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TRANSFORM 2021

Virtual Conference on the Digital Subsurface, 16–23 April

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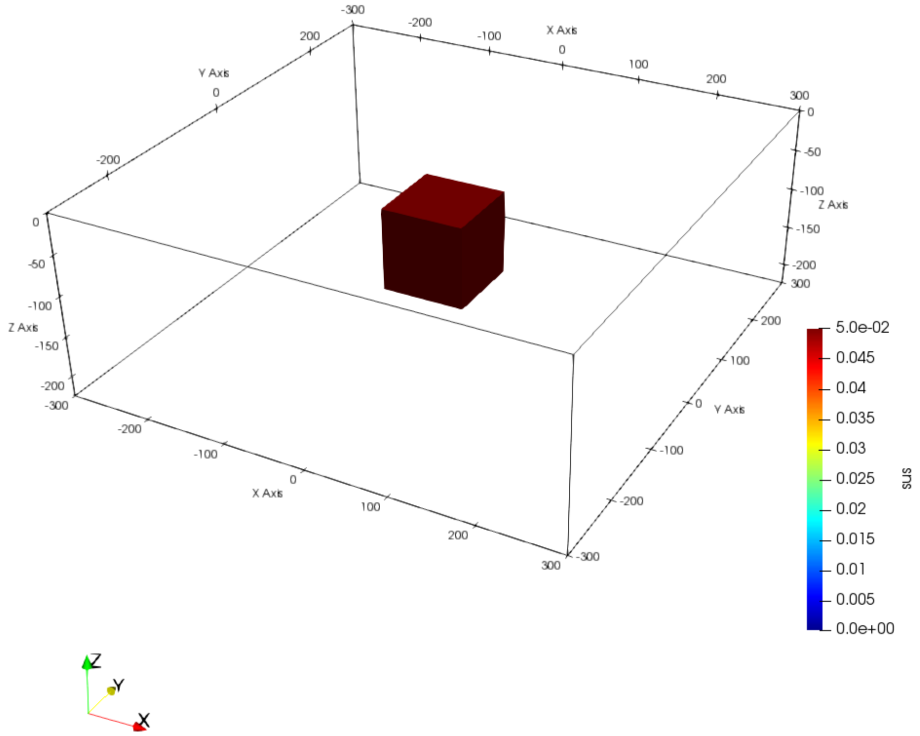
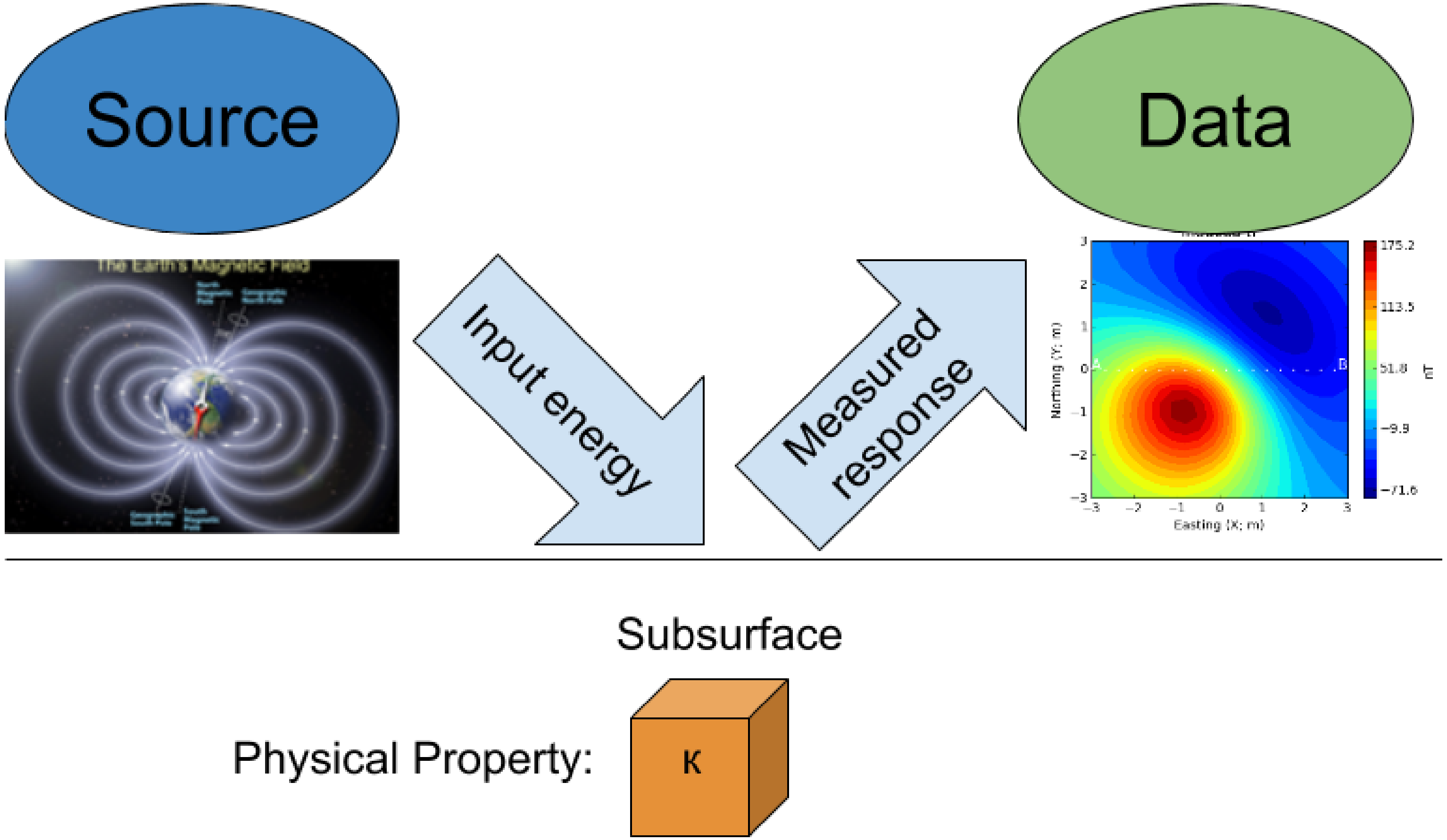
Outline

- Backgrounds: Magnetics
- Inversion Framework
- 1D Linear Inverse problem
- 3D Magnetic Inversion
- Including Geologic Information
- Summary

3D magnetic inversion

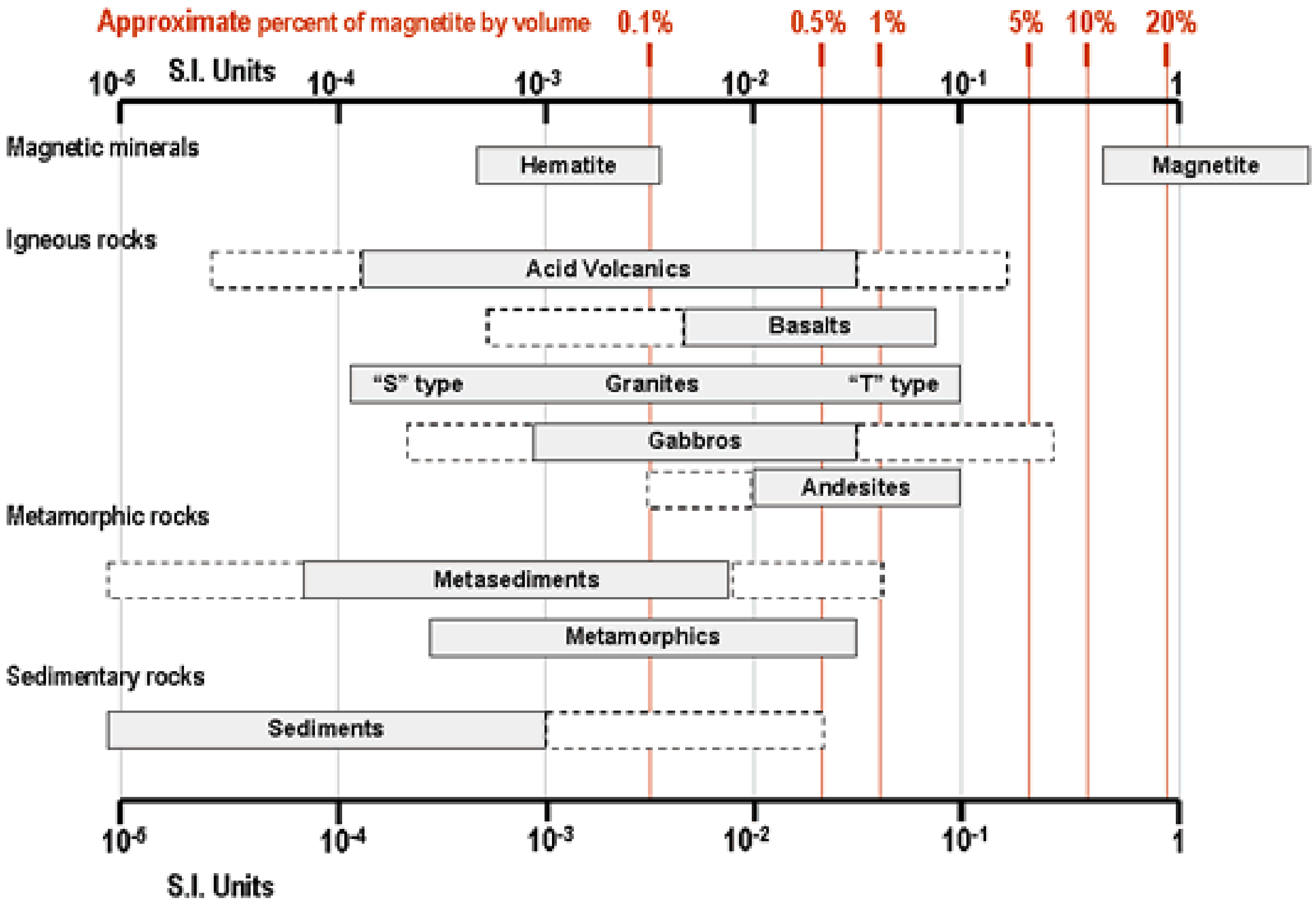


Survey: Magnetics



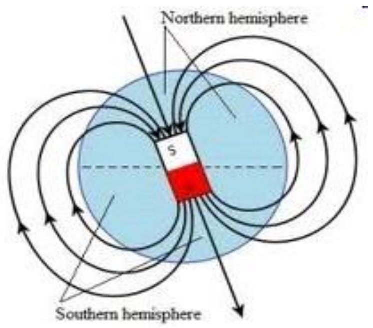
κ: Magnetic susceptibility

Magnetic susceptibility



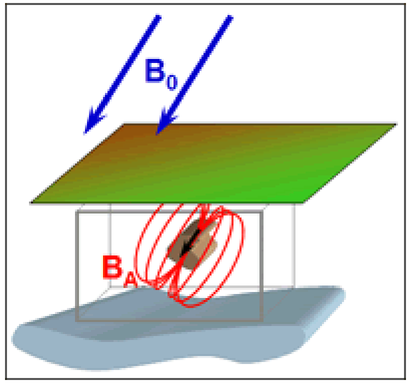
Adopted from Clark and Emerson, Exploration Geophysics (1991)

Magnetic surveying



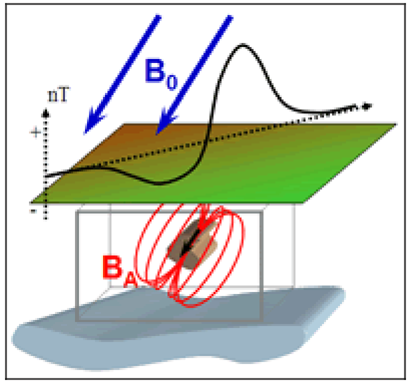
- Earth's magnetic field $\vec{B}_0$ is the source:
- Materials become magnetized

$$\vec{M} = \kappa \vec{H}_0 \text{ (magnetization)}$$
$$\vec{H}_0 = \vec{B}_0 / \mu_0$$



- Create anomalous magnetic field
- Measure total magnetic field: $|\vec{B}| = |\vec{B}_0 + \vec{B}_A|$
- Total field anomaly: $\triangle \vec{B} = |\vec{B}_0 + \vec{B}_A| - |\vec{B}_0|$

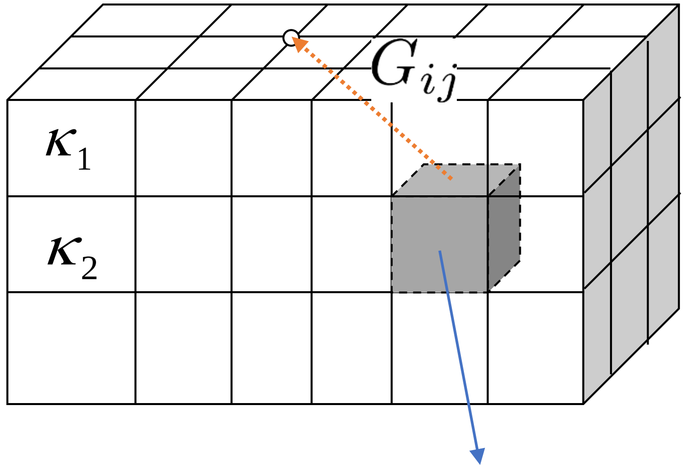
$$\triangle \vec{B} \simeq \vec{B}_A \cdot \hat{B}_0 \qquad \text{where} \quad \hat{B}_0 = \frac{\vec{B}_0}{|\vec{B}_0|}$$



