The livestream will begin shortly...

softwareunderground.org presents

TRANSFORM 2021

Virtual Conference on the Digital Subsurface, 16–23 April

supported by

TRANSFORM 2021

Thank You!

Matt Hall
Dieter Werthmuller
& Transform 2021 organizers

supported by

aws STUDIO X Agile*

Materials: *http://bit.ly/transform-2021-slides*

Slack: *swu.ng/slack > #t21-tue-inversion-for-geologists*

supported by

Inversion for geologists

Seogi Kang, Doug Oldenburg, Lindsey Heagy,
Dominique Fournier, Joe Capriotti & the SimPEG team

Collaborators

Doug

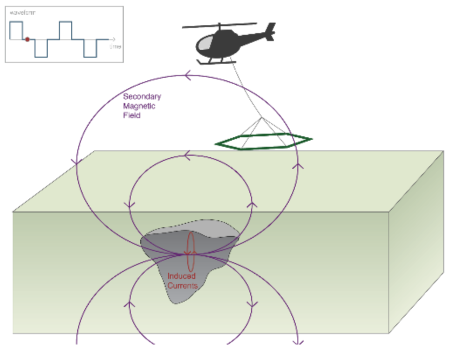
Lindsey

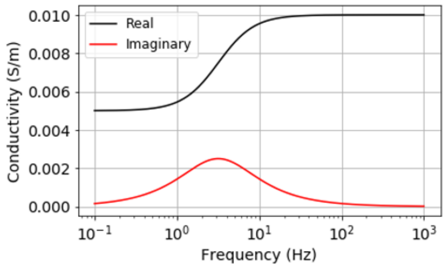
Dom

Joe

👋 hello (a bit about me)

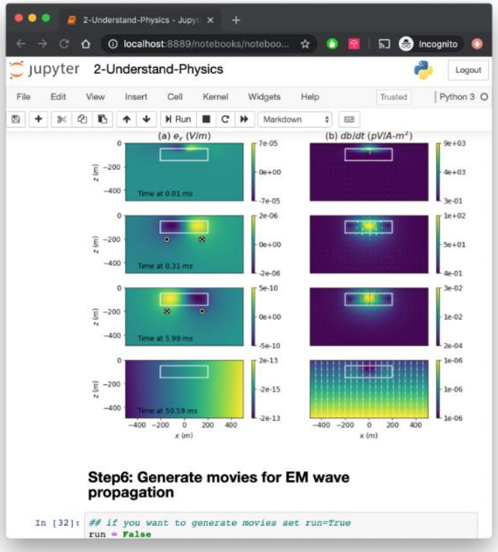
Computational EM geophysics





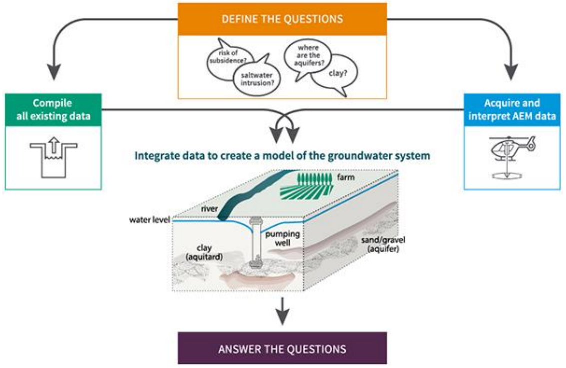
THE UNIVERSITY OF BRITISH COLUMBIA

Open-source software



simpeg

Groundwater science & management



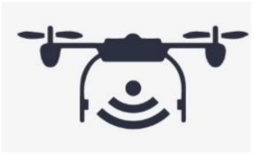
Stanford University

Challenging geoscience problems that we faced …

Increasing data volume and complexity

Airborne sensors

airborne geophysics



drone geophysics

Satellite sensors

Sentinel-2

GRACE

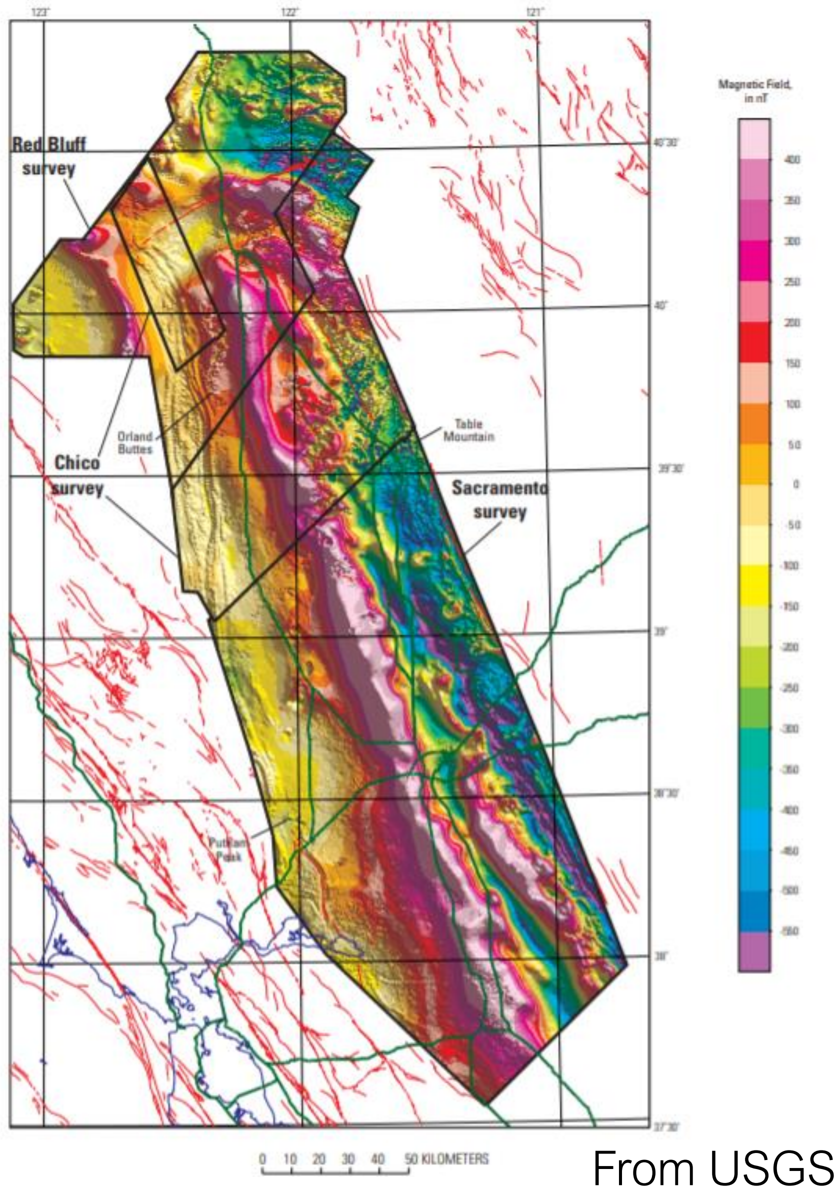
MODIS

Data are *publicly* available, but extracting *useful information* from these data are *challenging*

Airborne geophysics

- Potential fields
  - Magnetics
  - Gravity
- Electromagnetics
- (Ice) Radar

Increasing
Resolution



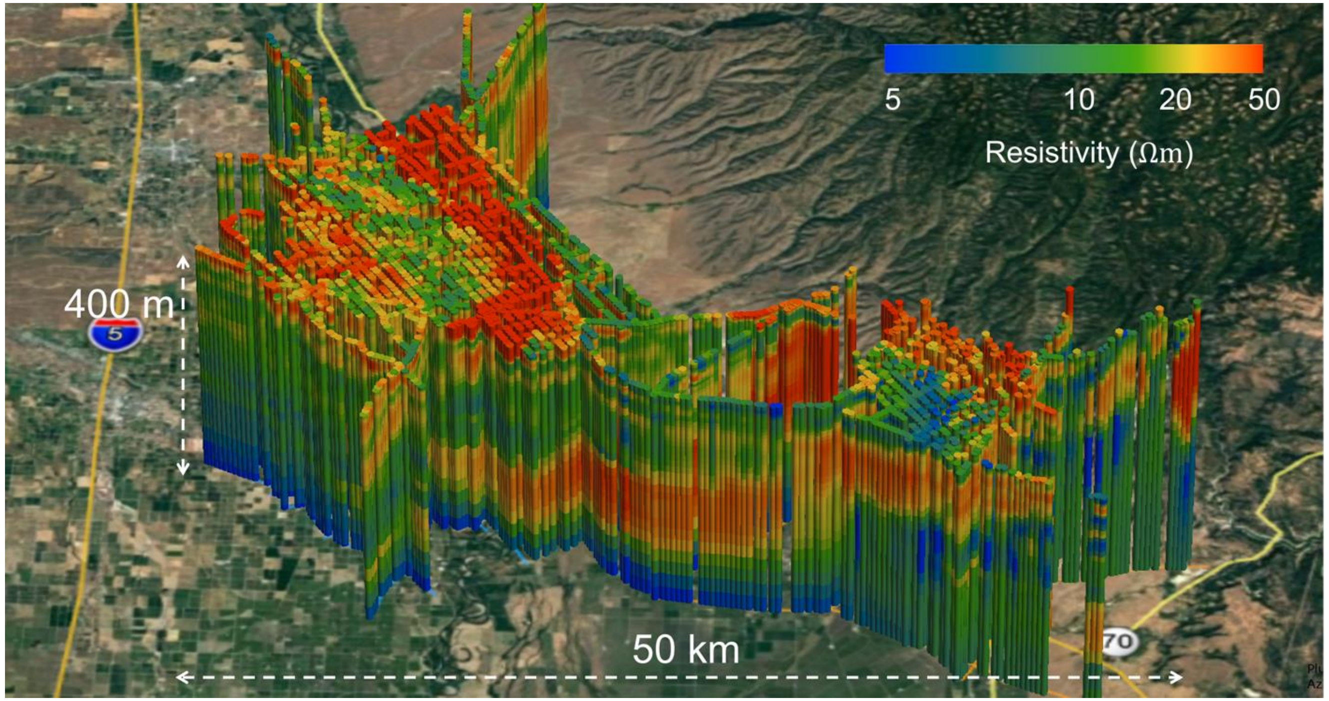
Hundreds of km

From USGS

Airborne geophysics

- Potential fields
  - Magnetics
  - Gravity
- Electromagnetics
- (Ice) Radar

Increasing Resolution

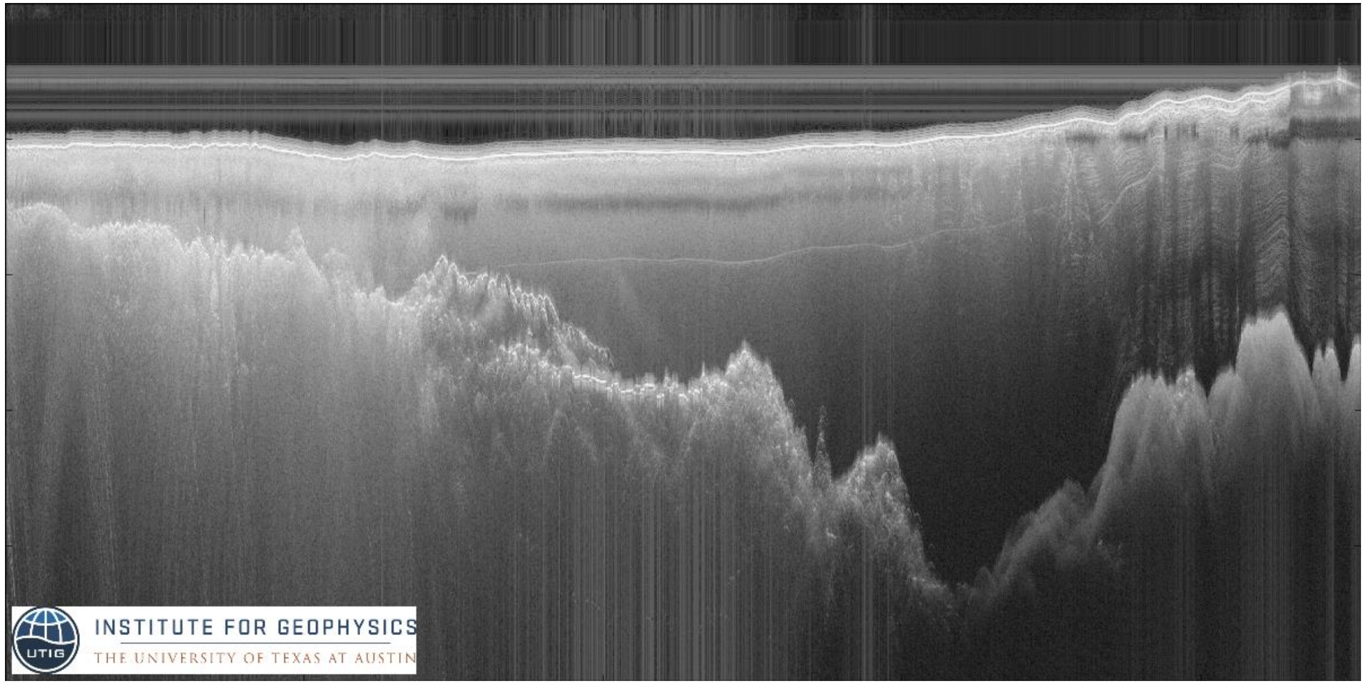


Kang et al. (2021)

Airborne geophysics

- Potential fields
  - Magnetics
  - Gravity
- Electromagnetics
- (Ice) Radar

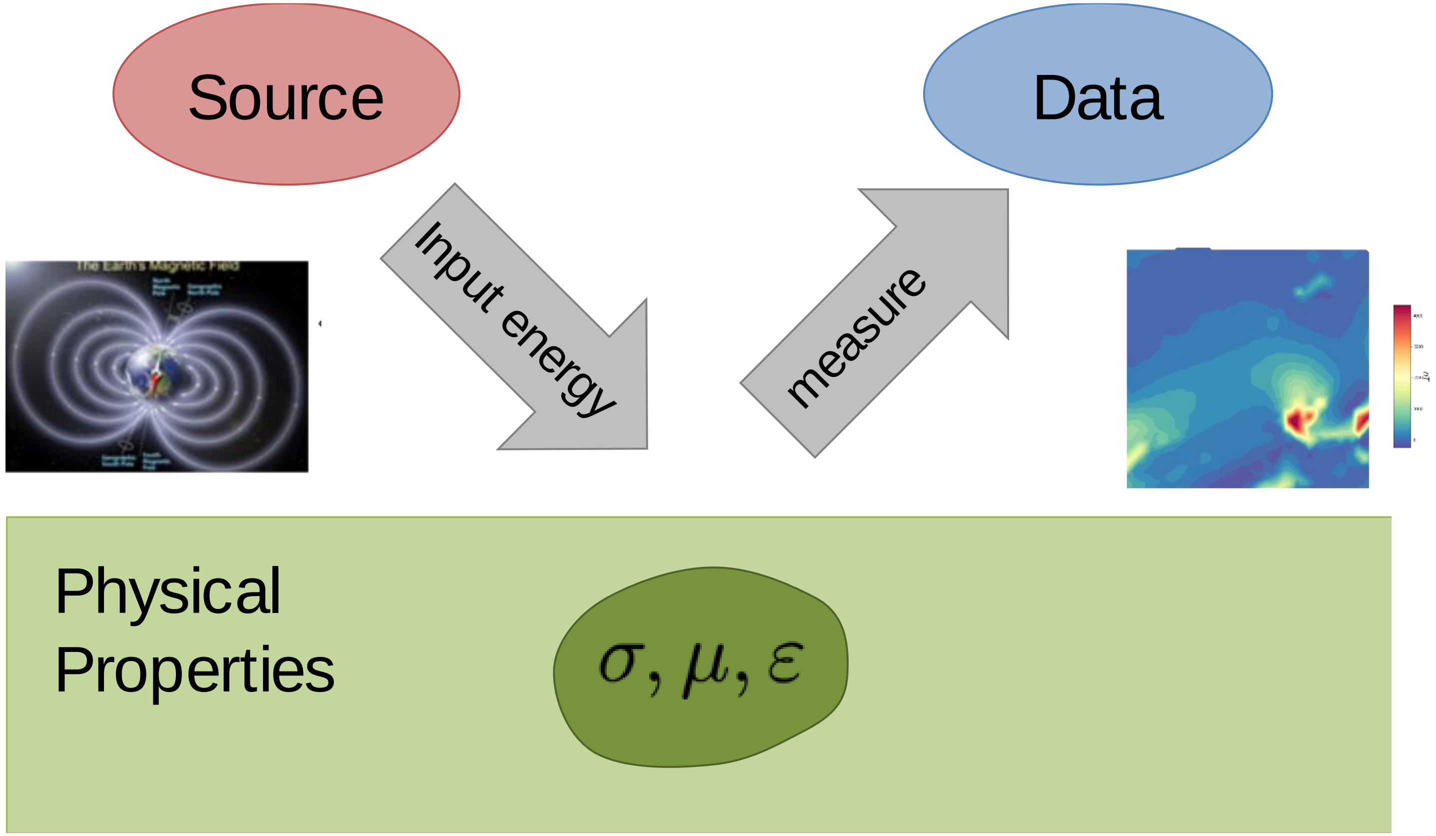
Increasing Resolution



Lindzey (2015)

Generic geophysical experiment?

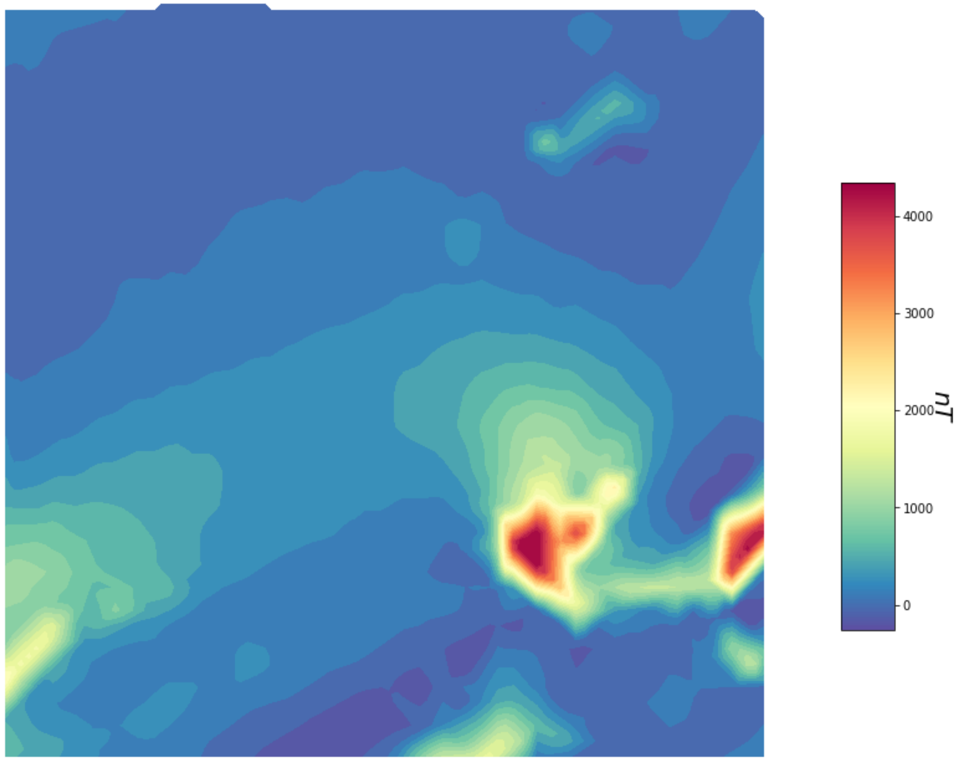
All require ways to see into the earth without direct sampling



An overarching question today is …

How do we find a subsurface model from the observed data in *a data-driven way*?

