Lecture 3

DC Resistivity: Basic Principles

GEOL 4397: Electromagnetic Methods for Exploration
GEOL 6398: Special Problems

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UNIVERSITY of HOUSTON
YOU ARE THE PRIDE
EARTH AND ATMOSPHERIC SCIENCES

Take attendance on CourseKey

Agenda

- Electrostatics
- Ohm's law
- Fundamentals of DC resistivity survey

Electrostatics

- Electric source charges are stationary

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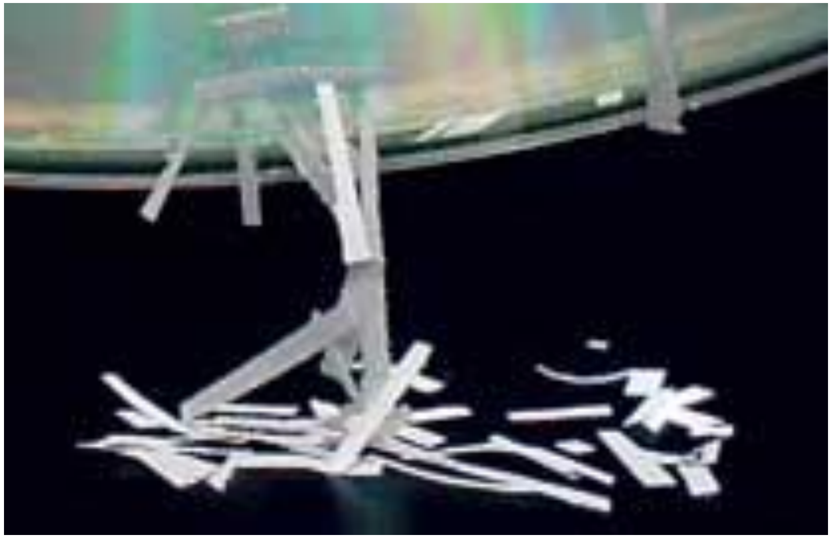
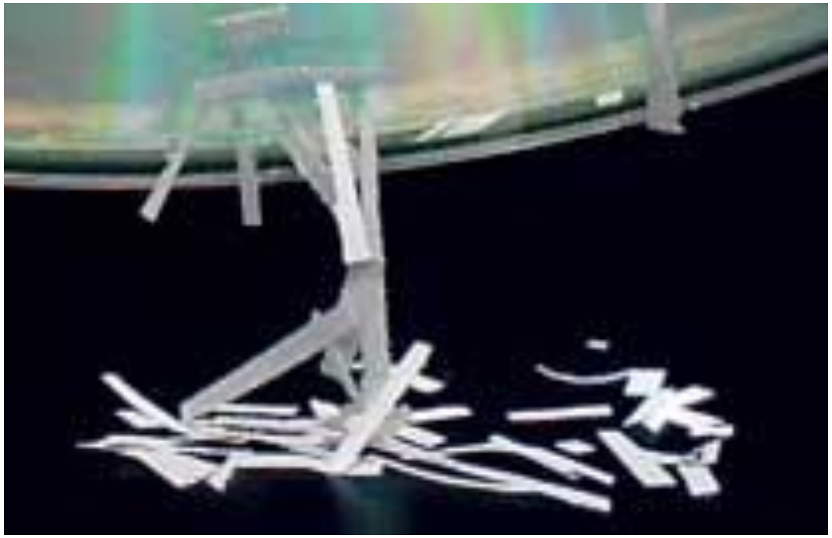


Paper strips attracted by a charged CD

https://en.wikipedia.org/wiki/Electrostatics

Electrostatics

- Electric source charges are stationary



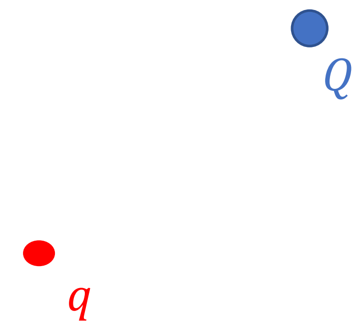
Paper strips attracted by a charged CD

Foam peanuts clinging to a cat's fur due to static electricity (caused by triboelectric effect).

https://en.wikipedia.org/wiki/Electrostatics

Coulomb's law

- The forces that electric charges exert on each other are described by Coulomb's law
- Based on experiments



ϵ_0: permittivity of free space

Coulomb's law

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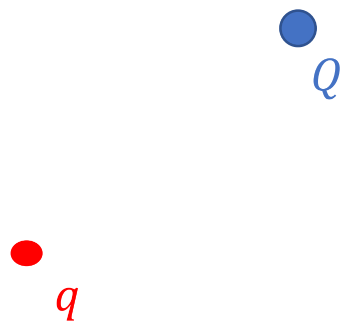
$$\boldsymbol{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}\hat{\boldsymbol{r}}$$



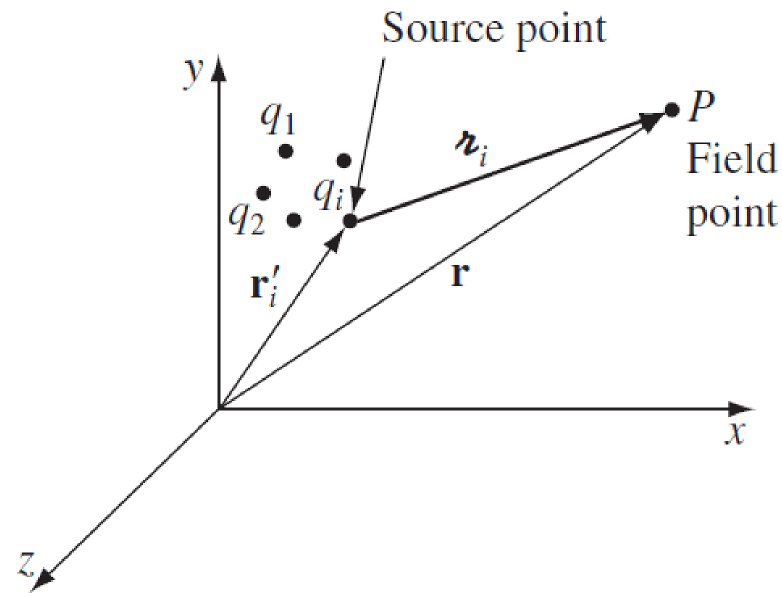
- The force is proportional to the product of the charges and inversely proportional to the square of their distance.

ϵ_0: permittivity of free space

Superposition

If we have several point charges q_1, q_2, …, q_n at distances r_1, r_2, …, r_n from Q

What is the total force on Q?



Griffiths, 4th edition, pp 61

Superposition

If we have several point charges q_1, q_2, ..., q_n at distances $r_1, r_2, \ldots, r_n$ from Q

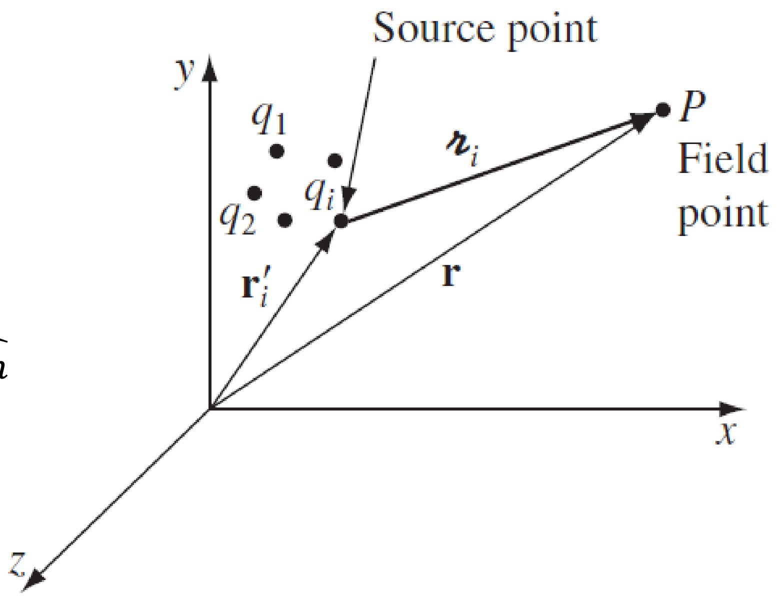
What is the total force on Q?

$$\boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \cdots \boldsymbol{F}_n$$
$$= \frac{1}{4\pi\epsilon_0}\frac{q_1 Q}{r_1^2}\widehat{\boldsymbol{r}_1} + \frac{1}{4\pi\epsilon_0}\frac{q_2 Q}{r_2^2}\widehat{\boldsymbol{r}_2} + \cdots + \frac{1}{4\pi\epsilon_0}\frac{q_n Q}{r_n^2}\widehat{\boldsymbol{r}_n}$$
$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{q_1}{r_1^2}\widehat{\boldsymbol{r}_1} + \frac{q_2}{r_2^2}\widehat{\boldsymbol{r}_2} + \cdots + \frac{q_n}{r_n^2}\widehat{\boldsymbol{r}_n}\right)$$

$$\boldsymbol{F} = Q\boldsymbol{E}$$

where

$$\boldsymbol{E} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\widehat{\boldsymbol{r}_i}$$



Griffiths, 4th edition, pp 61

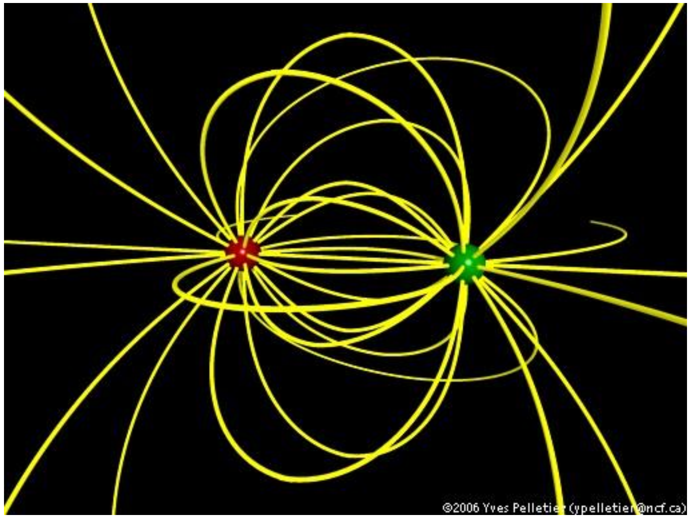
Electric field

- Electric field, $\boldsymbol{E}$, is a vector field
- Defined as the electrostatic force $\boldsymbol{F}$, divided by the magnitude of the test charge Q in Coulomb.

$$\boldsymbol{E} = \frac{\boldsymbol{F}}{Q}$$

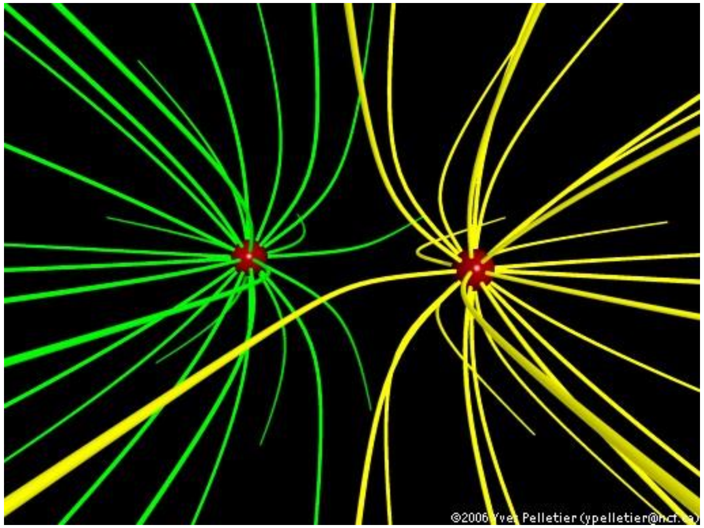
- You can think of electric field as a real physical entity, filling the space around electric charges.

