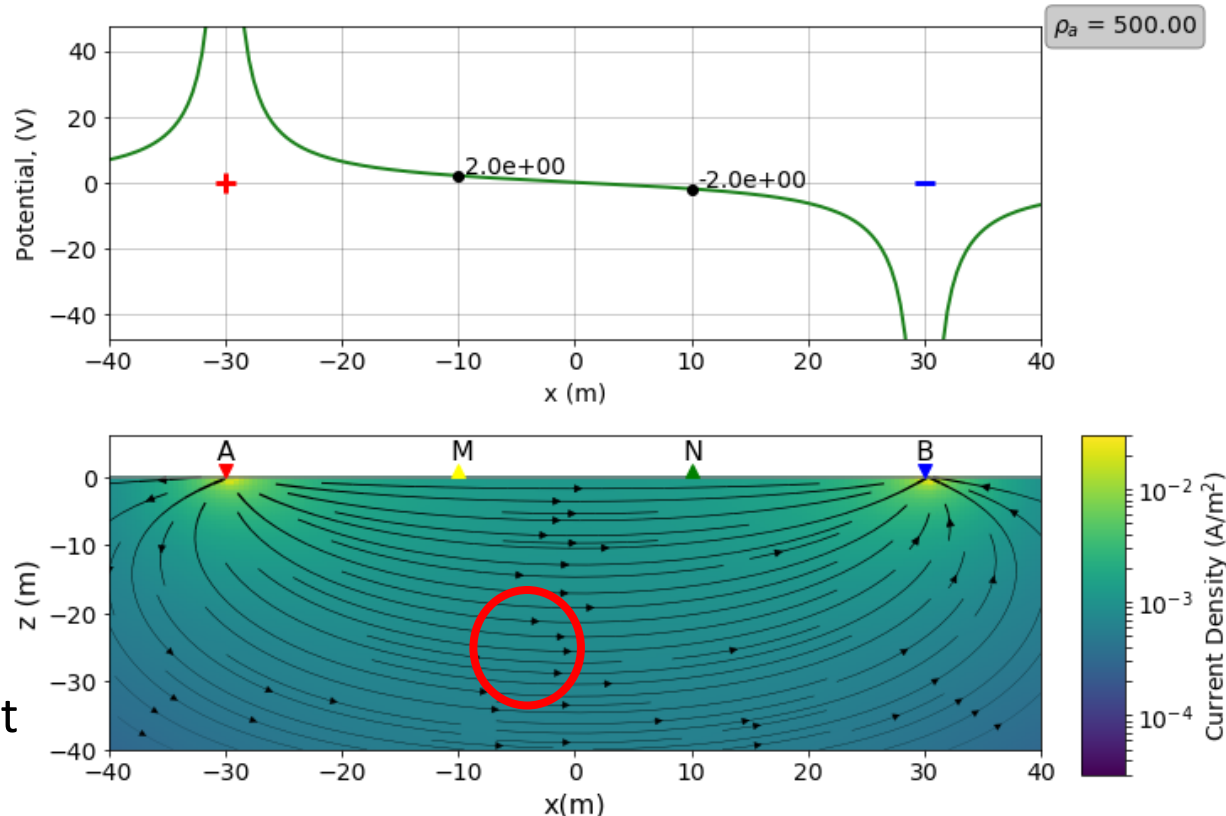
The only place where  $\frac{\partial \rho}{\partial t}$  is nonzero is at the current electrodes.

At all other locations,

$$\nabla \cdot \mathbf{J} = 0$$

That is, convergence is 0.

That is, all of the current that flows into a volume must be equal to the current flowing out of the volume.



Images generated using DC\_LayeredEarth.ipynb. A: -30 m, B: 30 m, M: -10 m, N = 10 m.  $\rho = 500 \Omega \cdot \text{m}$ .

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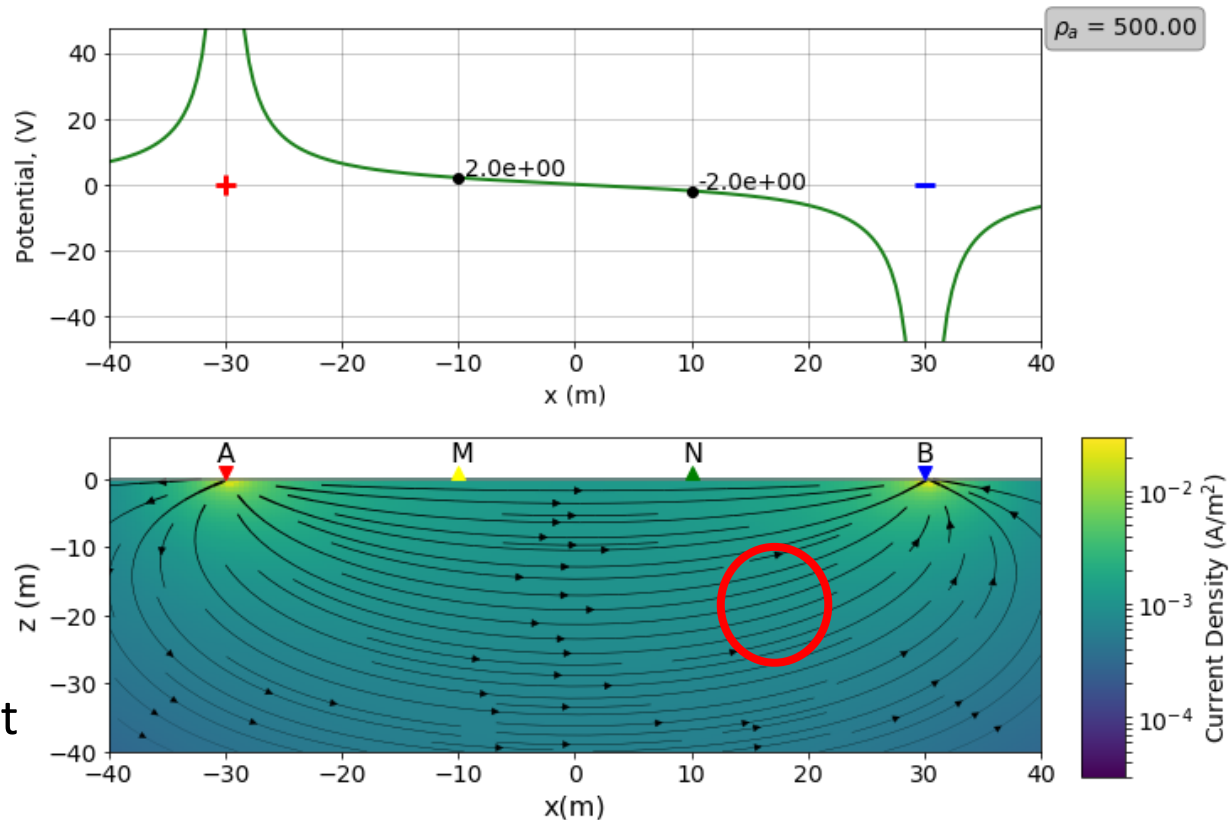
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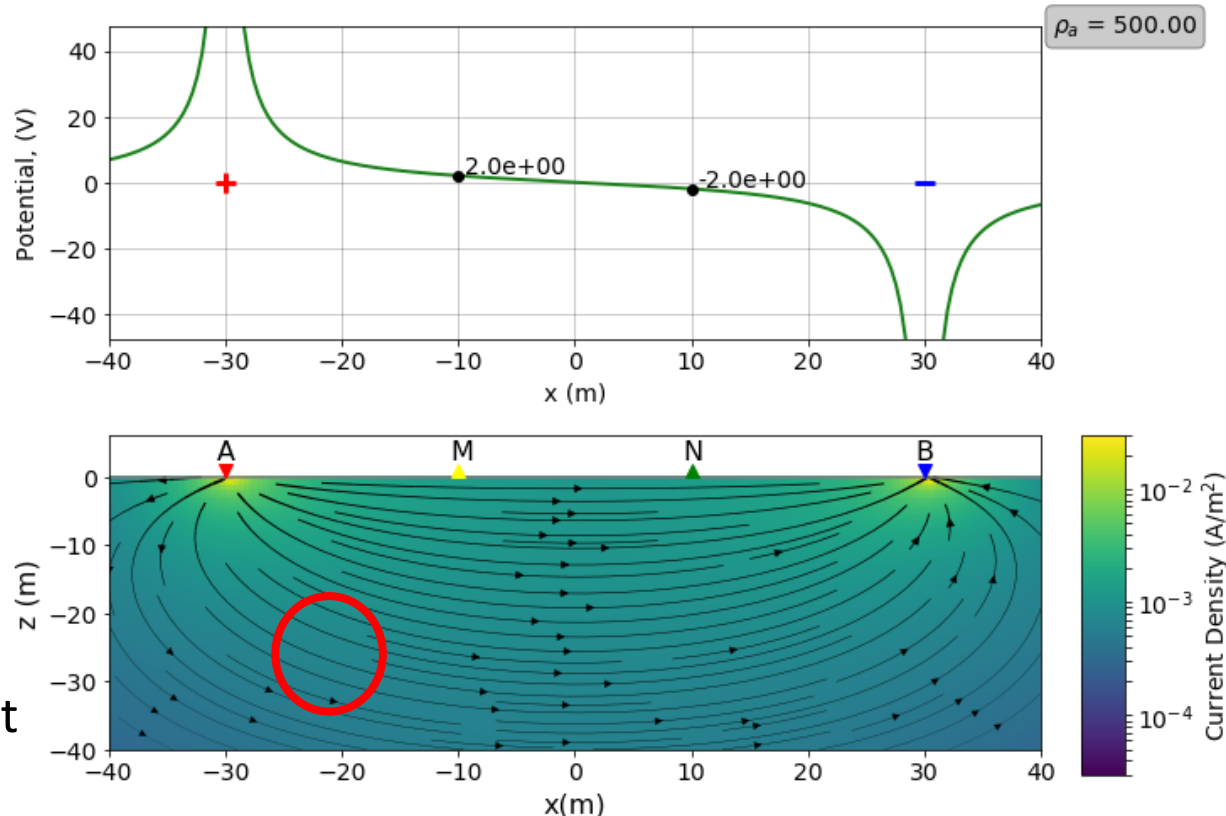
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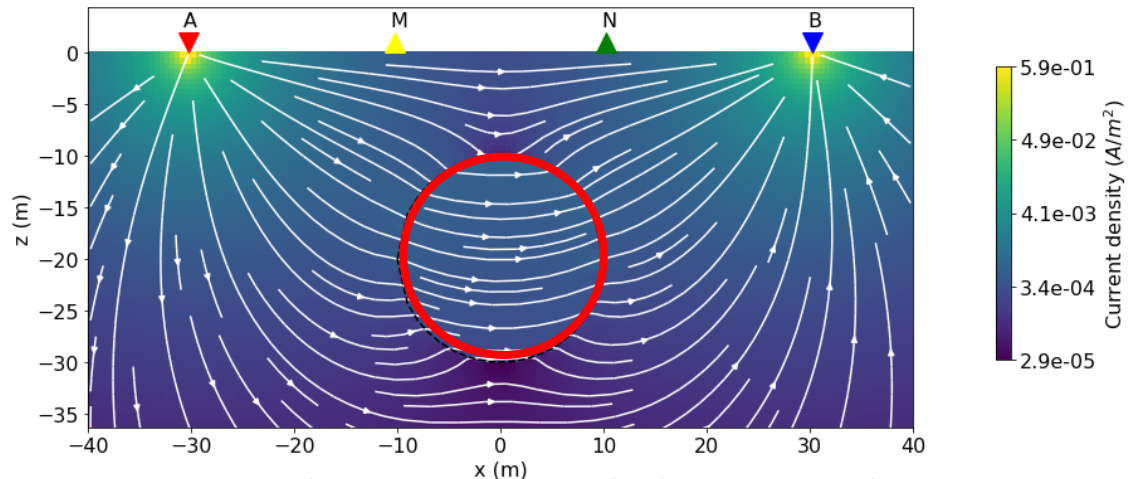
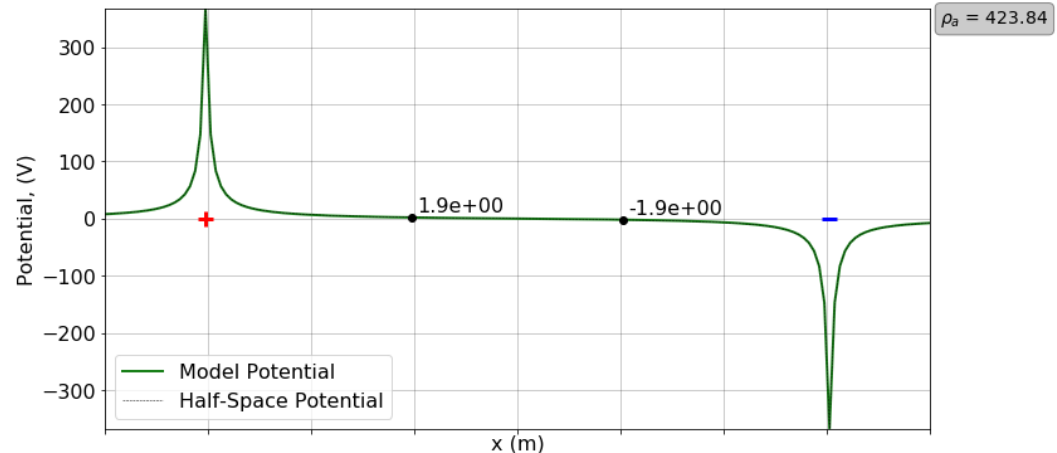
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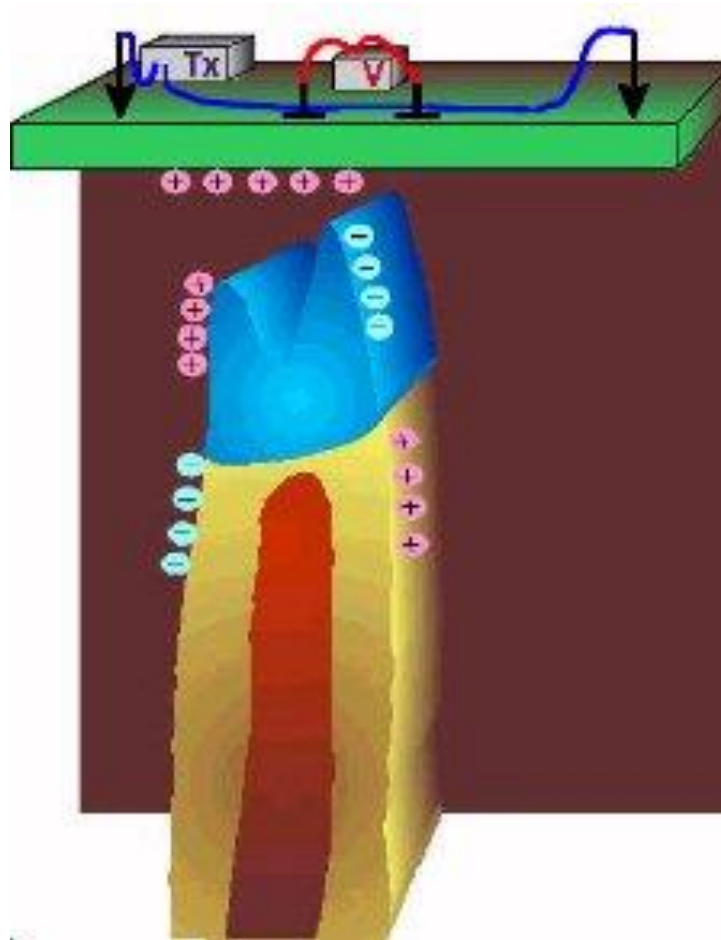
Images generated using DC\_Layer\_Cylinder\_2\_5D.ipynb. A: -30.25 m, B: 30.25 m, M: -10.25 m, N = 10.25 m.  $\rho_1 = 500 \Omega \cdot \text{m}$ ,  $\rho_2 = 50 \Omega \cdot \text{m}$ .