Forward modelling

- Discretize earth

$$\kappa_j \ (j = 1, \ldots, M)$$ susceptibility



V_j: j-th volume

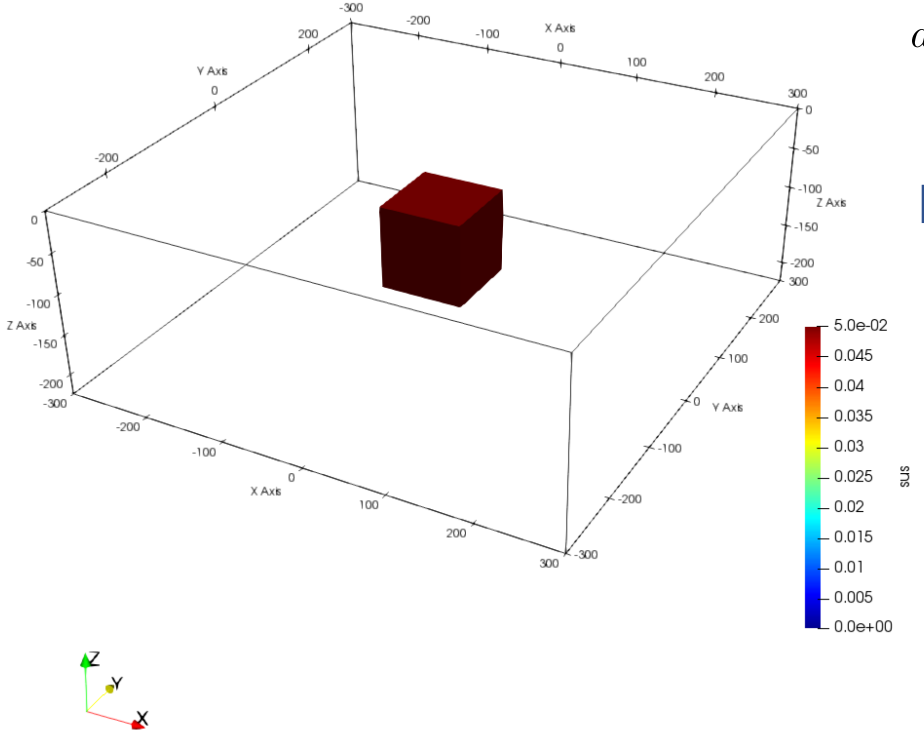
- Magnetic anomaly data are

$$d_i = \sum_{j=1}^{M} G_{ij} \kappa_j$$

$$\begin{cases} G_{ij} = \hat{B}_0 \cdot \left\{ \dfrac{\mu_0}{4\pi} \displaystyle\int_v \kappa \nabla\nabla \left(\dfrac{1}{r_i - r_j} \right) \, dV_j \right\} \cdot \hat{B}_0 \\ \hat{B}_0 = \dfrac{\vec{B}_0}{|\vec{B}_0|} \end{cases}$$

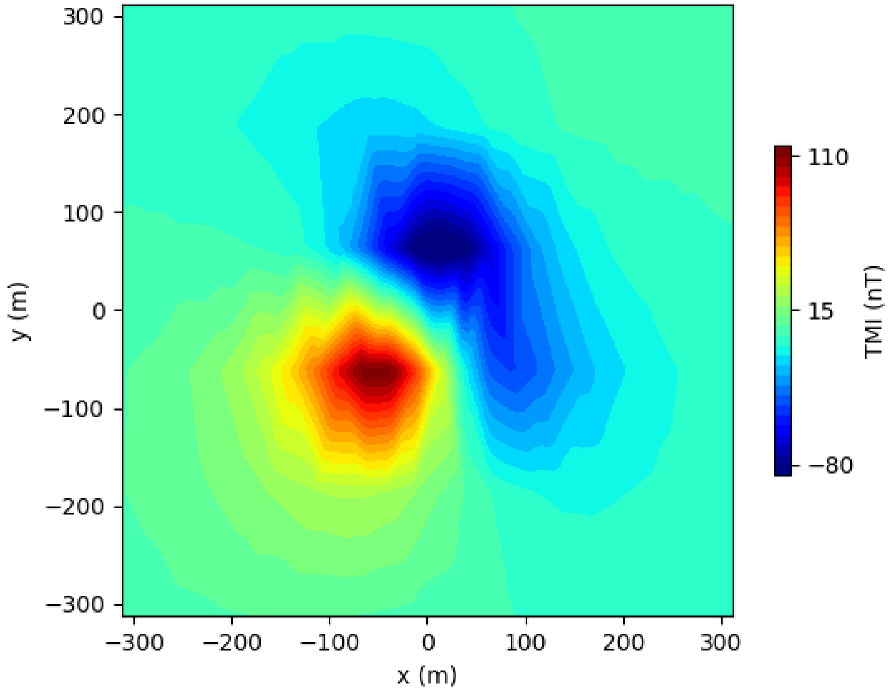
Forward modelling

Susceptibility model

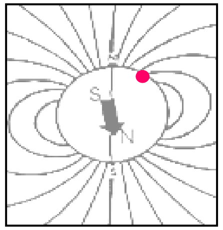


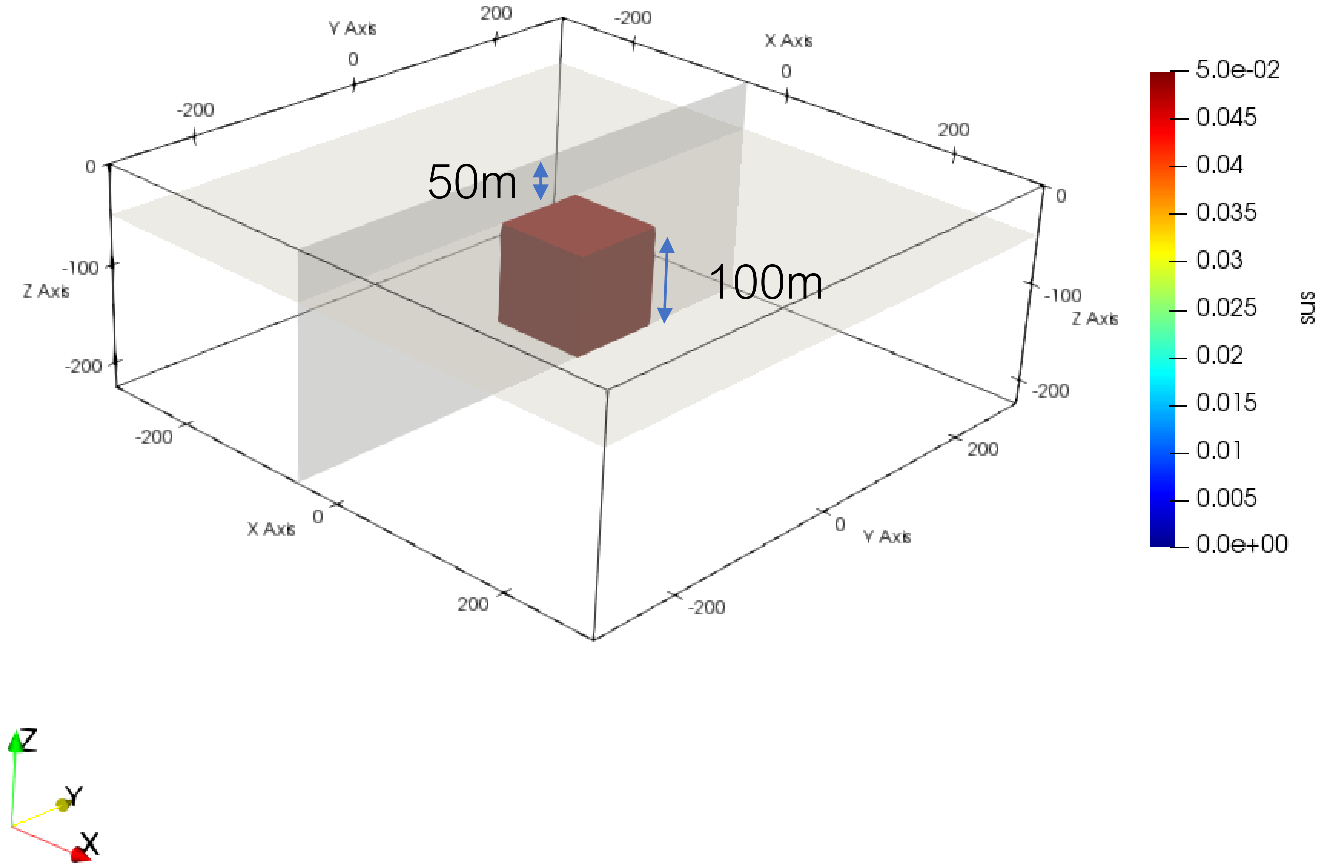
$$d_i = \sum_{j=1}^{M} G_{ij}\kappa_j$$

Magnetic data

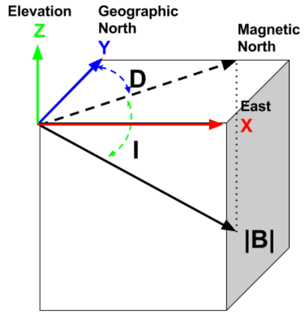


Synthetic susceptibility model





- Earth field
  - Inclination: 30°
  - Declination: 45 °
  - $|B_0|$ = 50,000 nT

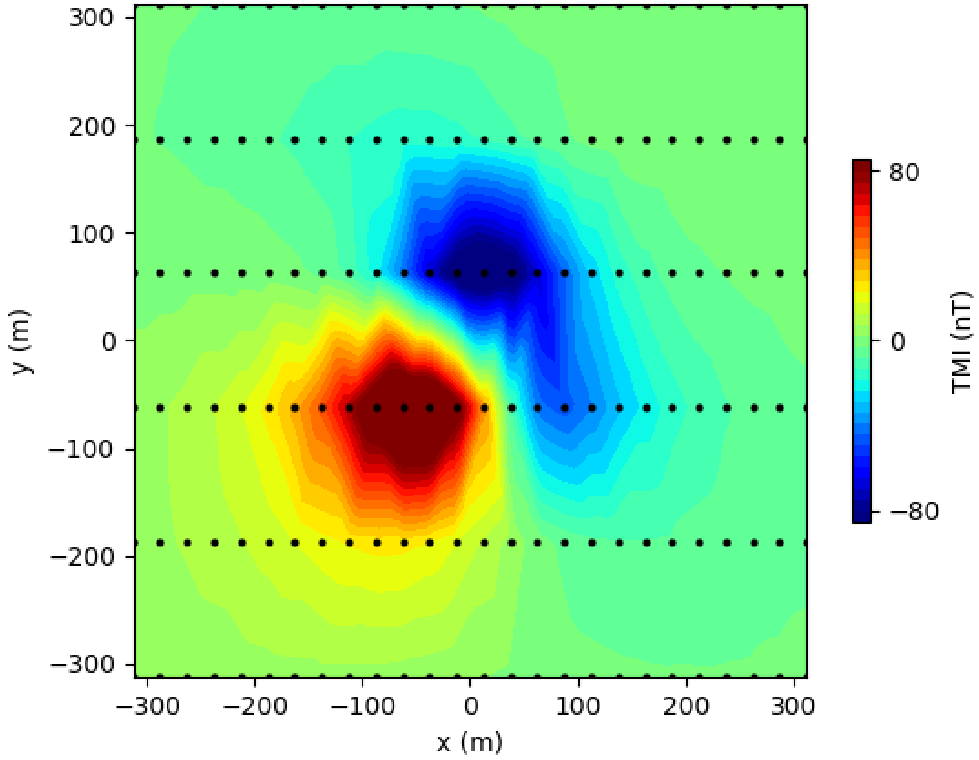


- Susceptible block
  - 100m x 100m x 100m block
  - Block susceptibility = 0.5
  - Block top = 50m

