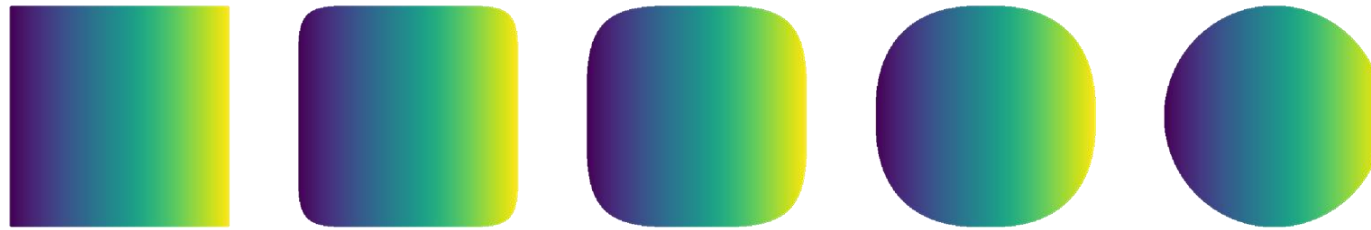


Take a break! We'll be back soon.

softwareunderground.org presents



# TRANSFORM 2021

Virtual Conference on the Digital Subsurface, 16–23 April

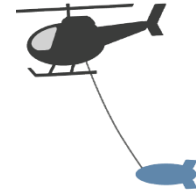


supported by

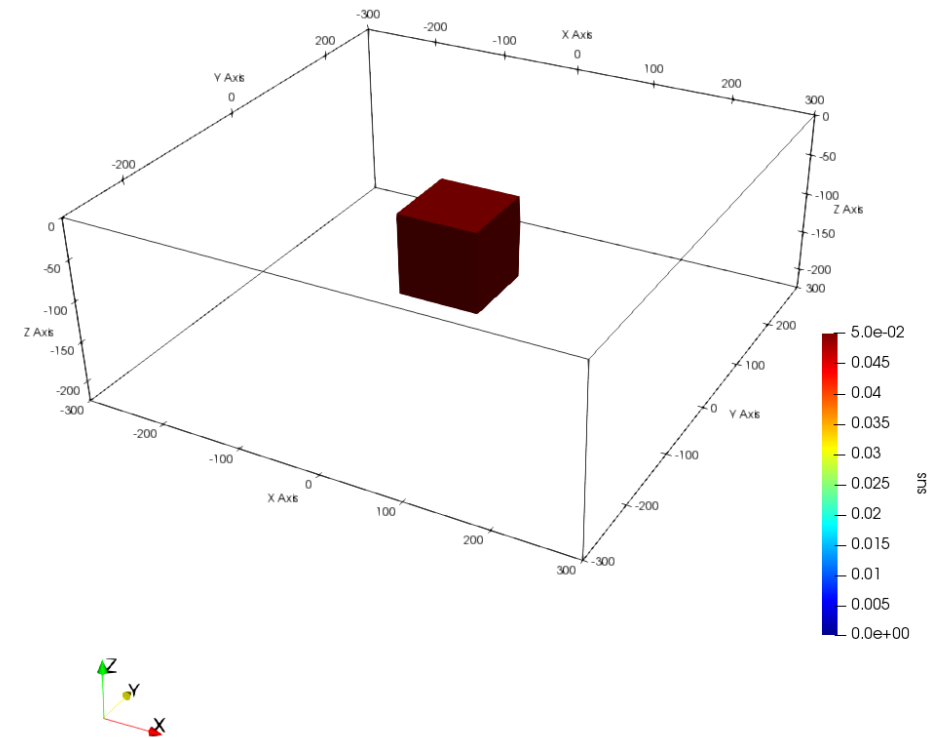


# Outline

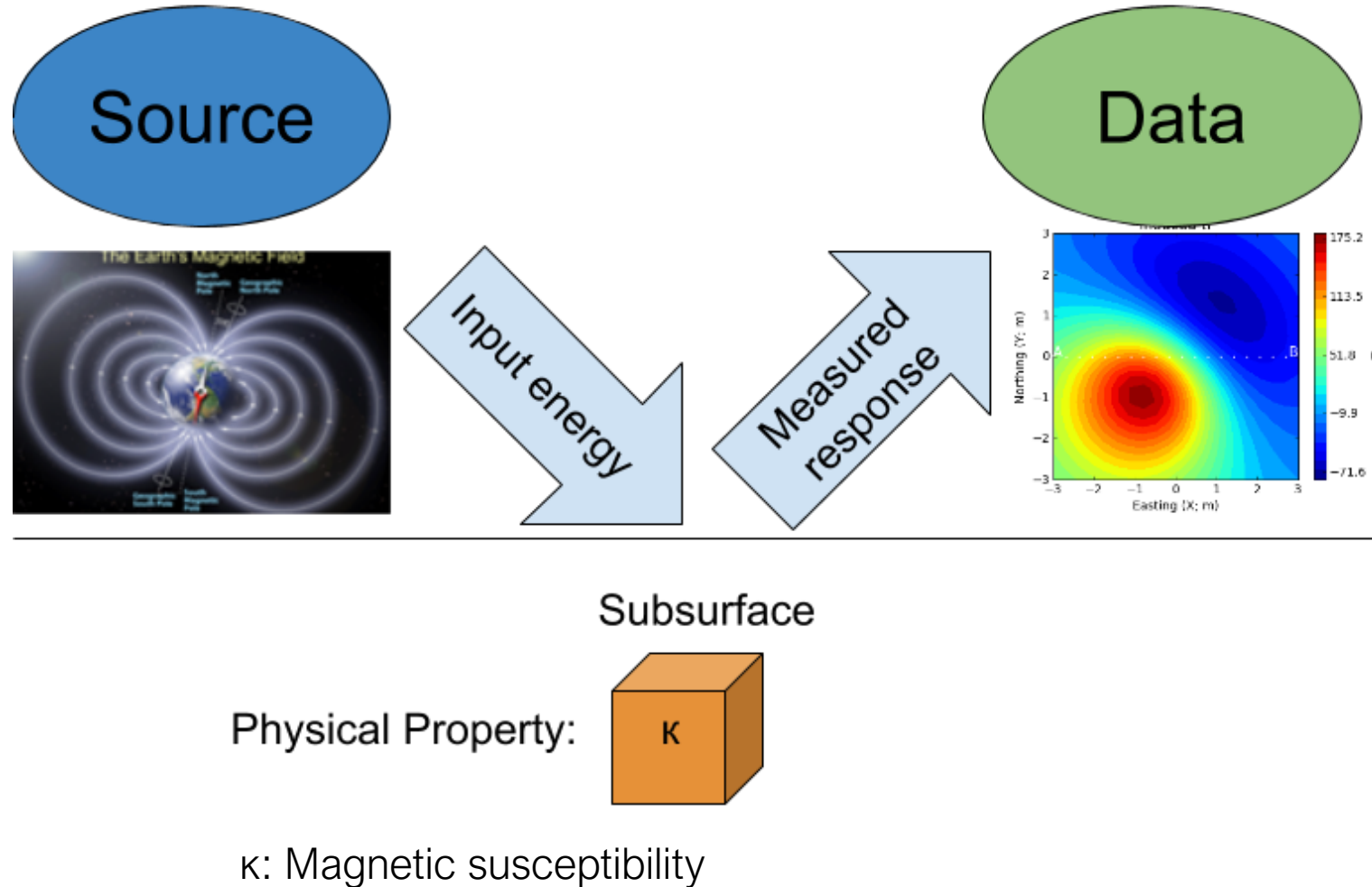
- Backgrounds: Magnetism
- Inversion Framework
- 1D Linear Inverse problem
- 3D Magnetic Inversion
- Including Other Information
- Summary



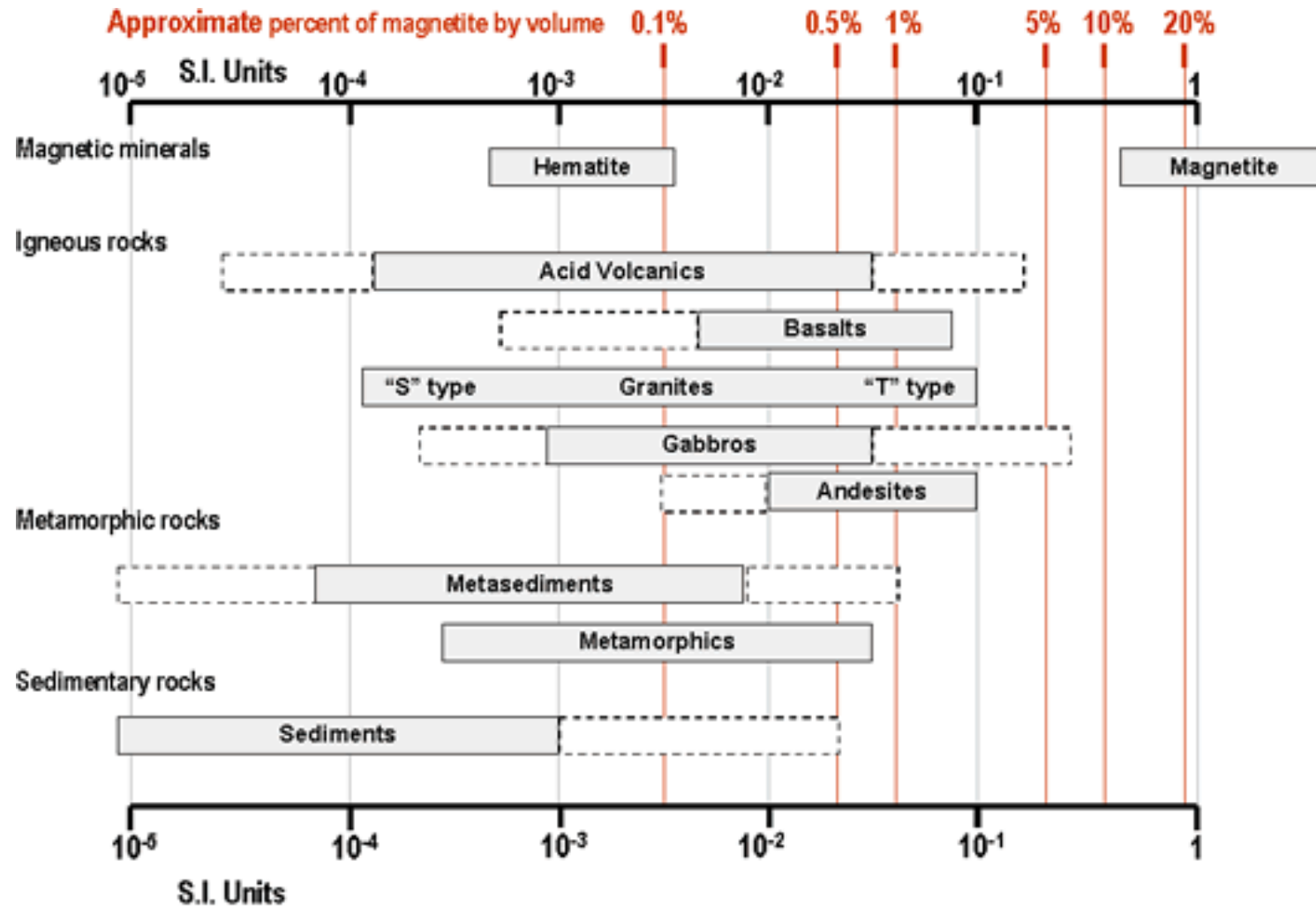
# 3D magnetic inversion



# Survey: Magnetics



# Magnetic susceptibility



# Magnetic surveying

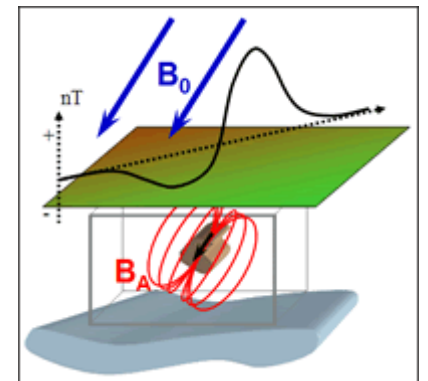
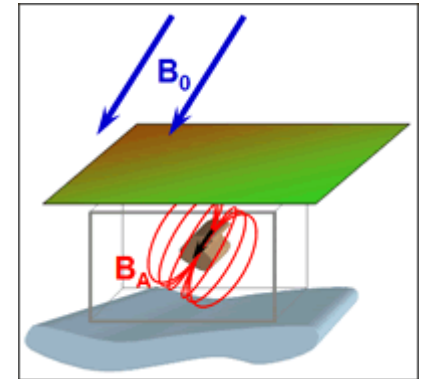
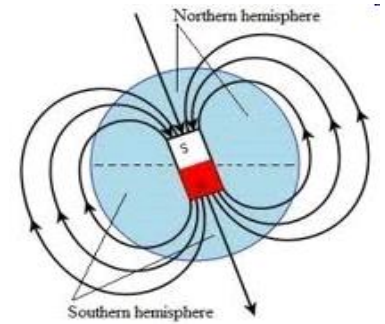
- Earth's magnetic field  $\vec{B}_0$  is the source:
- Materials become magnetized

$$\vec{M} = \kappa \vec{H}_0 \text{ (magnetization)}$$

$$\vec{H}_0 = \vec{B}_0 / \mu_0$$

- Create anomalous magnetic field
- Measure total magnetic field:  $|\vec{B}| = |\vec{B}_0 + \vec{B}_A|$

- Total field anomaly:  $\Delta \vec{B} = |\vec{B}_0 + \vec{B}_A| - |\vec{B}_0|$   
 $\Delta \vec{B} \simeq \vec{B}_A \cdot \hat{B}_0$  where  $\hat{B}_0 = \frac{\vec{B}_0}{|\vec{B}_0|}$



# Forward modelling

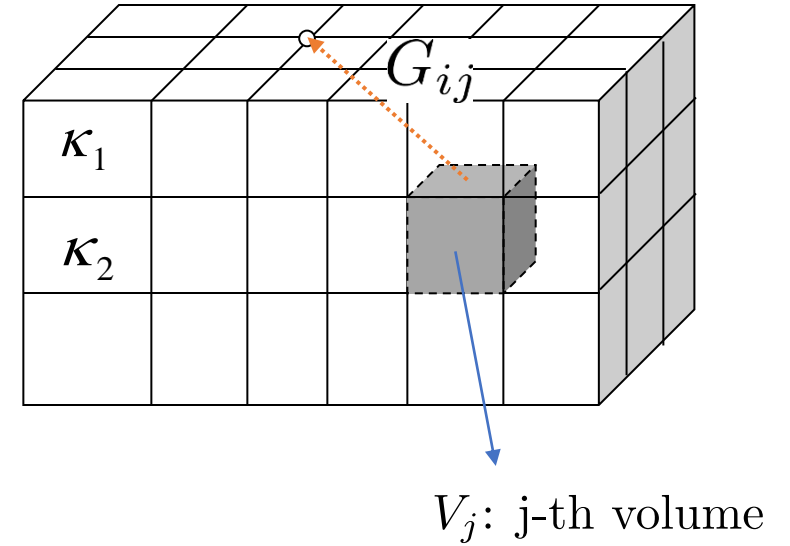
- Discretize earth

$\kappa_j$  ( $j = 1, \dots, M$ )      susceptibility

- Magnetic anomaly data are

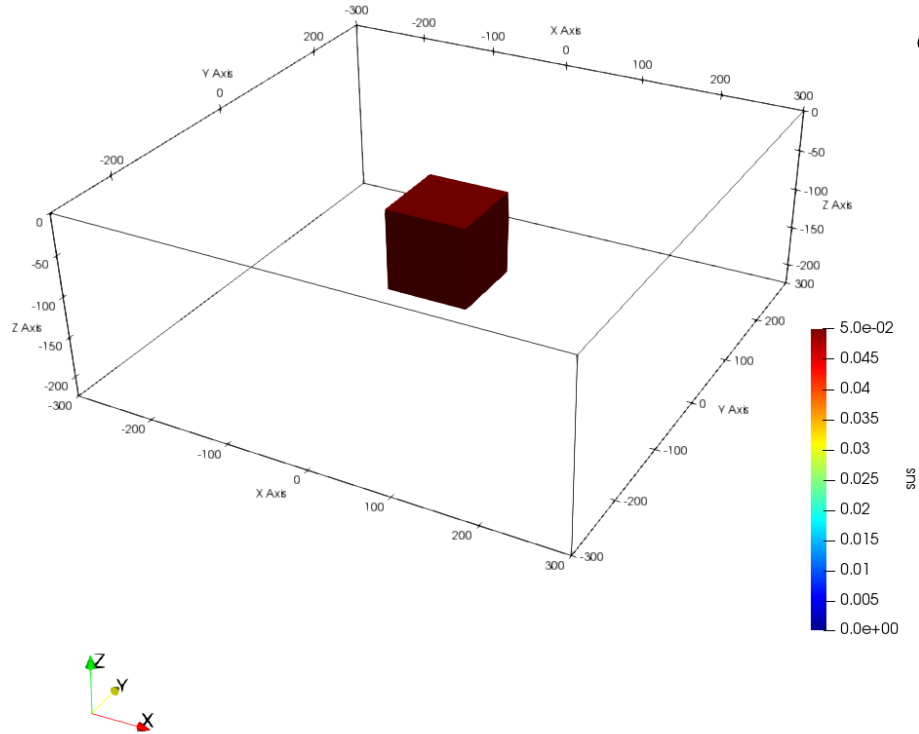
$$d_i = \sum_{j=1}^M G_{ij} \kappa_j$$

$$\begin{cases} G_{ij} = \hat{B}_0 \cdot \left\{ \frac{\mu_0}{4\pi} \int_v \kappa \nabla \nabla \left( \frac{1}{r_i - r_j} \right) dV_j \right\} \cdot \hat{B}_0 \\ \hat{B}_0 = \frac{\vec{B}_0}{|\vec{B}_0|} \end{cases}$$