# Agenda

- Recap
- Two current electrodes
- Apparent resistivity
- Understanding charges

# Understanding the charges



Credit: Doug Oldenburg, Seogi Kang and Linsey Heagy from UBC-GIF

# Some simple math

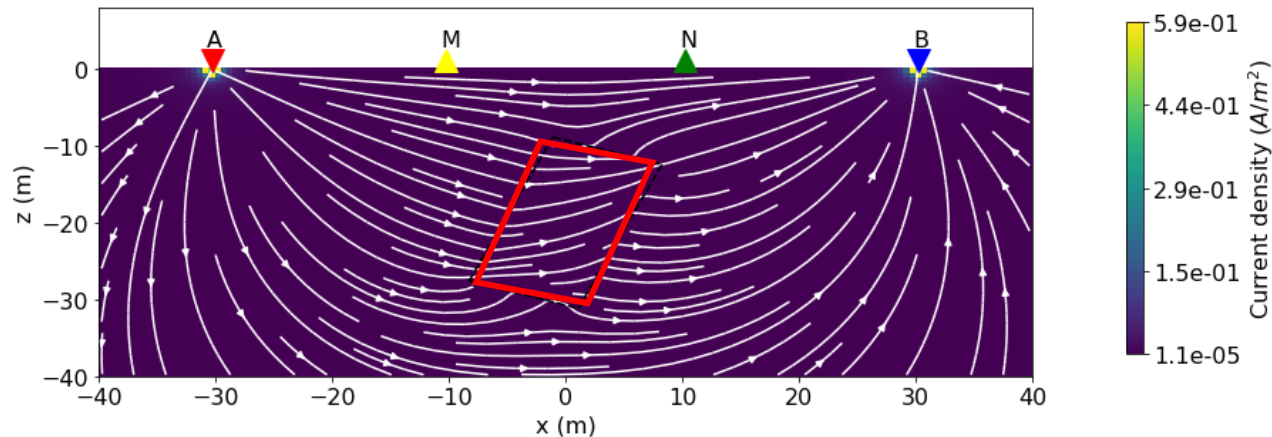
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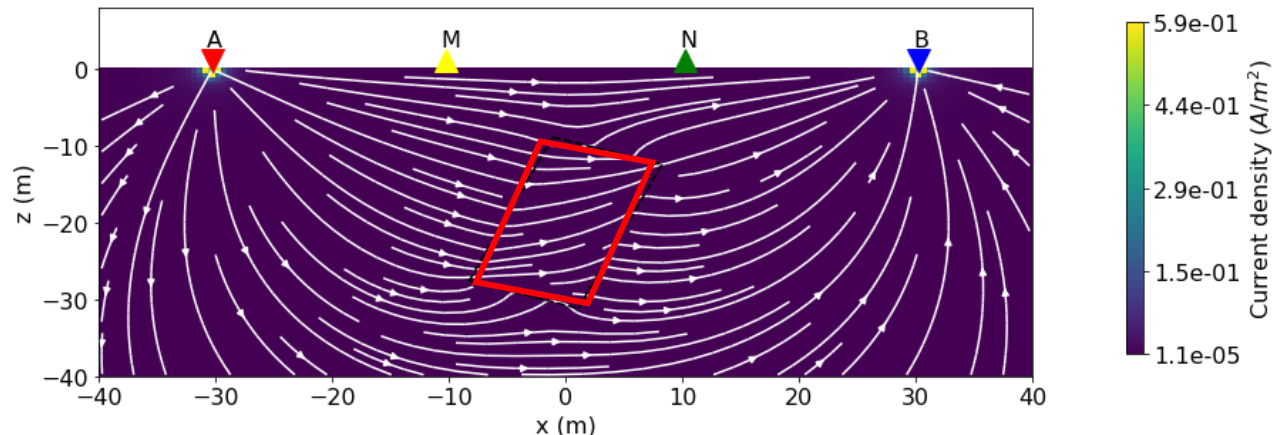


Images generated using DC\_Plate2\_5D.ipynb. A: -30.25 m, B: 30.25 m, M: -10.25 m, N = 10.25 m.  $\rho_1 = 500 \Omega \cdot \text{m}$ ,  $\rho_2 = 10 \Omega \cdot \text{m}$ .  $dx = 10$ ,  $dz = 20$ ,  $x_c = 0$ ,  $z_c = -20$ ,  $\theta = 21$ .

# How about the electric field?

$$\nabla \cdot \mathbf{J} = 0 \longrightarrow J_{1n} = J_{2n}$$

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Images generated using DC\_Plate2\_5D.ipynb. A: -30.25 m, B: 30.25 m, M: -10.25 m, N = 10.25 m.  $\rho_1 = 500 \Omega \cdot \text{m}$ ,  $\rho_2 = 10 \Omega \cdot \text{m}$ .  $dx = 10$ ,  $dz = 20$ ,  $x_c = 0$ ,  $z_c = -20$ ,  $\theta = 21$ .

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The normal component of electric field is discontinuous if there is resistivity contrast across some boundary.

