Lecture 16

Magnetotellurics

GEOL 4397: Electromagnetic Methods for Exploration

GEOL 6398: Special Problems

Jiajia Sun, Ph.D.

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UNIVERSITY of HOUSTON

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EARTH AND ATMOSPHERIC SCIENCES

Agenda

- Natural sources
- Quasi-static approximation
- Skin depth
- Apparent resistivity
- Phase

Man-made sources

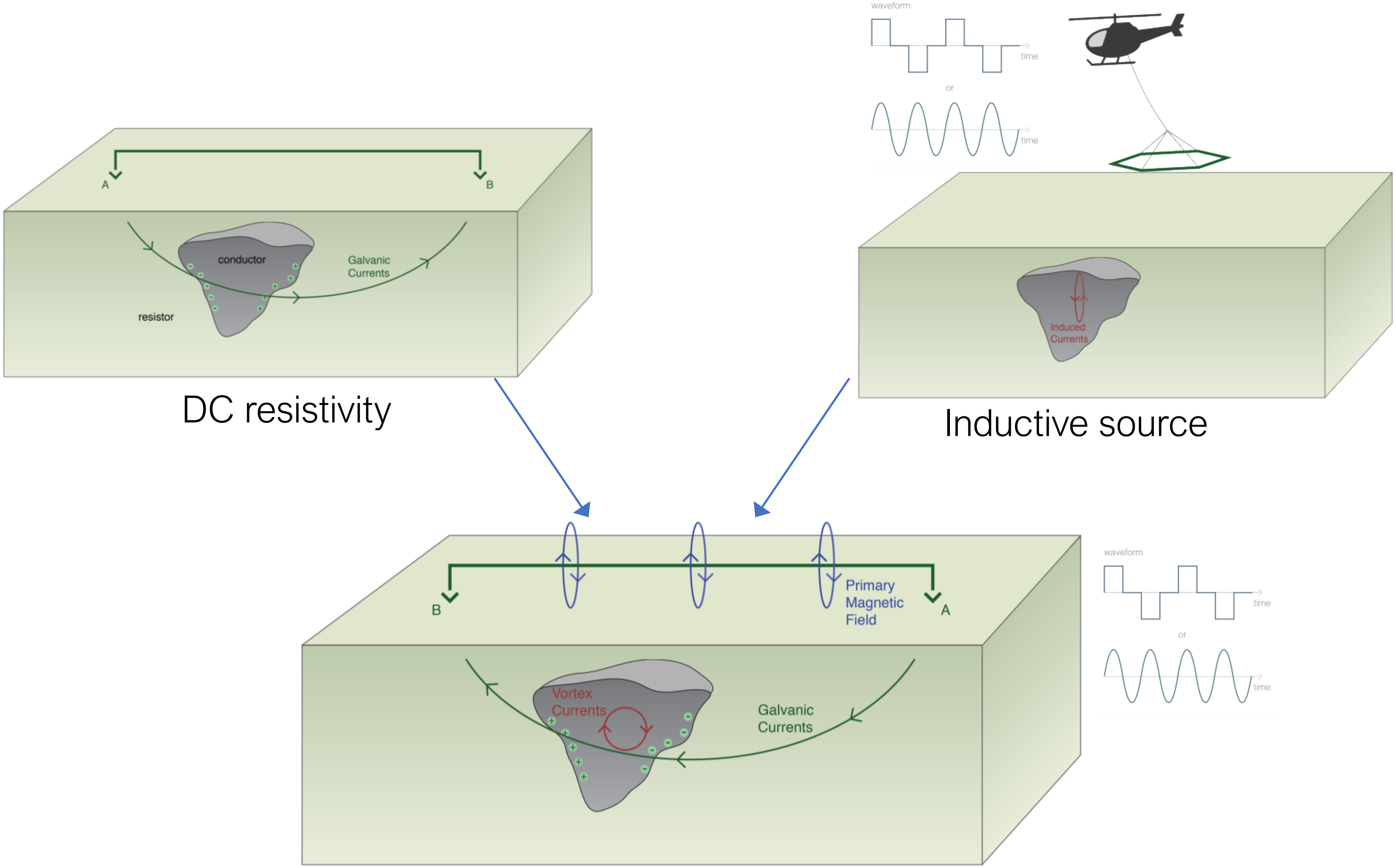

Image credit: Doug Oldenburg, Seogi Kang and Linsey Heagy from UBC-GIF

Natural sources: lightning

Lightning over Las Cruces, New Mexico
https://en.wikipedia.org/wiki/File:Lightning3.jpg

Benjamin Franklin's kite experiment showed that lightning is electrical in nature.

Lightning strike

- Sudden electrostatic discharge that occurs typically during a thunderstorm (between electrically charged regions of a cloud, between two clouds, or between a cloud and the ground).

https://en.wikipedia.org/wiki/Lightning

Natural sources: lightning

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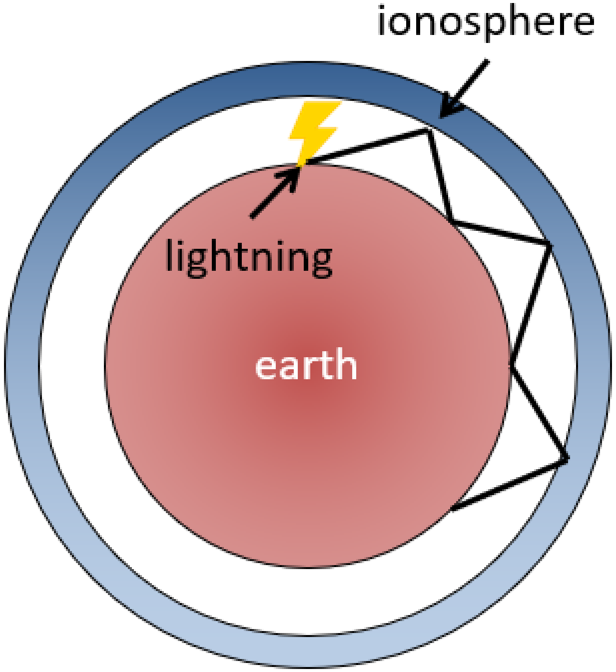
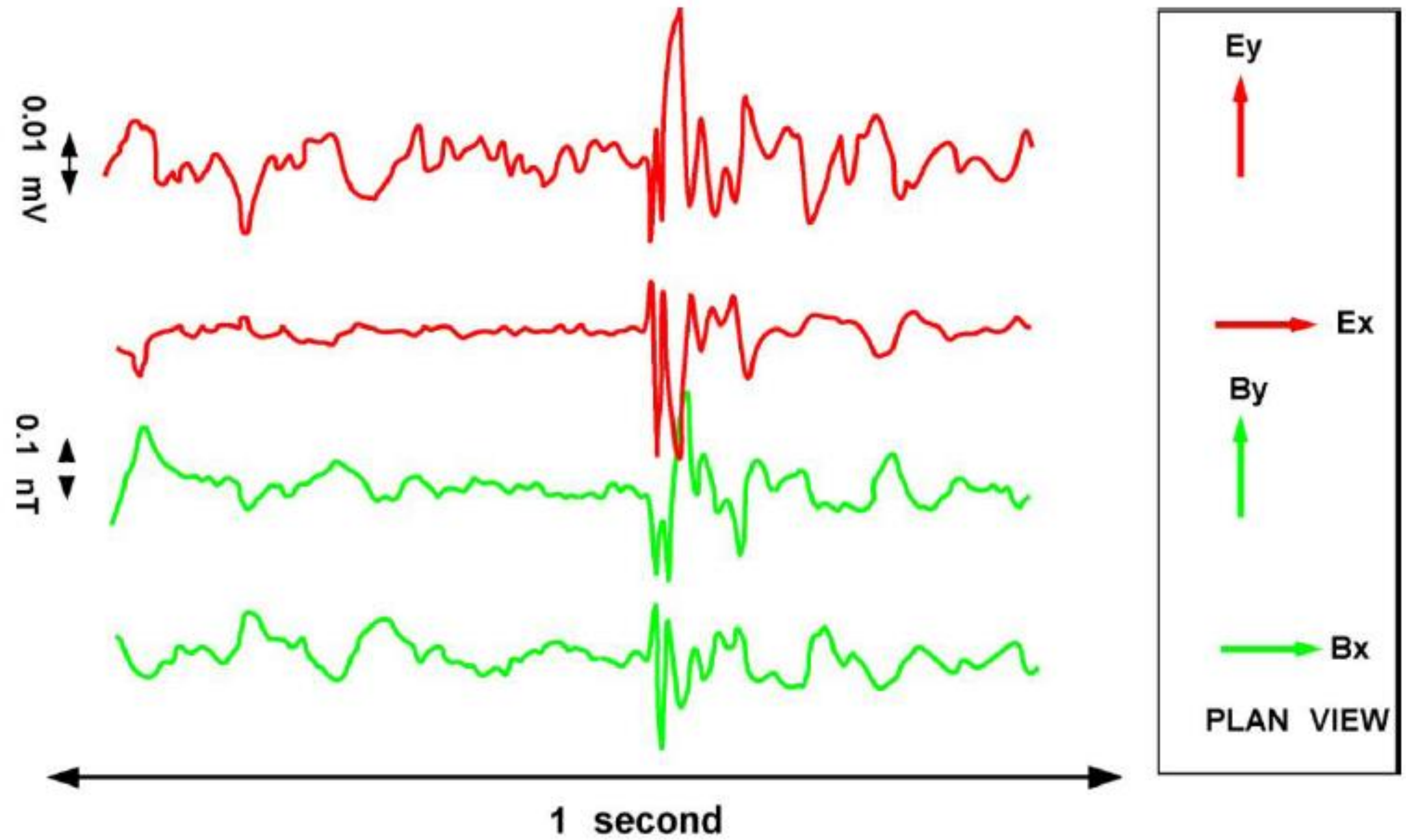


The EM fields generated by lightning events (otherwise known as sferics), propagate in a waveguide between the Earth's surface and the ionosphere (which is highly conductive). They fields travel far distances as plane waves.

https://em.geosci.xyz/content/geophysical_surveys/mt/index.html

Magnetotelluric signals from a lightning strike



MT data recorded at Carrizo Plain in California in 1994, during a study of the San Andreas Fault. It shows a typical “spheric” caused by a distant lightning strike that probably originated in the Amazon Basin. Data was recorded on an EMI MT-1 instrument.