Griffiths, 4th edition, pp 61



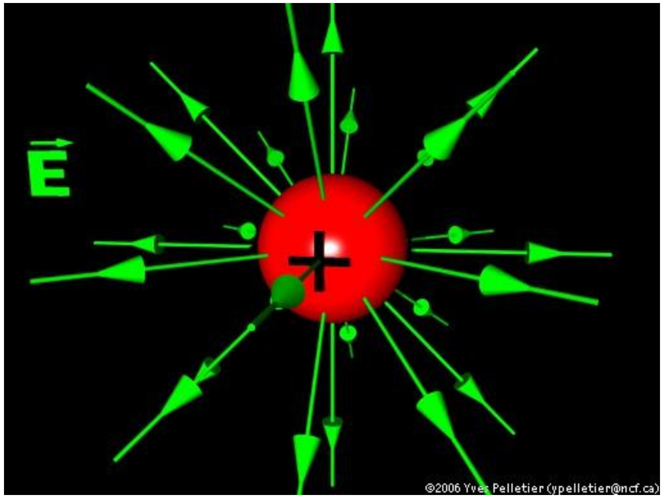
Electric field due to a dipole (i.e., two identical point charges of opposite signs)

http://web.ncf.ca/ch865/englishdescr/3DEFldDipole.html



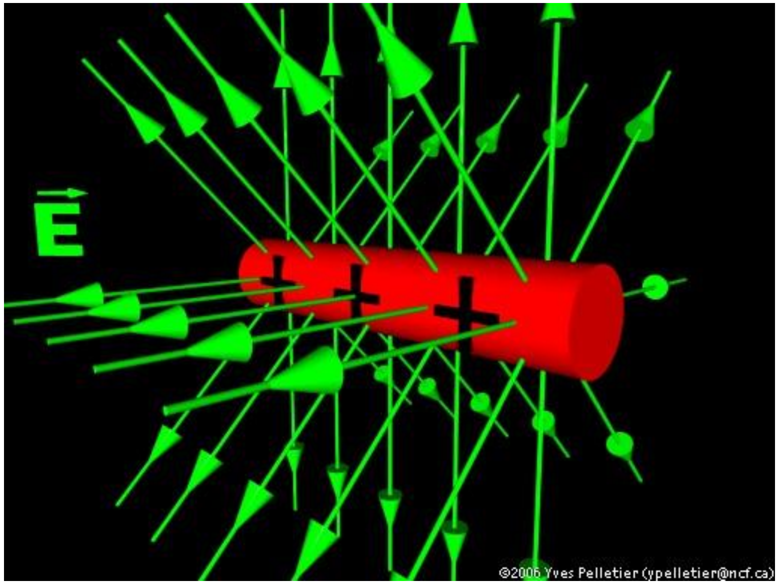
Electric field due to two identical point charges of the same signs

http://web.ncf.ca/ch865/englishdescr/3DEFldIdentCharges.html



Electric field due to a positively charged sphere

http://web.ncf.ca/ch865/englishdescr/EFldChargedSphere.html



Electric field due to a uniformly and positivity charged cylinder

http://web.ncf.ca/ch865/englishdescr/EFldChargedCylinder.html

More on electric field (divergence)

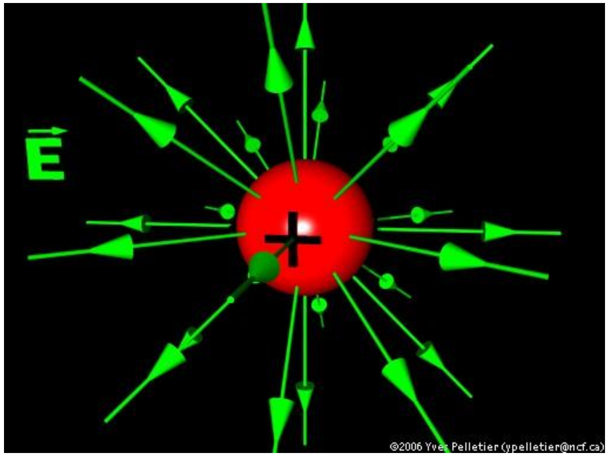
- Gauss's law for electric field

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$$

- To learn more, https://em.geosci.xyz/content/maxwell1_fundamentals/formative_laws/gauss_electric.html

More on electric field (curl)

- The curl of electric field



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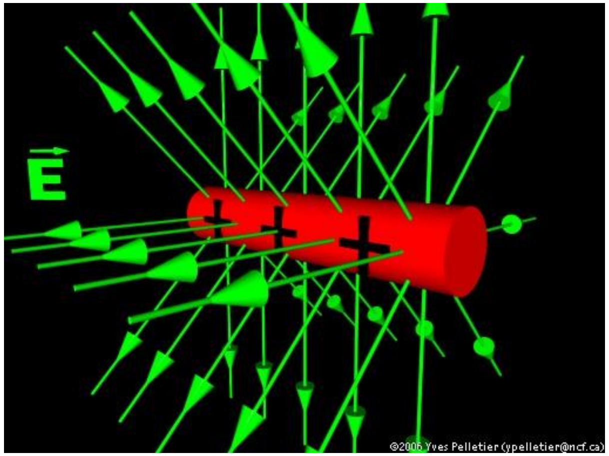

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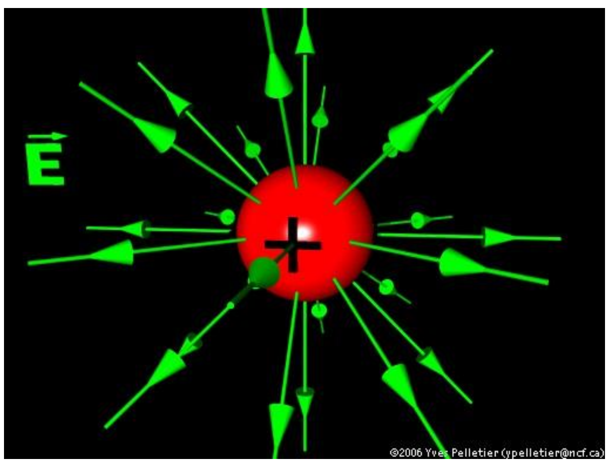
More on electric field

- The curl of electric field

$$\nabla \times \boldsymbol{E} = 0$$



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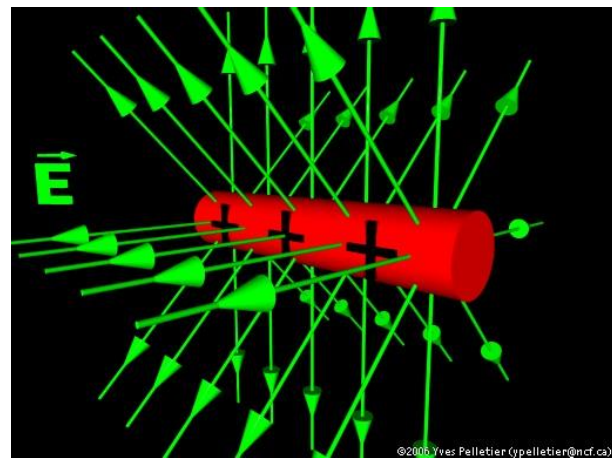

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Electric potential (optional material)

- Electric field $\boldsymbol{E}$ is very special kind of vector field because $\nabla \times \boldsymbol{E} = 0$
- Because of that, the line integral of $\boldsymbol{E}$ around any closed loop is 0 (that follows from Stokes' theorem)

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = 0$$

- Therefore, the line integral of $\boldsymbol{E}$ from point **a** point **b** is the same for all paths (i.e., independent of path).
- We can then define a function

$$V(\boldsymbol{r}) = \int_{o}^{r} \boldsymbol{E} \cdot d\boldsymbol{l}$$

- $V(\boldsymbol{r})$ depends only on the point $\boldsymbol{r}$. It is called **electric potential.**

Griffiths, 4th edition, pp 79

Electric potential

$$\boldsymbol{E} = \nabla V$$

Griffiths, 4th edition, pp 79

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Current

- An electric current is the flow of *electric charges*.
- In electric circuits, it is the flow of moving electrons in a wire. It can also be ions in an electrolyte.
- Current is measured as the amount of charge per unit time flowing across a surface.
- Unit: Ampere
- 1 Ampere = 1 Coulomb/second

Current density

- It is a vector
- Its magnitude is the amount of current per unit area of cross section.
- Its direction is in the direction of the current (i.e., the direction in which the charges move).

$$\boldsymbol{J} = \frac{d\boldsymbol{I}}{dA_{\perp}}$$

Electrical conductivity

- Electromagnetic methods are sensitive to electrical conductivity σ (Siemens/meter)

https://gpg.geosci.xyz/content/electromagnetics/electromagnetic_physical_properties.html

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- The inverse of conductivity is electrical resistivity, ρ (ohm·meters)