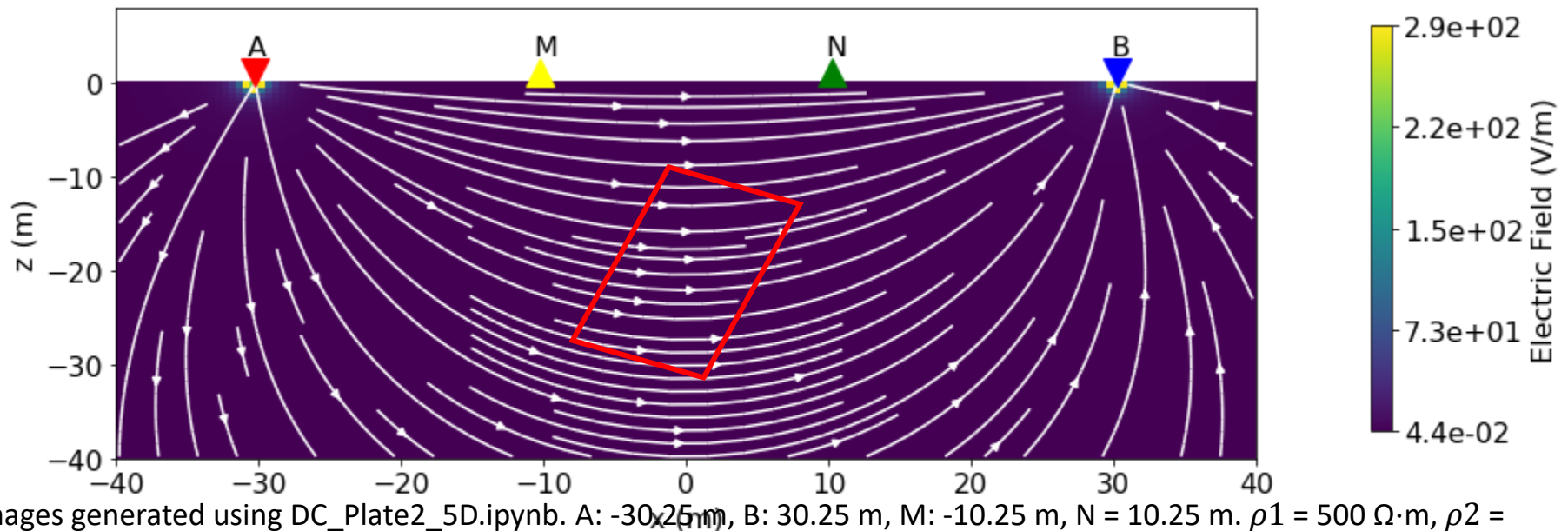
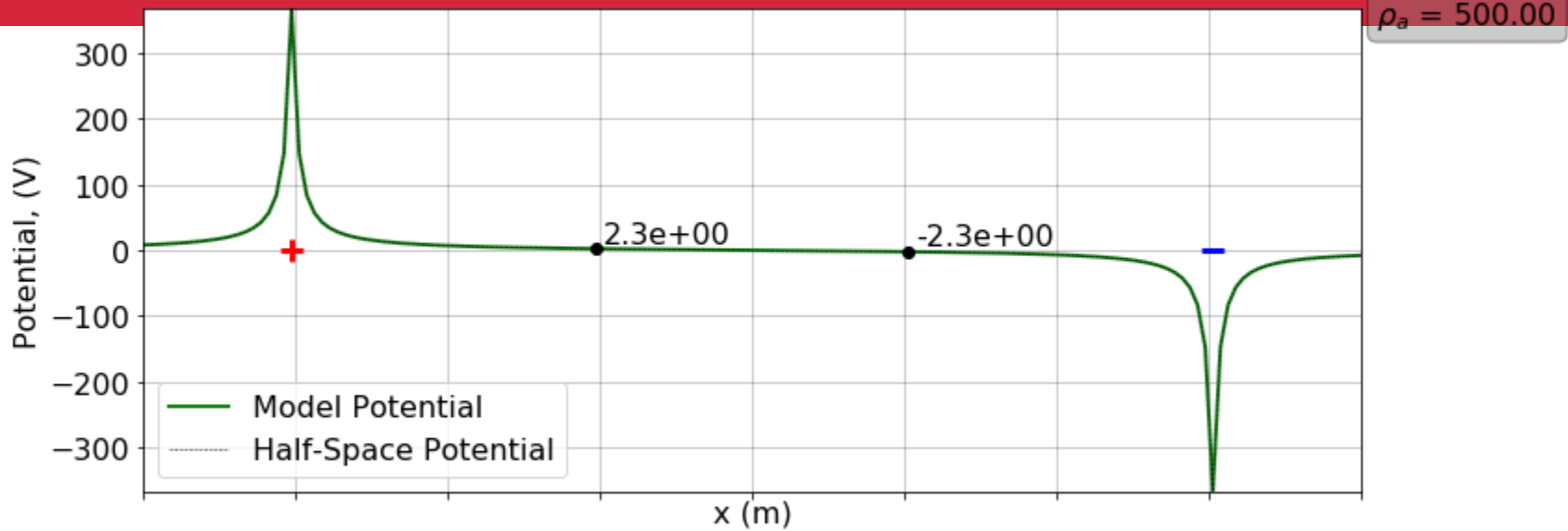
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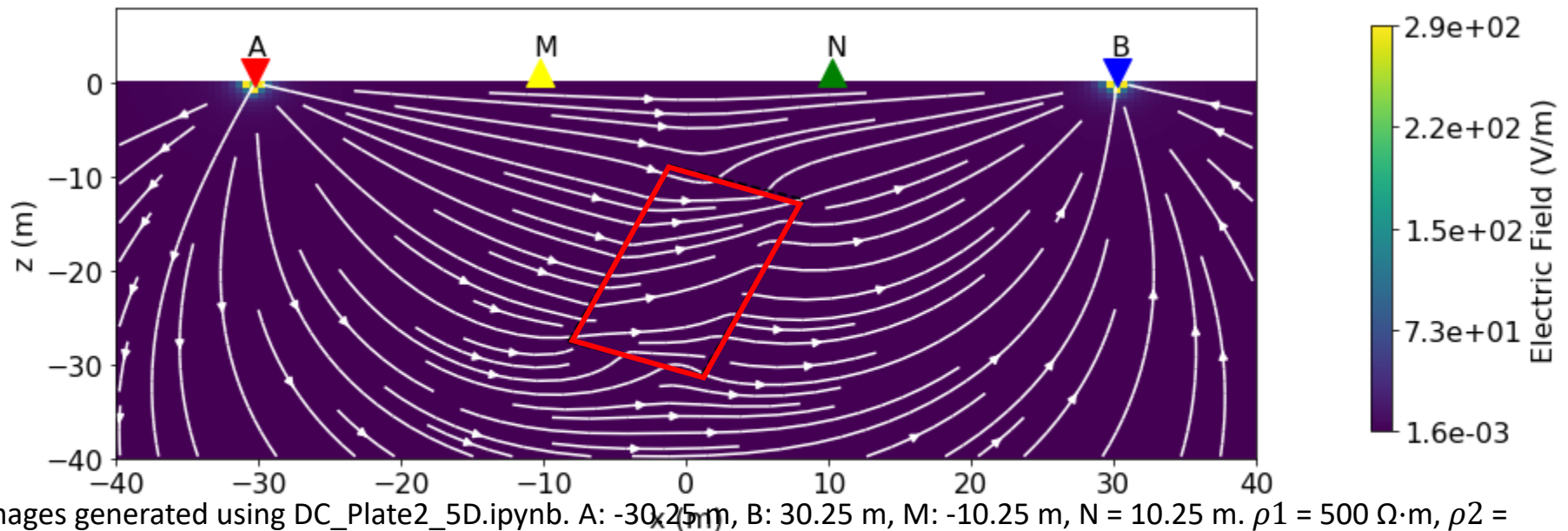
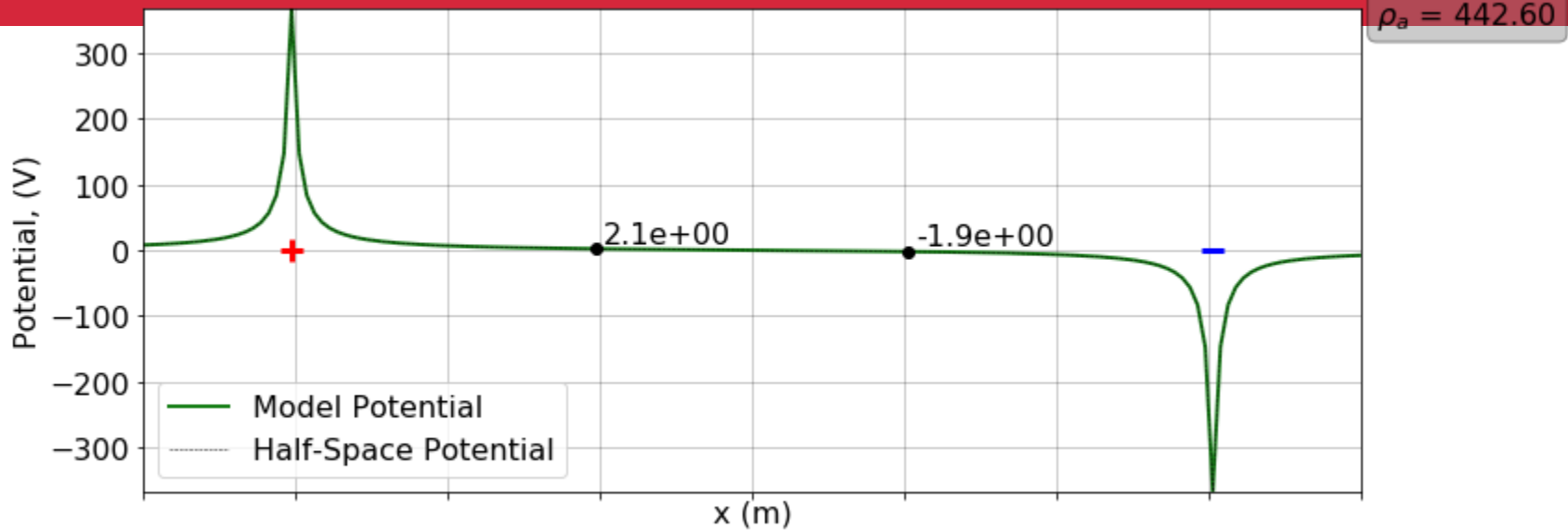
How about the tangential component of electric field?