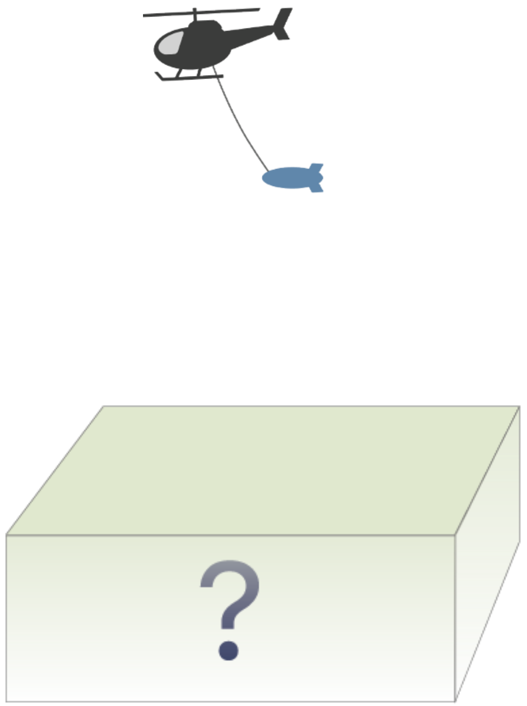
Observed data



A subsurface model

Outline

- Backgrounds: Magnetics
- Inversion Framework
- 1D Linear Inverse problem
- 3D Magnetic Inversion
- Including Geologic Information
- Summary



Python Packages that I am going to use today…

Numerical engine for geophysical simulation & inversion

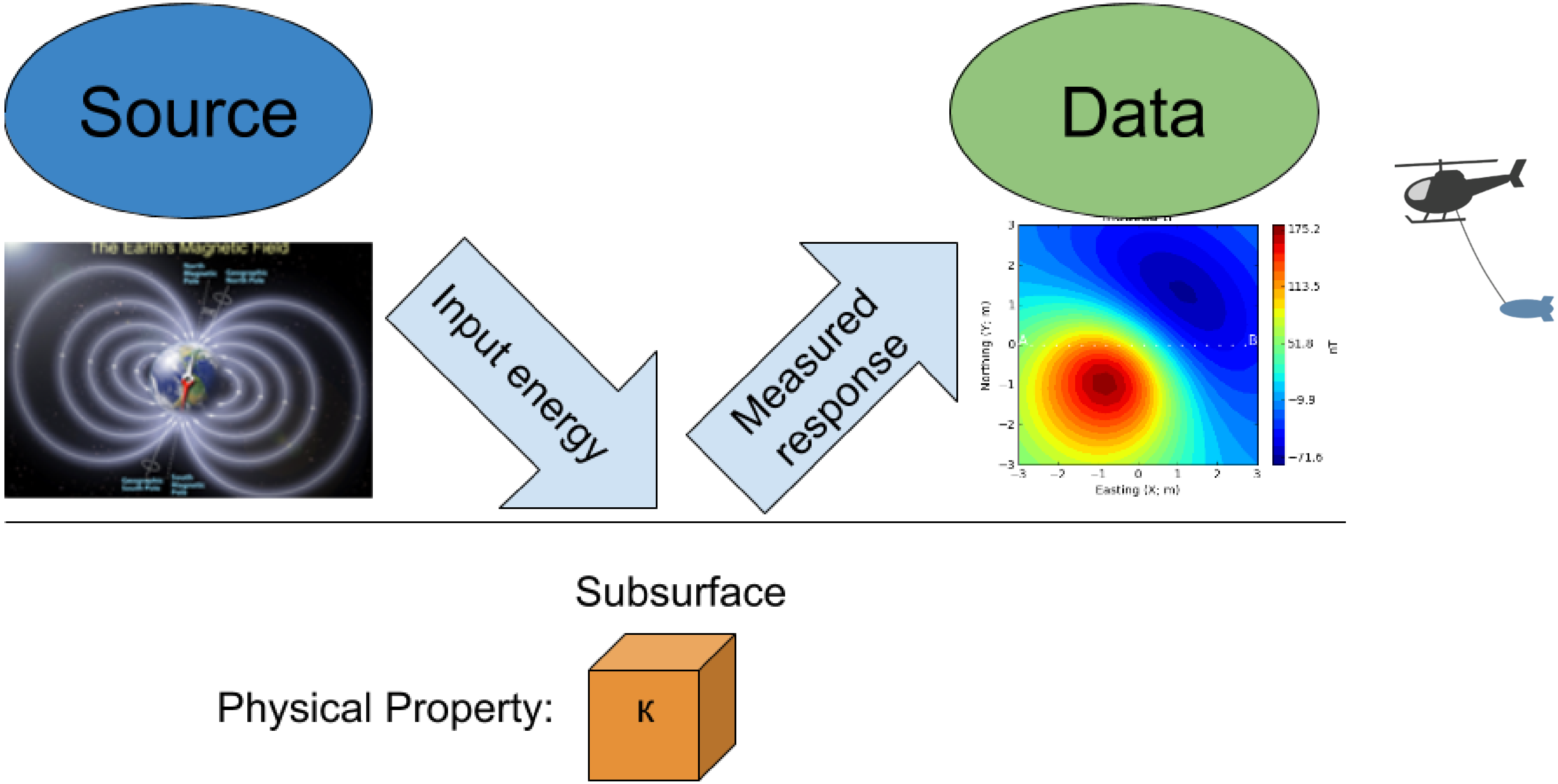
Geologic modelling

3D visualization

My intention of this lecture

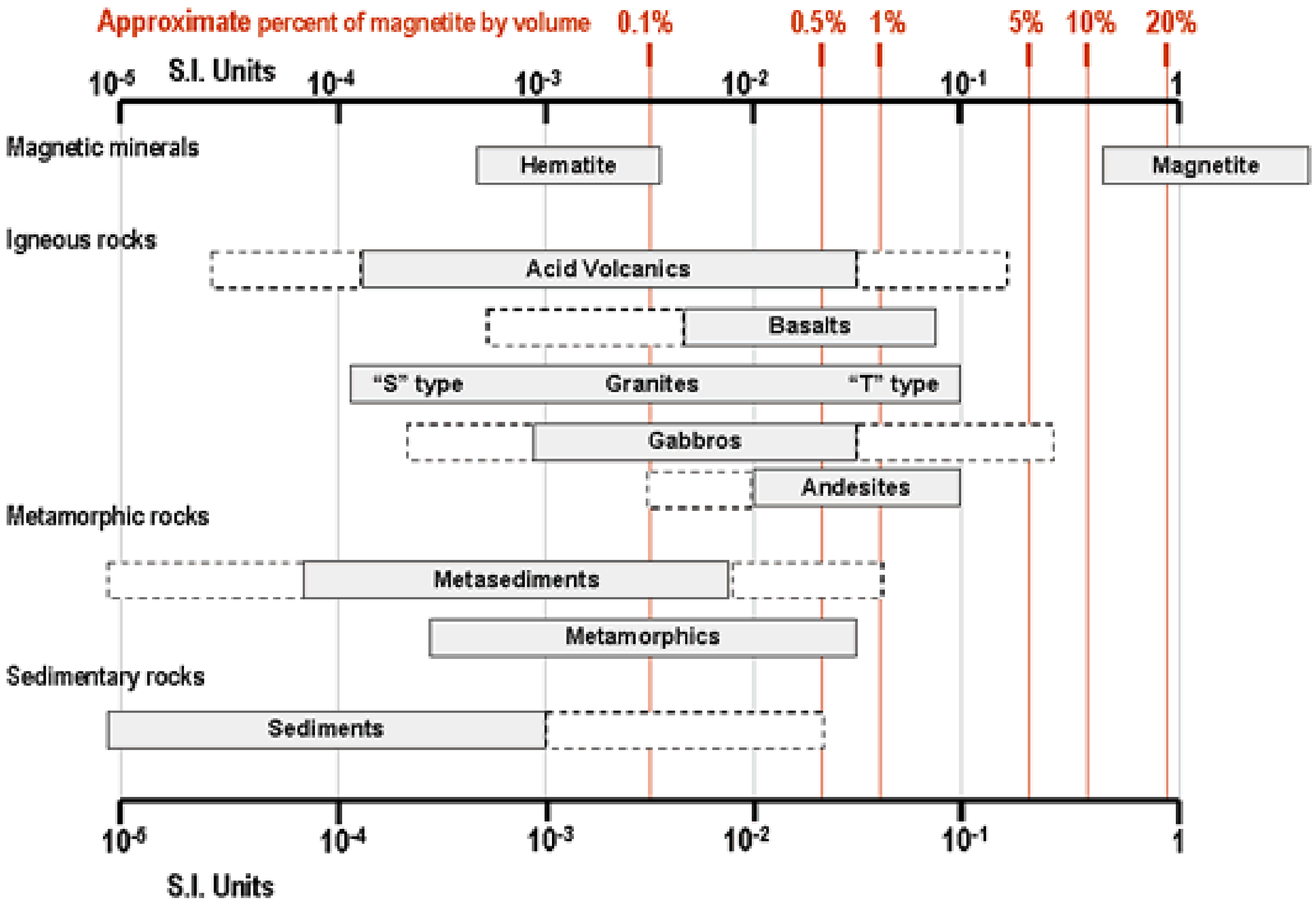
"*Not* for introducing how geophysical software packages work, *But* for *providing fundamental concepts of the inversion*"

Survey: Magnetics



κ: Magnetic susceptibility

Magnetic susceptibility



Adopted from Clark and Emerson, Exploration Geophysics (1991)

Magnetic surveying

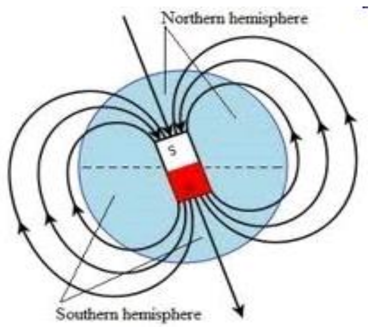
- Earth's magnetic field $\vec{B}_0$ is the source:
- Materials become magnetized

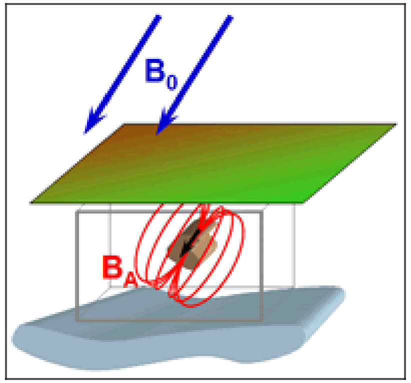
Magnetic Susceptibility

$$\vec{M} = \kappa \vec{H}_0 \ (\text{magnetization})$$

$$\vec{H}_0 = \vec{B}_0 / \mu_0$$

- Create anomalous magnetic field





Magnetic surveying

- Earth's magnetic field $\vec{B}_0$ is the source:
- Materials become magnetized

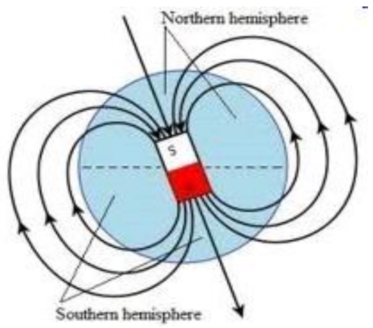
Magnetic Susceptibility

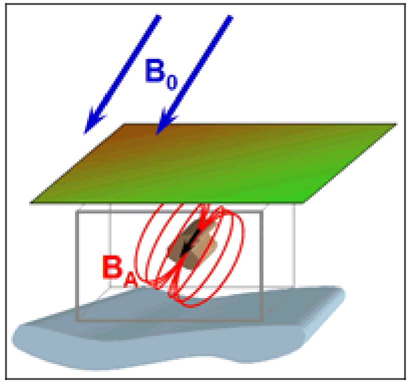
$$\vec{M} = \kappa \vec{H}_0 \ \text{(magnetization)}$$
$$\vec{H}_0 = \vec{B}_0/\mu_0$$

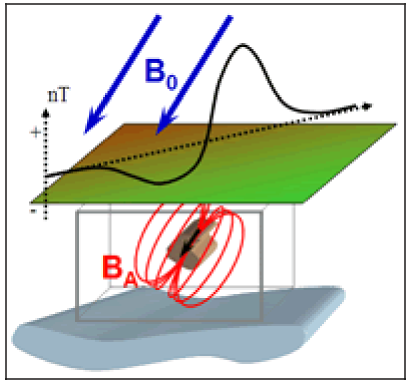
- Create anomalous magnetic field
- Measure total magnetic field: $|\vec{B}| = |\vec{B}_0 + \vec{B}_A|$
- Total field anomaly: $\triangle \vec{B} = |\vec{B}_0 + \vec{B}_A| - |\vec{B}_0|$

$$\triangle \vec{B} \simeq \vec{B}_A \cdot \hat{B}_0$$

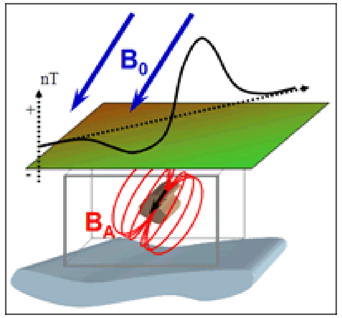
where $\hat{B}_0 = \dfrac{\vec{B}_0}{|\vec{B}_0|}$



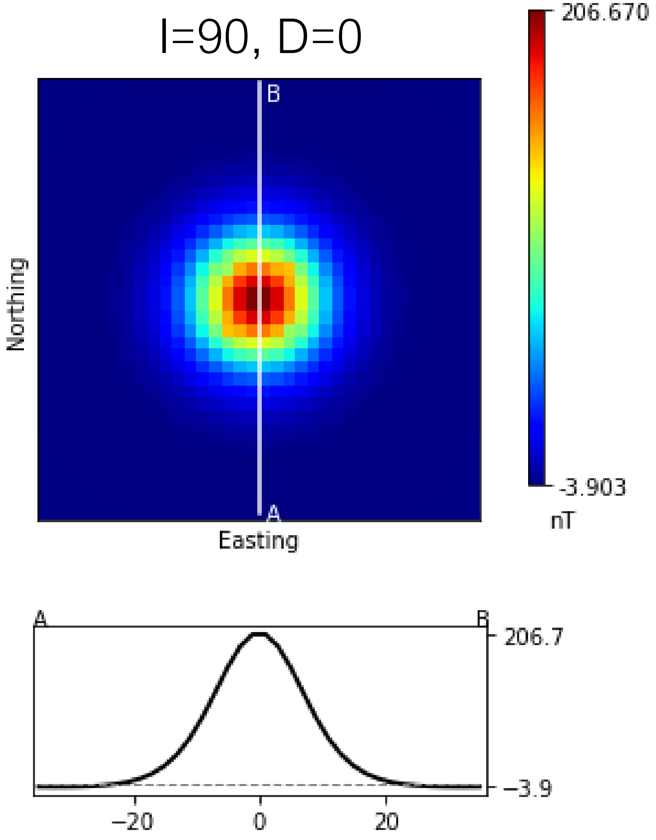




Magnetic data changes depending upon where you are

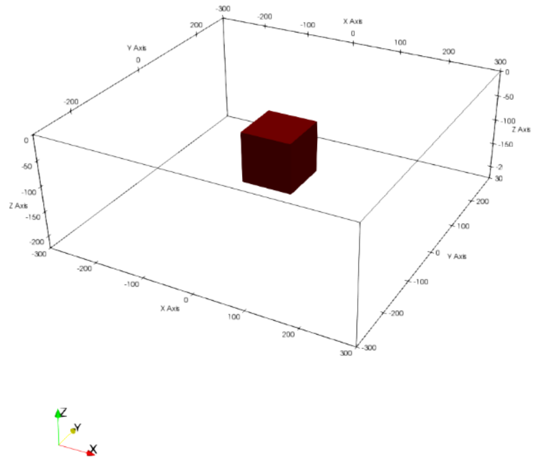


I=90, D=0

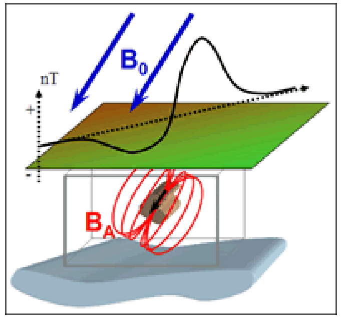


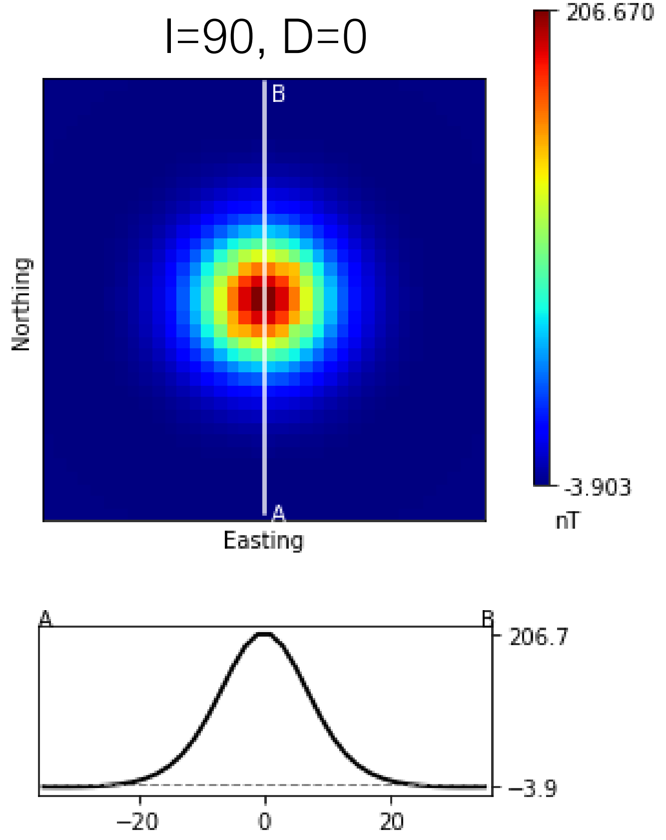
North pole

A prism in a homogeneous subsurface

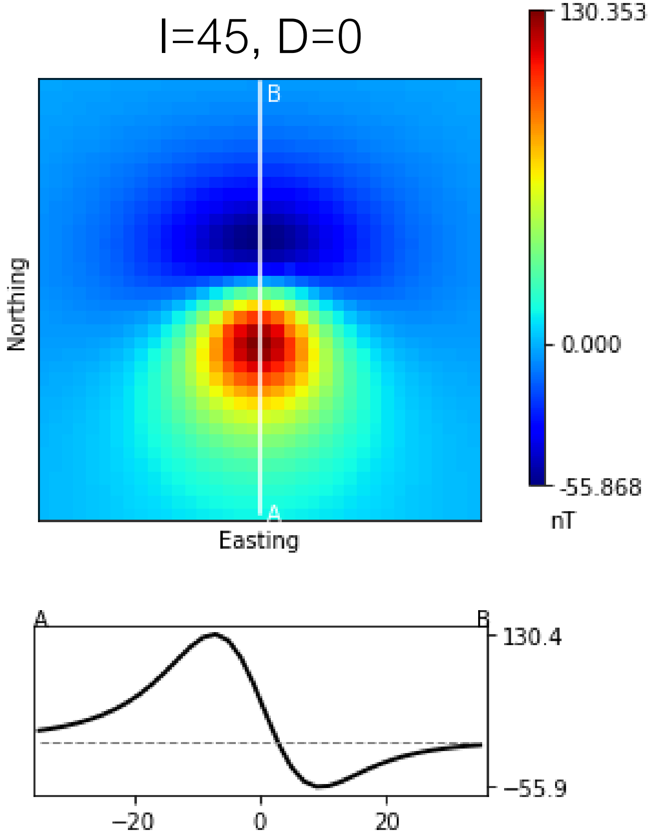


Magnetic data changes depending upon where you are

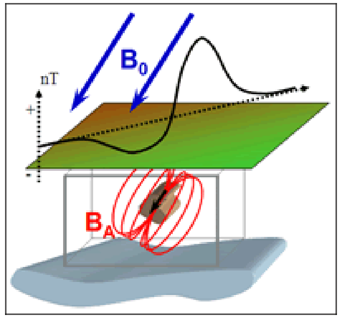


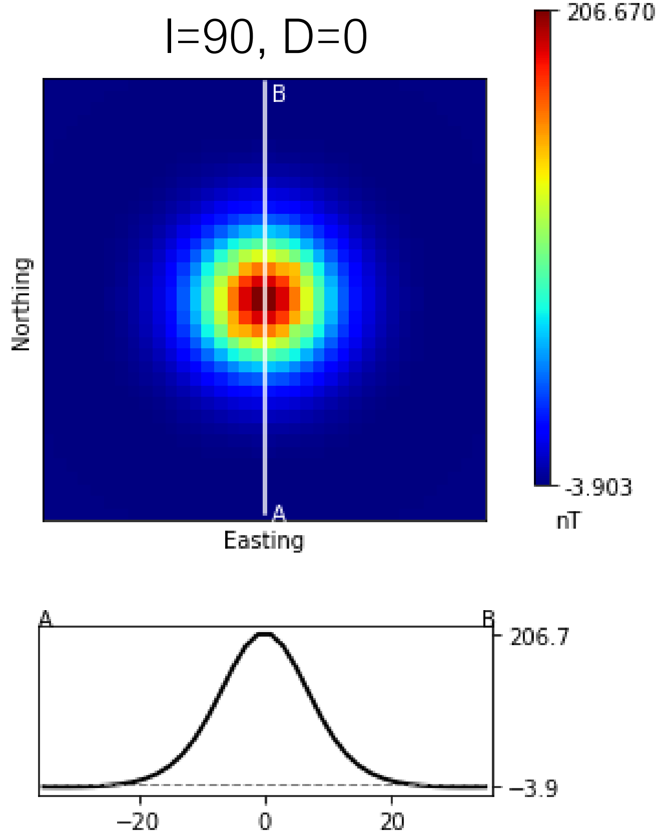


North pole

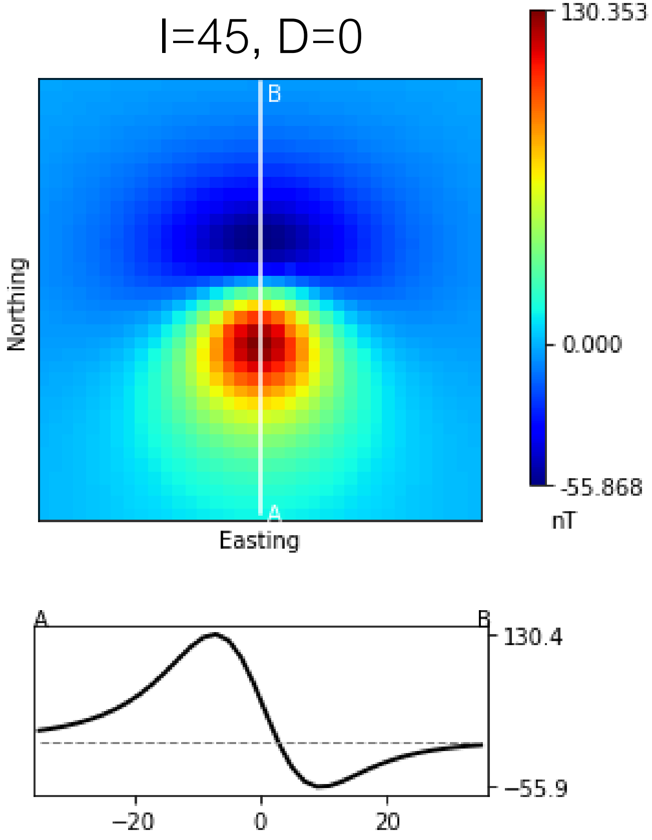


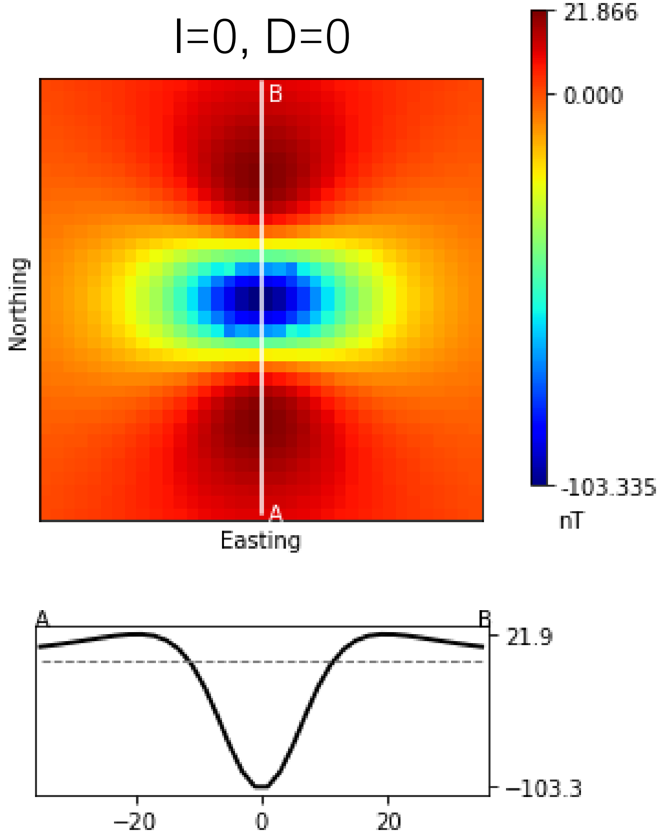
Magnetic data changes depending upon where you are





