

The livestream will begin shortly...

softwareunderground.org presents



TRANSFORM 2021

Virtual Conference on the Digital Subsurface, 16–23 April

supported by





TRANSFORM 2021

**Matt Hall
Dieter Werthmuller
& Transform 2021 organizers**

Thank You!



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Materials: <http://bit.ly/transform-2021-slides>

Slack: *swu.ng/slack > #t21-tue-inversion-for-geologists*



supported by



Inversion for geologists

Seogi Kang, Doug Oldenburg, Lindsey Heagy,
Dominique Fournier, Joe Capriotti & the SimPEG team



Collaborators

Doug



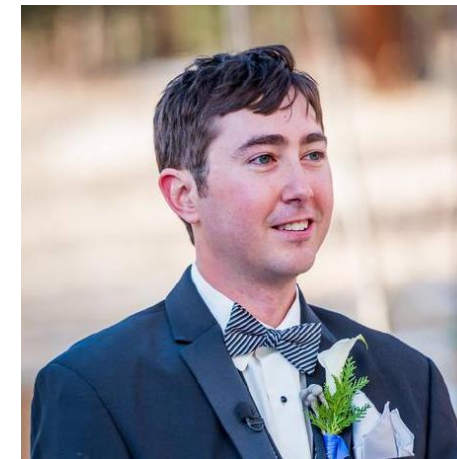
Lindsey



Dom



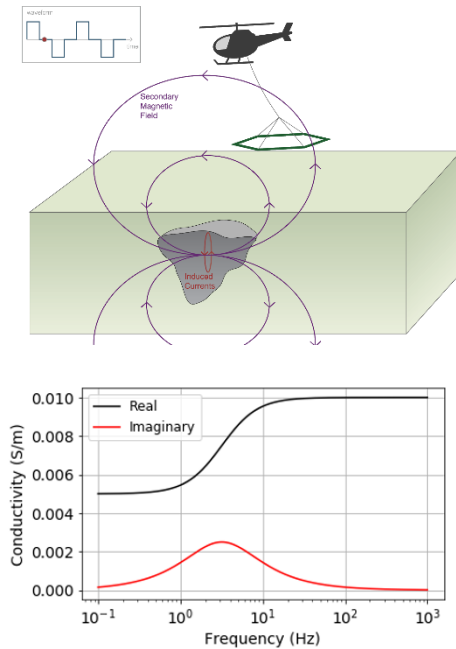
Joe





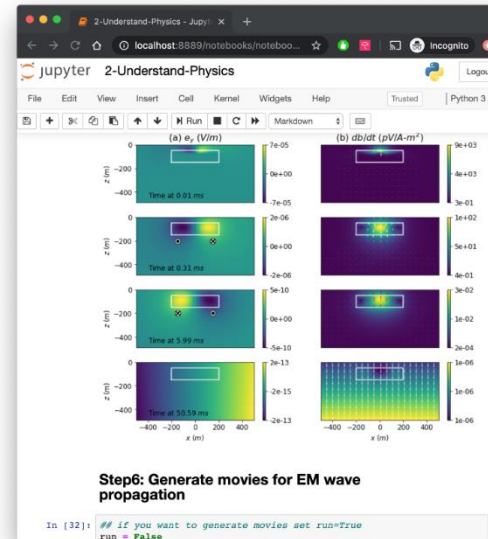
hello (a bit about me)

Computational EM geophysics

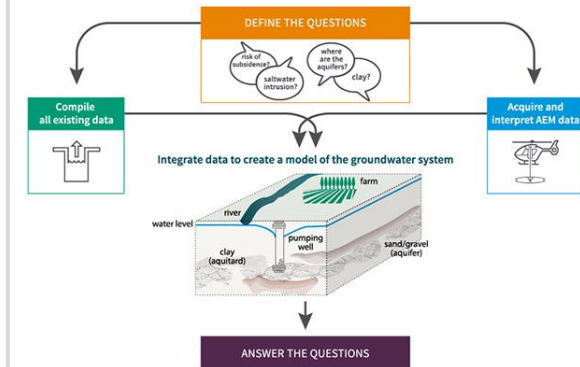


THE UNIVERSITY
OF BRITISH COLUMBIA

Open-source software



Groundwater science & management



Challenging geoscience problems that we faced ...



minerals



contaminants



water



geothermal



geotechnical



slope stability



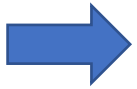
hydrocarbons



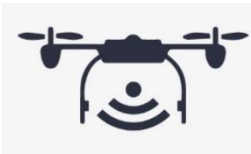
unexploded ordnance

Increasing data volume and complexity

Airborne sensors



airborne geophysics



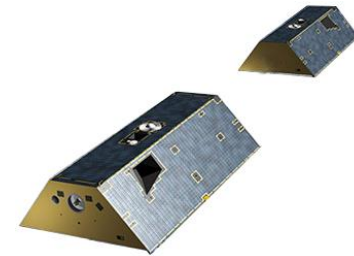
drone geophysics



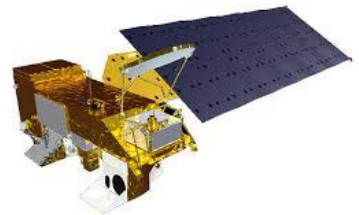
Satellite sensors



Sentinel-2



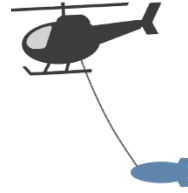
GRACE



MODIS

Data are publicly available, but extracting useful information from these data are challenging

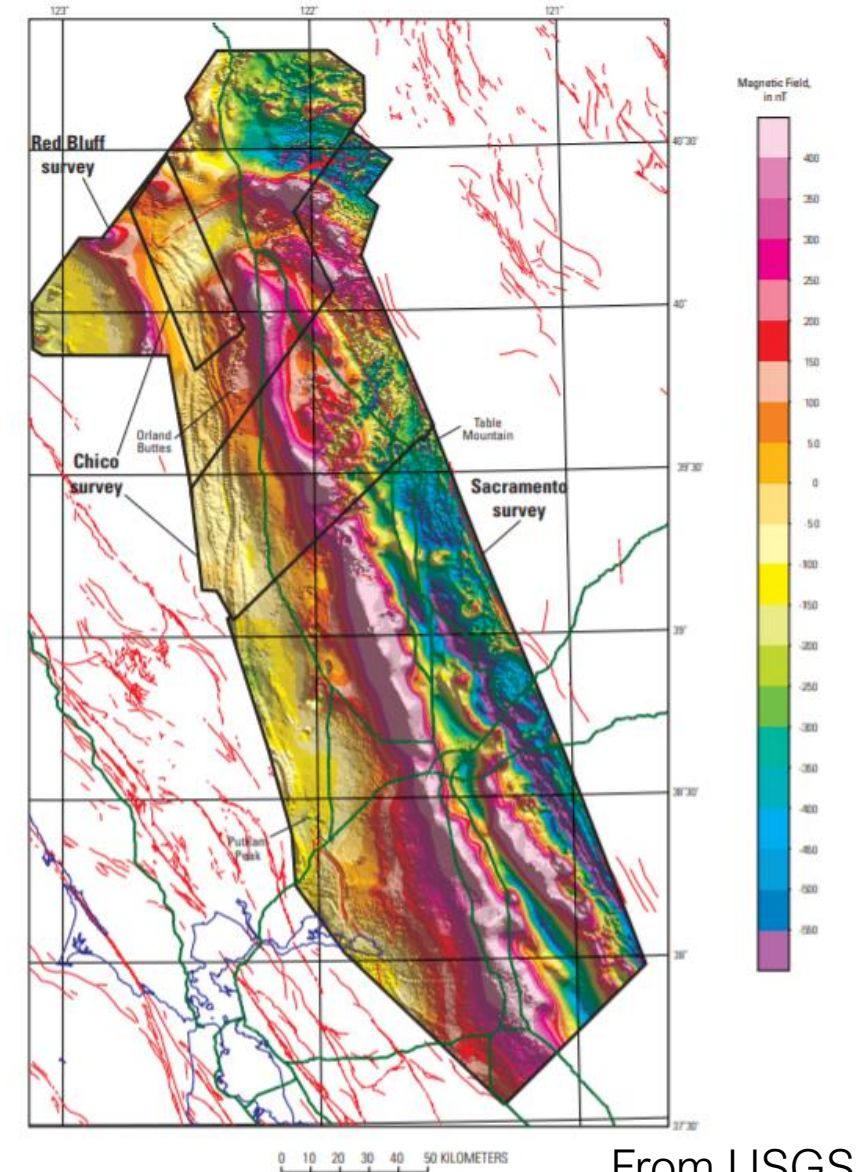
Airborne geophysics



- Potential fields
Magnetics
Gravity
- Electromagnetics
- (Ice) Radar

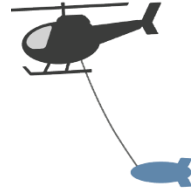


Increasing
Resolution



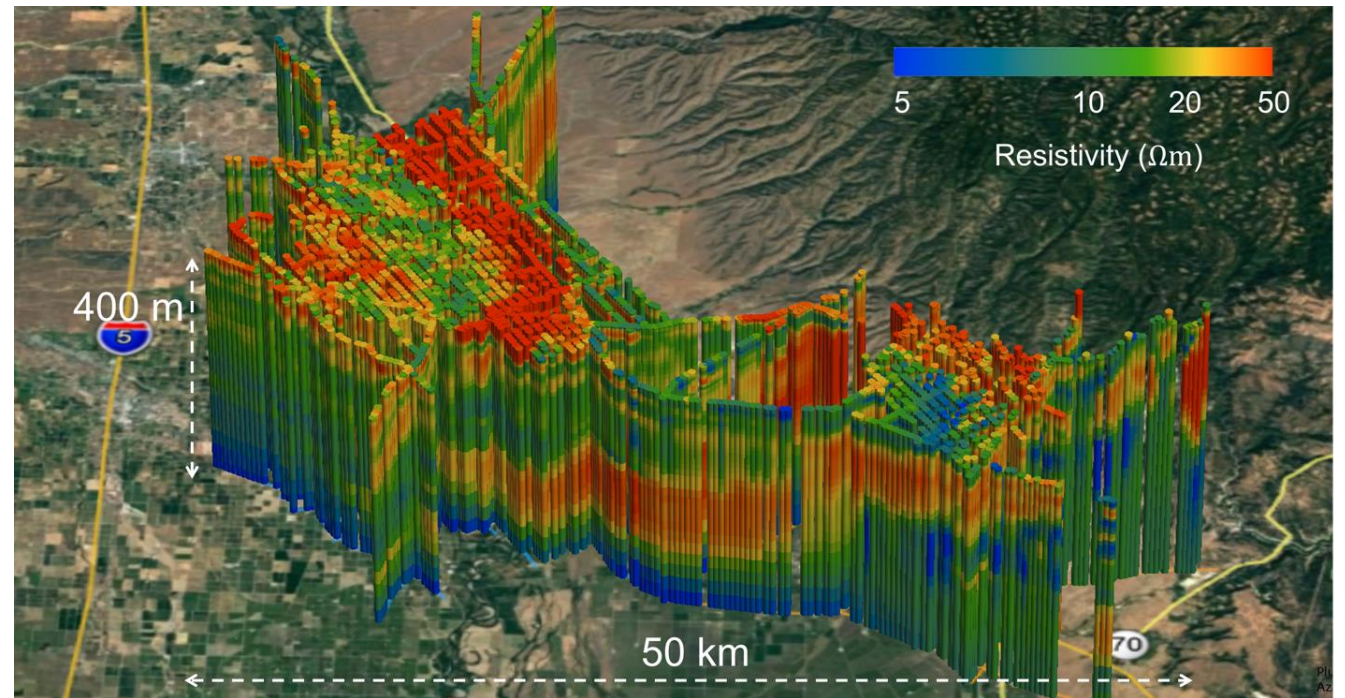
Hundreds of km

Airborne geophysics



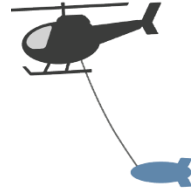
- Potential fields
Magnetics
Gravity
- Electromagnetics
- (Ice) Radar

Increasing
Resolution



Kang et al. (2021)

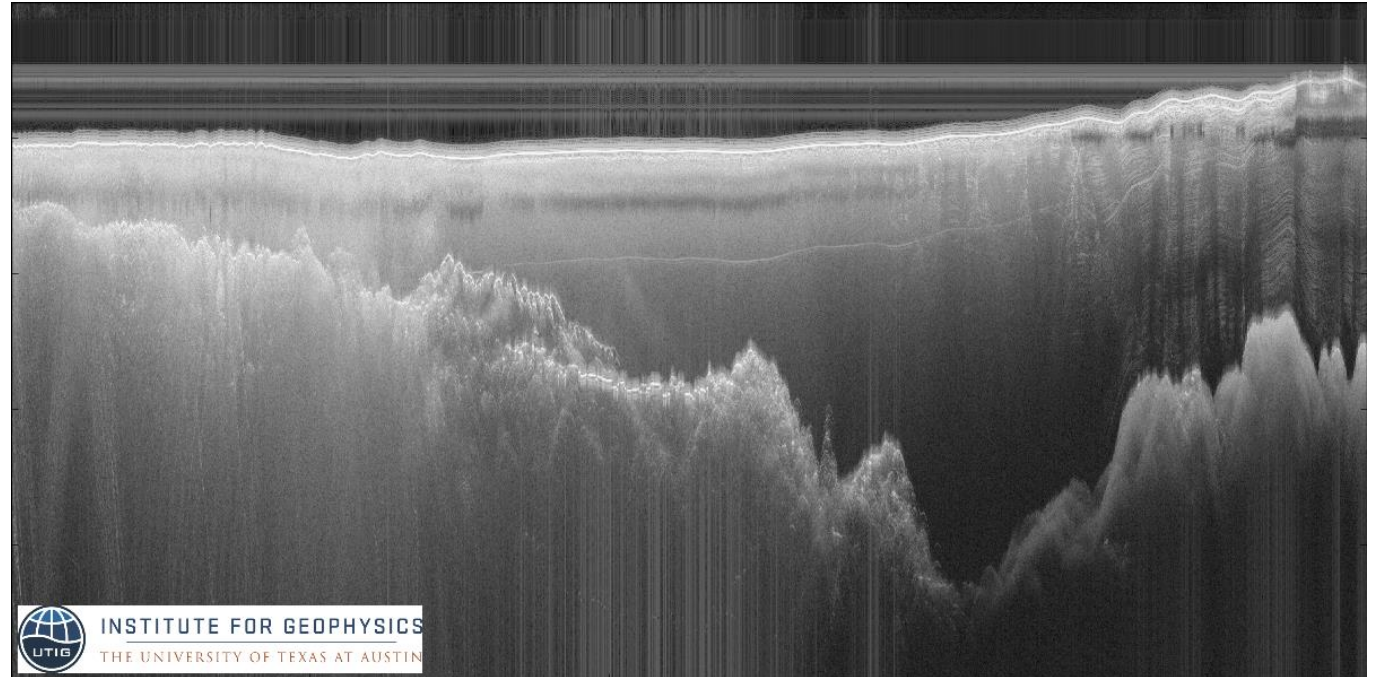
Airborne geophysics



- Potential fields
Magnetism
Gravity
- Electromagnetics
- (Ice) Radar

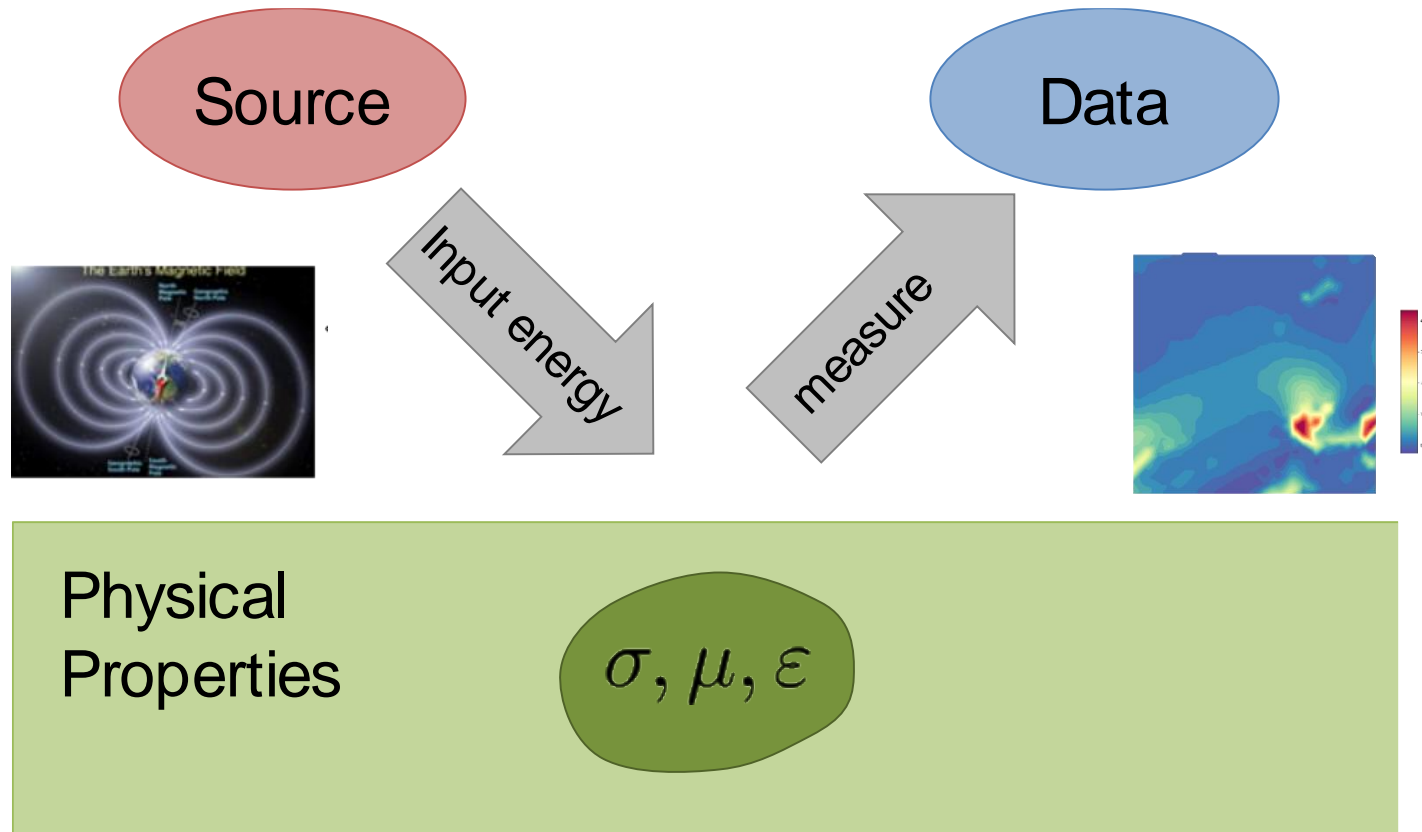


Increasing
Resolution



Generic geophysical experiment?

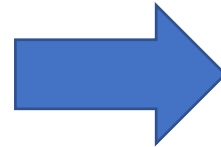
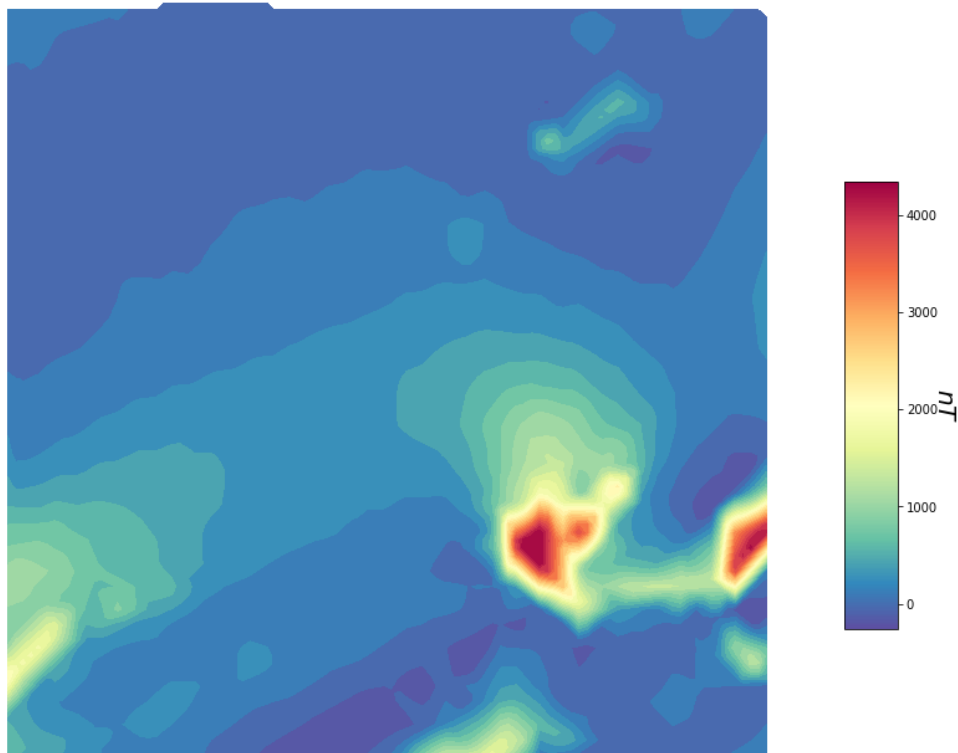
All require ways to see into the earth without direct sampling



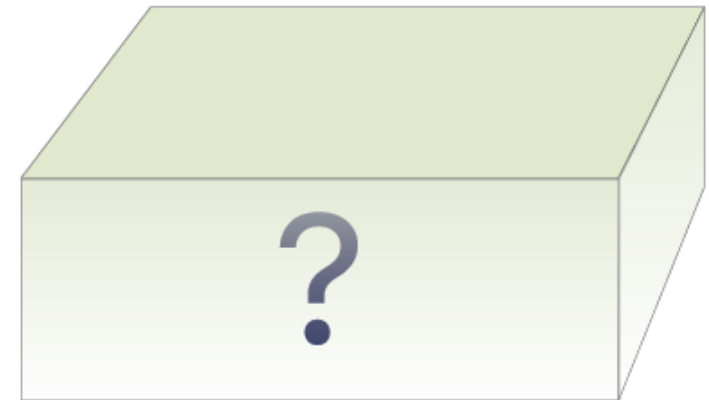
An overarching question today is ...