# Forward modelling

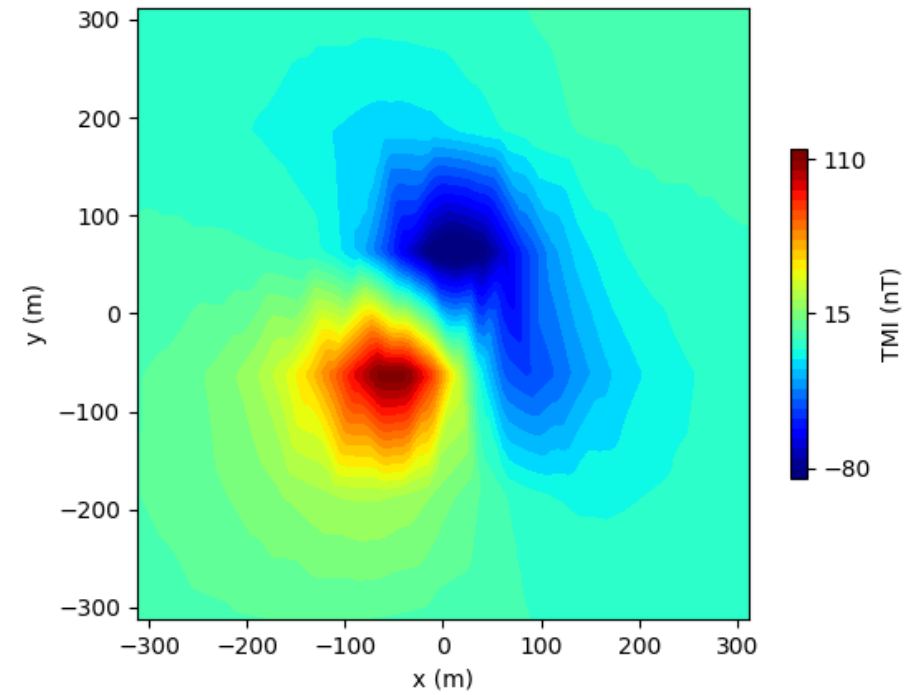
Susceptibility model



$$d_i = \sum_{j=1}^M G_{ij} \kappa_j$$

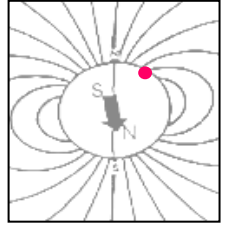
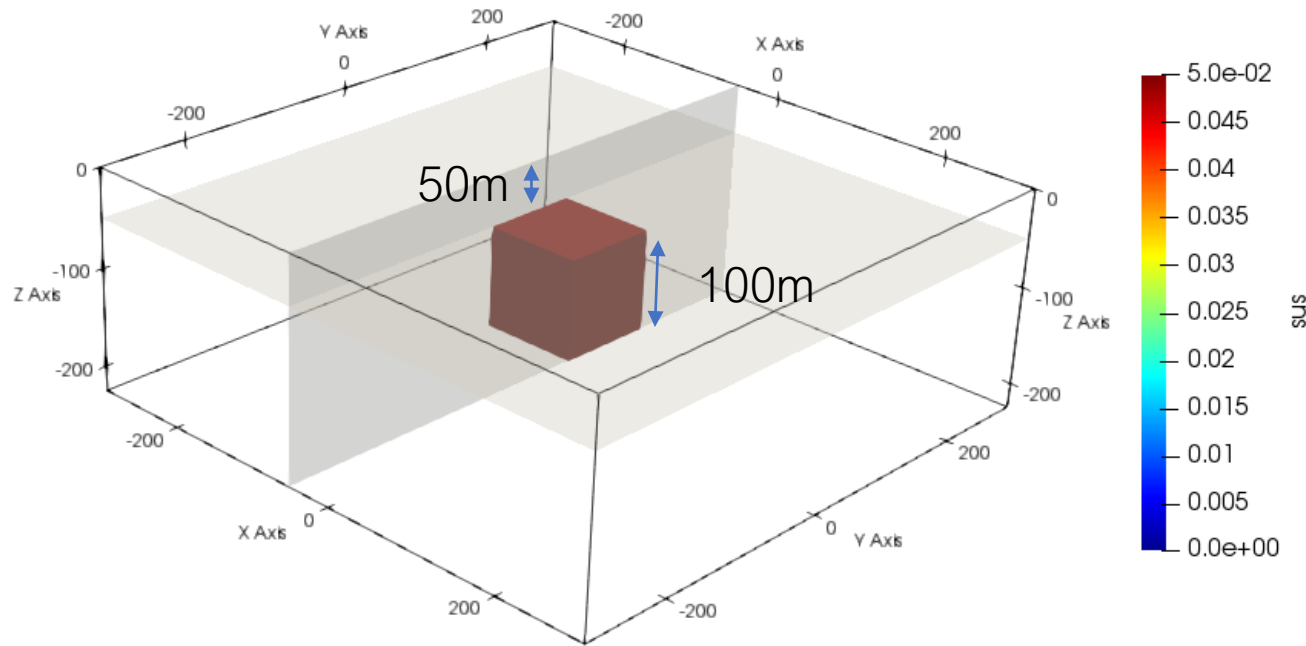


Magnetic data

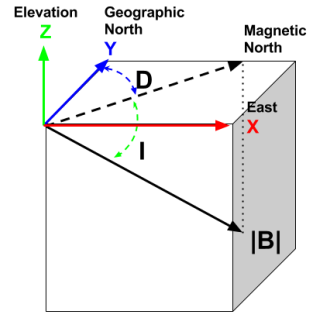




# Synthetic susceptibility model



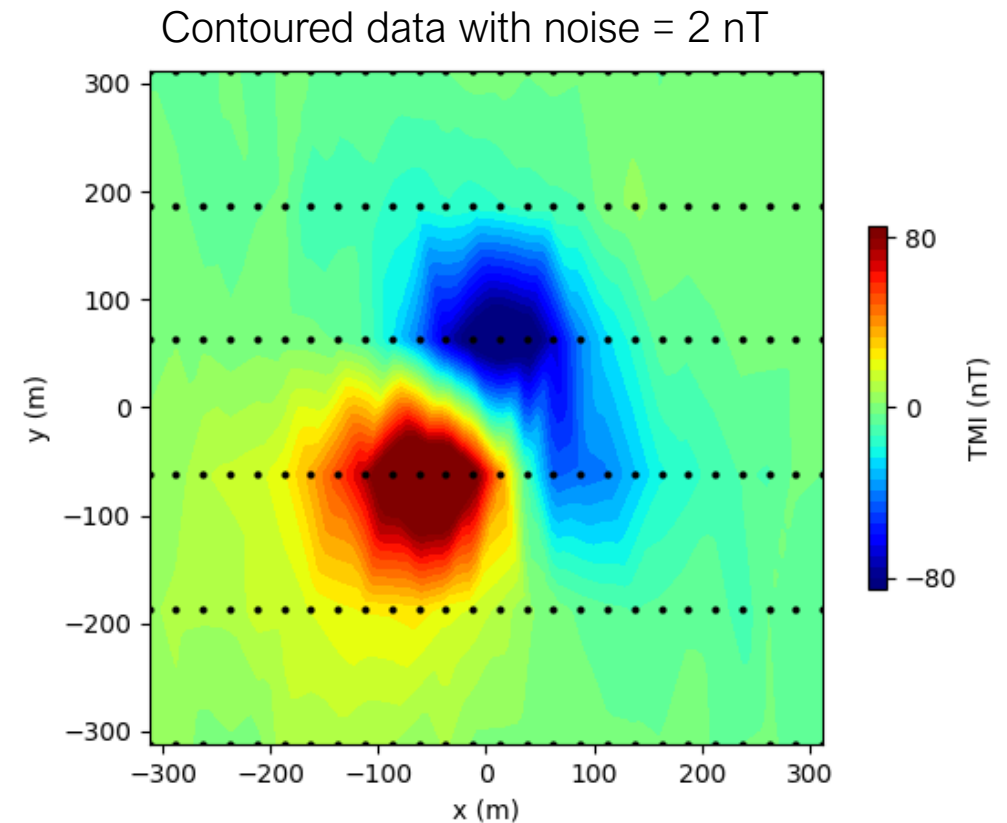
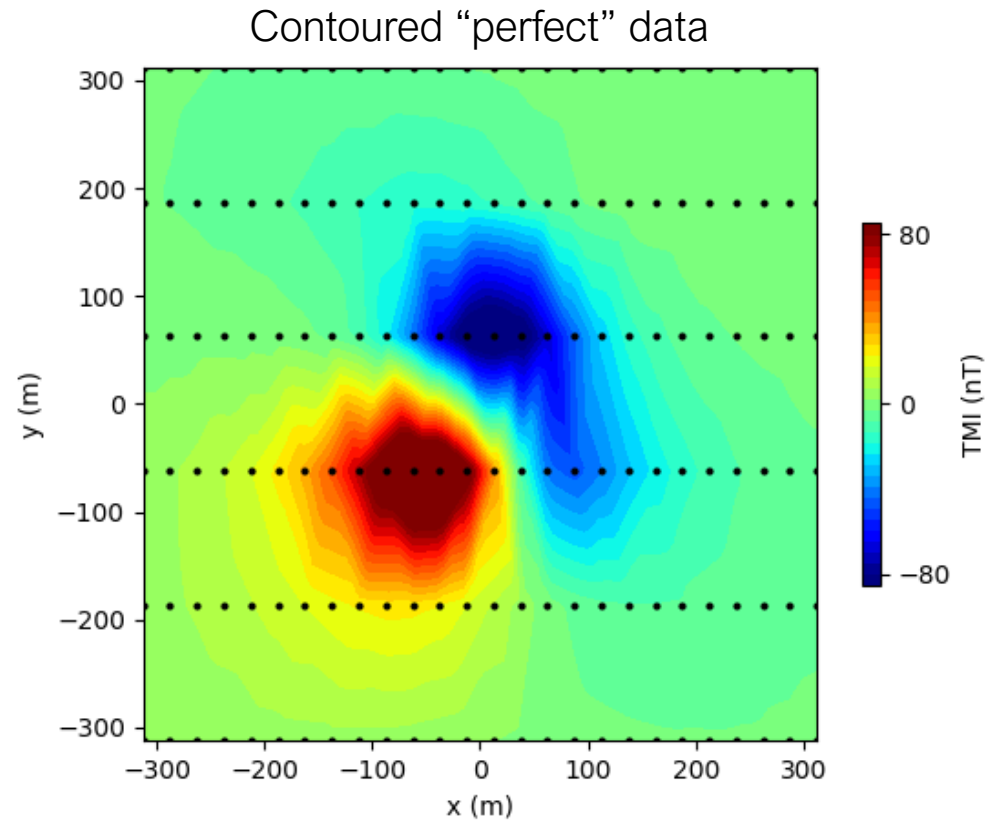
- Earth field
  - Inclination:  $30^\circ$
  - Declination:  $45^\circ$
  - $|B_0| = 50,000 \text{ nT}$
- Susceptible block
  - 100m x 100m x 100m block
  - Block susceptibility = 0.5
  - Block top = 50m



# Synthetic survey

Survey parameters:

- 100 m line spacing.
- 25 station spacing.
- N=156 (elevation= 2m)



# Solving inverse problem

Model objective function

$$\phi_m = \alpha_s \int_v w_s \left( \kappa - \kappa_{\text{ref}} \right)^2 dv + \alpha_x \int_v \left( \frac{d\kappa}{dx} \right)^2 dx + \alpha_y \int_v \left( \frac{d\kappa}{dy} \right)^2 dy + \alpha_z \int_v \left( \frac{d\kappa}{dz} \right)^2 dz$$

Data misfit

$$\phi_d = \sum_{j=1}^N \left( \frac{G_{ij} \kappa_j - d_j^{\text{obs}}}{\epsilon_j} \right)$$

Choose

$$\kappa_{\text{ref}} = 0, \alpha_s = 0.0001, \alpha_x = \alpha_y = \alpha_z = 1$$
$$L_x = \sqrt{\frac{\alpha_x}{\alpha_s}} = 100$$

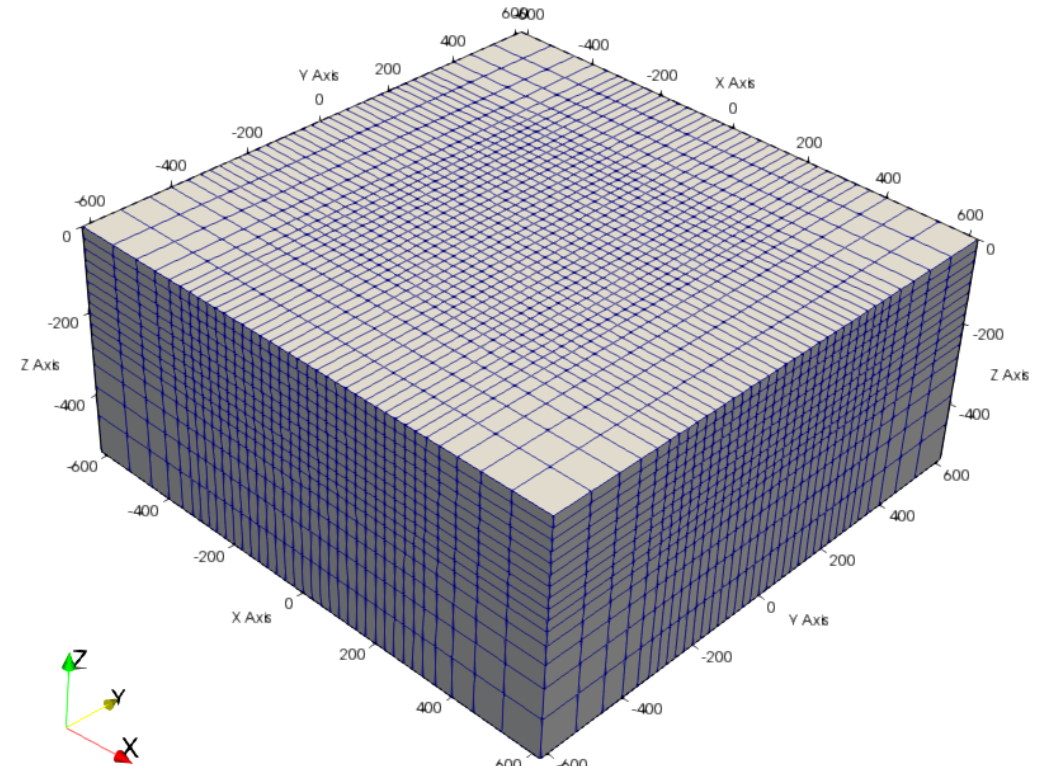
The Inverse problem is:

$$\text{minimize} \quad \phi = \phi_d + \beta \phi_m$$

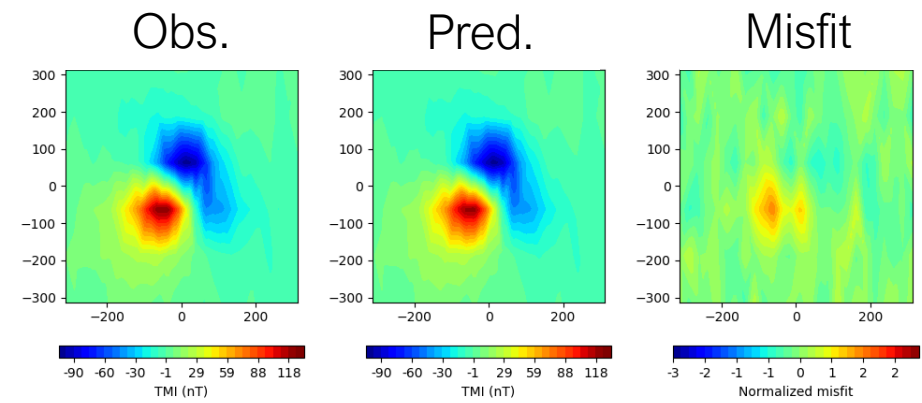
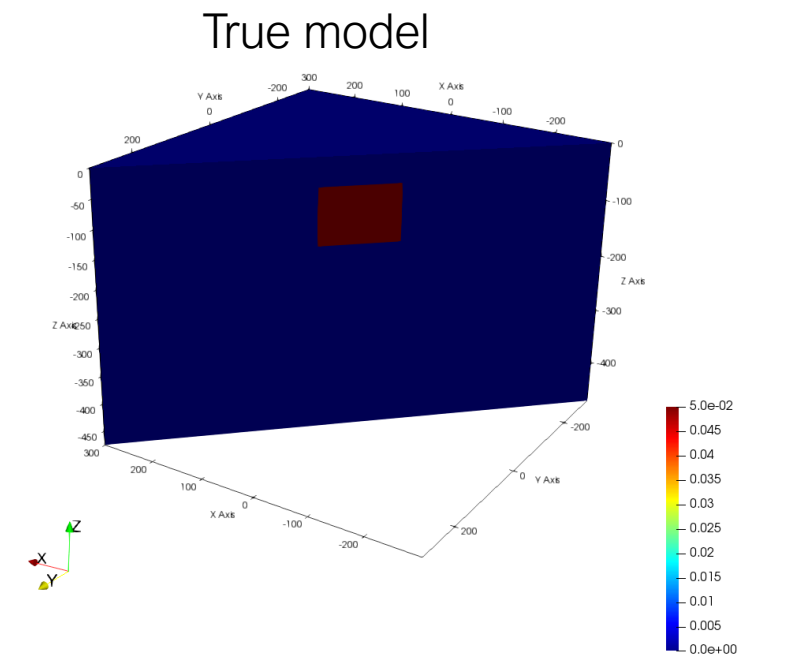
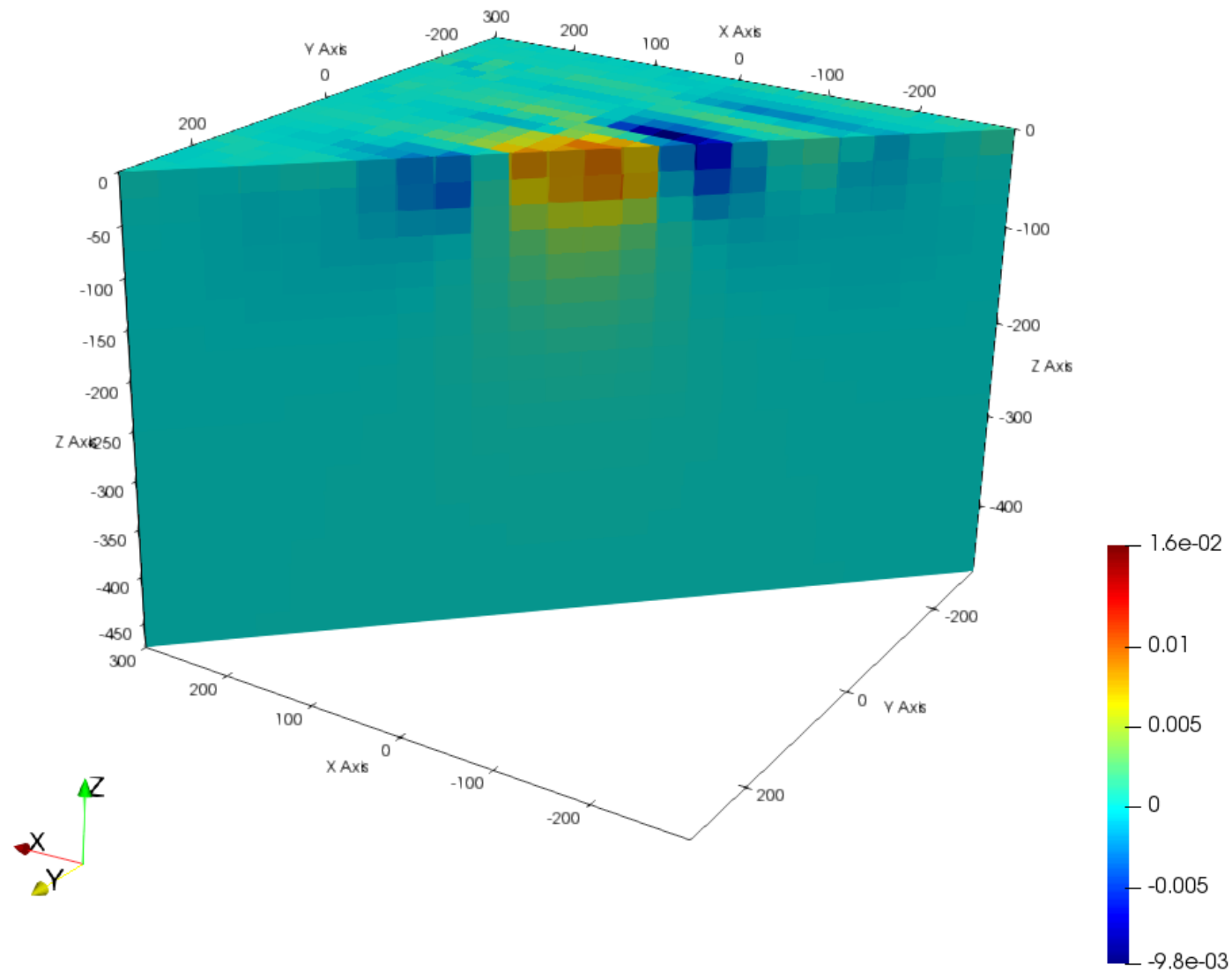
$$\text{find } \beta \text{ such that } \phi_d = \phi_d^* \text{ where } \phi_d = N$$

# Discretization

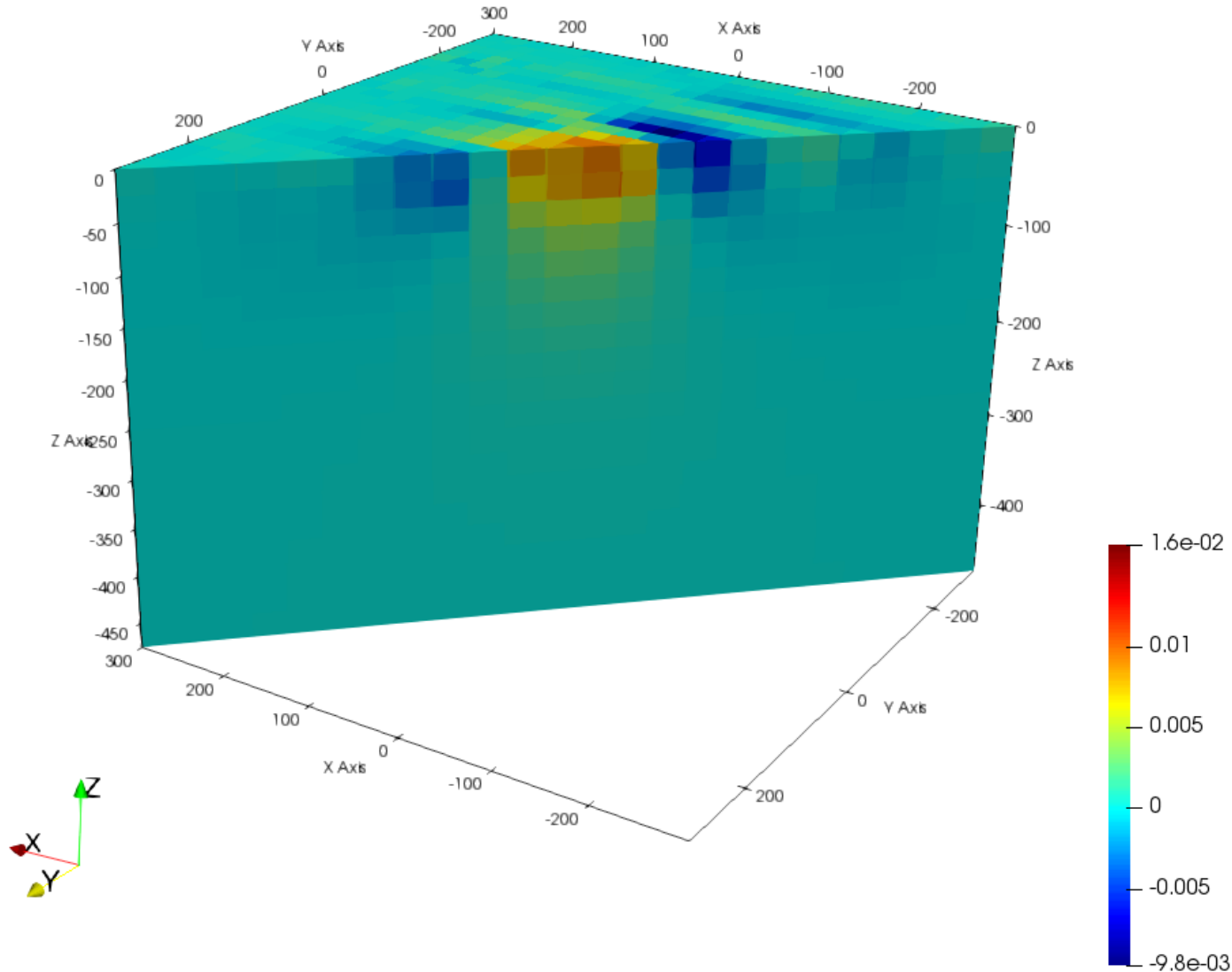
- Earth model for inversion:
  - $dx=dy=dz=25\text{m}$
  - N/S and E/W padding = 300m
  - Number of cells (M) = 19440
- Therefore:
  - No. data is  $N = 176$
  - No. unknowns is  $M = 11,492$



# Inversion results



# Inversion results



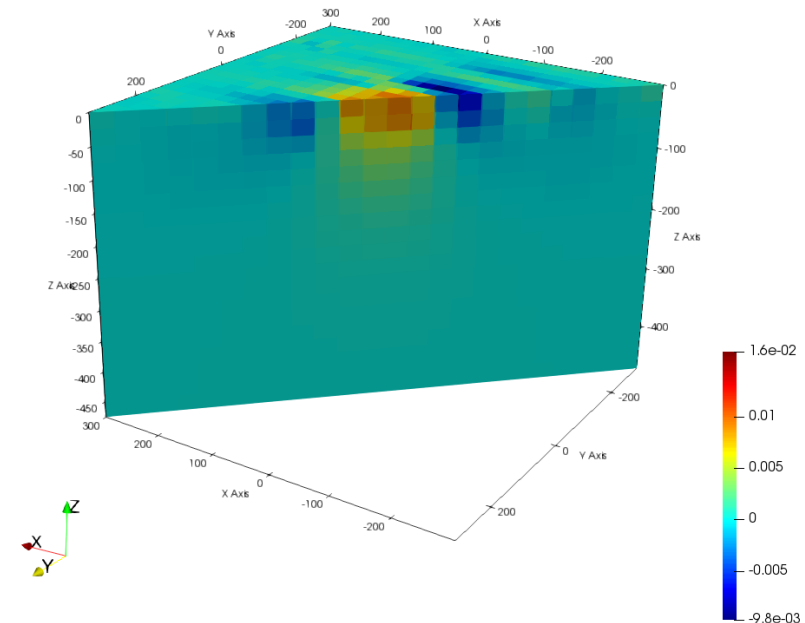
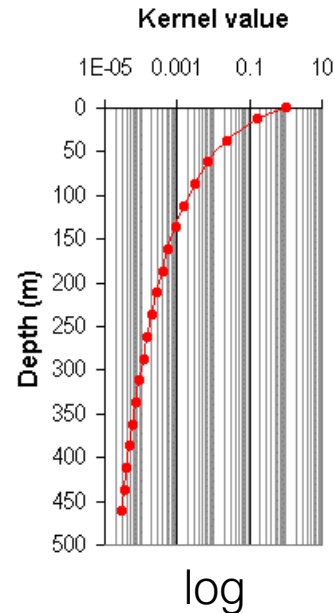
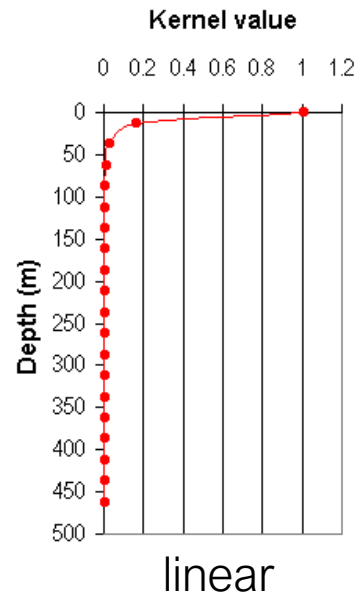
Two primary difficulties:

- Concentration of susceptibility is near the surface
- Regions of negative susceptibility

# What went wrong?

Fundamental non-uniqueness of all potential fields:

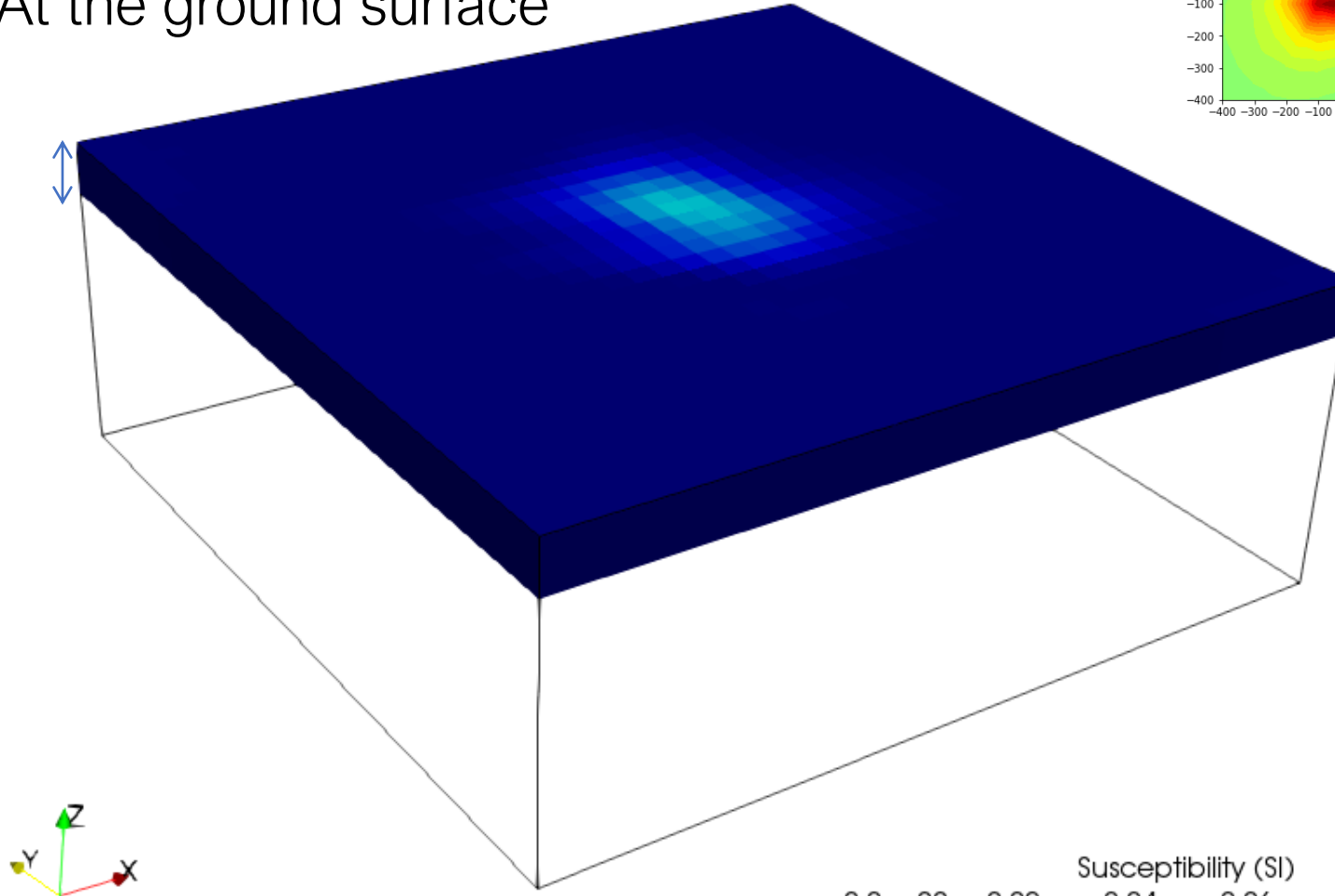
- As a consequence of Green's third identity ...
  - an observed magnetic field can be caused by a thin layer of susceptible material at any arbitrary depth
- The rapid decay of our kernels causes a concentration of  $\kappa$  near the surface to be a preferred solution.



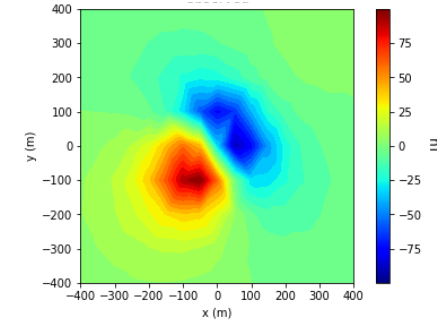
# Example of extreme non-uniqueness

At the ground surface

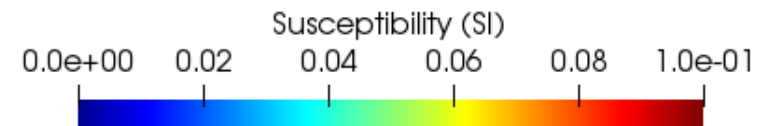
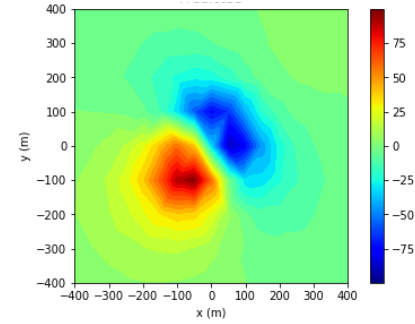
100 m