Credit: Martyn Unsworth, University of Alberta, 2013

Natural sources: magnetosphere

- Magnetosphere
- Region of space surrounding Earth in which charged particles are manipulated by Earth's magnetic field.

https://en.wikipedia.org/wiki/Magnetosphere

Natural sources: magnetosphere

- Magnetosphere
- Region of space surrounding Earth in which charged particles are manipulated by Earth's magnetic field.
- In the space close to Earth, the magnetic field resembles a magnetic dipole. Further out, field lines can be significantly distorted by solar wind (a stream of charged particles released from the upper atmosphere of the Sun).

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- Mitigating or blocking the effects of solar radiation or cosmic radiation, that also protects all living organisms from potentially detrimental and dangerous consequences.

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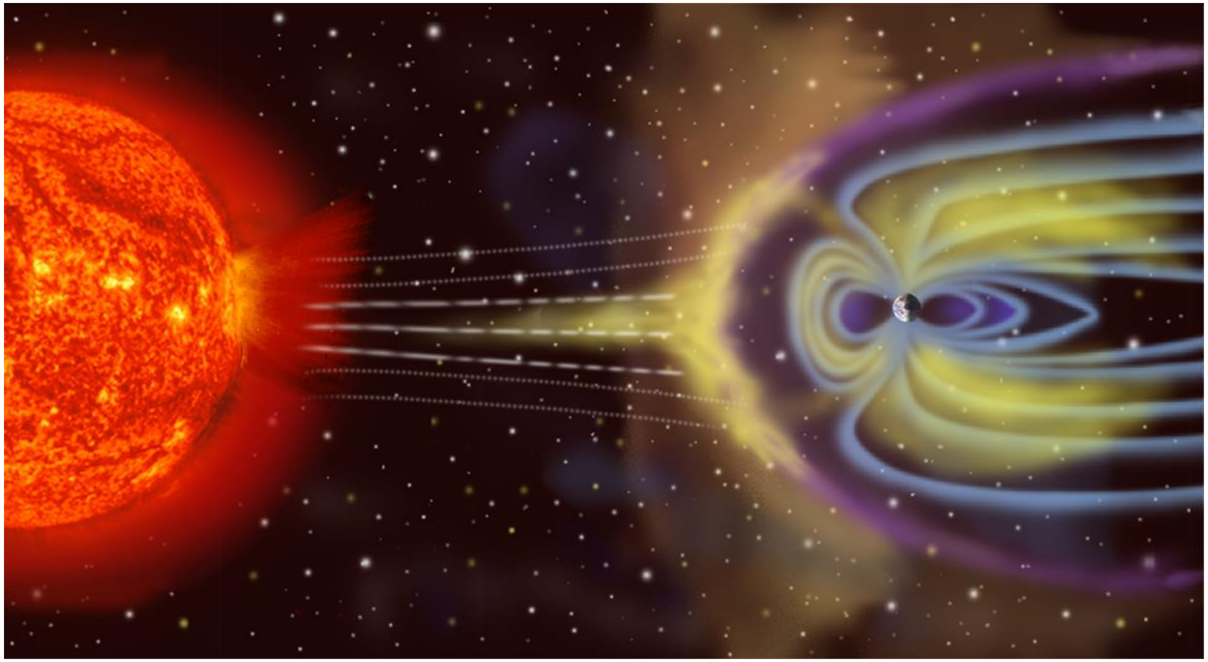
- Mitigating or blocking the effects of solar radiation or cosmic radiation, that also protects all living organisms from potentially detrimental and dangerous consequences.
- This term proposed by Thomas Gold in 1959 to explain how solar wind interacted with the Earth's magnetic field.

https://en.wikipedia.org/wiki/Magnetosphere



Artist's rendition of Earth's magnetosphere
https://en.wikipedia.org/wiki/Magnetosphere#/media/File:Magnetosphere_rendition.jpg

- Solar wind deflected by Earth's internal magnetic field to create the magnetosphere.
- Interaction between solar wind and Earth's magnetic field very complex.
- Changing magnetic fields from the magnetosphere can induce large electric currents in the ionosphere (a region of plasma with high electrical conductivity). Changes in these currents produce large changes in the magnetic field measured at the Earth's surface.

Credit: Martyn Unsworth, University of Alberta, 2013

Revisit Maxwell equations

$$\nabla \cdot \boldsymbol{d} = \rho_f$$ Gauss's law for electric fields

$$\nabla \cdot \boldsymbol{b} = 0$$ Gauss's law for magnetic fields

$$\nabla \times \boldsymbol{e} = -\frac{\partial \boldsymbol{b}}{\partial t}$$ Faraday's law

$$\nabla \times \mathbf{h} = \boldsymbol{j}_f + \frac{\partial \mathbf{d}}{\partial t}$$ Ampere-Maxwell equation

Constitutive relationships

$$\boldsymbol{j}_f = \sigma \boldsymbol{e} \qquad \boldsymbol{b} = \mu \boldsymbol{h} \qquad \boldsymbol{d} = \varepsilon \boldsymbol{e}$$

σ: electrical conductivity μ: magnetic permeability ε: dielectric permittivity

Revisit Maxwell equations

$$\nabla \cdot \boldsymbol{d} = \rho_f$$ Gauss's law for electric fields

$$\nabla \cdot \boldsymbol{b} = 0$$ Gauss's law for magnetic fields

$$\boxed{\begin{aligned} \nabla \times \boldsymbol{e} &= -\frac{\partial \boldsymbol{b}}{\partial t} \\ \nabla \times \mathbf{h} &= \boldsymbol{j}_f + \frac{\partial \mathbf{d}}{\partial t} \end{aligned}}$$

Faraday's law

Ampere-Maxwell equation

First order equations

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Revisit Maxwell equations

- First order equations

$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$

$$\nabla \times \mathbf{h} = \mathbf{j} + \frac{\partial \mathbf{d}}{\partial t}$$

$$\mathbf{j} = \sigma \mathbf{e}$$

$$\mathbf{b} = \mu \mathbf{h}$$

$$\mathbf{d} = \varepsilon \mathbf{e}$$

- Second order equations

$$\nabla^2 \mathbf{h} - \underbrace{\mu\sigma \frac{\partial \mathbf{h}}{\partial t}}_{\text{diffusion}} - \underbrace{\mu\epsilon \frac{\partial^2 \mathbf{h}}{\partial t^2}}_{\text{wave propagation}} = 0$$

* Same equation holds for E

From first order equations to second order equations (Optional)

Start from $\nabla \times \mathbf{h} = \boldsymbol{j}_f + \frac{\partial \mathbf{d}}{\partial t}$

Perform curl operation on both sides:

$$\nabla \times (\nabla \times \mathbf{h}) = \nabla \times (\boldsymbol{j}_f + \frac{\partial \mathbf{d}}{\partial t})$$

The left hand side: $\nabla \times (\nabla \times \mathbf{h}) = \nabla(\nabla \cdot \boldsymbol{h}) - \nabla^2 \boldsymbol{h} = -\nabla^2 \boldsymbol{h}$

The RHS term: $\nabla \times \left(\boldsymbol{j}_f + \frac{\partial \mathbf{d}}{\partial t}\right) = \nabla \times \boldsymbol{j}_f + \nabla \times \left(\frac{\partial \boldsymbol{d}}{\partial t}\right) = \nabla \times \boldsymbol{j}_f + \frac{\partial (\nabla \times \boldsymbol{d})}{\partial t}$

Recall $\boldsymbol{j}_f = \sigma \boldsymbol{e}$

Thus,

$$\nabla \times \boldsymbol{j}_f = \sigma \nabla \times \boldsymbol{e} = -\sigma \frac{\partial \boldsymbol{b}}{\partial t} = -\mu\sigma \frac{\partial \boldsymbol{h}}{\partial t}$$

$$\nabla \times \boldsymbol{d} = \varepsilon \nabla \times \boldsymbol{e} = -\varepsilon \frac{\partial \boldsymbol{b}}{\partial t} = -\mu\varepsilon \frac{\partial \boldsymbol{h}}{\partial t}$$

Thus, $\nabla^2 \boldsymbol{h} - \mu\sigma \frac{\partial \boldsymbol{h}}{\partial t} - \mu\varepsilon \frac{\partial^2 \boldsymbol{h}}{\partial t^2} = 0$

From time domain to frequency domain

$$\nabla^2 \mathbf{h} - \underbrace{\mu\sigma \frac{\partial \mathbf{h}}{\partial t}}_{\text{diffusion}} - \underbrace{\mu\epsilon \frac{\partial^2 \mathbf{h}}{\partial t^2}}_{\text{wave propagation}} = 0$$

Apply Fourier transform

$$\nabla^2 \boldsymbol{H} - i\omega\mu\sigma\boldsymbol{H} + \omega^2\mu\epsilon\boldsymbol{H} = 0$$

$$\boxed{\begin{array}{c} \nabla^2 \boldsymbol{H} + k^2\boldsymbol{H} = 0 \\ \text{where } k^2 = \omega^2\mu\epsilon - i\omega\mu\sigma \end{array}}$$