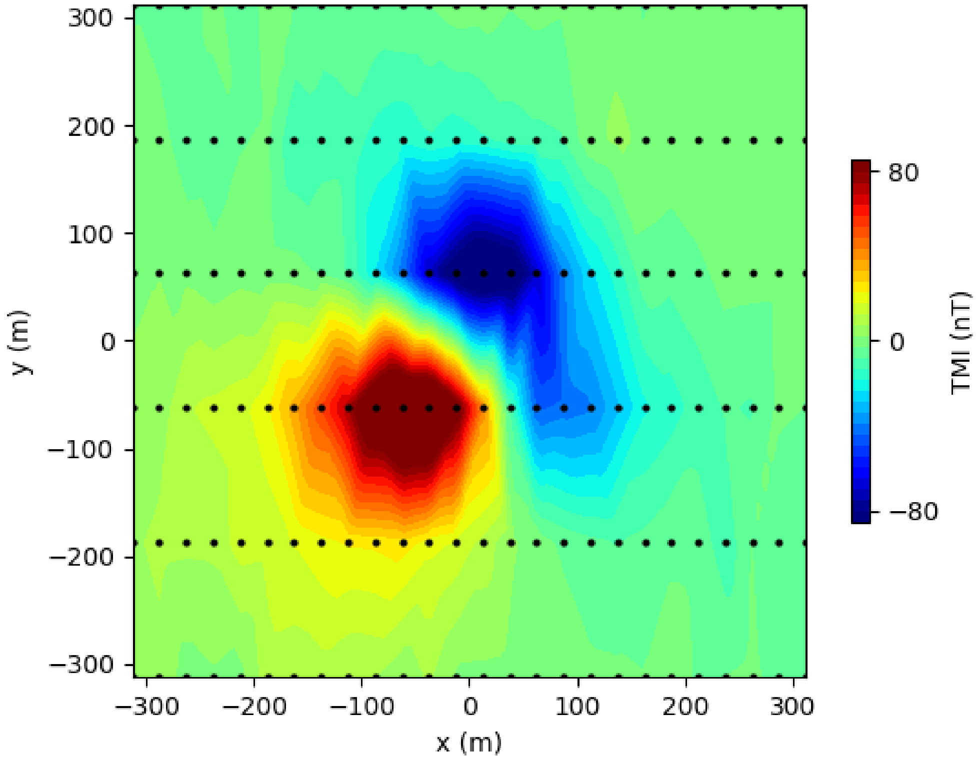
Synthetic survey

Survey parameters:
- 100 m line spacing.
- 25 station spacing.
- N=156 (elevation= 2m)

Contoured “perfect” data



Contoured data with noise = 2 nT



Solving inverse problem

Model objective function

$$\phi_m = \alpha_s \int_v w_s \Big(\kappa - \kappa_{\text{ref}}\Big)^2 dv + \alpha_x \int_v \Big(\frac{d\kappa}{dx}\Big)^2 dx + \alpha_y \int_v \Big(\frac{d\kappa}{dy}\Big)^2 dy + \alpha_z \int_v \Big(\frac{d\kappa}{dz}\Big)^2 dz$$

Data misfit $\phi_d = \sum_{j=1}^{N} \Big(\frac{G_{ij}\kappa_j - d_j^{obs}}{\epsilon_j}\Big)$

Choose $\kappa_{ref} = 0, \alpha_s = 0.0001, \alpha_x = \alpha_y = \alpha_z = 1$

$L_x = \sqrt{\frac{\alpha_x}{\alpha_s}} = 100$

The Inverse problem is:

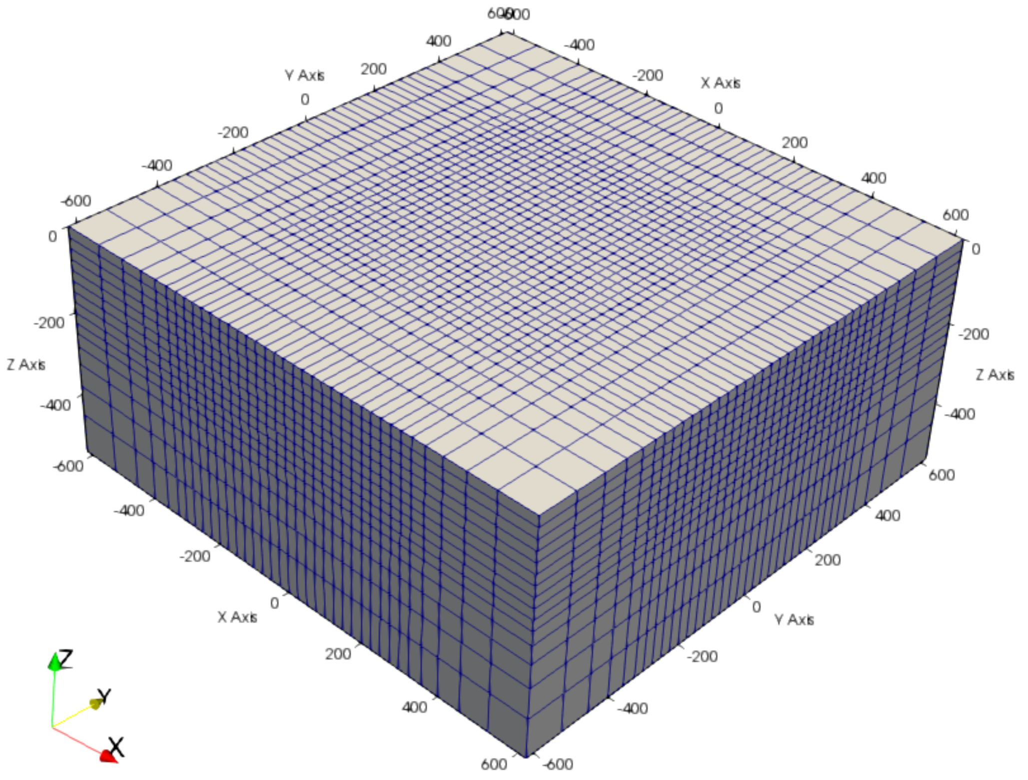
$$\text{minimize} \quad \phi = \phi_d + \beta\phi_m$$

$$\text{find } \beta \text{ such that } \phi_d = \phi_d^* \text{ where } \phi_d = N$$

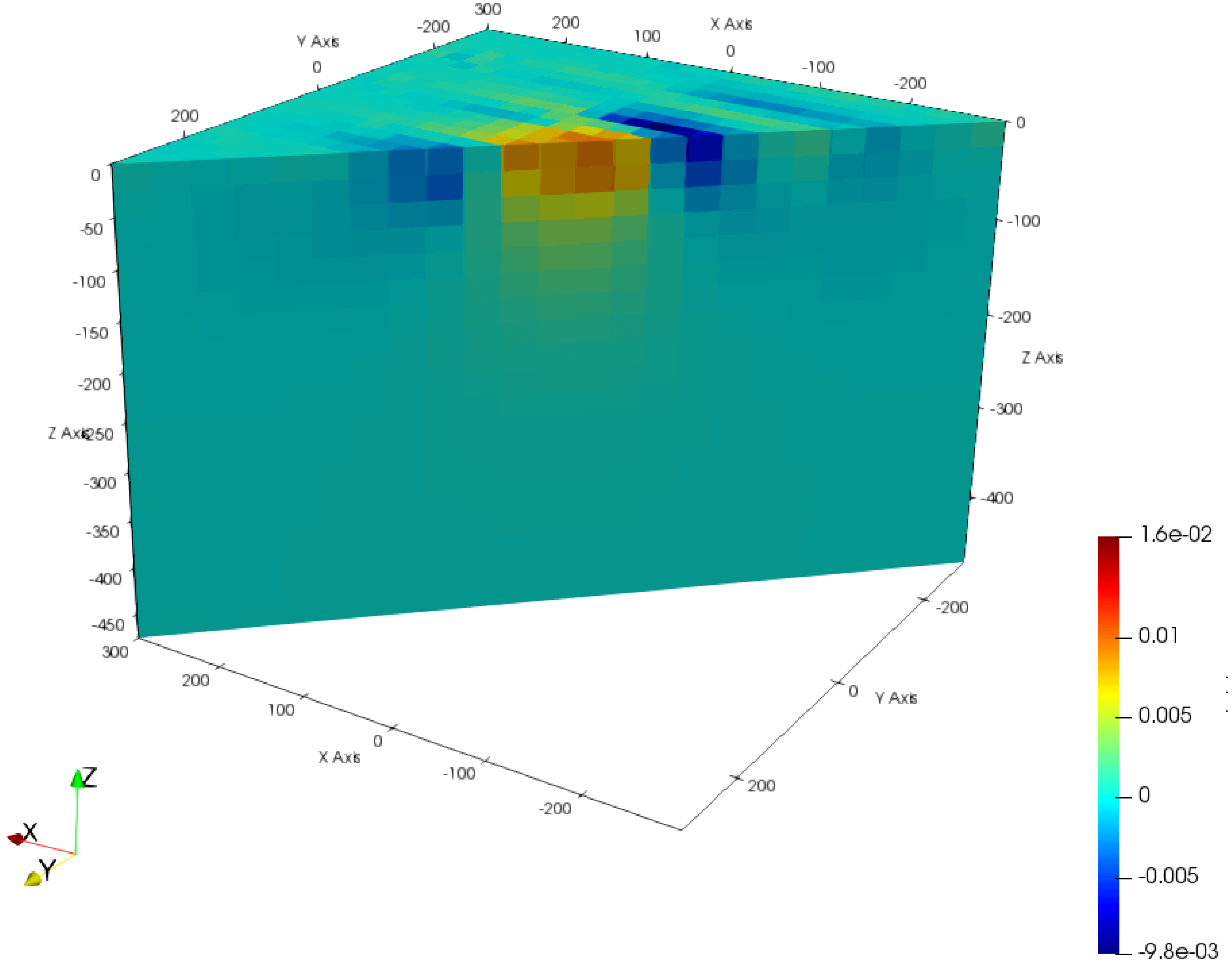
Discretization

- Earth model for inversion:
  - dx=dy=dz=25m
  - N/S and E/W padding = 300m
  - Number of cells (M) = 19440

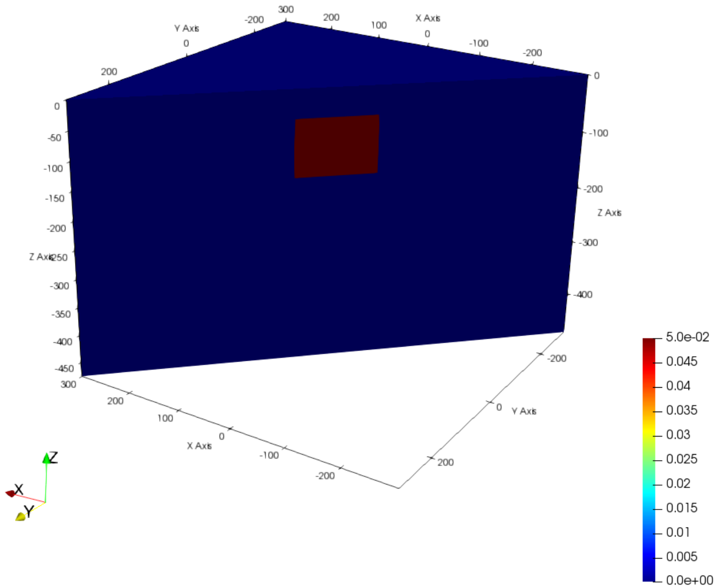
- Therefore:
  - No. data is N = 176
  - No. unknowns is M = 11,492

