Lecture 7
Review of Electrodynamics

GEOL 4397: Electromagnetic Methods for Exploration
GEOL 6398: Special Problems

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Sept. 13th, 2018

UNIVERSITY of HOUSTON
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EARTH AND ATMOSPHERIC SCIENCES

Announcement

- Please do not send me emails using Yahoo email accounts.

Agenda

- Biot-Savart law
- Ampere’s law
- Faraday’s law
- Lenz’s law
- Inductance

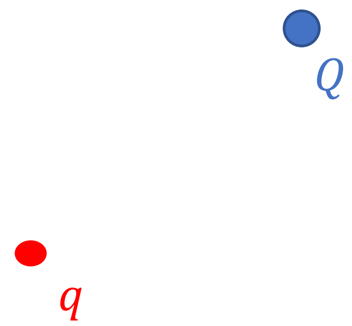
From charges to currents

- Stationary charges produce electric fields that are constant in time (hence the term electrostatics)

- Steady current produce magnetic fields that are constant in time (hence the term magnetostatics)

Coulomb's law

- The forces that electric charges exert on each other are described by Coulomb's law
- Based on experiments



$$\boldsymbol{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}\hat{\boldsymbol{r}}$$

- The force is proportional to the product of the charges and inversely proportional to the square of their distance.

$$\boldsymbol{E} = \frac{\boldsymbol{F}}{Q}$$

Griffiths, 4th edition, pp 61

ϵ_0: permittivity of free space

Biot-Savart law

Jean-Baptiste Biot
1774 – 1862

Félix Savart
1791 – 1841

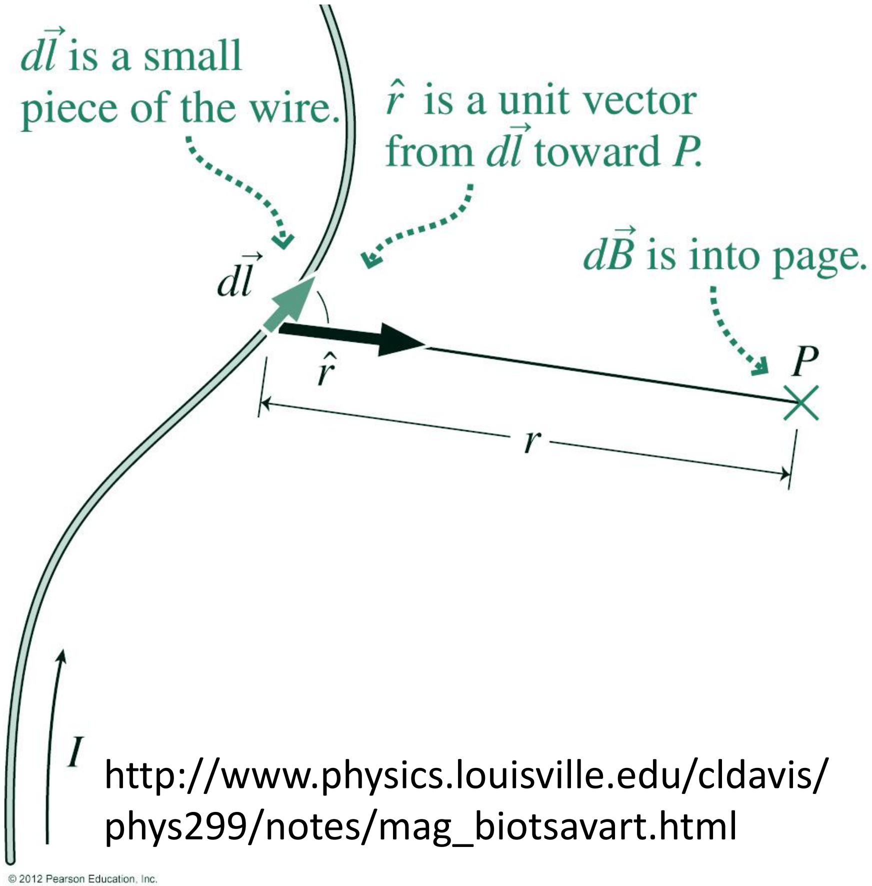
$$d\boldsymbol{B} = \frac{\mu_0}{4\pi}\frac{d\boldsymbol{l} \times \hat{\boldsymbol{r}}}{r^2}$$

μ_0 : the permeability of the vacuum (free space)

$d\boldsymbol{l}$: "*current element*" directed along the current in the wire

$\hat{\boldsymbol{r}}$: a unit vector from **dl** to where the **B** field is to be calculated

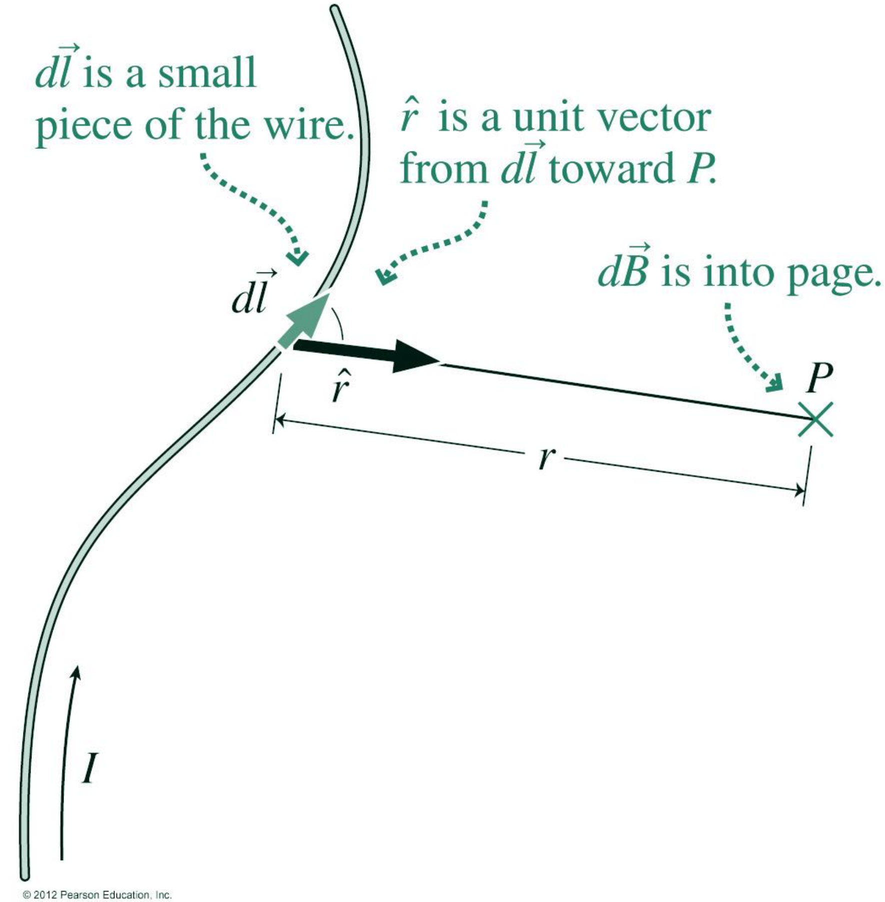
$d\boldsymbol{B}$ is at right angles to both $d\boldsymbol{l}$ and $\hat{\boldsymbol{r}}$



http://www.physics.louisville.edu/cldavis/phys299/notes/mag_biotsavart.html

Biot-Savart law

$$\boldsymbol{B} = \int \frac{\mu_0}{4\pi} \frac{d\boldsymbol{l} \times \hat{\boldsymbol{r}}}{r^2}$$



$\boldsymbol{B}$: magnetic field at P due to the steady current in the wire

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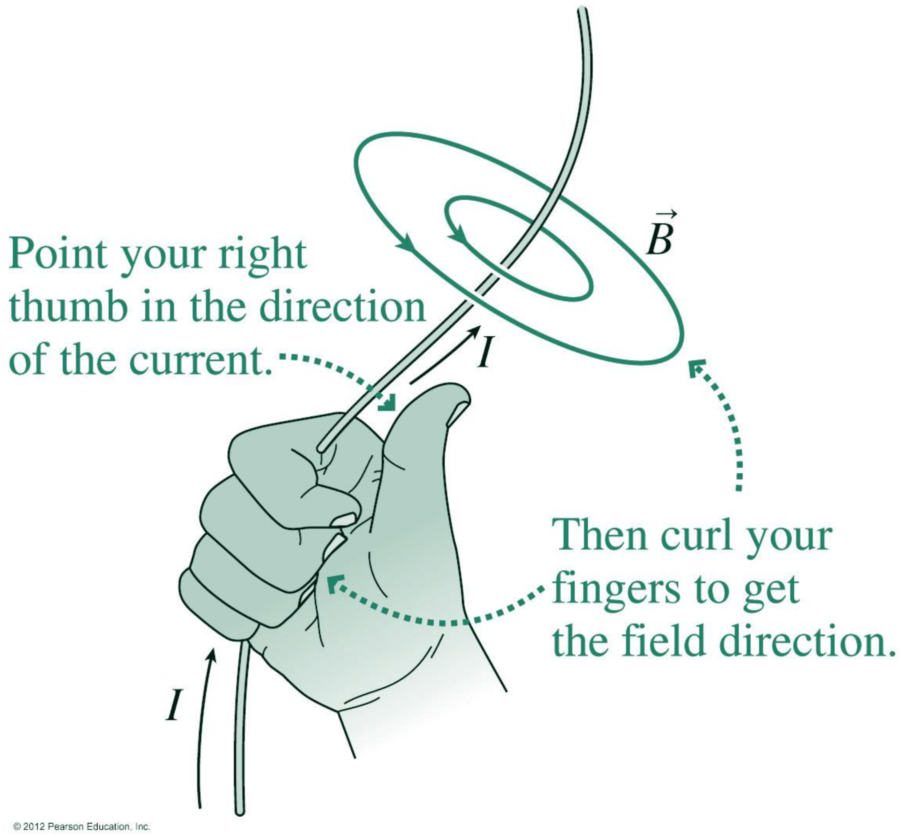