Quasi-static approximation

$$\nabla^2 \boldsymbol{H} + k^2 \boldsymbol{H} = 0$$

$$k^2 = \omega^2 \mu \epsilon - i\omega\mu\sigma$$

$$k^2 = \omega\mu(\omega\epsilon - i\sigma)$$

Quasi-static approximation

$$\nabla^2 \boldsymbol{H} + k^2 \boldsymbol{H} = 0$$

$$k^2 = \omega^2 \mu \epsilon - i\omega\mu\sigma$$

$$k^2 = \omega\mu(\omega\epsilon - i\sigma)$$

Let us compare $\omega\epsilon$ and σ

By computing $\frac{\omega\epsilon}{\sigma}$

Quasi-static approximation

$$\nabla^2 \boldsymbol{H} + k^2 \boldsymbol{H} = 0$$

$$k^2 = \omega^2 \mu \epsilon - i\omega\mu\sigma$$

$$k^2 = \omega\mu(\omega\epsilon - i\sigma)$$

Let us compare $\omega\epsilon$ and σ

By computing $\frac{\omega\epsilon}{\sigma}$

Compute $\frac{\omega\epsilon}{\sigma}$

$$\sigma = 10^{-4} S/m$$

$$f = 10^4 Hz$$

$$\epsilon = 8.85 \times 10^{-12} F/m$$

Quasi-static approximation

$$\nabla^2 \boldsymbol{H} + k^2 \boldsymbol{H} = 0$$

$$k^2 = \omega^2 \mu\epsilon - i\omega\mu\sigma$$

$$k^2 = \omega\mu(\omega\epsilon - i\sigma)$$

Let us compare $\omega\epsilon$ and σ

By computing $\frac{\omega\epsilon}{\sigma}$

Compute $\frac{\omega\epsilon}{\sigma}$

$$\sigma = 10^{-4} S/m$$

$$f = 10^4 Hz$$

$$\epsilon = 8.85 \times 10^{-12} F/m$$

Even with the above parameter values, $\frac{\omega\epsilon}{\sigma} \ll 1$

Therefore, $k^2 = -i\omega\mu\sigma$

Plane waves in a homogeneous media: frequency domain

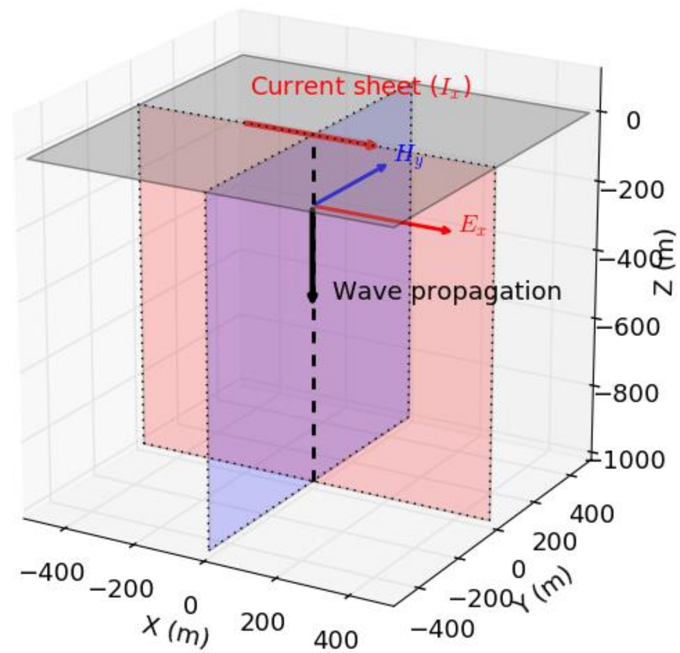
$$\nabla^2 \boldsymbol{H} + k^2 \boldsymbol{H} = 0$$

$$k^2 = -i\omega\mu\sigma$$

$$k = \sqrt{-i\omega\mu\sigma} = (1-i)\sqrt{\frac{\omega\mu\sigma}{2}}$$

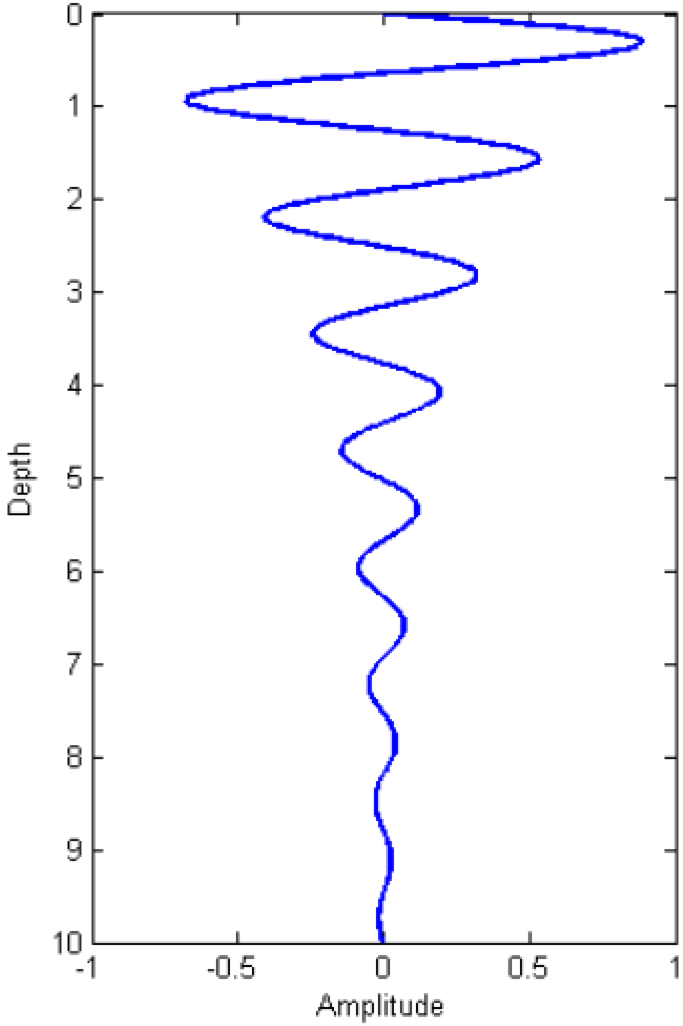
$$\equiv \alpha - i\beta$$

Plane wave solution

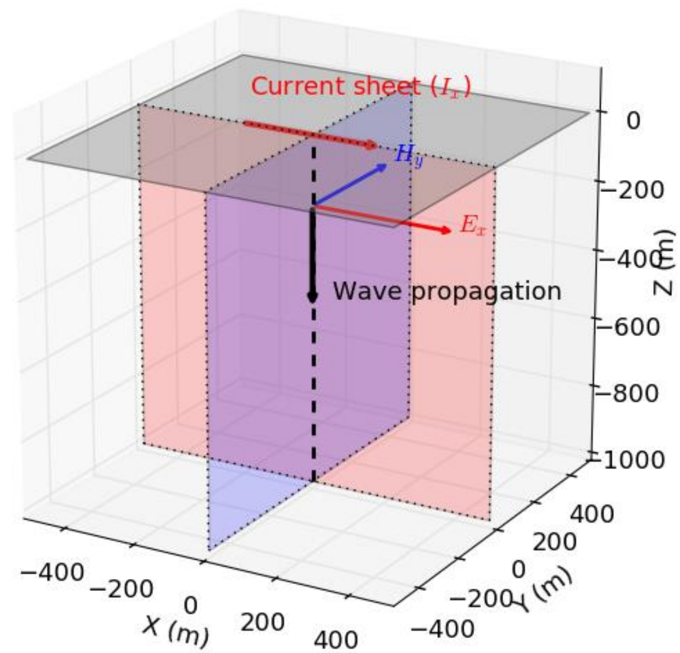


$$\mathbf{H} = \mathbf{H_0} e^{\underbrace{-\alpha z}_{\text{attenuation}}} e^{-i\underbrace{(\beta z - \omega t)}_{\text{phase}}}$$

Plane waves in a homogeneous media: frequency domain



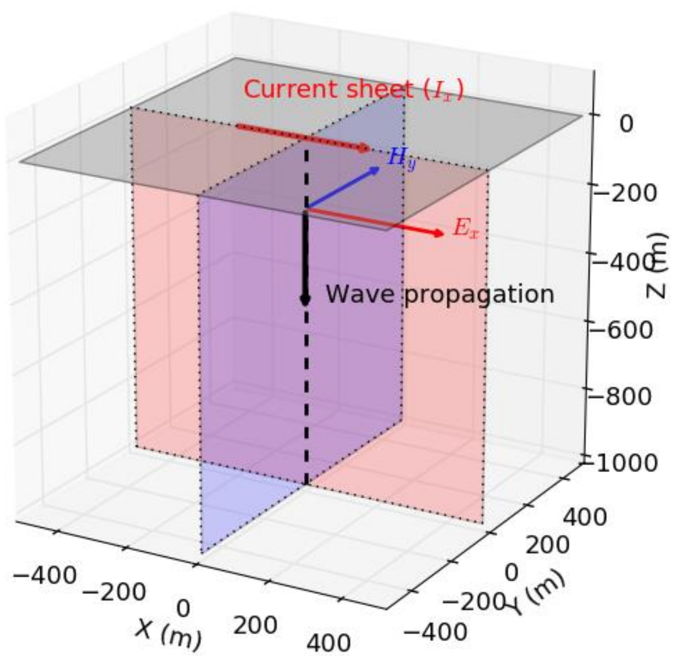
Plane wave solution

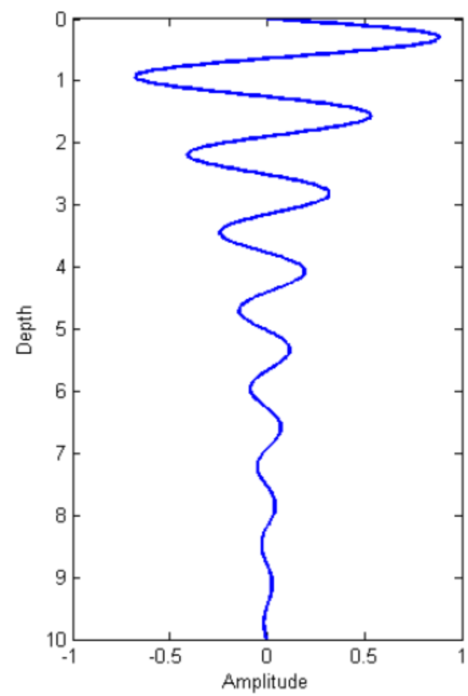


$$\mathbf{H} = \mathbf{H_0} \underbrace{e^{-\alpha z}}_{\text{attenuation}} e^{-i\underbrace{(\beta z - \omega t)}_{\text{phase}}}$$