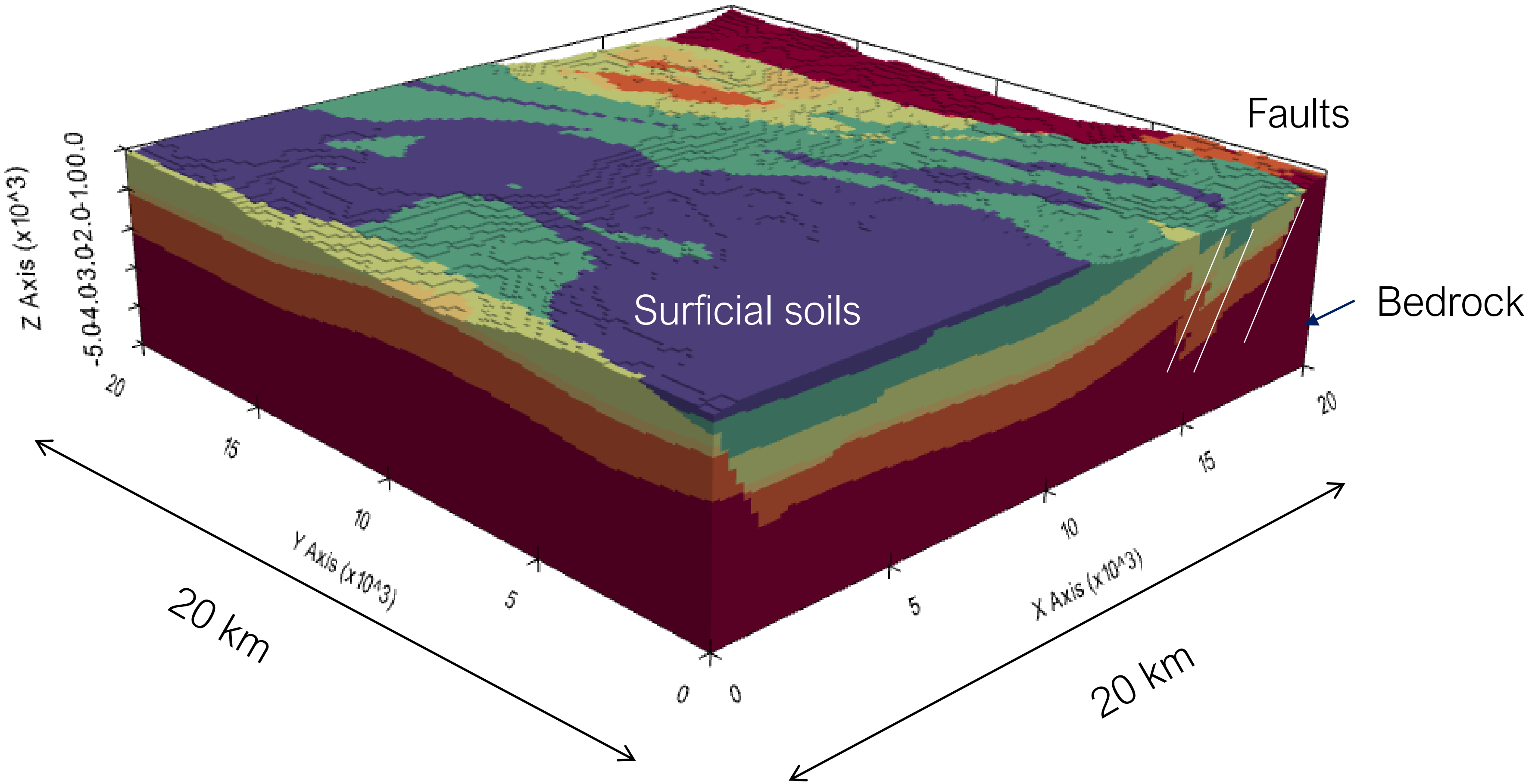
North pole



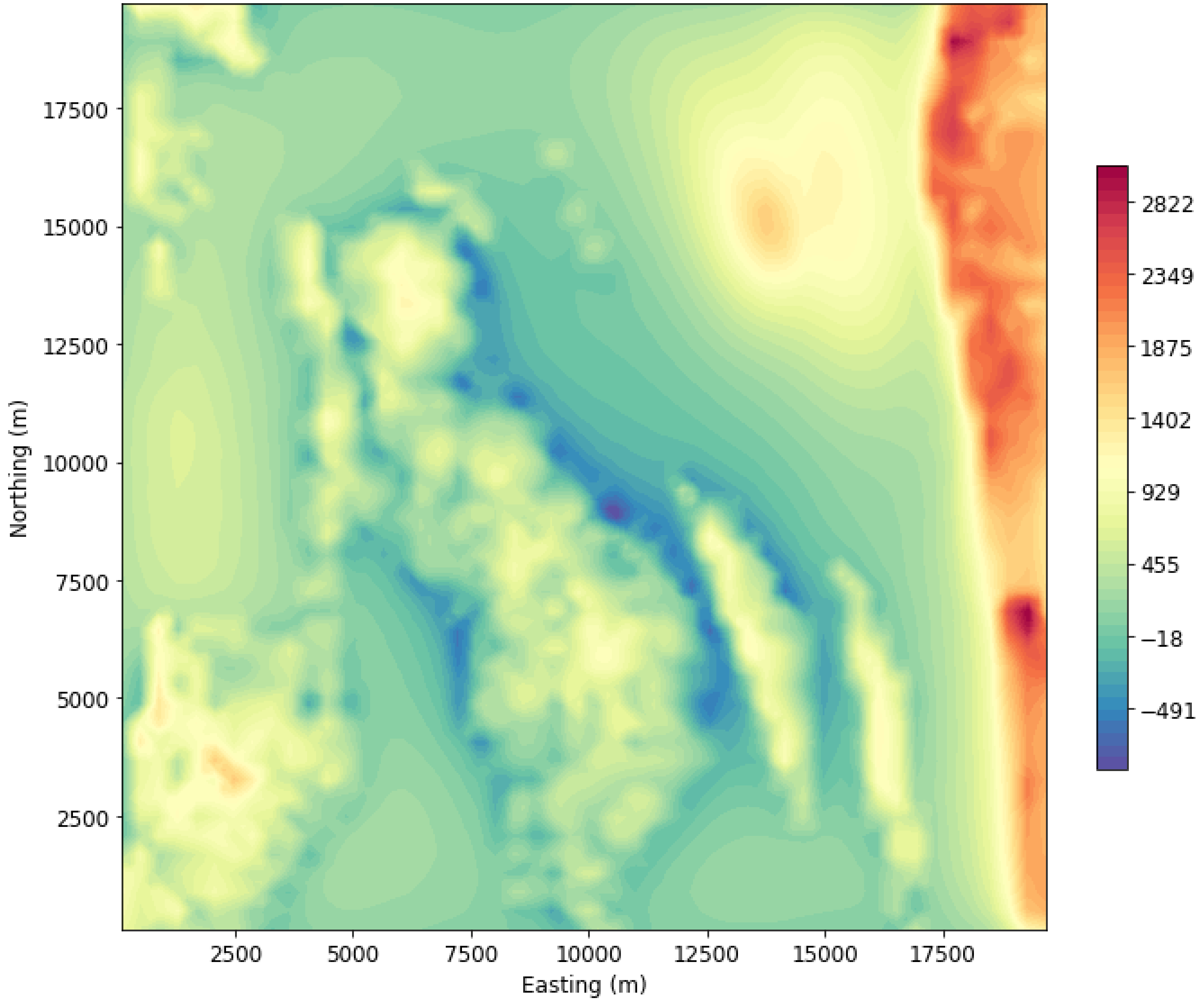


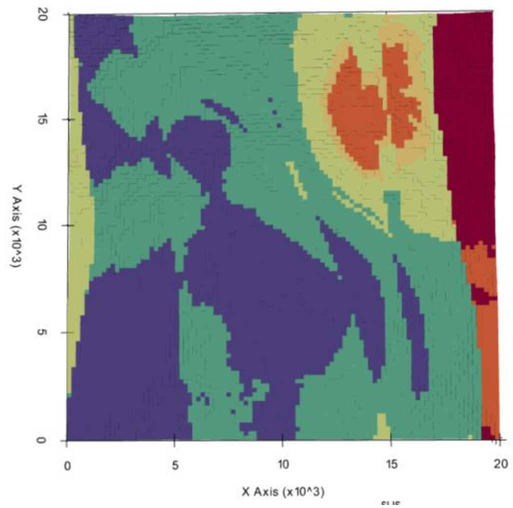
Equator

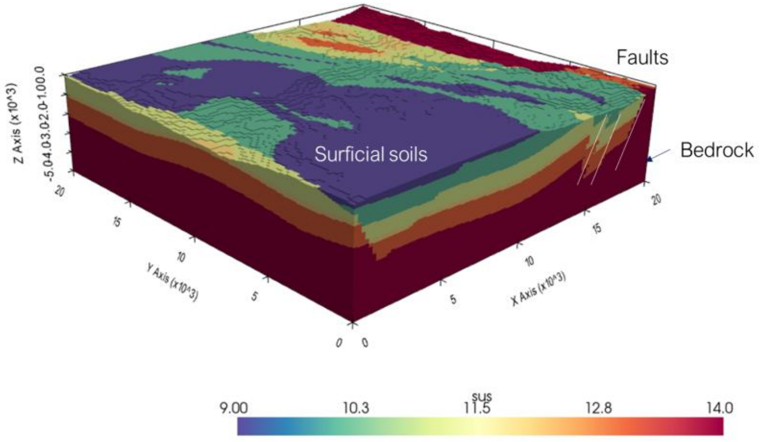
Subsurface structure is complex



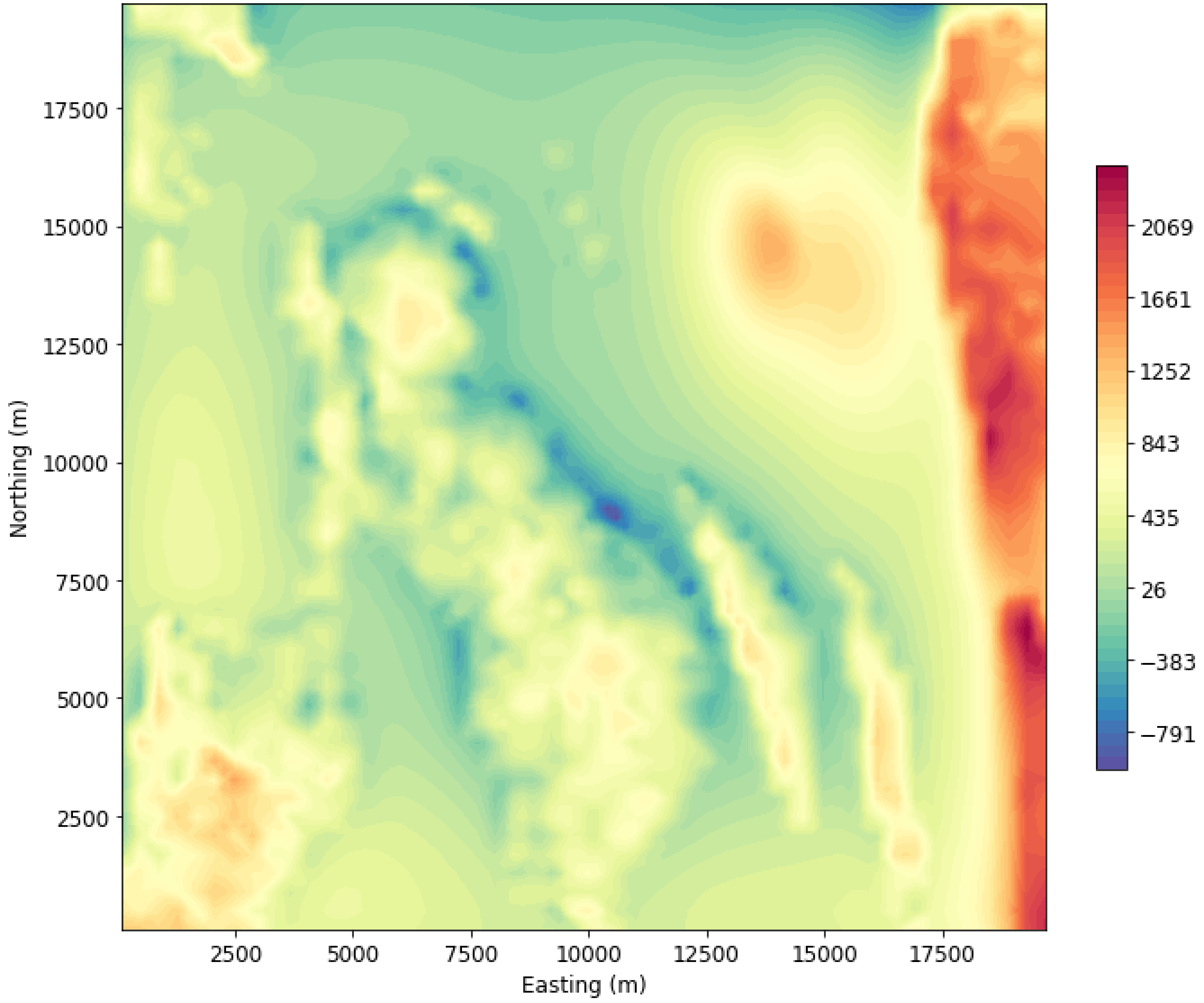
Measured magnetic data at I=90, D=0 (North pole)

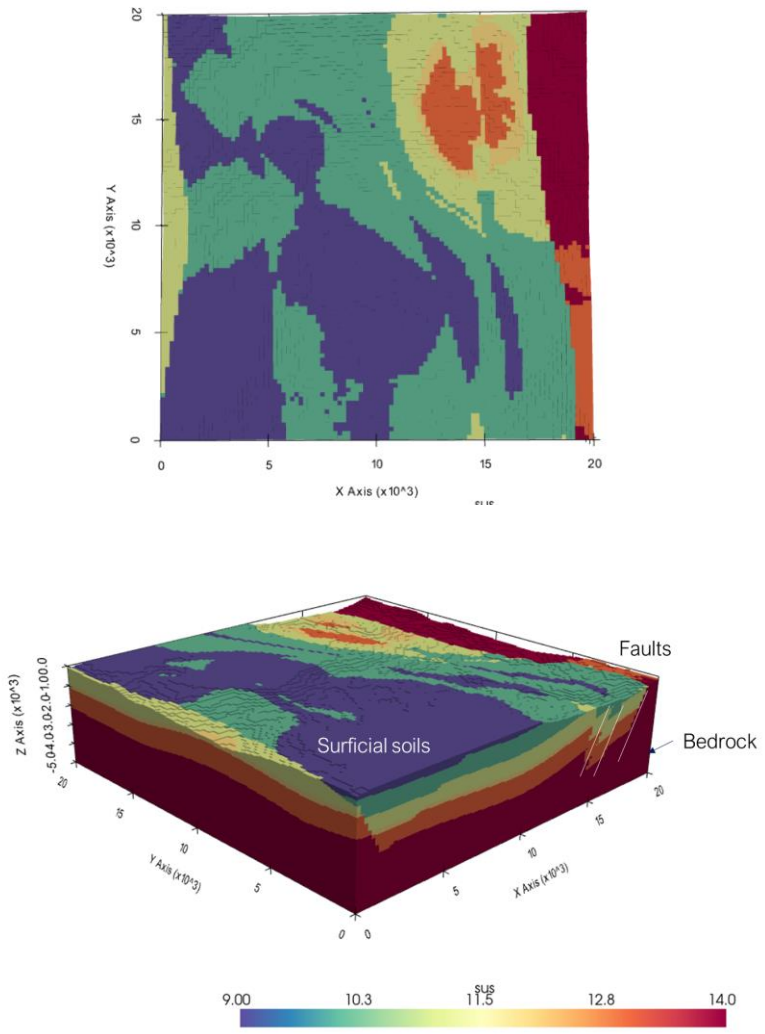




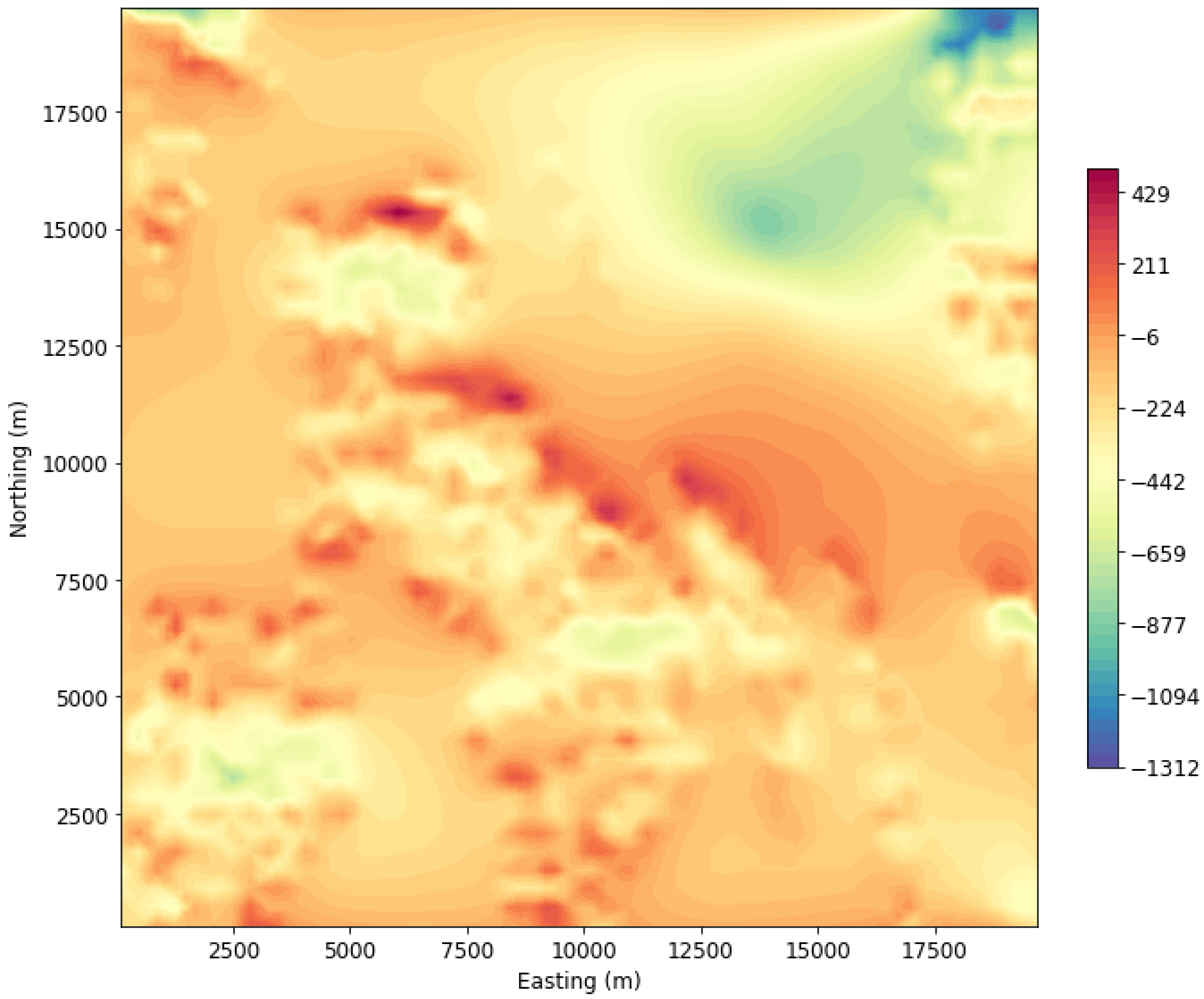


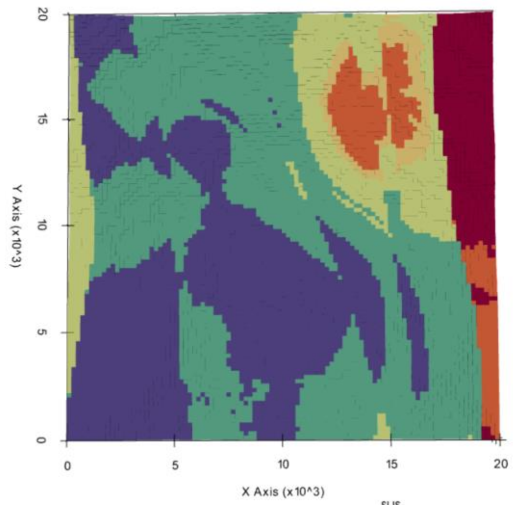
Measured magnetic data at I=66, D=-6 (California)