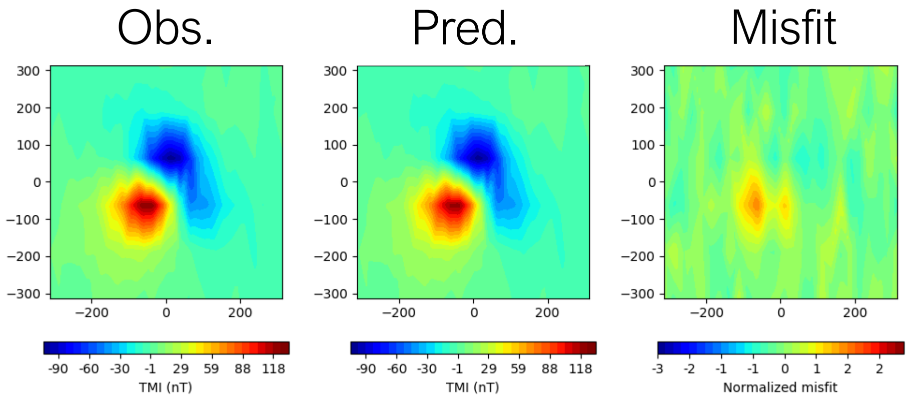


Inversion results

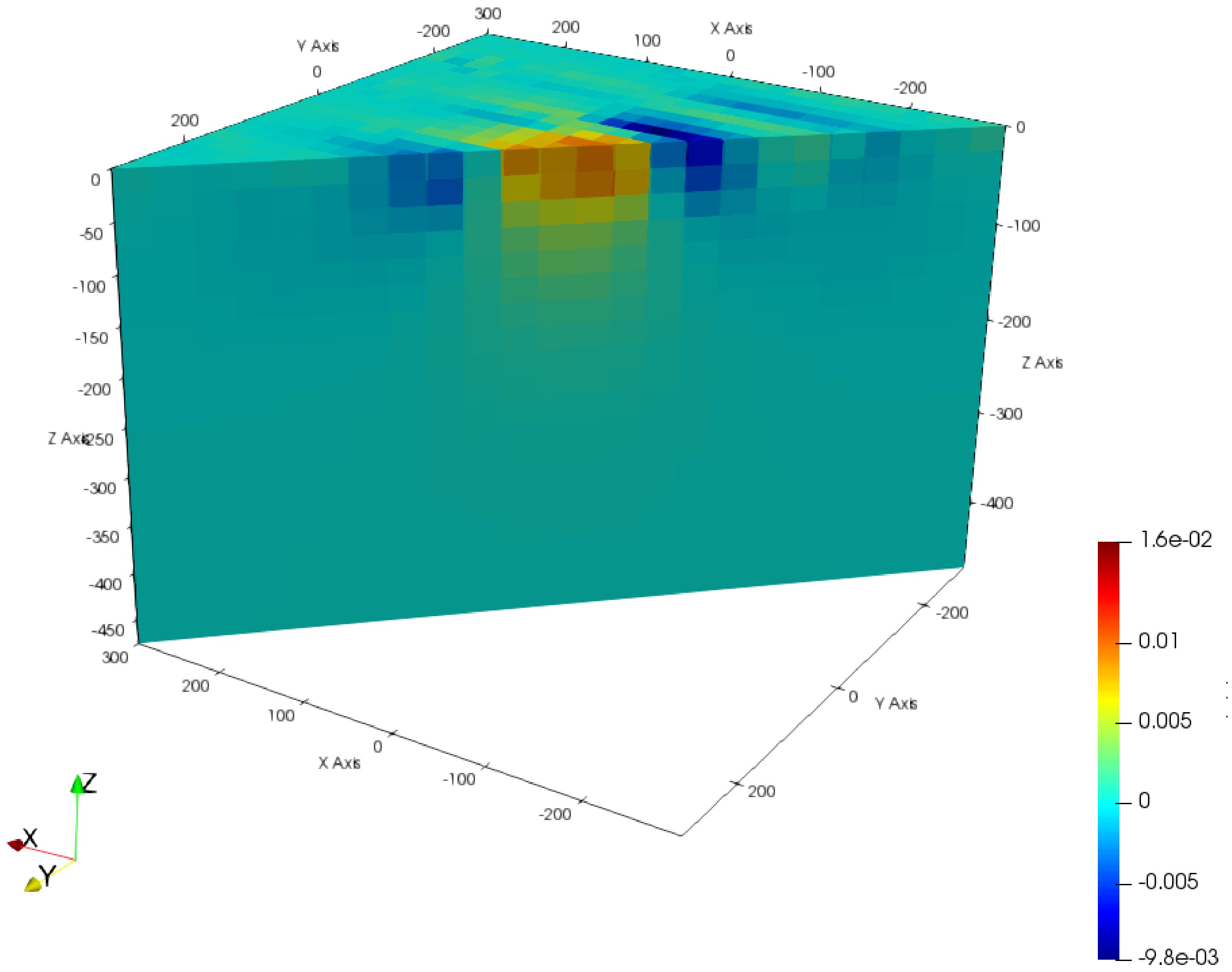


True model





Inversion results



Two primary difficulties:

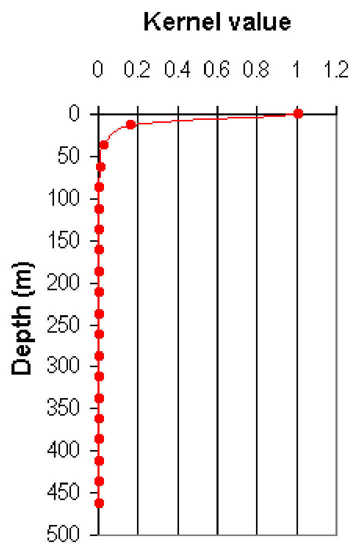
- Concentration of susceptibility is near the surface
- Regions of negative susceptibility

What went wrong?

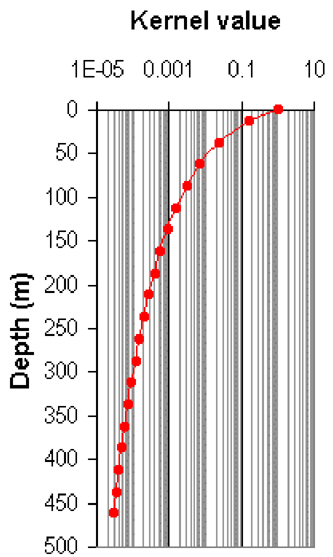
Fundamental non-uniqueness of all potential fields:

- As a consequence of Green's third identity ...
  - an observed magnetic field can be caused by a thin layer of susceptible material at any arbitrary depth

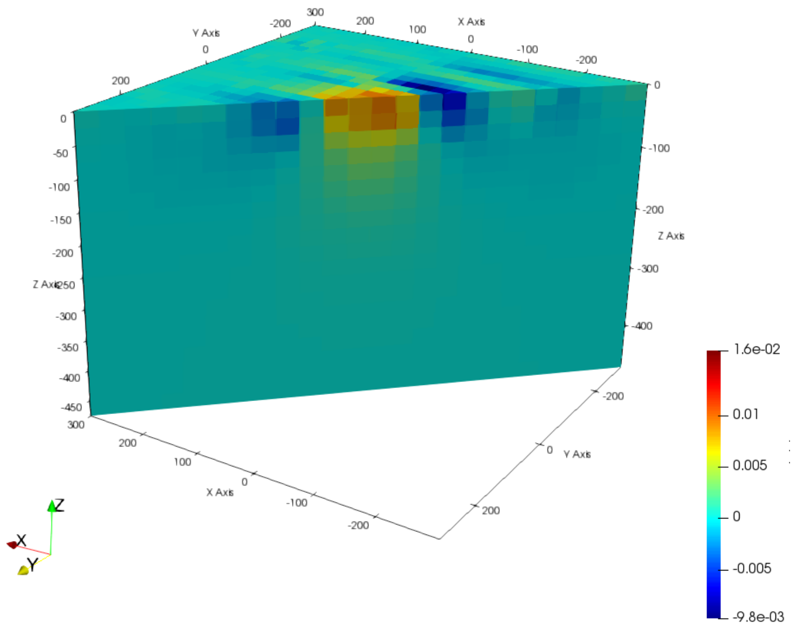
- The rapid decay of our kernels causes a concentration of κ near the surface to be a preferred solution.