How do we find a subsurface model from the observed data in a data-driven way?

Observed data

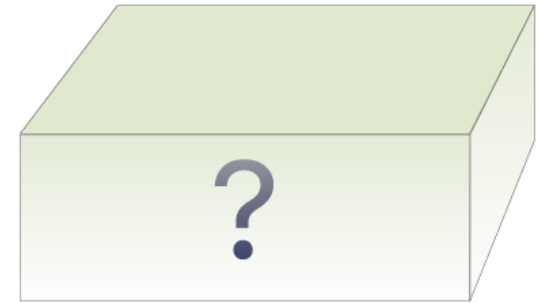
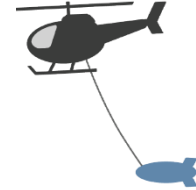


A subsurface model



Outline

- Backgrounds: Magnetism
- Inversion Framework
- 1D Linear Inverse problem
- 3D Magnetic Inversion
- Including Other Information
- Summary



Python Packages that I am going to use today...



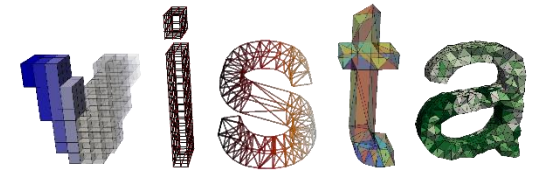
simpeg

Numerical engine for geophysical
simulation & inversion



GemPy

Geologic modelling

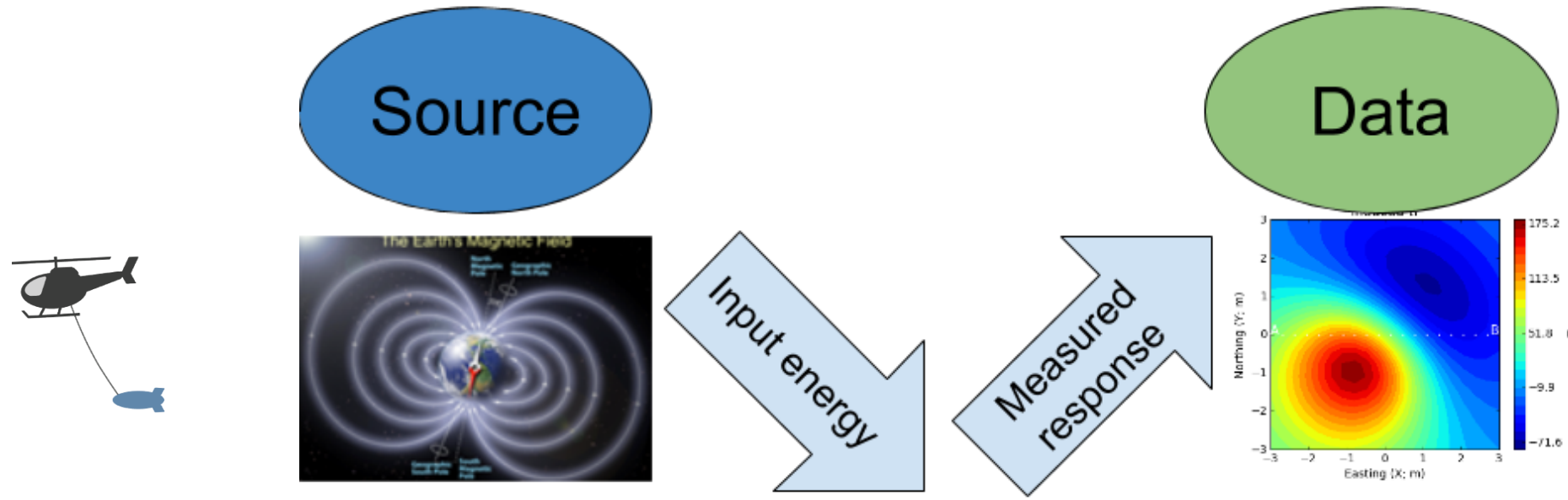


3D visualization

My intention of this lecture

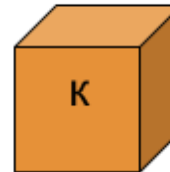
“*Not* for introducing how geophysical software packages work, *But* for providing fundamental concepts of the inversion”

Survey: Magnetics



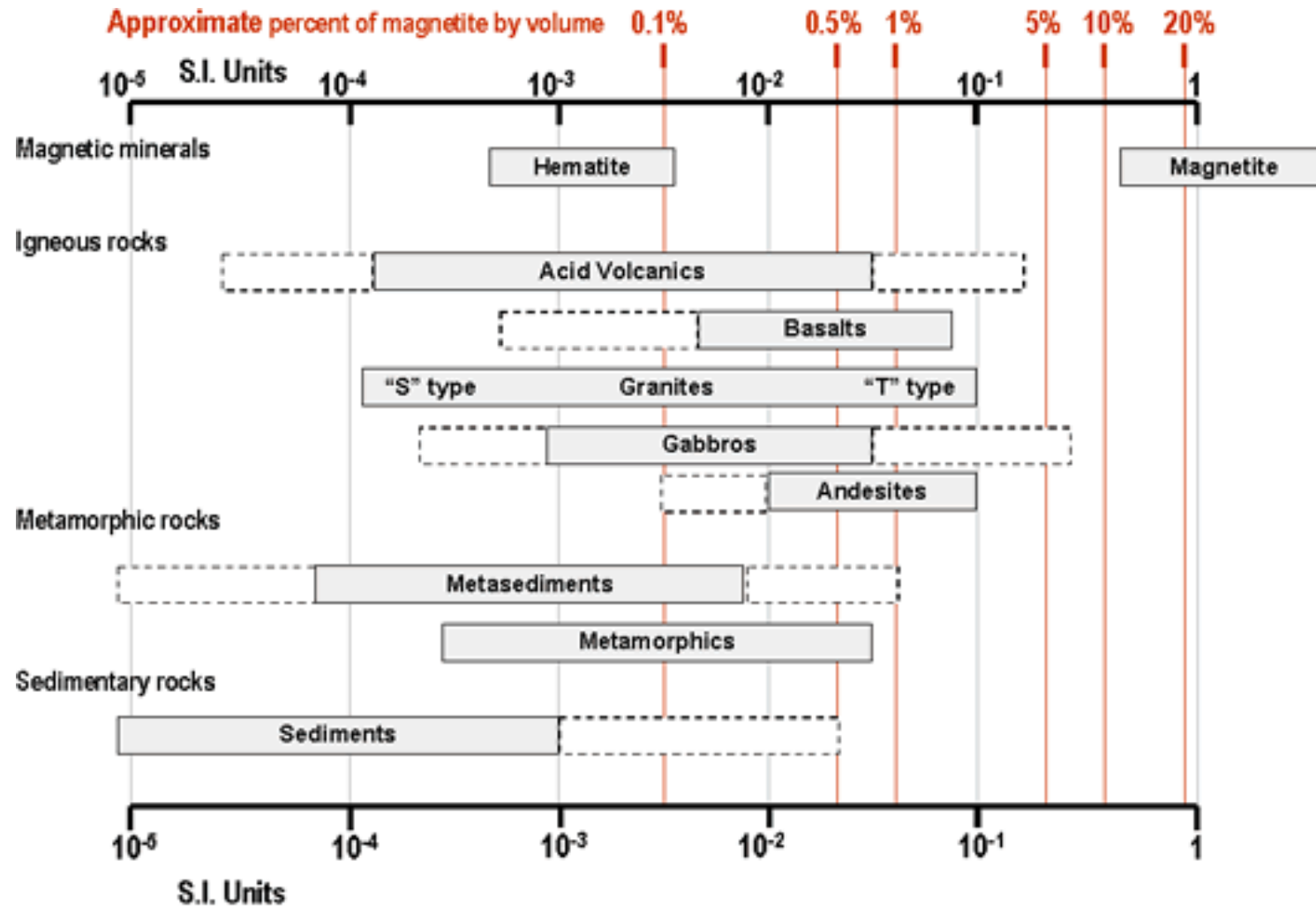
Subsurface

Physical Property:



κ : Magnetic susceptibility

Magnetic susceptibility



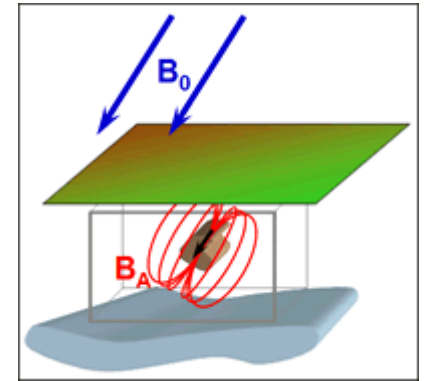
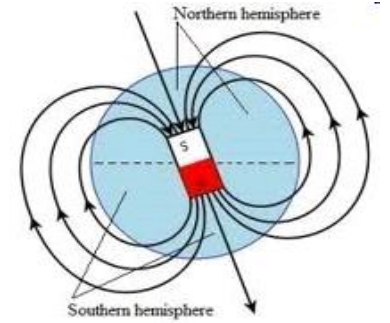
Magnetic surveying

- Earth's magnetic field \vec{B}_0 is the source:
- Materials become magnetized

$$\vec{M} = \kappa \vec{H}_0 \text{ (magnetization)}$$

$$\vec{H}_0 = \vec{B}_0 / \mu_0$$

- Create anomalous magnetic field



Magnetic surveying

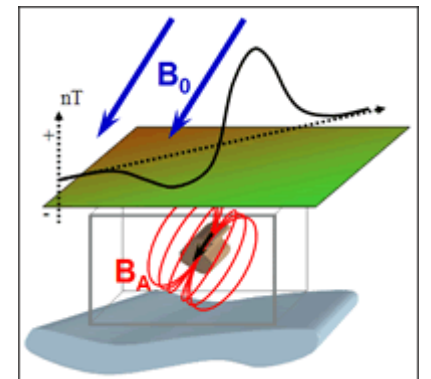
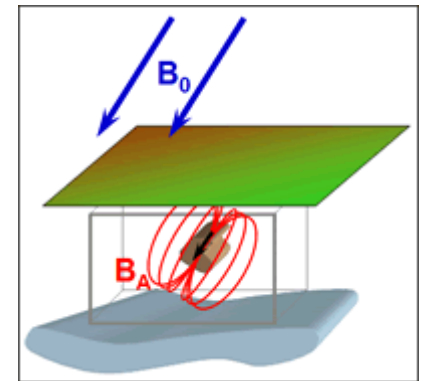
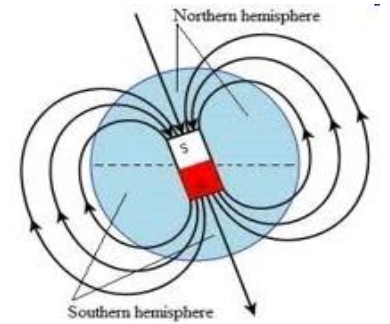
- Earth's magnetic field \vec{B}_0 is the source:
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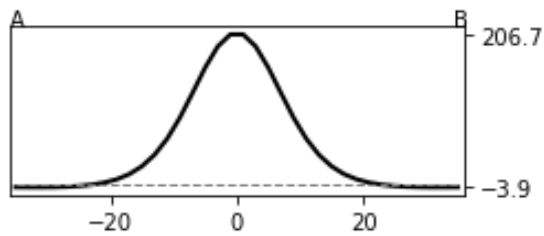
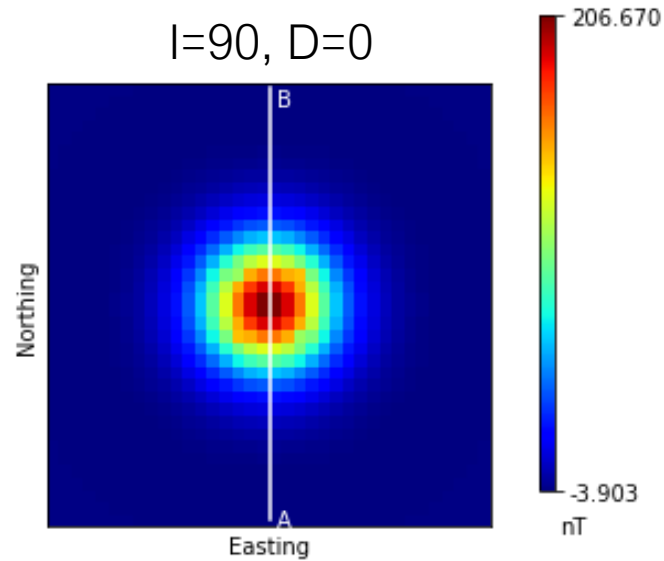
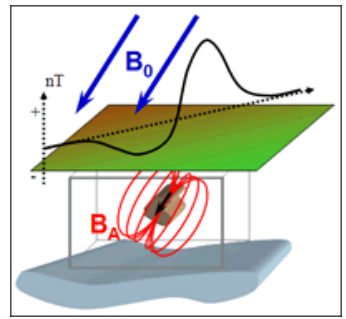
$$\vec{H}_0 = \vec{B}_0 / \mu_0$$

- Create anomalous magnetic field
- Measure total magnetic field: $|\vec{B}| = |\vec{B}_0 + \vec{B}_A|$

- Total field anomaly: $\Delta \vec{B} = |\vec{B}_0 + \vec{B}_A| - |\vec{B}_0|$
 $\Delta \vec{B} \simeq \vec{B}_A \cdot \hat{B}_0$ where $\hat{B}_0 = \frac{\vec{B}_0}{|\vec{B}_0|}$

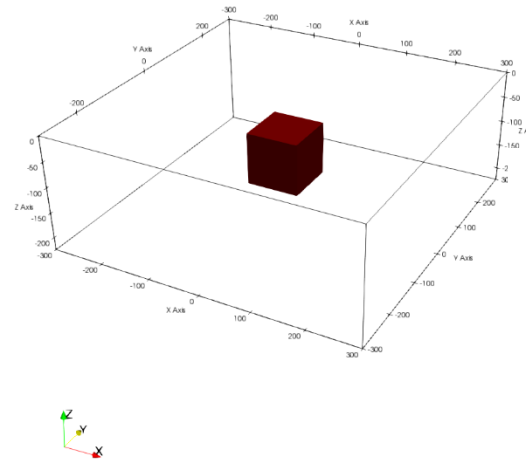


Magnetic data changes depending upon where you are

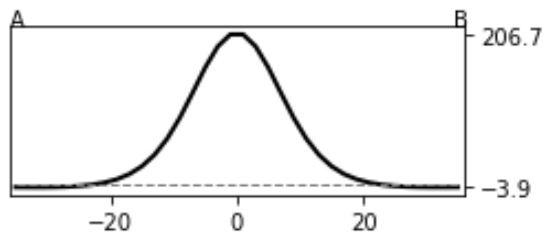
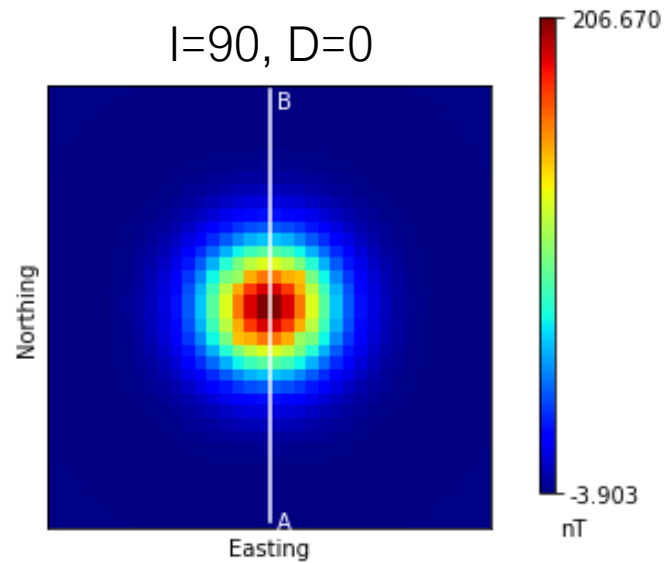
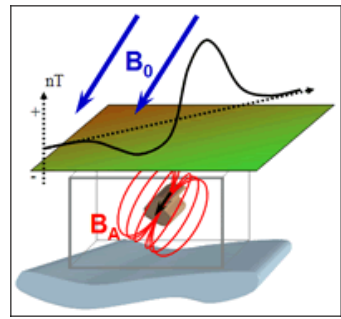


North pole

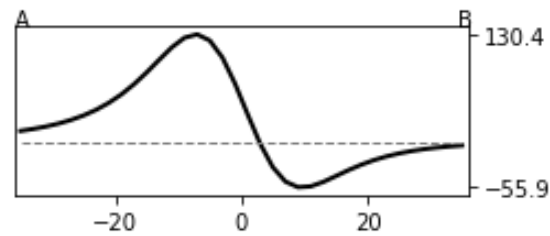
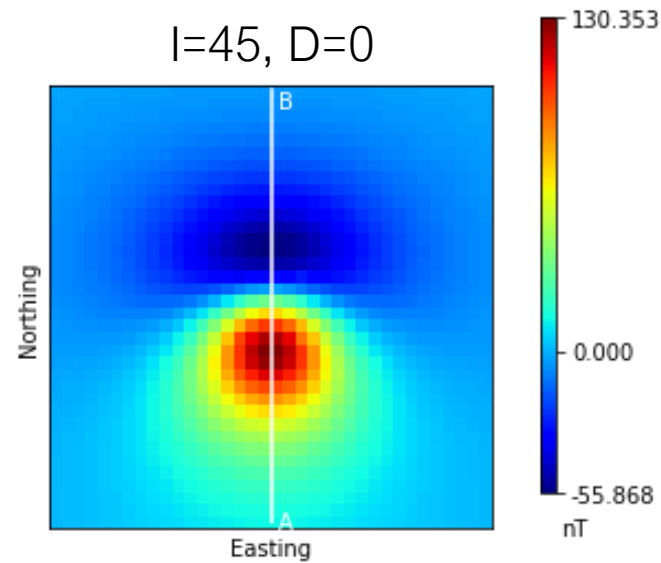
A prism in a homogeneous subsurface



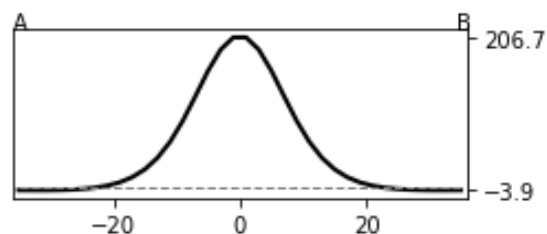
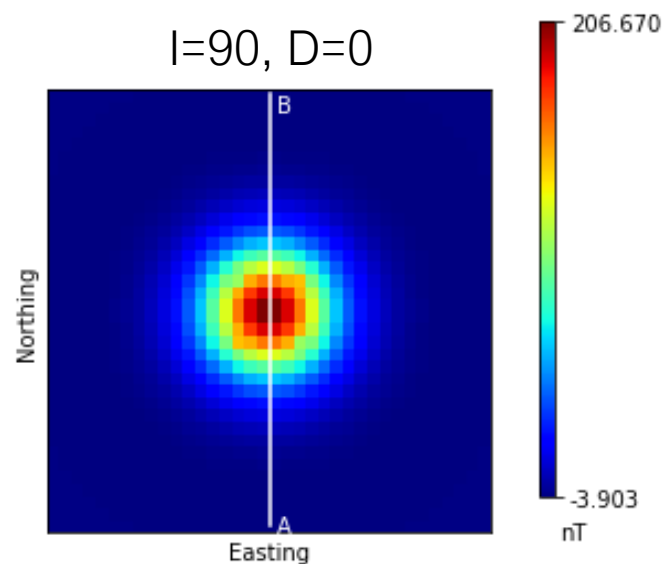
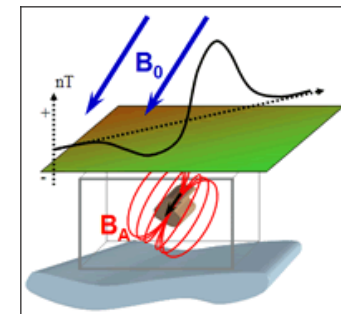
Magnetic data changes depending upon where you are