## Optional materials

- Short answer:
- Because  $\nabla \times \mathbf{E} = 0$ ,  $E_{1t} = E_{2t}$

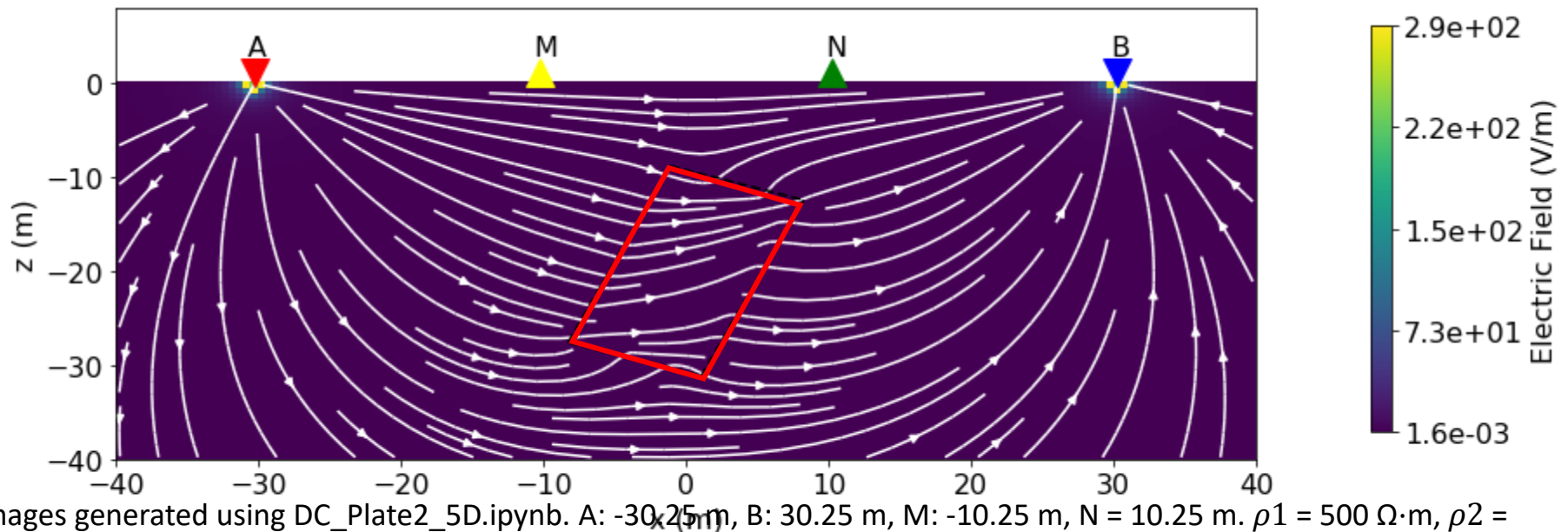


Images generated using DC\_Plate2\_5D.ipynb. A: -30.25 m, B: 30.25 m, M: -10.25 m, N: 10.25 m.  $\rho_1 = 500 \Omega \cdot m$ ,  $\rho_2 = 500 \Omega \cdot m$ .  $dx = 10$ ,  $dz = 20$ ,  $xc = 0$ ,  $zc = -20$ ,  $\theta = 21$ .



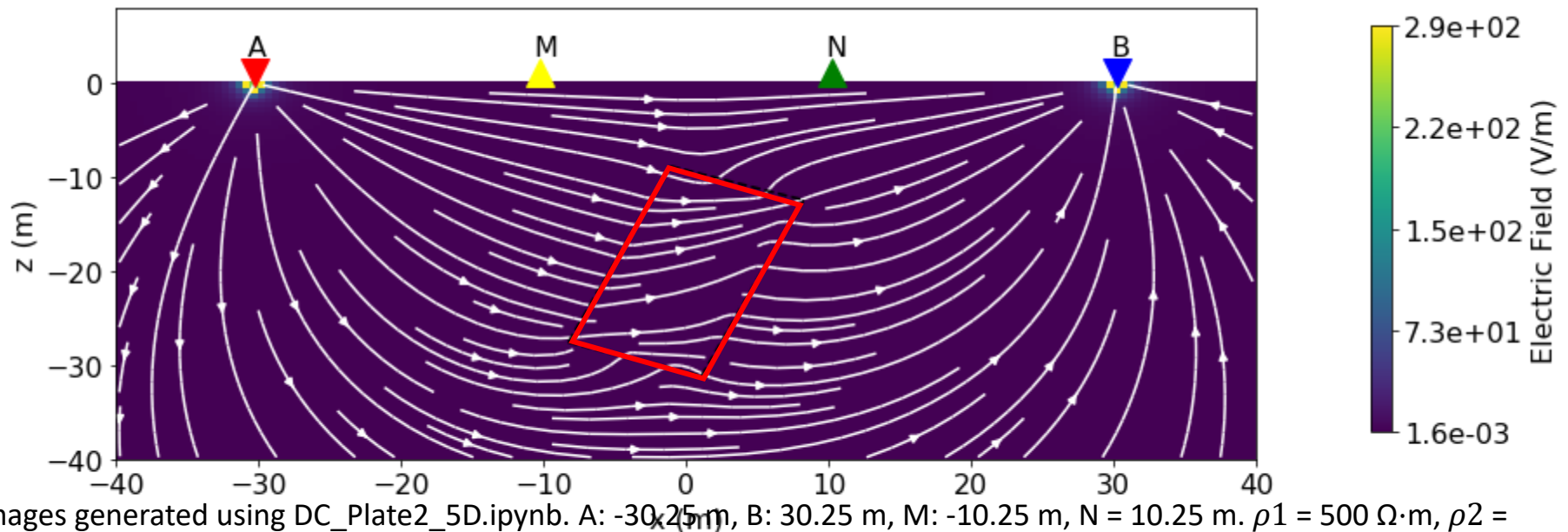
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- What could have happened?



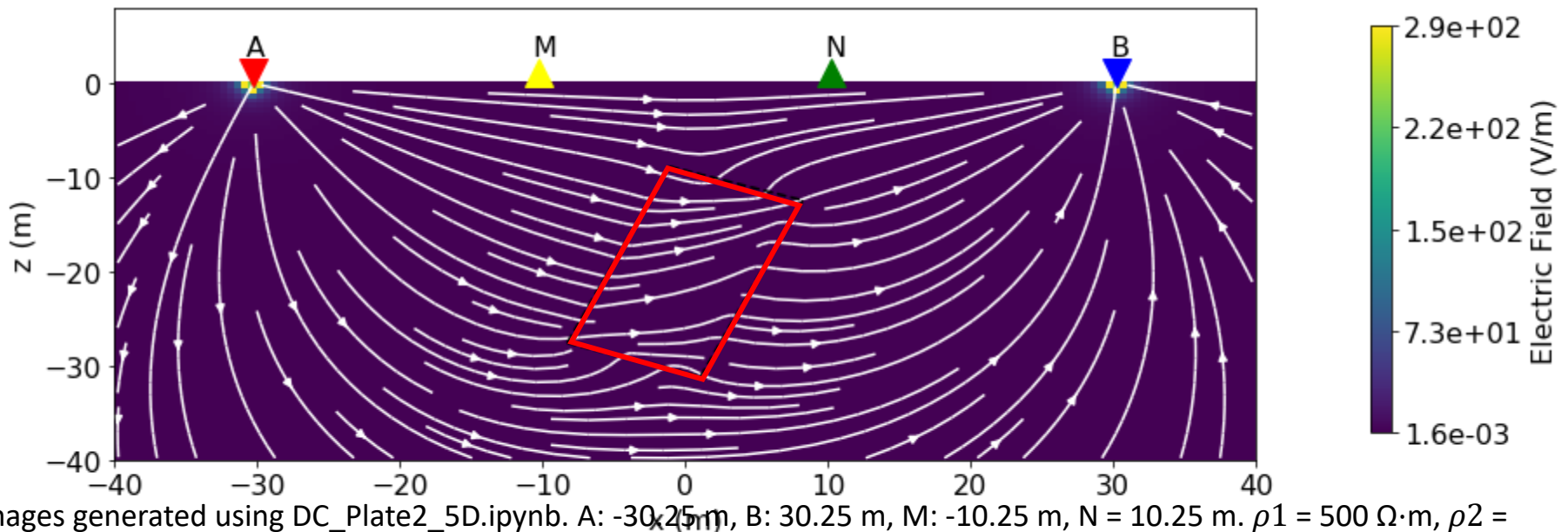
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$$\sigma_1 E_{1n} = \sigma_2 E_{2n}$$



- What could have happened?
- The electric field becomes discontinuous across the boundary.

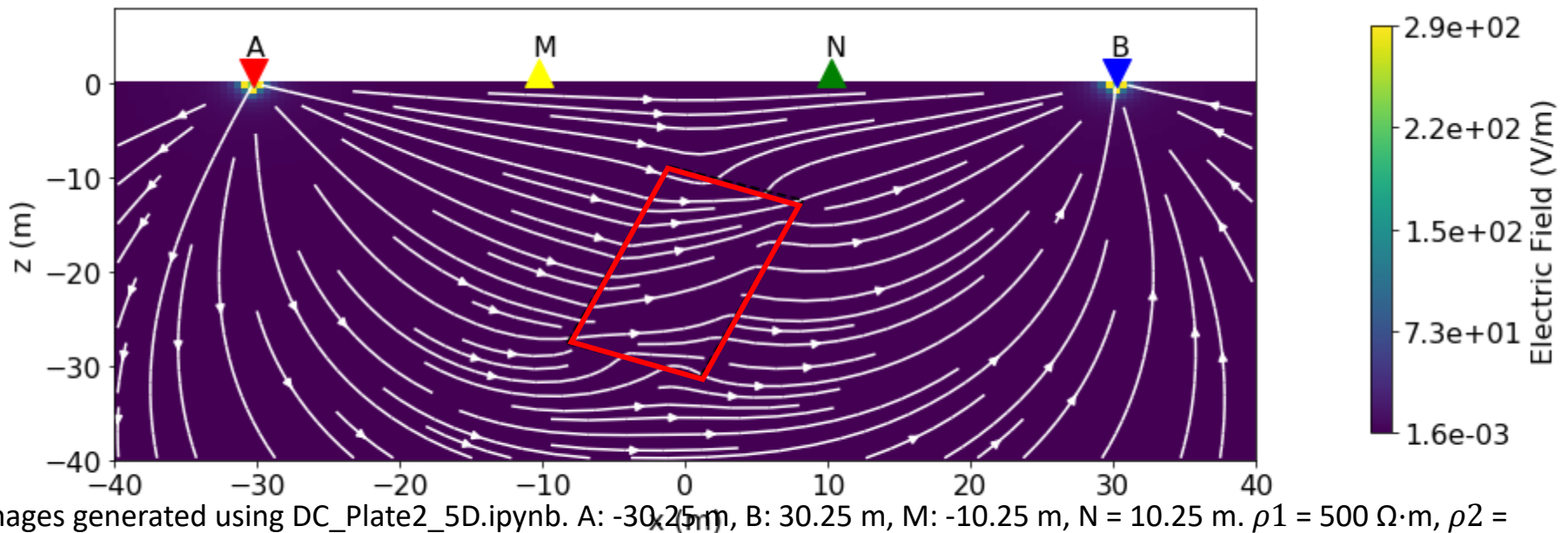
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- What could have happened?
- The electric field becomes discontinuous across the boundary.
- What causes the electric field to be discontinuous?