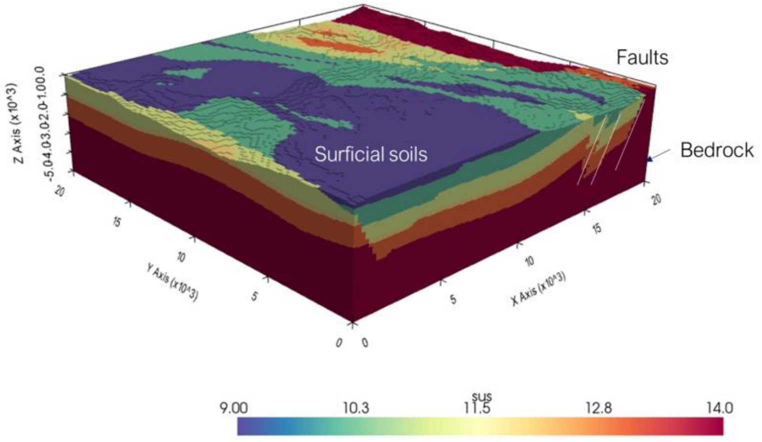




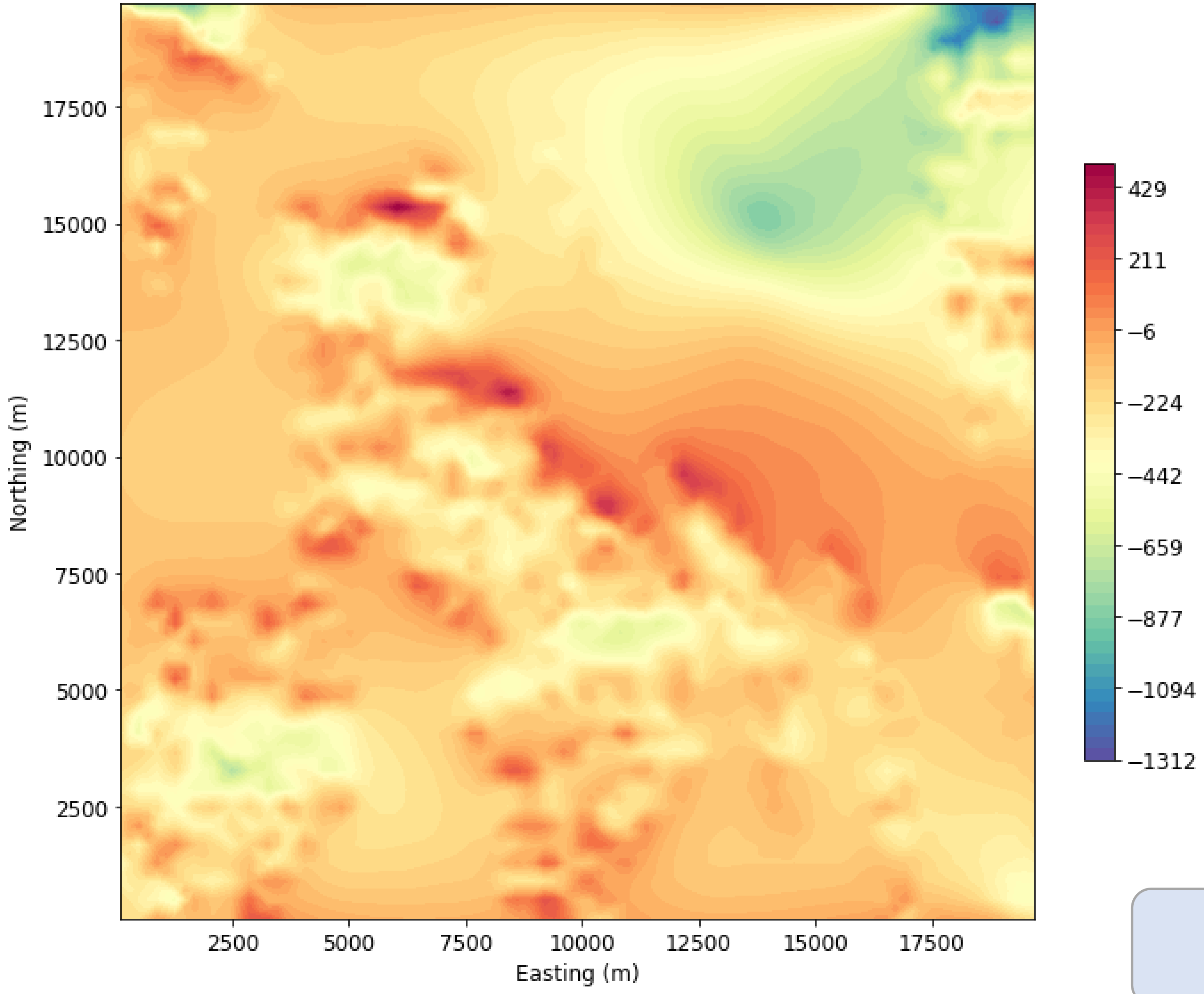
Measured magnetic data at I=0, D=0 (Equator)

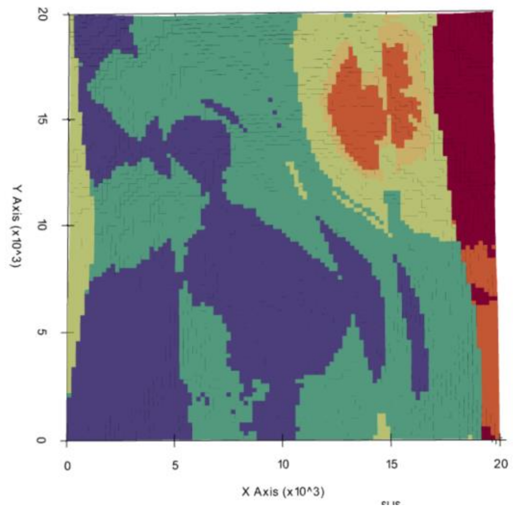


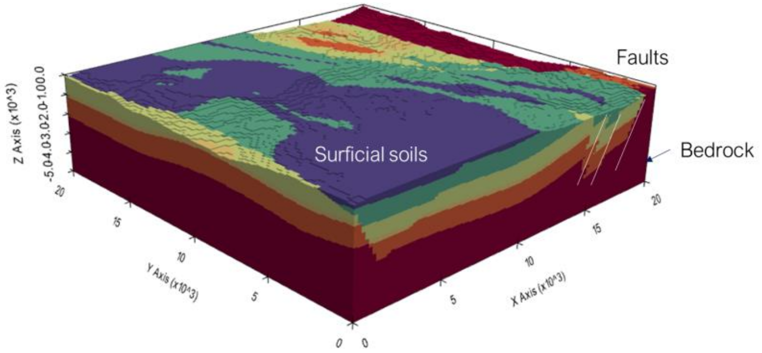




Measured magnetic data at I=20, D=0 (Equator)







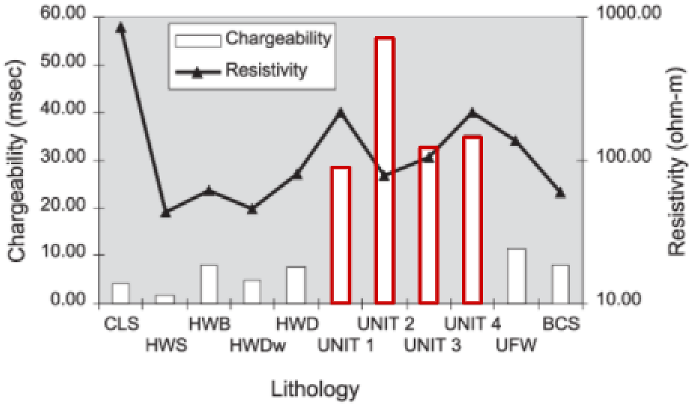
Measured data are large & complex

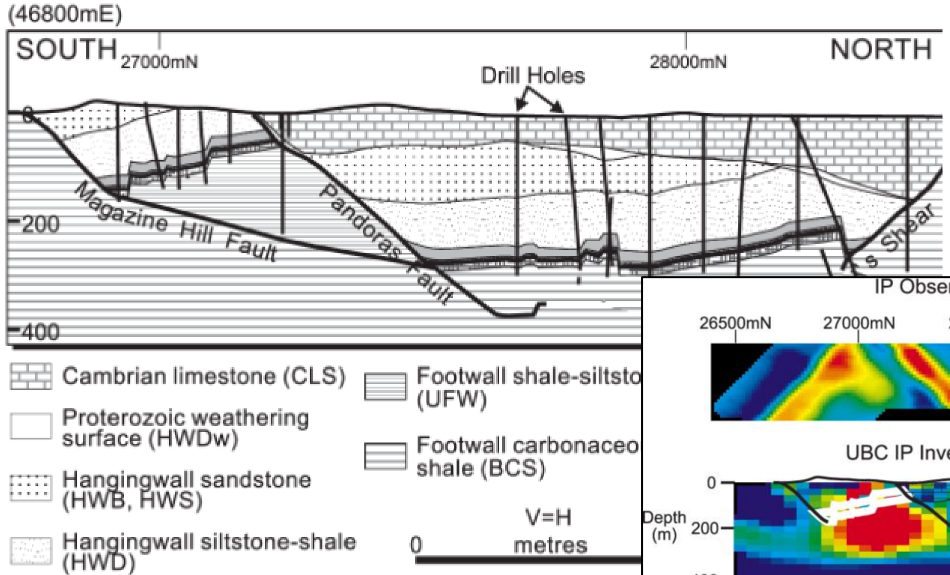
motivating field example

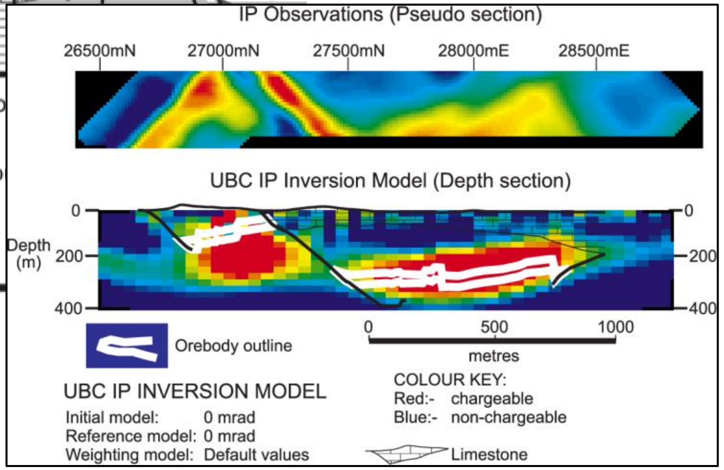
Transform 2020: Lindsey Heagy (DC/IP methods)

Century Deposit: geology + physical properties

- Resistivity: structure, input to IP
- Chargeability: Associated with mineralization





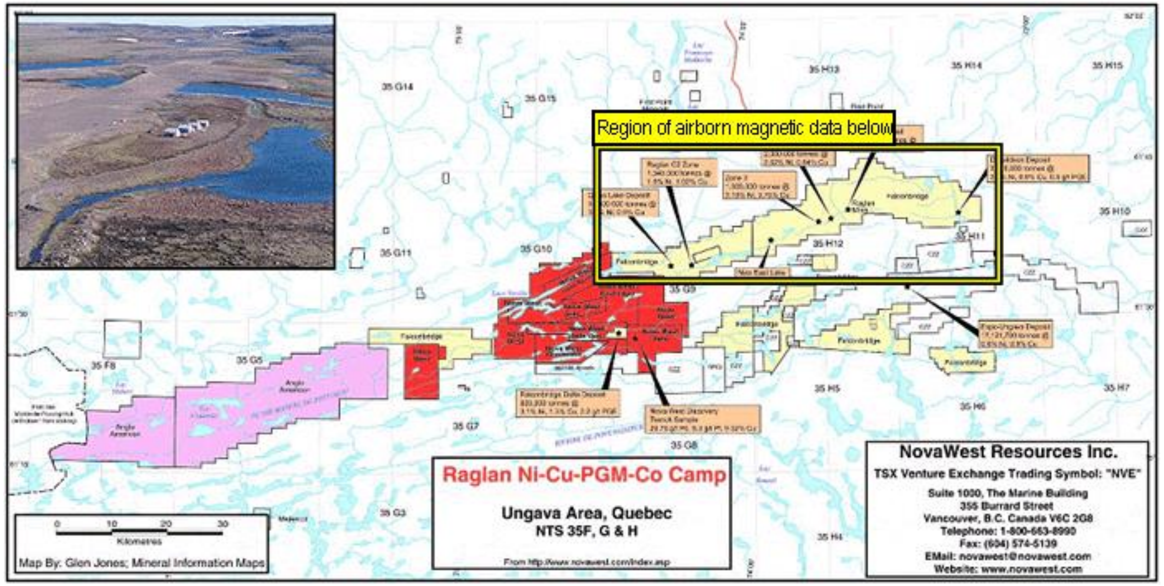


Reproduce the historic geophysical inversion results which made a high impact to the mining community

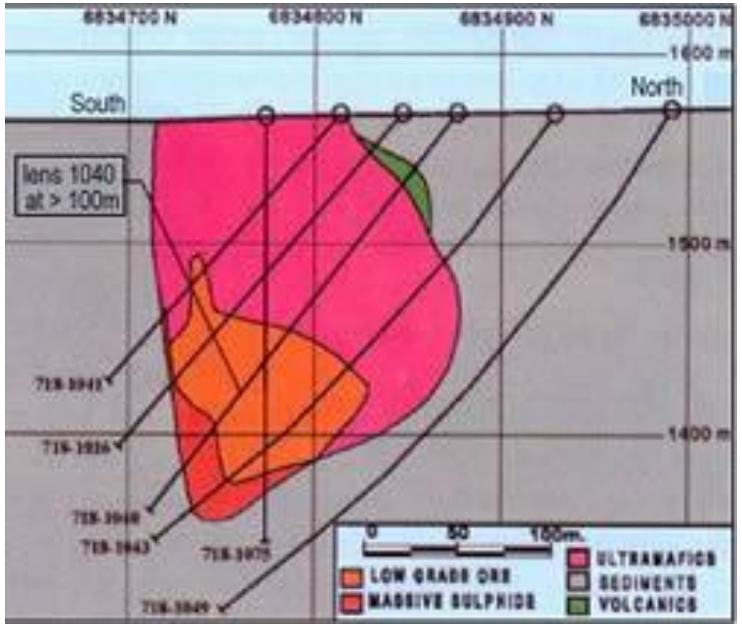
ground-based geophysics

Raglan Deposit: geology + physical properties

Location map (Northern Quebec, Canada)



Geologic section

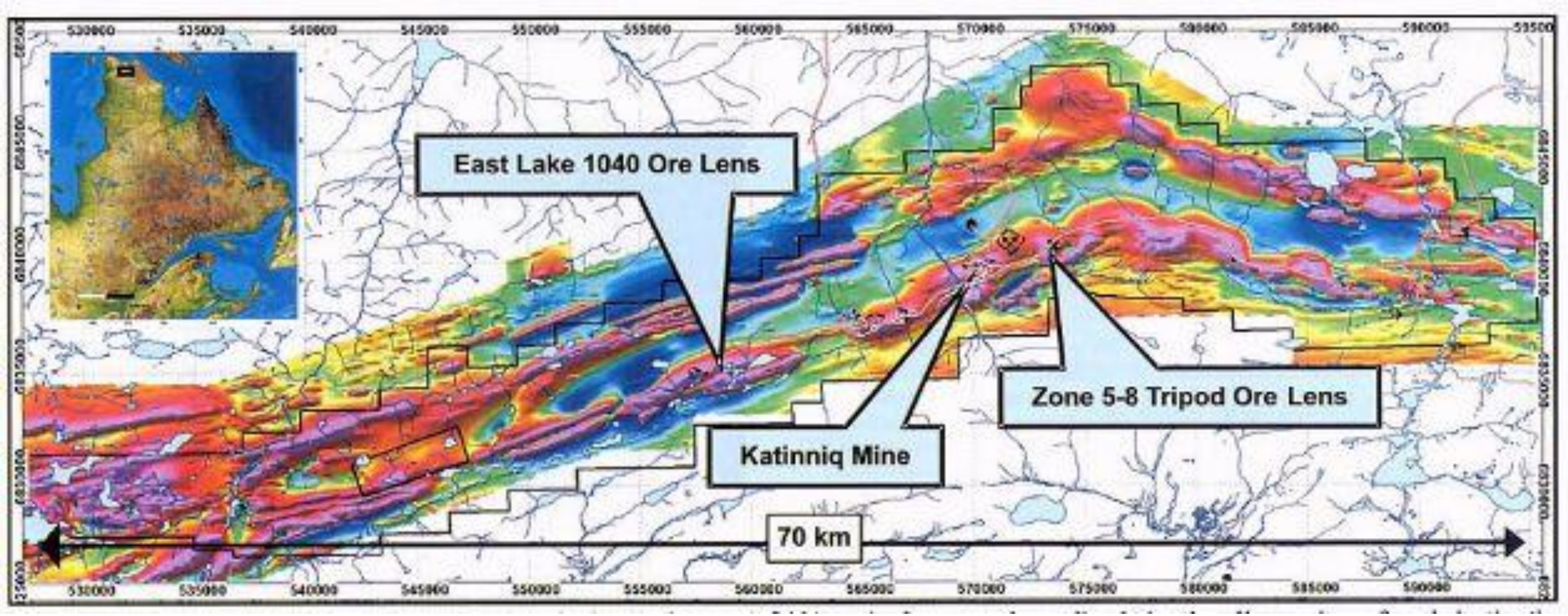


Physical properties

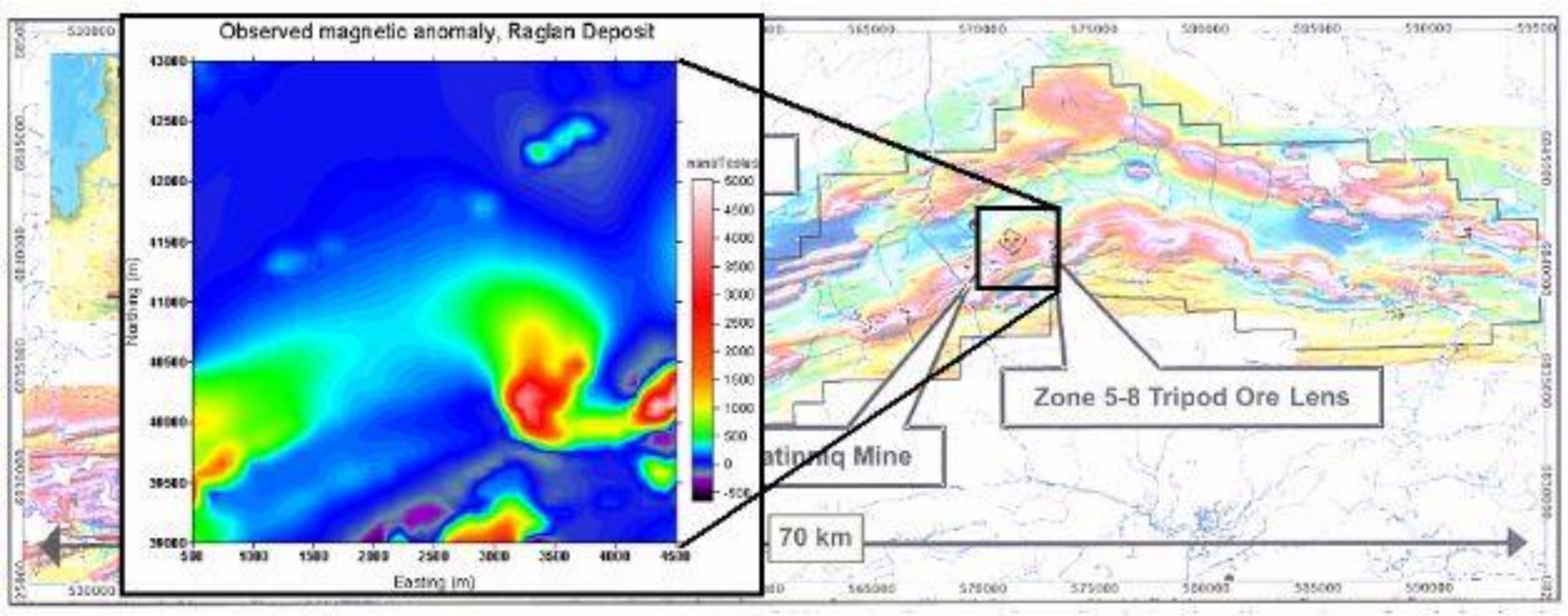
Seek for zones having a high susceptibility (~0.05 SI)

Grey rocks are host sediments.
Green rocks are volcanics.
Pink rocks are ultramafics (susceptibility 0.03 - 0.07 S.I.).
Orange rocks are low grade massive ore (susceptibility 0.03 - 0.07 S.I).
Red rocks are the primary massive sulphide ore (susceptibility 0.03 - 0.07 S.I.).