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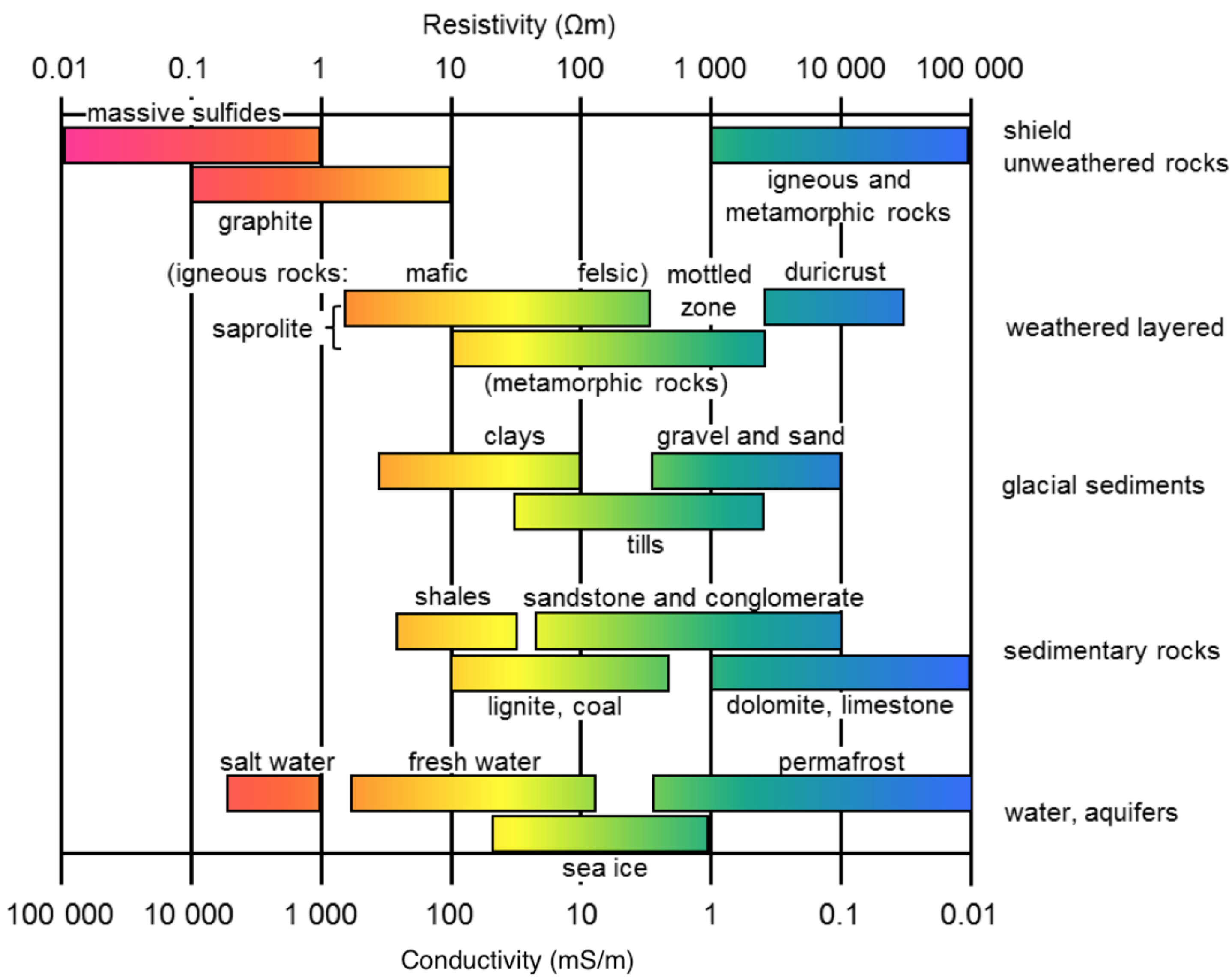
Electrical conductivity

- Electromagnetic methods are sensitive to electrical conductivity σ (Siemens/meter)
- The inverse of conductivity is electrical resistivity, ρ (ohm·meters)
- Electrical conductivity measures how easily the wire can transmit an electrical current.
- High values of resistivity imply that materials making up the wire is very resistant to the flow of electricity.

https://pburnley.faculty.unlv.edu/GEOL442_642/RES/NOTES/ResistivityNotes05Resistivity.html



Resistivity (Ωm)
0.01
0.1
1
10
100
1 000
10 000
100 000
massive sulfides
graphite
shield
unweathered rocks
igneous and
metamorphic rocks
(igneous rocks:
mafic
felsic)
mottled
zone
duricrust
saprolite
weathered layered
(metamorphic rocks)
clays
gravel and sand
glacial sediments
tills
shales
sandstone and conglomerate
sedimentary rocks
lignite, coal
dolomite, limestone
salt water
fresh water
permafrost
water, aquifers
sea ice
100 000
10 000
1 000
100
10
1
0.1
0.01
Conductivity (mS/m)

- Electrical resistivity (or, conductivity) is diagnostic
- We are interested in the subsurface resistivity.

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- We are interested in the subsurface resistivity.

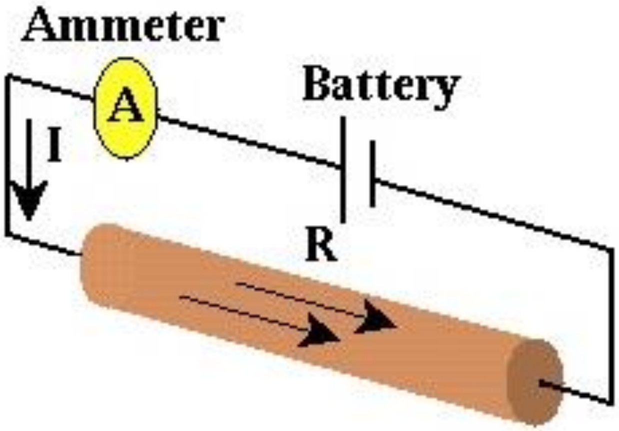
- Question: how to find out the resistivity distribution of the subsurface?

Ohm's law

- In 1827, Georg Ohm discovered an empirical relationship between the current flowing through a wire and the voltage required to drive that current.

https://en.wikipedia.org/wiki/Georg_Ohm

$$V = IR$$



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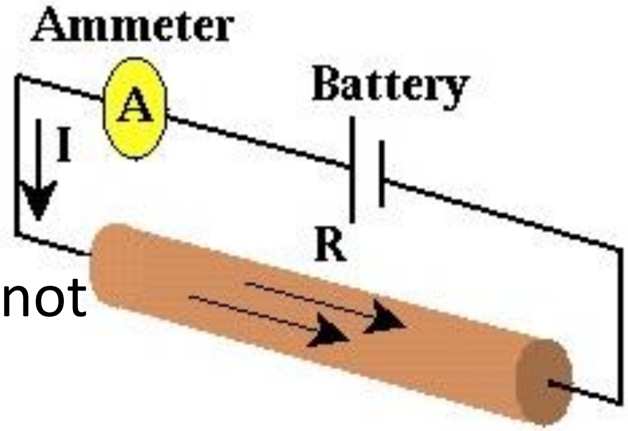
$$R = \frac{V}{I}$$



- Note that here R represents resistance, not the resistivity. In fact, resistivity $\rho = \frac{RA}{L}$

https://pburnley.faculty.unlv.edu/GEOL442_642/RES/NOTES/ResistivityNotes04Ohm.html

What does Ohm’s law tell us?

- Given current and voltage, we can estimate the resistance (which is related to resistivity).

Question

- Is it possible to apply this simple law to determine the electrical resitivities of the Earth materials in the subsurface?

Simple answer

- Yes!

Simple answer

- Yes!
- That is exactly what DC resistivity does.

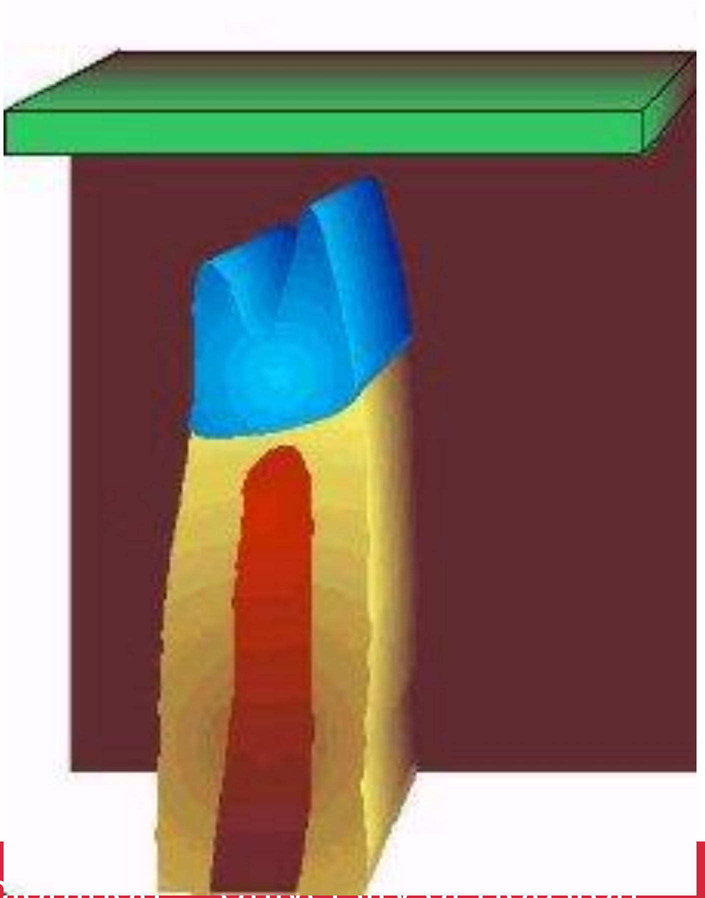
Basic Experiment

- **Target:**

  Ore body. Mineralized regions less resistive than host

Elura Orebody Electrical resistivities

Rock Type	*Ohm-m*
Overburden	12
Host rocks	200
Gossan	420
Mineralization (pyritic)	0.6
Mineralization (pyrrhotite)	0.6