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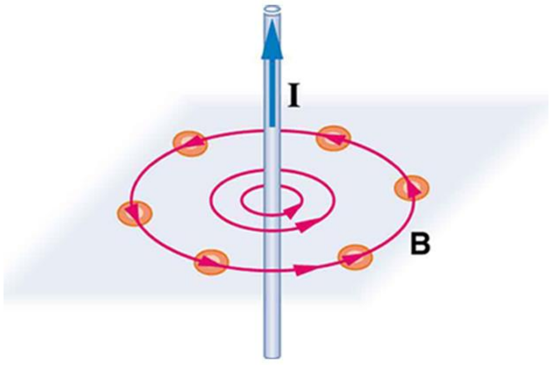

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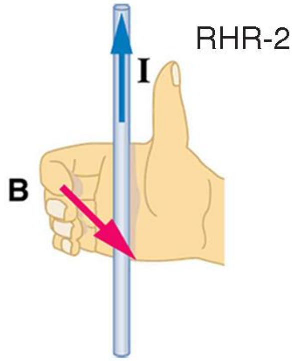
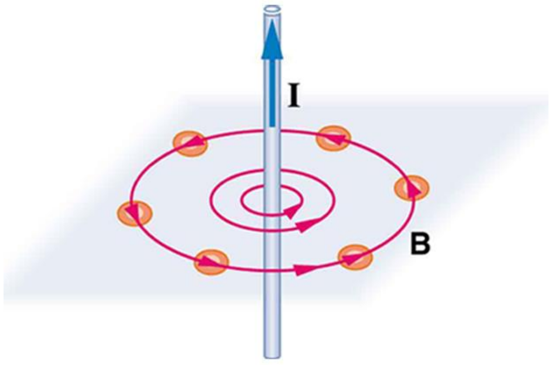


(b)

Magnetic field due to a long straight wire

https://opentextbc.ca/physicstestbook2/chapter/magnetic-fields-produced-by-currents-amperes-law/



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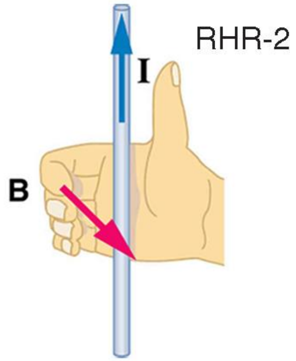

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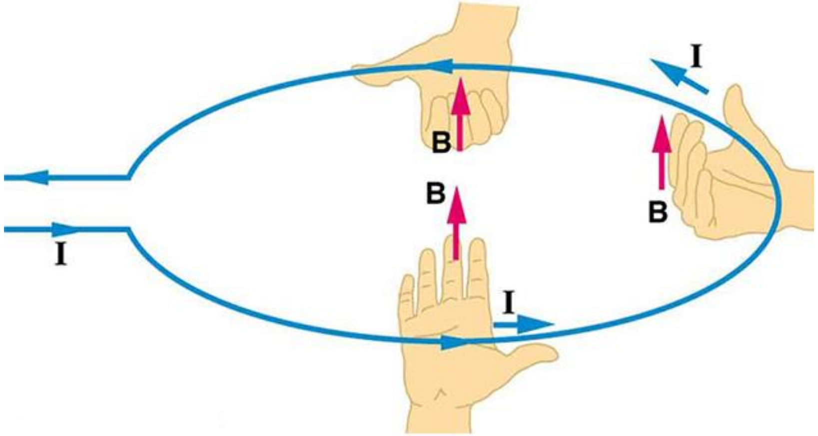

(a)

Note that the magnetic field due to a circular loop is similar to that of a bar magnet.

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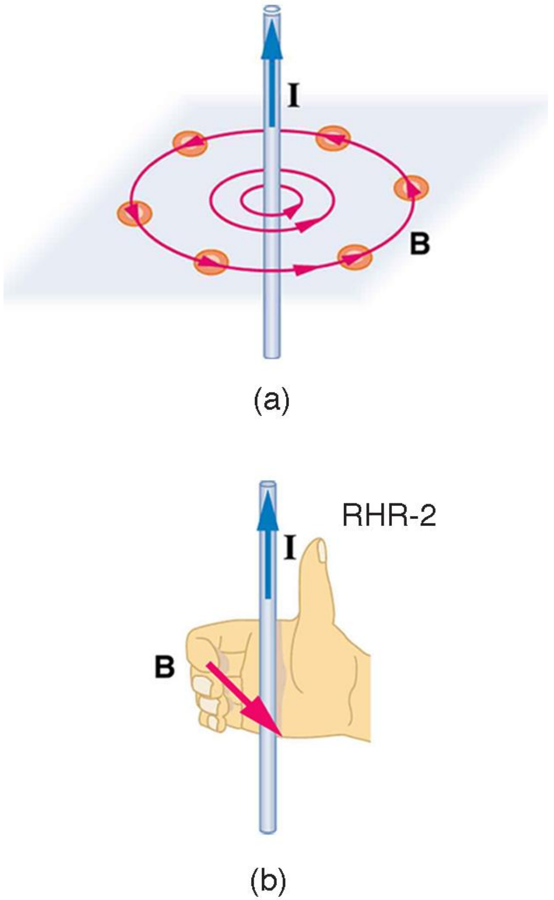

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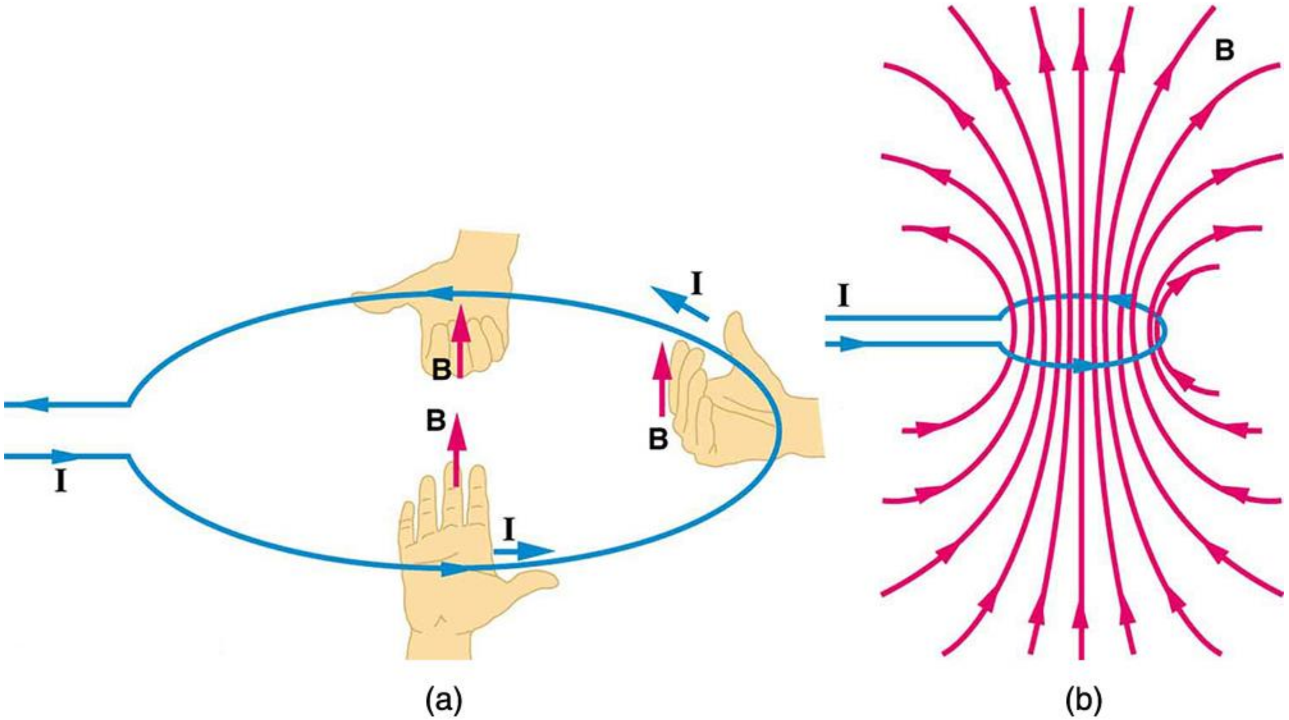

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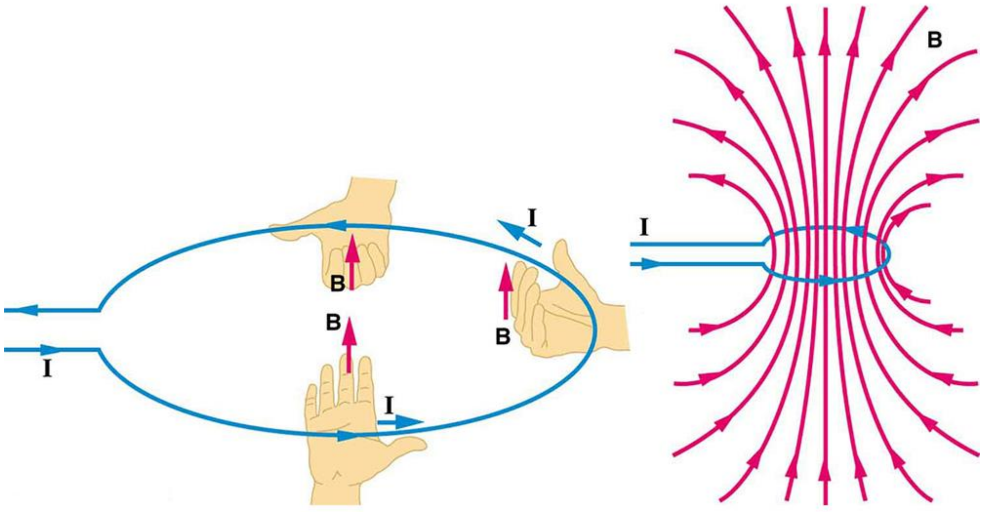