observed

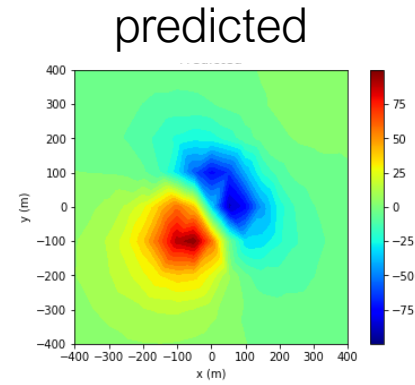
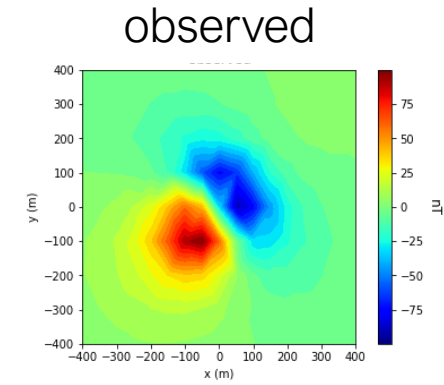
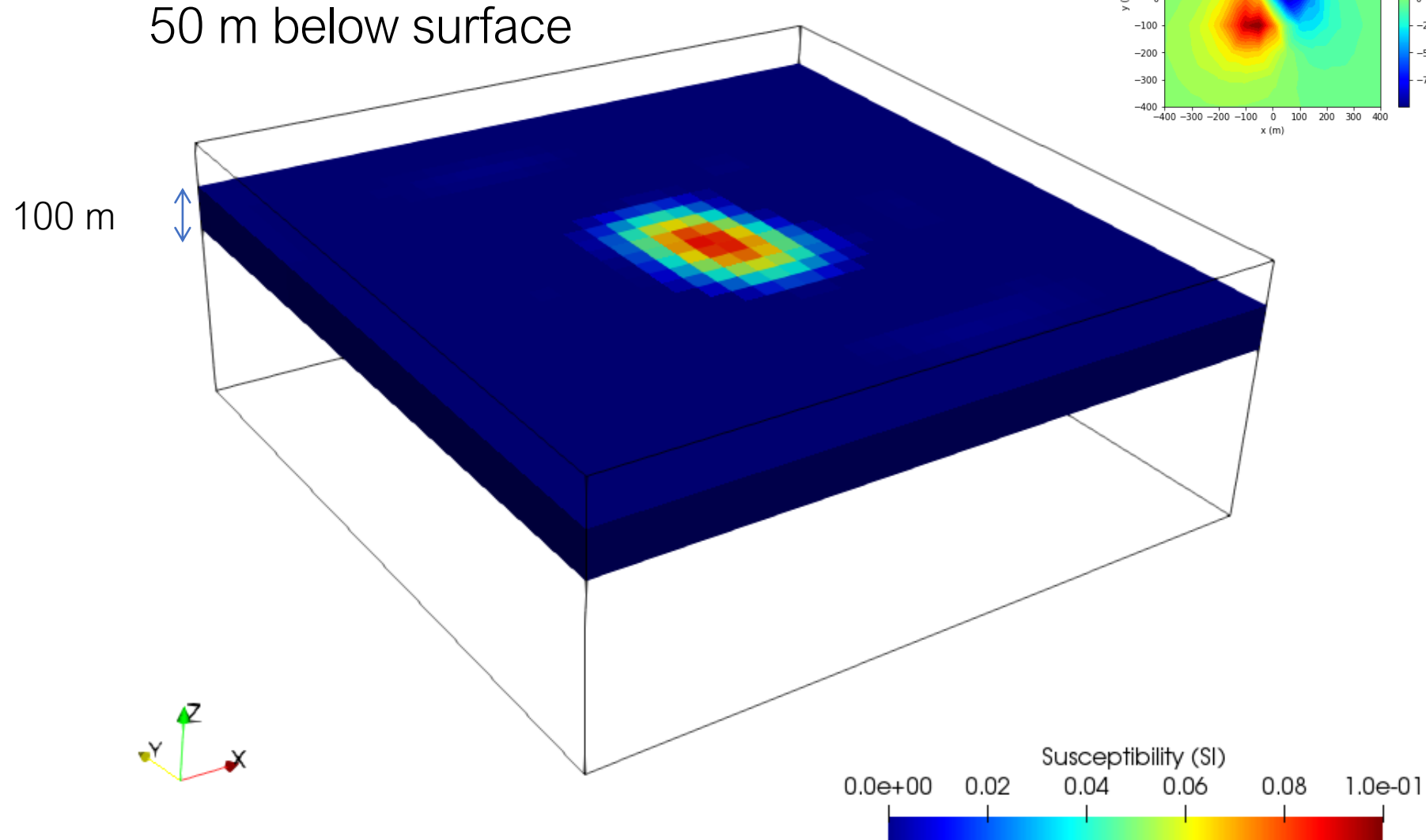


predicted





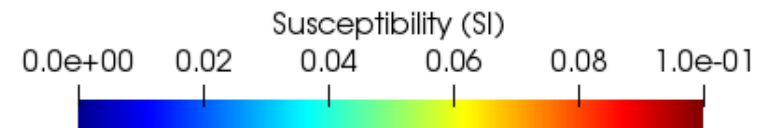
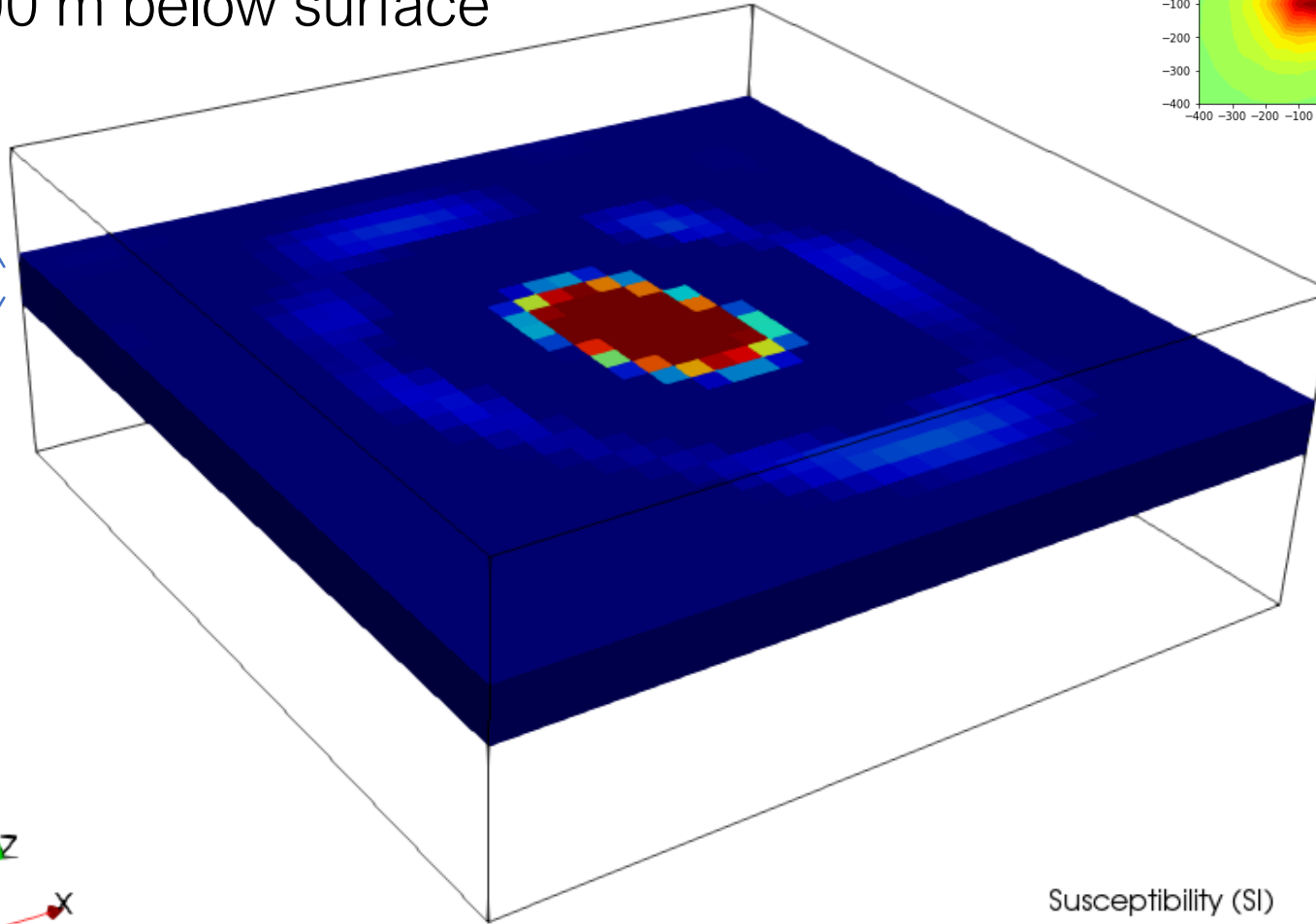
# Example of extreme non-uniqueness



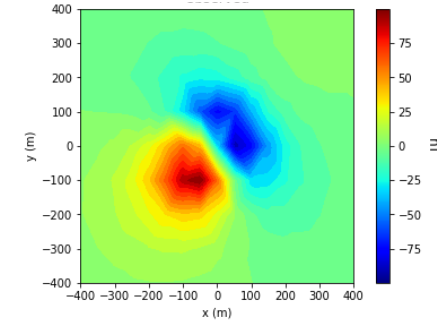
# Example of extreme non-uniqueness

100 m below surface

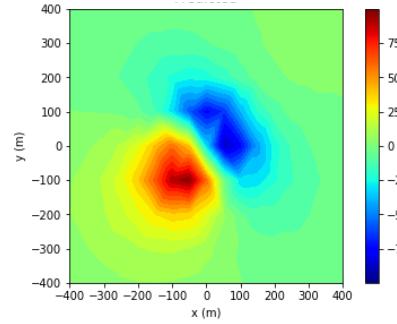
100 m



observed



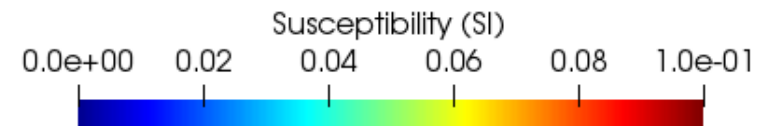
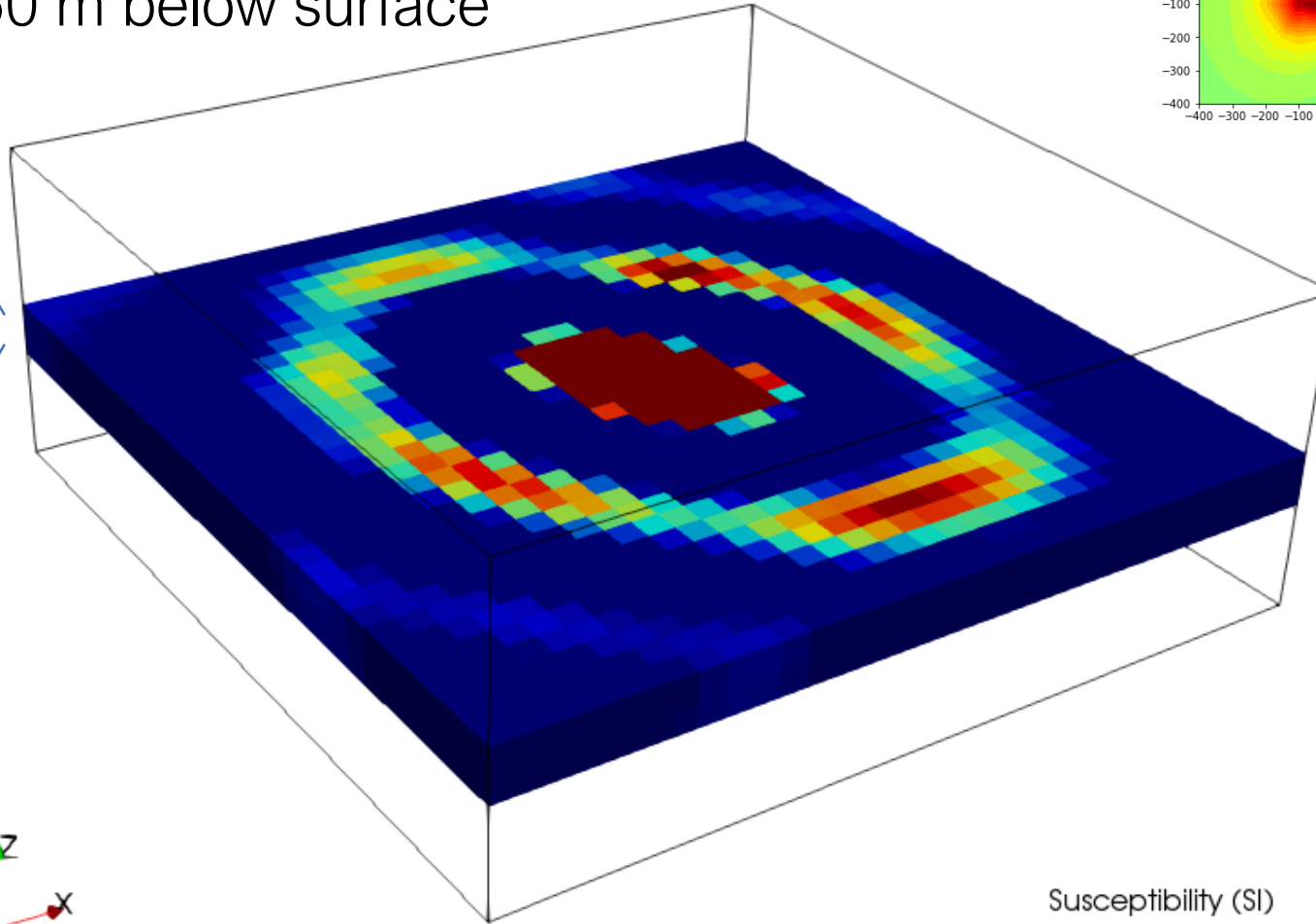
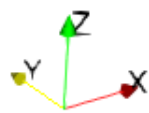
predicted



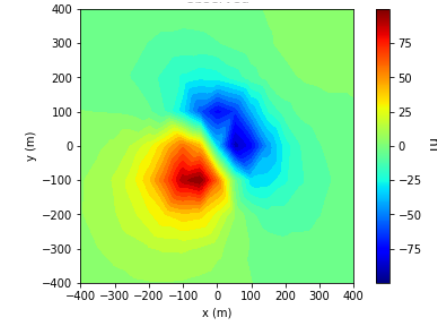
# Example of extreme non-uniqueness

150 m below surface

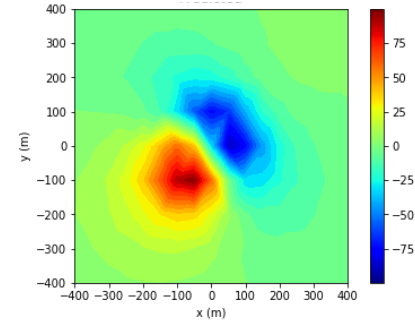
100 m



observed



predicted



# Inversion with sensitivity weighting

Model objective function

$$\phi_m = \alpha_s \int_v w_s \left( \kappa - \kappa_{\text{ref}} \right)^2 dv + \alpha_x \int_v w_x \left( \frac{d\kappa}{dx} \right)^2 dx + \alpha_y \int_v w_y \left( \frac{d\kappa}{dy} \right)^2 dy + \alpha_z \int_v w_z \left( \frac{d\kappa}{dz} \right)^2 dz$$

Data misfit

$$\phi_d = \sum_{j=1}^N \left( \frac{G_{ij} \kappa_j - d_j^{\text{obs}}}{\epsilon_j} \right)$$

$\{w_s, w_x, w_y, w_z\}$ : additional weightings

Choose  $\{w_s, w_x, w_y, w_z\} \propto \frac{1}{(z+z_0)^3}$

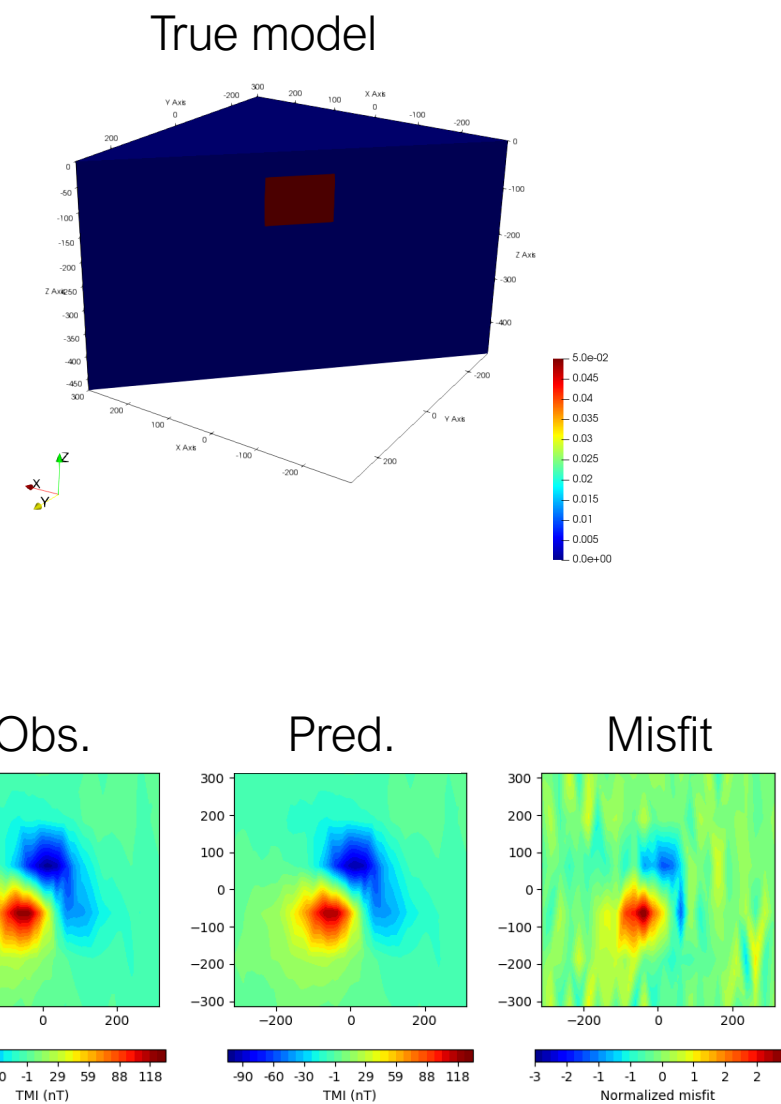
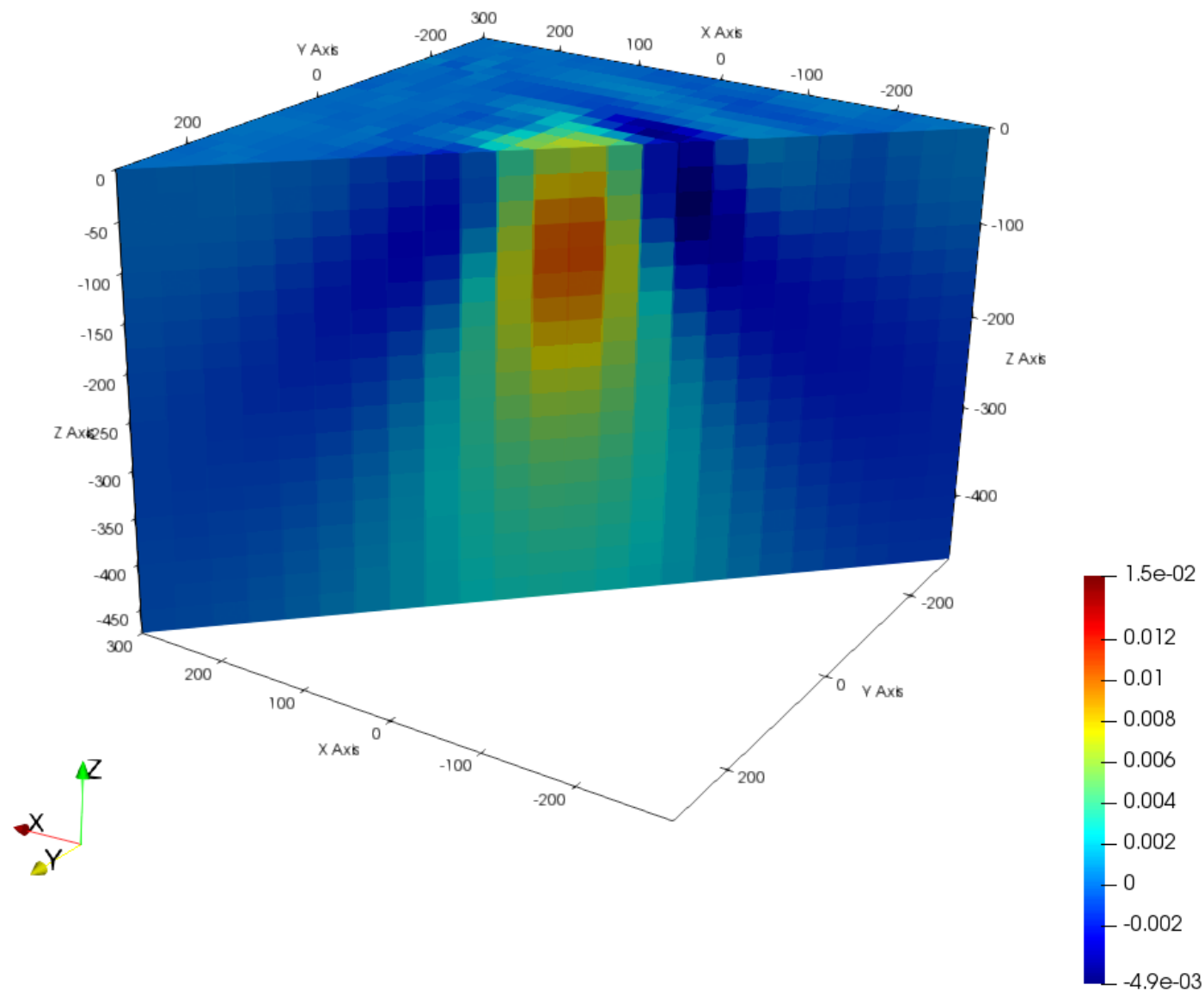
Allows cells at depth to contribute