Raglan Deposit: magnetic data

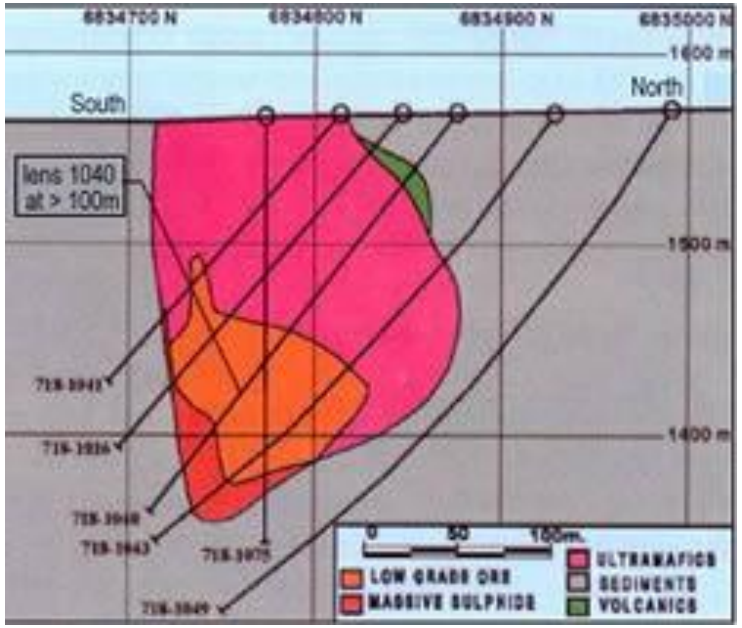


Raglan Deposit: magnetic data

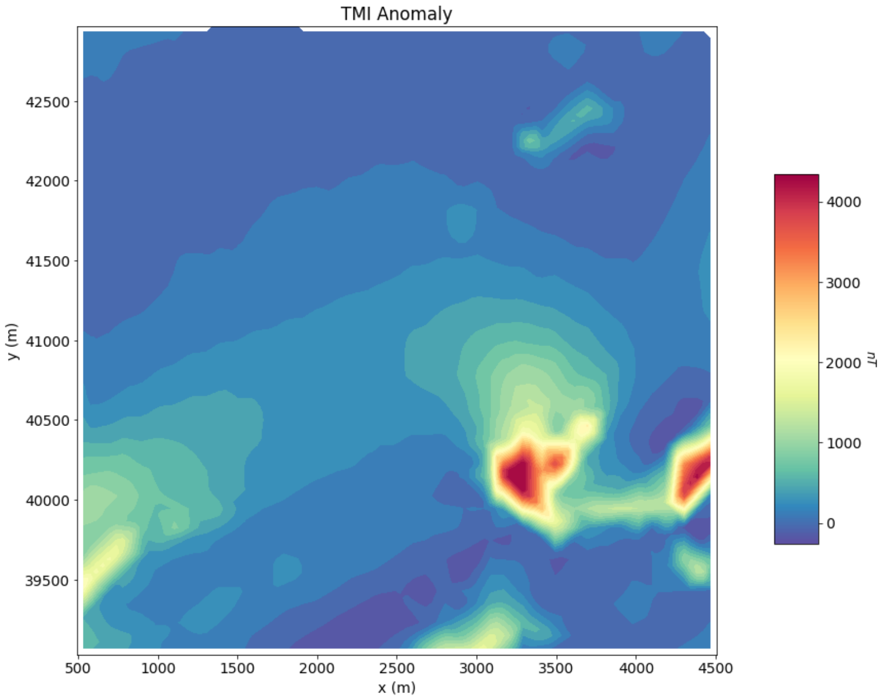


Initial conceptual model

Geologic section

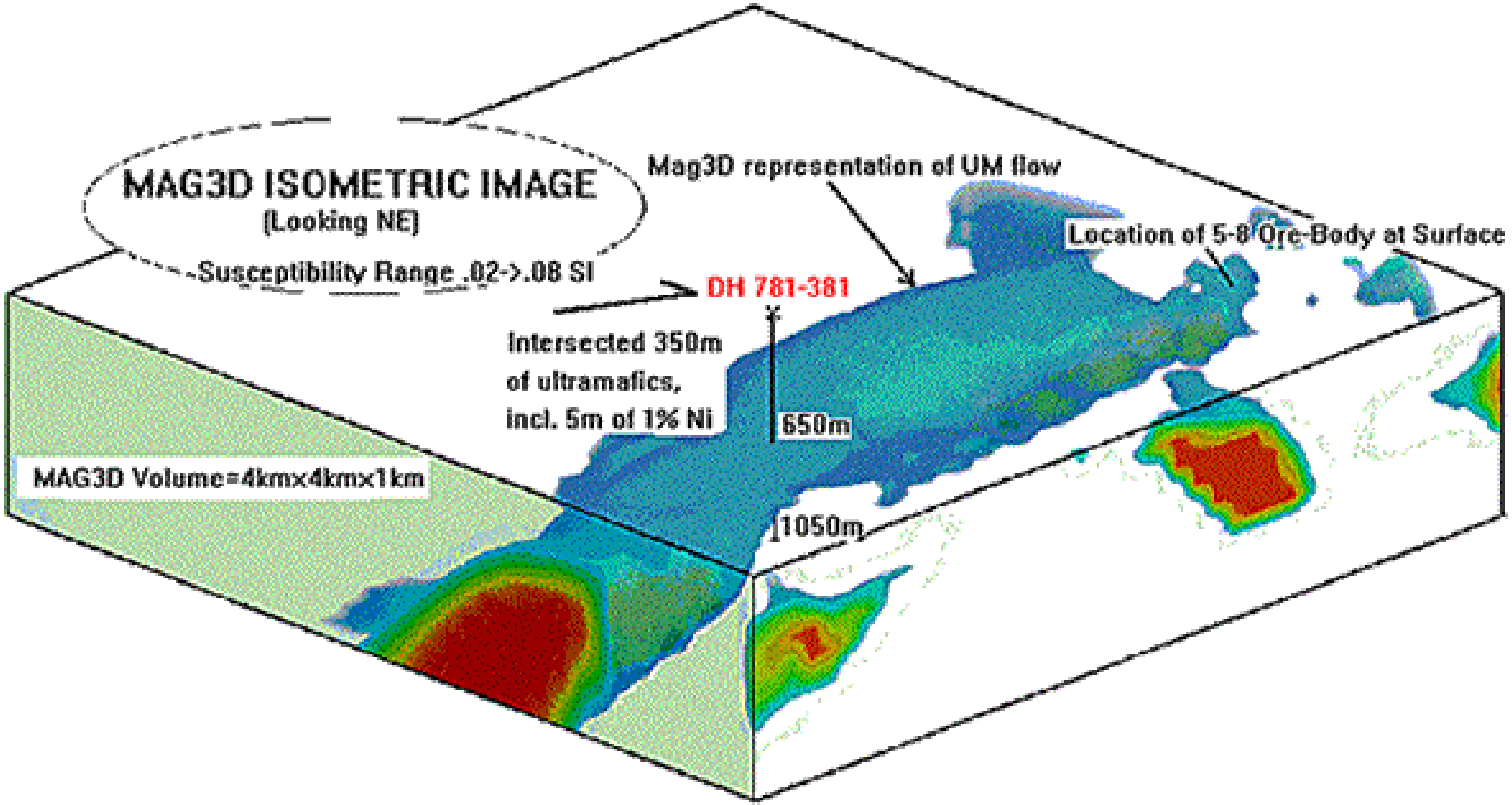


Magnetic data



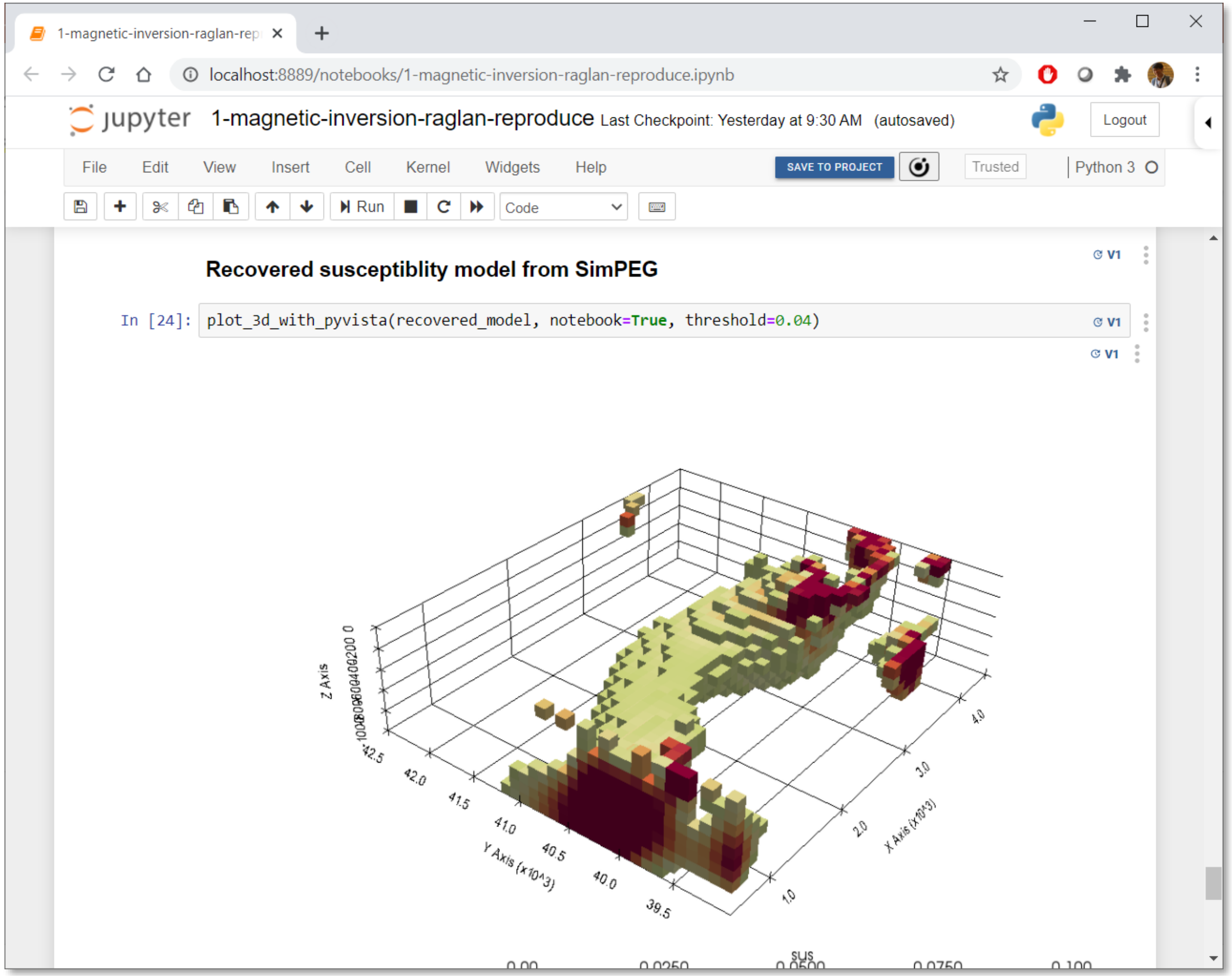
Initial conceptual model: two ultramafic pipes
Can make impact on drilling location and mineral reserve

Recovered susceptibility model from inversion



Changed the conceptual model: the two pipes are “connected”

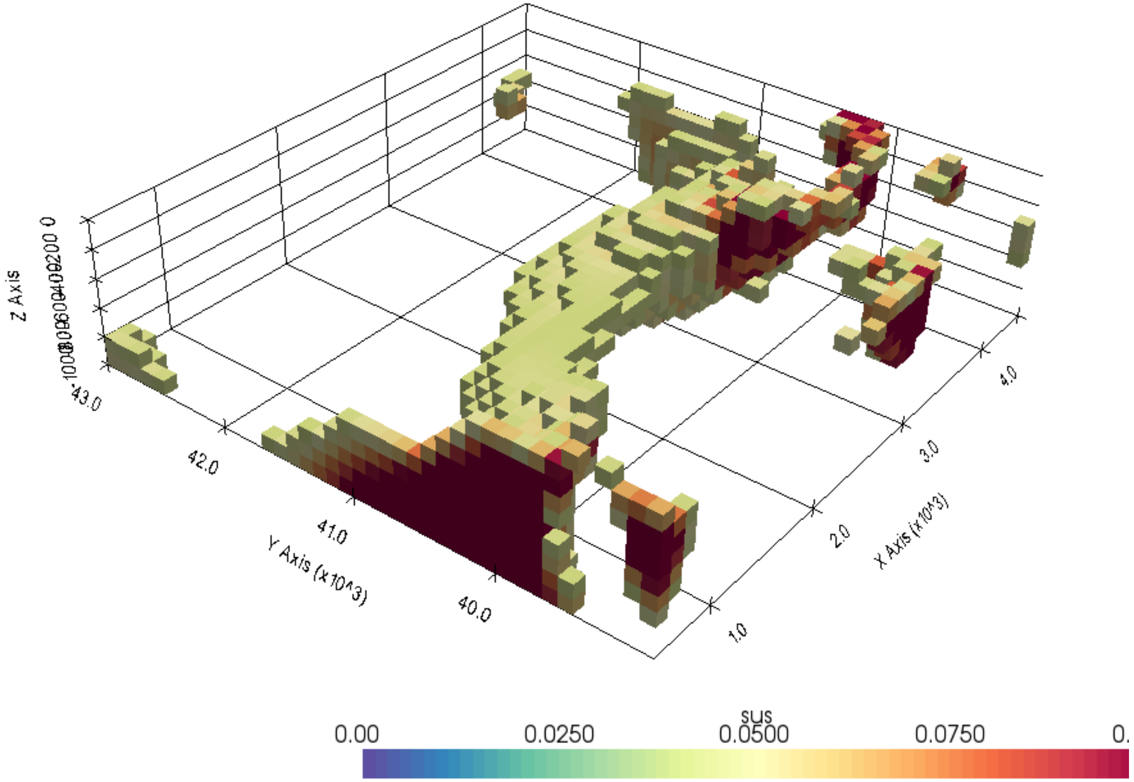
Can we reproduce this result?



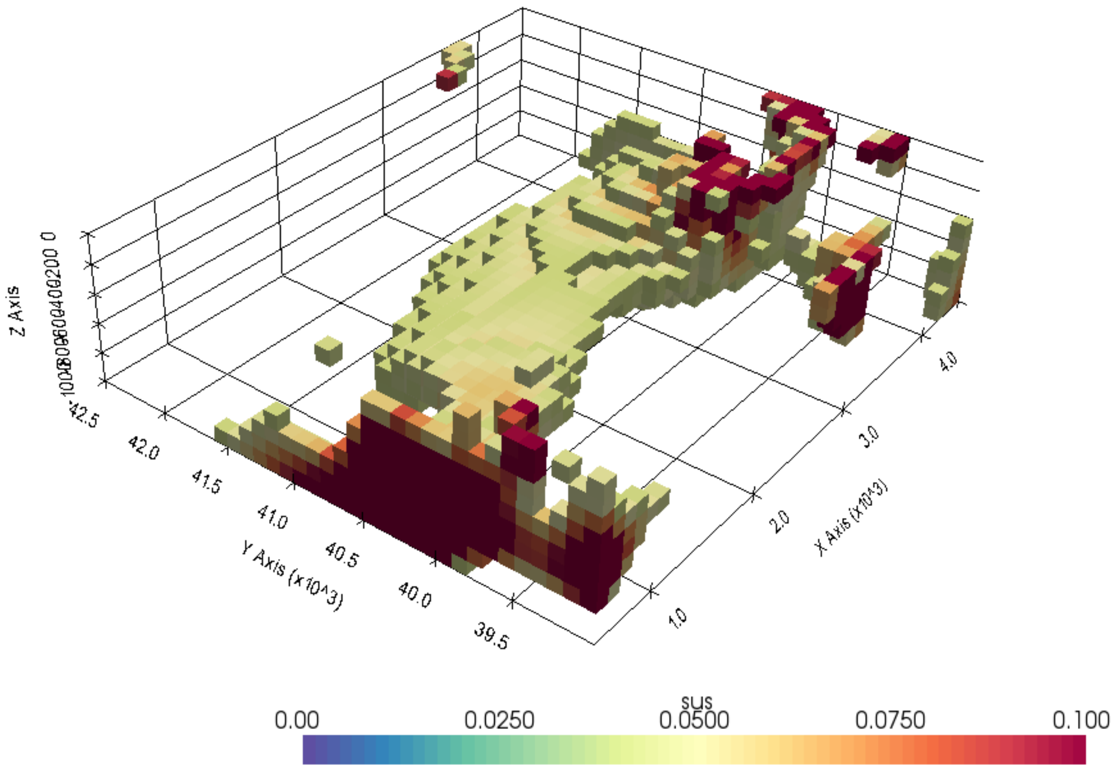
https://curvenote.com/@swung/inversion-for-geologists-transform-2021

Comparison

Model from 20 years ago (MAG3D)

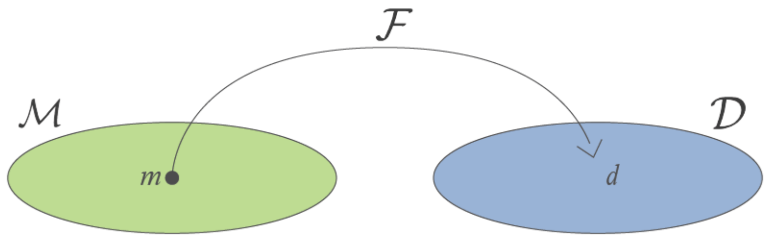


The recovered model (SimPEG)

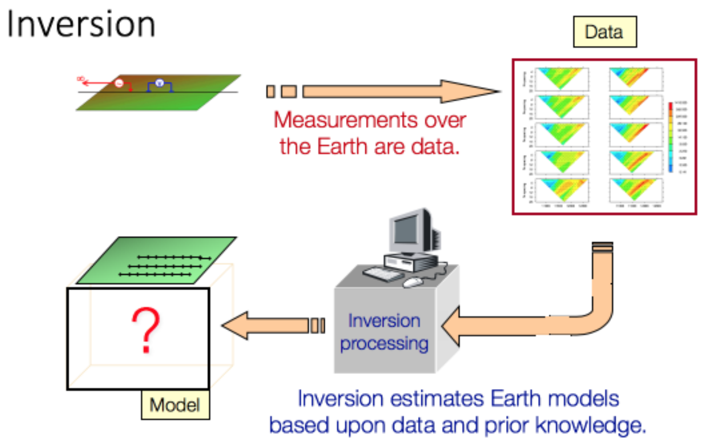


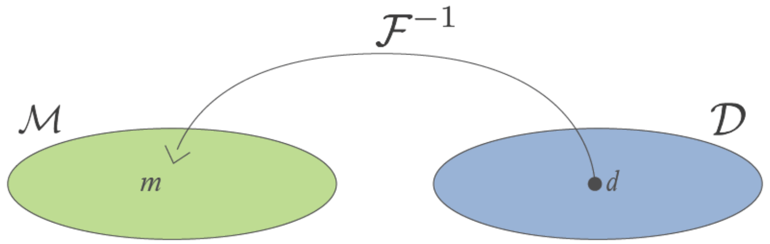
Our statement of the inverse problem

- Given observations: $d_j^{obs}, \quad j = 1, \ldots, N$
  - Uncertainties: ϵ_j
  - Ability for forward modelling: $\mathcal{F}[m] = d$

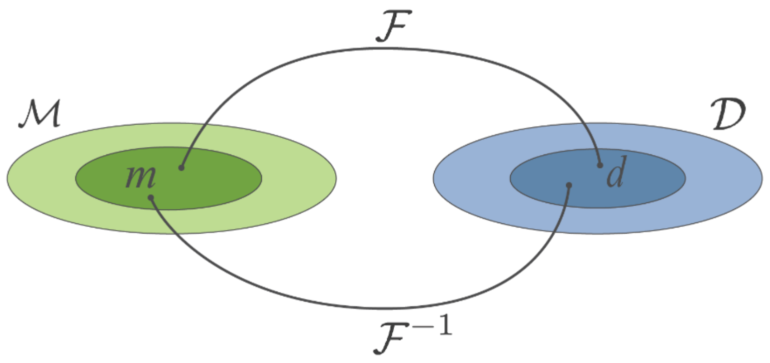


- Find the earth model that gave rise to the data.