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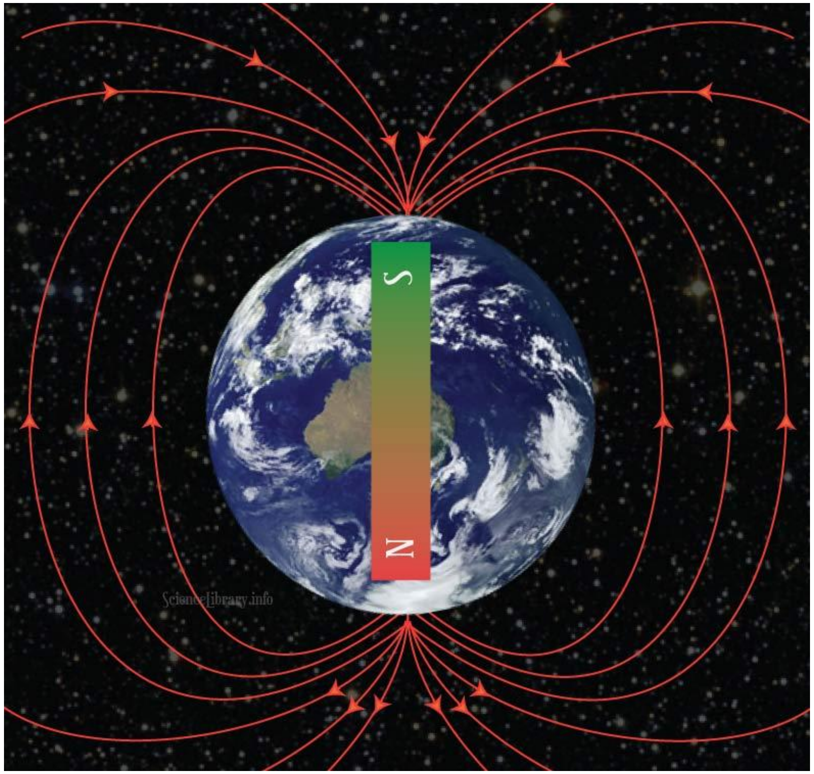

http://www.sciencelibrary.info/Magnets

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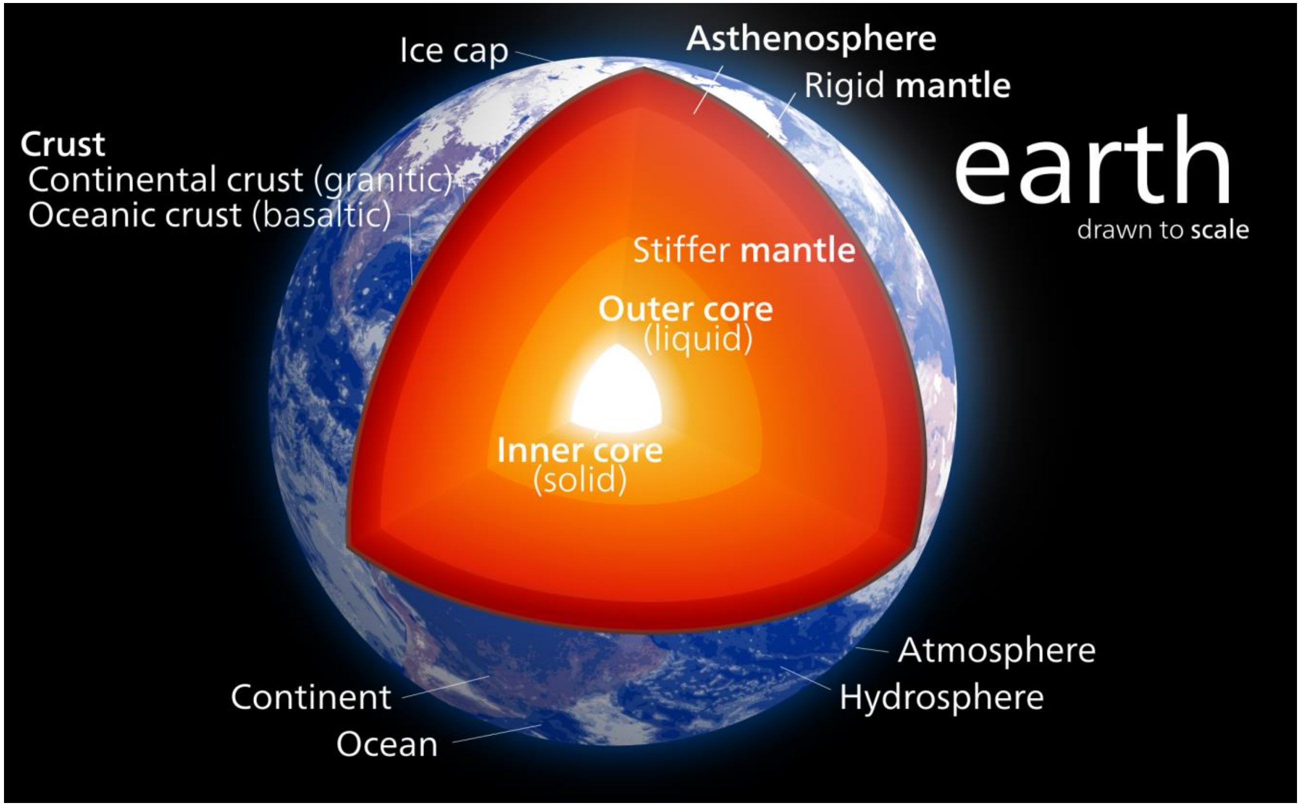
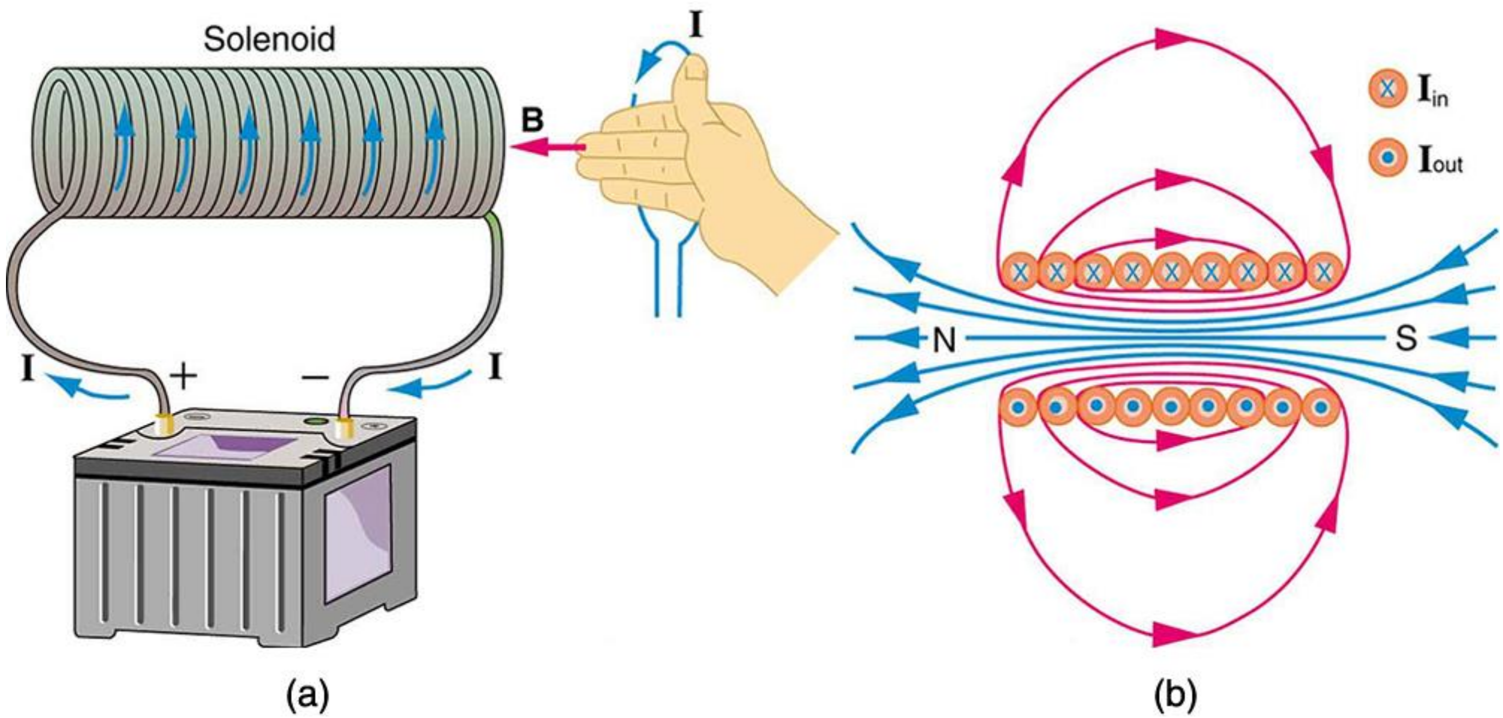


The **outer core** of the Earth is a fluid layer about 2,400 km (1,500 mi)[1] thick and composed of mostly iron and nickel that lies above Earth's solid inner core and below its mantle. The flow of liquid iron generates electric currents, which in turn produce magnetic fields.

https://en.wikipedia.org/wiki/Outer_core

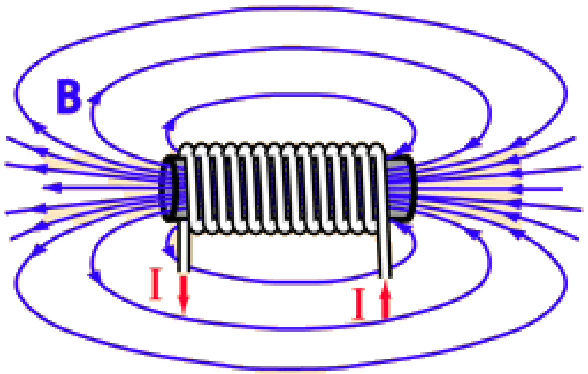
Magnetic Field of a Current-Carrying Solenoid



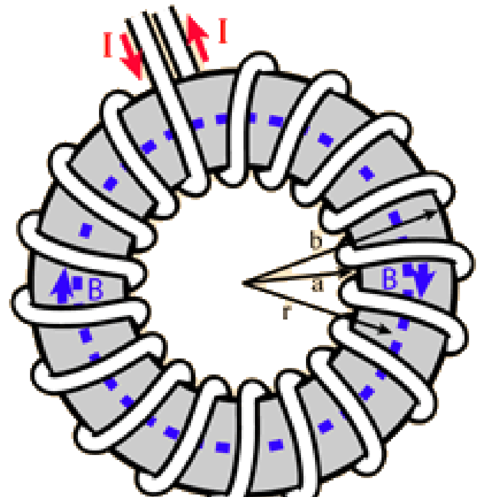
A **solenoid** is a long coil of wire (with many turns or loops, as opposed to a flat loop). Because of its shape, the field inside a solenoid can be very uniform (in both direction and magnitude), and also very strong. The field just outside the coils is nearly zero.

https://opentextbc.ca/physicstestbook2/chapter/magnetic-fields-produced-by-currents-amperes-law/

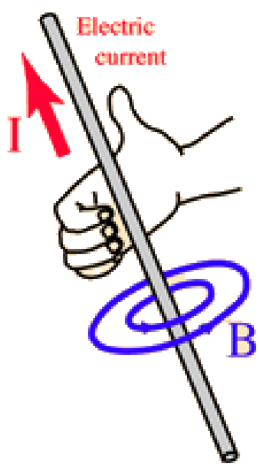
Examples



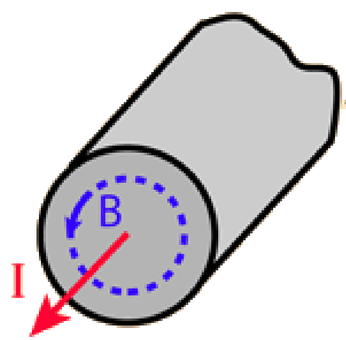
Magnetic field inside a long solenoic.