The Inverse problem is:

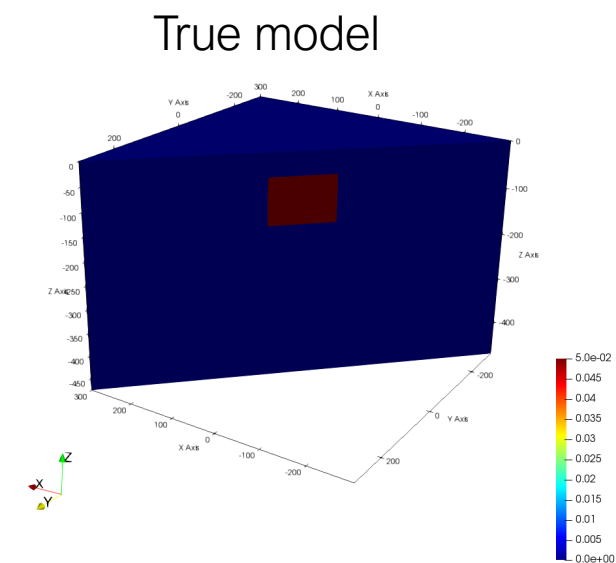
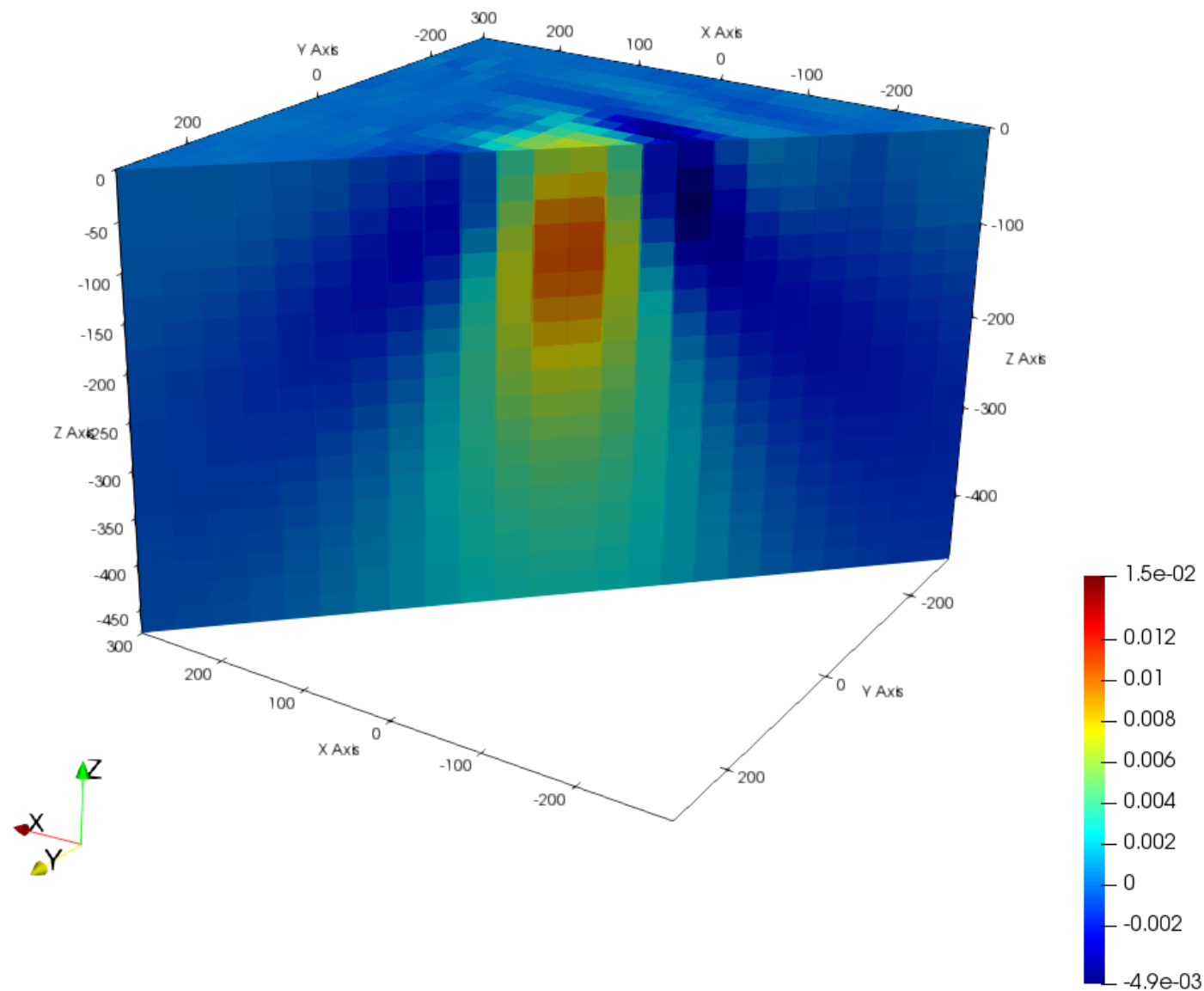
$$\text{minimize} \quad \phi = \phi_d + \beta \phi_m$$

$$\text{find } \beta \text{ such that } \phi_d = \phi_d^* \text{ where } \phi_d = N$$

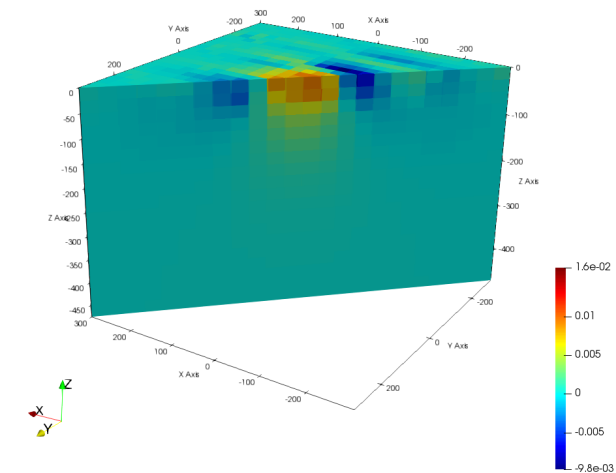
# Inversion with sensitivity weighting



# Inversion with sensitivity weighting

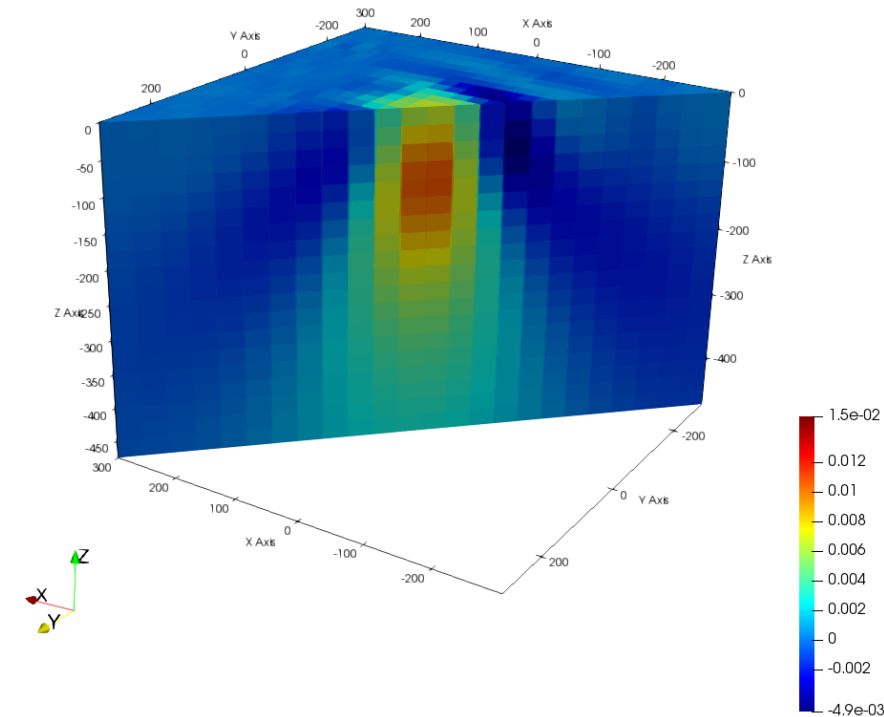


Without sensitivity weight



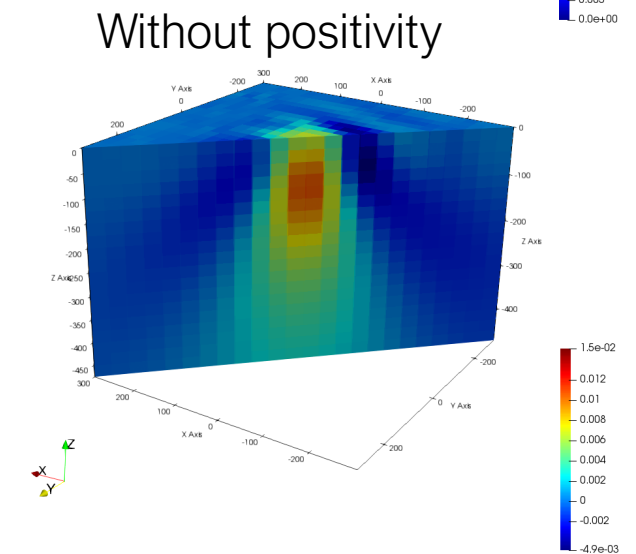
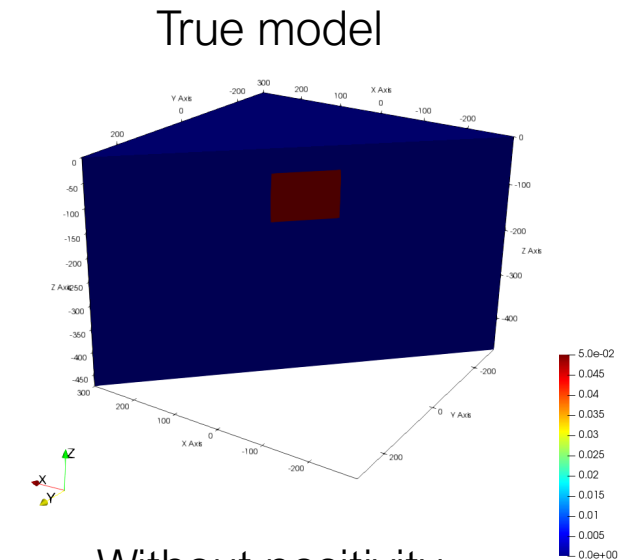
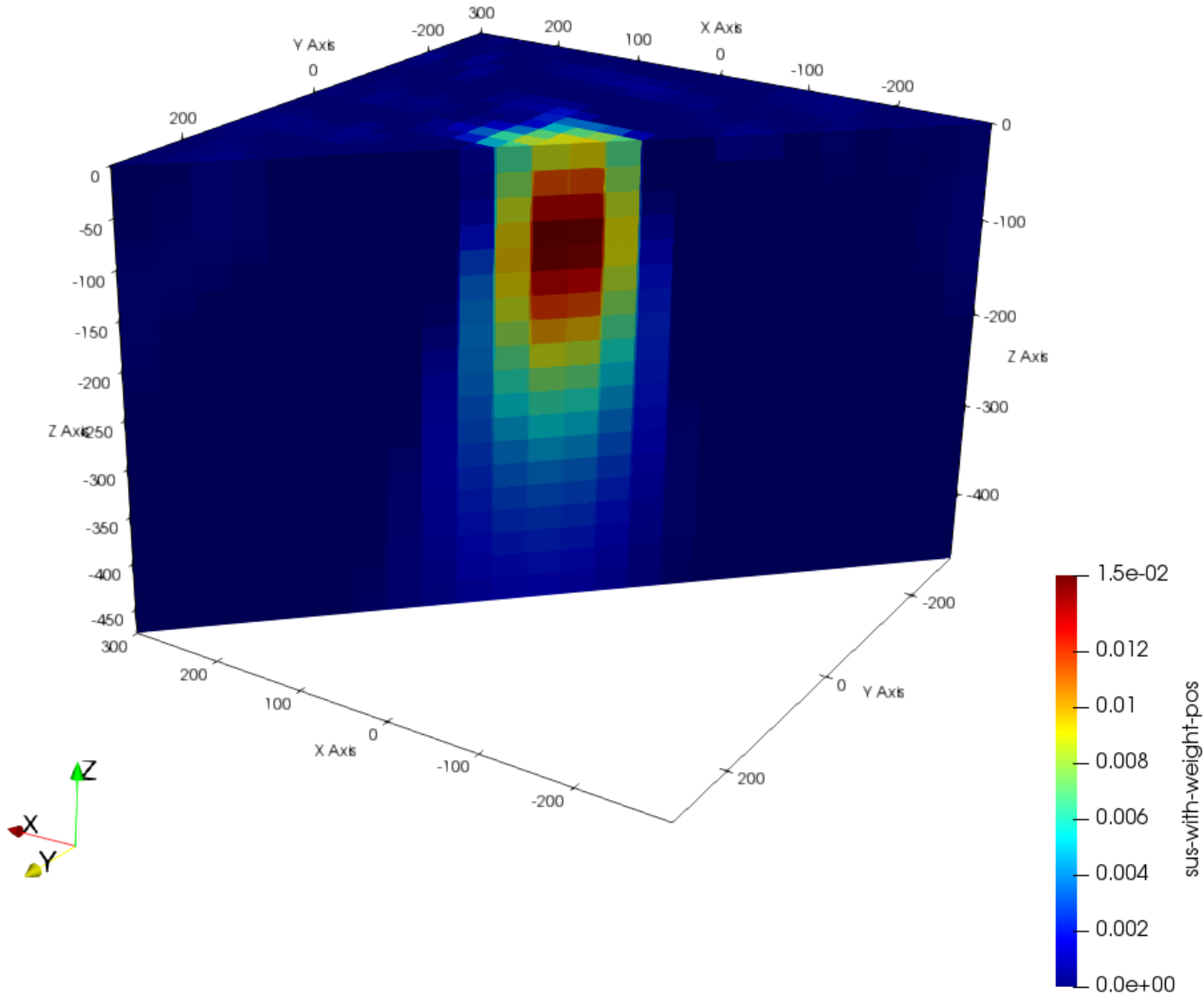
# Summary for sensitivity weighting

- Structure is no longer concentrated at surface.
- Main anomaly is at a reasonable depth.
- BUT:
  - Negative  $\kappa$  persists
  - There is a long tail extending down and out.
- Require positivity