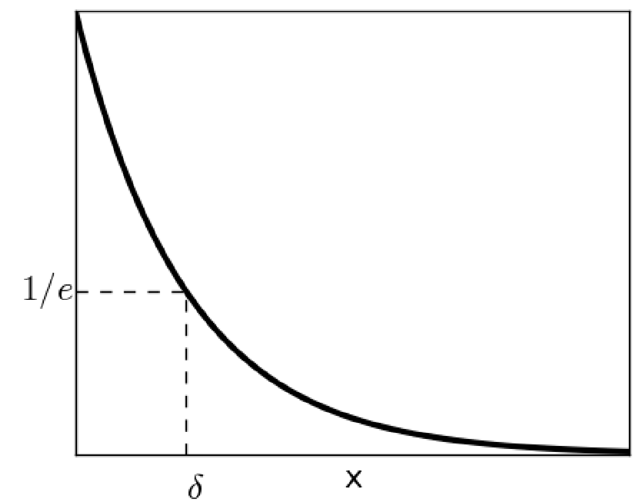
Skin depth

Plane wave solution





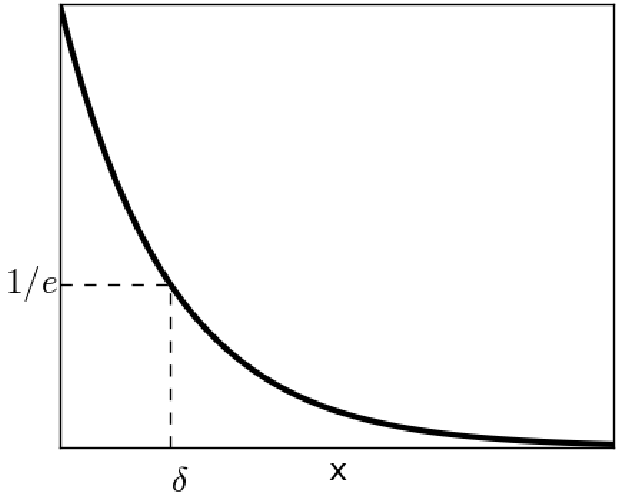
δ : skin depth



$$\mathbf{H} = \mathbf{H_0} \underbrace{e^{-\alpha z}}_{\text{attenuation}} \underbrace{e^{-i(\beta z - \omega t)}}_{\text{phase}}$$

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = 503\sqrt{\frac{1}{\sigma f}}$$

Skin depth



δ : skin depth

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = 503\sqrt{\frac{1}{\sigma f}}$$

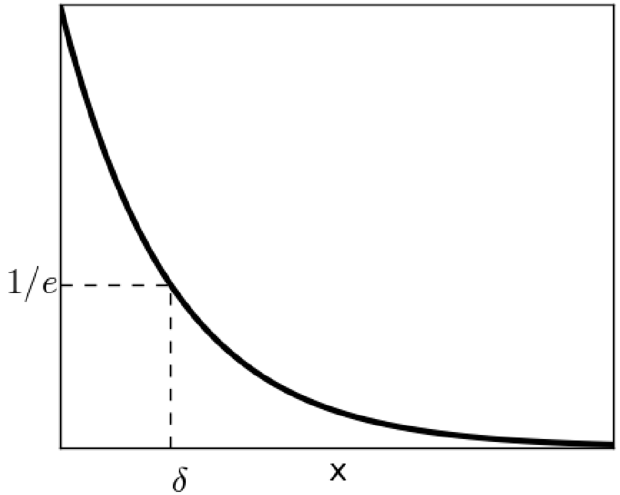
$$\mu = 4\pi \times 10^{-7} H/m$$

In-class exercise:

Calculate the skin depths

Type	σ (S/m)	δ(1 Hz)	δ(1 kHz)	δ(1 MHz)
Air	0			
Sea water	3.3			
Igneous	10^{-4}			
Sedimen ts	10^{-2}			

Skin depth



δ : skin depth

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = 503\sqrt{\frac{1}{\sigma f}}$$

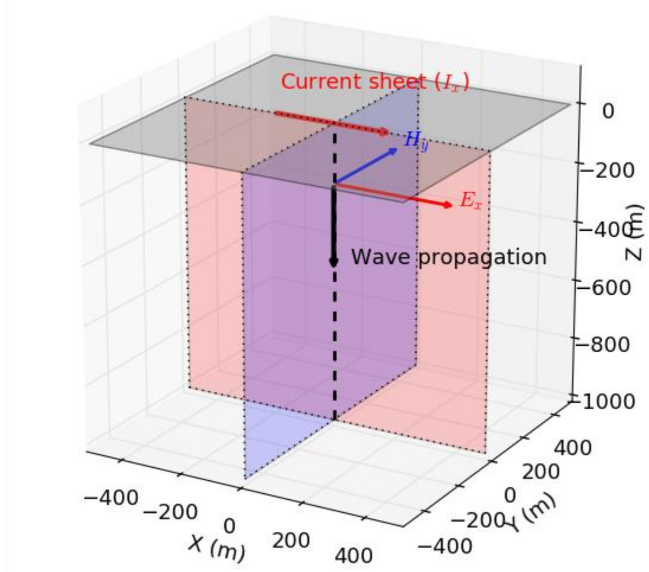
$$\mu = 4\pi \times 10^{-7} H/m$$

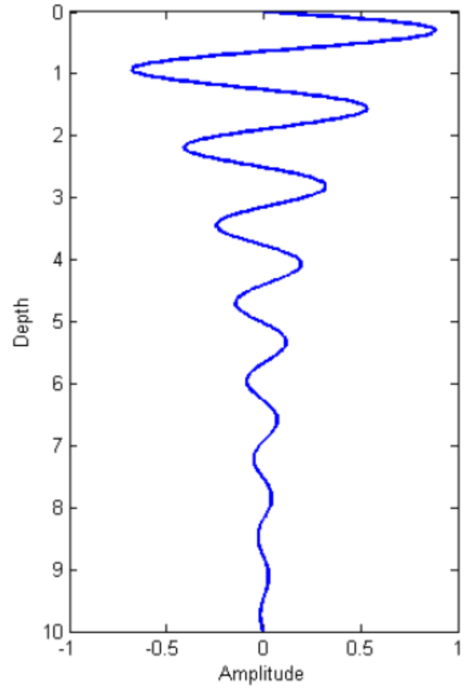
In-class exercise:

Calculate the skin depths

Type	σ (S/m)	δ(1 Hz)	δ(1 kHz)	δ(1 MHz)
Air	0	∞	∞	∞
Sea water	3.3	277	8.76	0.277
Igneous	10^{-4}	50300	1590	50.3
Sedimen ts	10^{-2}	5030	159	5.03

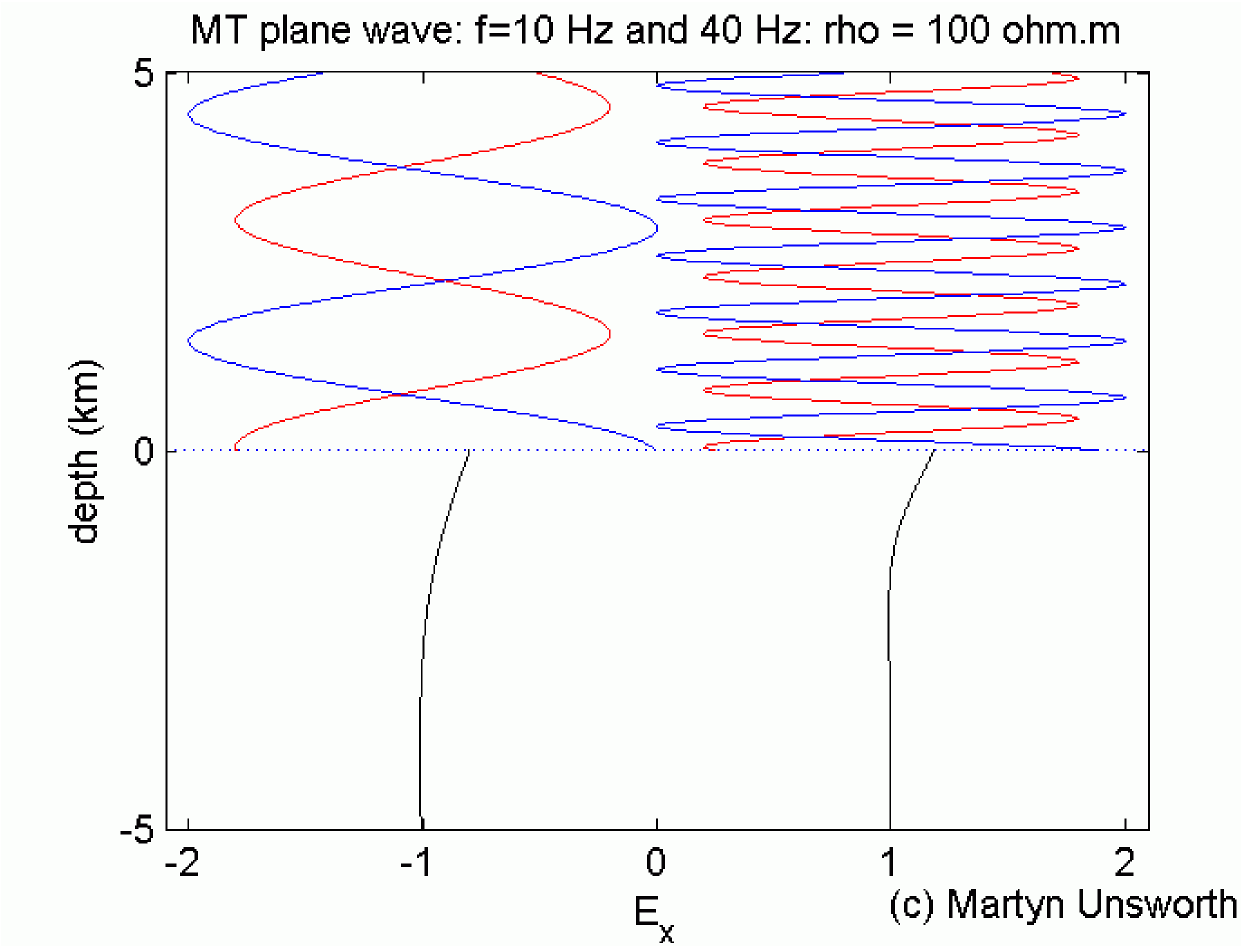
Oscillation





$$\mathbf{H} = \mathbf{H_0} \underbrace{e^{-\alpha z}}_{\text{attenuation}} \underbrace{e^{-i(\beta z - \omega t)}}_{\text{phase}}$$

- At any depth, if I measured the magnetic field as a function of time and plotted them up, it would be a sinusoidal waveform.
- At any time, the magnetic field in spatial domain is also sinusoidal but with decaying amplitudes.