Inverse problem



- Non-unique
- Ill-conditioned

The Inverse Problem is ill-posed

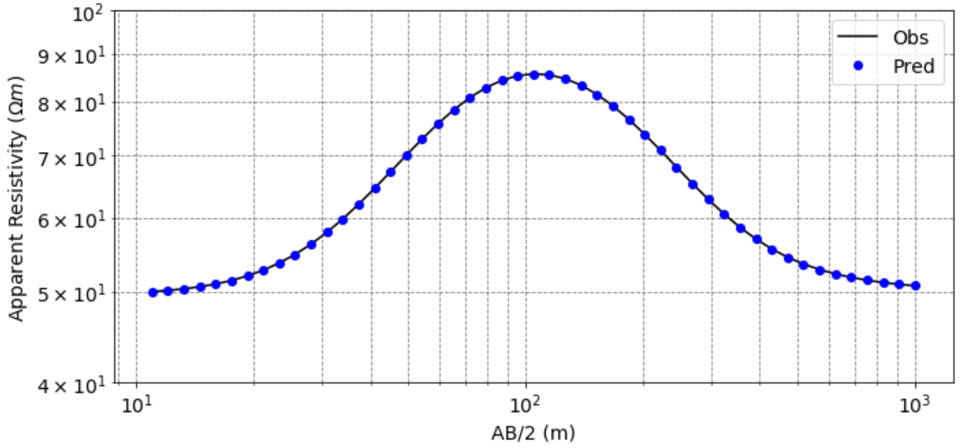
Any inversion approach must address these issues

Framework for the inverse problem

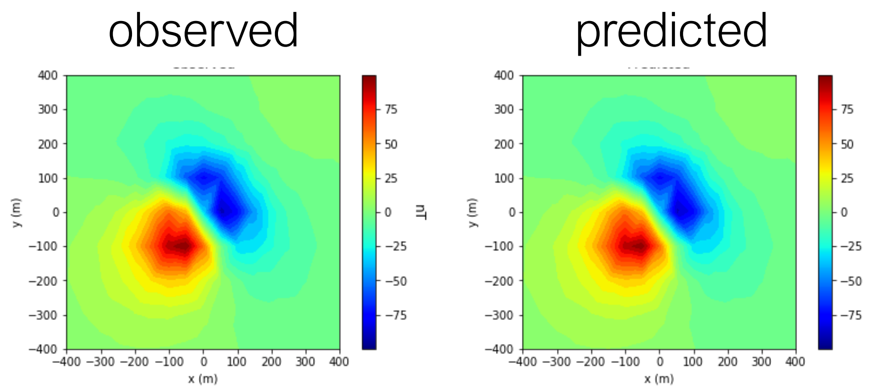
$$\text{minimize} \;\; \phi(m) = \phi_d(m) + \beta\phi_m(m)$$

ϕ_d: data misfit
ϕ_m: model norm
β: trade-off parameter

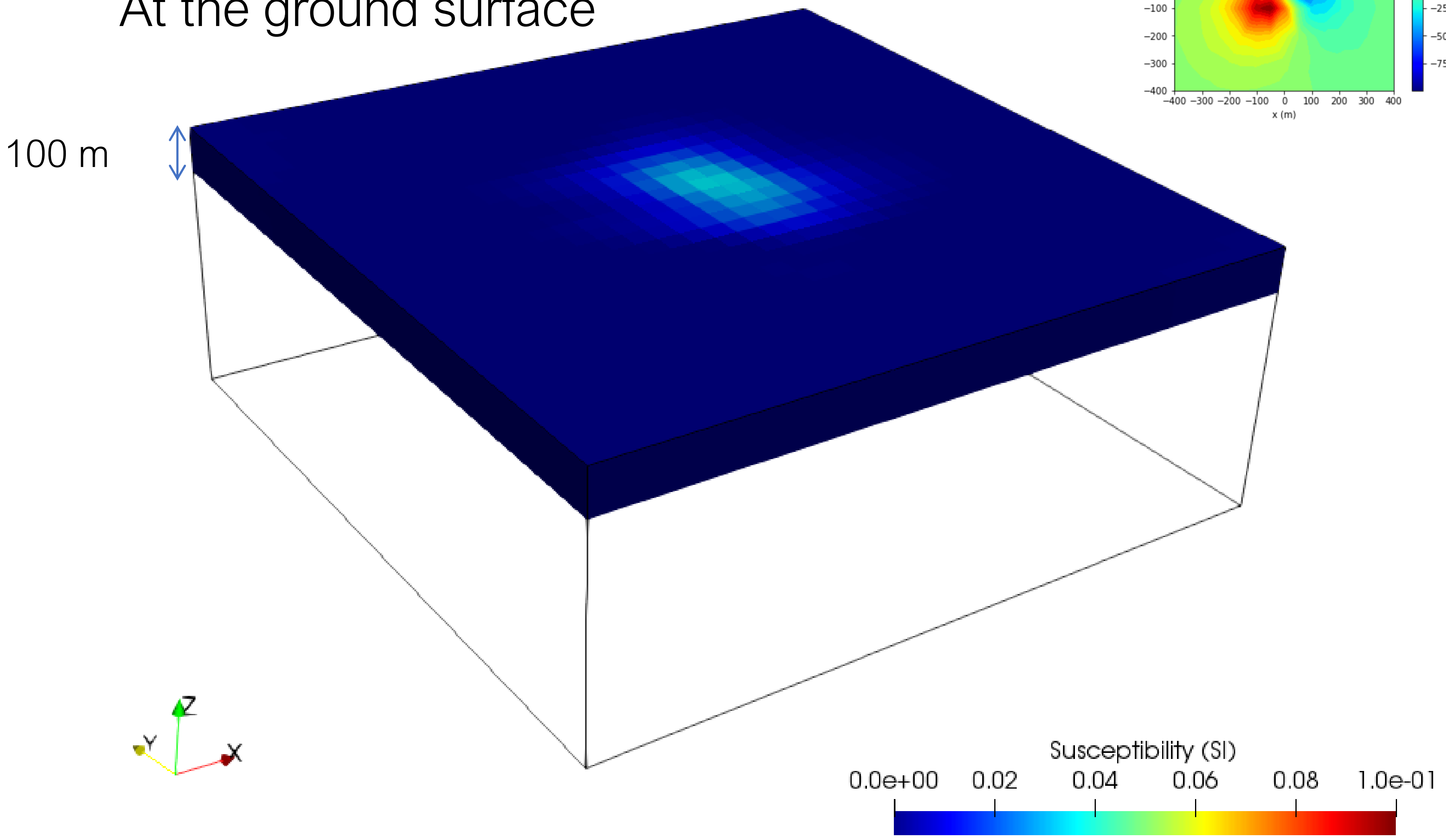
Find a model fitting the data



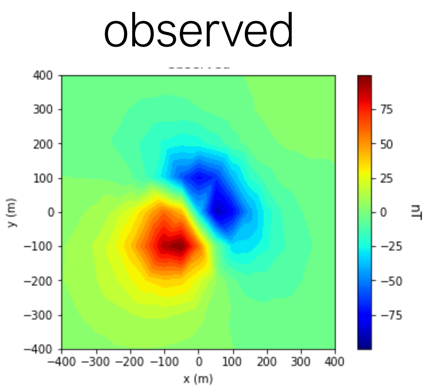
Example of extreme non-uniqueness

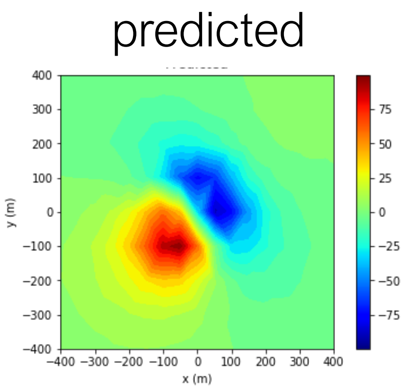


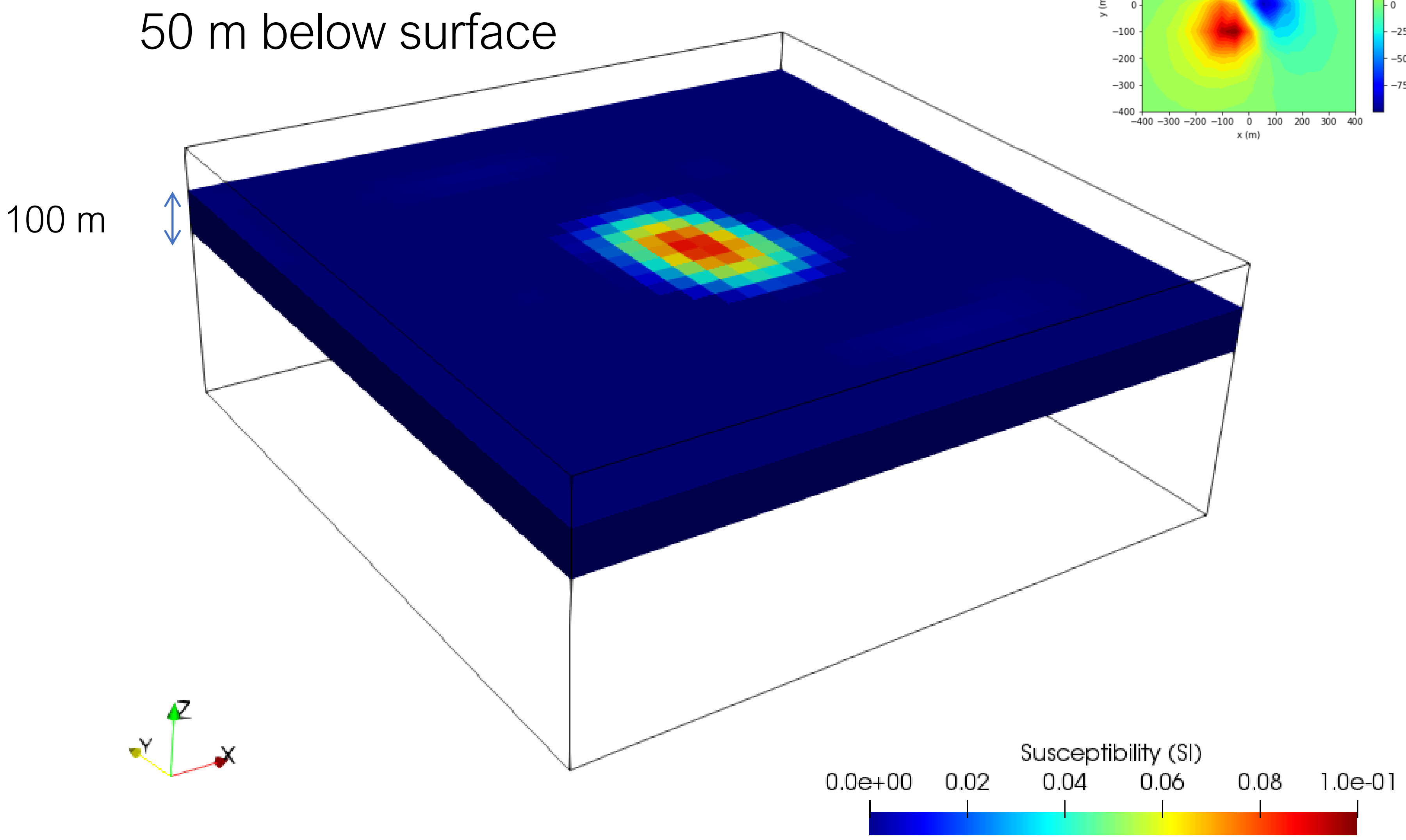
At the ground surface



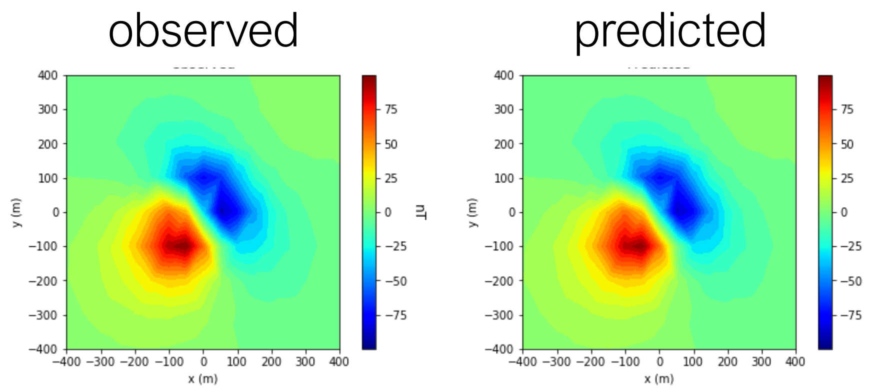
Example of extreme non-uniqueness



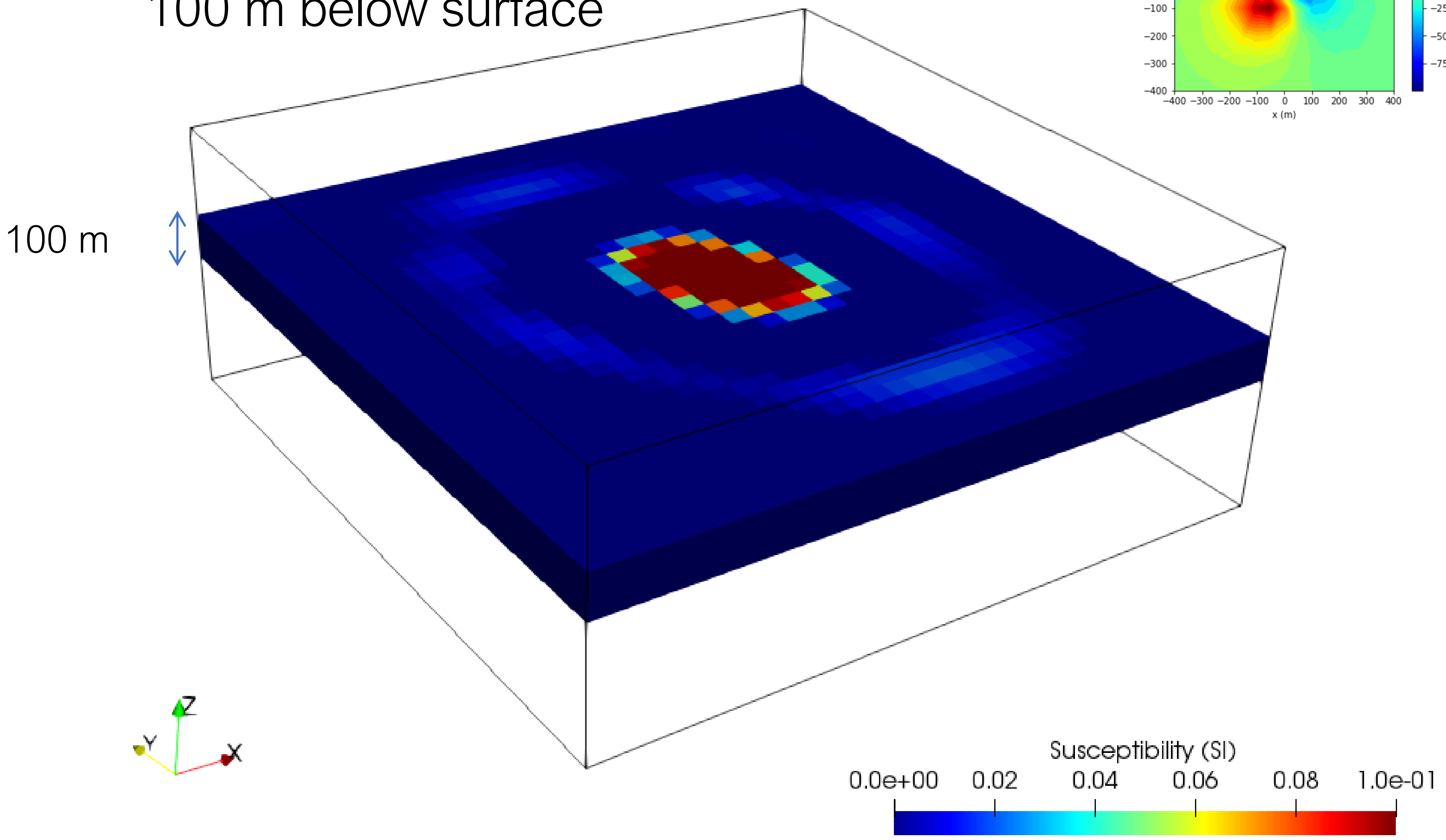




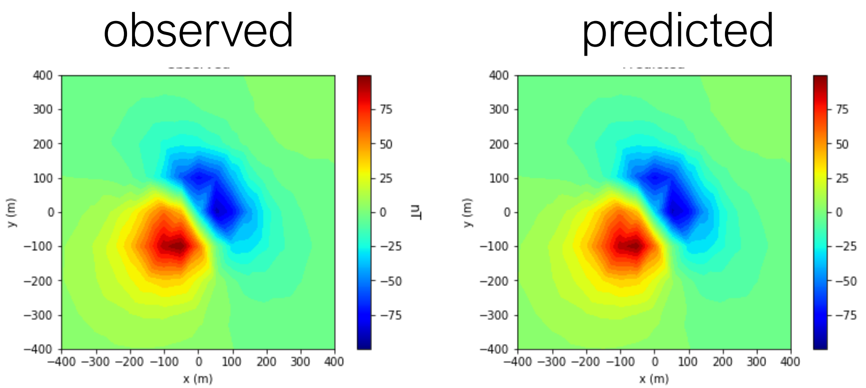
Example of extreme non-uniqueness

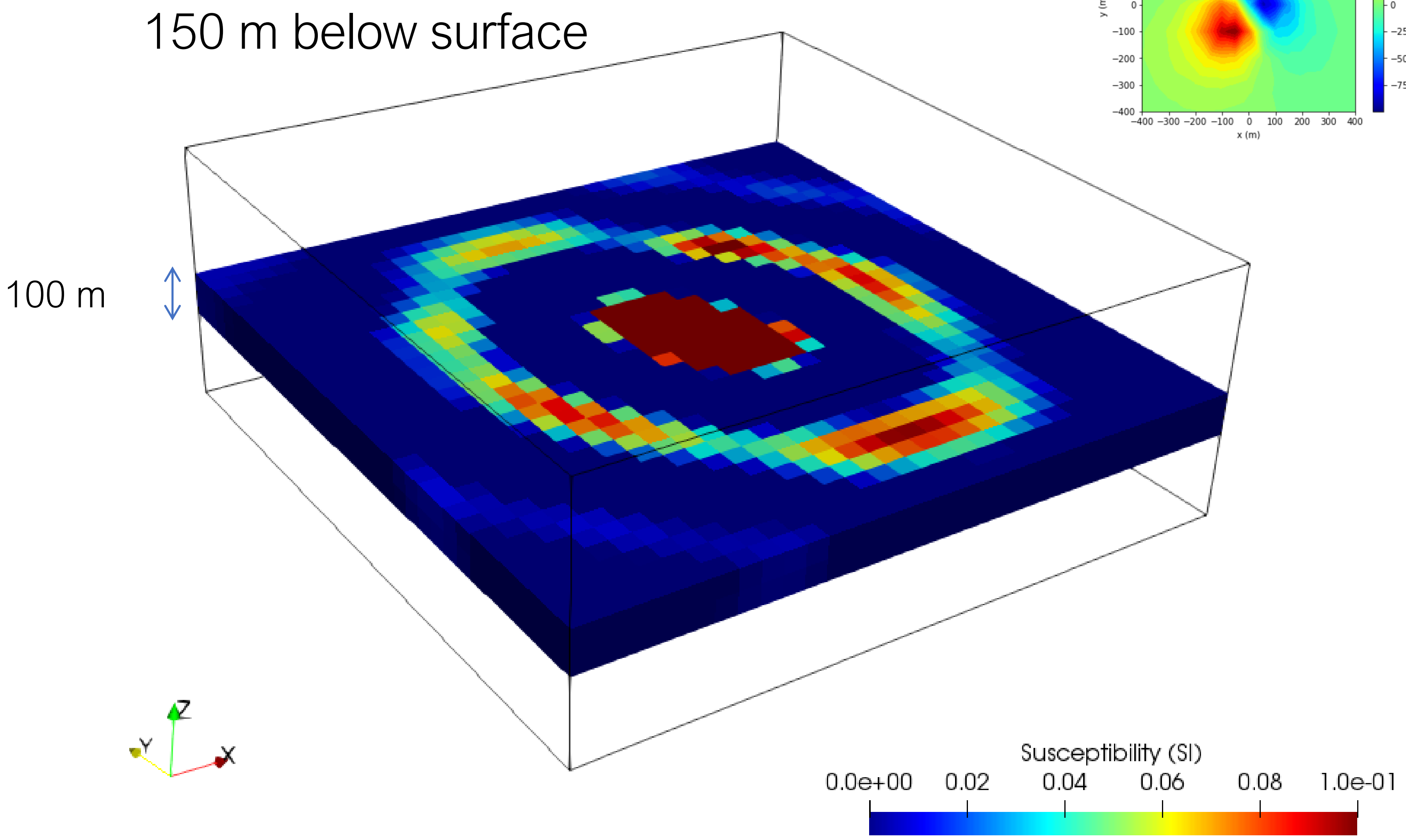


100 m below surface



Example of extreme non-uniqueness





Framework for the inverse problem

$$\text{minimize} \;\; \phi(m) = \underline{\phi_d}(m) + \beta \underline{\phi_m}(m)$$

ϕ_d: data misfit
ϕ_m: model norm
β: trade-off parameter

Find a model fitting the data

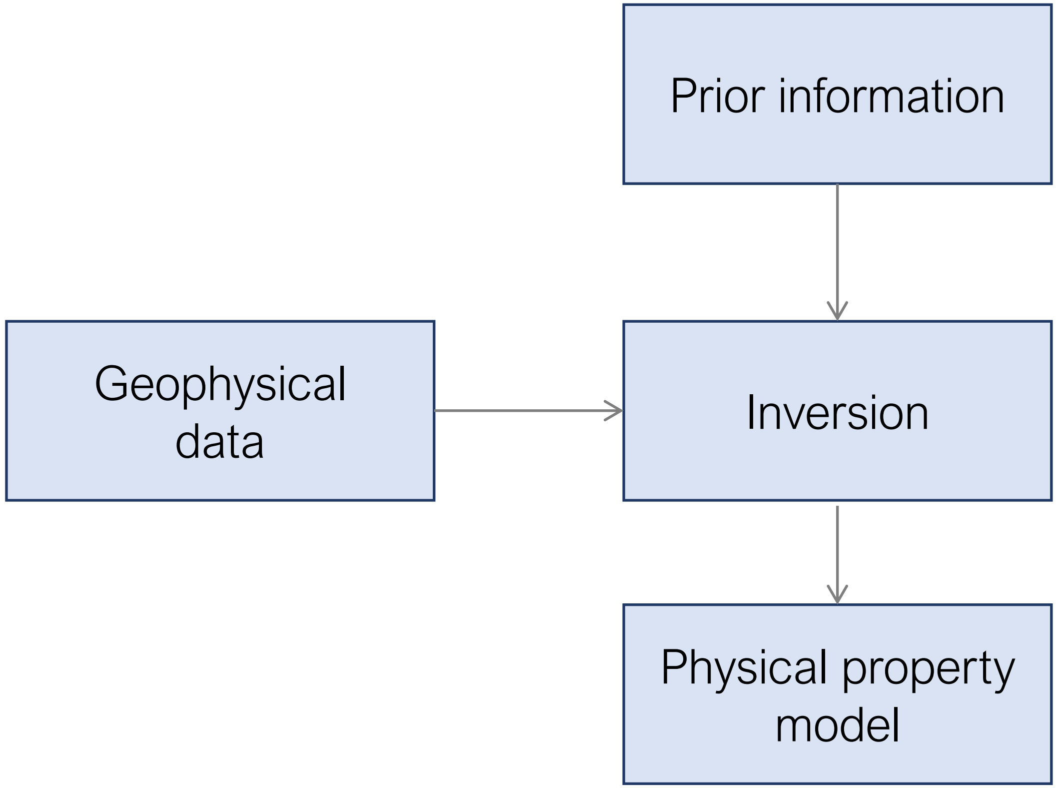
Find a model favoring prior knowledge

- drillers' logs
- geophysical logs (e.g., resistivity)
- spatial patterns
- average resistivity value of the region
- other geophysical data (e.g., seismic)

Constraining the inversion

What information is available?