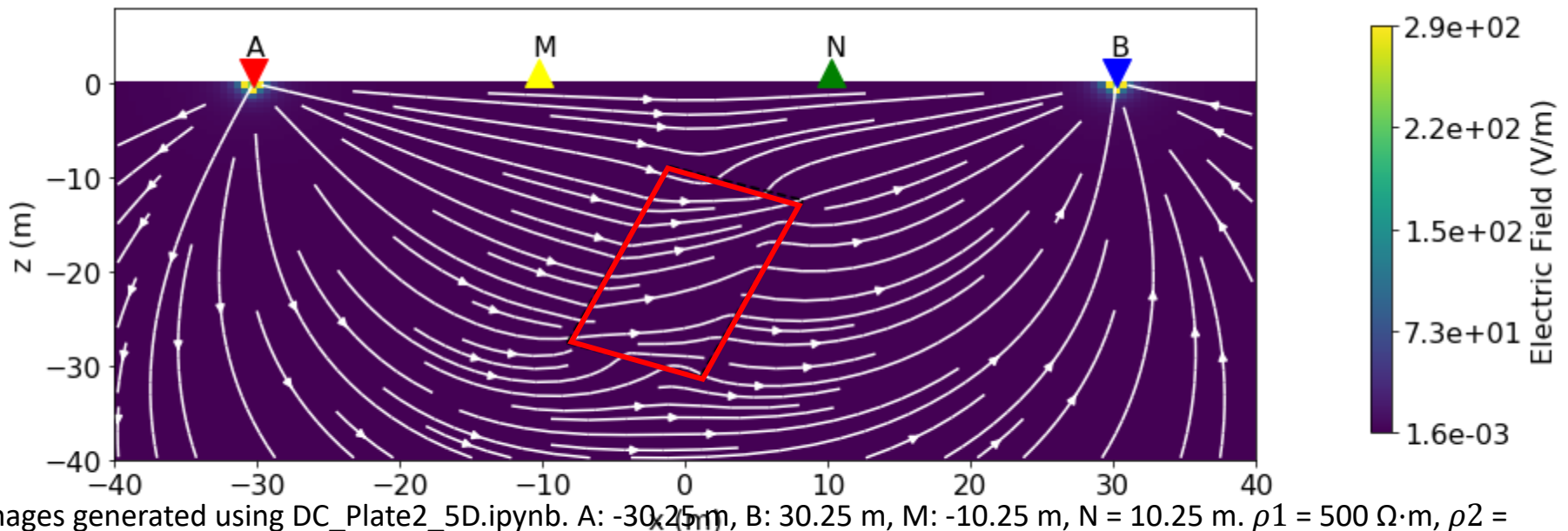
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Images generated using DC\_Plate2\_5D.ipynb. A: -30.25 m, B: 30.25 m, M: -10.25 m, N = 10.25 m.  $\rho_1 = 500 \Omega \cdot m$ ,  $\rho_2 = 10 \Omega \cdot m$ .  $dx = 10$ ,  $dz = 20$ ,  $xc = 0$ ,  $zc = -20$ ,  $\theta = 21$ .

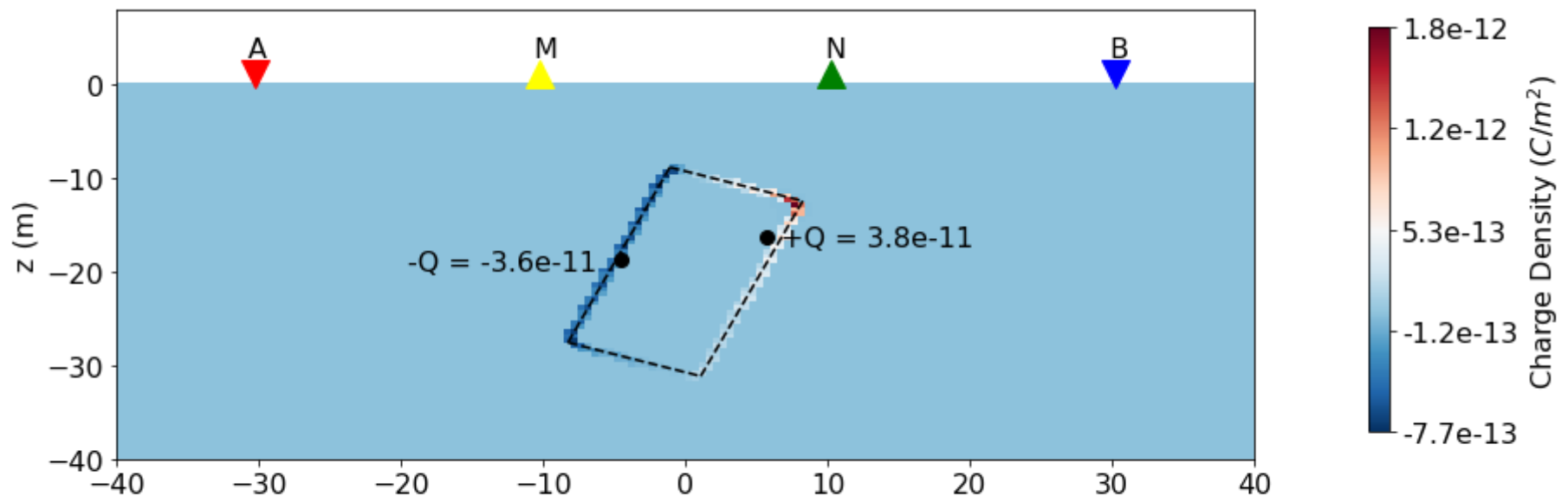
- What could have happened?
- The electric field becomes discontinuous across the boundary.
- What is the easiest way to make the electric field discontinuous?

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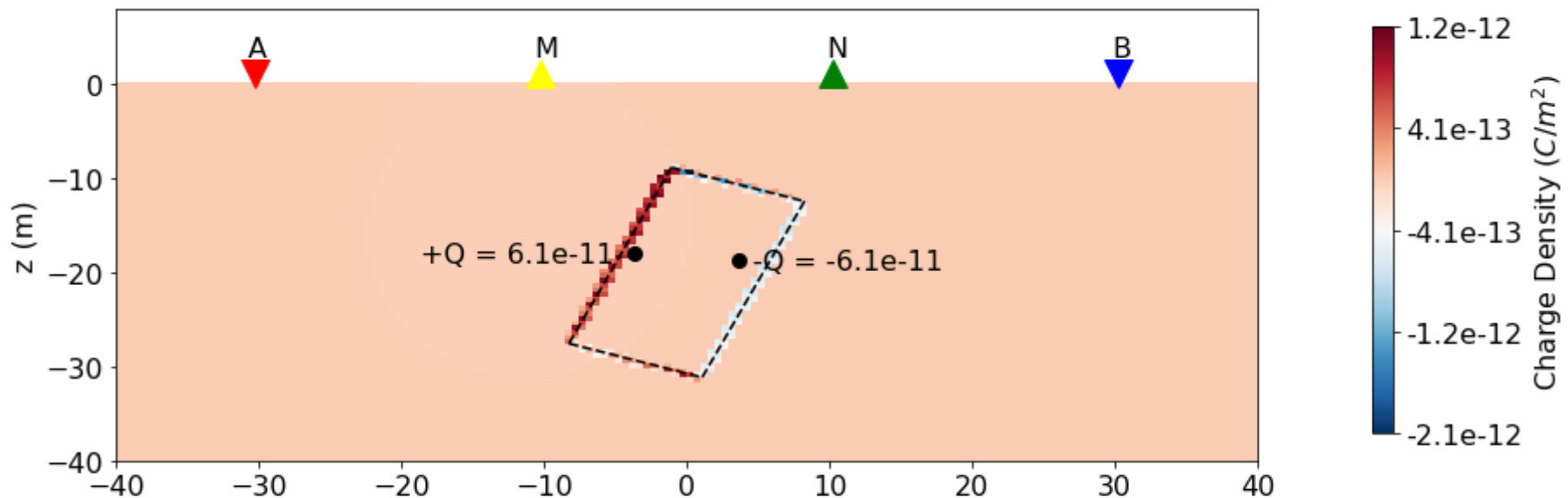


Images generated using DC\_Plate2\_5D.ipynb. A: -30.25 m, B: 30.25 m, M: -10.25 m, N: 10.25 m.  $\rho_1 = 500 \Omega \cdot m$ ,  $\rho_2 = 10 \Omega \cdot m$ .  $dx = 10$ ,  $dz = 20$ ,  $xc = 0$ ,  $zc = -20$ ,  $\theta = 21$ .

# How about a resistor?

- What is the easiest way to make the electric field discontinuous?

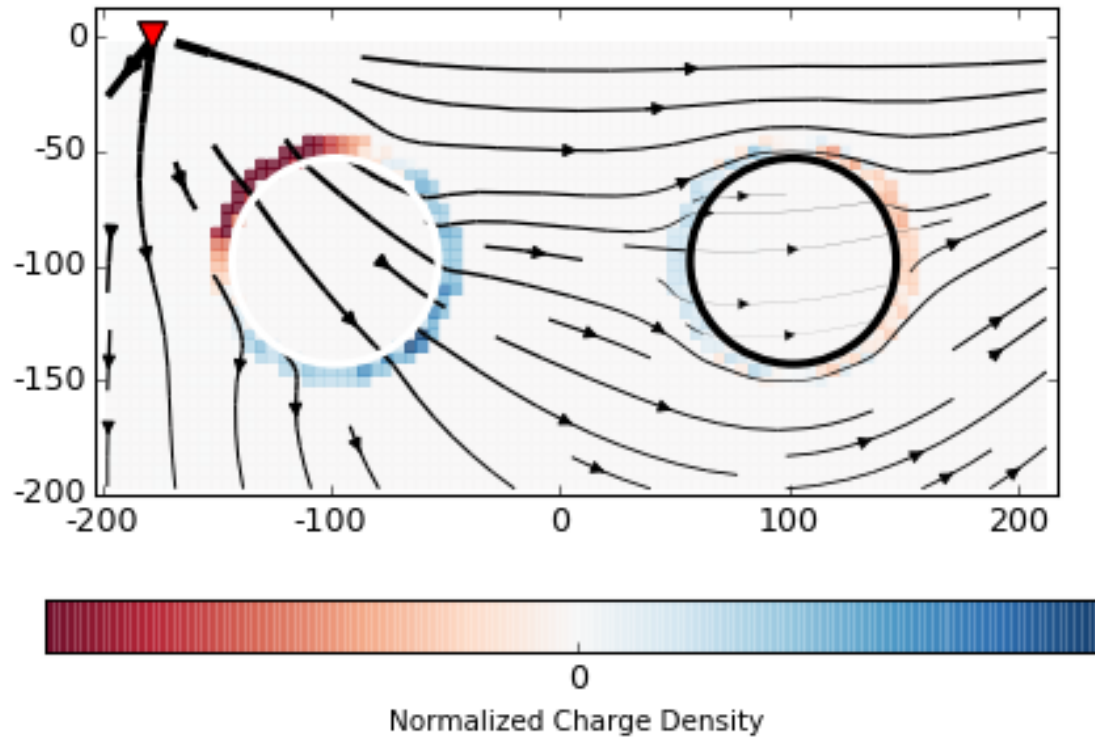
$$\sigma_1 E_{1n} = \sigma_2 E_{2n}$$



Images generated using DC\_Plate2\_5D.ipynb. A: -30.25 m, B: 30.25 m, M: -10.25 m, N: 10.25 m.  $\rho_1 = 500 \Omega \cdot \text{m}$ ,  $\rho_2 = 5000 \Omega \cdot \text{m}$ .  $dx = 10$ ,  $dz = 20$ ,  $xc = 0$ ,  $zc = -20$ ,  $\theta = 21$ .