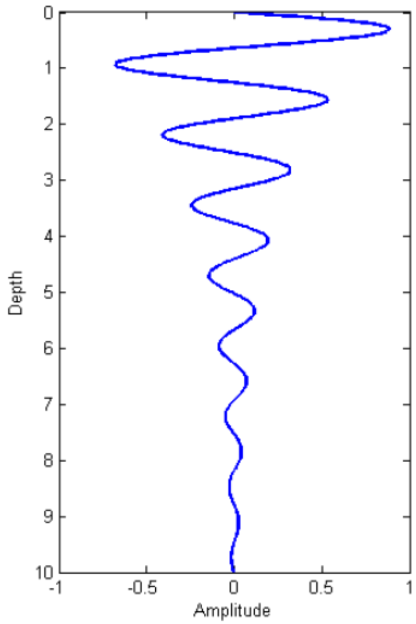
These EM waves travel downwards (blue) to the Earth's surface and most of the energy is reflected upwards (red). Some energy enters the Earth and diffuses downwards. The depth of penetration in the Earth is controlled by the skin depth phenomena. Depth of penetration decreases as the EM signal frequency increases.

https://sites.ualberta.ca/~unsworth/MT/MT.html

How to estimate the subsurface resistivity?

$$\mathbf{H} = \mathbf{H_0} \underbrace{e^{-\alpha z}}_{\text{attenuation}} \underbrace{e^{-i(\beta z - \omega t)}}_{\text{phase}}$$

$$\alpha = \beta = \sqrt{\frac{\omega \mu \sigma}{2}}$$



- Recall that the amplitude at any depth is a function of conductivity.

How to estimate the subsurface resistivity?

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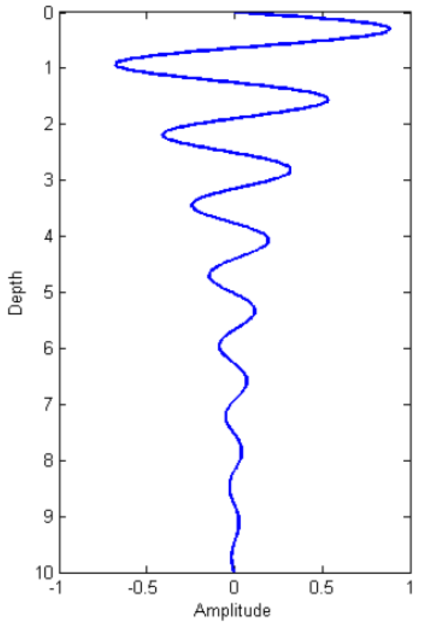
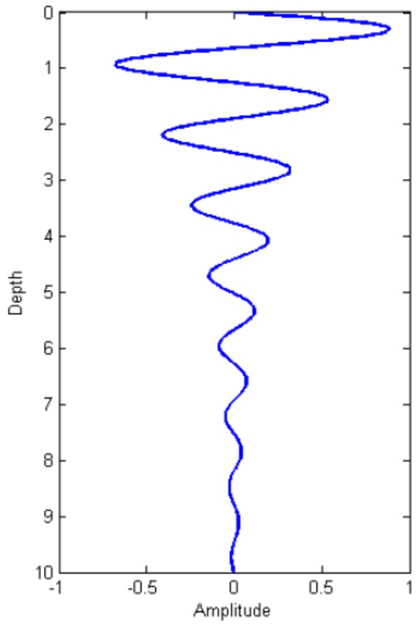


- Recall that the amplitude at any depth is a function of conductivity.
- So, we could measure magnetic field at some depth, and calculate the conductivity based on the measured amplitude of the waveform.

How to estimate the subsurface resistivity?

$$\mathbf{H} = \mathbf{H_0} e^{\underbrace{-\alpha z}_{\text{attenuation}}} e^{\underbrace{-i(\beta z - \omega t)}_{\text{phase}}}$$

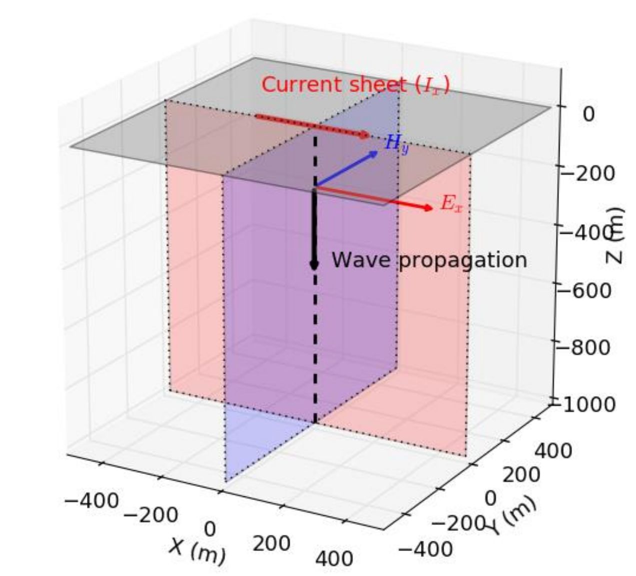
$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$



- Recall that the amplitude at any depth is a function of conductivity.
- So, we could measure magnetic field at some depth, and calculate the conductivity based on the measured amplitude of the waveform.
- What is wrong with this approach?

How to estimate the subsurface resistivity?

$$\mathbf{H} = \mathbf{H_0} e^{-\alpha z} e^{-i(\beta z - \omega t)}$$



Suppose **E** field is polarized in x direction

Then we only have E_x, H_y component

$$E_x = E_x(0) e^{-\alpha z} e^{-i(\beta z - \omega t)}$$
$$H_y = H_y(0) e^{-\alpha z} e^{-i(\beta z - \omega t)}$$

Recall that $\nabla \times \boldsymbol{H} = \sigma \boldsymbol{E}$

For 1D, it becomes

$$-\frac{\partial H_y}{\partial z} = \sigma E_x$$

Therefore, $(\alpha + i\beta) H_y(0) e^{-\alpha z} e^{-i(\beta z - \omega t)} = \sigma E_x(0) e^{-\alpha z} e^{-i(\beta z - \omega t)}$

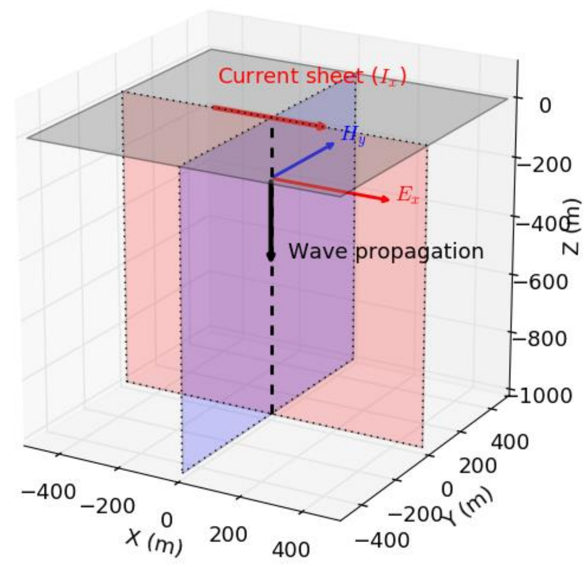
Therefore,

$$\frac{E_x(0)}{H_y(0)} = \frac{a + i\beta}{\sigma} = \sqrt{\frac{\omega\mu}{2\sigma}} + i\sqrt{\frac{\omega\mu}{2\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} e^{i\frac{\pi}{4}}$$

How to estimate the subsurface resistivity?

Suppose **E** field is polarized in x direction
Then we only have E_x, H_y component

$$E_x = E_x(0)e^{-\alpha z}e^{-i(\beta z-\omega t)}$$
$$H_y = H_y(0)e^{-\alpha z}e^{-i(\beta z-\omega t)}$$



Therefore,

$$\frac{E_x(0)}{H_y(0)} = \frac{a+i\beta}{\sigma} = \sqrt{\frac{\omega\mu}{2\sigma}} + i\sqrt{\frac{\omega\mu}{2\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}e^{i\frac{\pi}{4}}$$

Define $Z(\omega) = \frac{E_x(0)}{H_y(0)} = \sqrt{\frac{\omega\mu}{\sigma}}e^{i\frac{\pi}{4}}$

Then $|Z(\omega)| = \sqrt{\frac{\omega\mu}{\sigma}}$

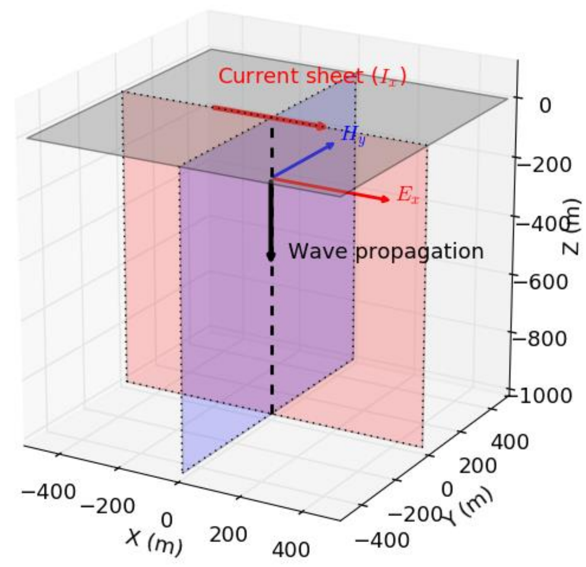
$$|Z(\omega)|^2 = \frac{\omega\mu}{\sigma}$$