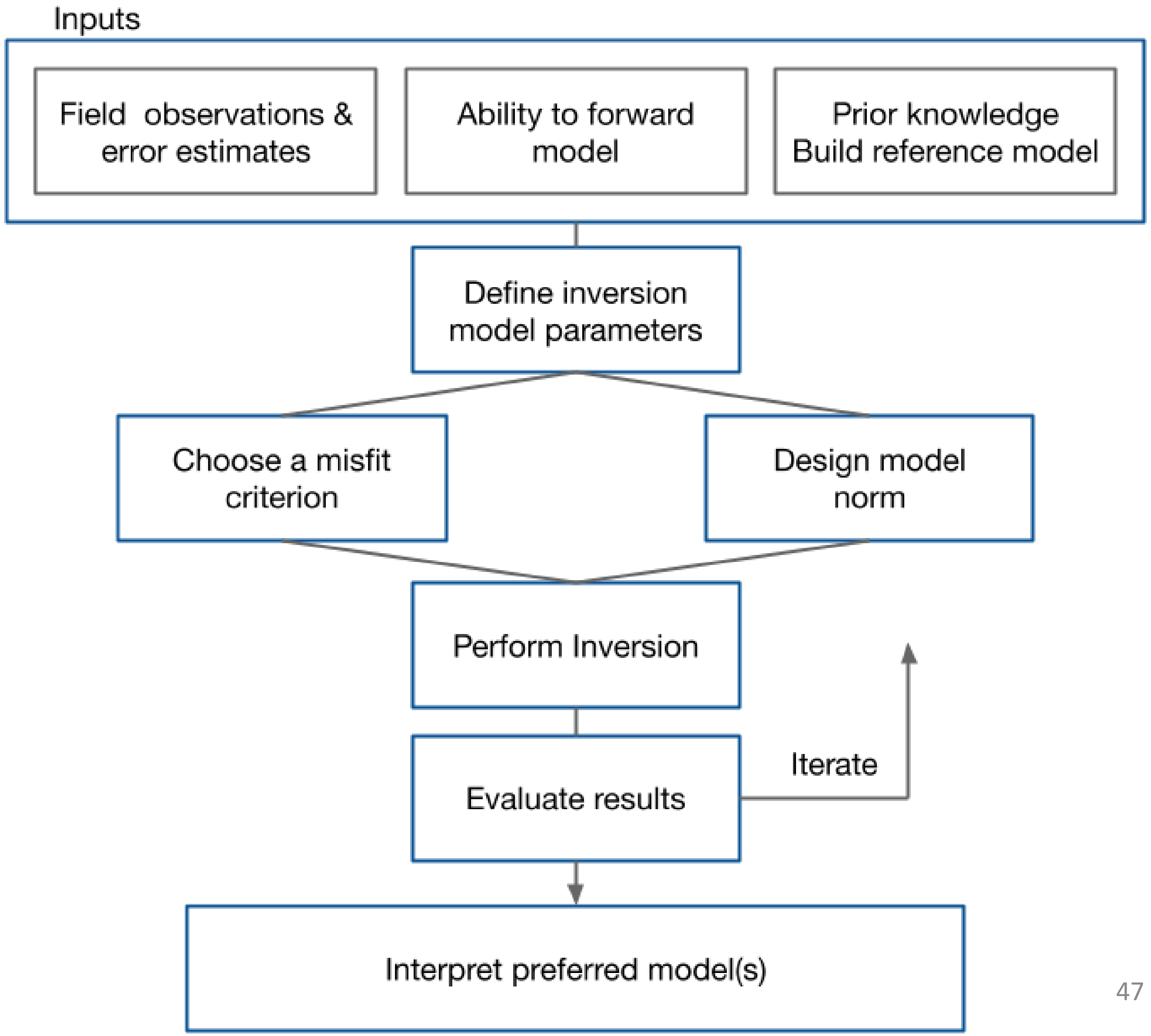
- Geologic structure
- Geologic constraints
- Reference model
- Bounds
- Multiple data sets
- Physical property measurements



How do we achieve our goal?

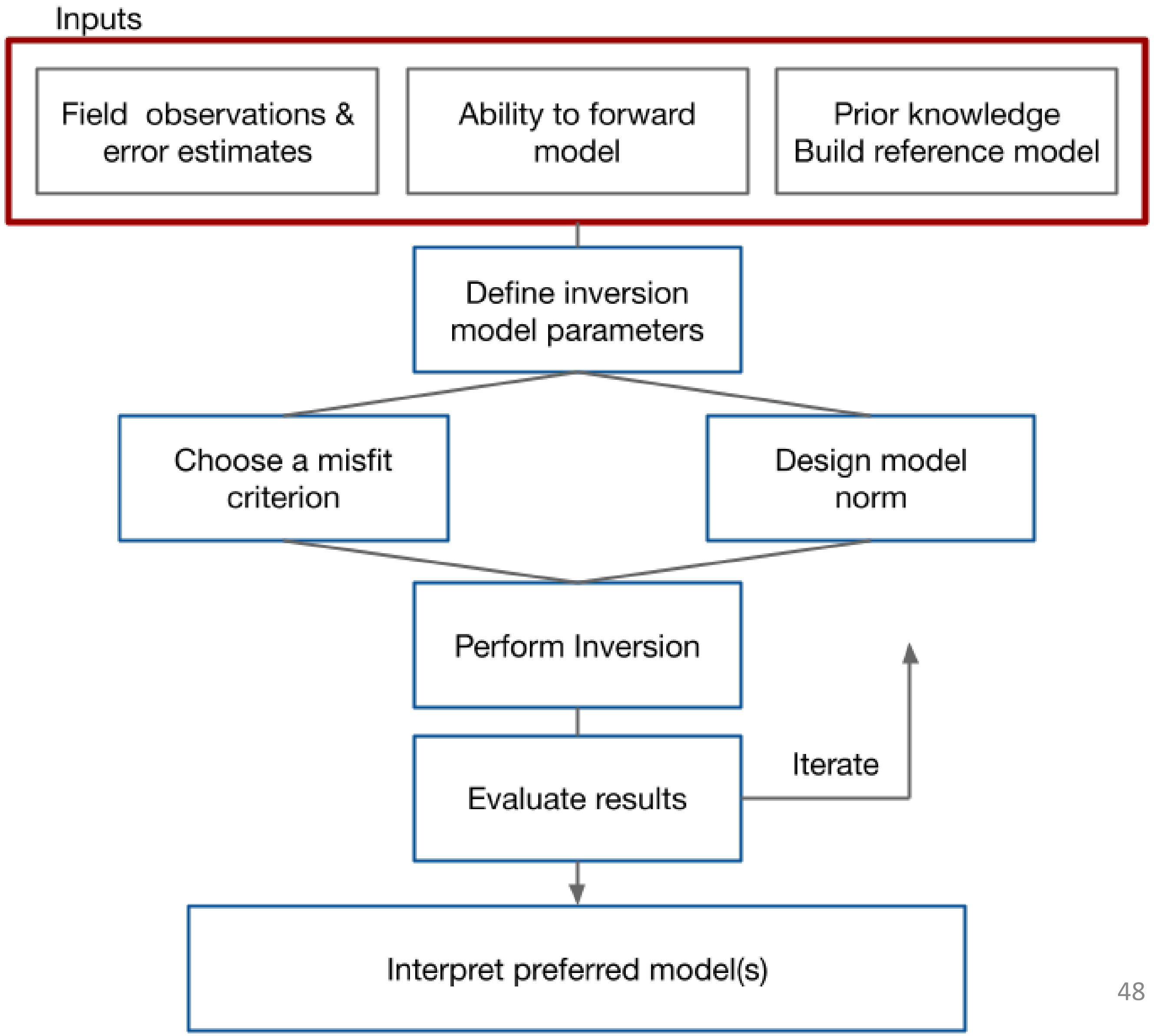
Flow chart for the inverse problem

- Many components to achieving a quality result
- Success is in the details
- Evaluate each step in the box critically before going on



Starting up

- Survey and observations
- What processing has been done?
- Normalization of data
- Ability for forward model
- Assemble geologic, petrophysical information
- Build a reference model
- What is the question you want answered from the inversion?



Inversion model parameters

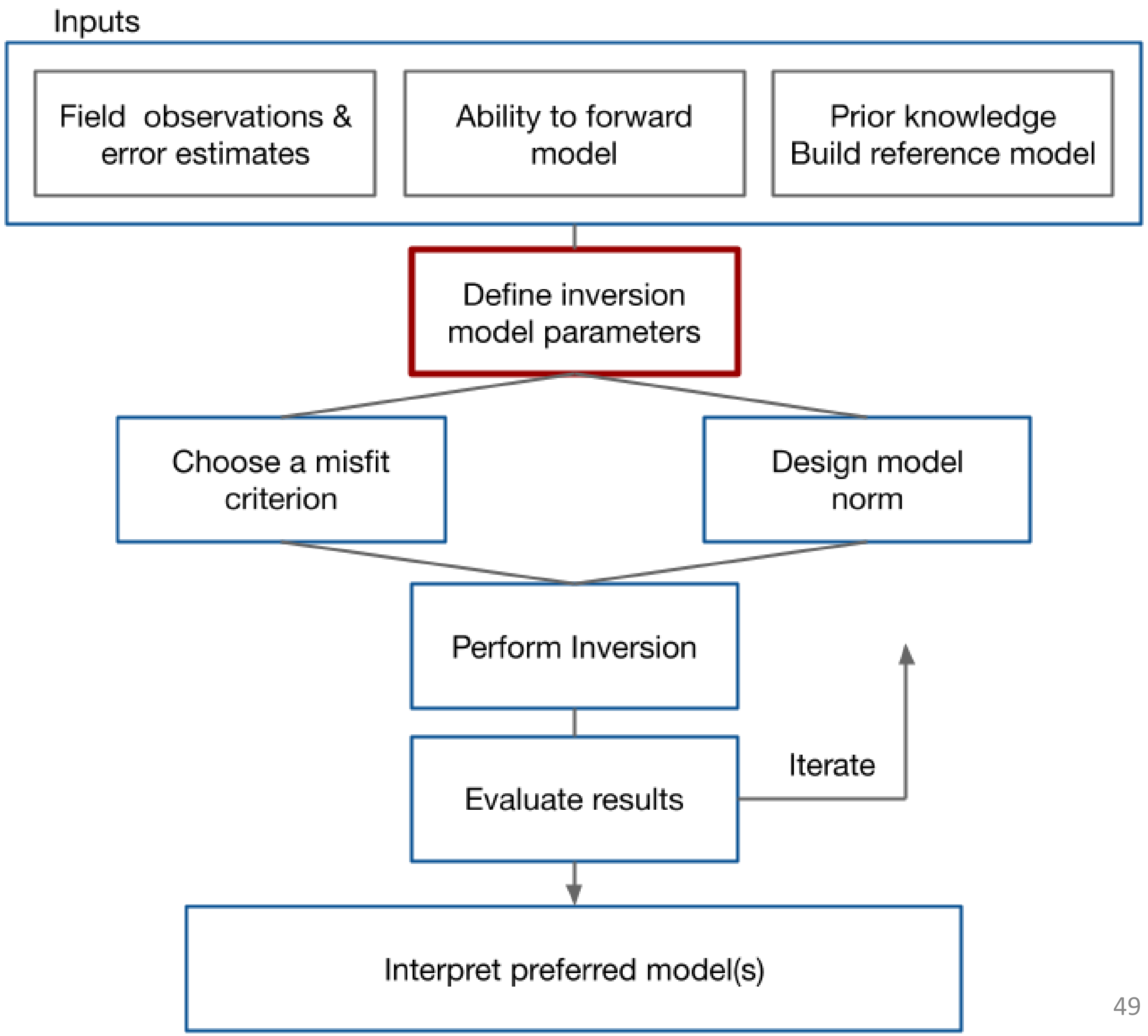
- In the forward problem

$$d = \mathcal{F}[m]$$

m is our sought function

(susceptibility, density, ….)

- Inverse problem: we have options (e.g., subsurface, parametric ….)

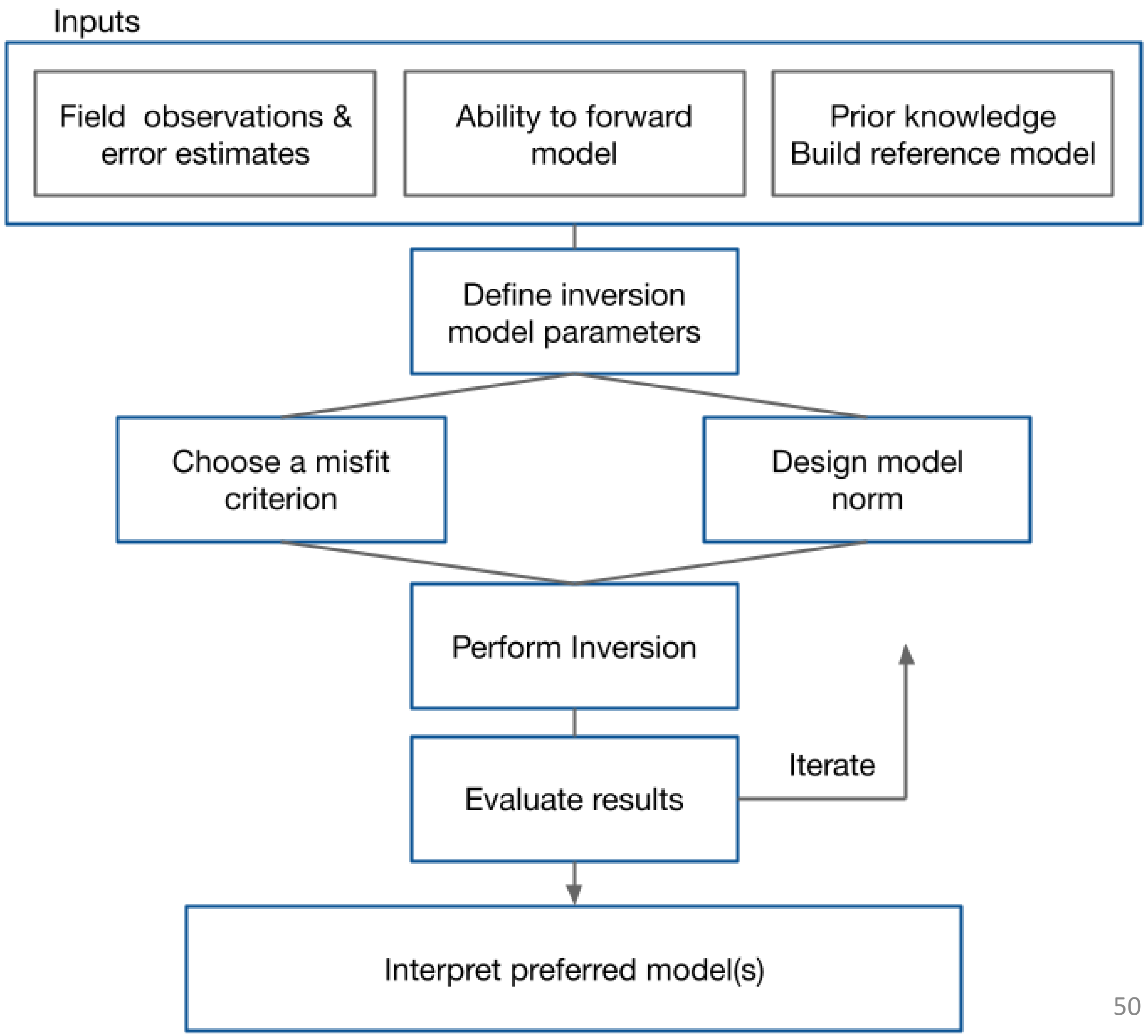


Inversion as an optimization problem

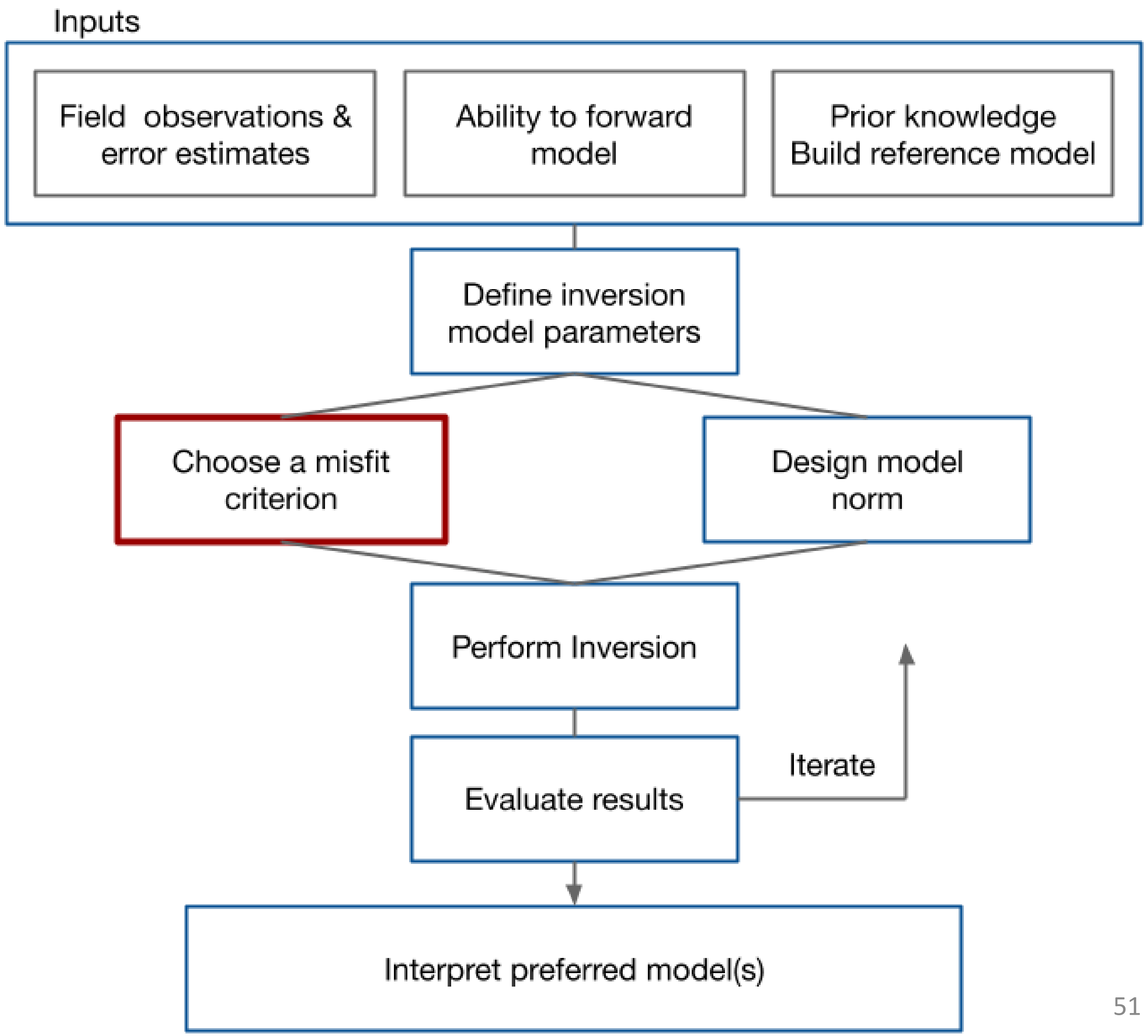
- Find a single "best" solution by solving optimization

$$\text{minimize} \quad \phi = \phi_d + \beta\phi_m$$
$$\text{subject to} \quad m_L \leq m \leq m_U$$

$$\begin{aligned} \phi_d &: \text{ data misfit} \\ \phi_m &: \text{ model norm} \\ \beta &: \text{ trade-off parameter} \\ m_L, m_U &: \text{ lower and upper bounds} \end{aligned}$$