# Test

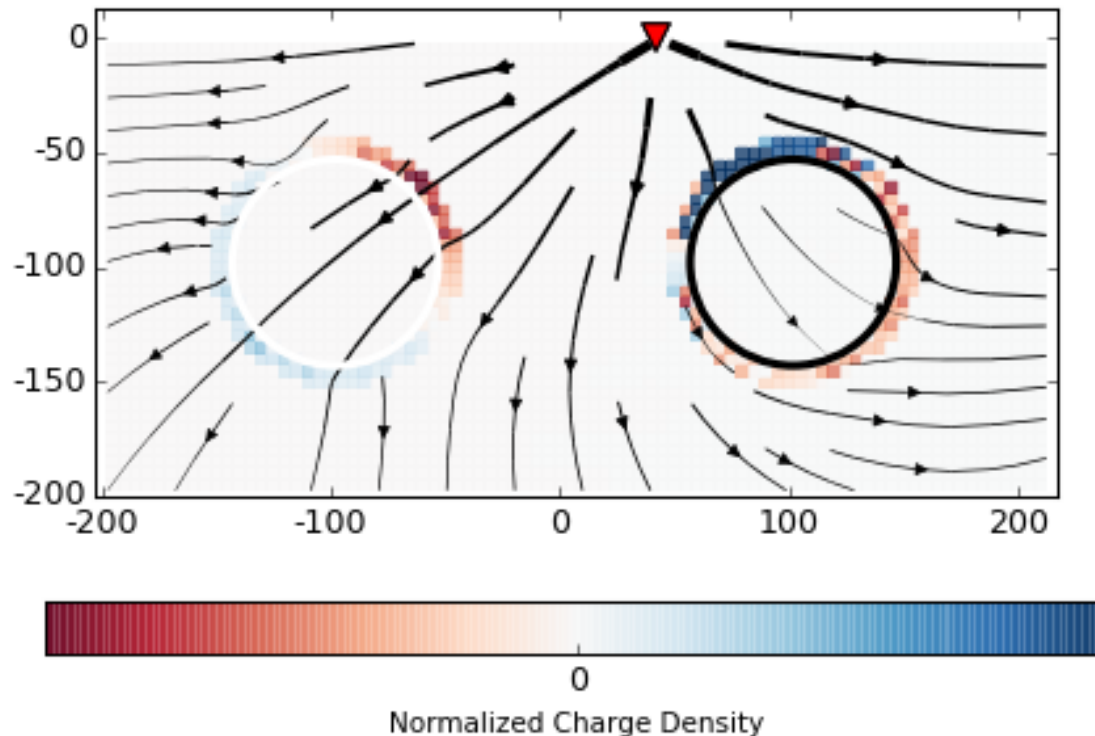
Can you tell which one is conductive and which one is resistive?



[https://gpg.geosci.xyz/content/DC\\_resistivity/DC\\_basic\\_principles\\_heterogeneous\\_earth.html](https://gpg.geosci.xyz/content/DC_resistivity/DC_basic_principles_heterogeneous_earth.html)

# Test

Now that you have figured out which one is conductive and which is resistive. What do you think the charges will look like for the following scenario?

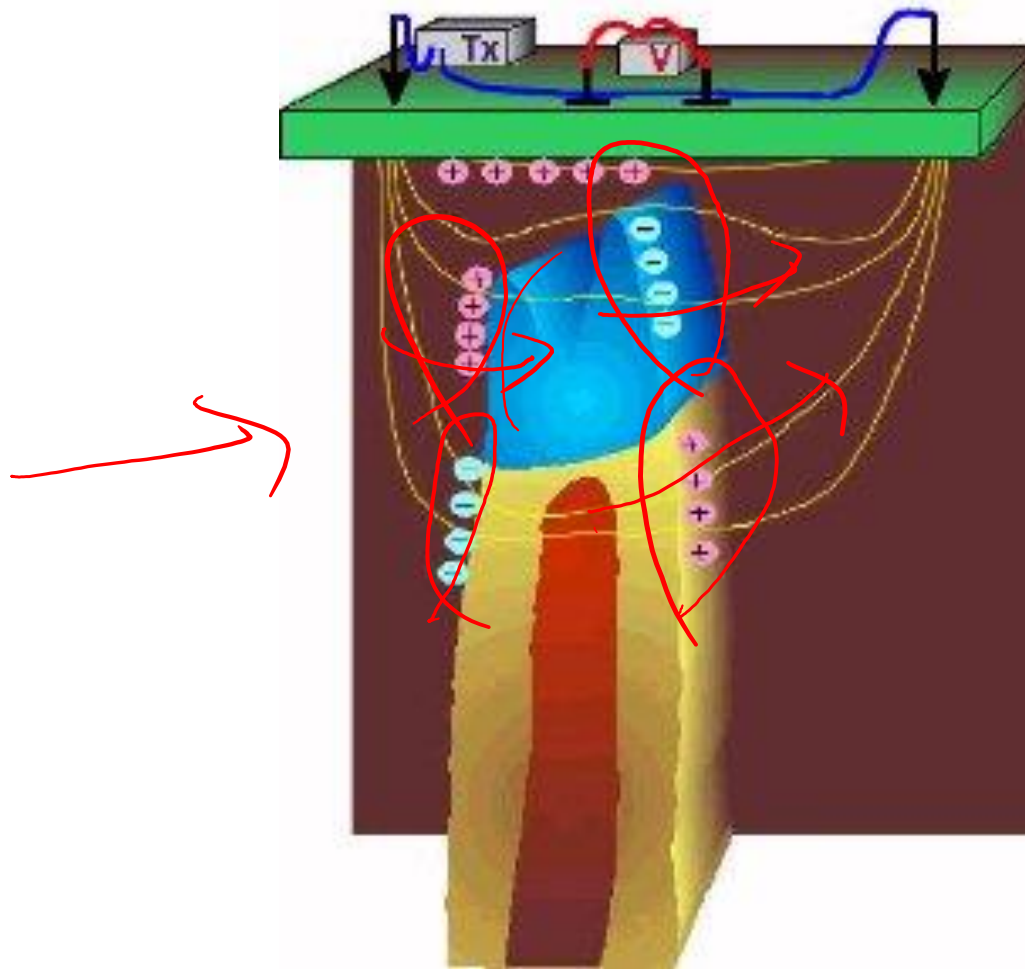


[https://gpg.geosci.xyz/content/DC\\_resistivity/DC\\_basic\\_principles\\_heterogeneous\\_earth.html](https://gpg.geosci.xyz/content/DC_resistivity/DC_basic_principles_heterogeneous_earth.html)

# Optional reading materials

- [https://gpg.geosci.xyz/content/DC resistivity/DC basic principles heterogeneous earth.html](https://gpg.geosci.xyz/content/DC_resistivity/DC_basic_principles_heterogeneous_earth.html)

# Understanding the charges



Elura Orebody Electrical resistivities

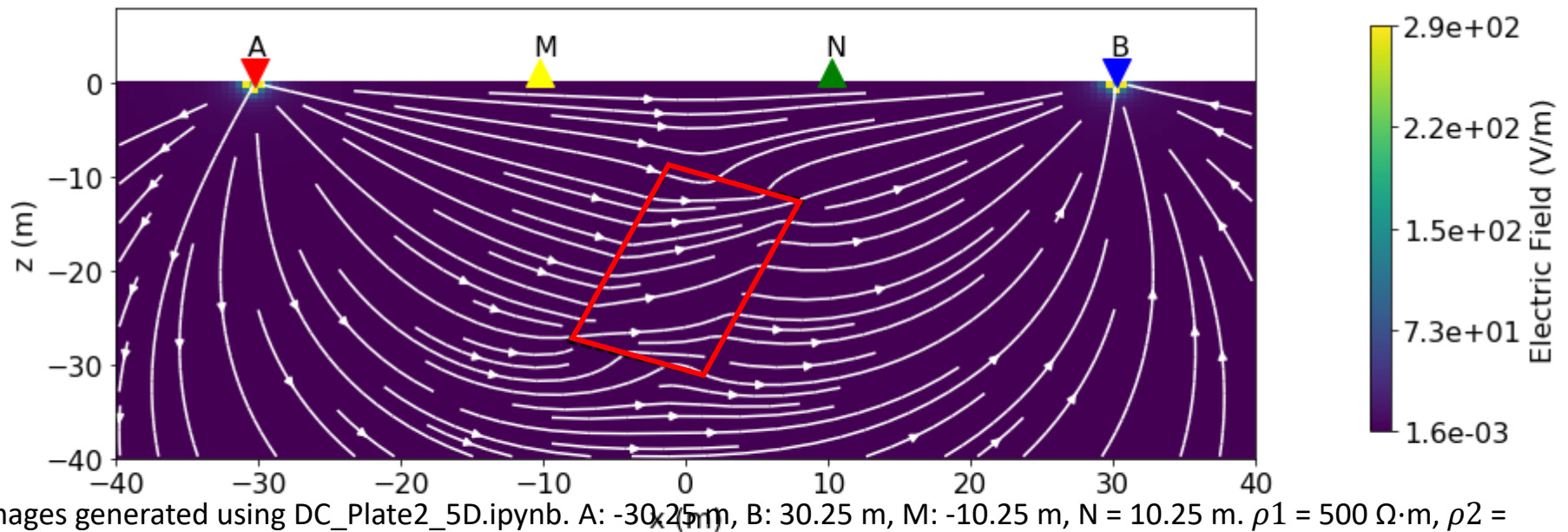
Rock Type	Ohm-m
Overburden	12
Host rocks	200
Gossan	420
Mineralization (pyritic)	0.6
Mineralization (pyrrhotite)	0.6