Credit: Doug Oldenburg, Seogi Kang and Linsey Heagy from UBC-GIF

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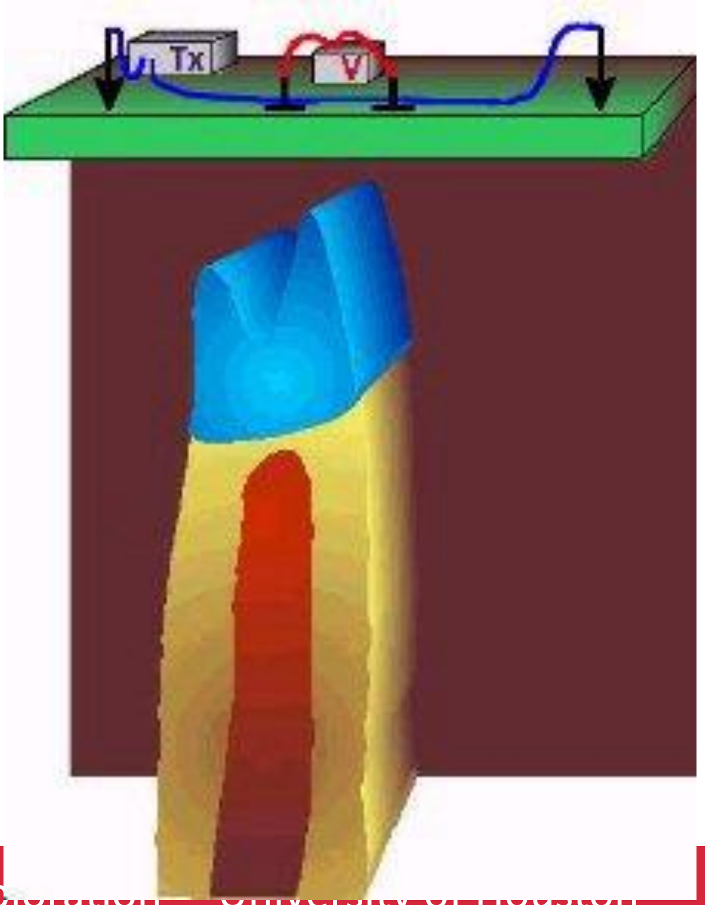

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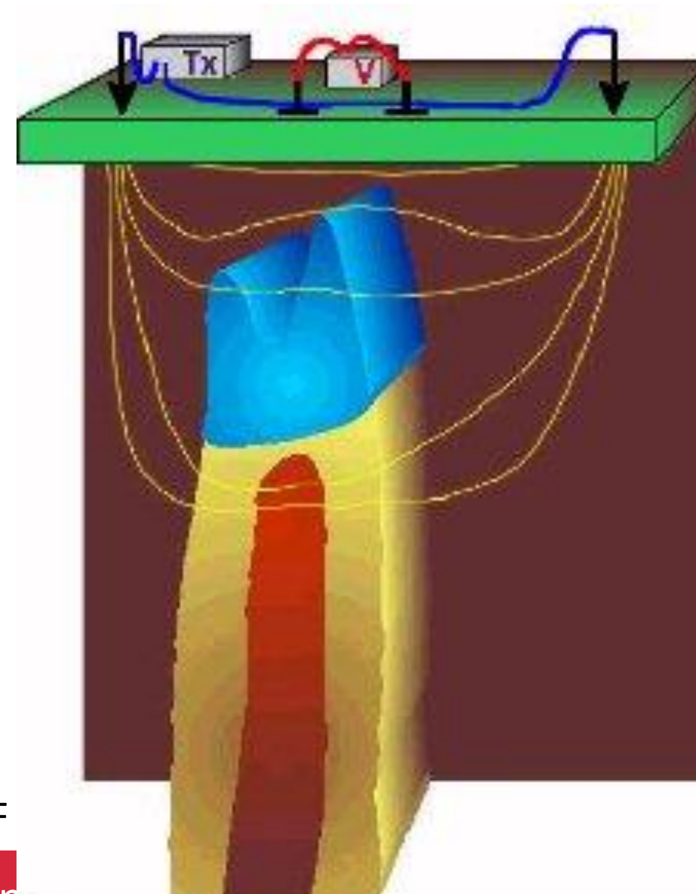

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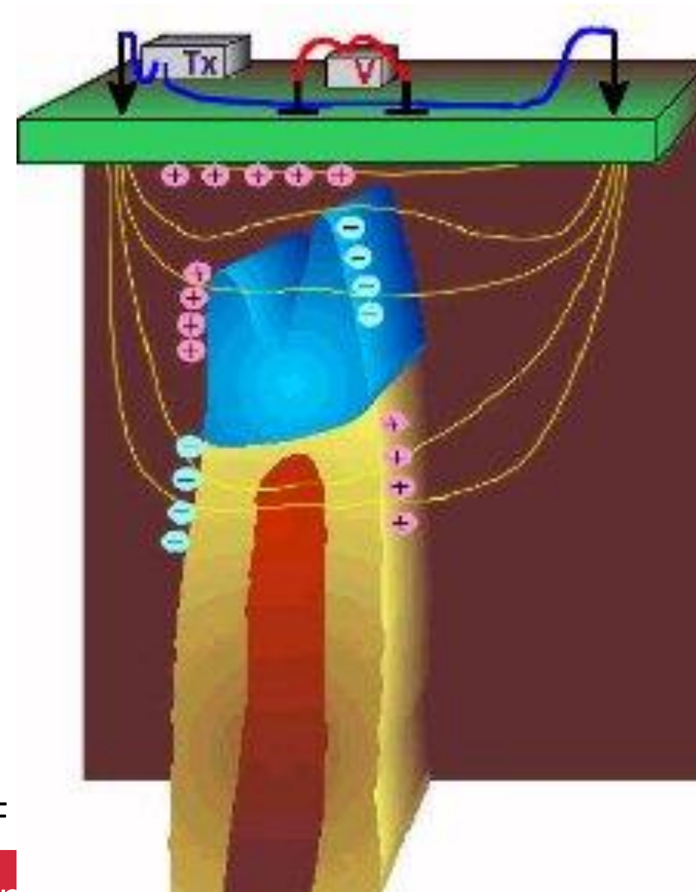
- **Charges:**
  Build up at interfaces



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Basic Experiment

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- **Setup:**
  Tx: Current electrodes
  Rx: Potential electrodes
- **Currents:**
  Preferentially flow through conductors
- **Charges:**
  Build up at interfaces
- **Potentials:**
  Associated with the charges are measured at the surface

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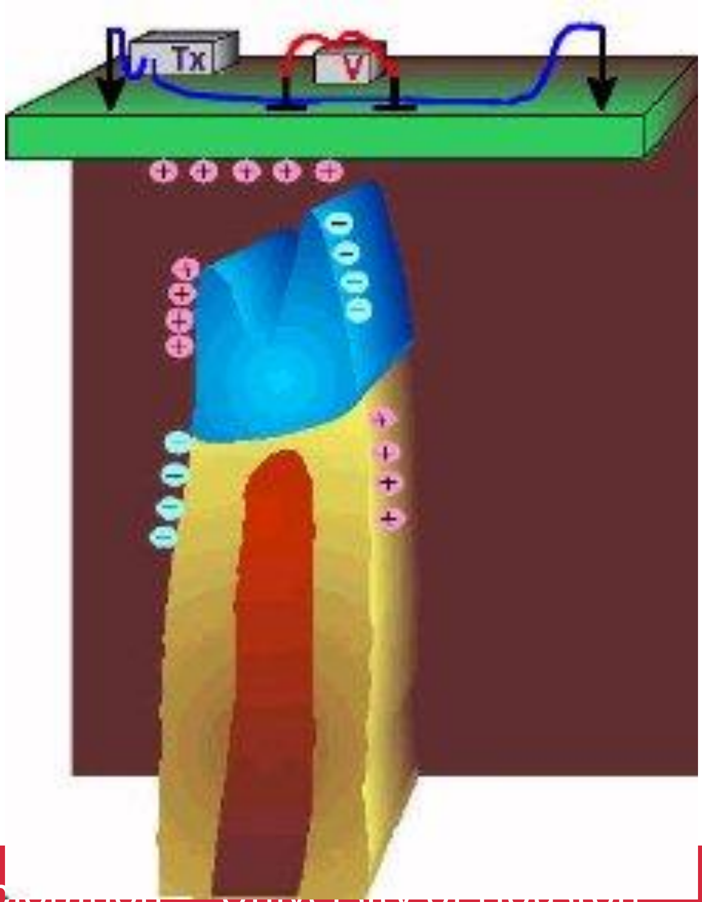

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How do we obtain resistivity?

Homogenous halfspace

- A battery (or a generator) is connected to the Earth by wires and two electrodes.
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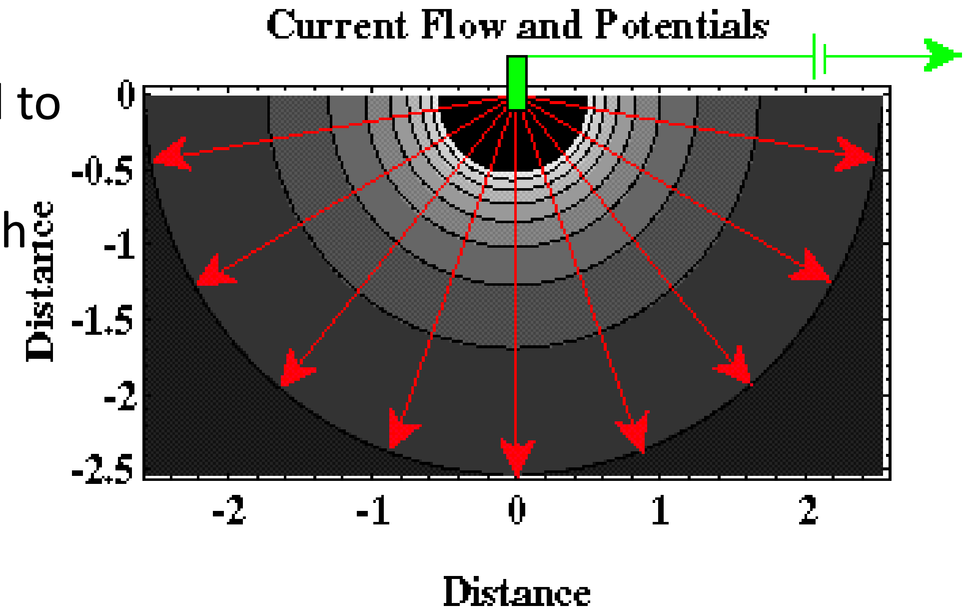

https://pburnley.faculty.unlv.edu/GEOL442_642/RES/NOTES/ResistivityNotes07CD.html

Red: current
Contour lines: equipotential.