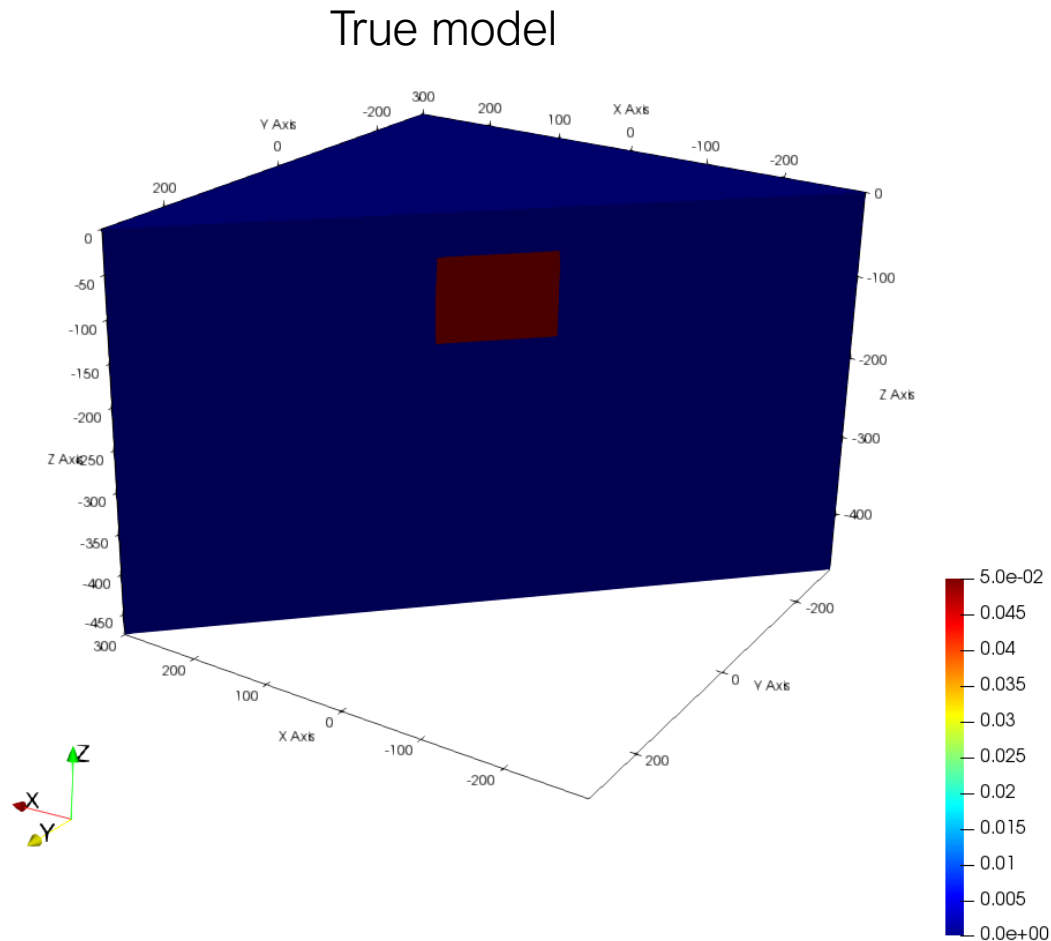
# 3D magnetic inversion:

- depth (or sensitivity) weighting
- positivity (bounds):  $\mathbf{m} \geq 0$





# Think about the spatial character of the rue model



Most of model parameters are zero

Susceptible block and background has sharp boundaries

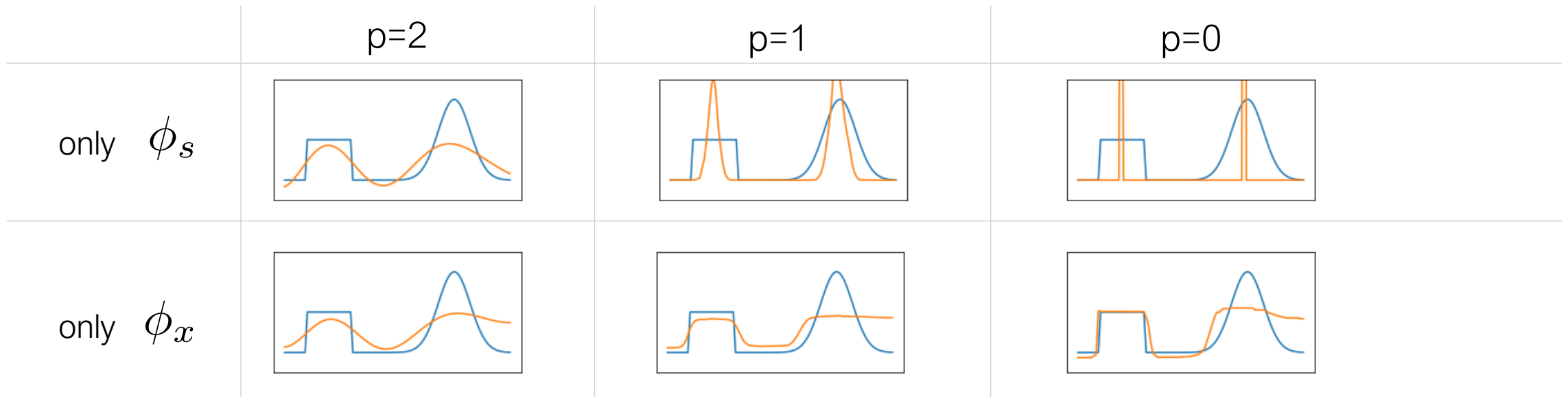
Model norm:

$$\phi_m = \alpha_s \int_v w_s \left( \kappa - \kappa_{\text{ref}} \right)^2 dv +$$
$$\alpha_x \int_v w_x \left( \frac{d\kappa}{dx} \right)^2 dx + \alpha_y \int_v w_y \left( \frac{d\kappa}{dy} \right)^2 dy + \alpha_z \int_v w_z \left( \frac{d\kappa}{dz} \right)^2 dz$$

# General character

$$\phi_m = \sum_{i=1}^M |m_i|^p v_i$$

- Geometric character
  - $p=2$ : all elements close to zero
  - $p=1$ : sparse solution, # of non-zero elements are  $\leq$  # of data
  - $p=0$ : minimum support, model with the fewest number of elements
- 1D problem



# Magnetic inversion with Lp norms

Model objective function

$$\phi_m = \alpha_s \int_v w_s \left| \kappa - \kappa_{\text{ref}} \right|^{p_s} dv + \alpha_x \int_v w_x \left| \frac{d\kappa}{dx} \right|^{p_x} dx + \alpha_y \int_v w_y \left| \frac{d\kappa}{dy} \right|^{p_y} dy + \alpha_z \int_v w_z \left| \frac{d\kappa}{dz} \right|^{p_z} dz$$

Data misfit

$$\phi_d = \sum_{j=1}^N \left( \frac{G_{ij} \kappa_j - d_j^{\text{obs}}}{\epsilon_j} \right)$$

Fournier and Oldenburg (2019)

The Inverse problem is:

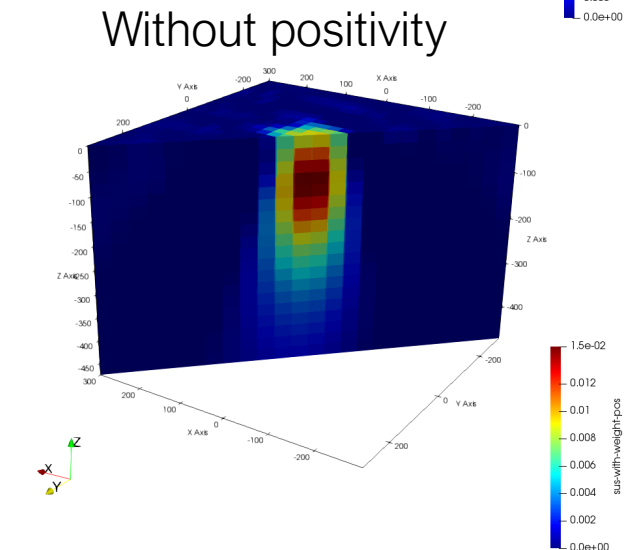
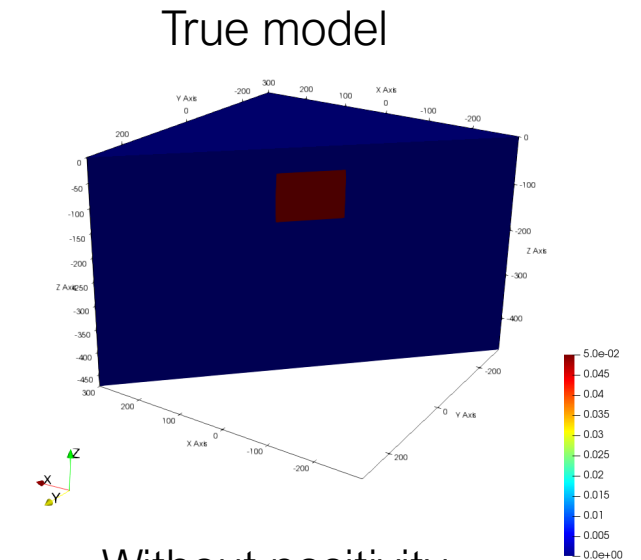
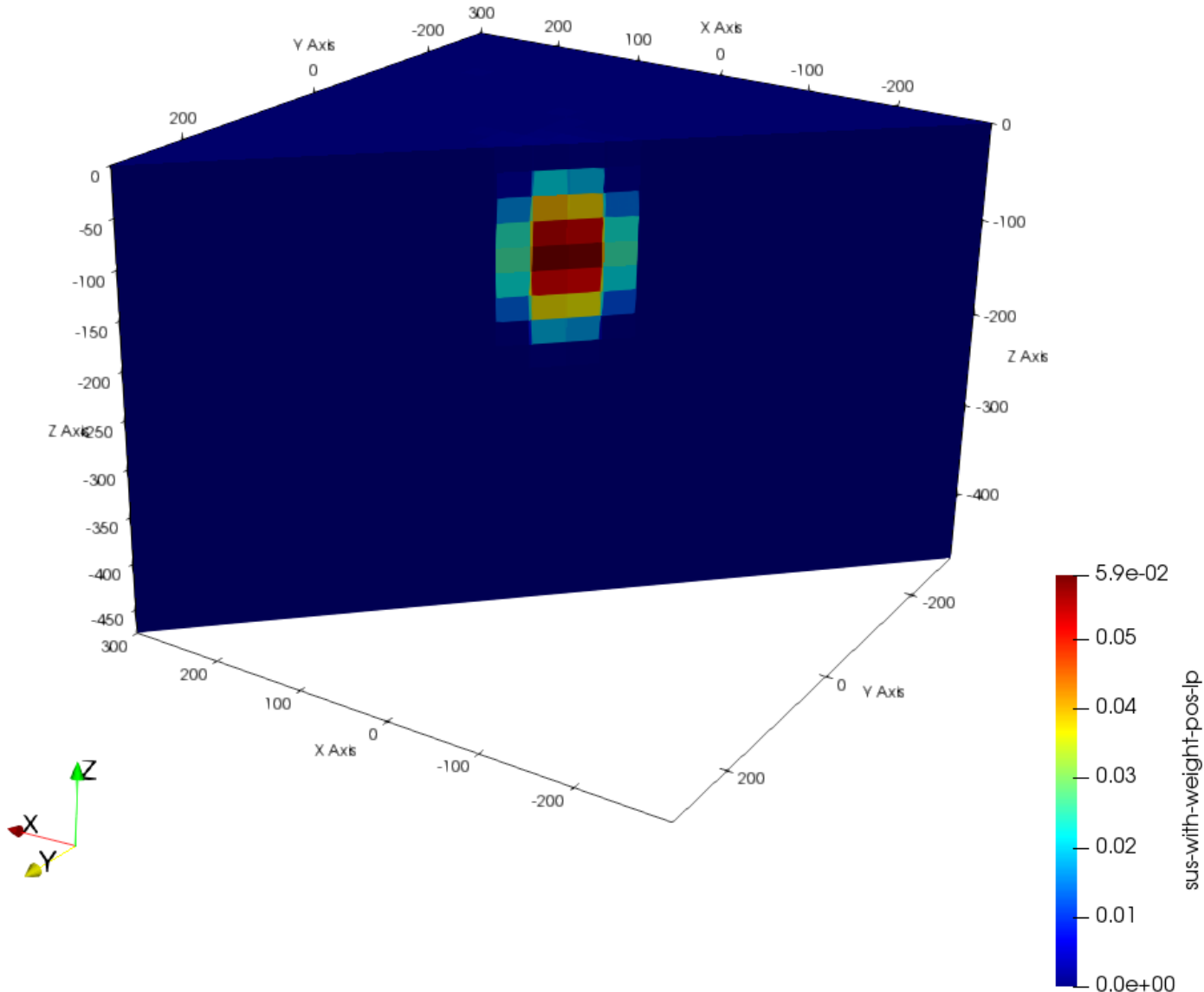
$$\text{minimize} \quad \phi = \phi_d + \beta \phi_m$$

$$\text{find } \beta \text{ such that } \phi_d = \phi_d^* \text{ where } \phi_d = N$$

$$\text{subject to } \kappa \geq 0$$

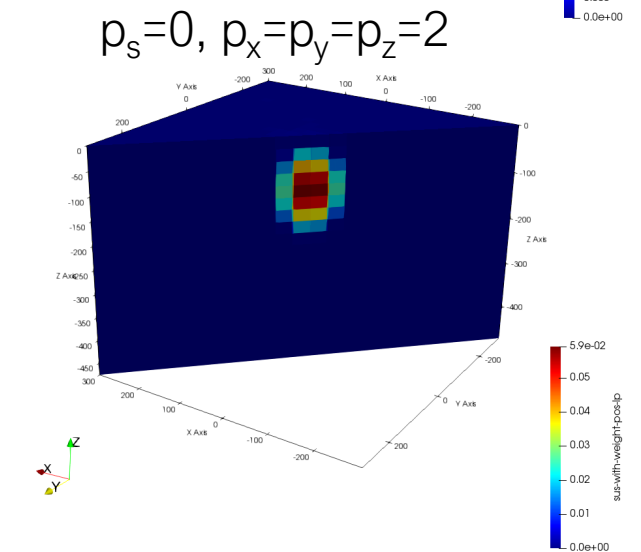
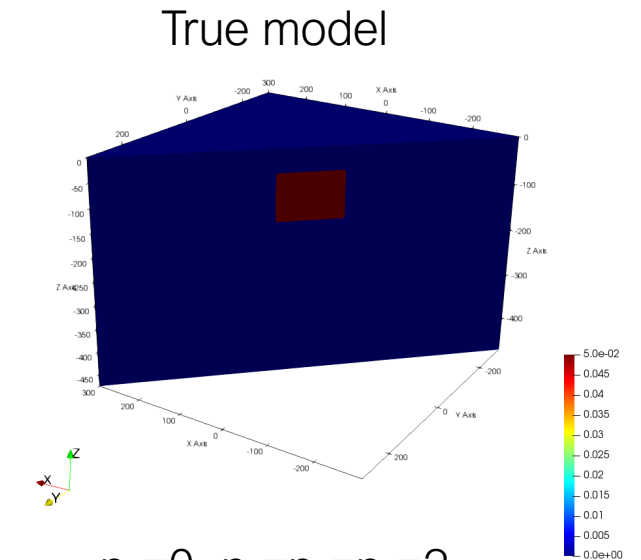
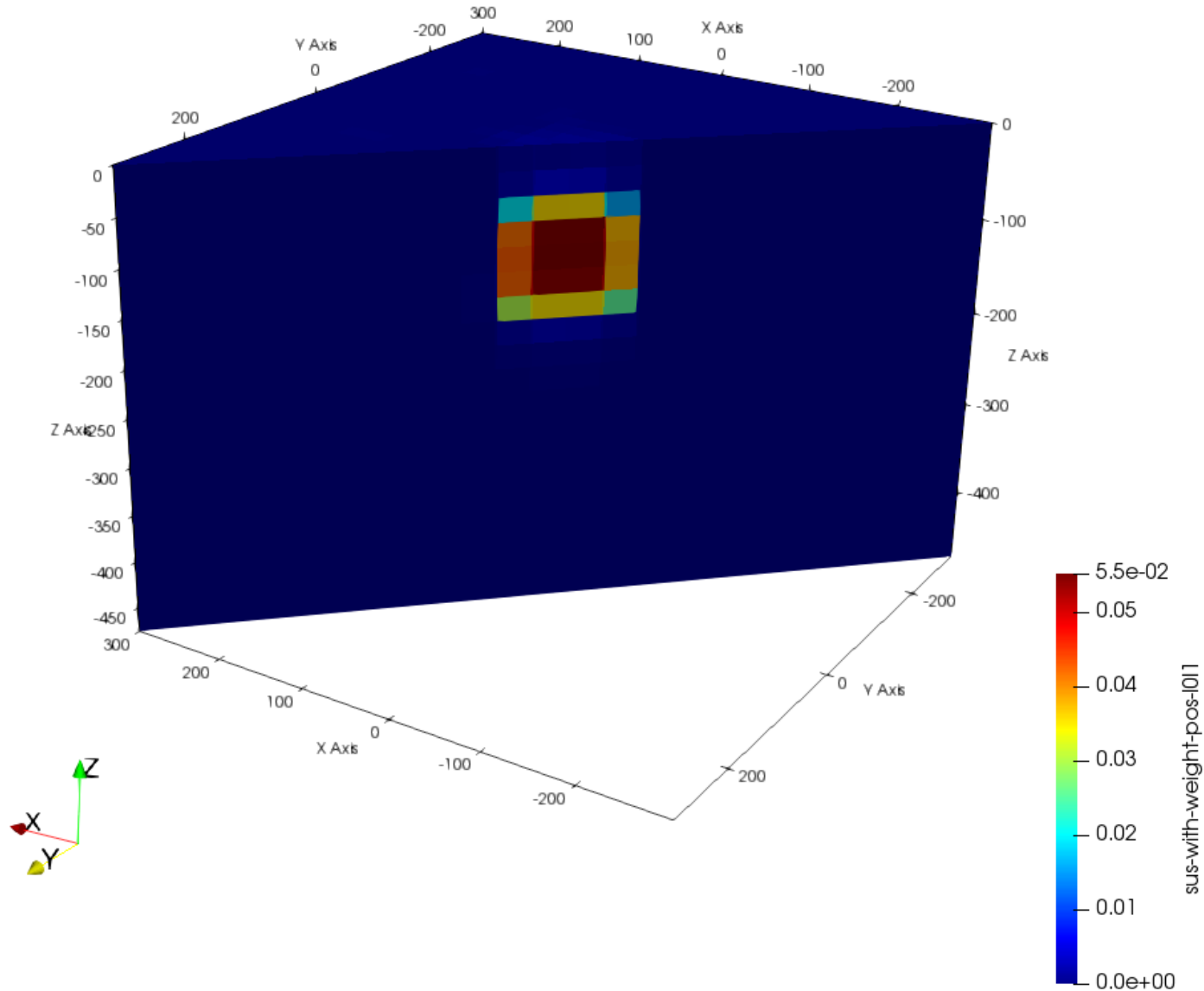
# 3D magnetic inversion:

- depth (or sensitivity) weighting
- positivity (bounds):  $\mathbf{m} \geq 0$
- $L_p$  norm ( $\mathbf{p}_s=0$ ,  $p_x=p_y=p_z=2$ )



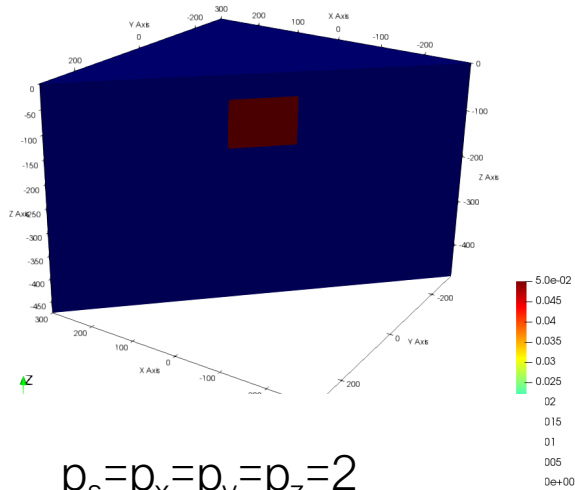
# 3D magnetic inversion:

- depth (or sensitivity) weighting
- positivity (bounds):  $\mathbf{m} \geq 0$
- $L_p$  norm ( $p_s=0$ ,  $p_x=p_y=p_z=1$ )

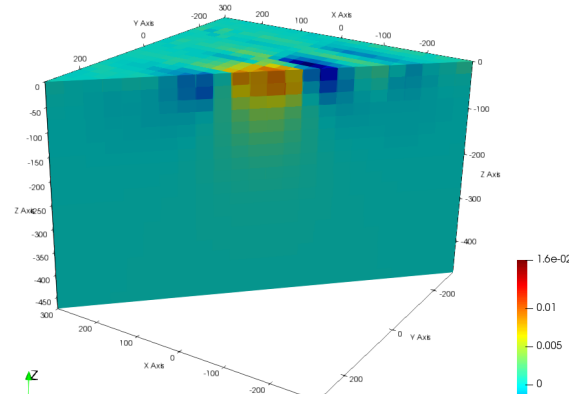


# Summary: magnetic inversion

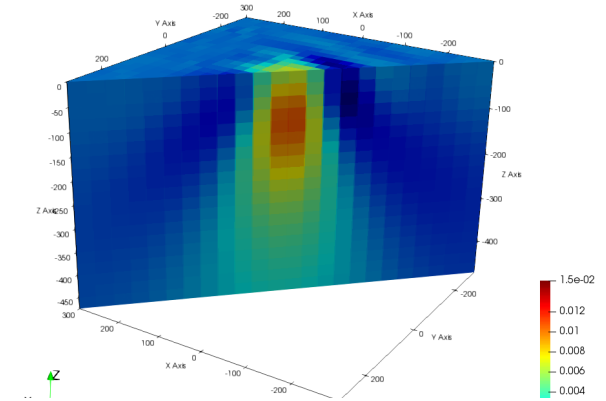
True model



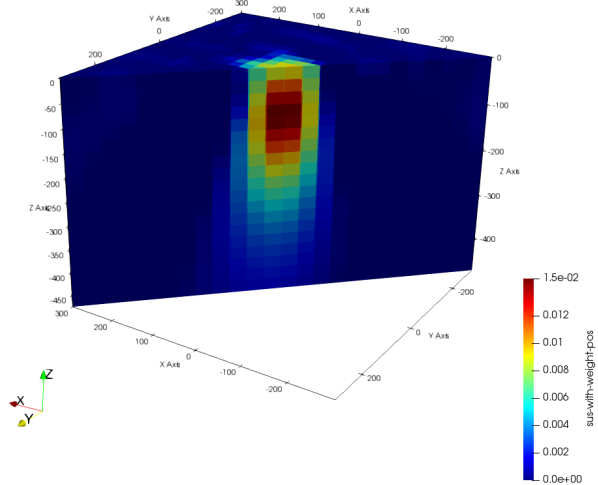
No weight



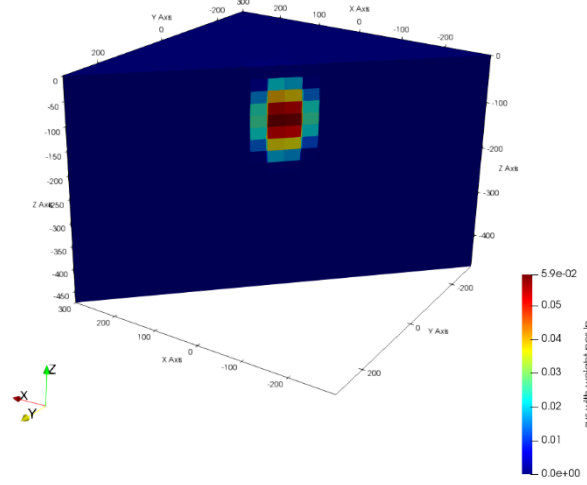
Sensitivity weight



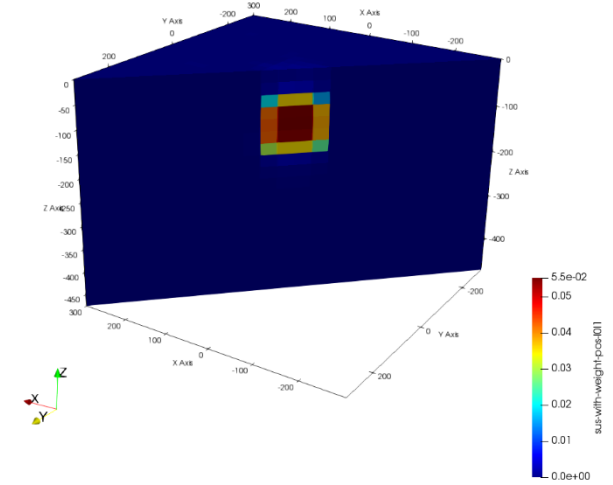
$p_s=p_x=p_y=p_z=2$



$p_s=0, p_x=p_y=p_z=2$

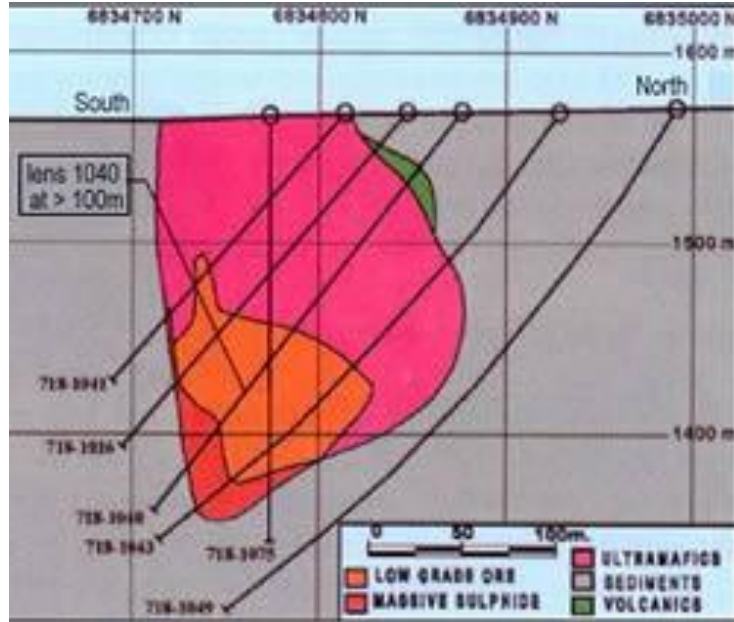


$p_s=0, p_x=p_y=p_z=1$

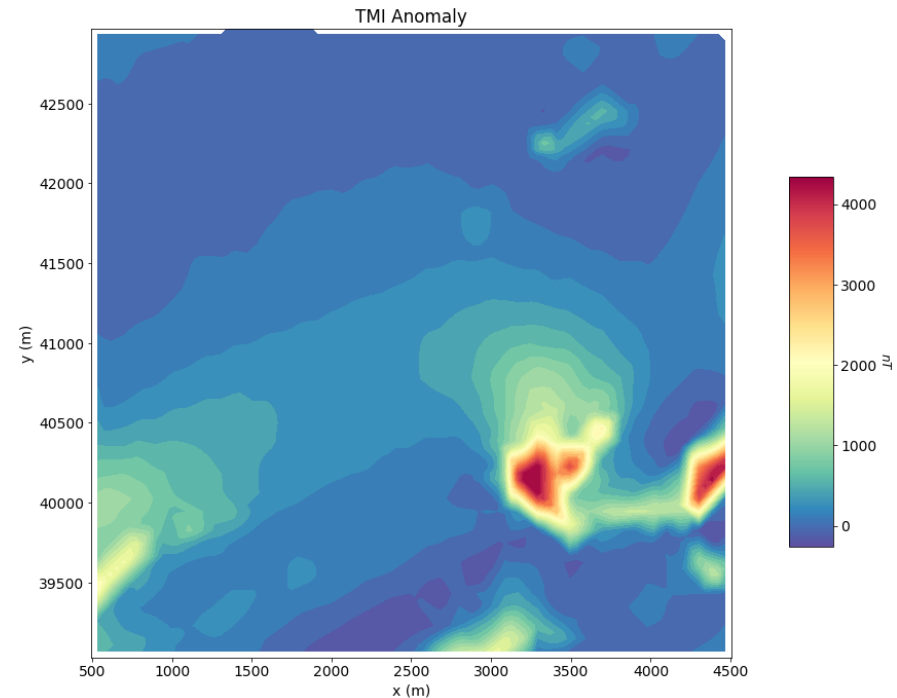


# Revisit the case history

Geologic section



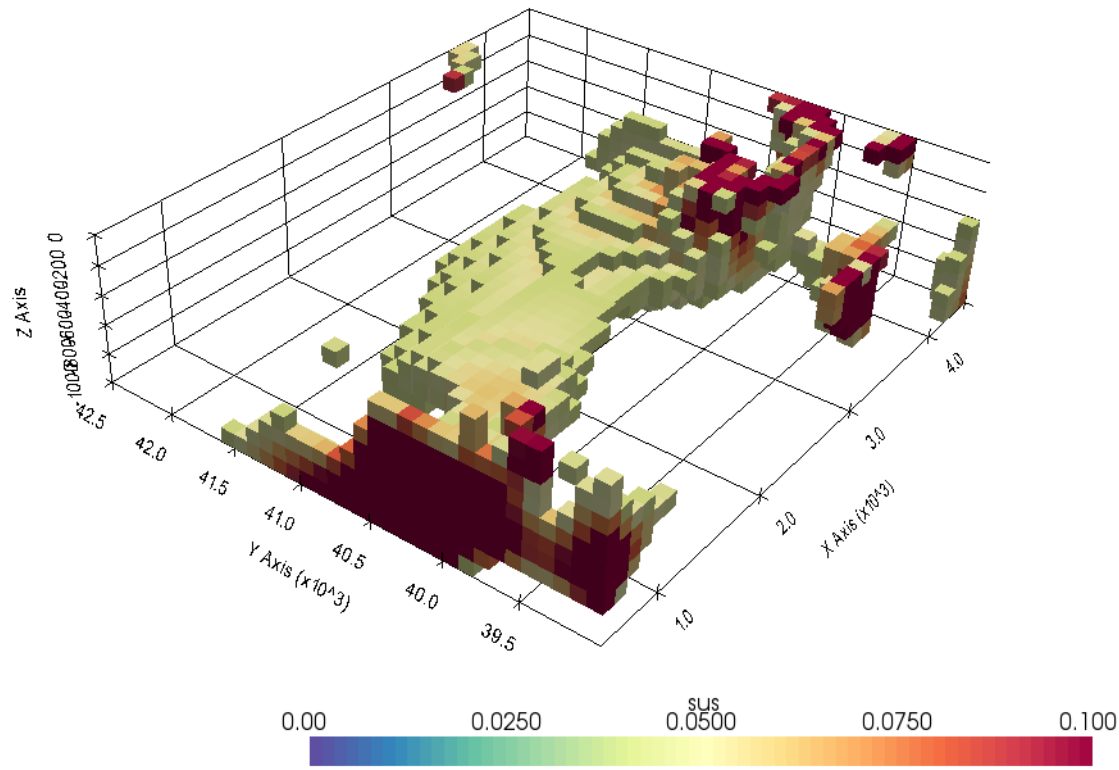
Magnetic data



Initial conceptual model: two ultramafic pipes  
Can make impact on drilling location and mineral reserve

# Magnetic inversion

The recovered model (SimPEG)



Model norm:

$$\phi_m = \alpha_s \int_v w_s \left( \kappa - \kappa_{\text{ref}} \right)^2 dv + p_s$$

$$\alpha_x \int_v w_x \left( \frac{d\kappa}{dx} \right)^2 dx + \alpha_y \int_v w_y \left( \frac{d\kappa}{dy} \right)^2 dy + \alpha_z \int_v w_z \left( \frac{d\kappa}{dz} \right)^2 dz$$

What if the target body is expected to be compact?

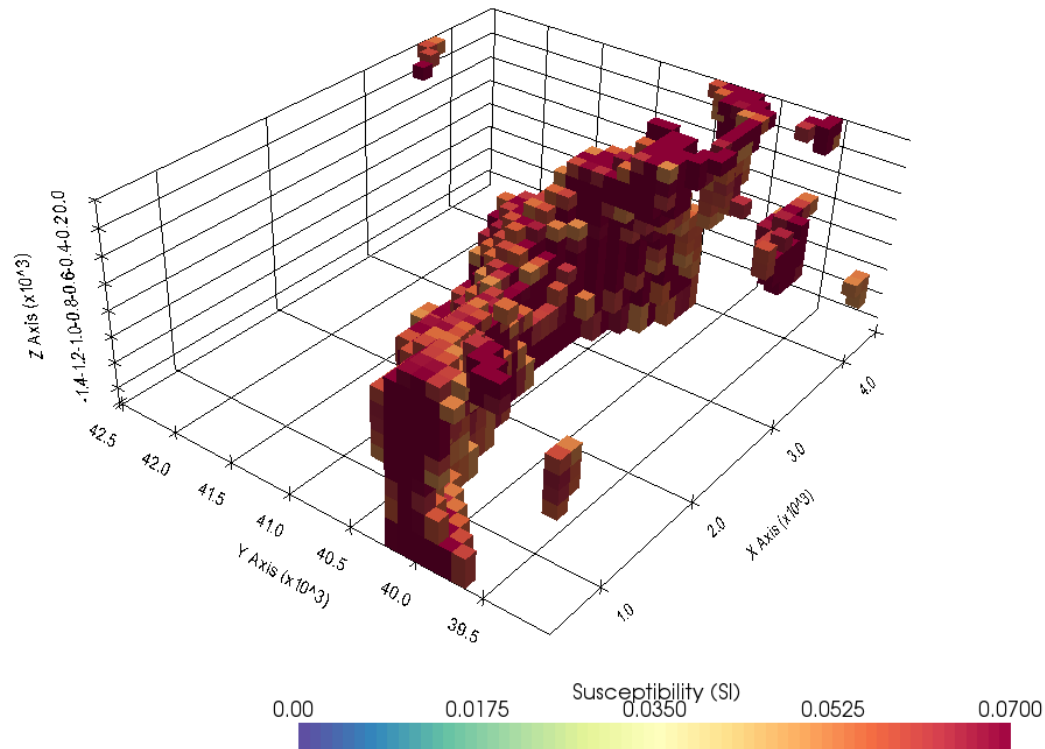
Use a sparse norm (e.g.,  $p_s=0$ )



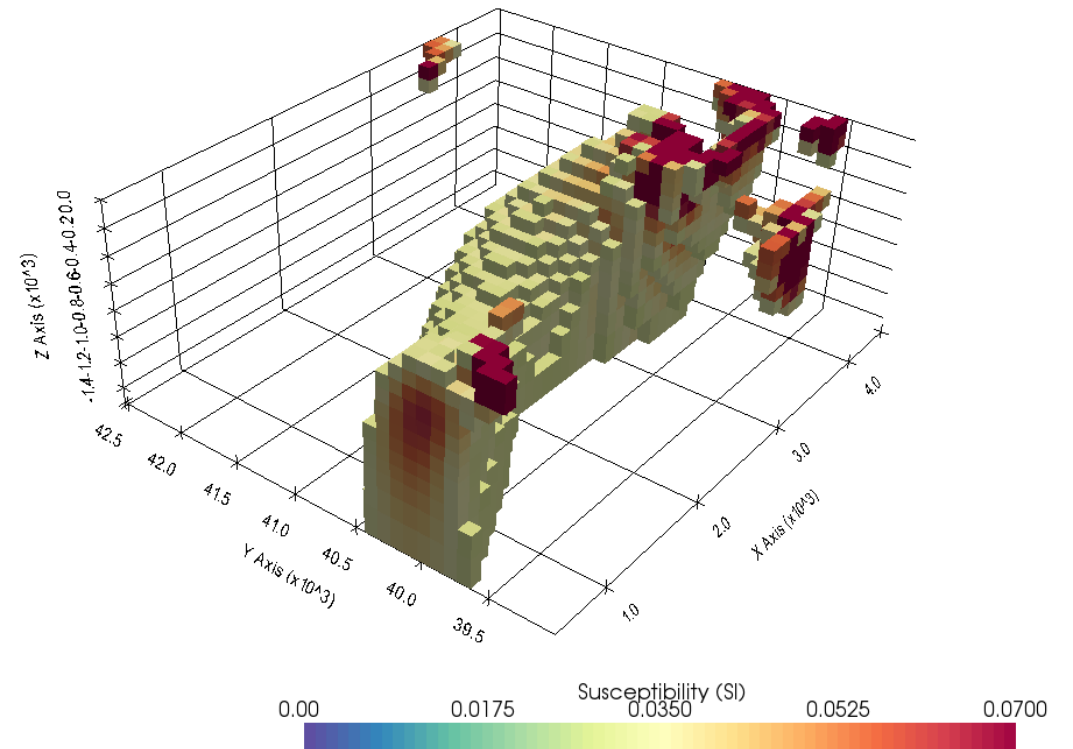
# Demo: $L_p$ -norm inversion of the Raglan magnetic data