Credit: Doug Oldenburg, Seogi Kang and Linsey Heagy from UBC-GIF



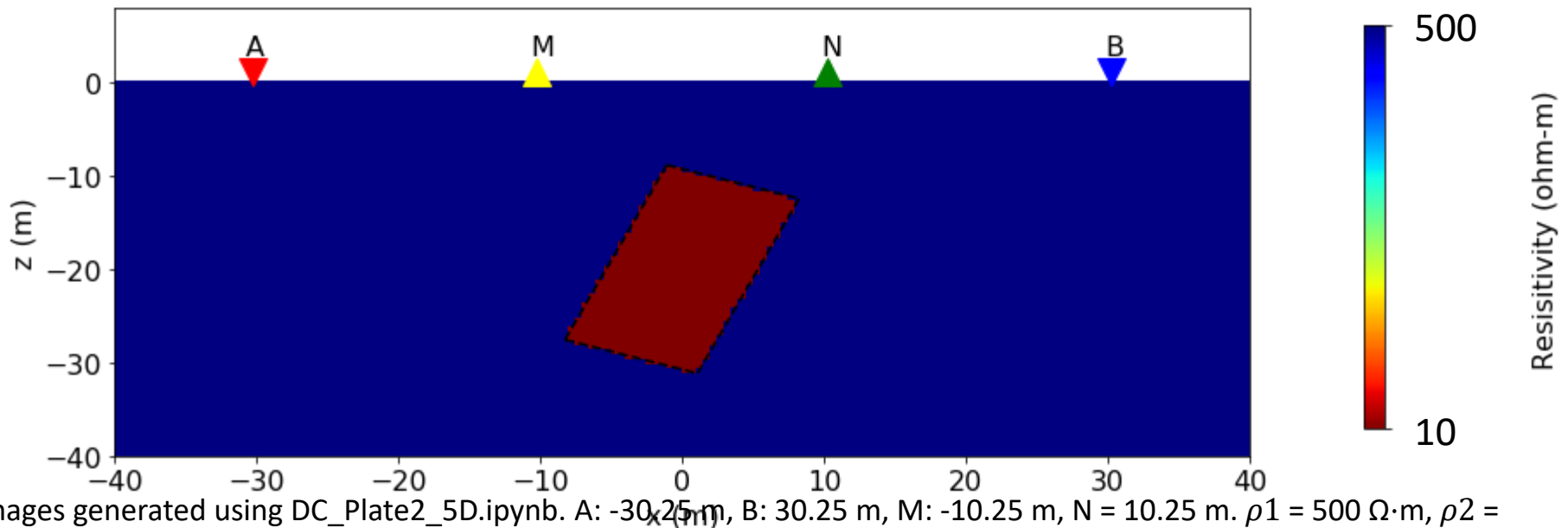
The rest are optional materials

# Understanding the change in directions



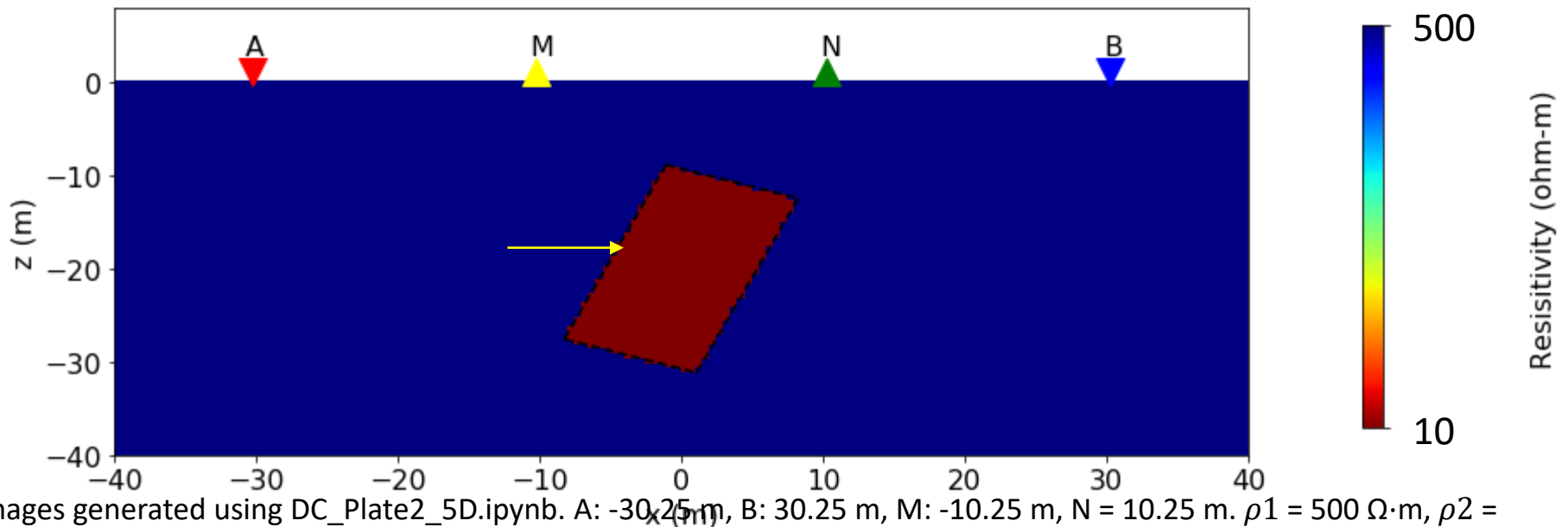
Images generated using DC\_Plate2\_5D.ipynb. A: -30.25 m, B: 30.25 m, M: -10.25 m, N = 10.25 m.  $\rho_1 = 500 \Omega \cdot \text{m}$ ,  $\rho_2 = 10 \Omega \cdot \text{m}$ .  $dx = 10$ ,  $dz = 20$ ,  $xc = 0$ ,  $zc = -20$ ,  $\theta = 21$ .

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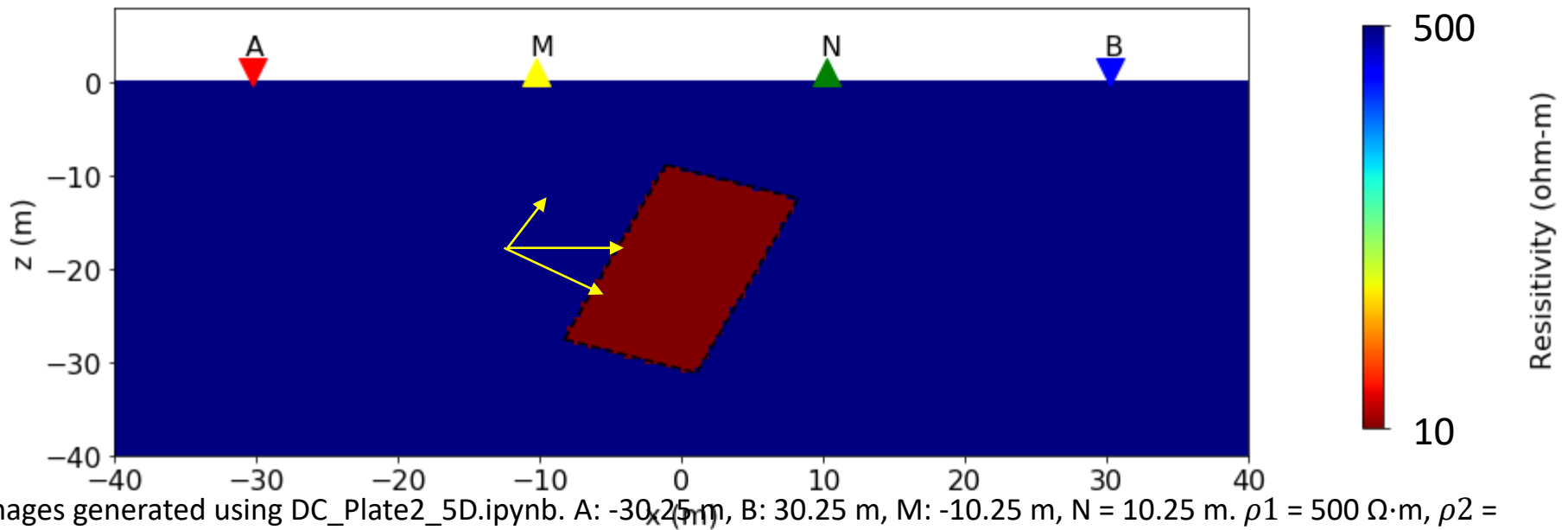
Images generated using DC\_Plate2\_5D.ipynb. A: -30.25 m, B: 30.25 m, M: -10.25 m, N = 10.25 m.  $\rho_1 = 500 \Omega \cdot \text{m}$ ,  $\rho_2 = 10 \Omega \cdot \text{m}$ .  $dx = 10$ ,  $dz = 20$ ,  $xc = 0$ ,  $zc = -20$ ,  $\theta = 21$ .