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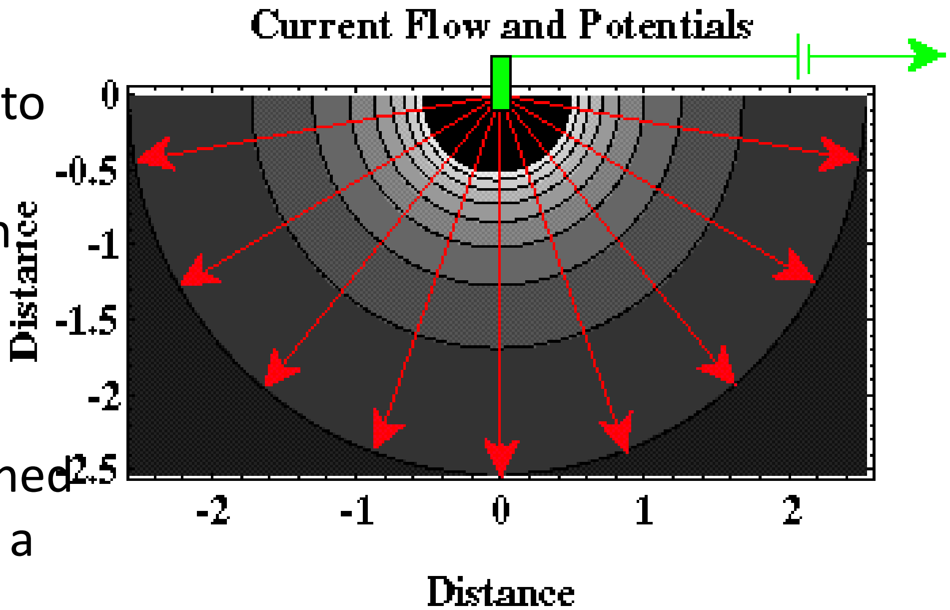

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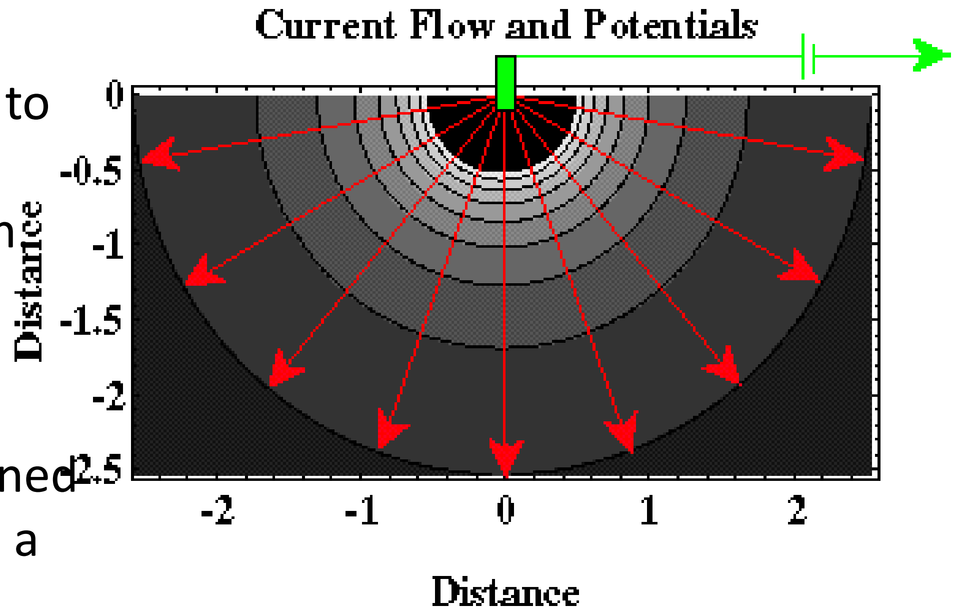

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Question:

- How to obtain resistivity?

Simple answer for uniform halfspace

$$\rho = \frac{2\pi r V}{I}$$

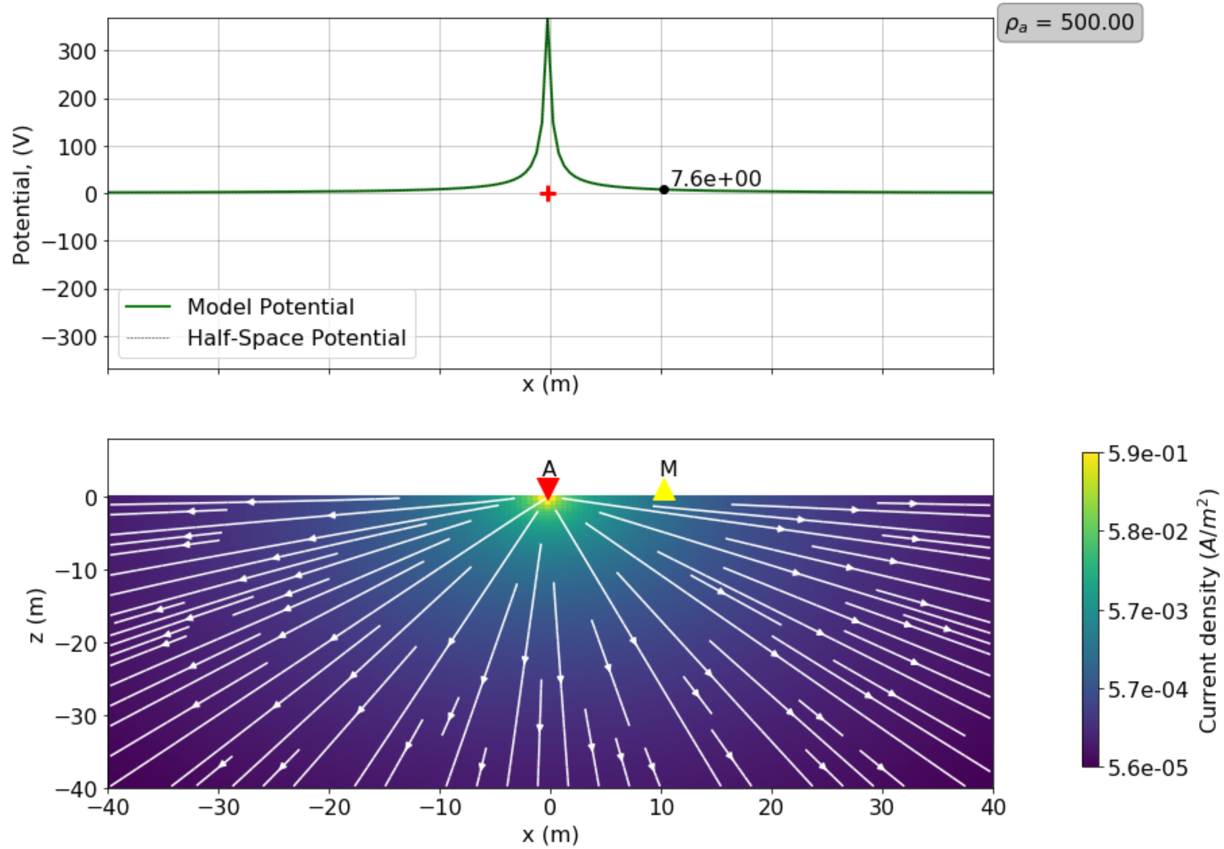


Image generated using DC_Plate2_5D. Pole-Pole. A -0.25 m, M 10.25 m. DC_Layer_Cylinder_2_5D would also do.

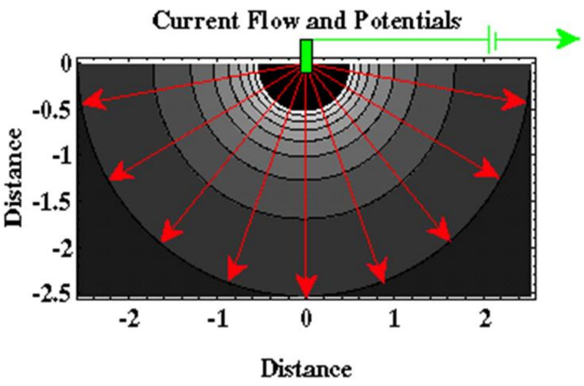
Special thanks to Thibaut Astic from UBC-GIF

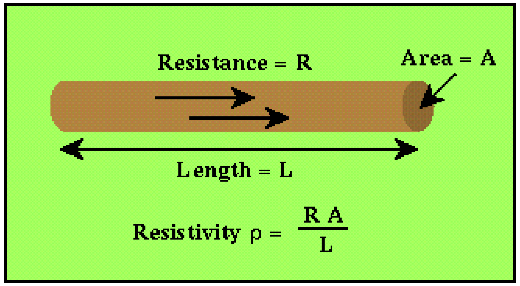
Credit: Doug Oldenburg, Seogi Kang and Linsey Heagy from UBC-GIF

Think about Ohm's law

$$V = IR$$

Note that R is resistance





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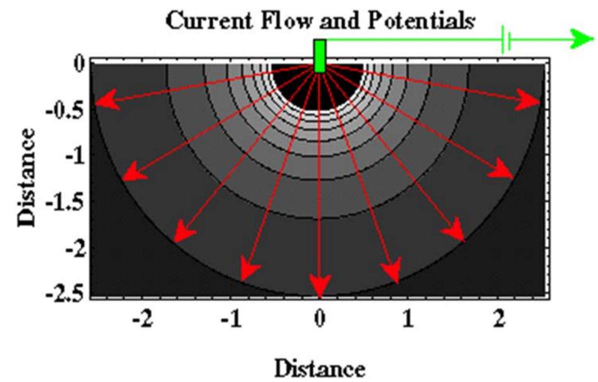

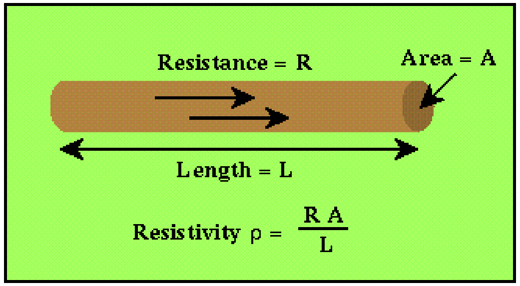

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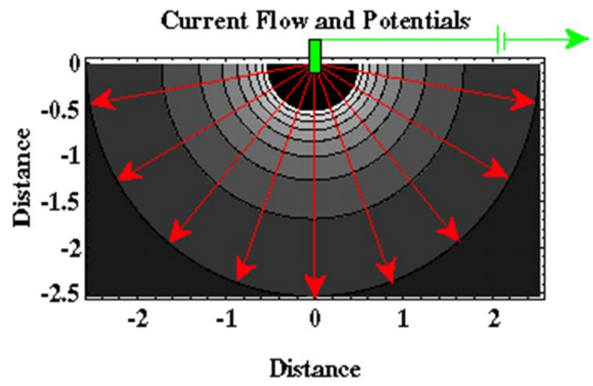