Magnetic field inside a toroidal coil.



Magnetic field from a long straight wire.



Magnetic field inside a conductor.

http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/amplaw.html

From Biot-Savart law to Ampere's law

- Biot-Savart law

$$\boldsymbol{B} = \int \frac{\mu_0}{4\pi} \frac{d\boldsymbol{l} \times \hat{\boldsymbol{r}}}{r^2}$$

- Ampere's law

$$\nabla \times \boldsymbol{B} = \mu_0 \mathbf{j}$$

To learn more about how to derive Ampere's law from Biot-Savart law, please refer to Page 229-233 in David J. Grifiths' book (Introudution to Electrodynamics, Fourth Edition)

- So, we have learned that currents produce magnetic field.

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- Can magnetic field generate currents?

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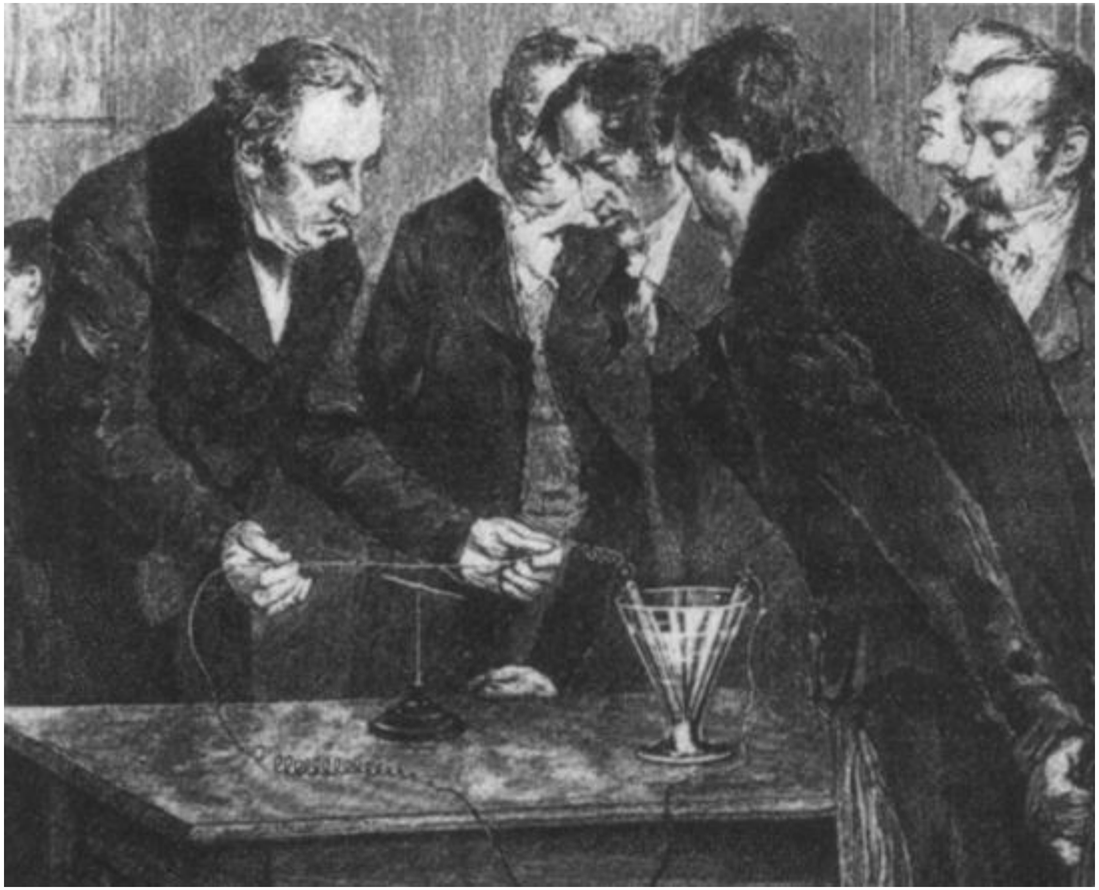
- Can magnetic field generate currents?

- This is actually the question asked by scientists shortly after Oersted discovered currents produce magnetic fields in 1820.

Oersted’s discovery in 1820



http://www.sciencelibrary.info/Magnets

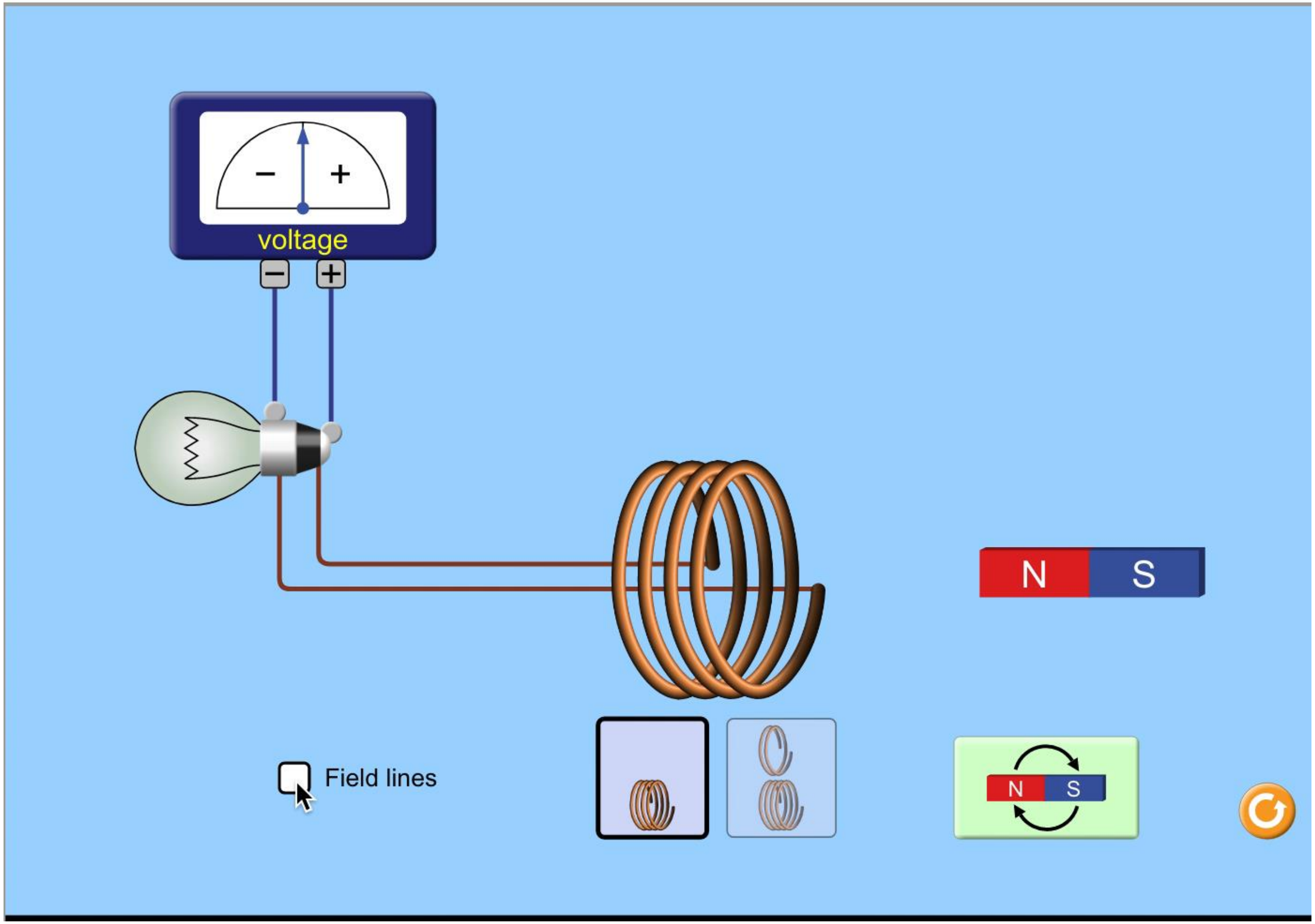
- The answer is YES!

- In 1831, Michael Faraday and Joseph Henry independently demonstrated that magnetic fields can produce currents.