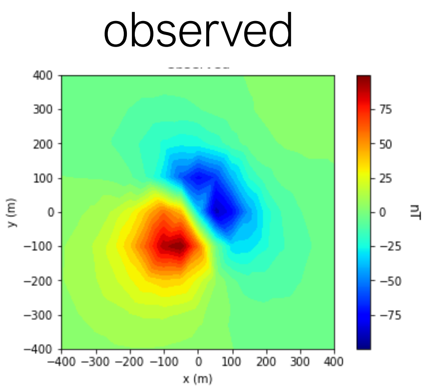
linear

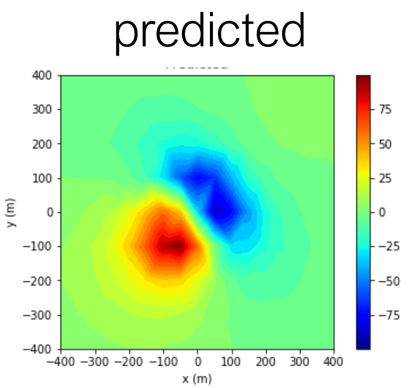


log

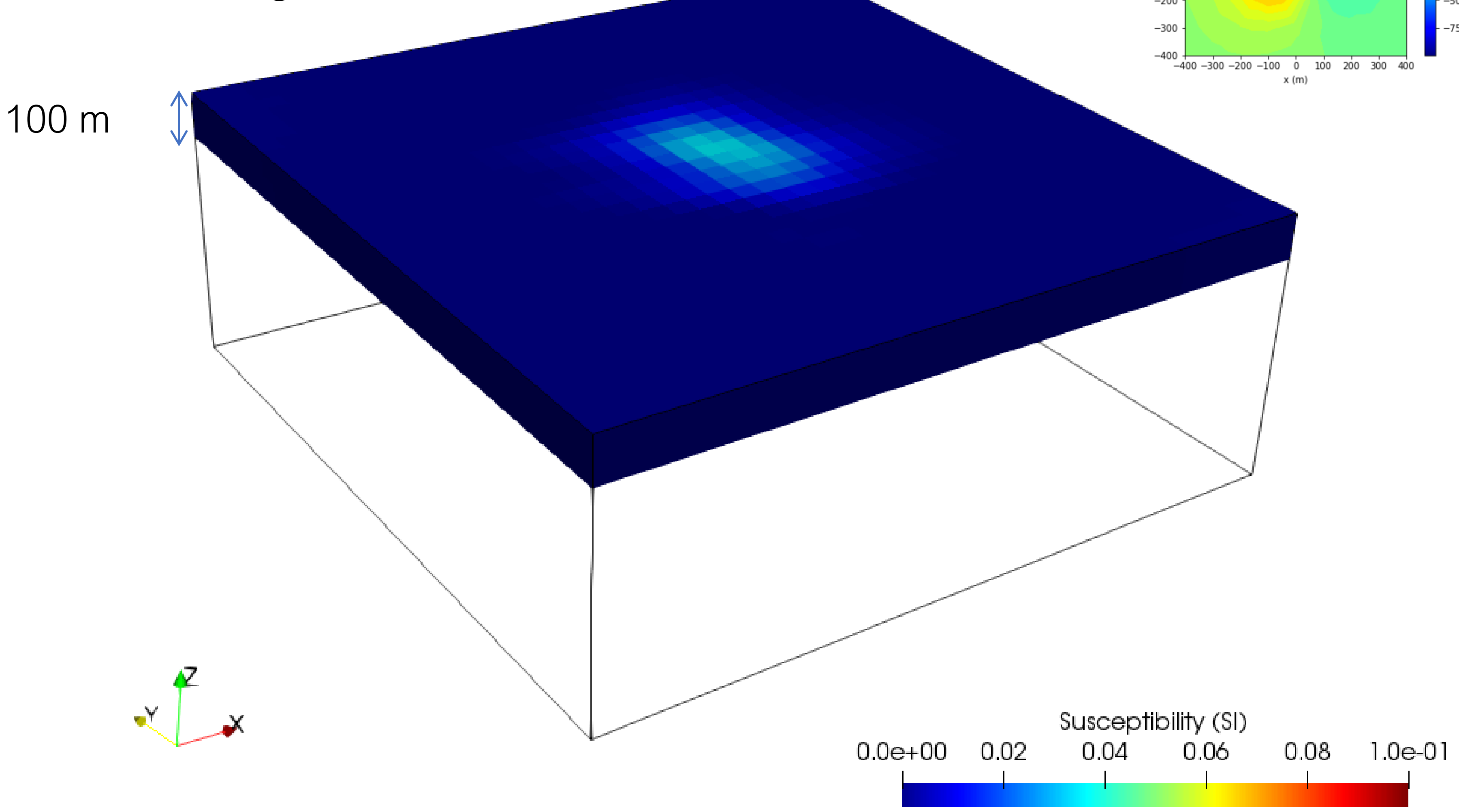


Example of extreme non-uniqueness

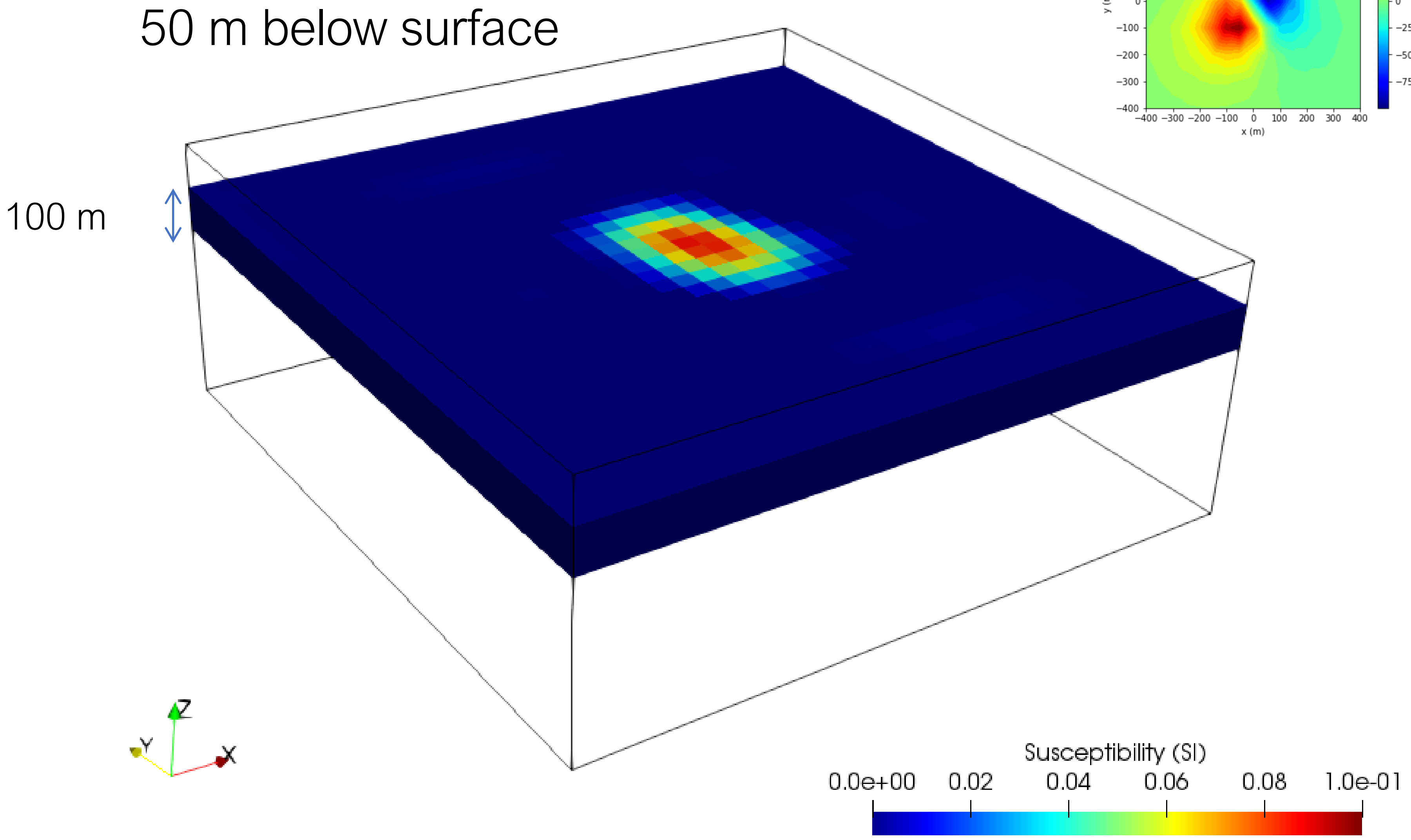


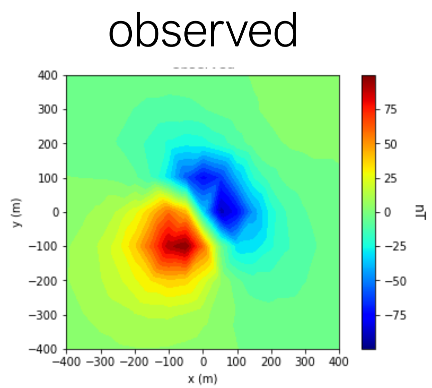


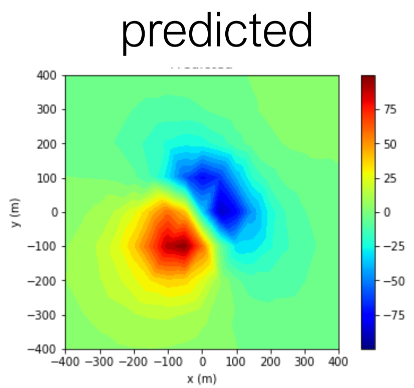
At the ground surface



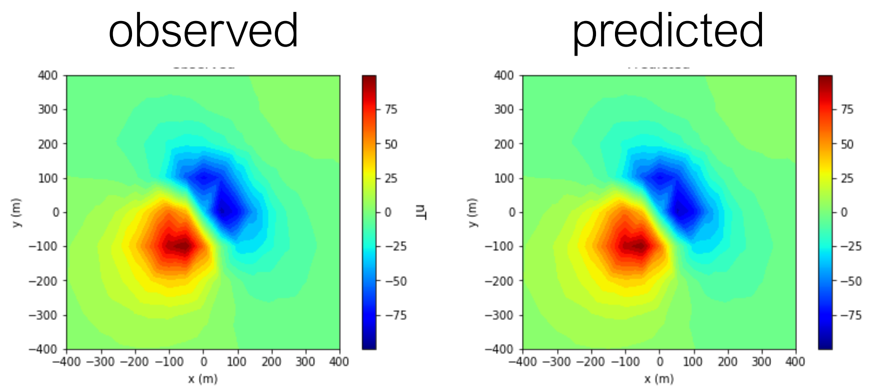
Example of extreme non-uniqueness



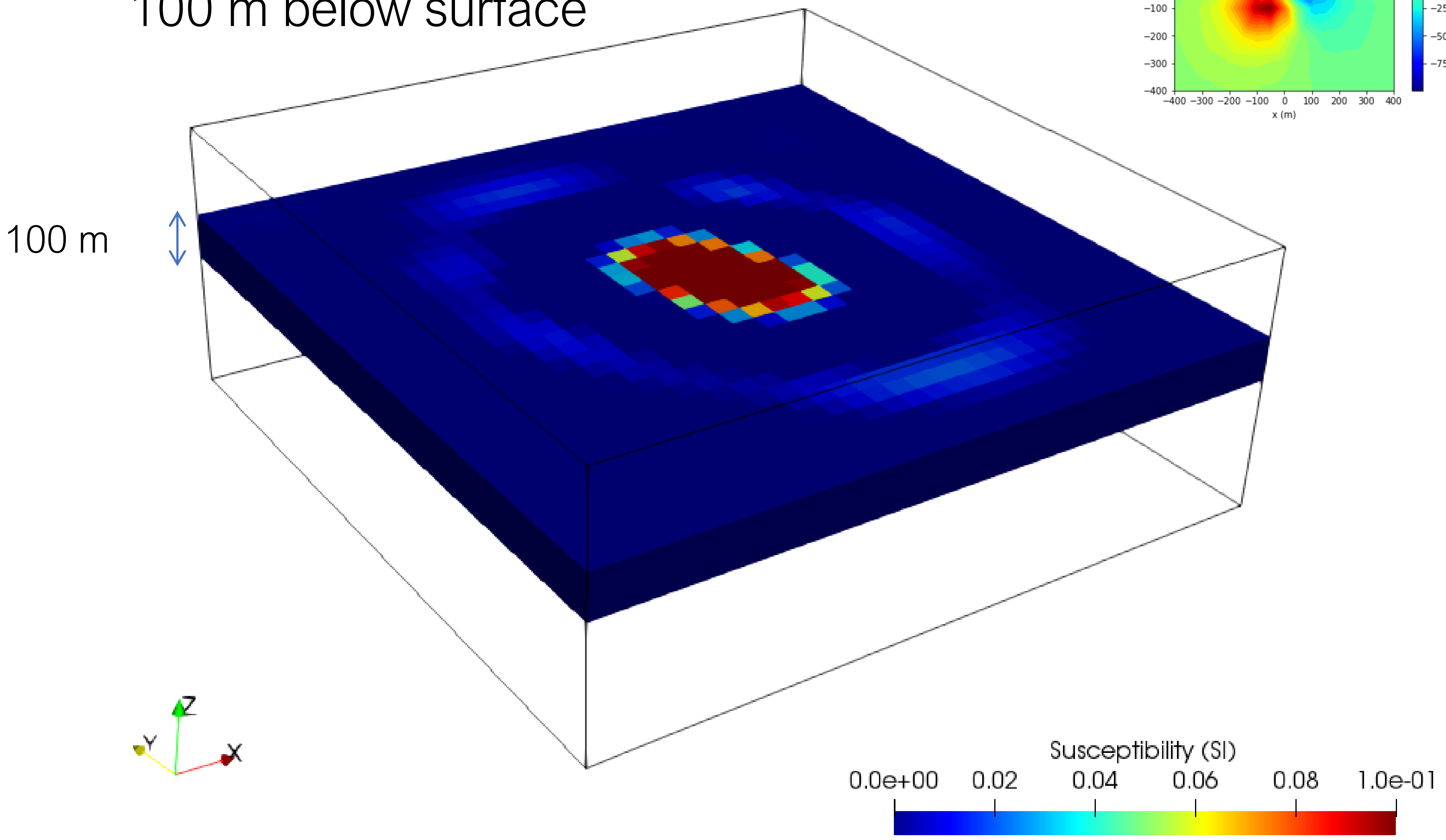




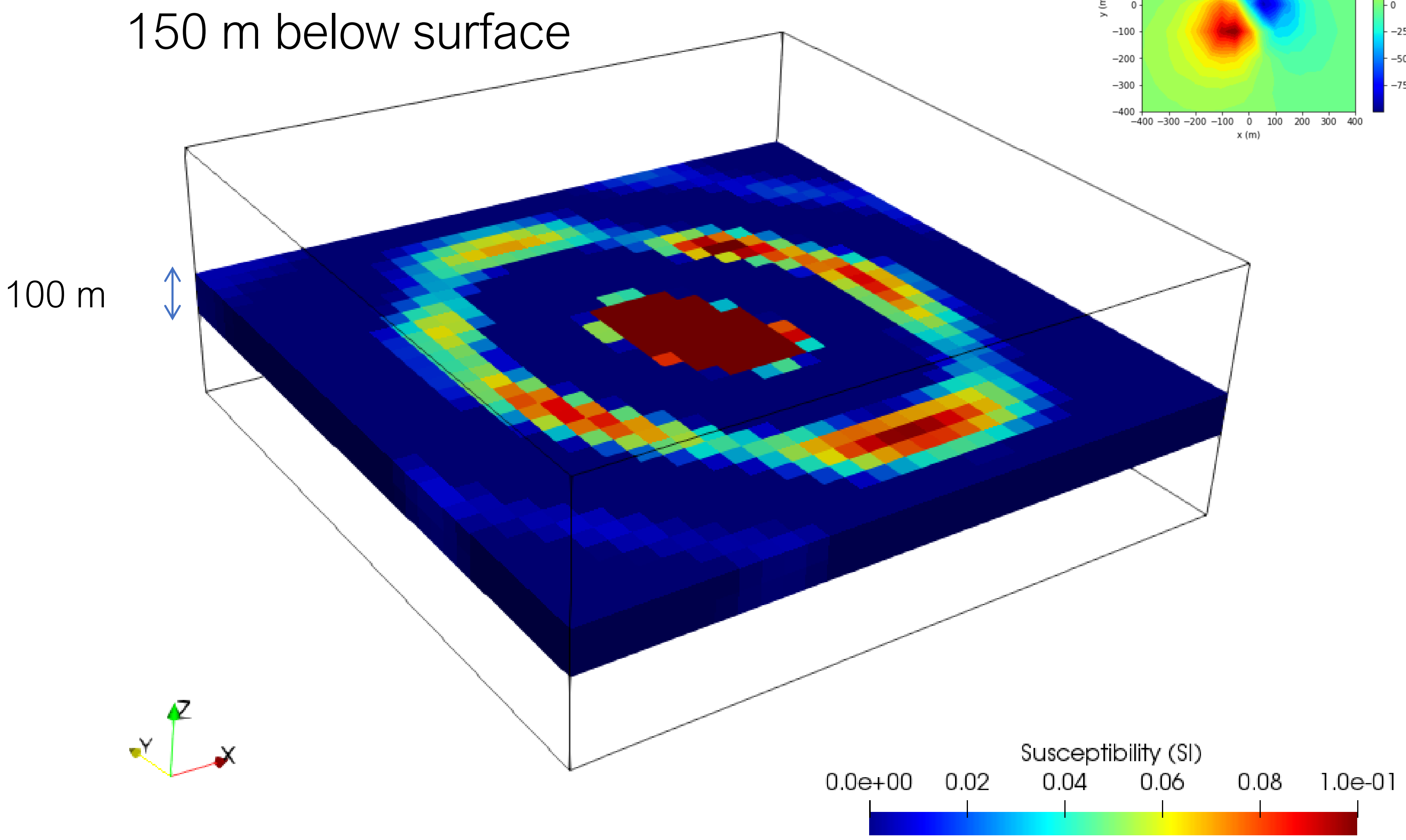
Example of extreme non-uniqueness

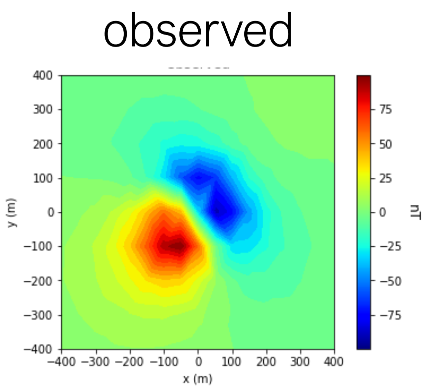


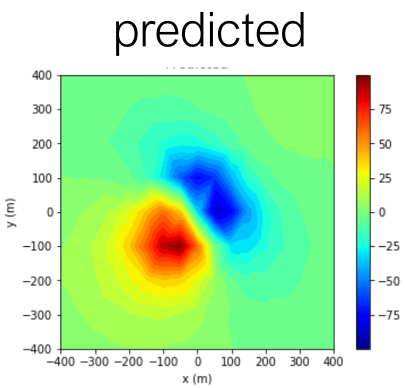
100 m below surface



Example of extreme non-uniqueness







Inversion with sensitivity weighting

Model objective function

$$\phi_m = \alpha_s \int_v w_s\left(\kappa - \kappa_{\text{ref}}\right)^2 dv + \alpha_x \int_v w_x\left(\frac{d\kappa}{dx}\right)^2 dx + \alpha_y \int_v w_y\left(\frac{d\kappa}{dy}\right)^2 dy + \alpha_z \int_v w_z\left(\frac{d\kappa}{dz}\right)^2 dz$$

Data misfit $\quad \phi_d = \sum_{j=1}^{N}\left(\frac{G_{ij}\kappa_j - d_j^{obs}}{\epsilon_j}\right)$