# Summary: Lp-norm inversion

Lp-norm inversion



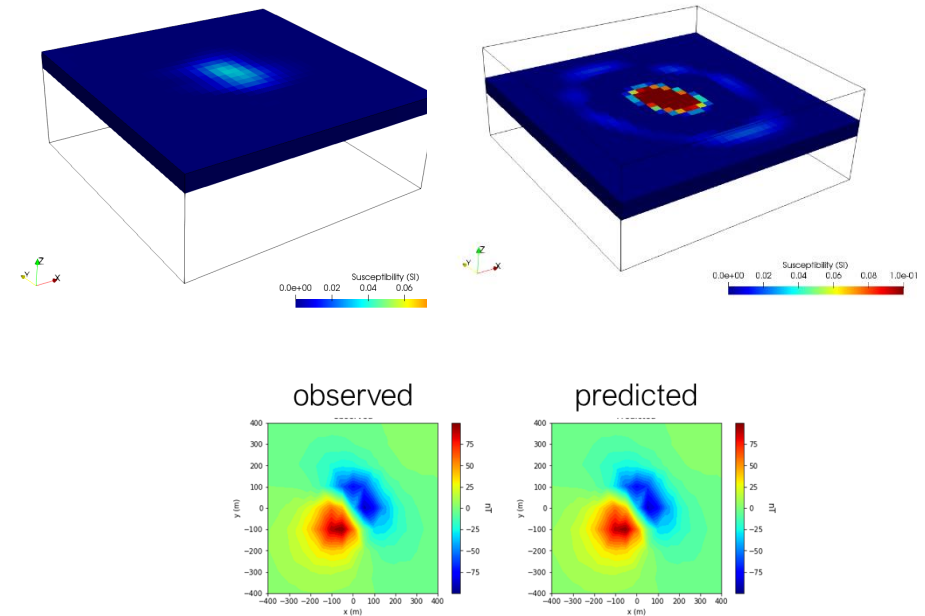
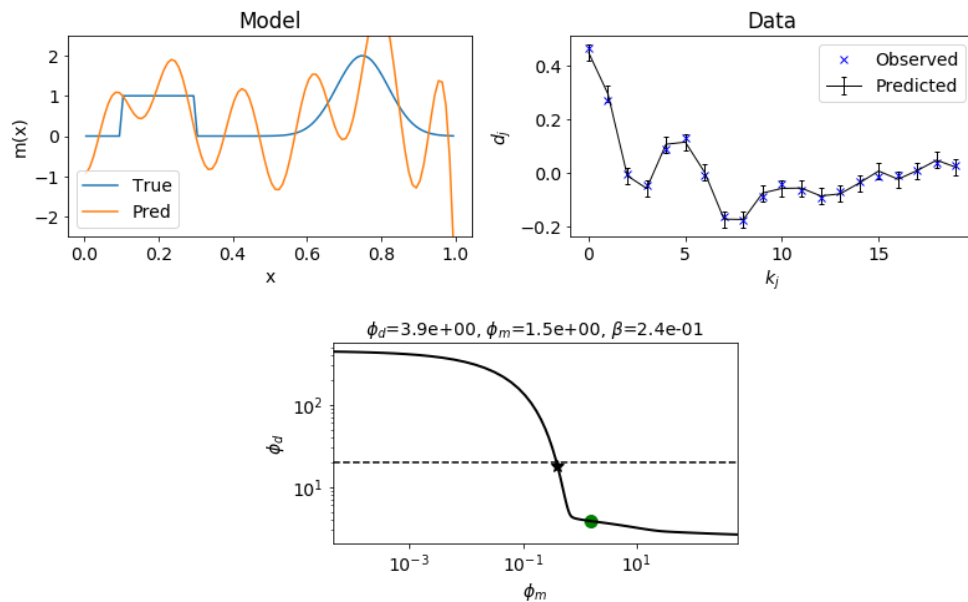
L2-norm inversion



Beneficial for reserve estimate

# Summary

- Increasing complexity & volume of data
  - Increasing needs of data-driven approaches
- Fitting the observed data is not an enough condition!

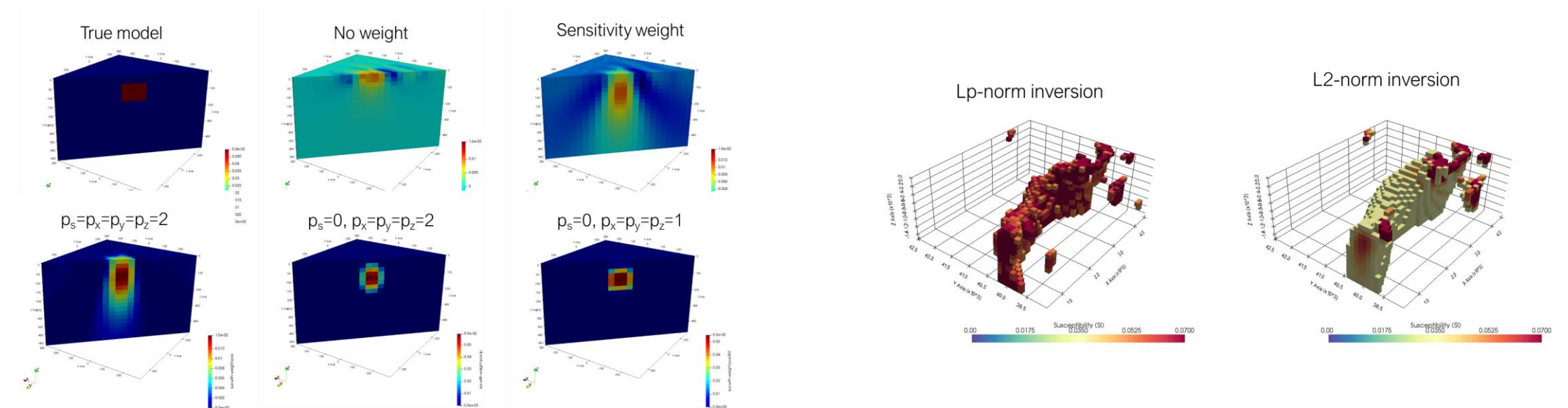


# Summary

- An inversion framework can play an important role
  - Data-driven + Physics-driven + Prior knowledge

minimize  $\phi(m) = \phi_d(m) + \beta \phi_m(m)$

$\phi_d$ : data misfit  
 $\phi_m$ : model norm  
 $\beta$ : trade-off parameter



# Multiple airborne geophysics

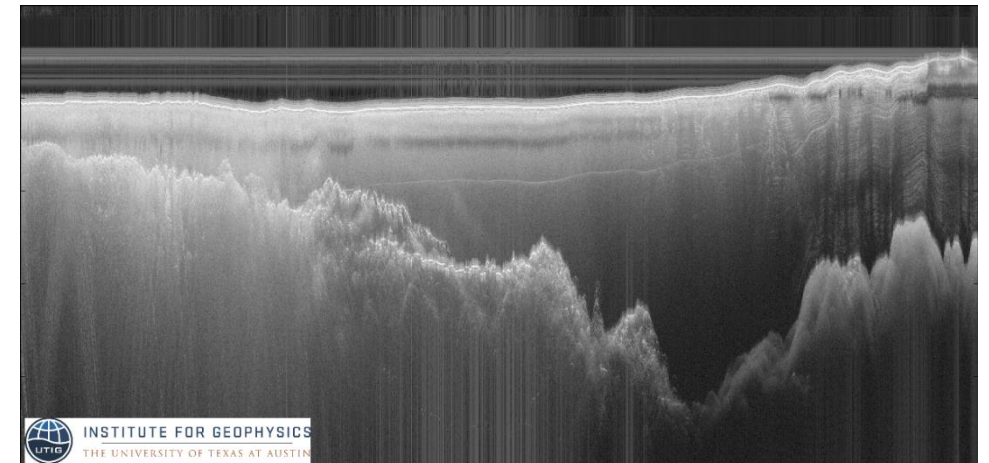
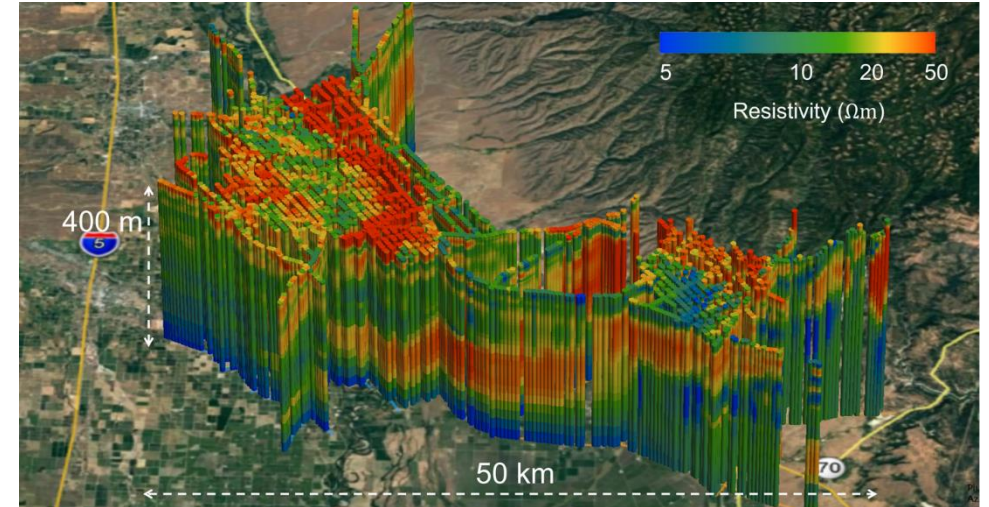
- Potential fields

Magnetics

Gravity

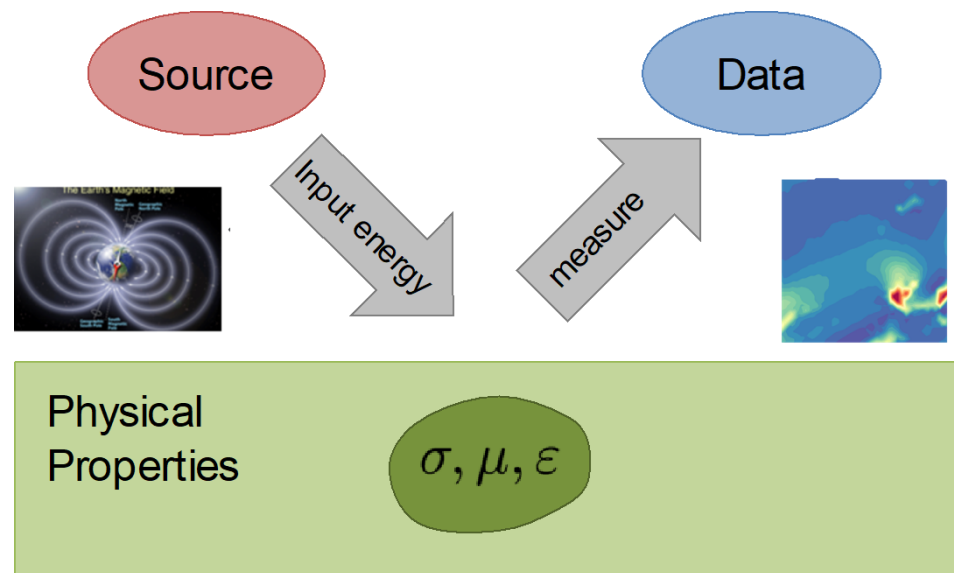
- Electromagnetics

- (Ice) Radar



Increasing  
Resolution

But in a generic level, they are very similar ...



minimize  $\phi(m) = \phi_d(m) + \beta\phi_m(m)$

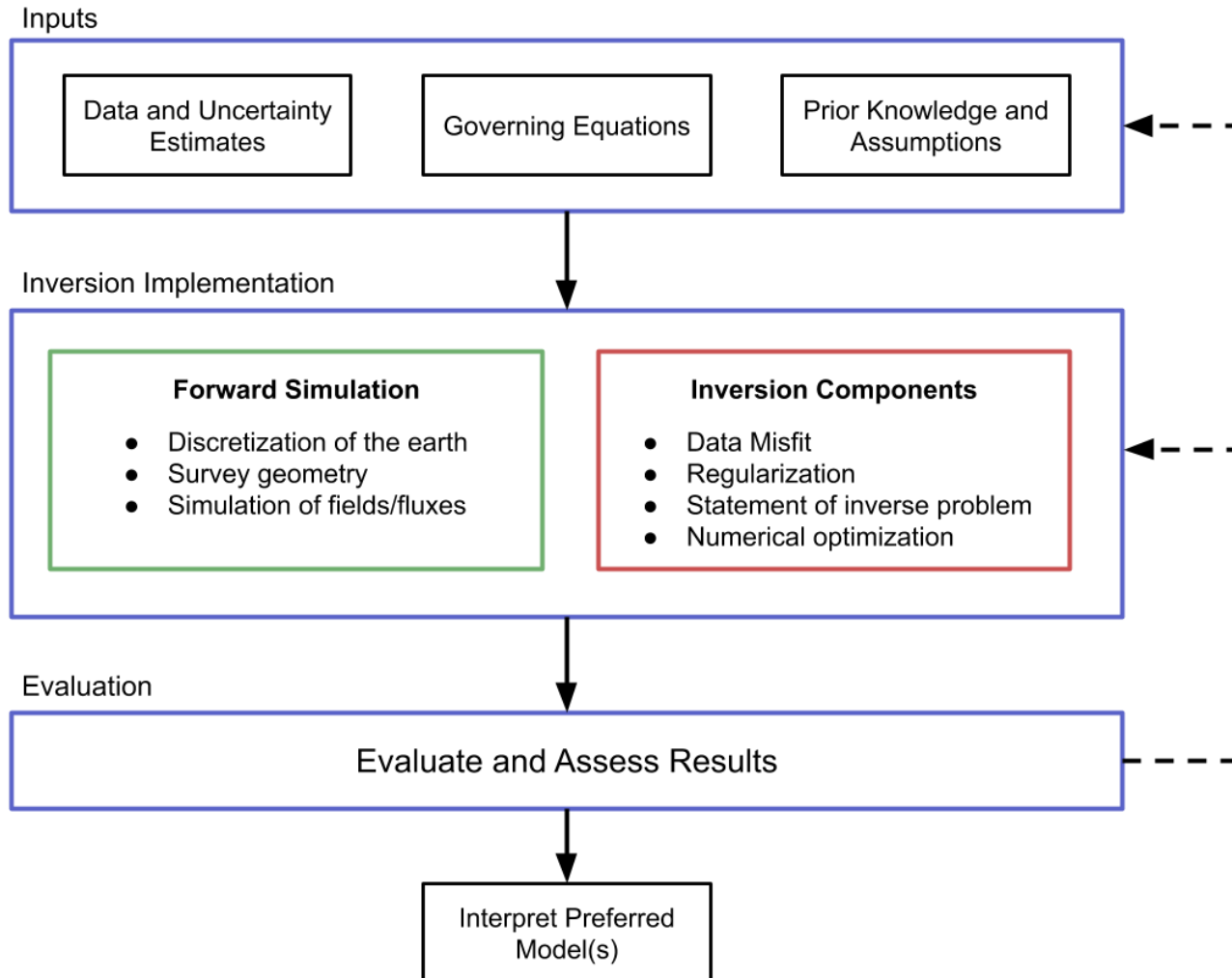
$\phi_d$ : data misfit  
 $\phi_m$ : model norm  
 $\beta$ : trade-off parameter

# SimPEG provides ..

## A common & modular inversion framework



- Gravity
- Magnetics
- Direct current resistivity
- Induced polarization
- Electromagnetics
- Fluid flow





# There are many other software packages

- And they are growing!



.....





# Open data



## Challenging geoscience problems



# Open data



Inversion framework can provide a “data-driven” approach

## Challenging geoscience problems





# Thank you!

SimPEG: <https://www.simpeg.xyz>

Notebooks: <https://curvenote.com/@swung/inversion-for-geologists-transform-2021>

Github: <https://github.com/simpeg/transform-2021-simpeg>

banesullivan



bsmithyman



dccowan



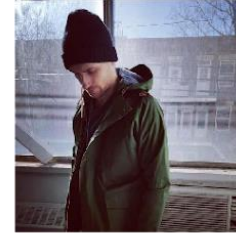
dougoldenburg



dwfmarchant



fourndo



fwkoch



grosenkj



jcapriot



JKutt



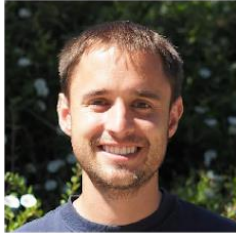
lacmajedrez



lheagy



micmitch



prisae



rowanc1



sdevriese



sgkang



thast



The livestream has ended.

softwareunderground.org presents



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