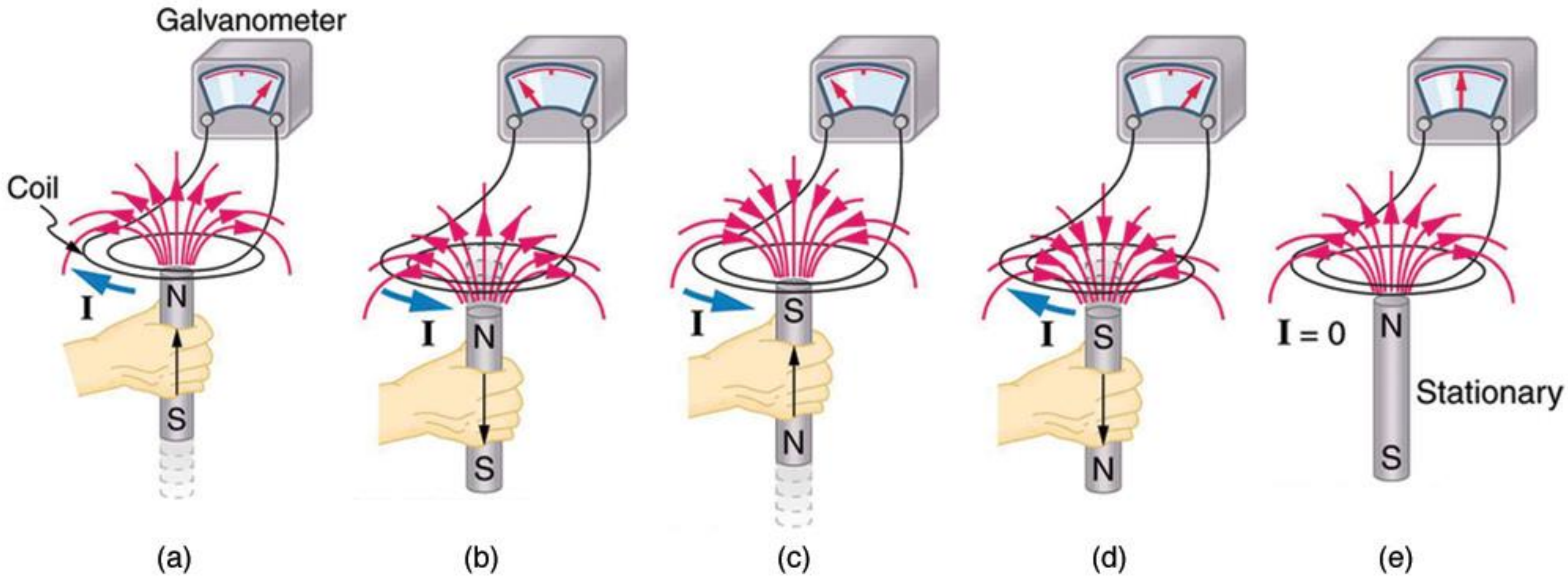
- Electromagnetic induction

- Faraday's law

EM induction

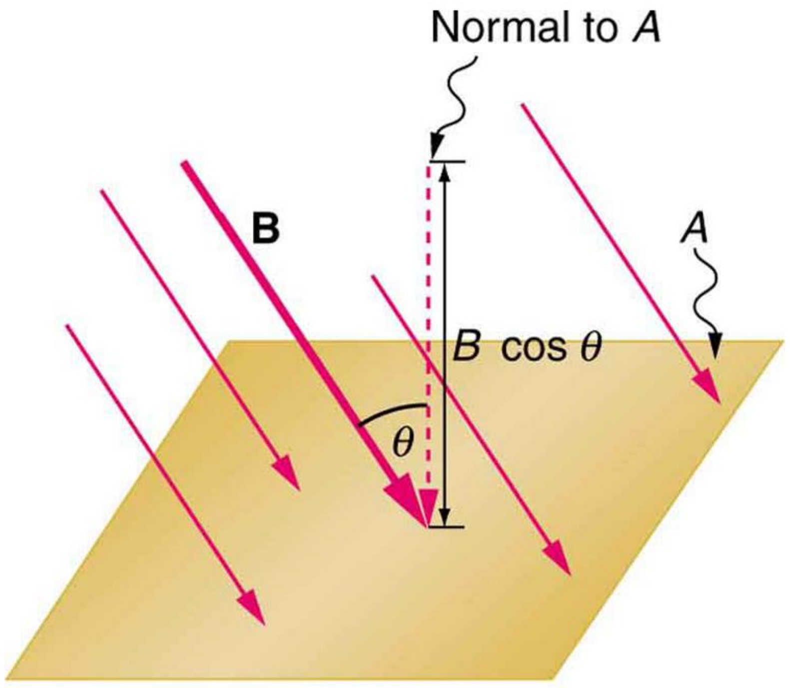


EM induction

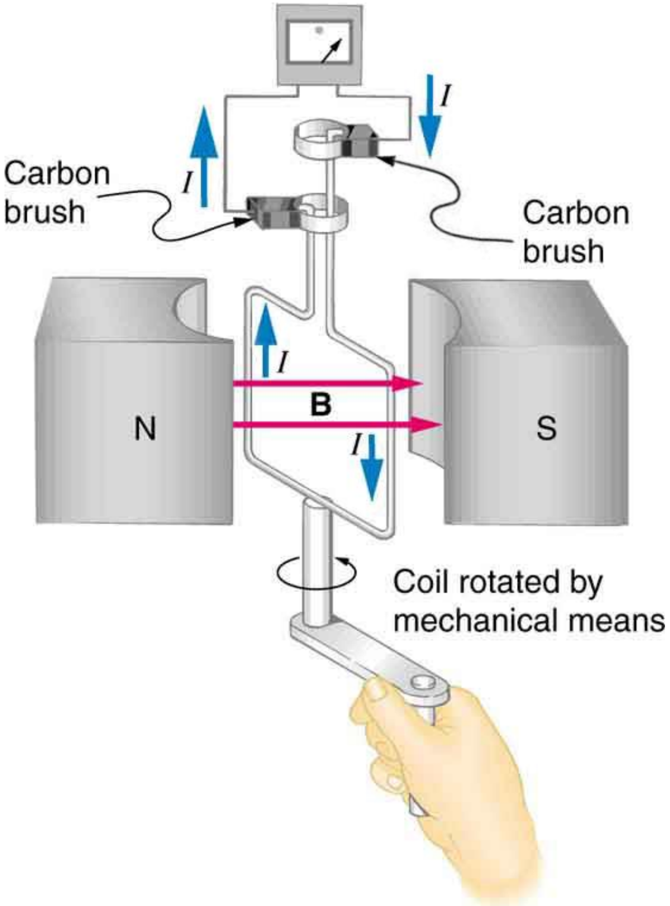


https://opentextbc.ca/physicstestbook2/chapter/induced-emf-and-magnetic-flux/#import-auto-id1169738076827

Magnetic flux



$$\Phi = BA\cos\theta = B_{\perp}A$$



https://opentextbc.ca/physicstestbook2/chapter/induced-emf-and-magnetic-flux/#import-auto-id1169738076827

Electromotive Force (emf)

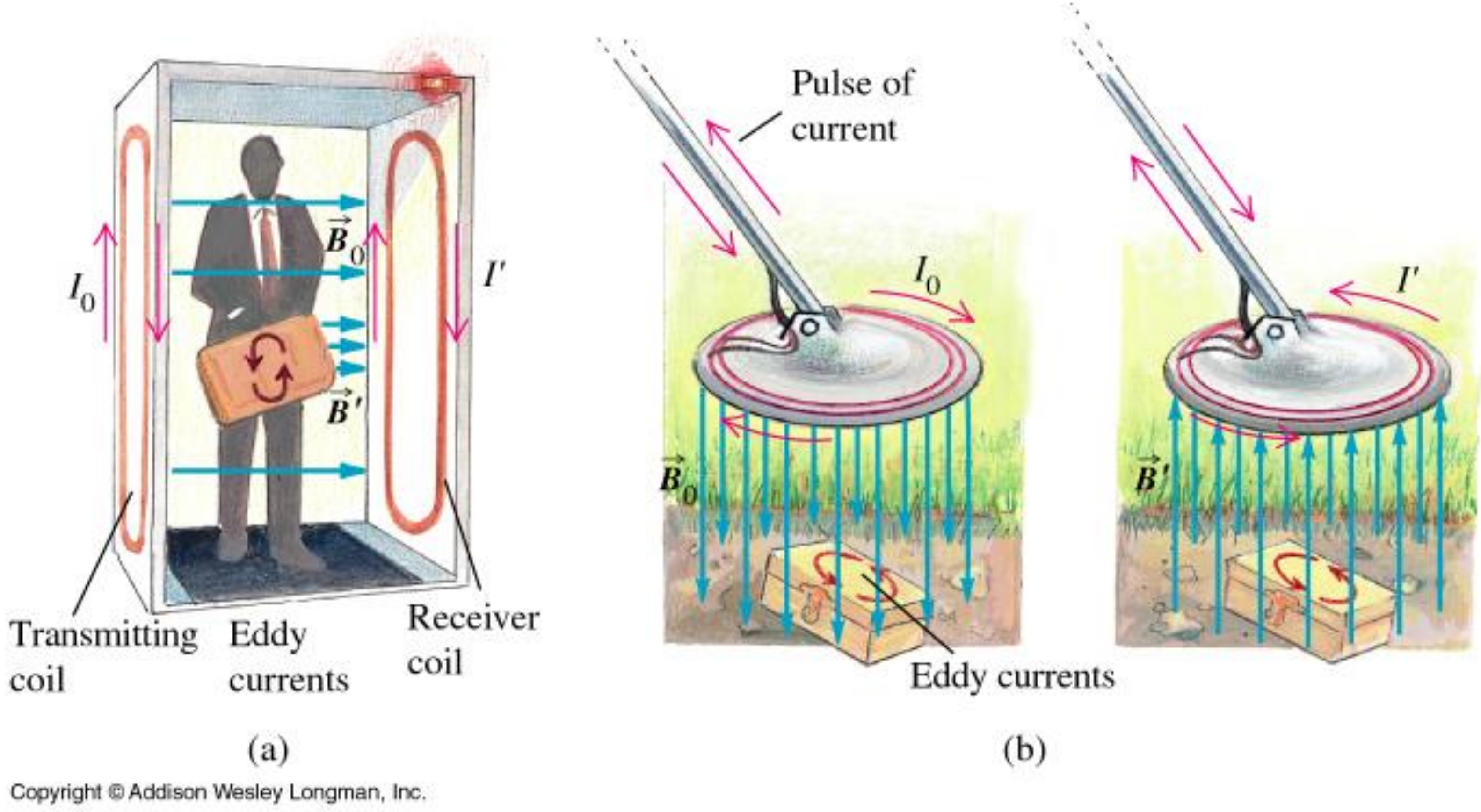
- The force that drives currents to flow in a wire or in a conductive body
- Unit: Volts

$$\varepsilon = -\frac{d\phi}{dt}$$

where ϕ is magnetic flux.

- Any change in magnetic flux ϕ induces an emf. This process is called electromagnetic induction.
- An emf, when applied to a conductor, generates currents.