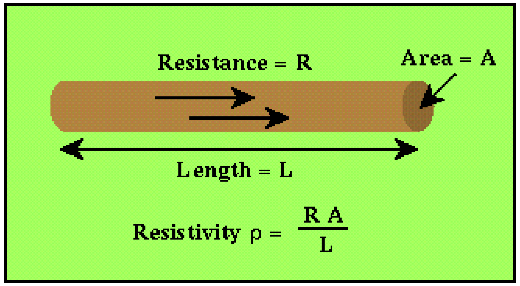

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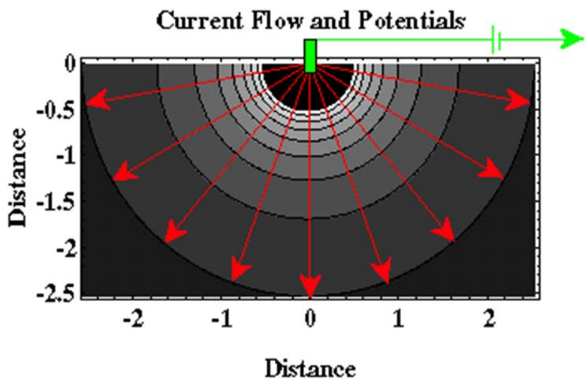

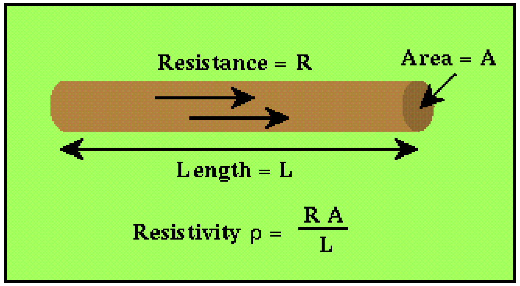

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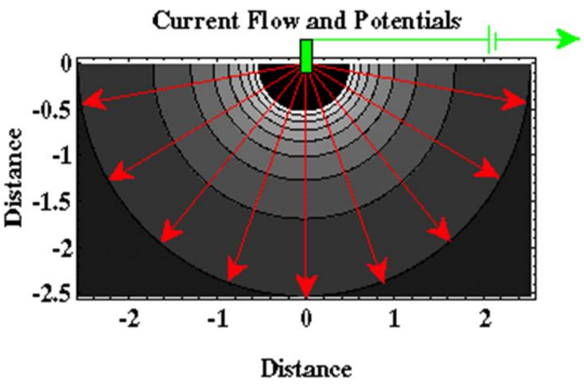
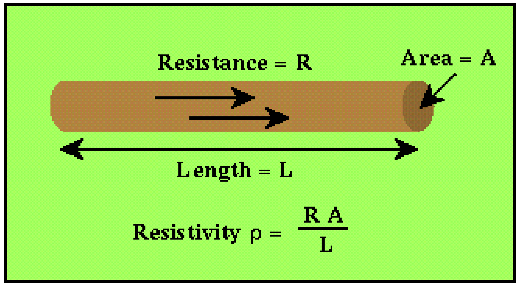
What is the L and A for our DC survey?





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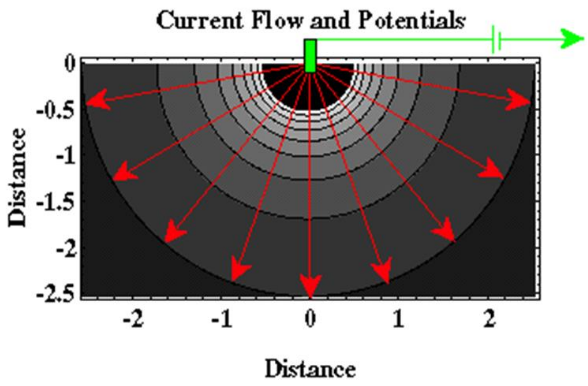
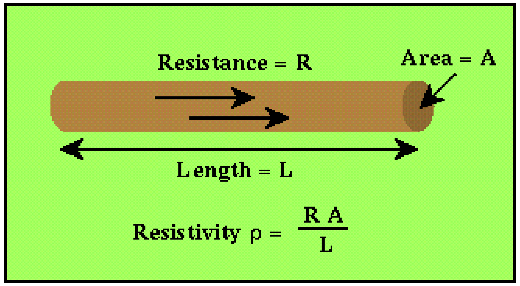
What is the L and A for our DC survey?

$$L = r$$
$$A = 2\pi r^2$$





https://pburnley.faculty.unlv.edu/GEOL442_642/RES/NOTES/ResistivityNotes05Resistivity.html

Think about Ohm’s law

$$V = IR$$

Note that R is resistance

But we want to relate potential to resistivity

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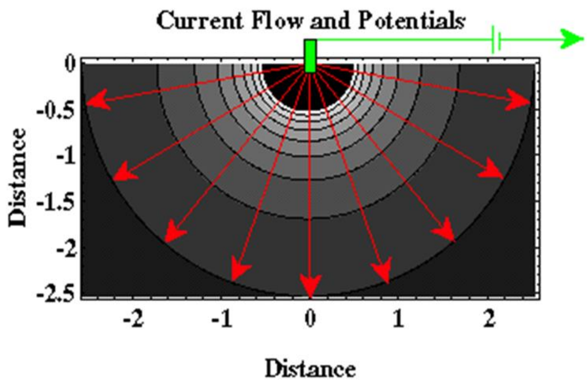
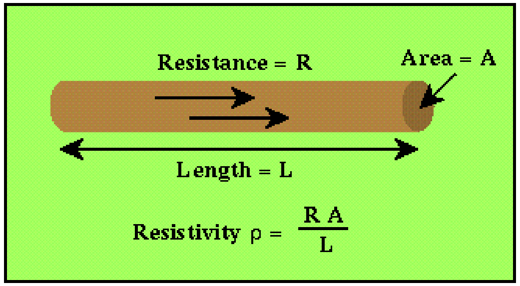
Therefore, $R = \frac{\rho L}{A} = \frac{\rho}{2\pi r}$





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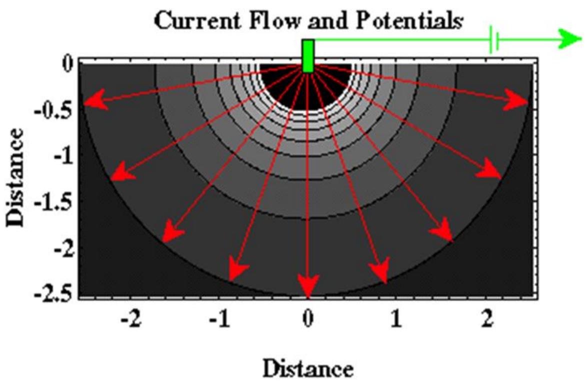
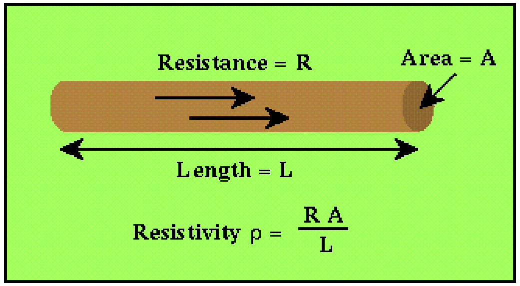
$$A = 2\pi r^2$$

Therefore, $R = \frac{\rho L}{A} = \frac{\rho}{2\pi r}$

Therefore, $V = IR = \frac{\rho I}{2\pi r}$





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Simple answer for uniform halfspace

$$V = \frac{\rho I}{2\pi r}$$

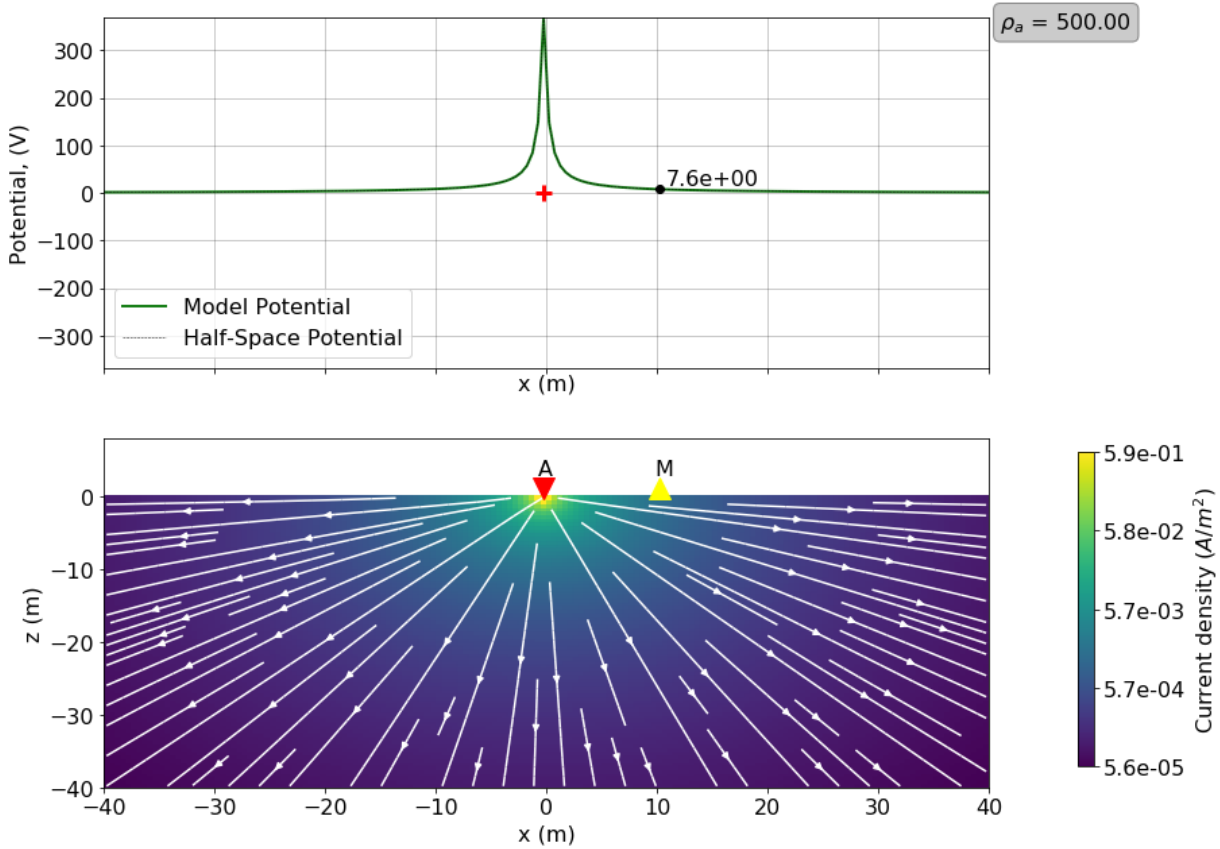


Image generated using DC_Plate2_5D. Pole-Pole. A -0.25 m, M 10.25 m. DC_Layer_Cylinder_2_5D would also do.