$\{w_s, w_x, w_y, w_z\}$: additional weightings

Choose $\{w_s, w_x, w_y, w_z\} \propto \frac{1}{(z+z_0)^3}$

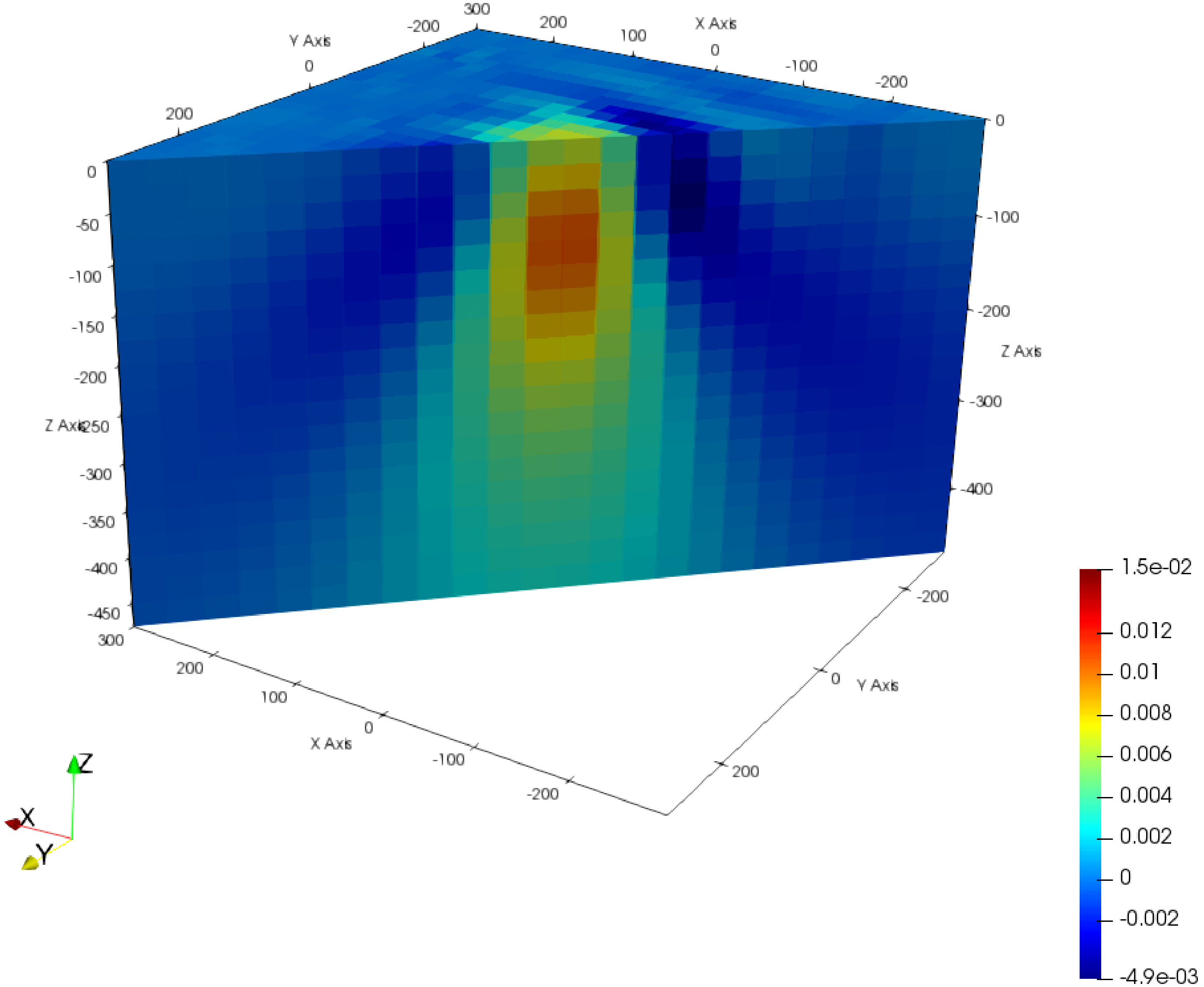
Allows cells at depth to contribute

The Inverse problem is:

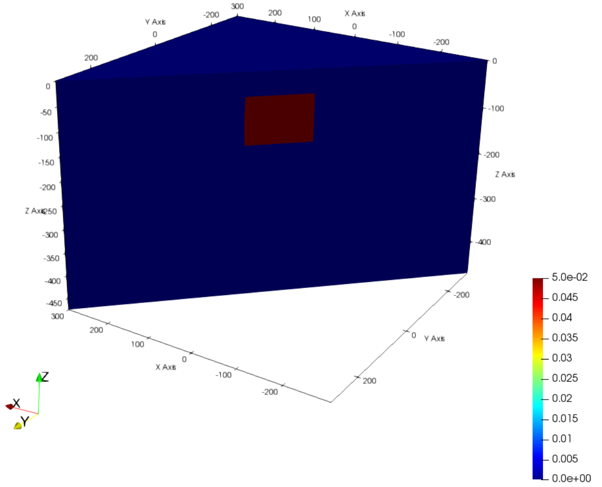
$$\text{minimize} \quad \phi = \phi_d + \beta\phi_m$$

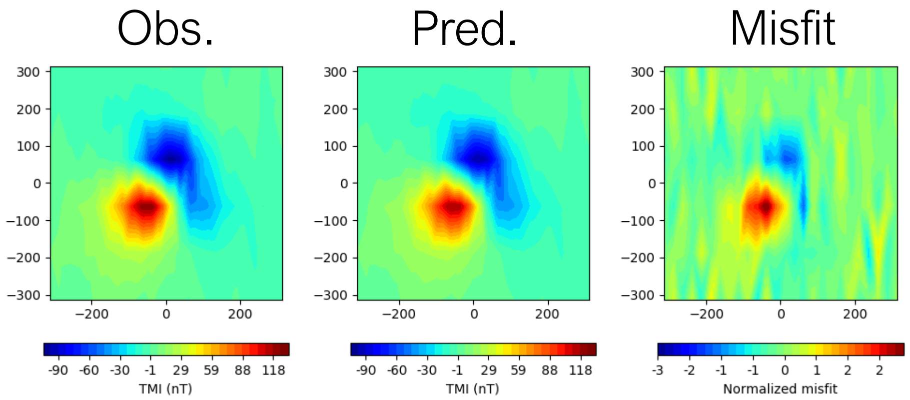
$$\text{find } \beta \text{ such that } \phi_d = \phi_d^* \text{ where } \phi_d = N$$