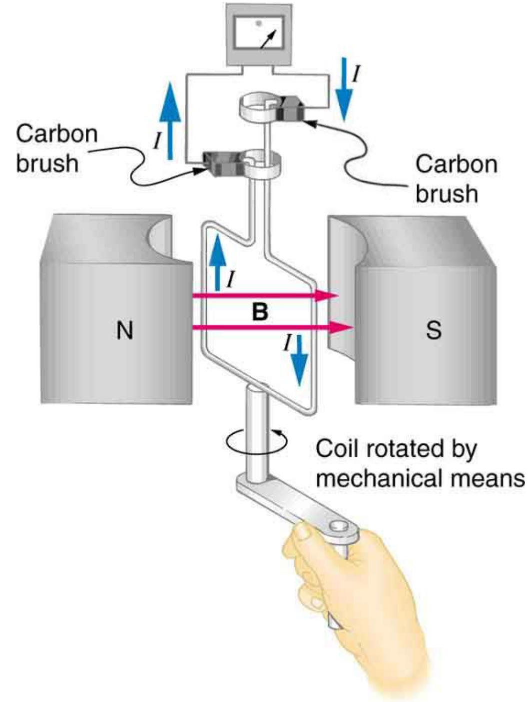
Application examples of EM induction



https://gpg.geosci.xyz/content/electromagnetics/electromagnetic_basic_principles.html

Example of EM induction: Wind energy


Wind turbines in the Thames Estuary in the UK



Wind pushes the blades of the turbine, spinning a shaft attached to magnets. The magnets spin around a conductive coil, inducing an electric current in the coil, and eventually feeding the electrical grid.

https://opentextbc.ca/physicstestbook2/chapter/introduction-to-electromagnetic-induction-ac-circuits-and-electrical-technologies/

More applications of EM inductions

- currents induced by magnetic fields (i.e., EM induction) are essential to our technological society.
- The ubiquitous generator, e.g, in automobiles, on bicycles, in nuclear power plants, etc.
- Pickup coils in electric guitars
- Transformers (https://en.wikipedia.org/wiki/Transformer)
- certain microphones
- airport security gates
- …

https://opentextbc.ca/physicstestbook2/chapter/introduction-to-electromagnetic-induction-ac-circuits-and-electrical-technologies/

Faraday's law

$$\varepsilon = -\frac{d\phi}{dt}$$

A changing magnetic field will produce a voltage in a coil, causing currents to flow. This voltage is known as the induced electromotive force (EMF).

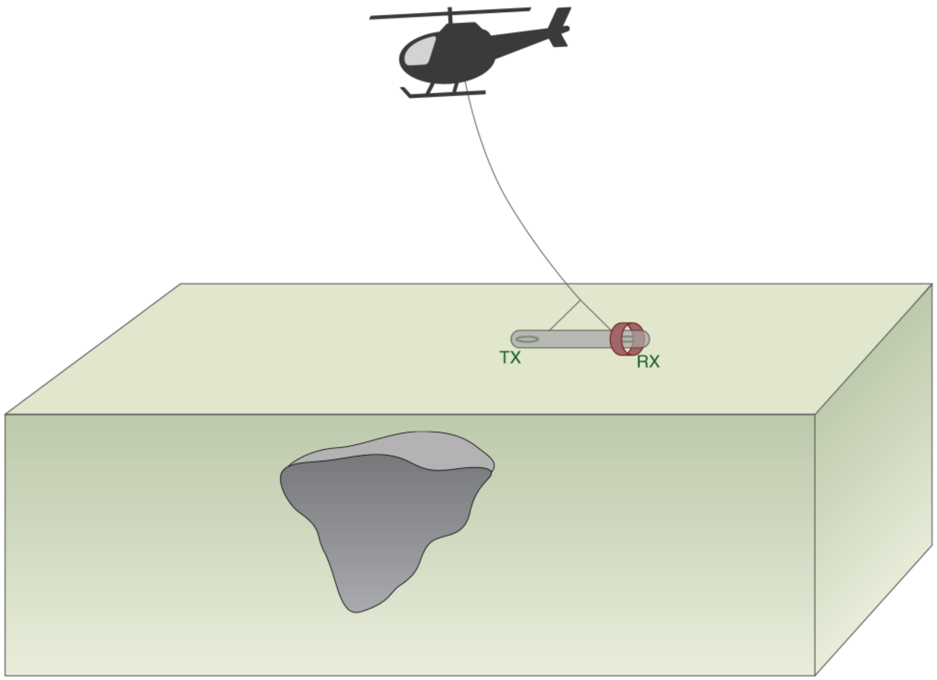
$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$

Faraday's law in differential form

See Page 313 in Griffiths' book (4th edition)for derivations.

Basic Experiment for Airborne EM

- **Setup:**
  - transmitter and receiver are in a towed bird



Credit: Doug Oldenburg, Seogi Kang and Linsey Heagy from UBC-GIF

Basic Experiment for Airborne EM

- **Setup:**
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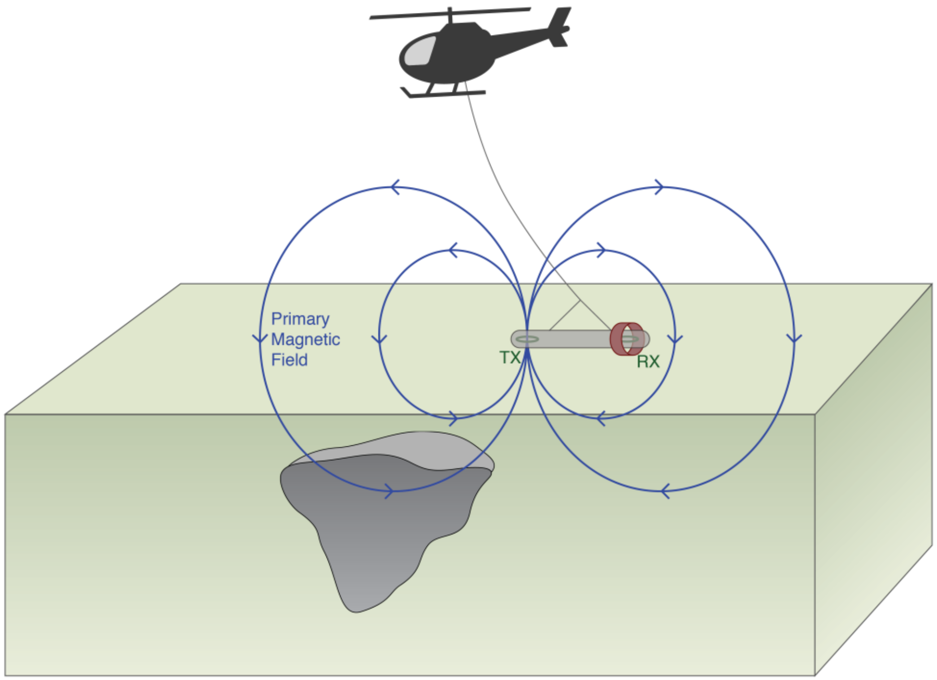
- **Primary:**
  - Transmitter produces a primary magnetic field



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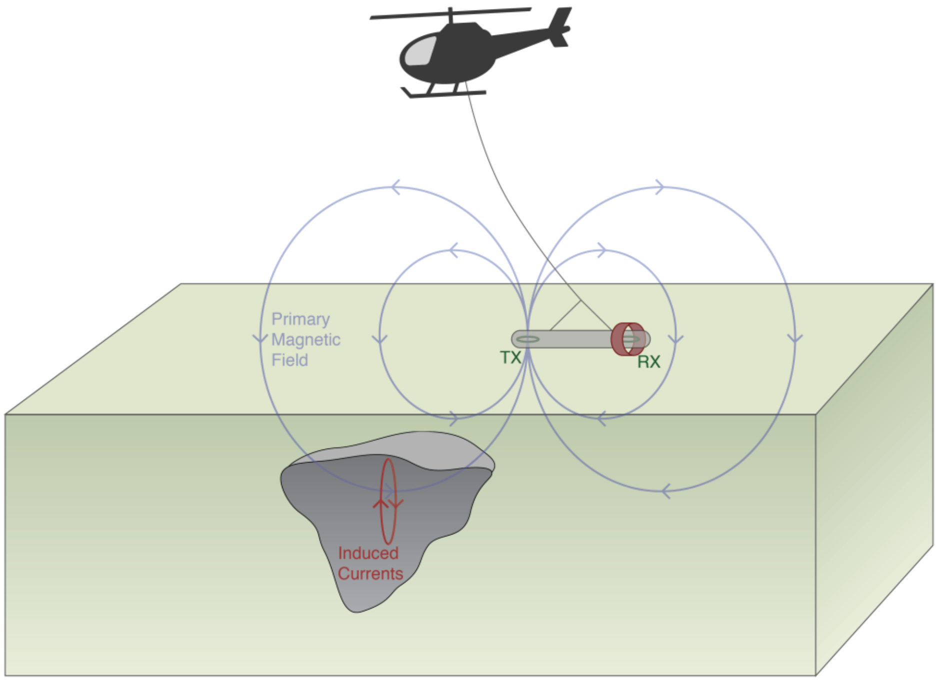
- **Induced Currents:**
  - Time varying magnetic fields generate electric fields everywhere and currents in conductors



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- **Primary:**
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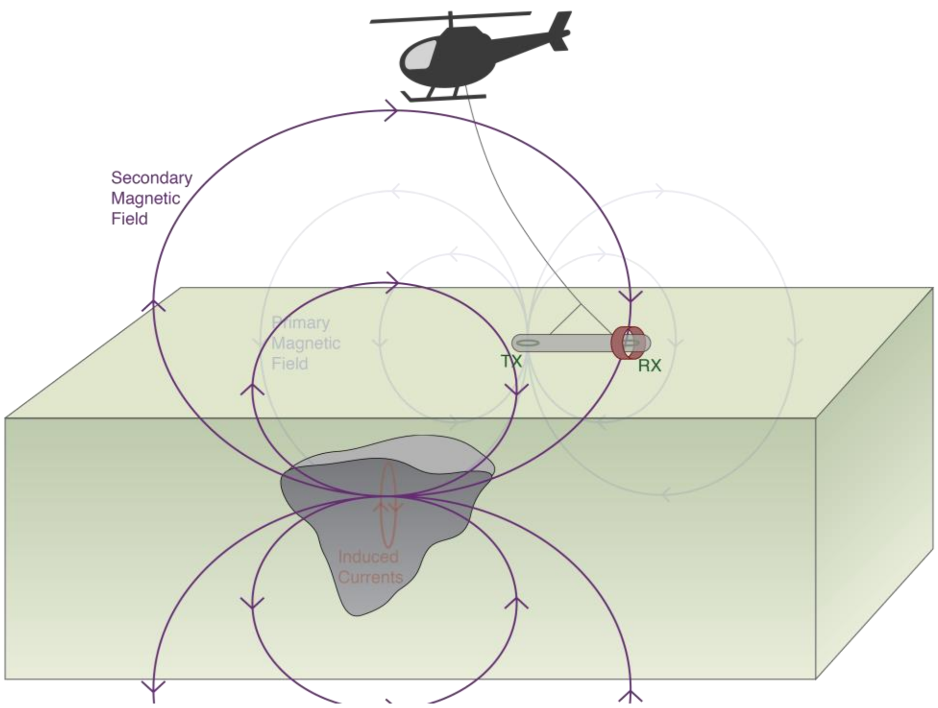
- **Secondary Fields:**
  - The induced currents produce a secondary magnetic field.