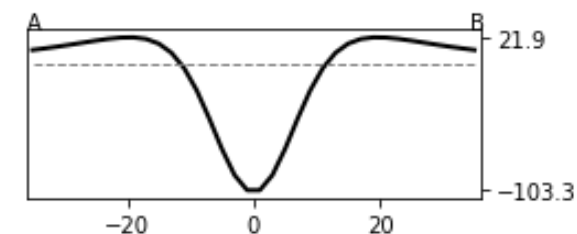
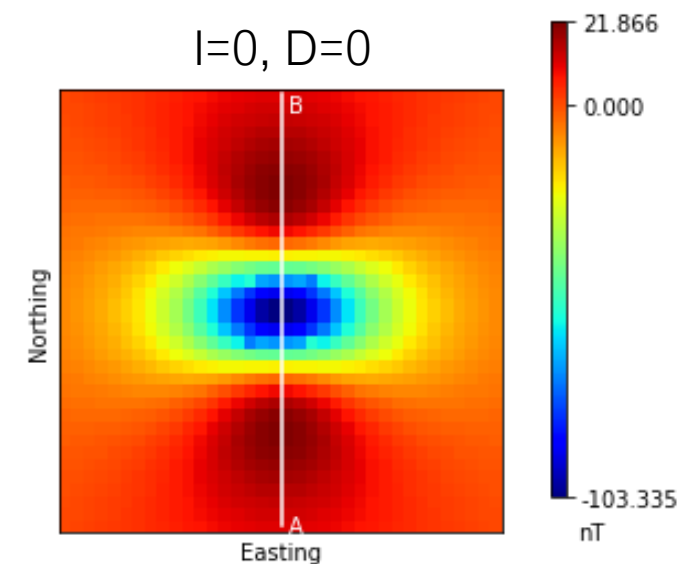
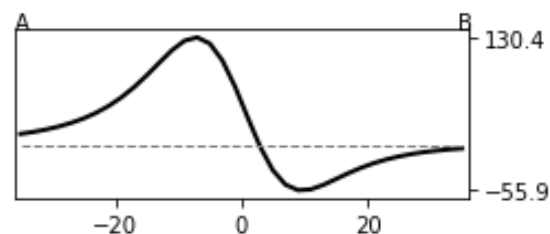
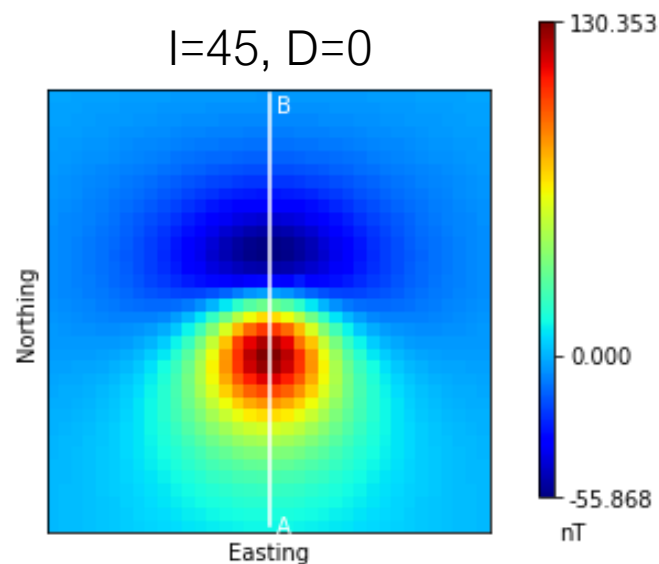
North pole



Magnetic data changes depending upon where you are

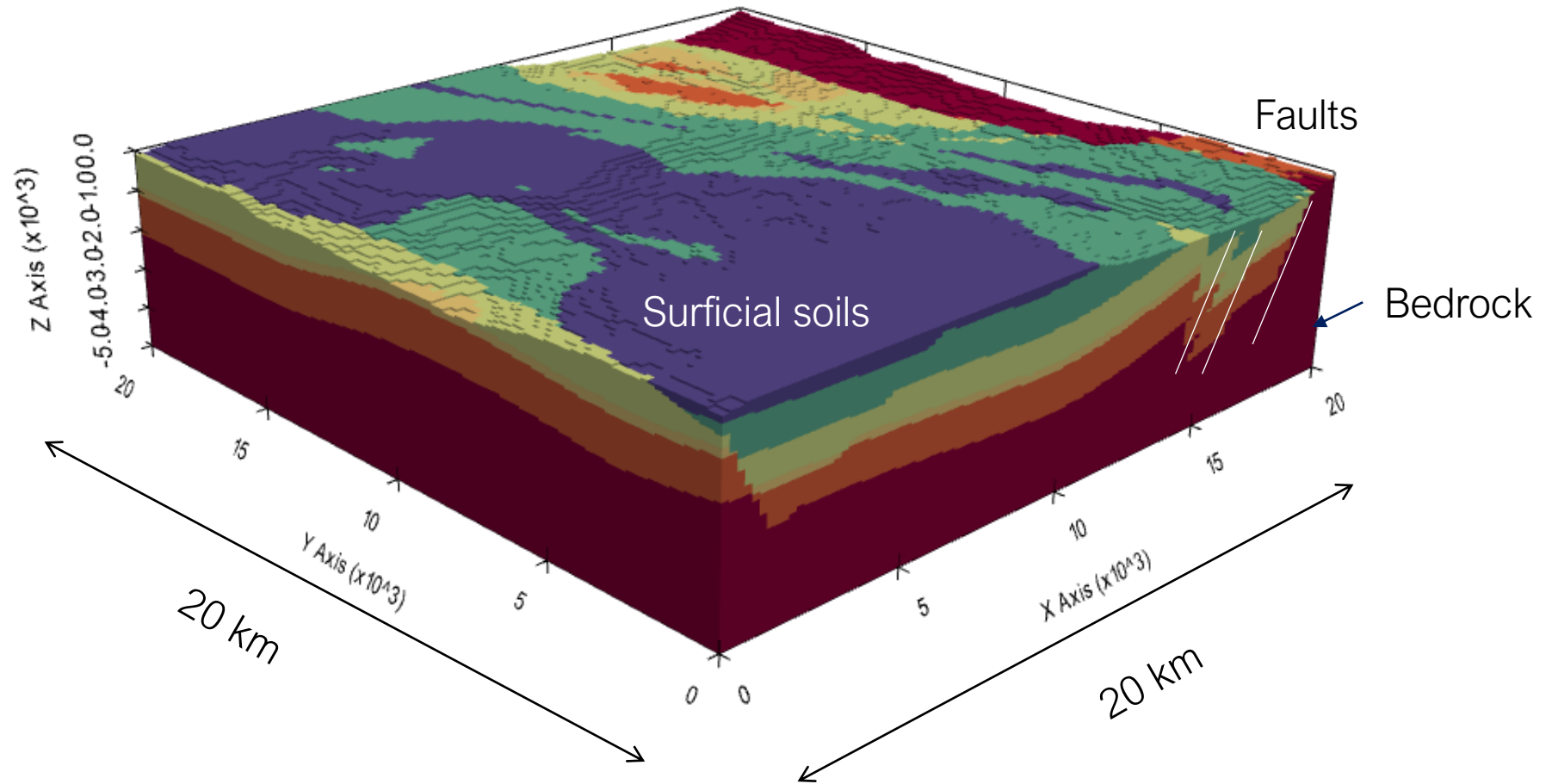


North pole

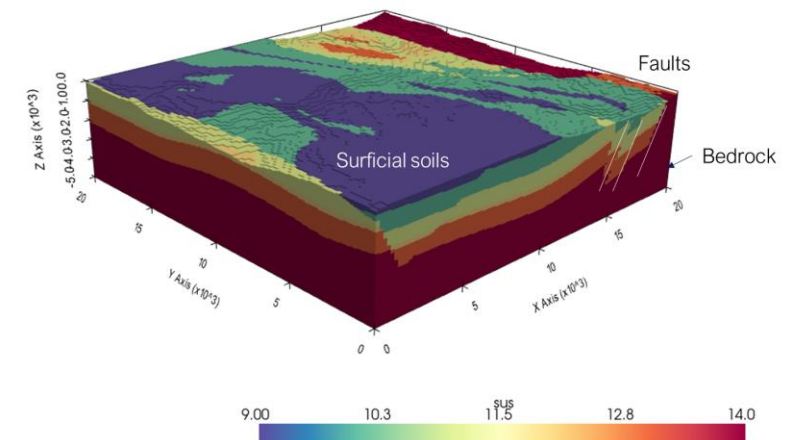
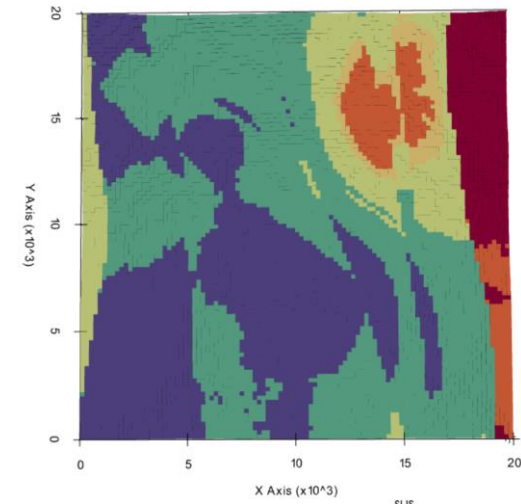
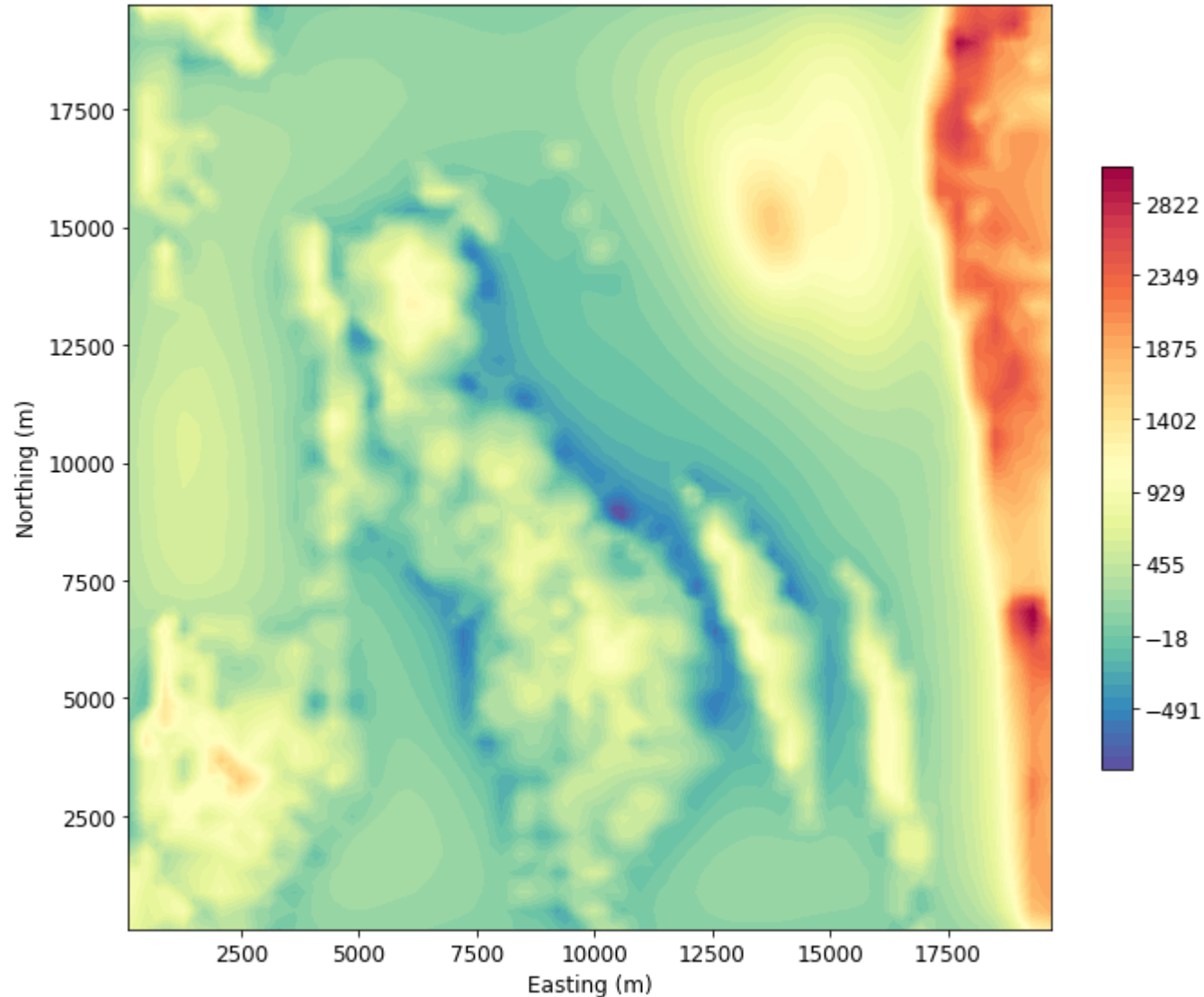


Equator

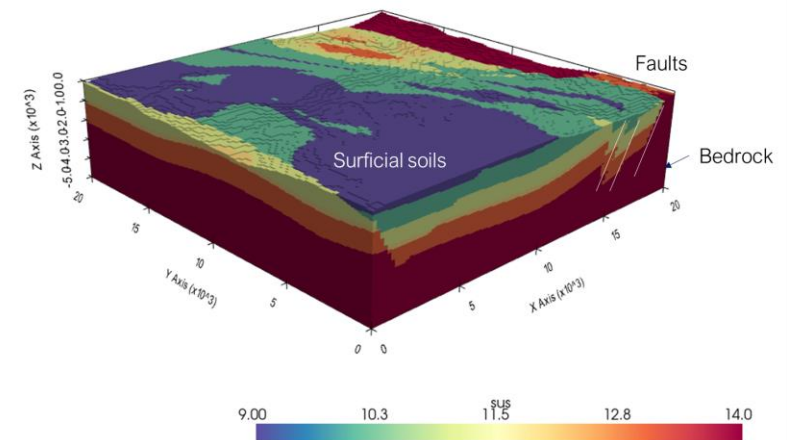
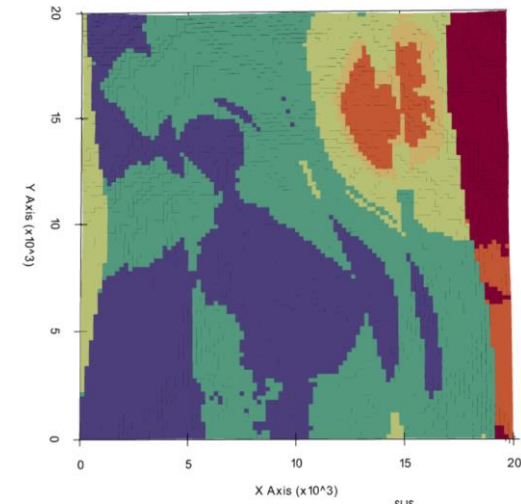
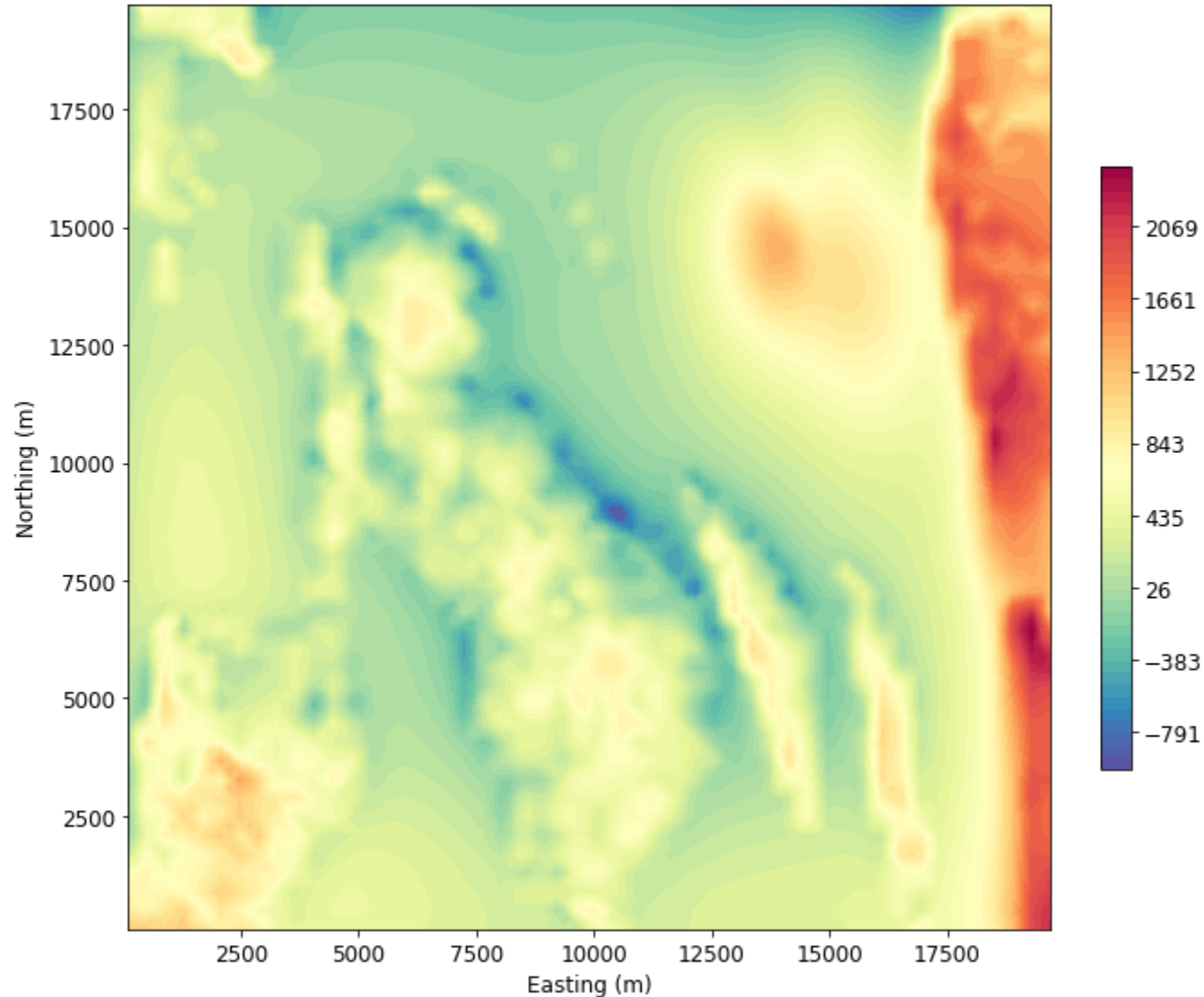
Subsurface structure is complex