Flow chart for the Inverse problem



Dealing with uncertainties

Observed datum

$$d_j^{obs} = F_j(m) + n_j$$

Noise n_j includes

- Modelling errors
  - dimensionality errors (1D v. 3D)
  - incomplete physics
  - discretization errors
- Noise on data
  - instrument / sensor noise
  - survey parameter errors
  - wind …

True statistics of "noise" is complicated.
In practice, assume errors are Gaussian
$\mathcal{N}(0, \epsilon_j)$

Dealing with uncertainties

Consider $\quad$ random variable, $x_j \in \mathcal{N}(0,1)$

Define $$\chi_N^2 = \sum_{j=1}^{N} x_j^2$$

Chi-squared statistic with N degrees of freedom

$$\begin{cases} \text{Expected value: } E(\chi_N^2) = N \\ \text{Variance: } \mathrm{Var}(\chi_N^2) = 2N \\ \text{Standard deviation: } \mathrm{std}(\chi_N^2) = \sqrt{2N} \end{cases}$$

Misfit function

Crucial steps for any misfit:
(1) Specify the metric used
(2) Determine target misfit

We use L_2 norm (least squares statistic)

Define data misfit: $\phi_d = \sum_{j=1}^{N} \left(\frac{F_j(m) - d^{obs}}{\epsilon_j} \right)^2$

Define $\mathbf{W}_d = \mathbf{diag}(1/\epsilon_1, \ldots, 1/\epsilon_N)$

$$\phi_d = \|\mathbf{W}_d(F[\mathbf{m}] - \mathbf{d}^{obs})\|_2^2$$

$$E[\phi_d] \simeq N$$

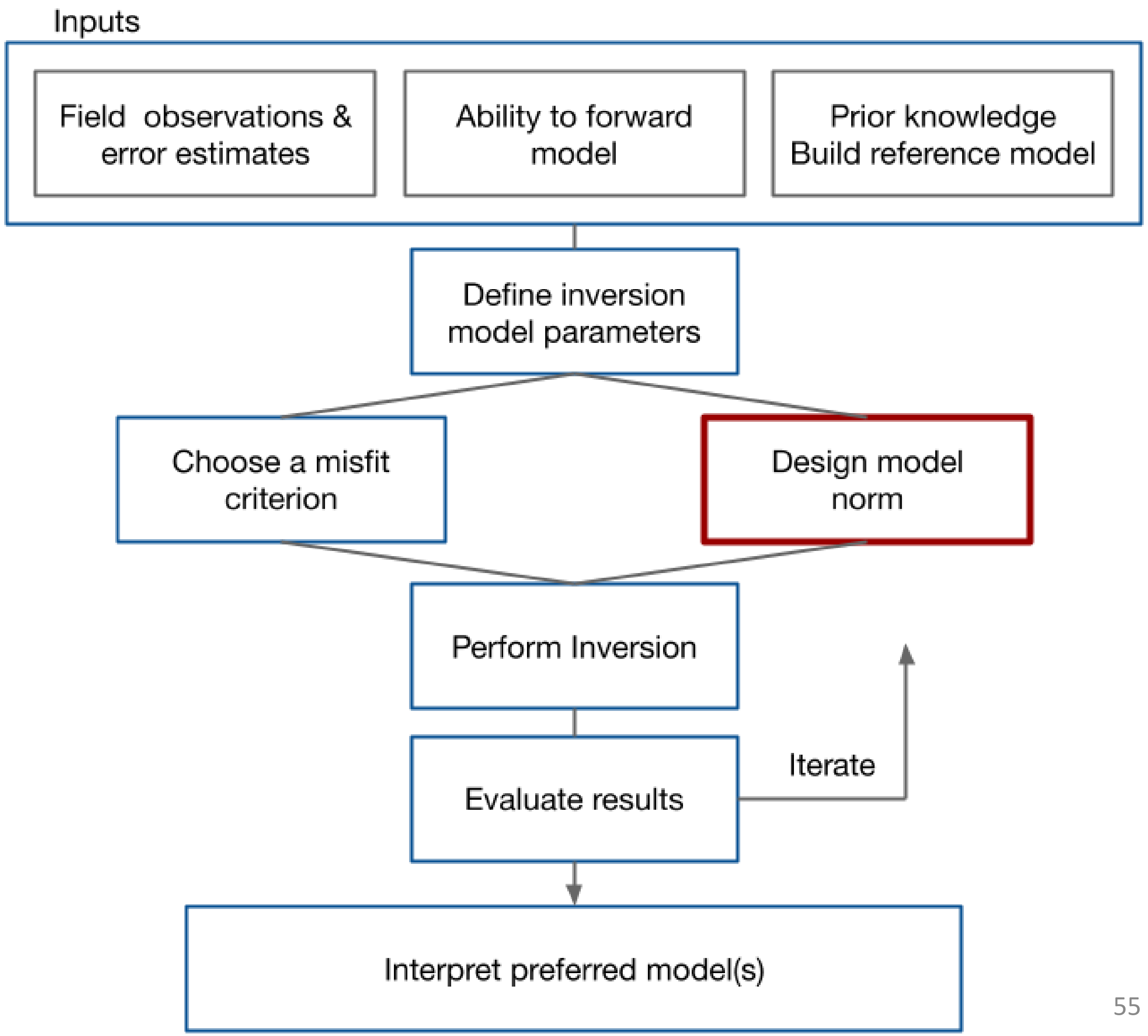
ϕ_d is now a χ^2_N variable

<u>Reality</u>: we do not know uncertainties

<u>Try</u>:

$$\epsilon_j = \%|d_j^{obs}| + \text{floor}$$

Flow chart for the Inverse problem



Model norms

First define our model norms as functions and then discretize

Smallest model:

$$\phi_m = \int (m - m_{ref})^2 dx$$

Flattest model:

$$\phi_m = \int \left(\frac{dm}{dx}\right)^2 dx$$

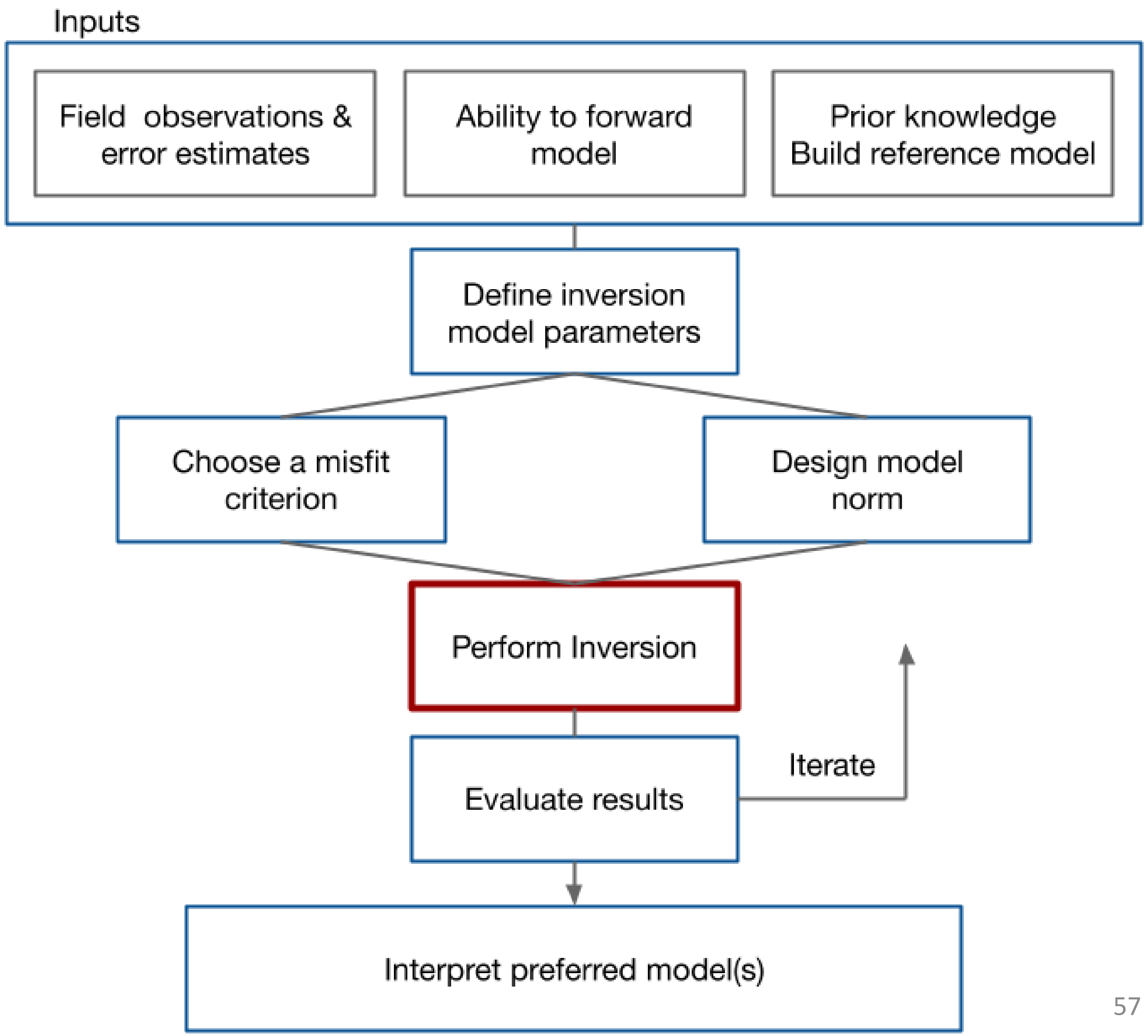
Combination:

$$\phi_m = \alpha_s \int (m - m_{ref})^2 dx + \alpha_x \int \left(\frac{dm}{dx}\right)^2 dx$$

Discretize:

$$\phi_m = \alpha_s \|\mathbf{W}_s(\mathbf{m} - \mathbf{m}_{ref})\|_2^2 + \alpha_x \|\mathbf{W}_x(\mathbf{m})\|_2^2$$

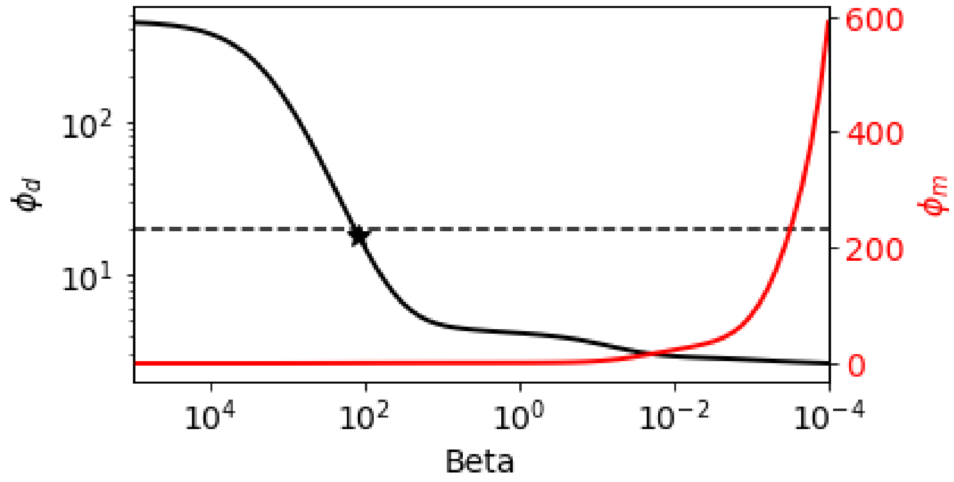
Flow chart for the Inverse problem



Role of beta

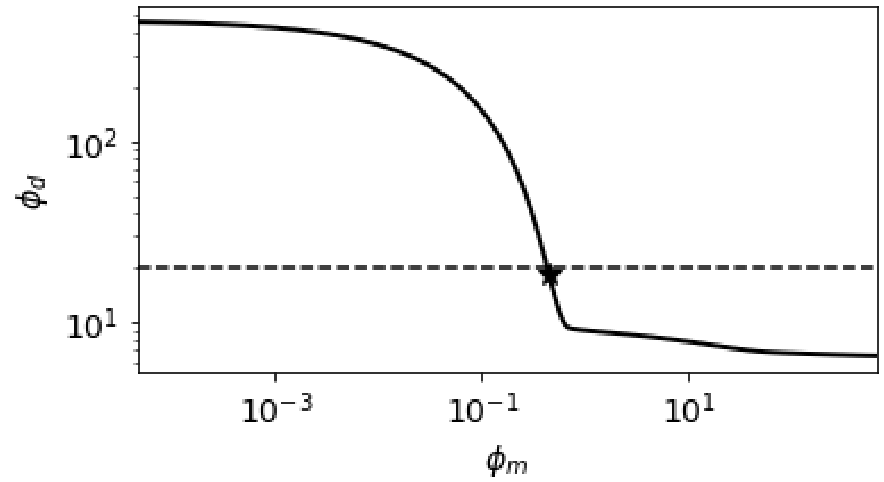
$$\phi(m) = \phi_d(m) + \beta\phi_m(m)$$

$$\beta \to 0 \quad : \qquad \phi \sim \phi_d$$
$$\beta \to \infty : \qquad \phi \sim \phi_m$$



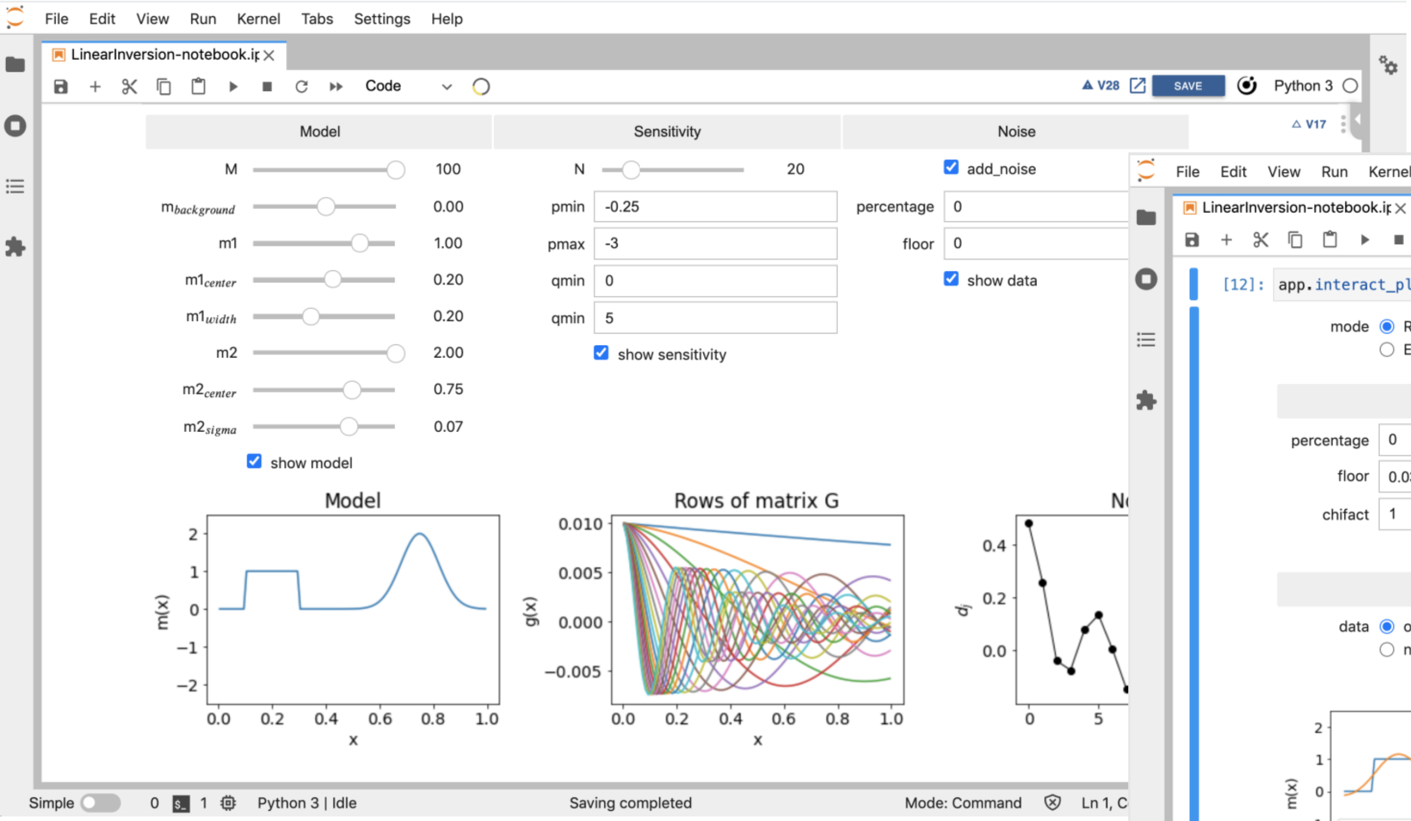
Tikhonov Curve

- Desired misfit $\phi_d^* \simeq N$
- Choose β such that $\phi_d(m) = \phi_d^*$

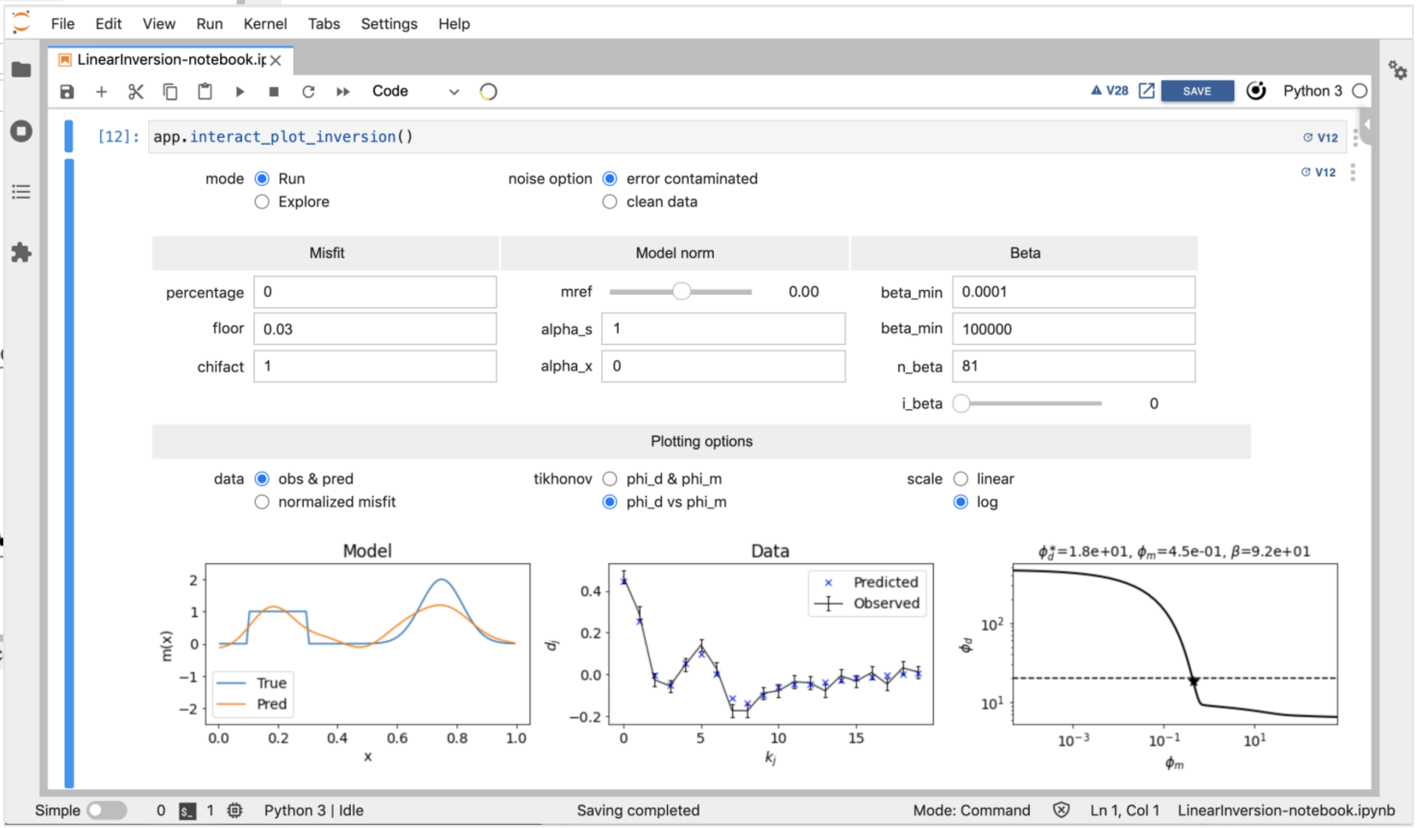


Demo: Linear Inversion App

Develop survey



Run inversion



https://curvenote.com/@swung/inversion-for-geologists-transform-2021