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- Faraday's law
- Lenz's law
- Inductance

Faraday's law of induction

$$\varepsilon = -\frac{d\phi}{dt}$$

$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$

shows that any variation in the magnetic flux produces an electromotive force (emf, EE). This emf creates electrical currents within those bodies which are subjected to the time varying flux. The amplitude of the induced current is dependent on the strength of the emf and the conductivity of the material, while the direction of the induced current is characterized by Lenz's Law

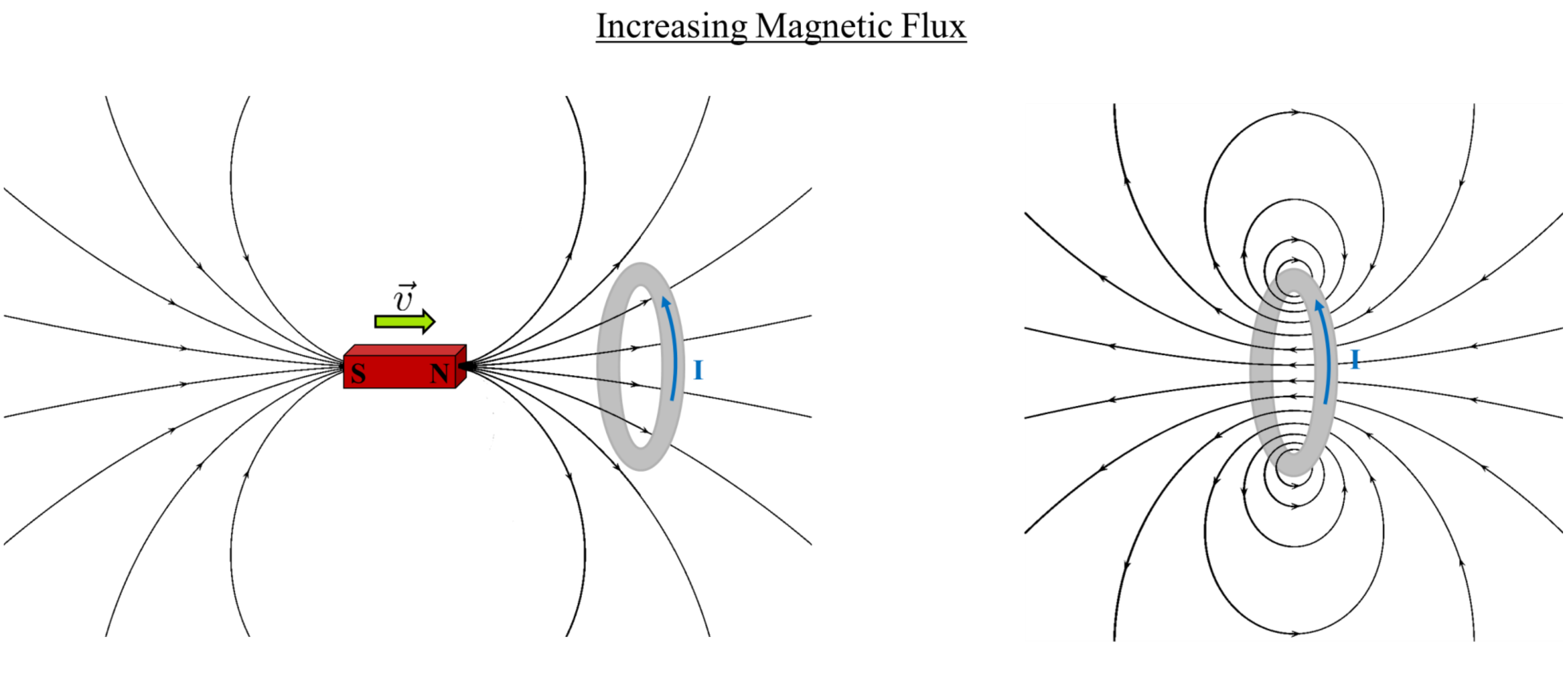
https://em.geosci.xyz/content/maxwell1_fundamentals/formative_laws/lenz.html

Lenz's law

- Lenz's Law states that the induced current will flow in such a direction that its secondary or induced magnetic fields act to oppose the observed change in magnetic flux.
- Simply put, "nature abhors a change in flux" so the induced current flows in such a manner to try to cancel out the change
- This is the reason for the negative sign in Faraday's Law

https://em.geosci.xyz/content/maxwell1_fundamentals/formative_laws/lenz.html

Lenz's law



(a) (b)

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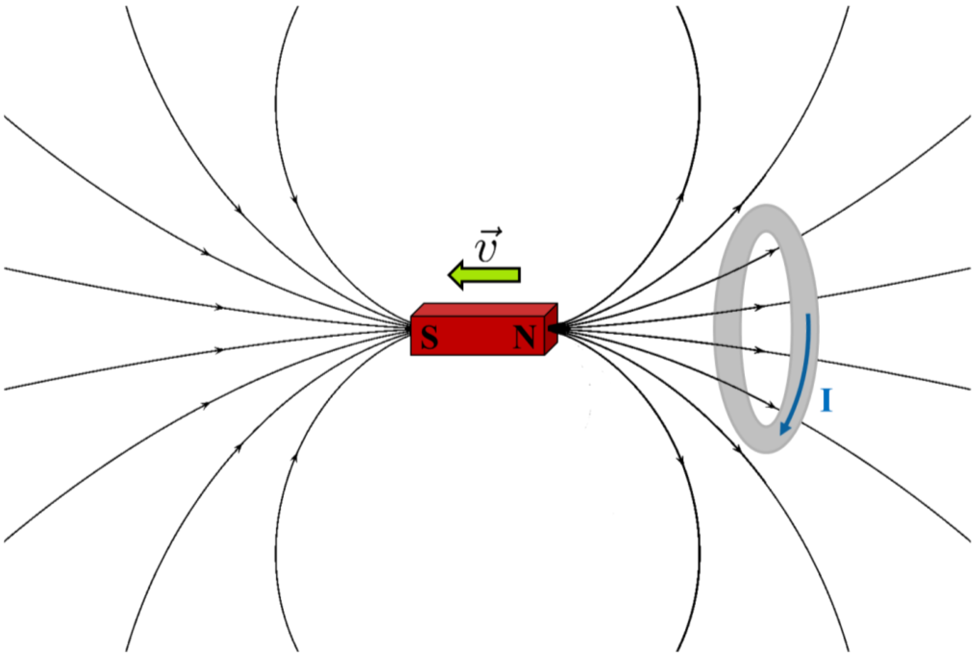
Lenz’s law

Decreasing Magnetic Flux



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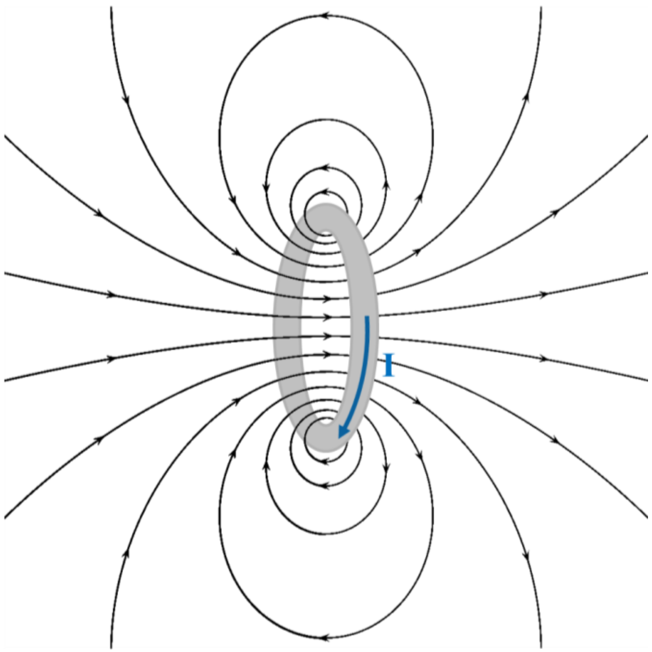

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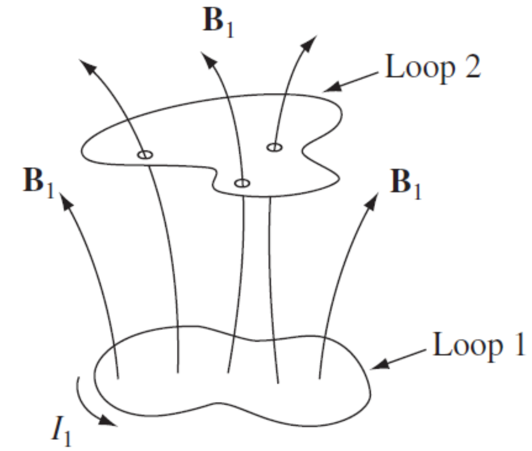
Inductance

- Imagine two loops of wire at rest
- If we run a steady current I_1 around loop 1, it produces a magnetic field $\boldsymbol{B}_1$



Griffiths, 4th edition, pp 322

Inductance

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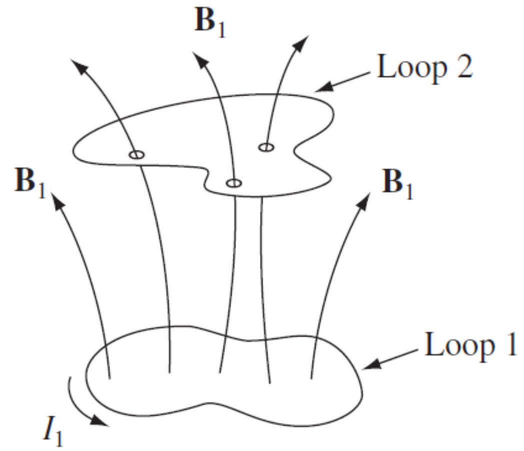