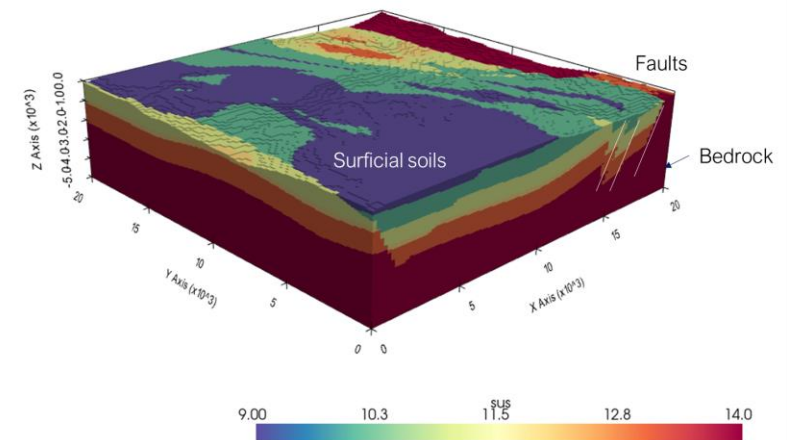
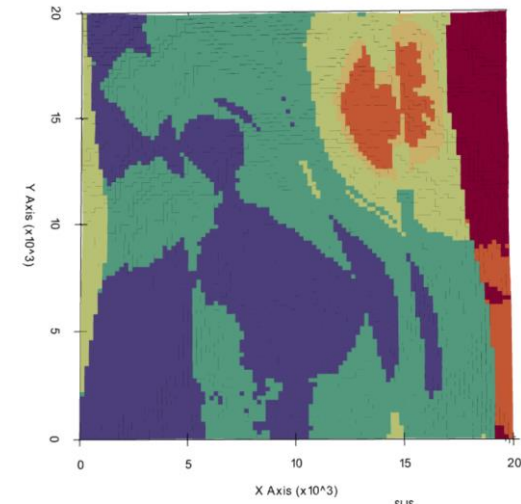
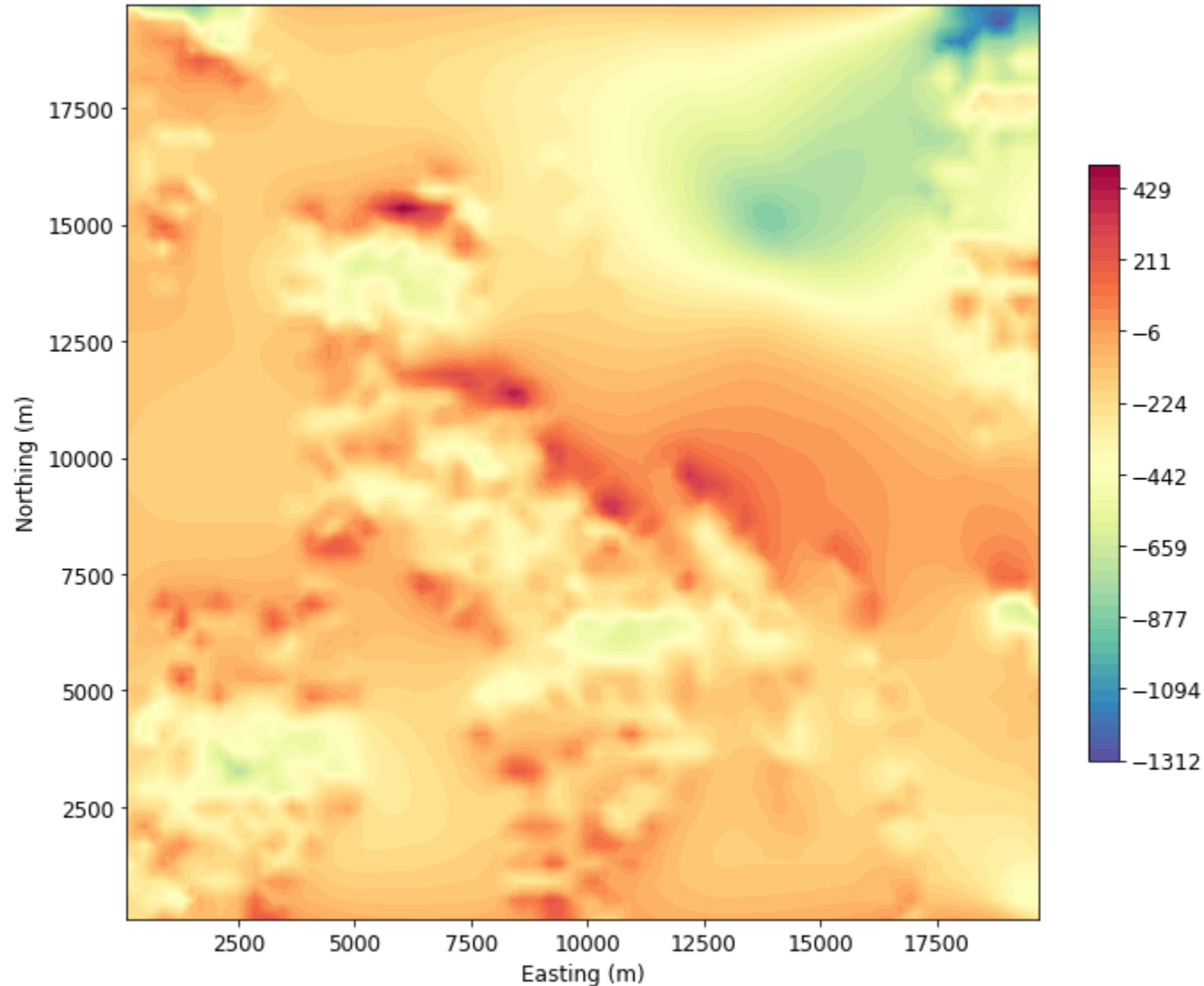
Measured magnetic data at $I=90$, $D=0$ (North pole)



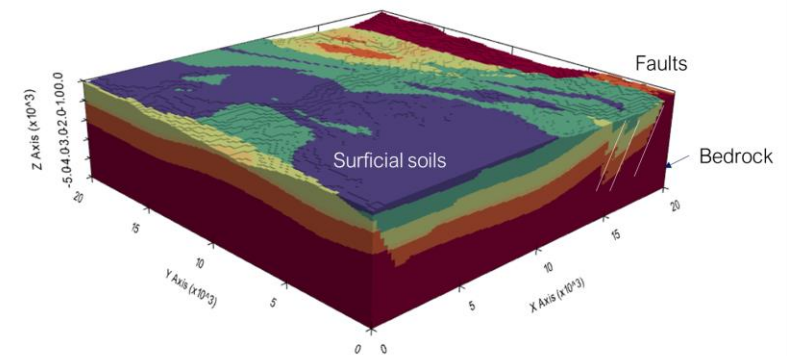
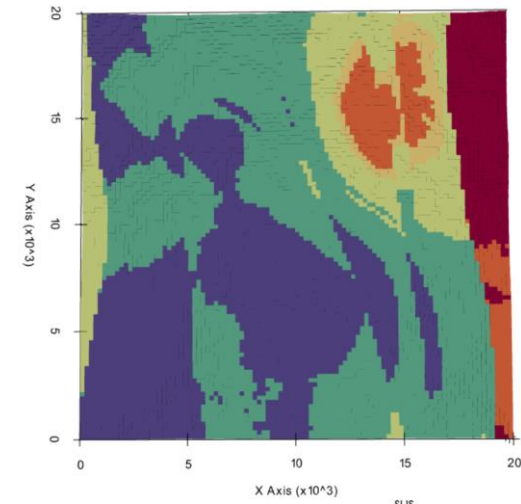
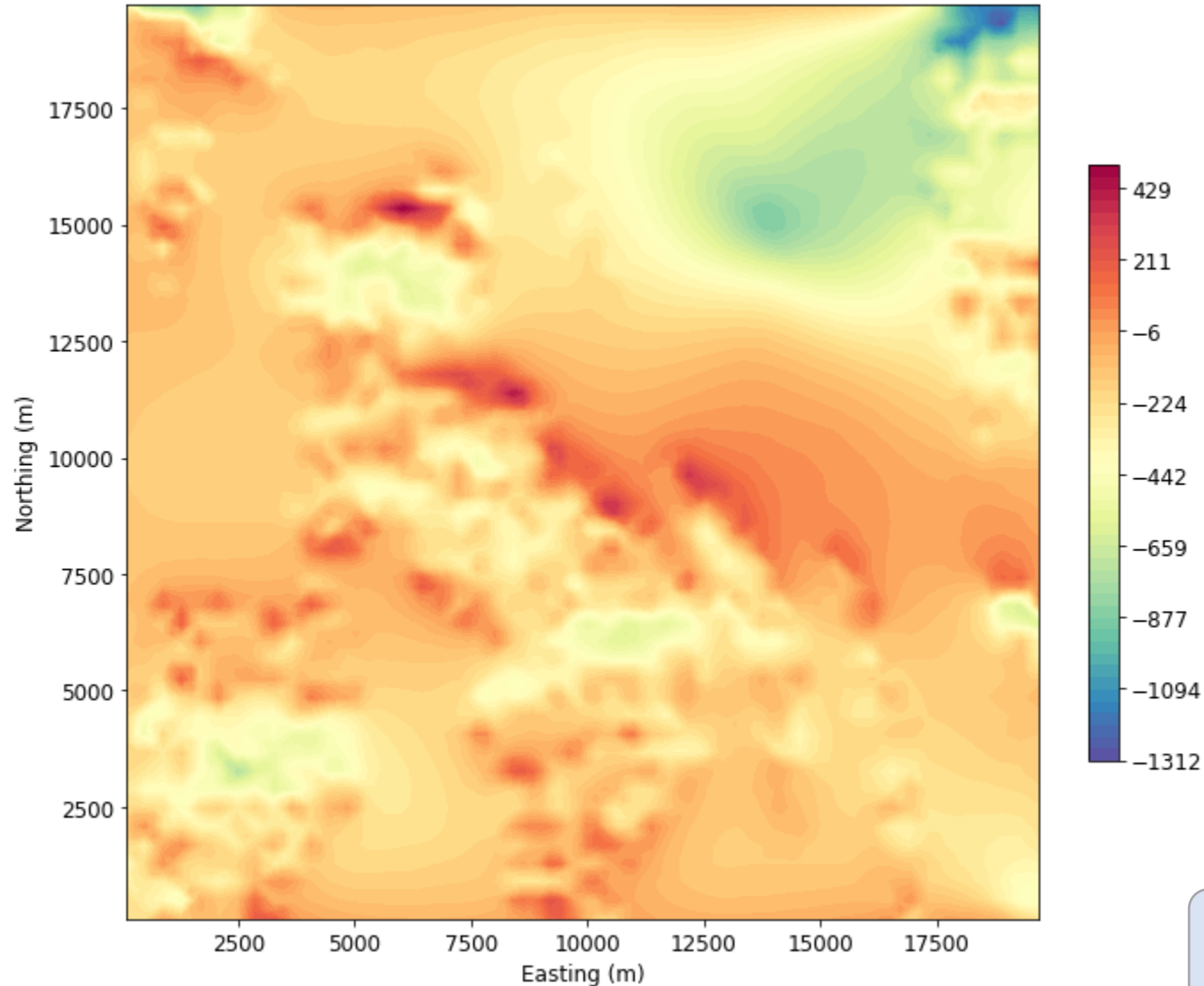
Measured magnetic data at I=66, D=-6 (California)



Measured magnetic data at $I=0$, $D=0$ (Equator)



Measured magnetic data at $I=20$, $D=0$ (Equator)



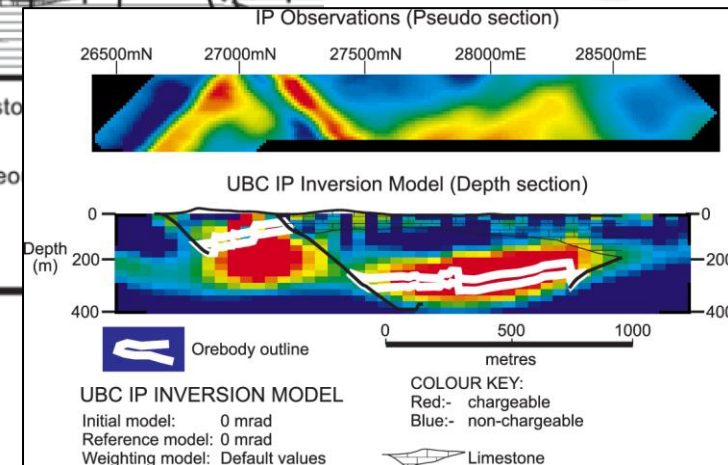
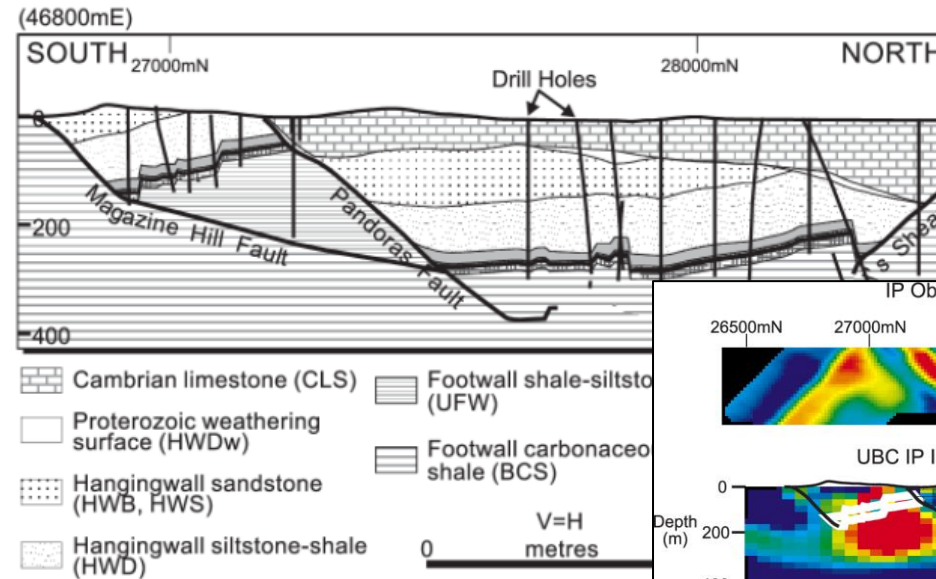
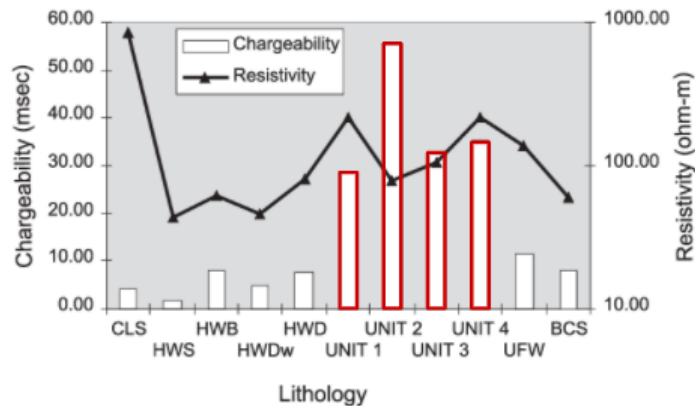
Measured data are large & complex

motivating field example

Transform 2020: Lindsey Heagy (DC/IP methods)

Century Deposit: geology + physical properties

- Resistivity: structure, input to IP
- Chargeability: Associated with mineralization

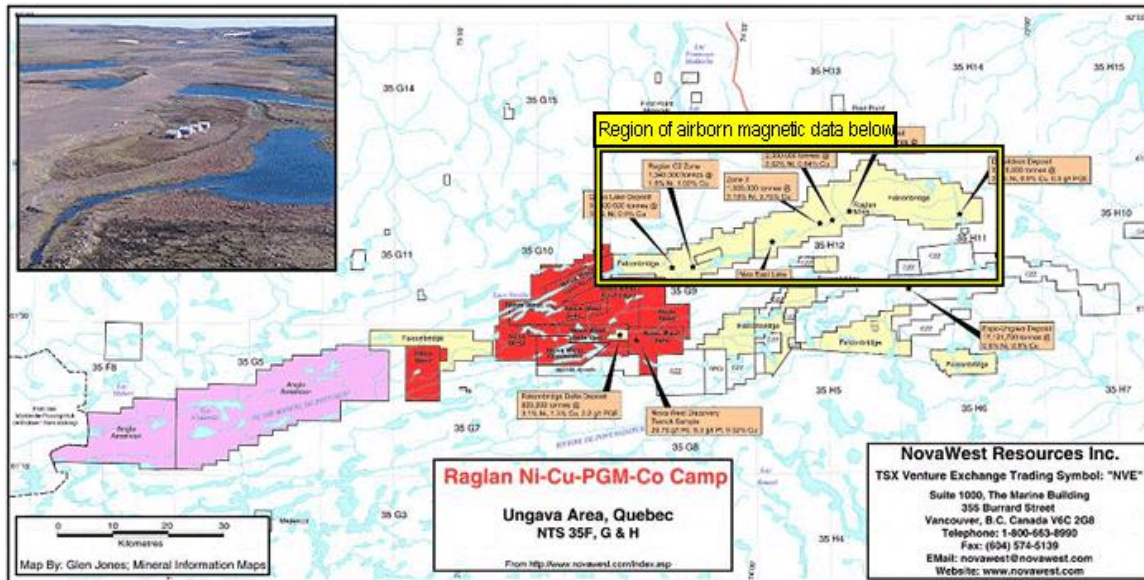


Reproduce the historic geophysical inversion results which made a high impact to the mining community

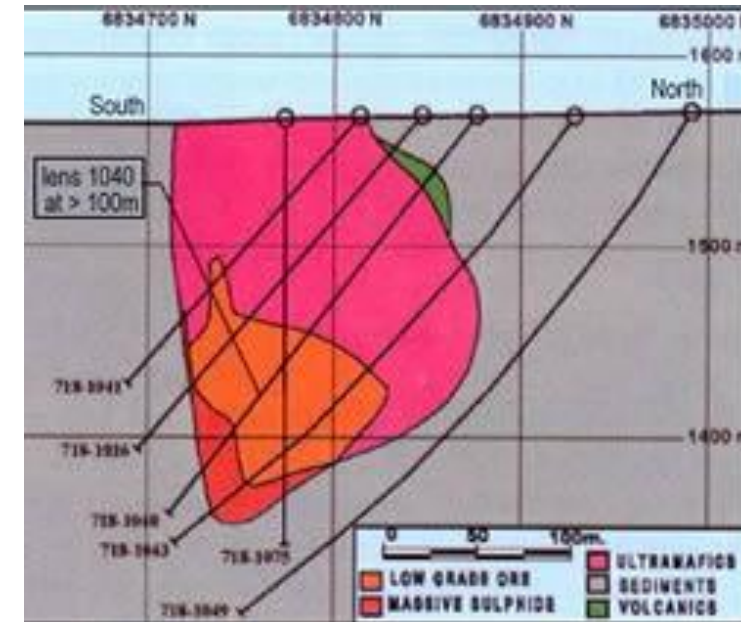
ground-based geophysics

Raglan Deposit: geology + physical properties

Location map (Northern Quebec, Canada)



Geologic section



Physical properties

Grey rocks are host sediments.

Green rocks are volcanics.

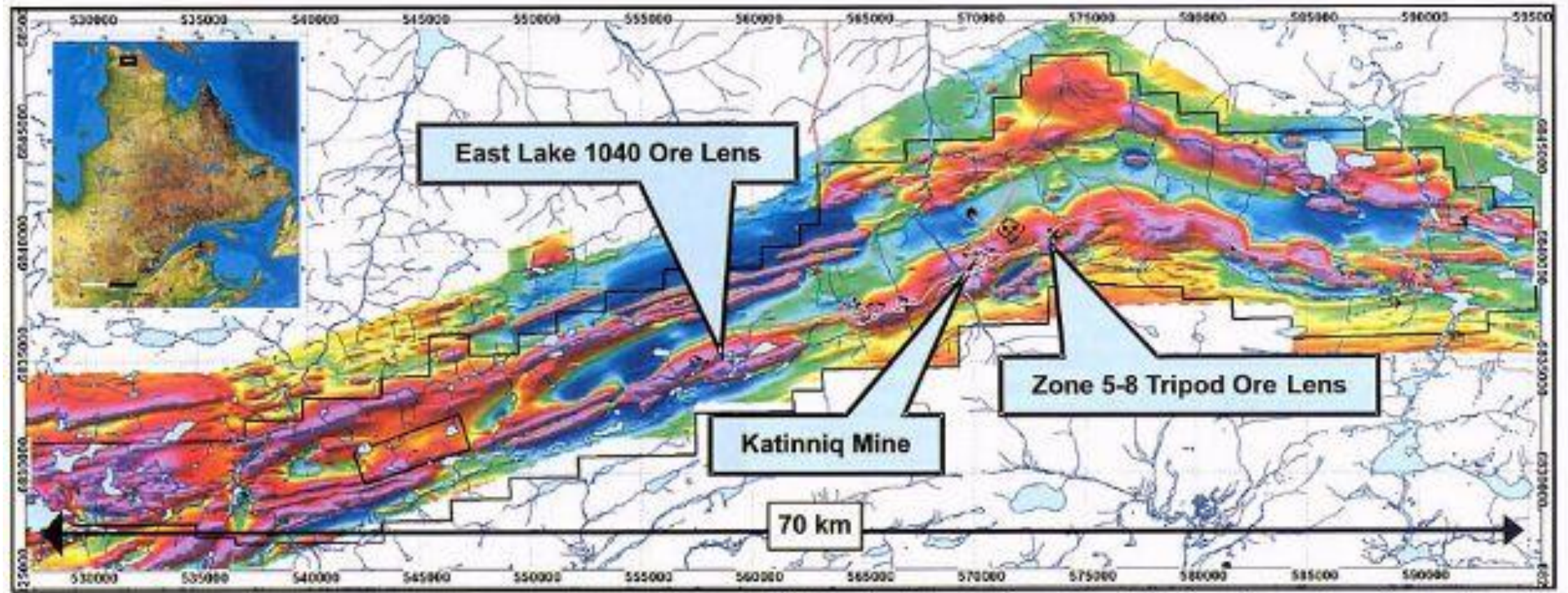
Pink rocks are ultramafics (susceptibility 0.03 - 0.07 S.I.).