Then, $\rho = \frac{1}{\sigma} = \frac{1}{\omega\mu}|Z(\omega)|^2$

How to estimate the subsurface resistivity?

$$\rho(\omega) = \frac{1}{\sigma} = \frac{1}{\omega\mu}|Z(\omega)|^2$$

Where $$Z(\omega) = \frac{E_x(0,w)}{H_y(0,w)}$$



$Z(\omega)$ is termed impedance

From impedance, we can estimate the subsurface resistivity
For a homogeneous Earth, it gives the true resistivity
For a non-homogeneous Earth, it is apparent resistivity

Apparent resistivity

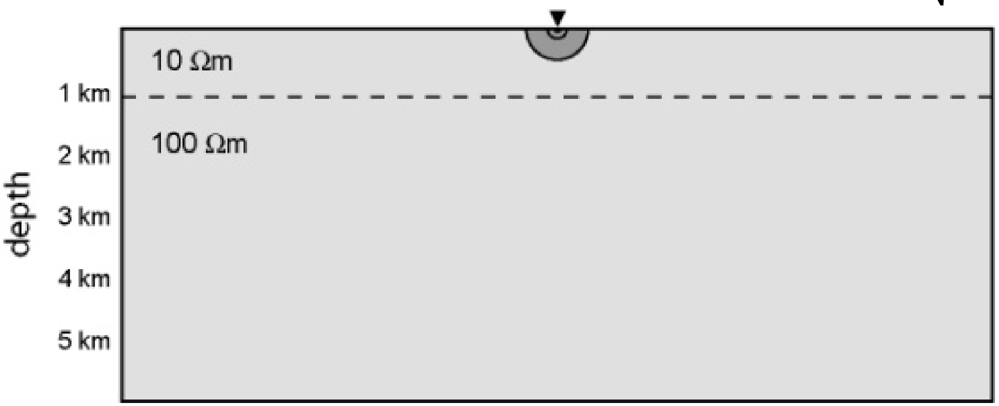
- Can be considered as average resistivity of the Earth down to skin depth

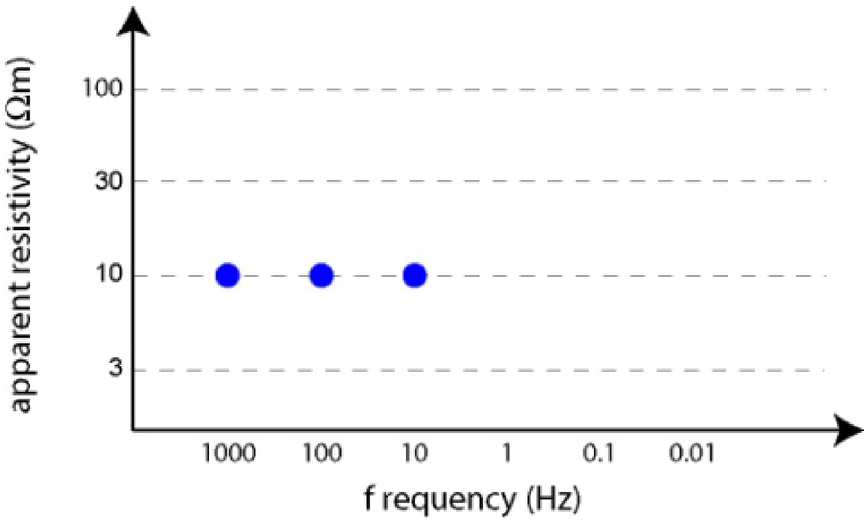
$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = 503\sqrt{\frac{1}{\sigma f}}$$

- At lower frequency, it is the average resistivity of the shallower part of the Earth
- At higher frequency, it is the average resistivity of the Earth down to a greater depth.

Apparent resistivity: $f = 10$ Hz

$$\delta = 503\sqrt{\frac{1}{\sigma f}} = 500\, m$$



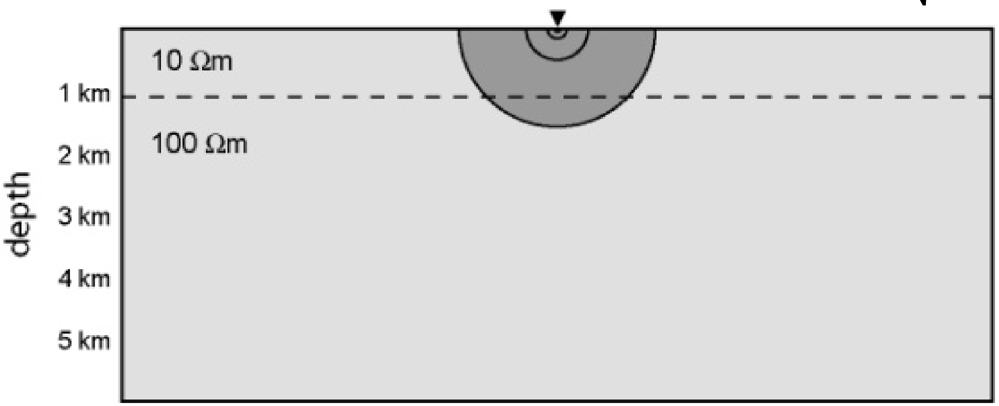


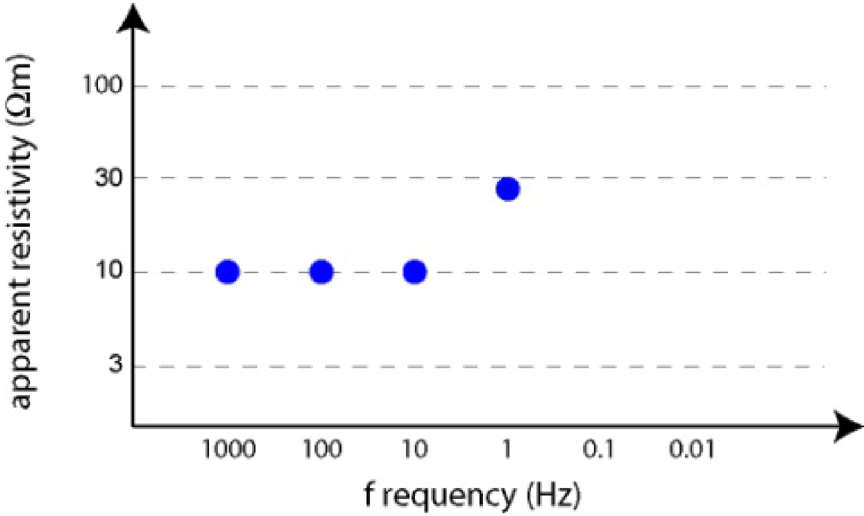
- At higher frequency, the skin depth is much smaller than the thickness of the layer
- Average resistivity is simply the resistivity of the upper layer

Credit: Martyn Unsworth, University of Alberta, 2013

Apparent resistivity: $f = 1$ Hz

$$\delta = 503\sqrt{\frac{1}{\sigma f}} = 1580\,m$$



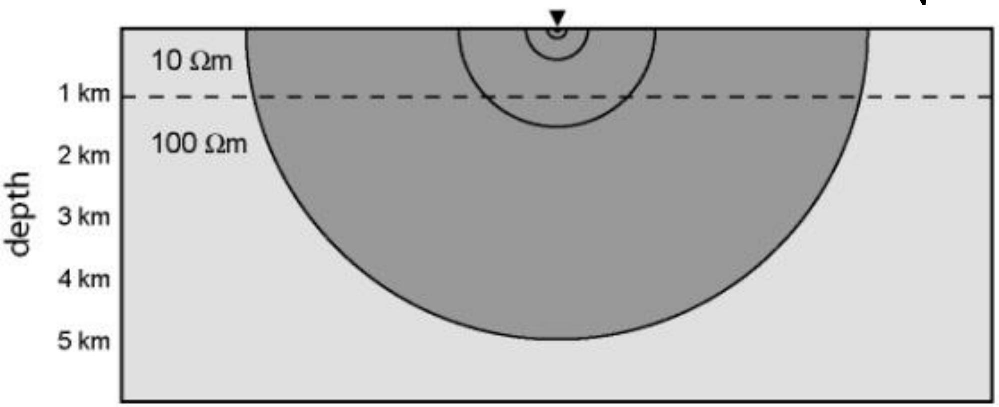


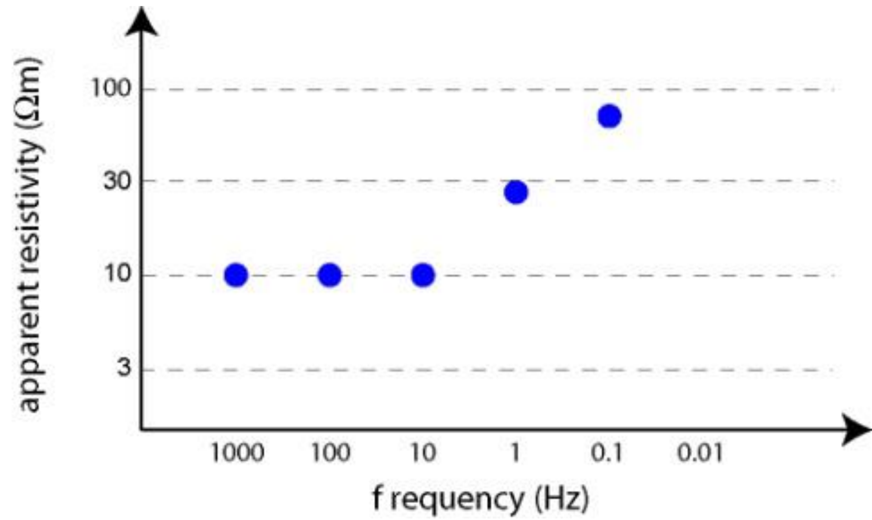
- The EM signals are now just sampling the lower layer.
- Average resistivity begins to increase as the lower resistive layer is being sampled.