Griffiths, 4th edition, pp 322

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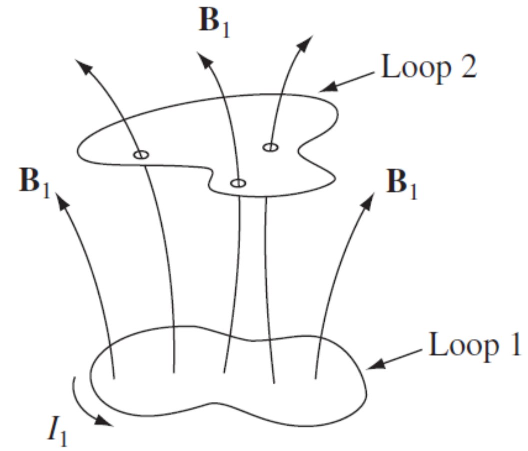

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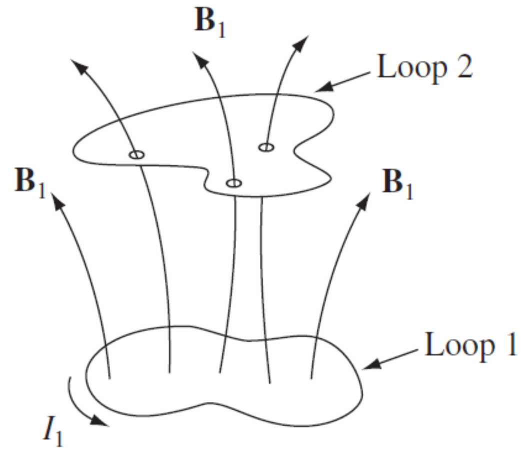
- Some of the magnetic field lines pass through loop 2



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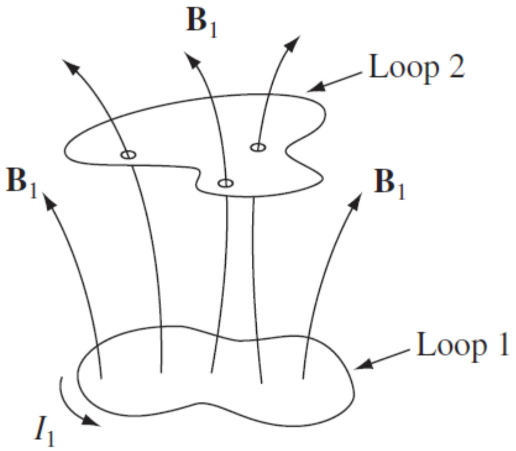
- The flux of $\boldsymbol{B}_1$ through loop 2 is

$$\Phi_2 = \int \boldsymbol{B}_1 \cdot d\boldsymbol{a}_2$$



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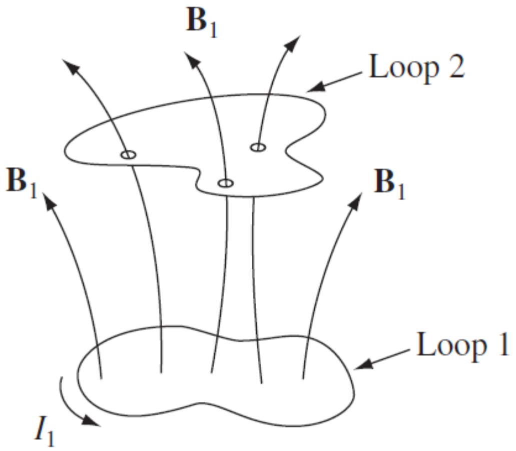

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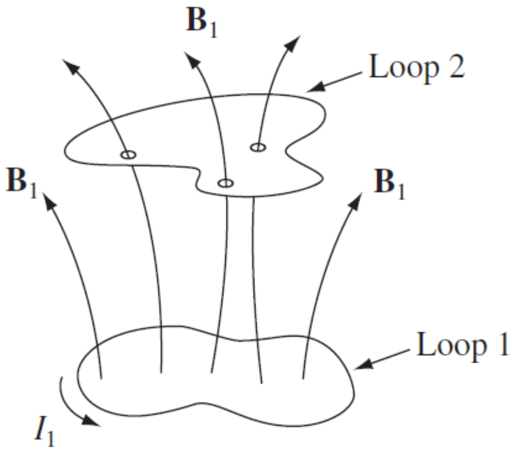


which is also proportional to the current I_1

- Therefore, $\Phi_2 = M_{21} I_1$

where M_{21} is the constant of proportionality known as the mutual inductance of the two loops

Griffiths, 4th edition, pp 322

Mutual inductance (optional)

- We derive a formula for M_{21} using vector potential and Stokes' theorem

$$\Phi_2 = \int \boldsymbol{B}_1 \cdot d\boldsymbol{a}_2 = \int (\nabla \times \boldsymbol{A}_1) \cdot d\boldsymbol{a}_2 = \oint \boldsymbol{A}_1 \cdot d\boldsymbol{I}_2$$

- According to eq 5.66 in Griffiths' book (fourth edition)

$$\boldsymbol{A}_1 = \frac{\mu_0 I_1}{4\pi} \oint \frac{d\boldsymbol{I}_1}{r}$$

- Therefore,

$$\Phi_2 = \frac{\mu_0 I_1}{4\pi} \oint \left(\oint \frac{d\boldsymbol{I}_1}{r} \right) \cdot d\boldsymbol{I}_2$$

- Therefore,

$$M_{21} = \frac{\mu_0}{4\pi} \oint \oint \frac{d\boldsymbol{I}_1 \cdot d\boldsymbol{I}_2}{r}$$

Griffiths, 4th edition, pp 322

Mutual inductance

Observations:

- M_{21} is a purely geometrical quantity, having to do with the sizes, shapes, and relative positions of the two loops

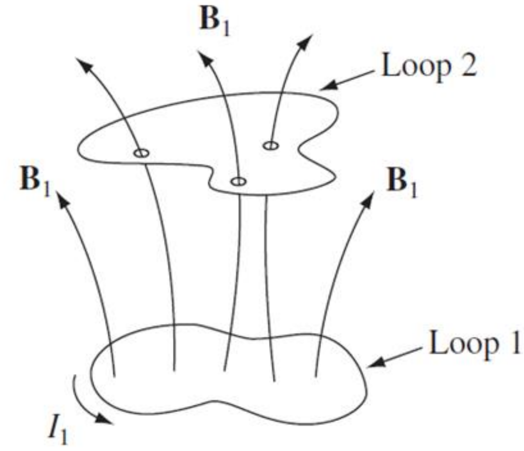
- $M_{21} = M_{12}$

  Whatever the shapes and positions of the loops, the flux through loop 2 due to current I in loop 1 is identical to the flux through loop 1 due to current I in loop 2.

  We can then drop the subscripts, and call them both M.

Griffiths, 4th edition, pp 323

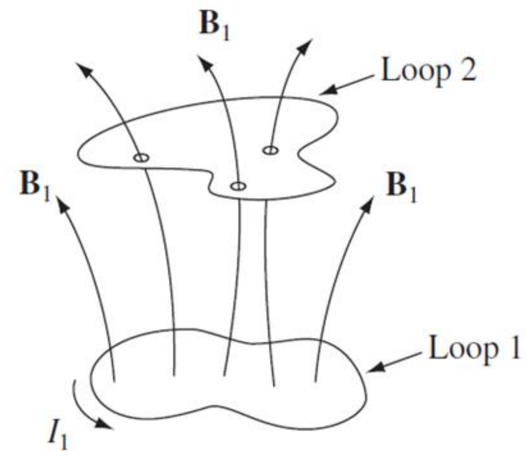
EMF



- If we vary the current in loop 1, the flux through loop 2 will vary accordingly.

Griffiths, 4th edition, pp 324

EMF

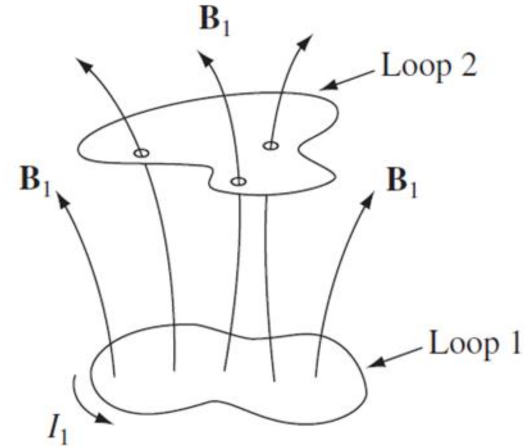


- If we vary the current in loop 1, the flux through loop 2 will vary accordingly.
- According to Faraday's law, the changing flux will induce an emf in loop 2

$$\varepsilon_2 = -\frac{d\Phi_2}{dt} = -M\frac{dI_1}{dt}$$

Griffiths, 4th edition, pp 324

EMF



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- Here we assume that the currents change slowly enough that the system can be considered quasi-static, and Biot-Savart law still holds.
- This is the EMF induced in one circuit by a current flowing in another circuit (amazing!).

Griffiths, 4th edition, pp 324

Self inductance

- Changing current in loop 1 not only induces an emf in loop 2, but also induces an emf in itself.

Griffiths, 4th edition, pp 324

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- The flux through loop 1 is proportional to the current

$$\Phi = LI$$

Griffiths, 4th edition, pp 324

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Griffiths, 4th edition, pp 324

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Griffiths, 4th edition, pp 324

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- The flux through loop 1 is proportional to the current

$$\Phi = LI$$

- L: self inductance (or simply inductance)
- Only depends on the geometry (size and shape) of the loop.
- If the current changes, the emf induced in the loop is

$$\varepsilon = -L\frac{dI}{dt}$$

Griffiths, 4th edition, pp 324

Back EMF

- The minus sign in the previous equation reflects Lenz's law, which says that the emf is in such a direction as to oppose any change in current.

Griffiths, 4th edition, pp 325

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Griffiths, 4th edition, pp 325

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Griffiths, 4th edition, pp 325

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- The minus sign in the previous equation reflects Lenz's law, which says that the emf is in such a direction as to oppose any change in current.
- Therefore, it is also called back emf.
- Whenever you try to alter the current in a wire, you must fight against this back emf.
- Inductance plays somewhat the same role in electric circuits that mass plays in mechanical system: The greater L is, the harder it is to change the current, just as the larger the mass, the harder it is to change an object's velocity.

Griffiths, 4th edition, pp 325