Orange rocks are low grade massive ore (susceptibility 0.03 - 0.07 S.I.).

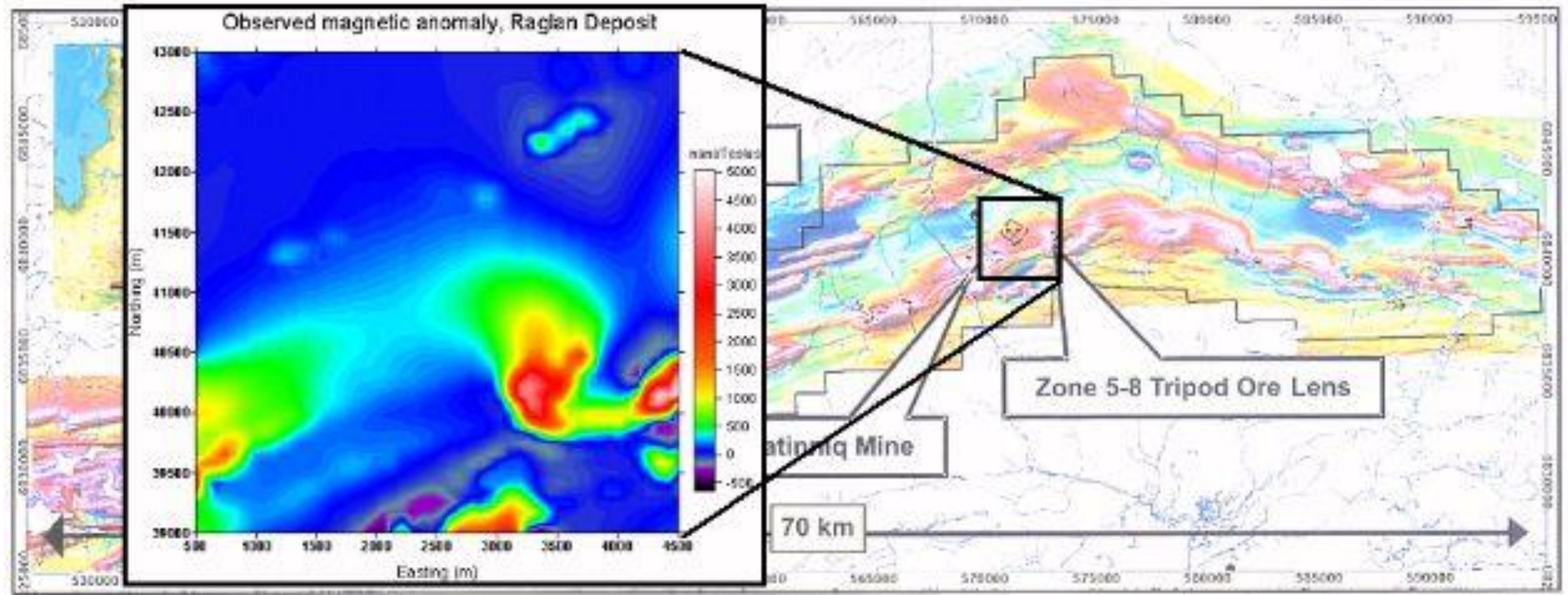
Red rocks are the primary massive sulphide ore (susceptibility 0.03 - 0.07 S.I.).

Seek for zones having a high susceptibility (~ 0.05 SI)

Raglan Deposit: magnetic data

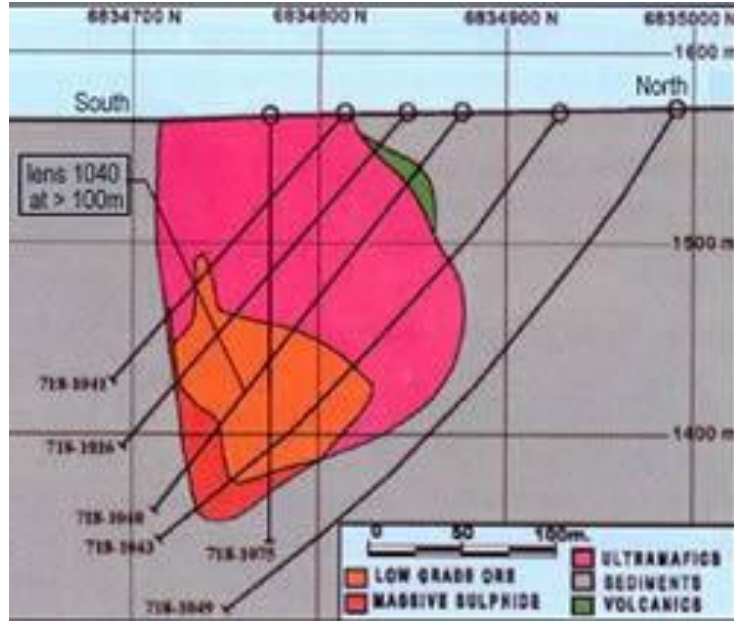


Raglan Deposit: magnetic data

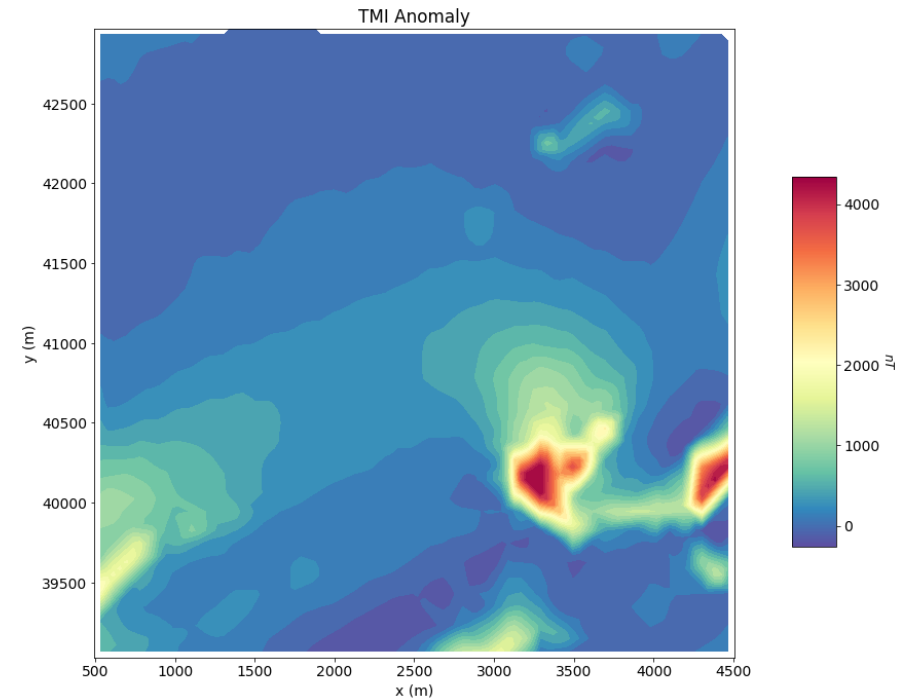


Initial conceptual model

Geologic section

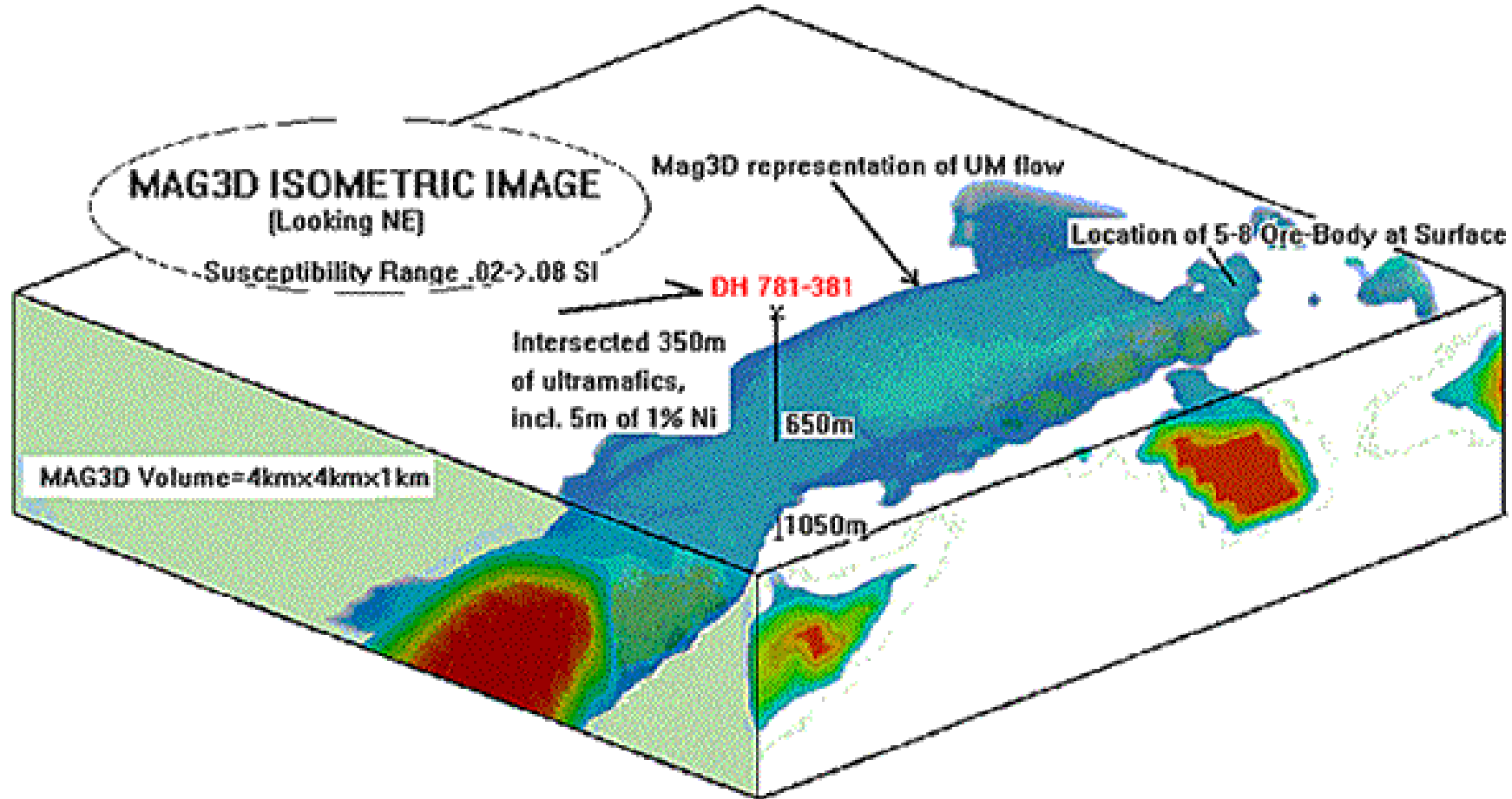


Magnetic data



Initial conceptual model: two ultramafic pipes
Can make impact on drilling location and mineral reserve

Recovered susceptibility model from inversion

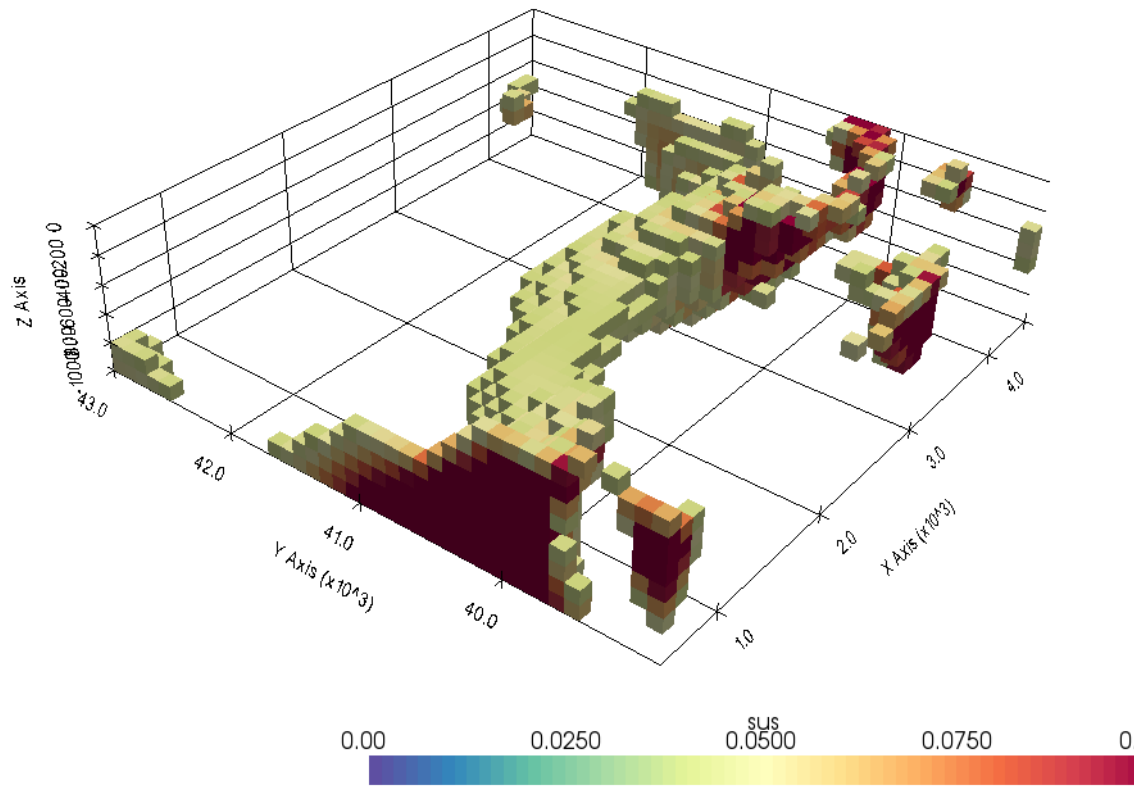


Changed the conceptual model: the two pipes are “connected”

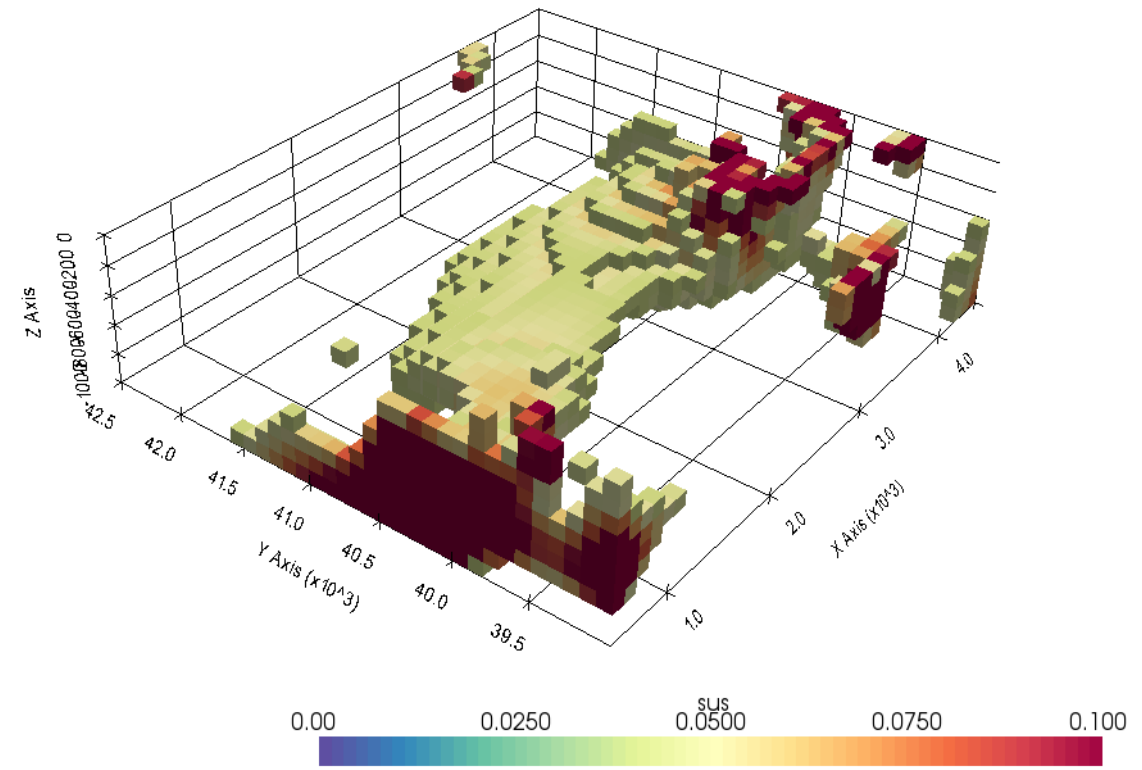
A quick demo: “can we reproduce this result?”

Comparison

Model from 20 years ago (MAG3D)

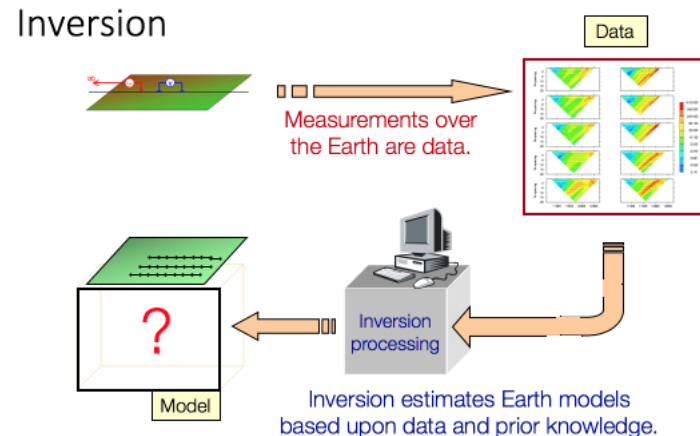
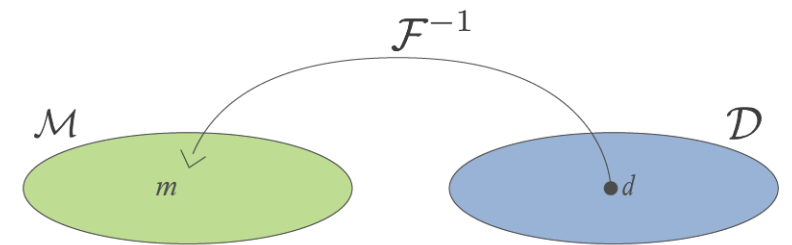
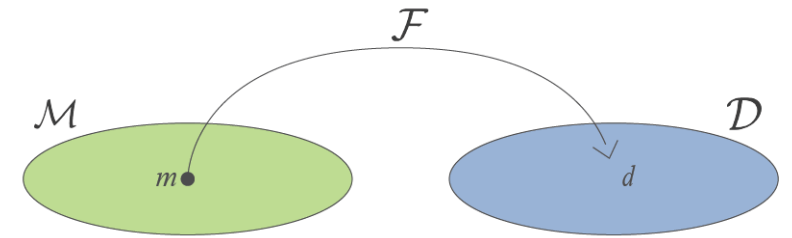


The recovered model (SimPEG)

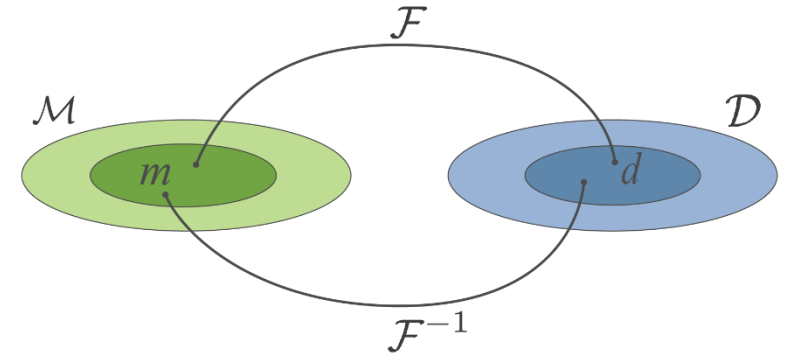


Our statement of the inverse problem

- Given observations: d_j^{obs} , $j = 1, \dots, N$
 - Uncertainties: ϵ_j
 - Ability for forward modelling: $\mathcal{F}[m] = d$
- Find the earth model that gave rise to the data.