Credit: Martyn Unsworth, University of Alberta, 2013

Apparent resistivity: $f = 0.1$ Hz

$$\delta = 503\sqrt{\frac{1}{\sigma f}} = 5000\,m$$



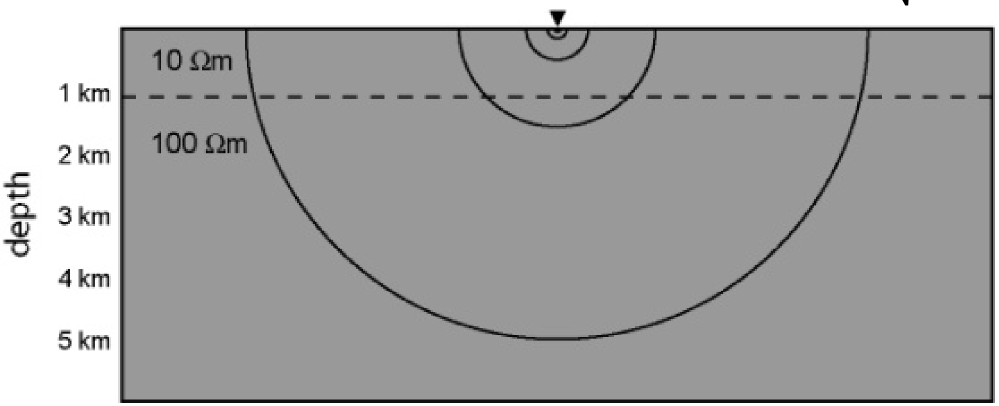


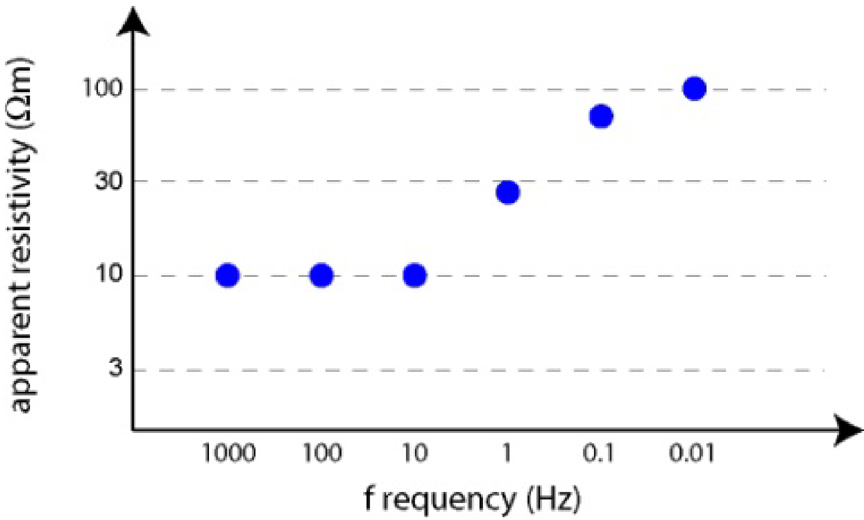
- The region that is sampled by EM signals is dominated by the lower layer
- Therefore, apparent resistivity is close to 100 Ωm.

Credit: Martyn Unsworth, University of Alberta, 2013

Apparent resistivity: $f = 0.01$ Hz

$$\delta = 503\sqrt{\frac{1}{\sigma f}} = 15800\ m$$





- Apparent resistivity approaches 100 Ωm asymptotically.

Credit: Martyn Unsworth, University of Alberta, 2013

Phase

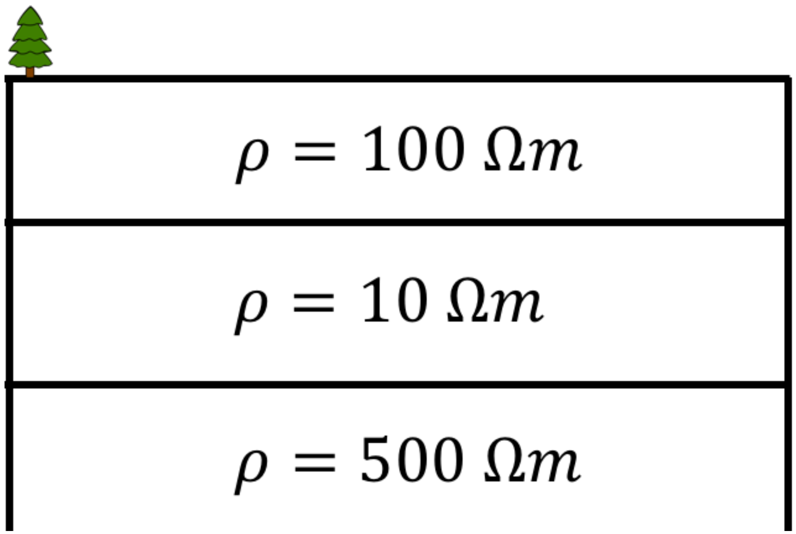
$$Z(\omega) = \frac{E_x(0,w)}{H_y(0,w)} = \sqrt{\frac{\omega\mu}{\sigma}}\, e^{i\frac{\pi}{4}}$$

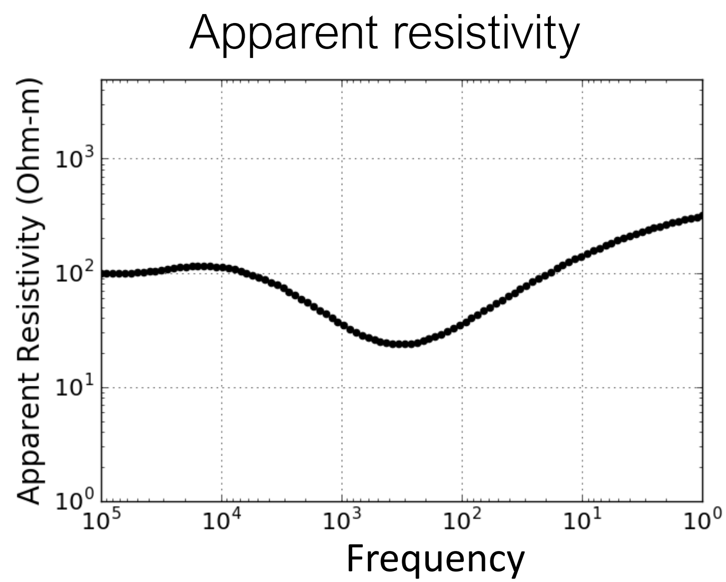
The phase $\phi(w) = \frac{\pi}{4}$

For homogeneous Earth, the phase is $\frac{\pi}{4}$

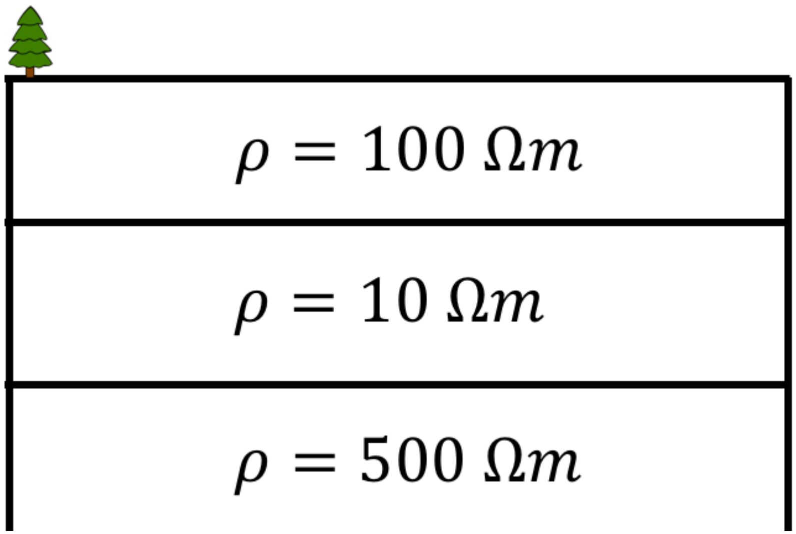
We can also plot the phase as a function of frequency (or period)