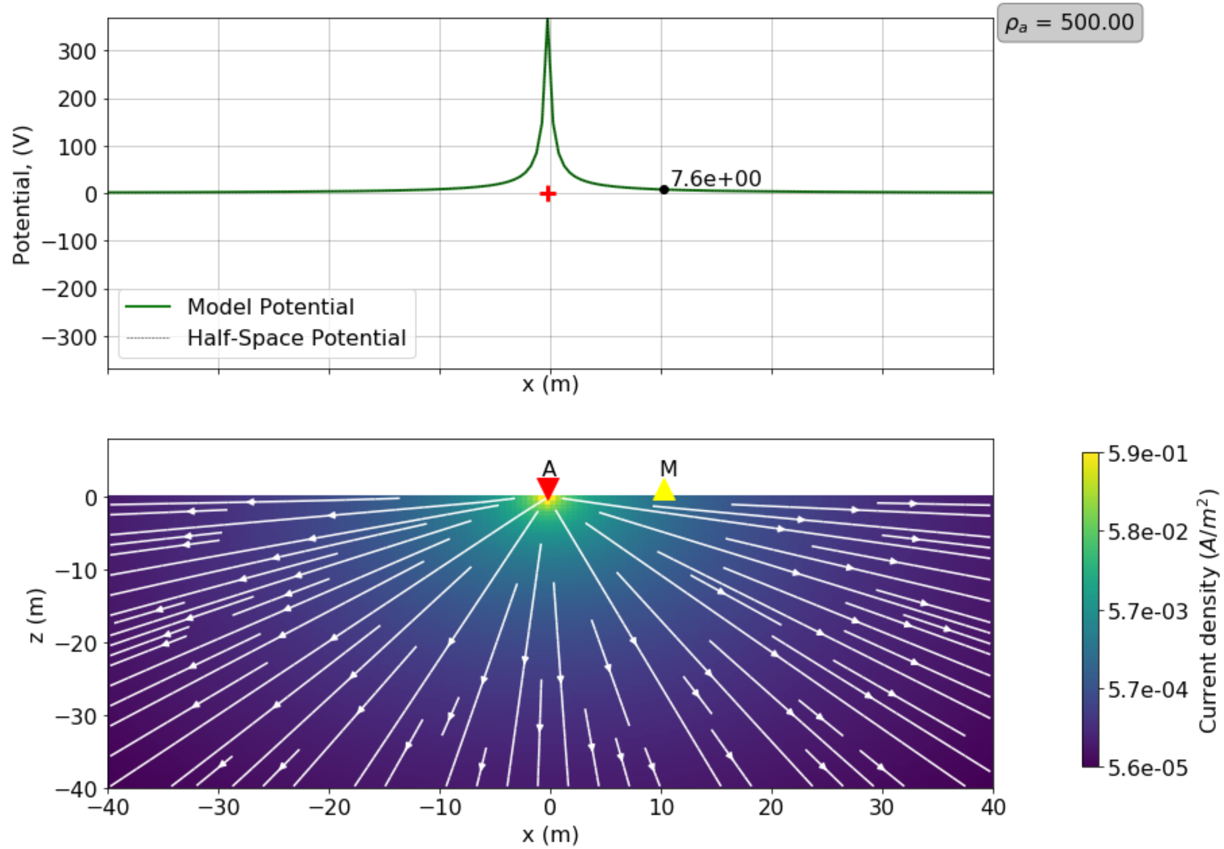
Special thanks to Thibaut Astic from UBC-GIF

Credit: Doug Oldenburg, Seogi Kang and Linsey Heagy from UBC-GIF

Simple answer for uniform halfspace

$$V = \frac{\rho I}{2\pi r}$$

$$\rho = \frac{2\pi r V}{I}$$



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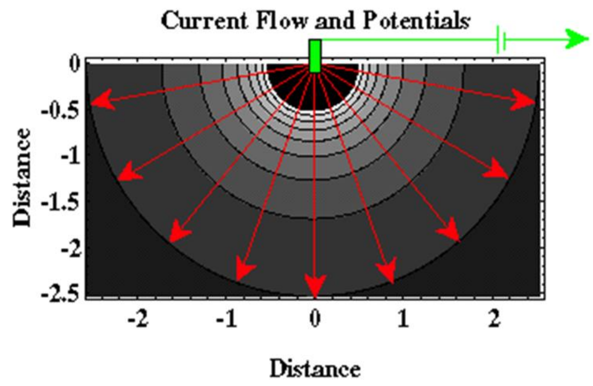
Some math (optional)



- Start from Ohm's law

$$\boldsymbol{J} = \sigma \boldsymbol{E}$$

- Thus,

$$\boldsymbol{E} = \frac{\boldsymbol{J}}{\sigma} = \rho \boldsymbol{J}$$

- What is $\boldsymbol{J}$?

$$\boldsymbol{J} = \frac{I}{2\pi r^2}\hat{\boldsymbol{r}}$$

- Then,

$$\boldsymbol{E} = \rho \boldsymbol{J} = \frac{\rho I}{2\pi r^2}\hat{\boldsymbol{r}}$$

- Remember that potential is defined as the work done by the electric field to move a unit of positive charge from a reference point (e.g., infinity) to a specific point inside the field.

- Therefore,

$$V = \int_{\infty}^{P} \boldsymbol{E} \cdot d\boldsymbol{l}$$

$$= -\int_{\infty}^{P} \frac{\rho I}{2\pi r^2}\hat{\boldsymbol{r}} d\boldsymbol{r}$$

$$= -\int_{\infty}^{P} \frac{\rho I}{2\pi r^2} dr$$

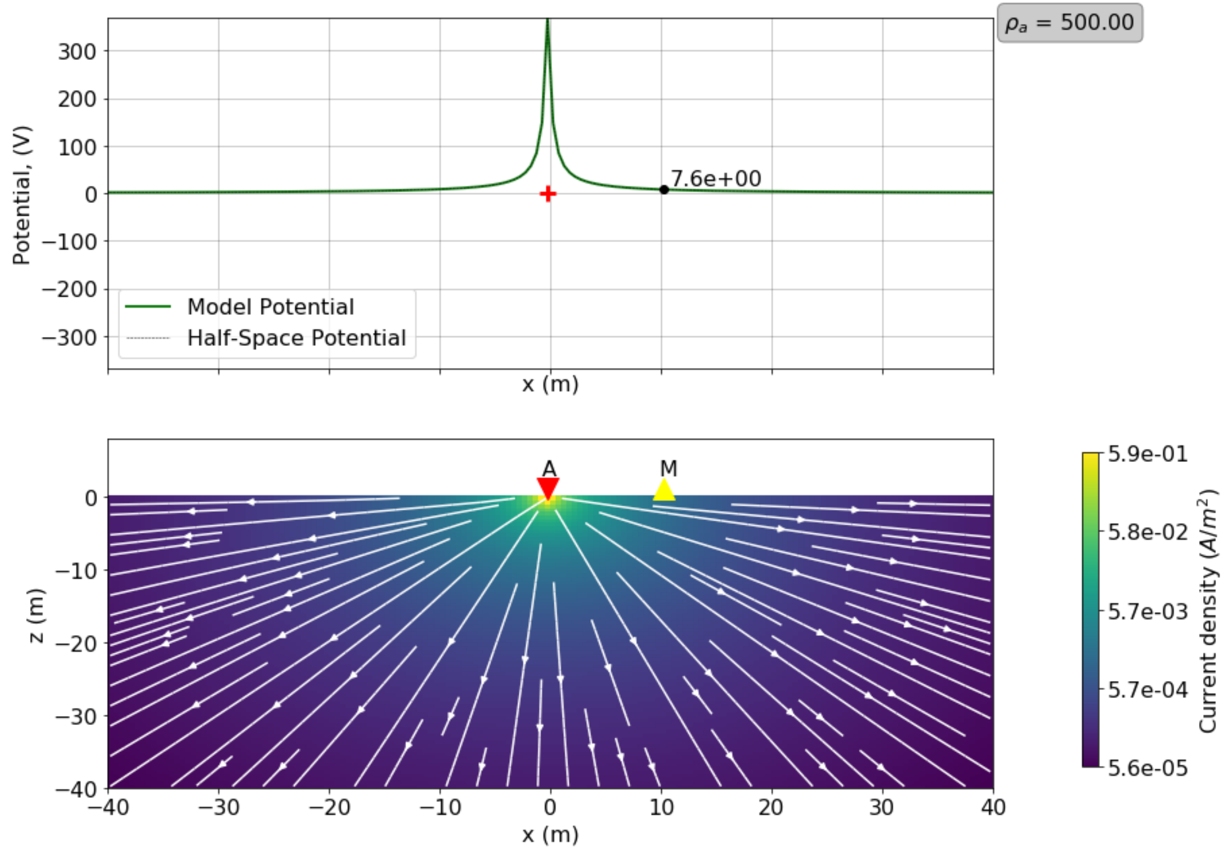
$$= \frac{\rho I}{2\pi r}$$

A summary thus far

$$V = \frac{\rho I}{2\pi r}$$

$$\rho = \frac{2\pi r V}{I}$$

If we know current I
then, measure the potential value at any location
We can derive the resistivity of the Earth!!!



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A general formulation (optional)

- Ohm's law

$$\boldsymbol{J} = \sigma \boldsymbol{E}$$

- Electric potential

$$\boldsymbol{E} = \nabla V$$

- Continuity equation

$$\nabla \cdot \boldsymbol{J} = -\frac{\partial \rho}{\partial t}$$

where ρ is free charge density

- Therefore,

$$\nabla \cdot (\sigma \nabla V) = -\frac{\partial \rho}{\partial t}$$

- Remember that we want to relate current I, conductivity σ and electric potential V

$$\nabla \cdot (\sigma \nabla V) = -\frac{I}{\Delta v}\delta(\boldsymbol{r} - \boldsymbol{r}_s)$$

- Applies to 3D inhomogeneous Earth

Optional materials on continuity equation

- Start from Ampere-Maxwell equation

$$\nabla \times \mathbf{h} = \boldsymbol{j}_f + \frac{\partial \mathbf{d}}{\partial t}$$

- Vector identity

$$\nabla \cdot (\nabla \times \mathbf{h}) = 0$$

- Apply divergence to both sides of Amphere-Maxwell equation

$$\nabla \cdot \left(\boldsymbol{j}_f + \frac{\partial \mathbf{d}}{\partial t} \right) = 0$$

$$\frac{\partial (\nabla \cdot \boldsymbol{d})}{\partial t} = -\nabla \cdot \boldsymbol{j}_f$$

- Apply Gauss's law for electric field

$$\nabla \cdot \boldsymbol{j}_f = -\frac{\partial \rho_f}{\partial t}$$

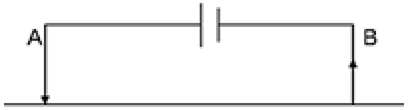
Optional materials on continuity equation

$$\nabla \cdot \boldsymbol{j}_f = -\frac{\partial \rho_f}{\partial t}$$

- A different derivation is given in Griffiths, 4th edition, pp 222
- This is called continuity equation
- It is the precise mathematical statement of local charge conservation.
- The left side of equation is the divergence of the electric current density. It measures how much current is flowing out.
- If divergence is positive, then more charge is exiting than entering a specified volume. Then, the amount of charge within the volume must be decreasing. This is exactly what the right side of the equation measures – how much electric charge is leaving or accumulating in a volume.