Inverse problem



- Non-unique
- Ill-conditioned



The Inverse Problem is ill-posed

Any inversion approach must address these issues

Framework for the inverse problem

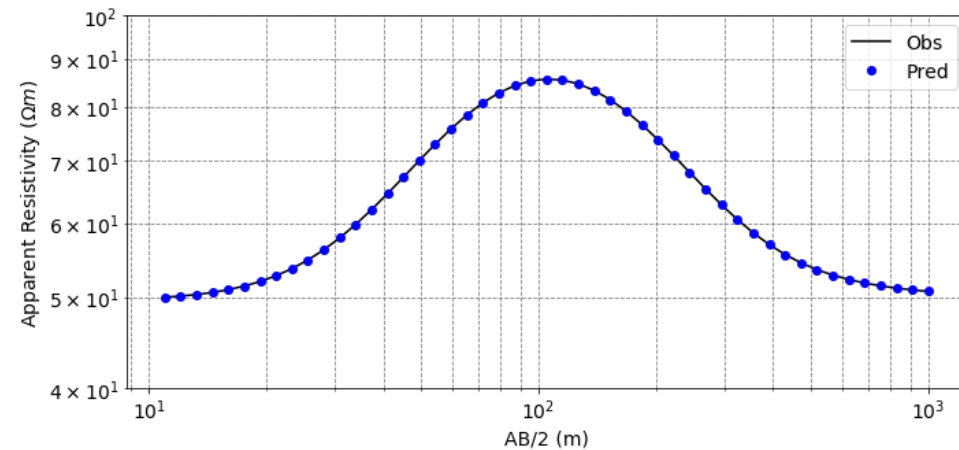
$$\text{minimize } \phi(m) = \phi_d(m) + \beta \phi_m(m)$$

ϕ_d : data misfit

ϕ_m : model norm

β : trade-off parameter

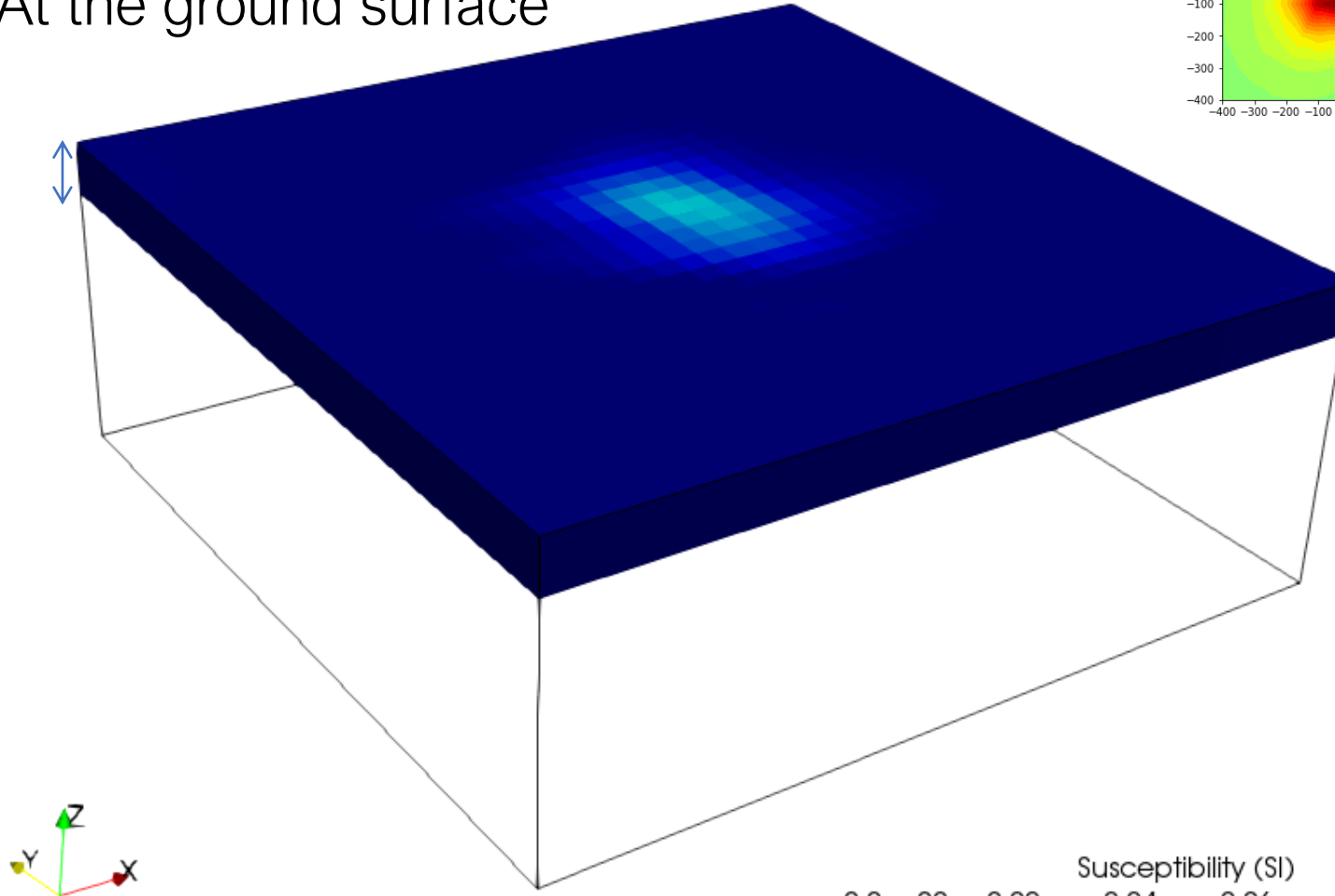
Find a model fitting the data



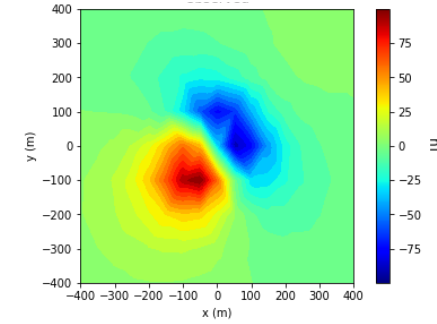
Example of extreme non-uniqueness

At the ground surface

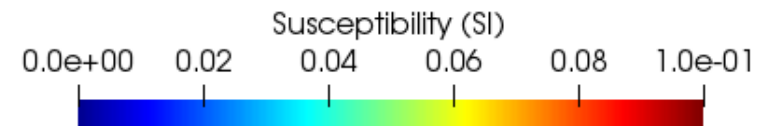
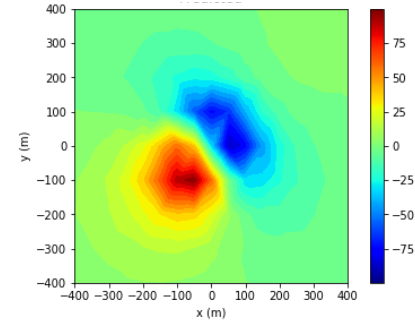
100 m



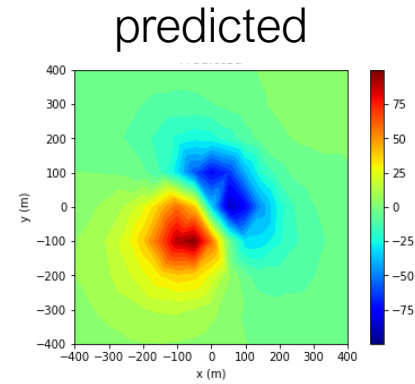
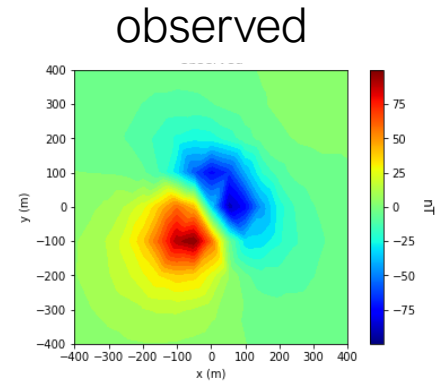
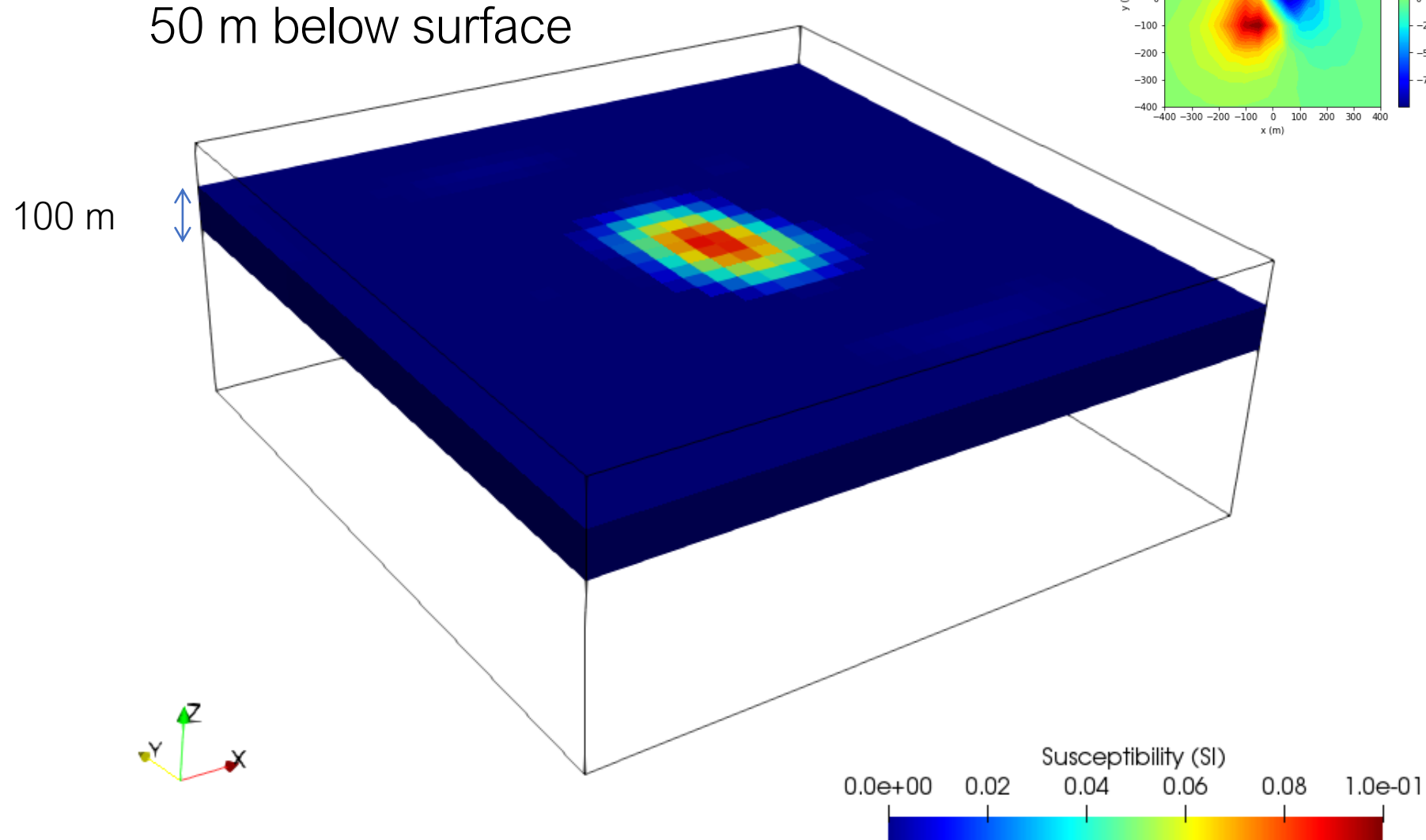
observed



predicted



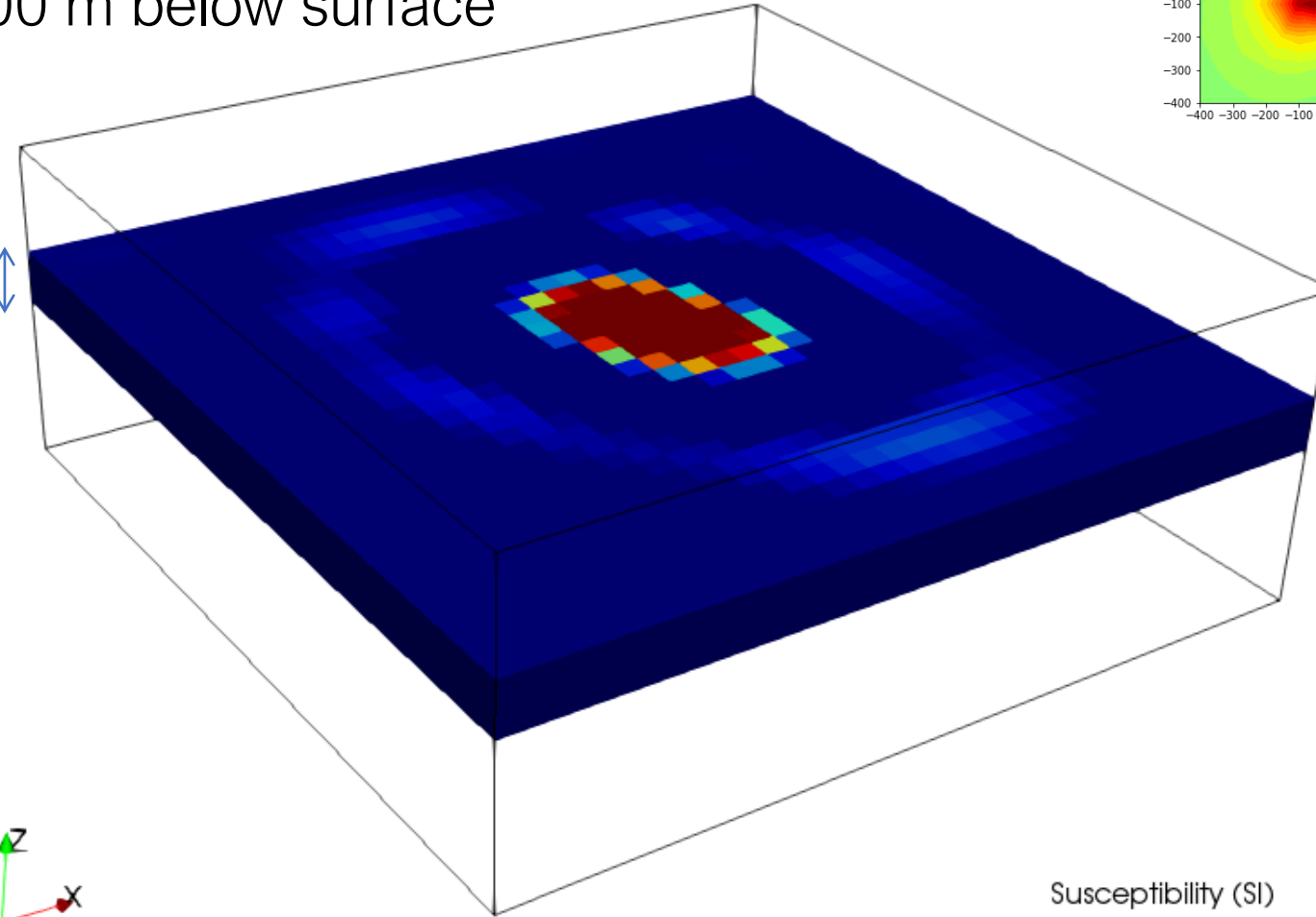
Example of extreme non-uniqueness



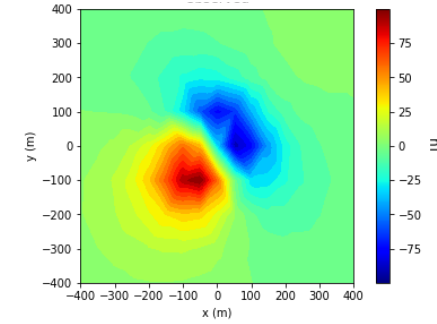
Example of extreme non-uniqueness

100 m below surface

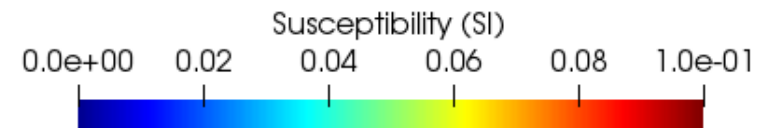
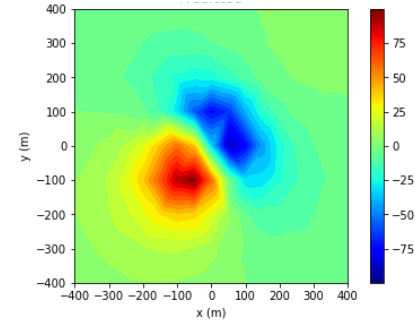
100 m



observed



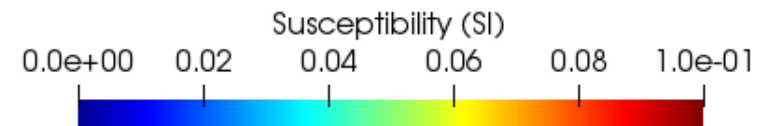
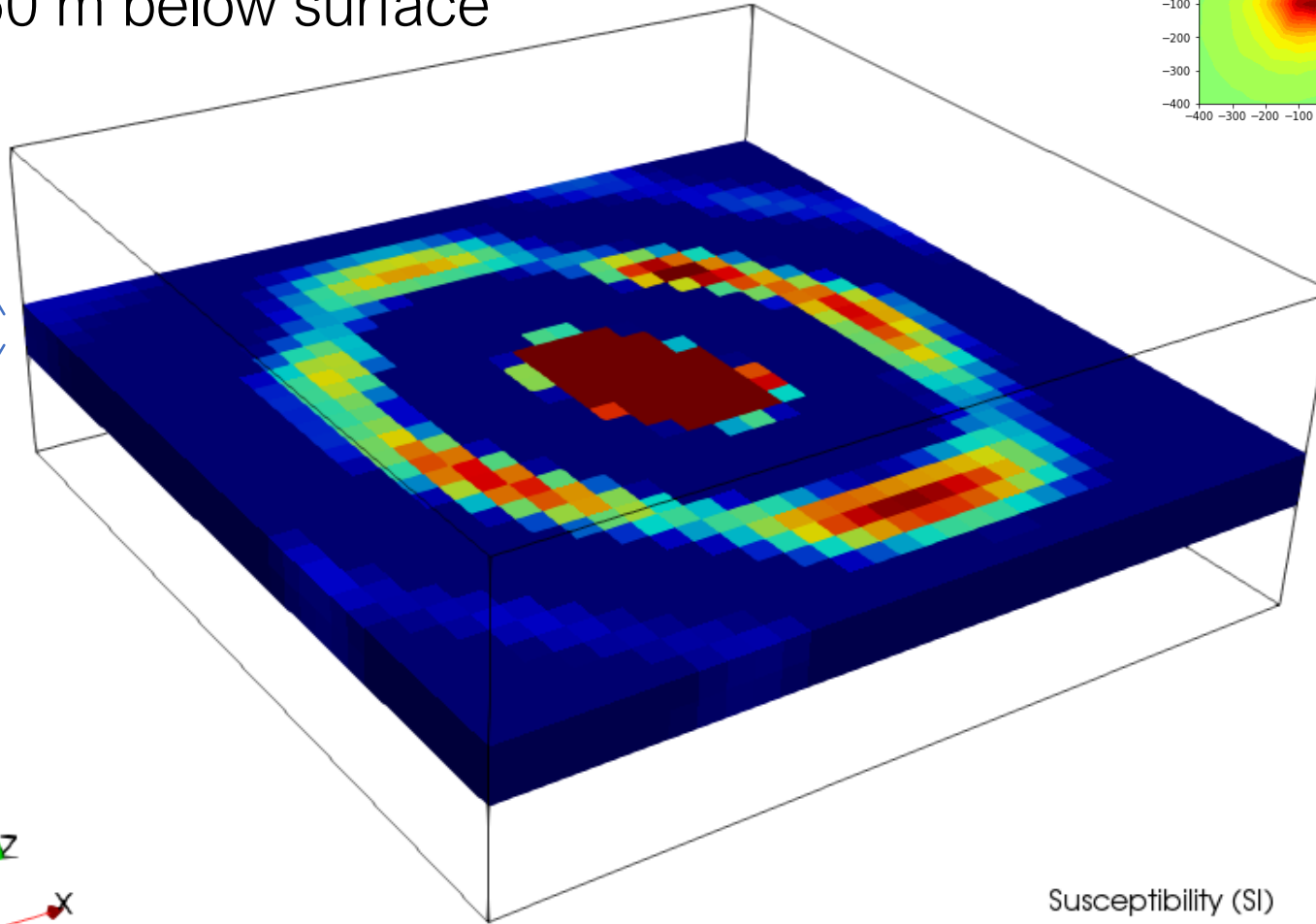
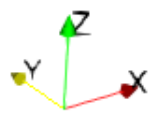
predicted