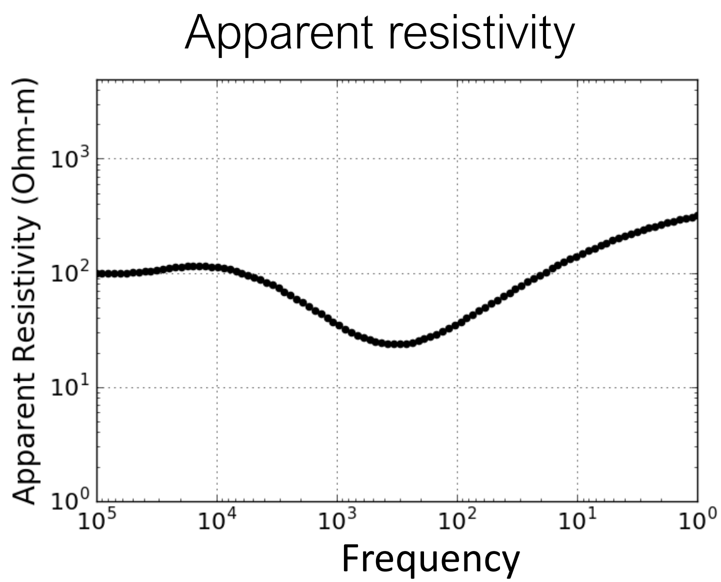
Three layer model





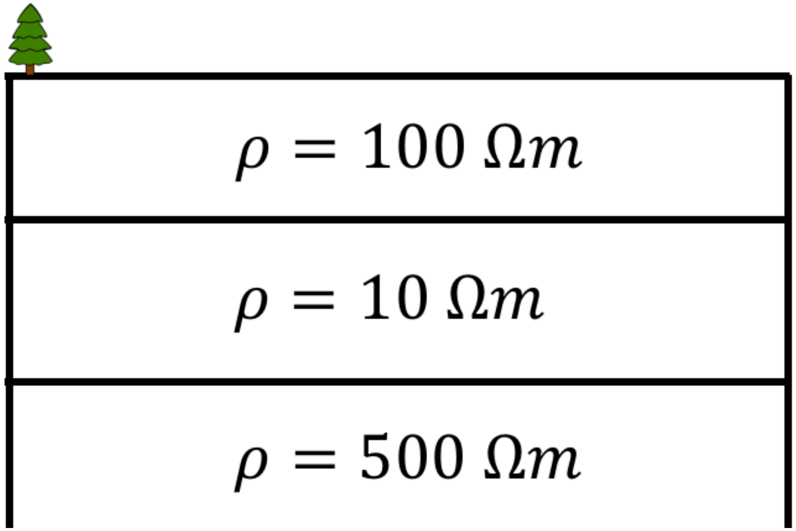
Three layer model

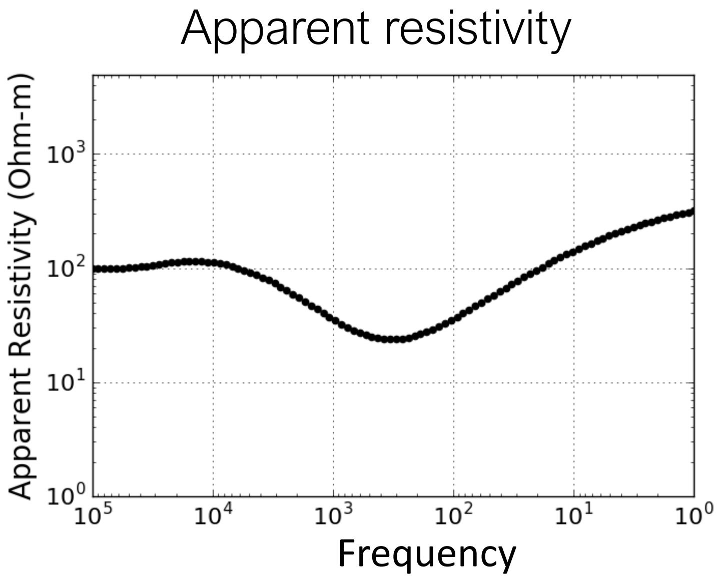




How about phase?

Three layer model: phase



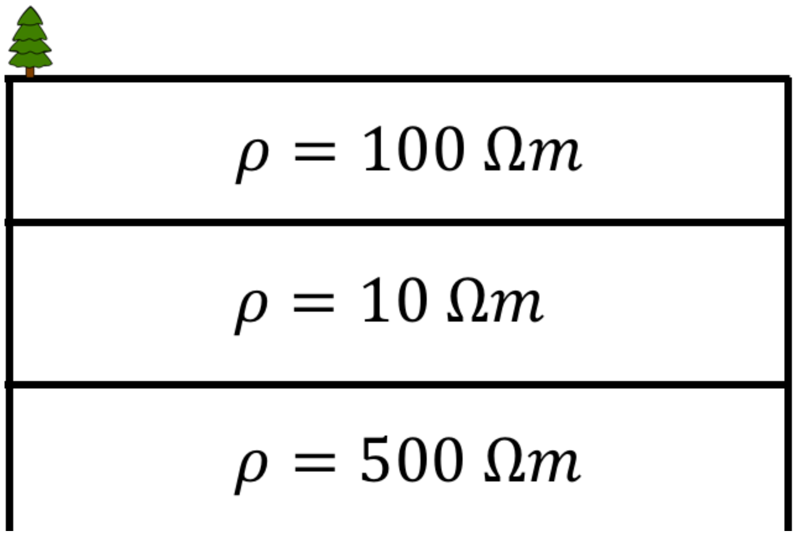


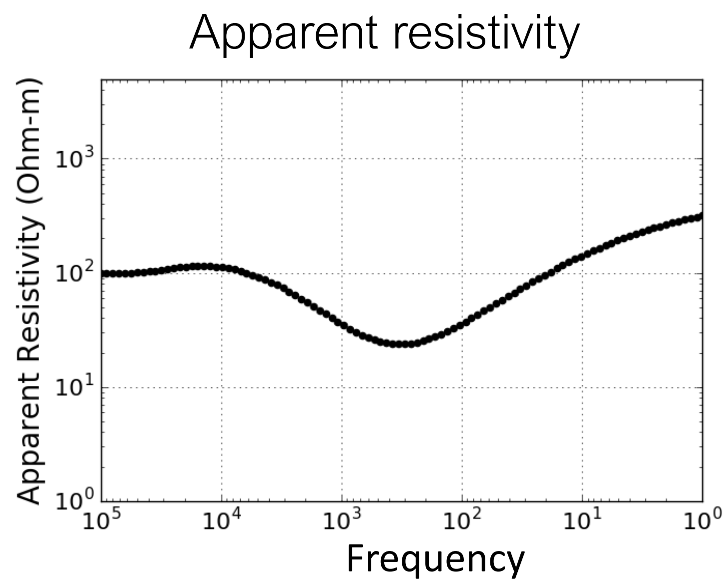
At very high frequency, the MT signals see a homogeneous Earth. Therefore, the phase is $\frac{\pi}{4}$.

At very low frequency, the lower halfspace dominates, and the MT signals see more or less a homogeneous Earth. Therefore, the phase is also $\frac{\pi}{4}$.

How about phase at intermediate frequencies?

Three layer model: phase





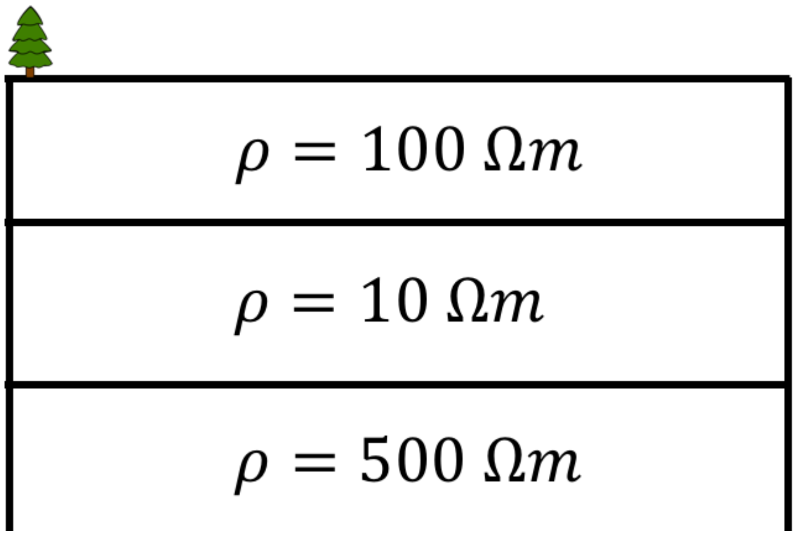
How about phase at intermediate frequencies?

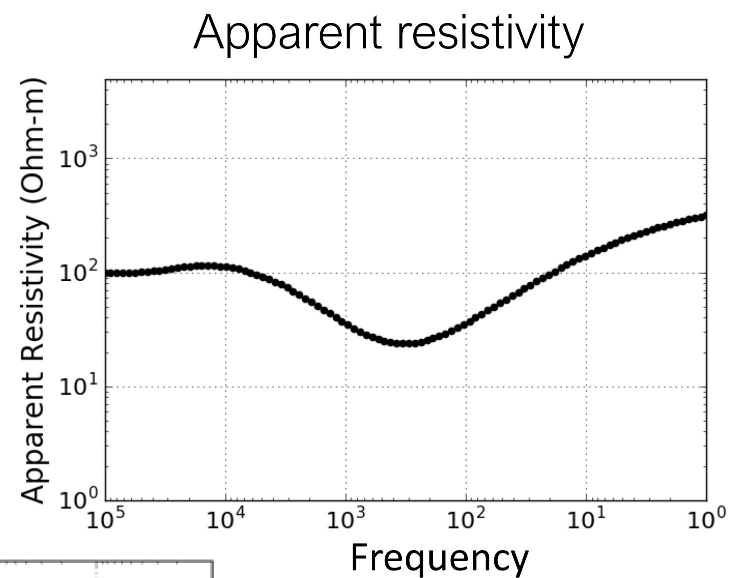
$\phi \approx \frac{\pi}{4}(1 - \frac{\partial \log(\rho_a)}{\partial \log(T)})$, where T is the period of the EM signal

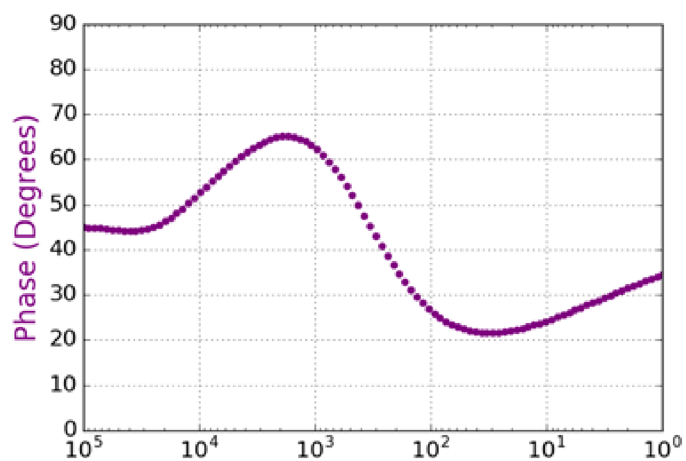
Therefore, if ρ_a increases with T, the derivative term is positive, and the phase $\phi < \frac{\pi}{4}$

If ρ_a decreases with T, the derivative term is negative, and the phase $\phi > \frac{\pi}{4}$

Three layer model: phase







Summary

- MT sounding curve reflects the subsurface resistivity and its change with depth.
  - Higher frequency reflects shallower part
  - Lower frequency reflects deeper part.
- Phase is also sensitive to change in resistivity with depth.
  - When resistivity increases with depth, the phase is smaller than $\frac{\pi}{4}$
  - When resistivity decrease with depth, the phase is larger than $\frac{\pi}{4}$