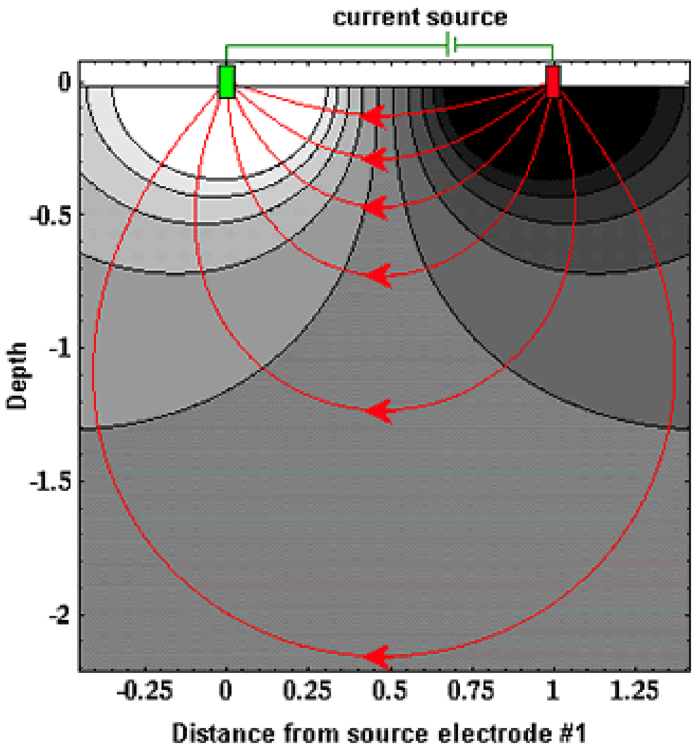
http://maxwells-equations.com/equations/continuity.php

Two electrode current sources



- Recall, for single current electrode,

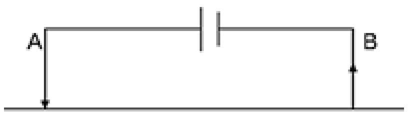
$$V = IR = \frac{\rho I}{2\pi r}$$



Observation:

- Current flows along the curved paths connecting the two electrodes

Two electrode current sources



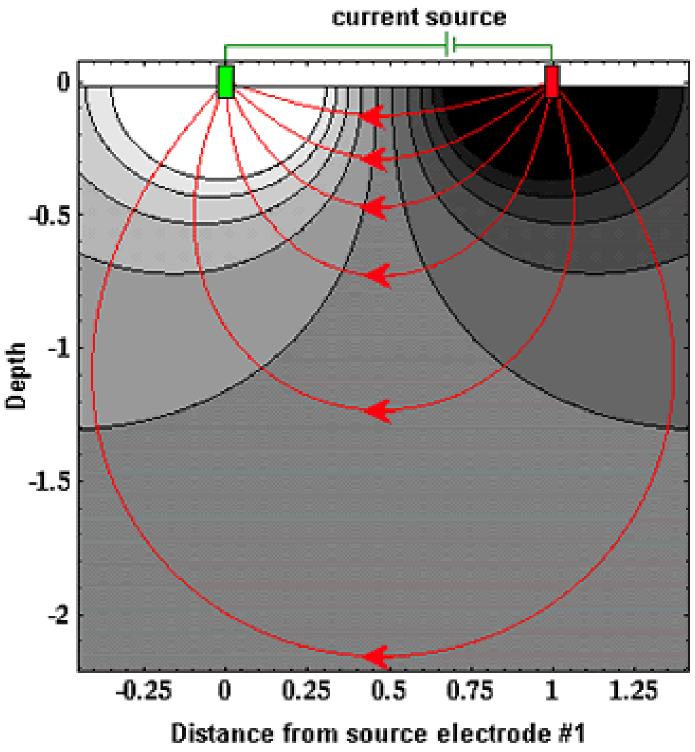
- Recall, for single current electrode,

$$V = IR = \frac{\rho I}{2\pi r}$$

- Following the superposition principle (Griffiths, 4th edition, pp 82), the potential at position M due to two current electrodes, A and B is,

$$V = \frac{\rho I}{2\pi}\left(\frac{1}{r_{AM}} - \frac{1}{r_{BM}}\right)$$

- Note that we still assume a homogeneous earth.



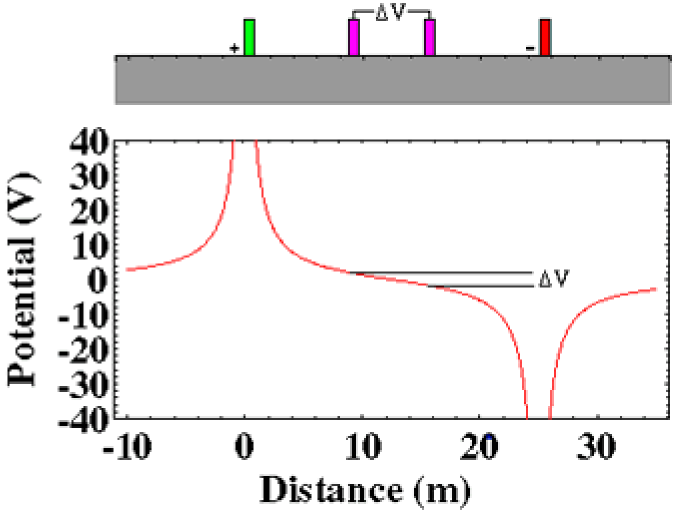
Observation:

- Current flows along the curved paths connecting the two electrodes

Voltage

- What we measure in practice is voltage (using voltmeter)
- That is, potential difference

What is the potential difference between potential electrodes M and N?



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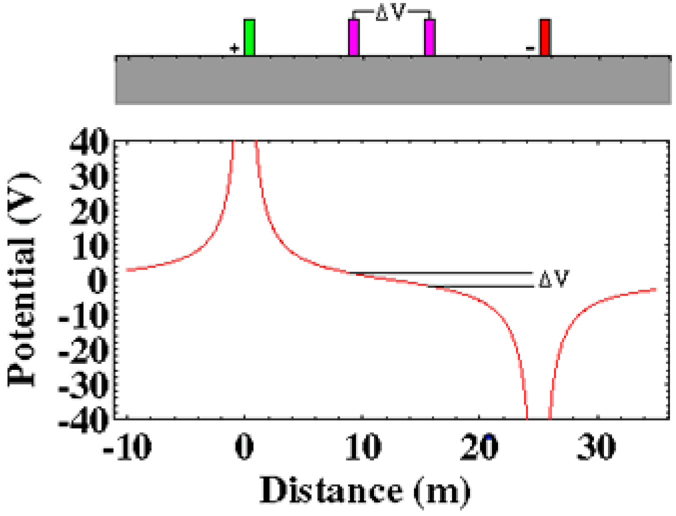

What is the potential difference between potential electrodes M and N?

$$V_M = \frac{\rho I}{2\pi}\left(\frac{1}{r_{AM}} - \frac{1}{r_{BM}}\right)$$

Voltage

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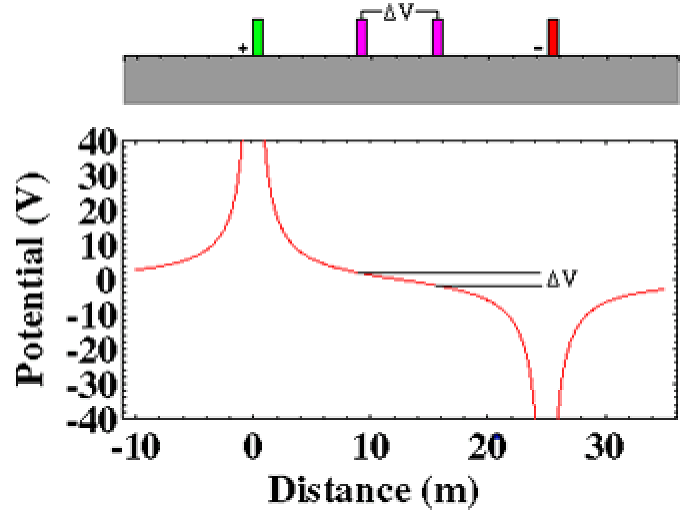

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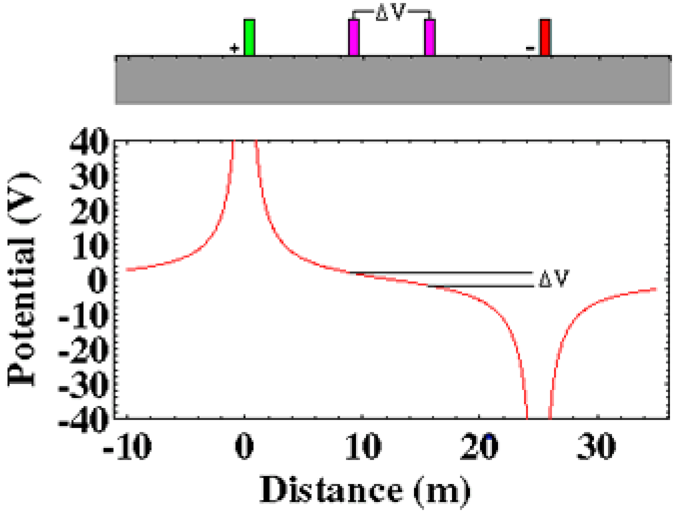

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$$V_N = \frac{\rho I}{2\pi}\left(\frac{1}{r_{AN}} - \frac{1}{r_{BN}}\right)$$

$$\Delta V_{MN} = V_M - V_N = \rho I \frac{1}{2\pi}\left(\frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}}\right)$$

From voltage to resistivity

- Remember that, we want to estimate resistivity from the measured voltage

$$\Delta V_{MN} = V_M - V_N = \rho I \frac{1}{2\pi} (\frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}})$$

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$$\rho = \frac{\Delta V_{MN}}{IG}$$