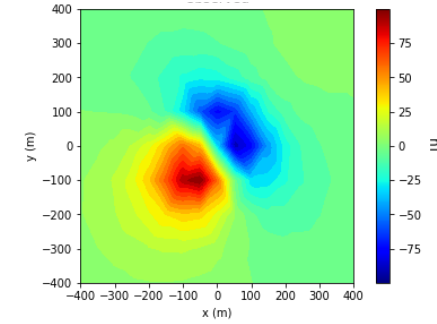
Example of extreme non-uniqueness

150 m below surface

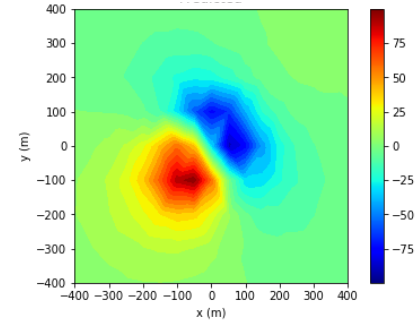
100 m



observed



predicted



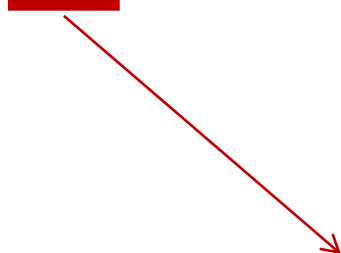
Framework for the inverse problem

minimize $\phi(m) = \phi_d(m) + \beta \phi_m(m)$

ϕ_d : data misfit
 ϕ_m : model norm
 β : trade-off parameter



Find a model fitting the data



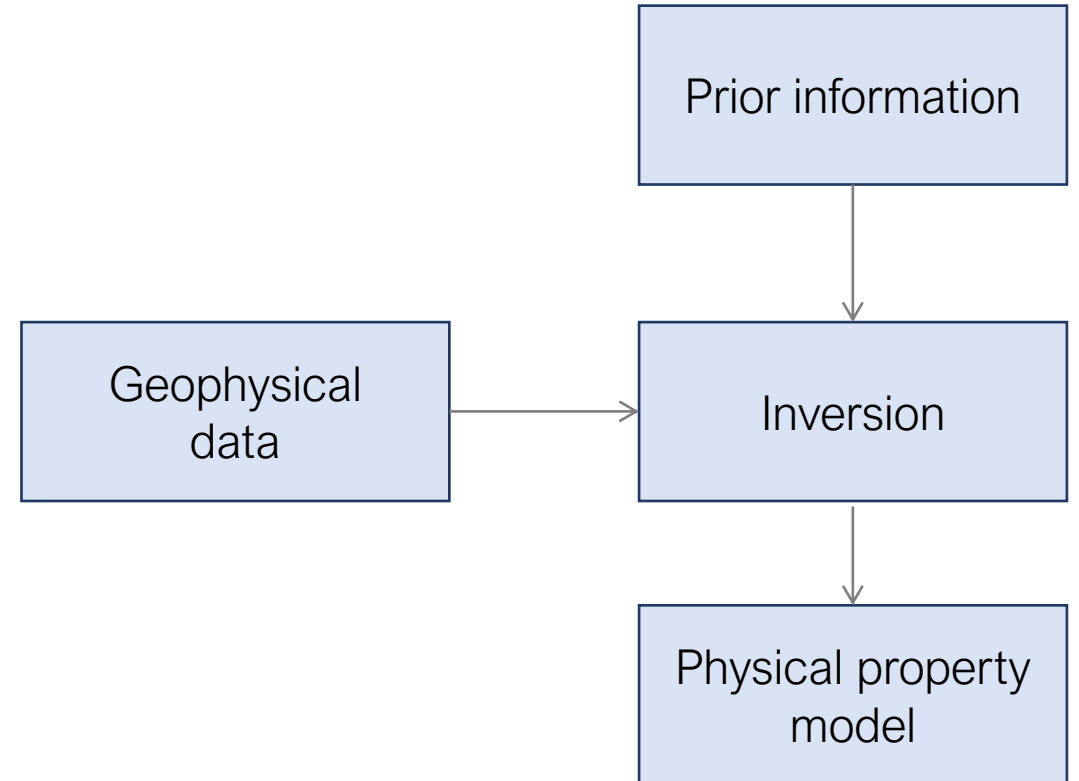
Find a model favoring prior knowledge

- drillers' logs
- geophysical logs (e.g., resistivity)
- spatial patterns
- average resistivity value of the region
- other geophysical data (e.g., seismic)

Constraining the inversion

What information is available?

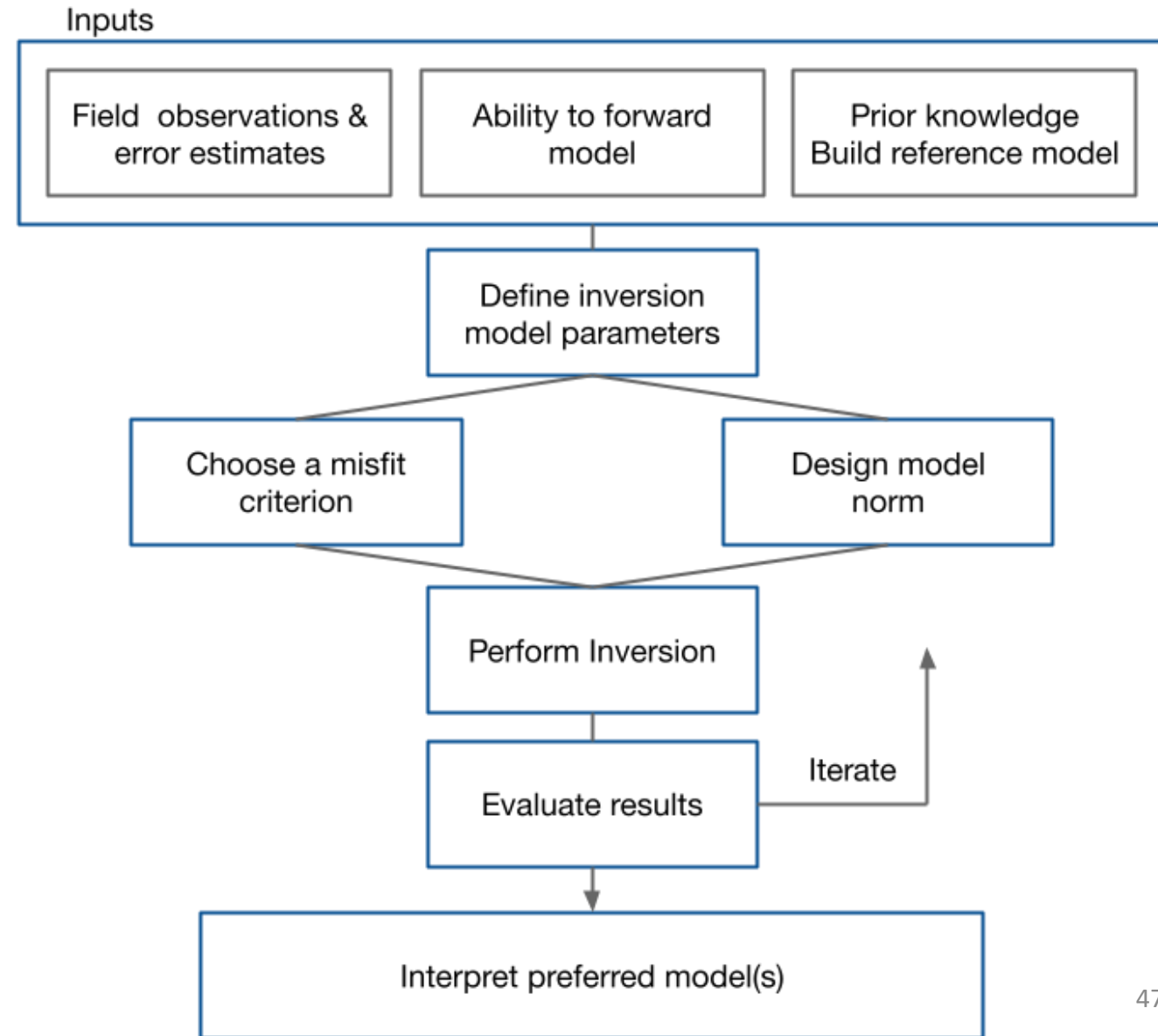
- Geologic structure
- Geologic constraints
- Reference model
- Bounds
- Multiple data sets
- Physical property measurements



How do we achieve our goal?

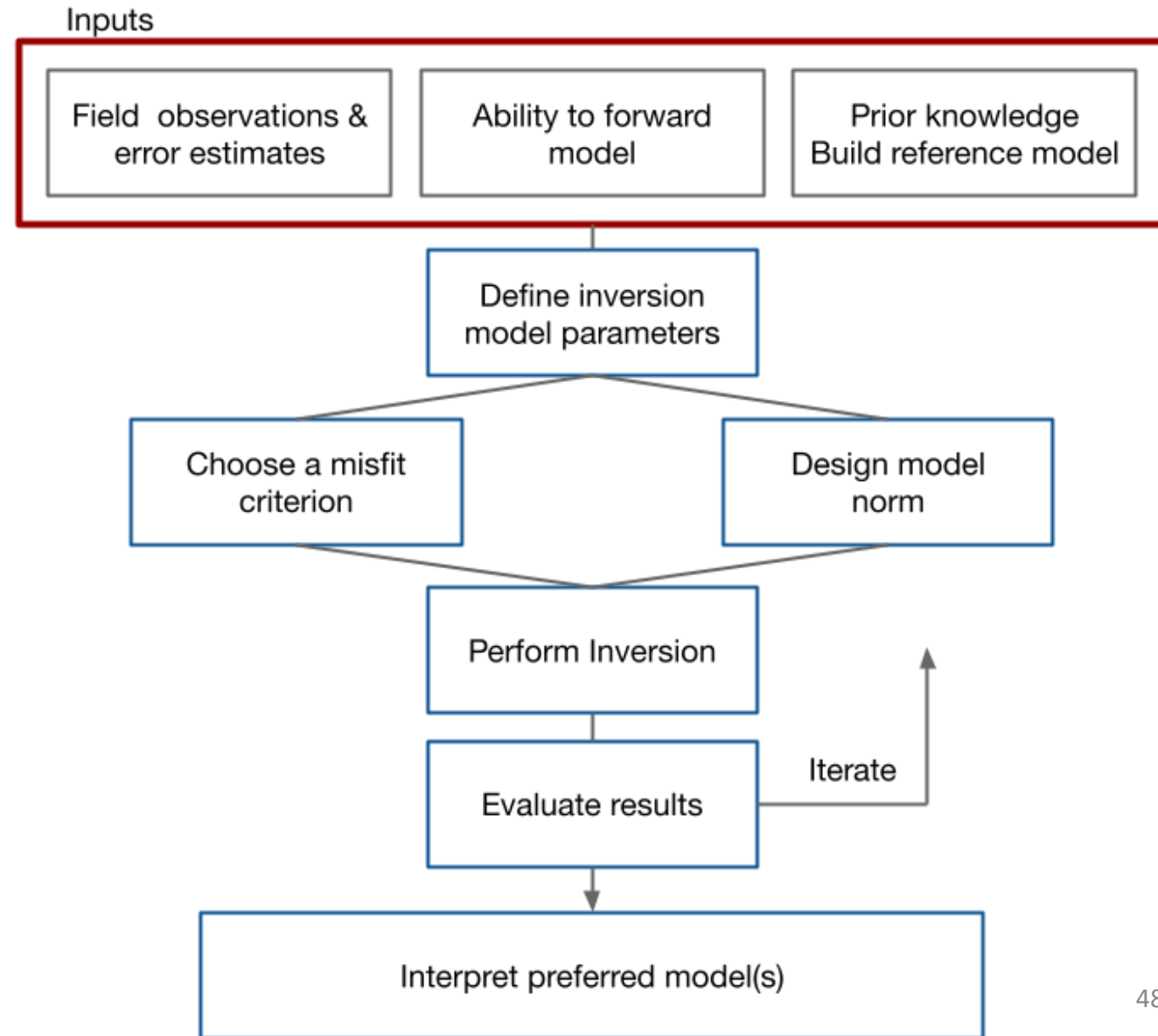
Flow chart for the inverse problem

- Many components to achieving a quality result
- Success is in the details
- Evaluate each step in the box critically before going on



Starting up

- Survey and observations
- What processing has been done?
- Normalization of data
- Ability for forward model
- Assemble geologic, petrophysical information
- Build a reference model
- What is the question you want answered from the inversion?



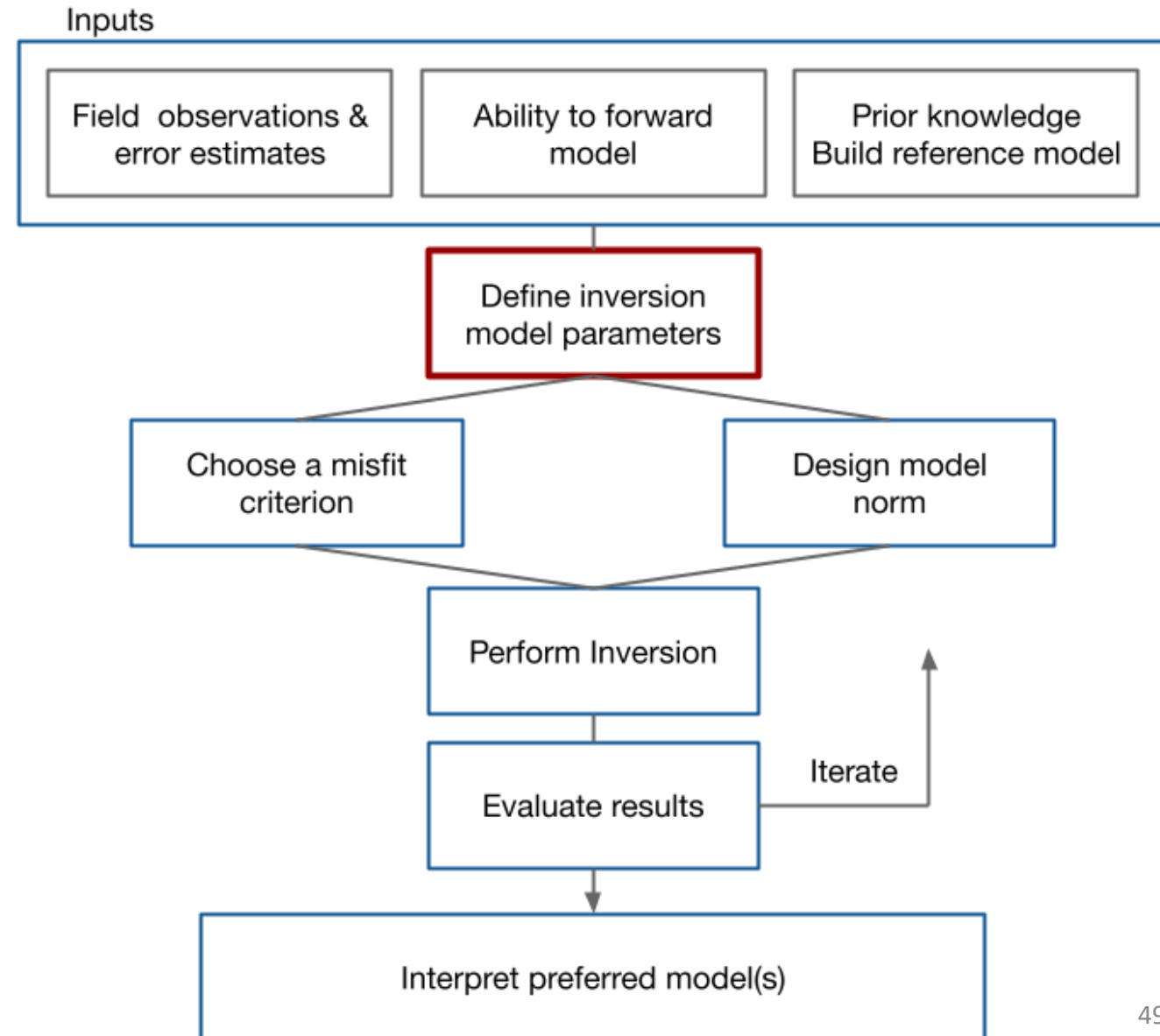
Inversion model parameters

- In the forward problem

$$d = \mathcal{F}[m]$$

m is our sought function
(susceptibility, density,)

- Inverse problem: we have options
(e.g., subsurface, parametric)



Inversion as an optimization problem

- Find a single “best” solution by solving optimization

$$\text{minimize} \quad \phi = \phi_d + \beta \phi_m$$

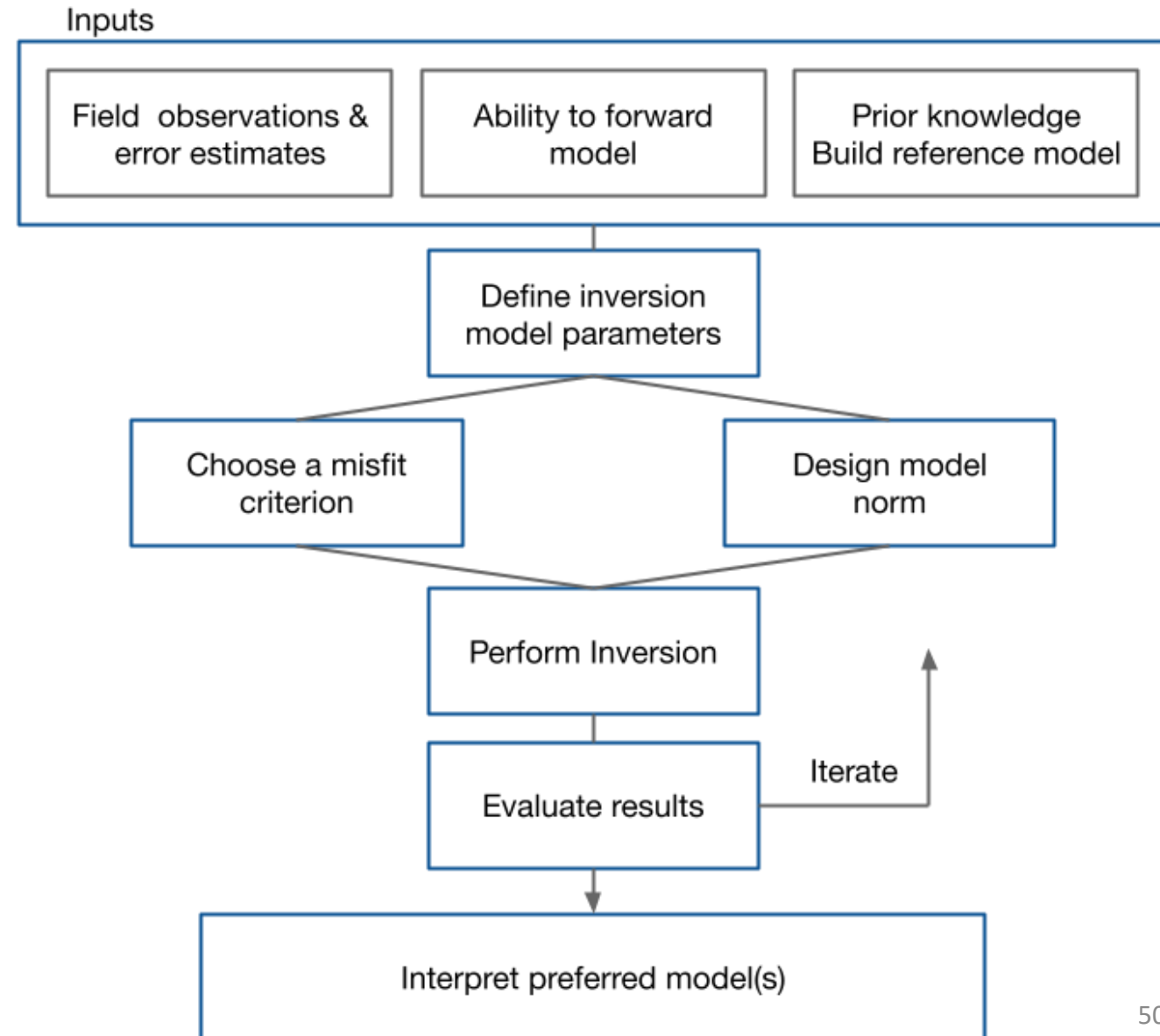
$$\text{subject to} \quad m_L \leq m \leq m_U$$