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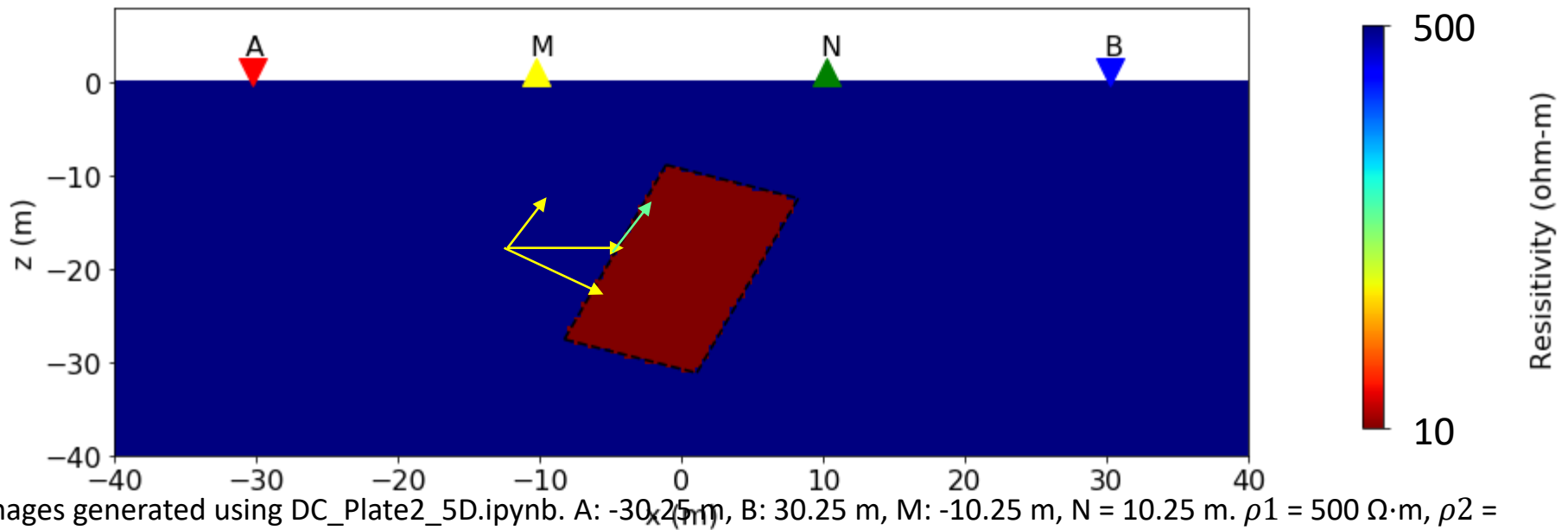
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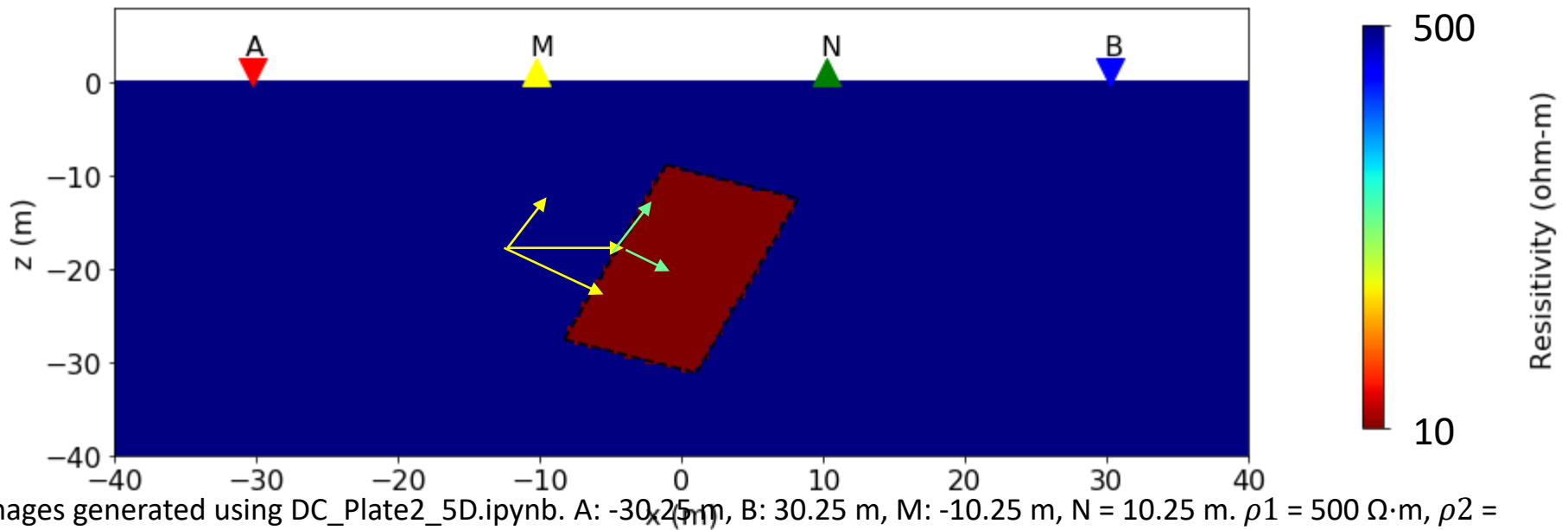
Images generated using DC\_Plate2\_5D.ipynb. A: -30.25 m, B: 30.25 m, M: -10.25 m, N = 10.25 m.  $\rho_1 = 500 \Omega \cdot \text{m}$ ,  $\rho_2 = 10 \Omega \cdot \text{m}$ .  $dx = 10$ ,  $dz = 20$ ,  $xc = 0$ ,  $zc = -20$ ,  $\theta = 21$ .

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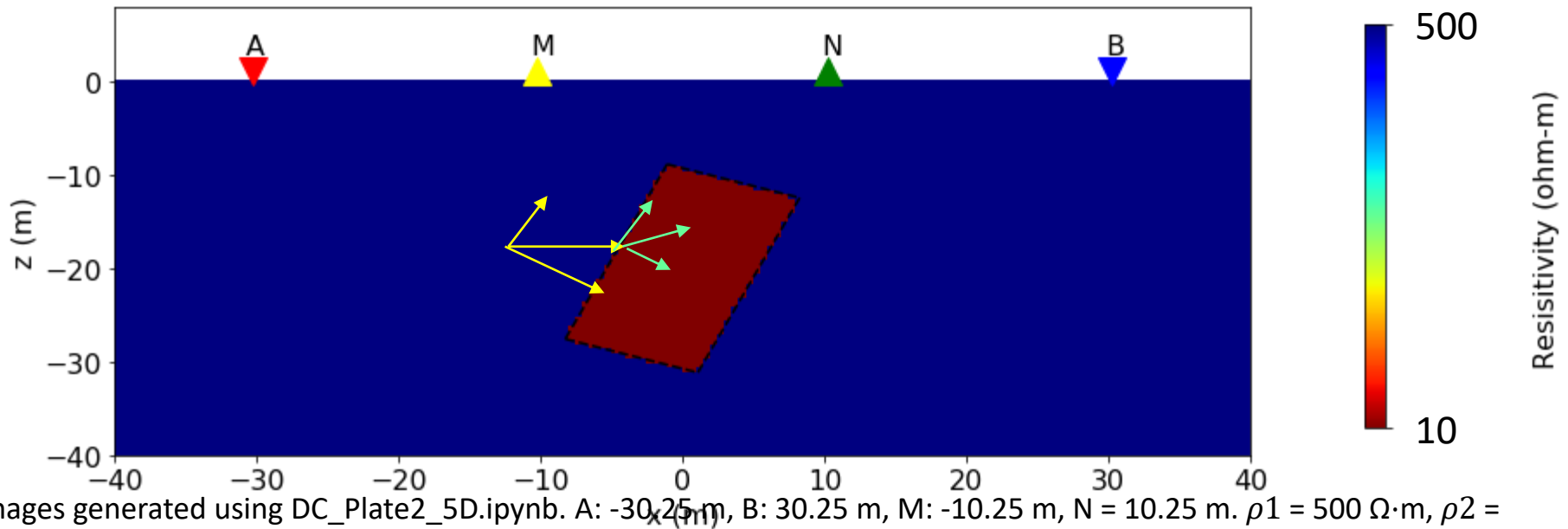
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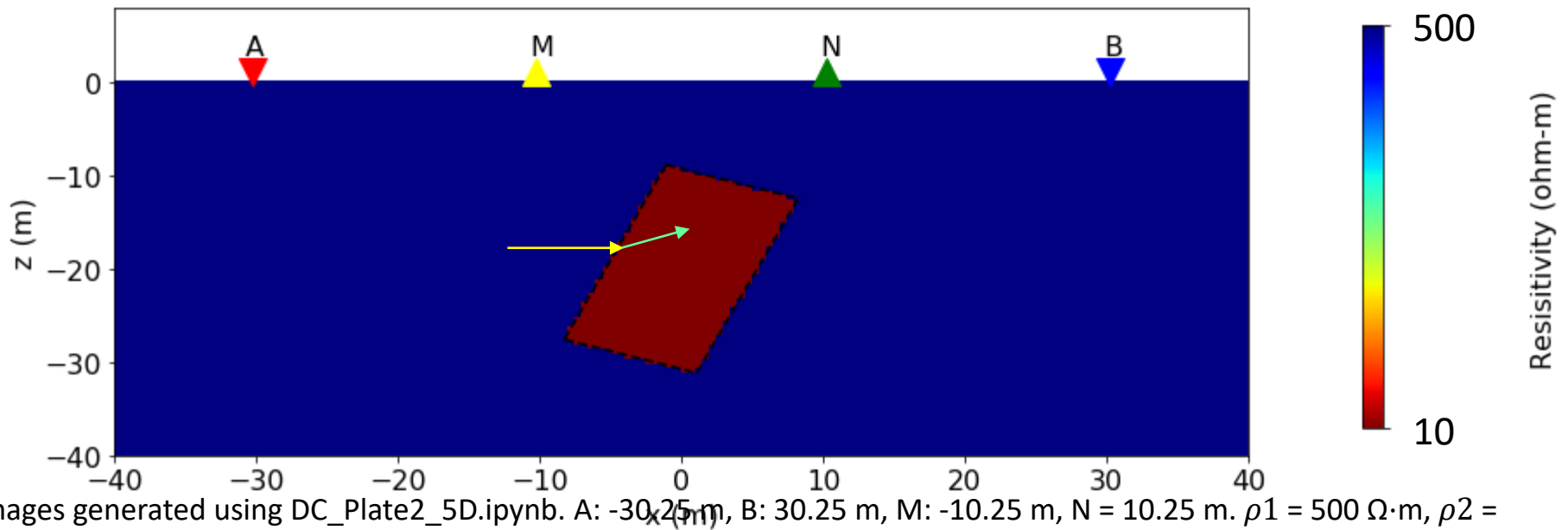
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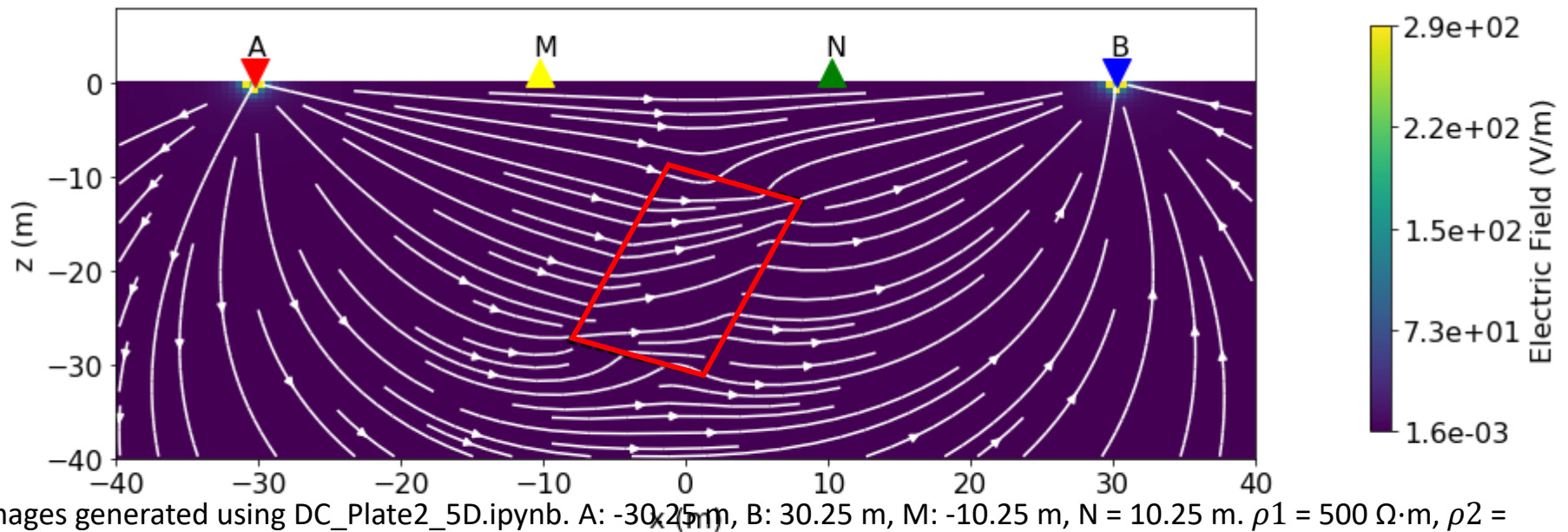


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