ϕ_d : data misfit

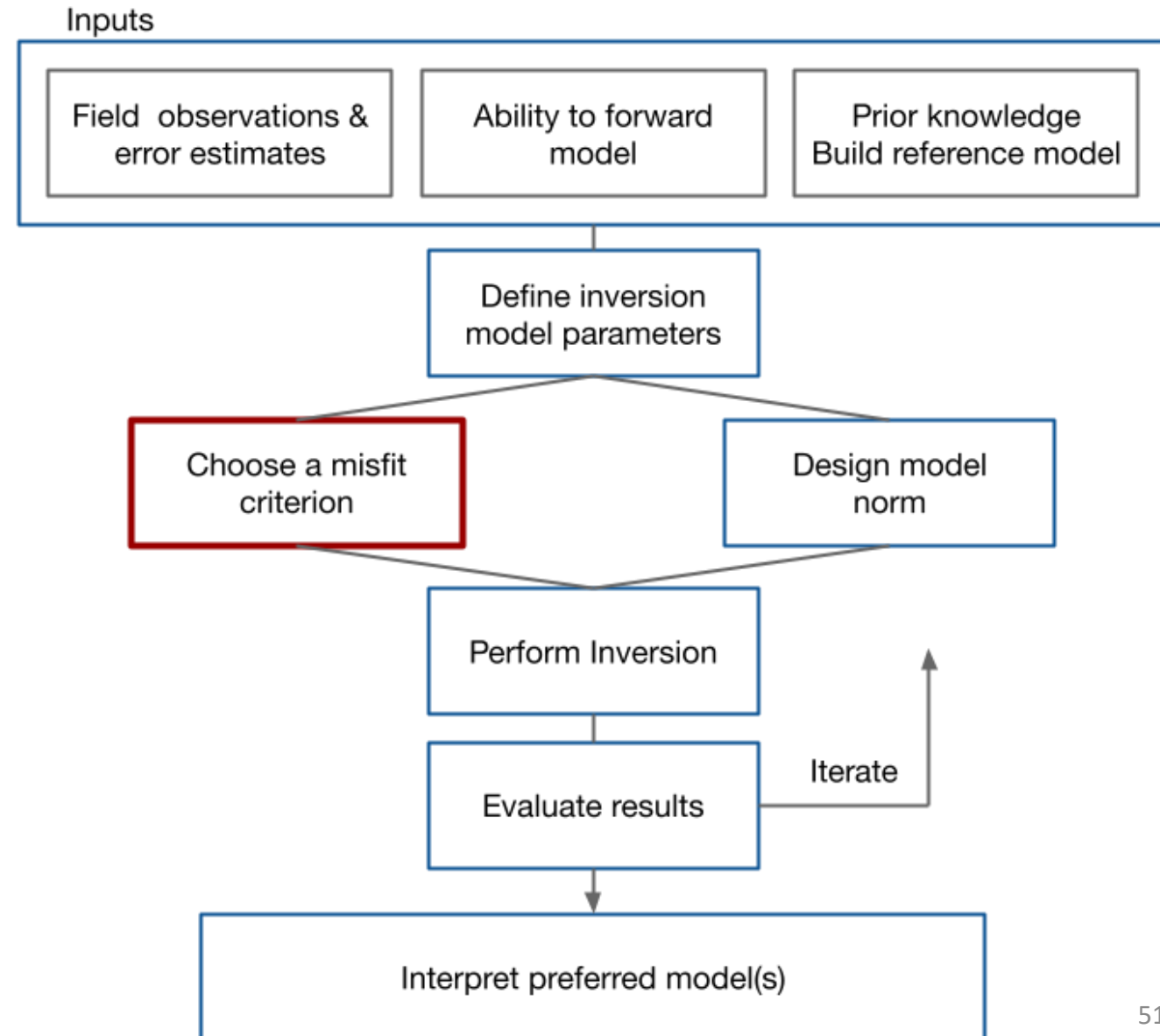
ϕ_m : model norm

β : trade-off parameter

m_L, m_U : lower and upper bounds



Flow chart for the Inverse problem



Dealing with uncertainties

Observed datum

$$d_j^{obs} = F_j(m) + n_j$$

Noise n_j includes

- Modelling errors
 - dimensionality errors (1D v. 3D)
 - incomplete physics
 - discretization errors
- Noise on data
 - instrument / sensor noise
 - survey parameter errors
 - wind ...

True statistics of “noise” is complicated.
In practice, assume errors are Gaussian

$$\mathcal{N}(0, \epsilon_j)$$

Dealing with uncertainties

Consider random variable, $x_j \in \mathcal{N}(0, 1)$

Define $\chi_N^2 = \sum_{j=1}^N x_j^2$ Chi-squared statistic with N degrees of freedom

$$\left\{ \begin{array}{l} \text{Expected value: } E(\chi_N^2) = N \\ \text{Variance: } \text{Var}(\chi_N^2) = 2N \\ \text{Standard deviation: } \text{std}(\chi_N^2) = \sqrt{2N} \end{array} \right.$$

Misfit function

Crucial steps for any misfit:

- (1) Specify the metric used
- (2) Determine target misfit

We use L_2 norm (least squares statistic)

Define data misfit: $\phi_d = \sum_{j=1}^N \left(\frac{F_j(m) - d^{obs}}{\epsilon_j} \right)^2$

Define $\mathbf{W}_d = \mathbf{diag}(1/\epsilon_1, \dots, 1/\epsilon_N)$

$$\phi_d = ||\mathbf{W}_d(F[\mathbf{m}] - \mathbf{d}^{obs})||_2^2$$

$$E[\phi_d] \simeq N$$

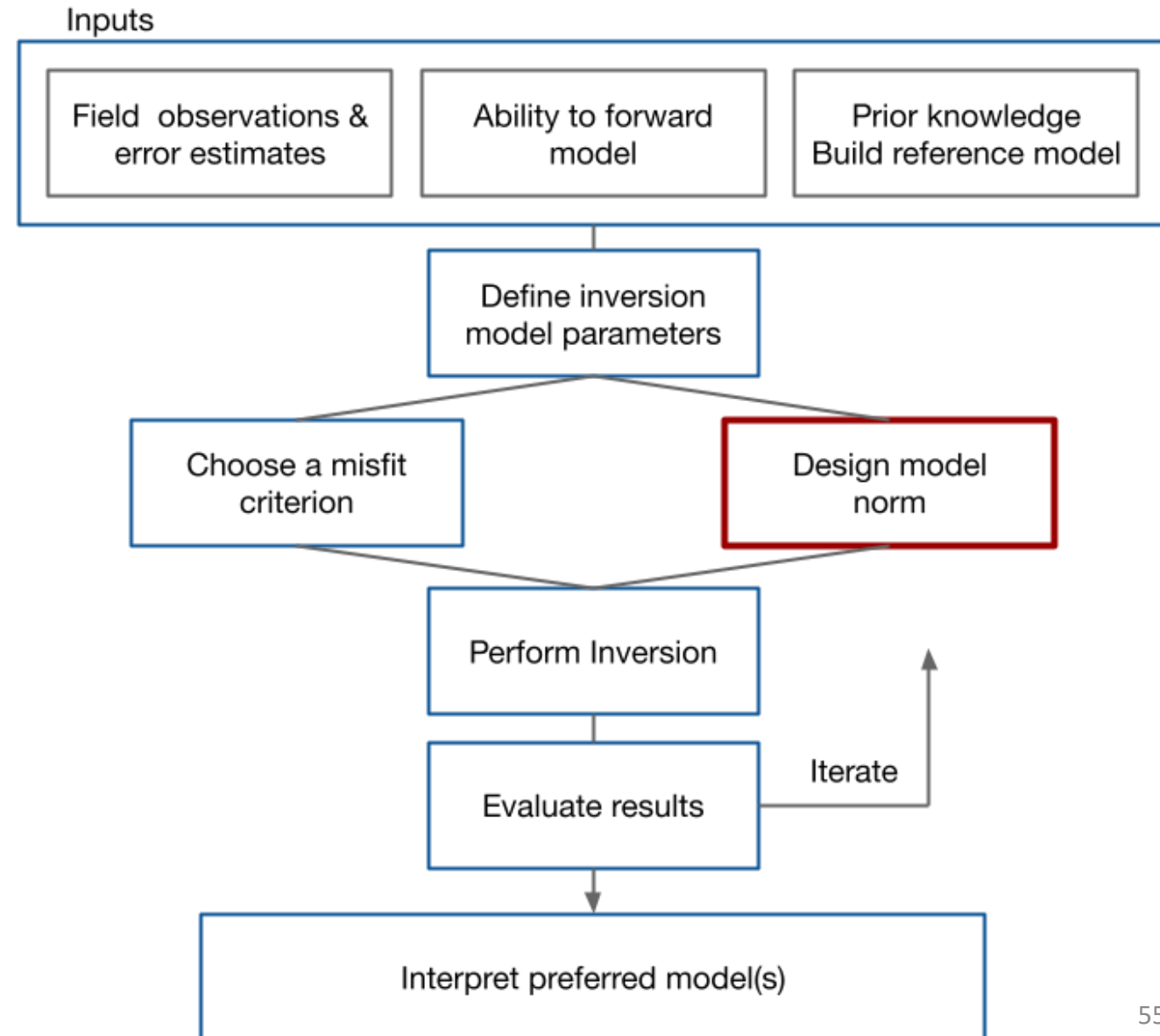
ϕ_d is now a χ^2_N variable

Reality: we do not know uncertainties

Try:

$$\epsilon_j = \%|d_j^{obs}| + \text{floor}$$

Flow chart for the Inverse problem



Model norms

First define our model norms as functions and then discretize

Smallest model:

$$\phi_m = \int (m - m_{ref})^2 dx$$

Flattest model:

$$\phi_m = \int \left(\frac{dm}{dx} \right)^2 dx$$

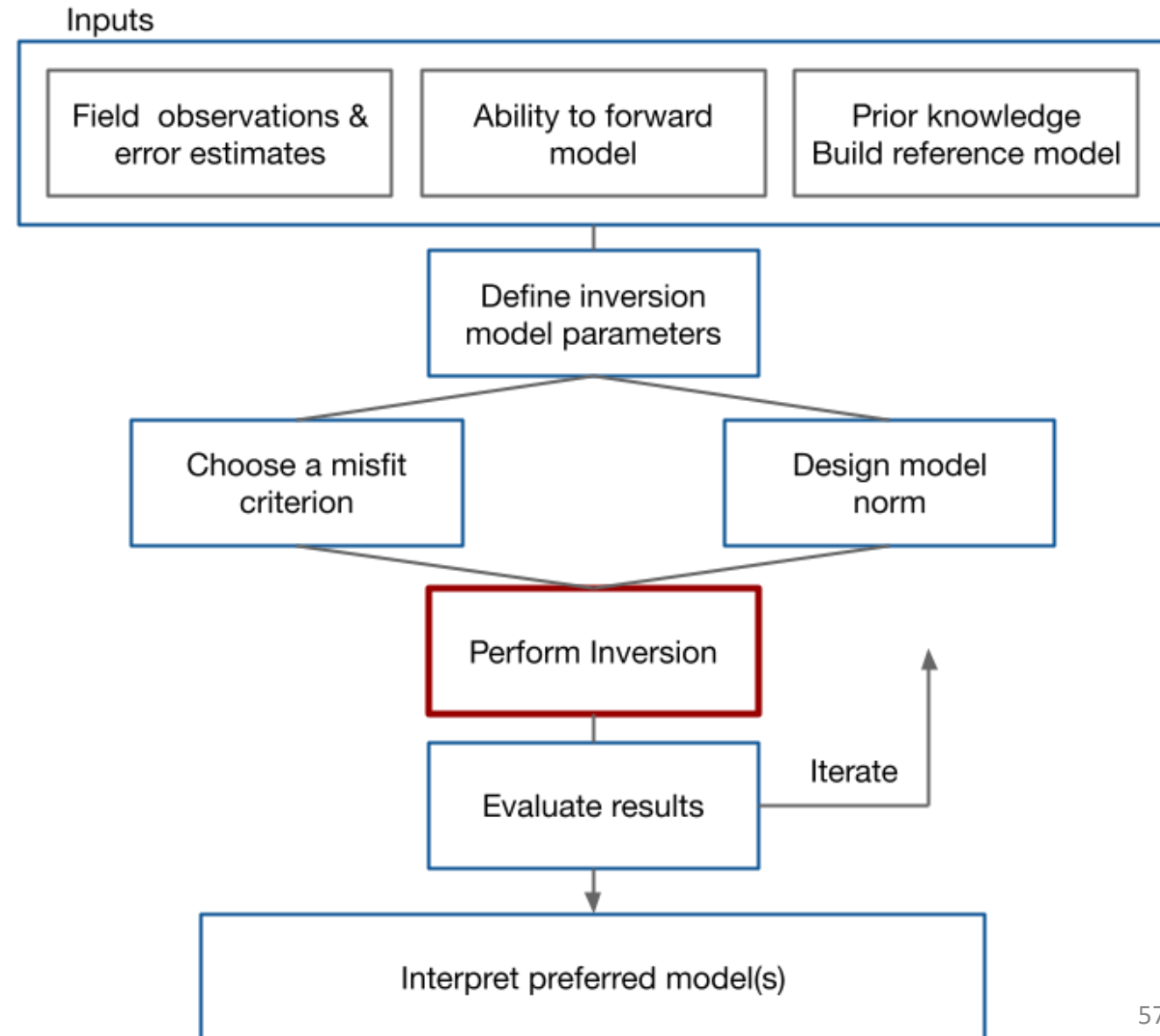
Combination:

$$\phi_m = \alpha_s \int (m - m_{ref})^2 dx + \alpha_x \int \left(\frac{dm}{dx} \right)^2 dx$$

Discretize:

$$\phi_m = \alpha_s \|\mathbf{W}_s(\mathbf{m} - \mathbf{m}_{ref})\|_2^2 + \alpha_x \|\mathbf{W}_x(\mathbf{m})\|_2^2$$

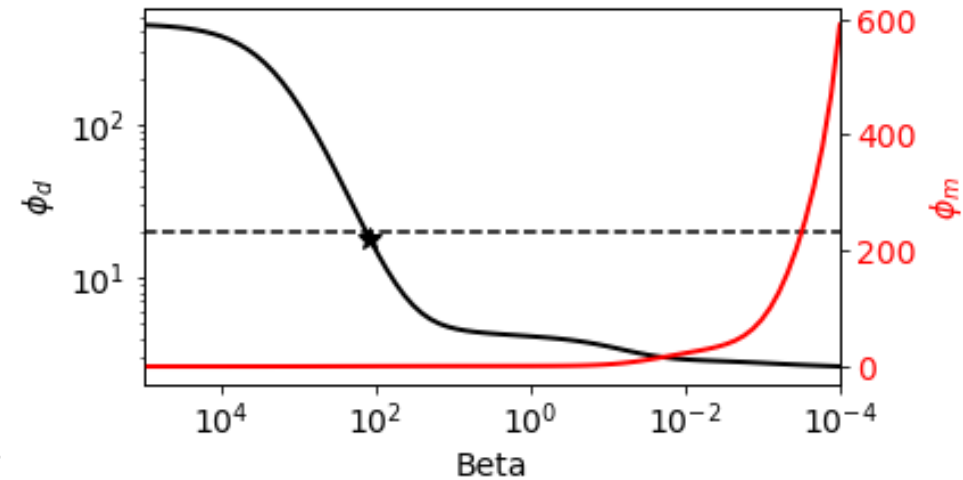
Flow chart for the Inverse problem



Role of beta

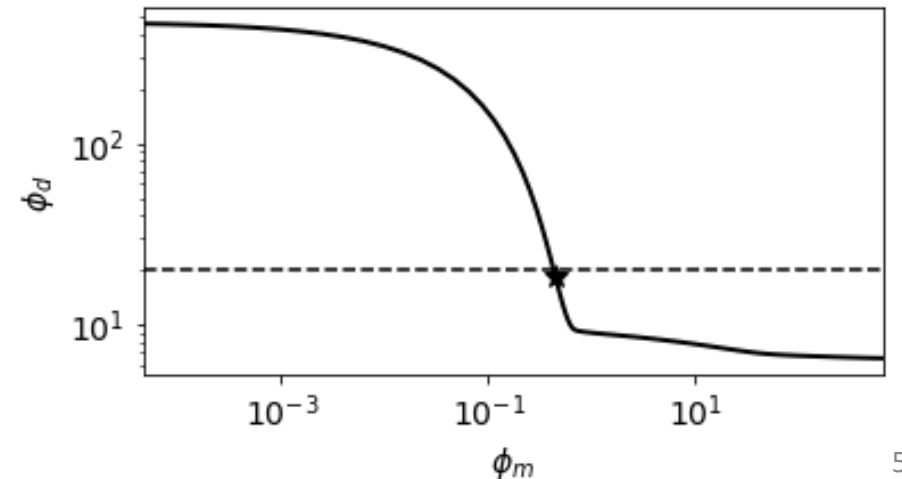
$$\phi(m) = \phi_d(m) + \beta\phi_m(m)$$

$$\begin{aligned}\beta \rightarrow 0 & : \quad \phi \sim \phi_d \\ \beta \rightarrow \infty & : \quad \phi \sim \phi_m\end{aligned}$$



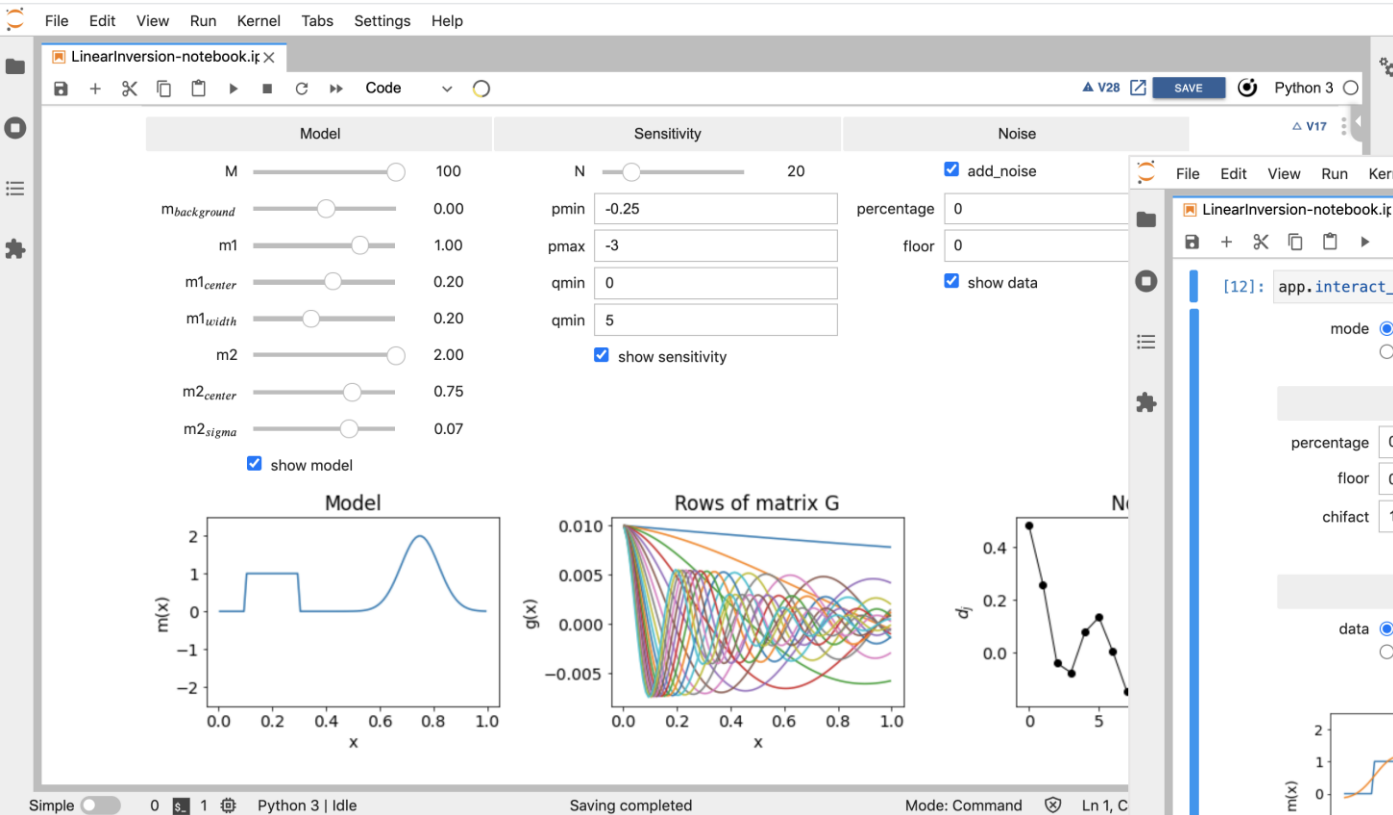
Tikhonov Curve

- Desired misfit $\phi_d^* \simeq N$
- Choose β such that $\phi_d(m) = \phi_d^*$



Demo: Linear Inversion App

Develop survey



Run inversion