Inversion with sensitivity weighting

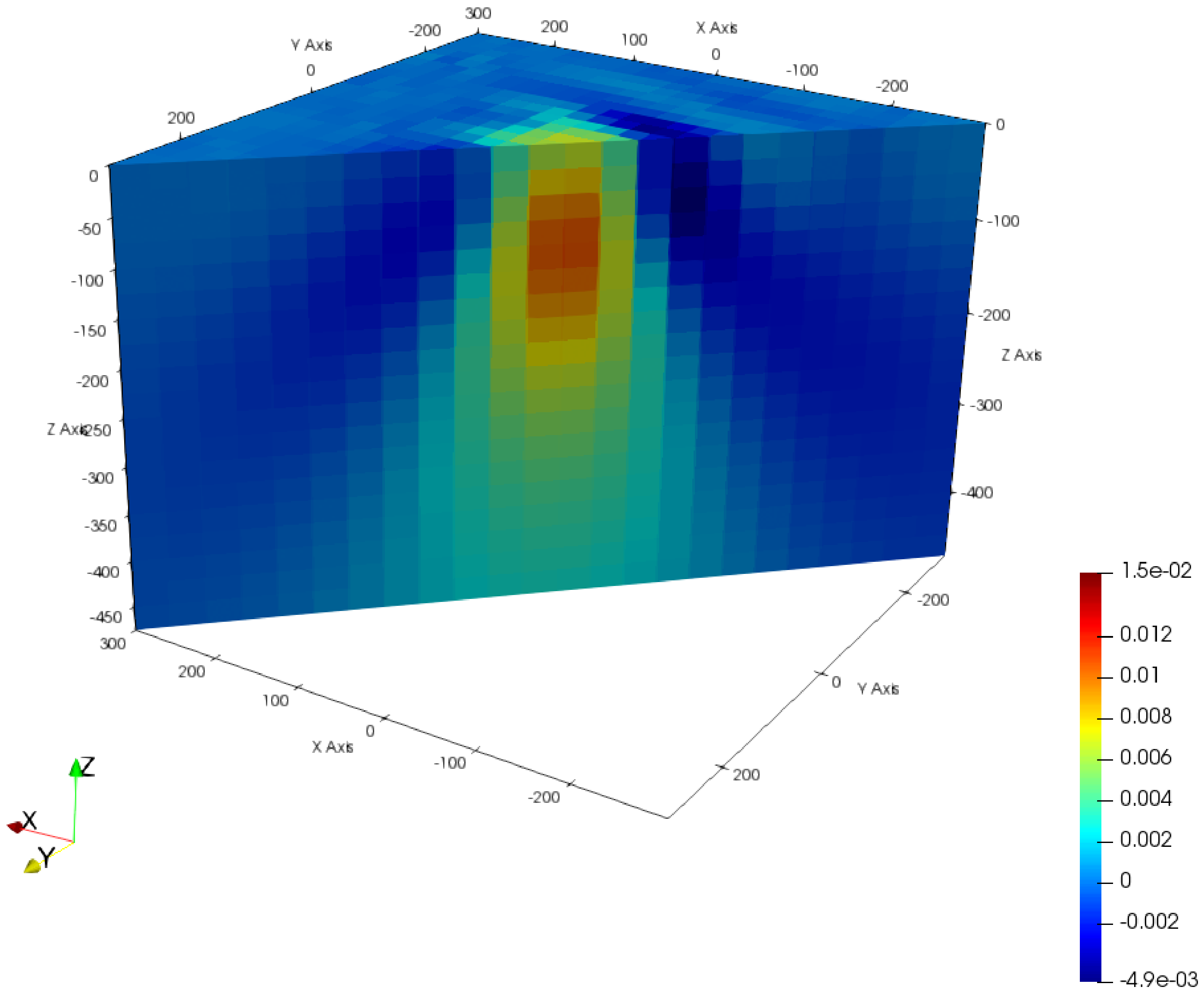


True model

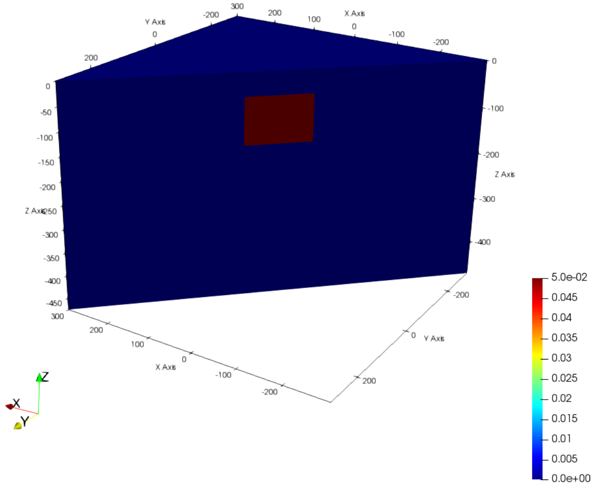




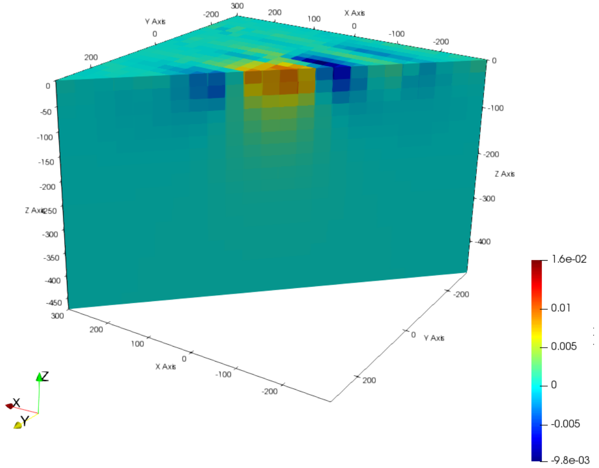
Inversion with sensitivity weighting



True model

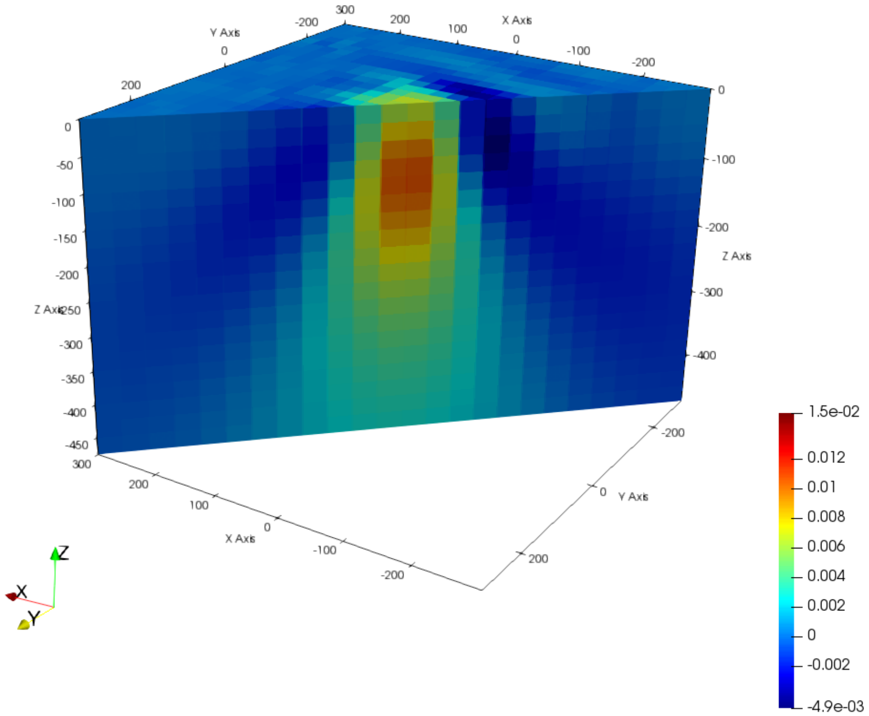


Without sensitivity weight



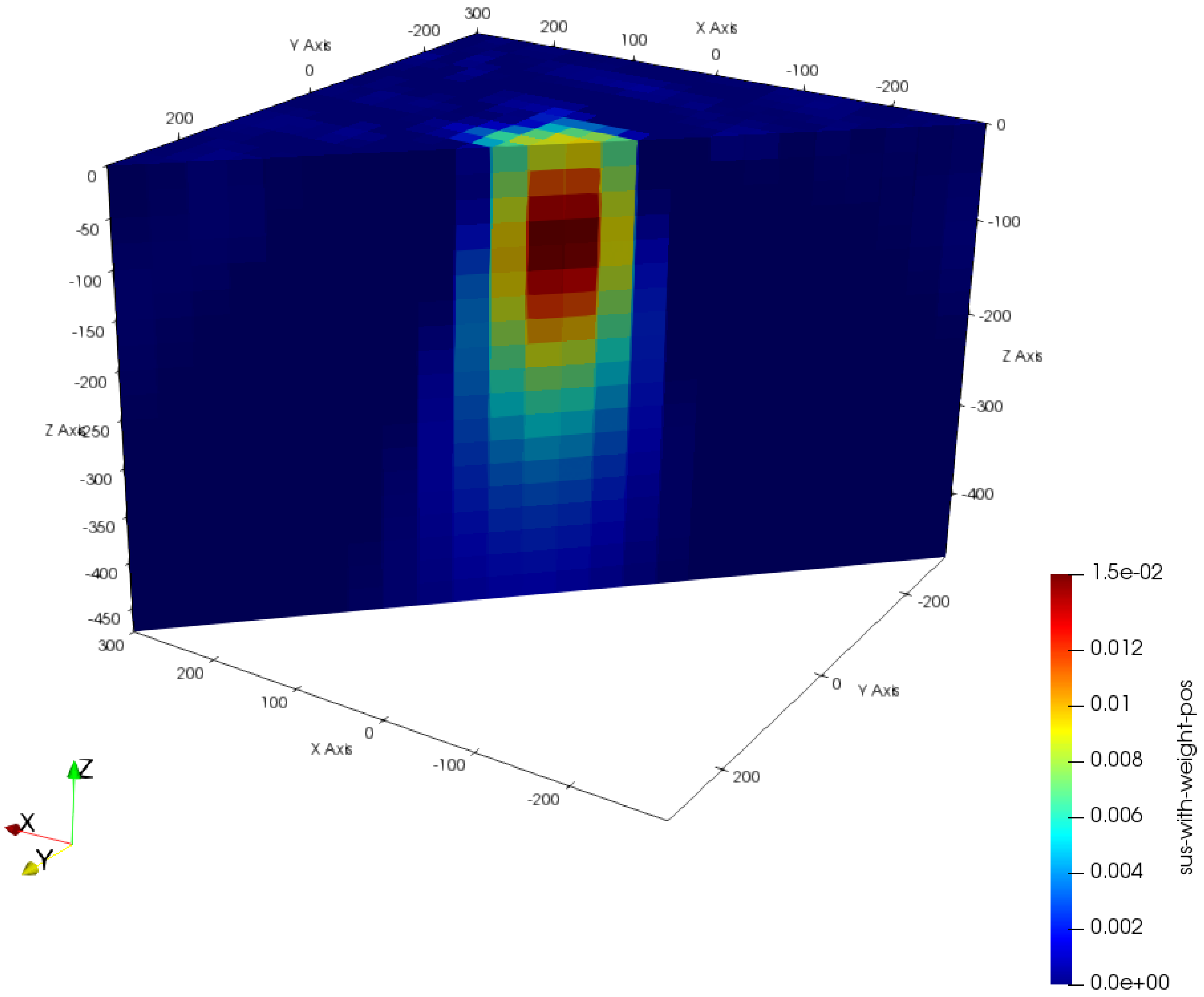
Summary for sensitivity weighting

- Structure is no longer concentrated at surface.
- Main anomaly is at a reasonable depth.
- BUT:
  - Negative κ persists
  - There is a long tail extending down and out.
- Require positivity

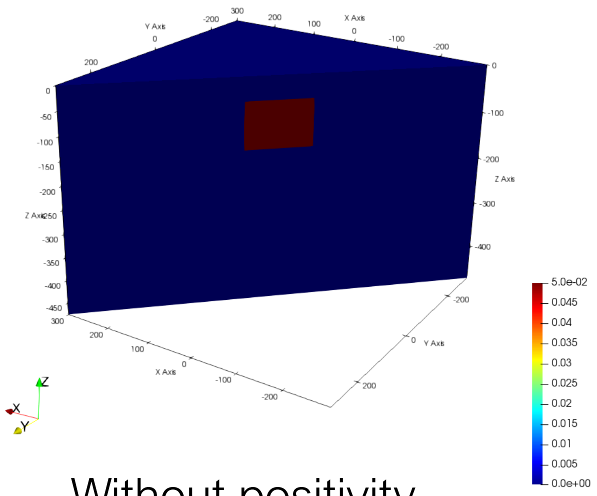


3D magnetic inversion:

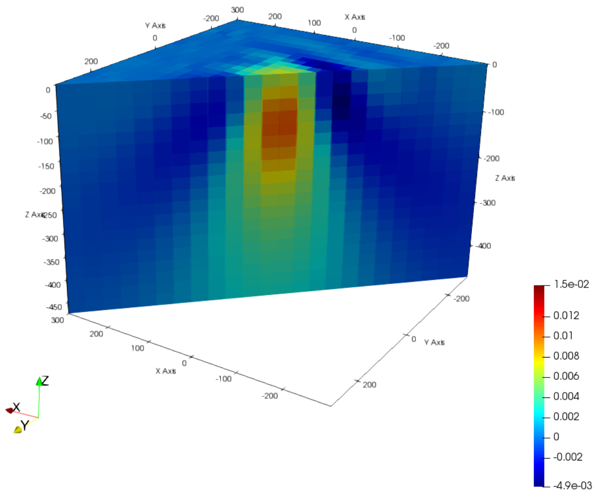
- depth (or sensitivity) weighting
- positivity (bounds): $\mathbf{m} \geq 0$



True model

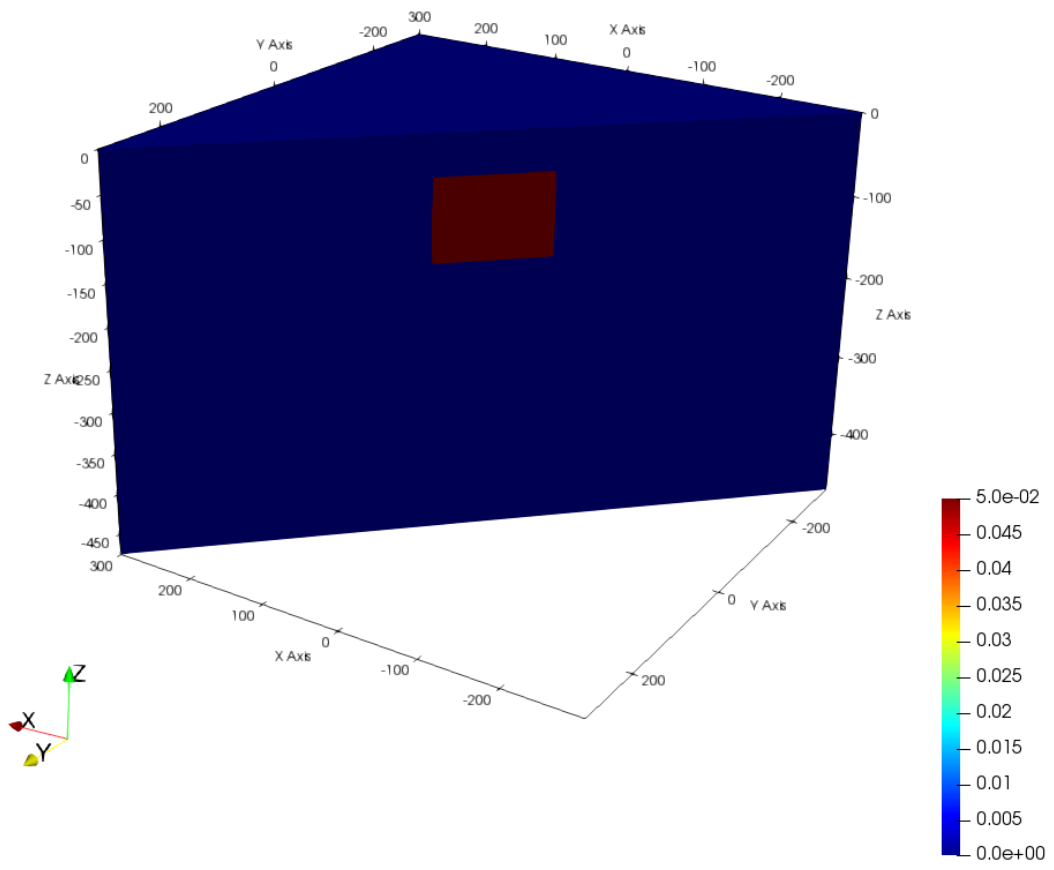


Without positivity



Think about the spatial character of the true model

True model



Most of model parameters are zero

Susceptible block and background has sharp boundaries

Model norm:

$$\phi_m = \alpha_s \int_v w_s \left(\kappa - \kappa_{\text{ref}}\right)^2 dv +$$

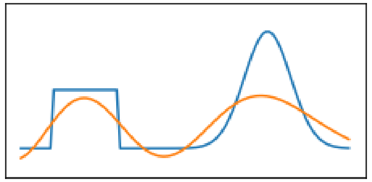
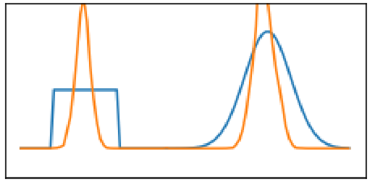
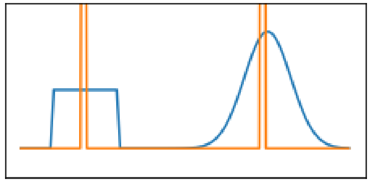
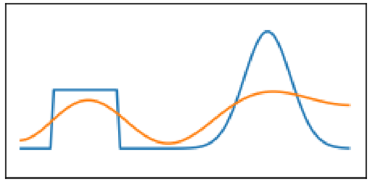
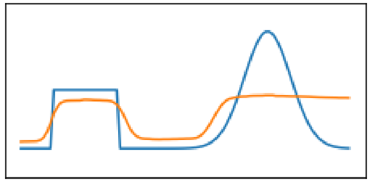
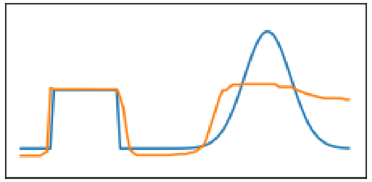
$$\alpha_x \int_v w_x \left(\frac{d\kappa}{dx}\right)^2 dx + \alpha_y \int_v w_y \left(\frac{d\kappa}{dy}\right)^2 dy + \alpha_z \int_v w_z \left(\frac{d\kappa}{dz}\right)^2 dz$$

General character

$$\phi_m = \sum_{i=1}^{M} |m_i|^p v_i$$

- Geometric character
  - p=2: all elements close to zero
  - p=1: sparse solution, # of non-zero elements are ≤ # of data
  - p=0: minimum support, model with the fewest number of elements

- 1D problem

	p=2	p=1	p=0
only ϕ_s			
only ϕ_x			

Magnetic inversion with Lp norms

Model objective function

$$\phi_m = \alpha_s \int_v w_s \left|\kappa - \kappa_{\text{ref}}\right|^{p_s} dv + \alpha_x \int_v w_x \left|\frac{d\kappa}{dx}\right|^{p_x} dx + \alpha_y \int_v w_y \left|\frac{d\kappa}{dy}\right|^{p_y} dy + \alpha_z \int_v w_z \left|\frac{d\kappa}{dz}\right|^{p_z} dz$$

Data misfit $$\phi_d = \sum_{j=1}^{N} \left(\frac{G_{ij}\kappa_j - d_j^{obs}}{\epsilon_j}\right)$$

Fournier and Oldenburg (2019)

The Inverse problem is:

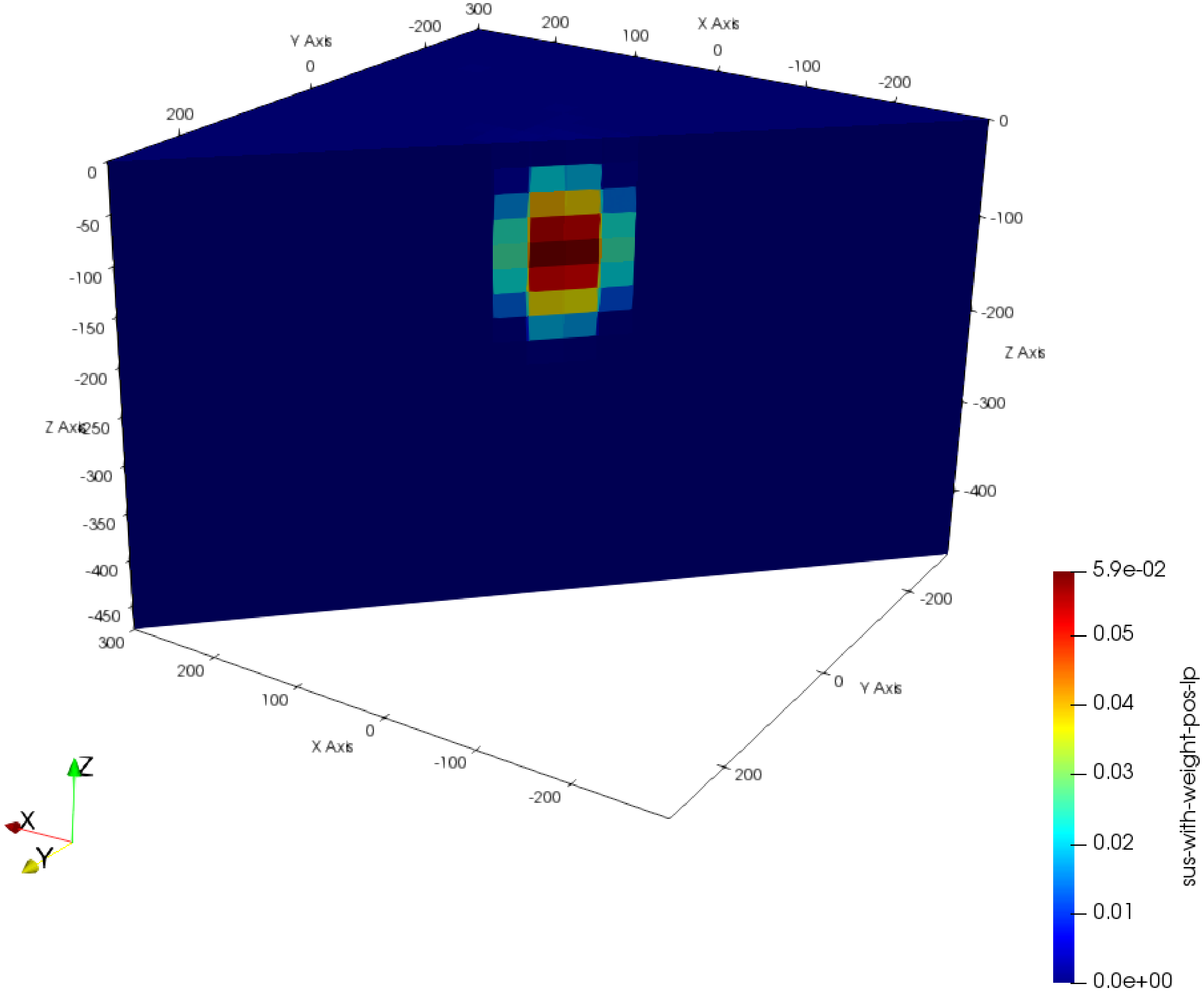
$$\text{minimize} \quad \phi = \phi_d + \beta\phi_m$$

$$\text{find } \beta \text{ such that } \phi_d = \phi_d^* \text{ where } \phi_d = N$$

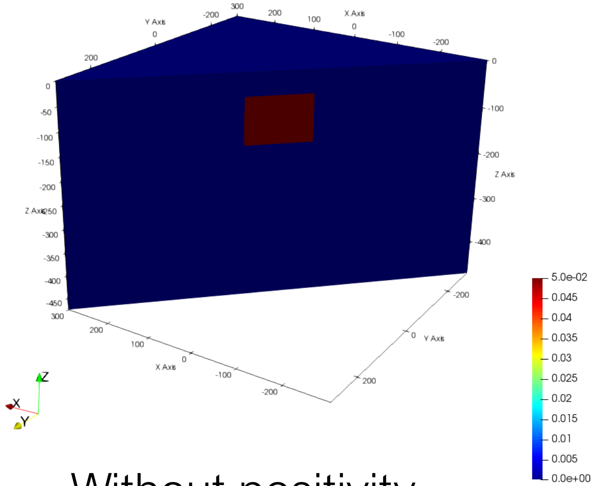
$$\text{subject to } \kappa \geq 0$$

3D magnetic inversion:

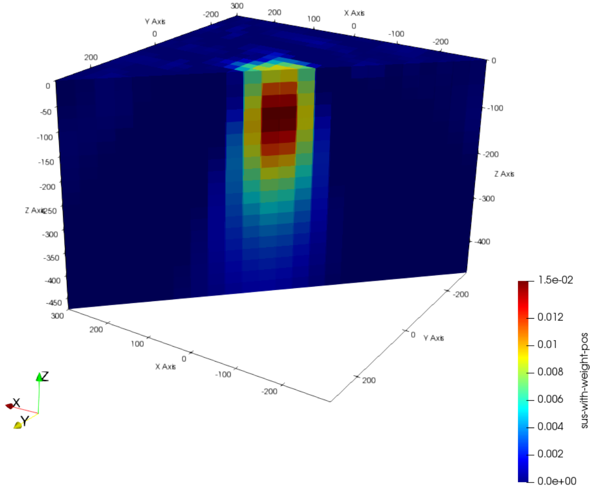
- depth (or sensitivity) weighting
- positivity (bounds): $\mathbf{m} \geq 0$
- L_p norm ($\mathbf{p_s=0}$, $p_x=p_y=p_z=2$)



True model

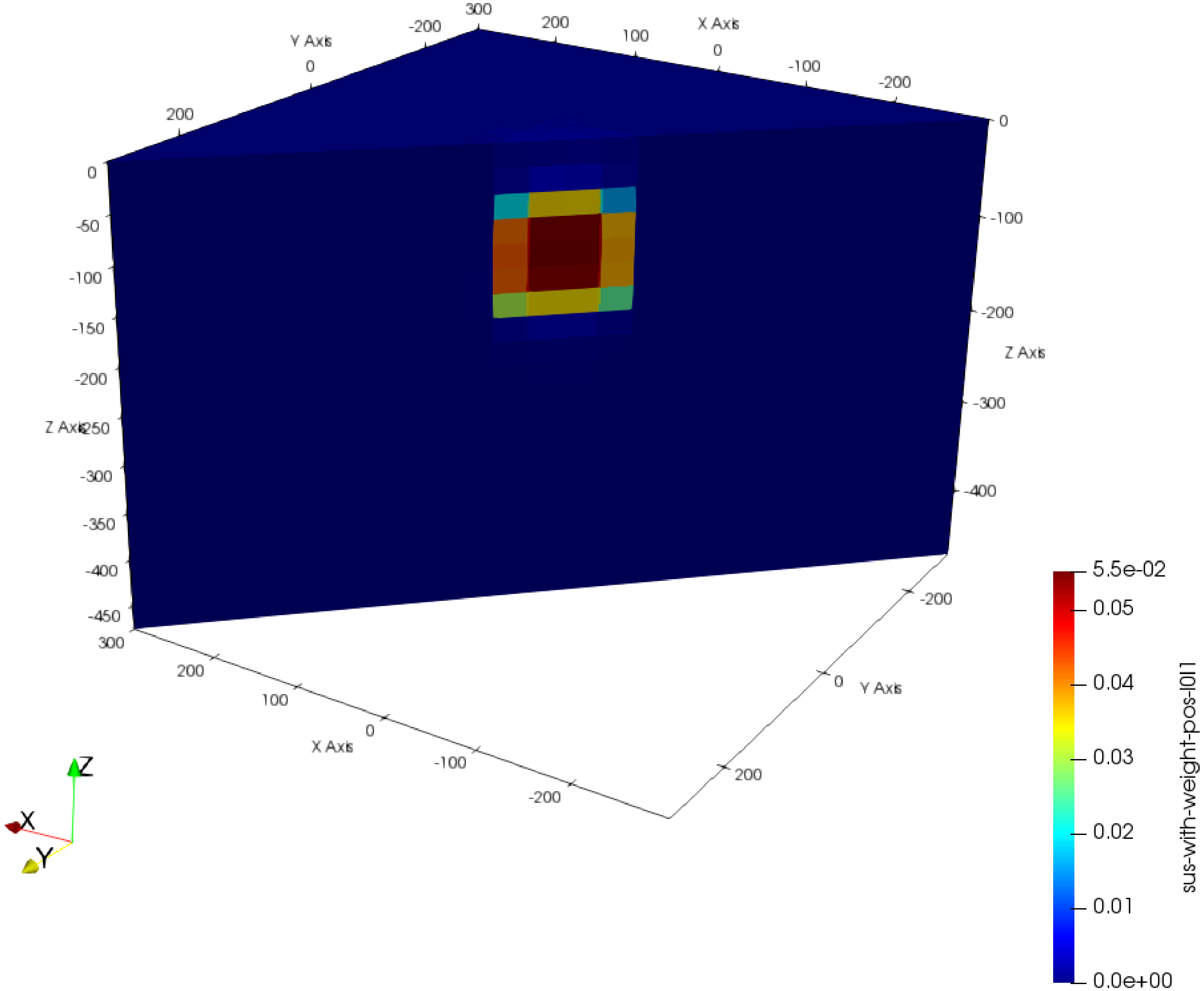


Without positivity

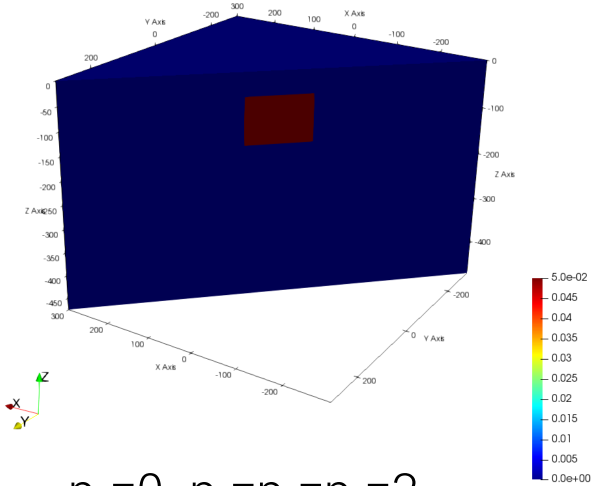


3D magnetic inversion:

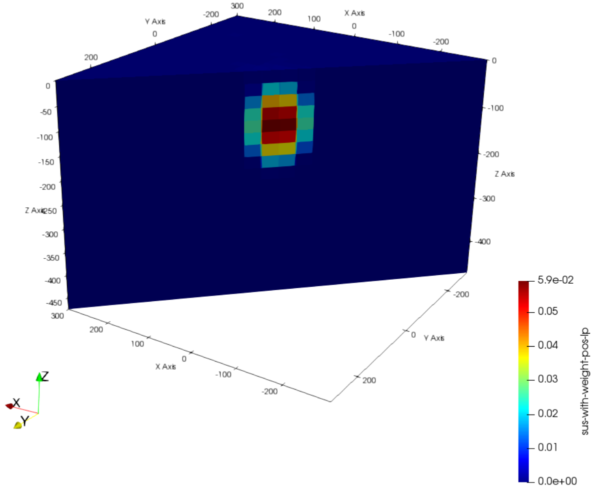
- depth (or sensitivity) weighting
- positivity (bounds): $\mathbf{m} \geq 0$
- L_p norm (p_s=0, $\mathbf{p_x}$=$\mathbf{p_y}$=$\mathbf{p_z}$=1)



True model

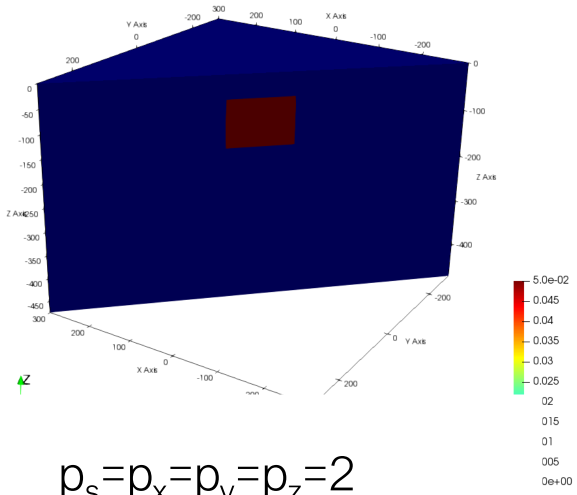


p_s=0, p_x=p_y=p_z=2

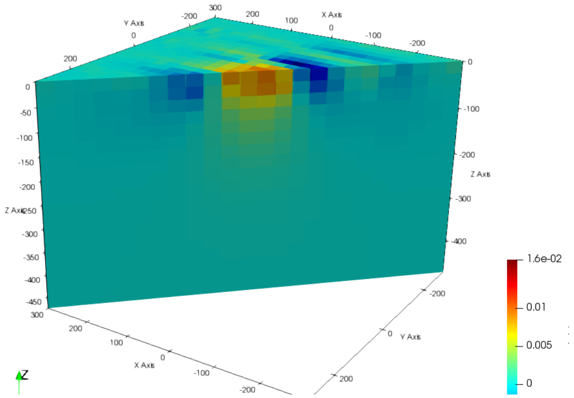


Summary: magnetic inversion

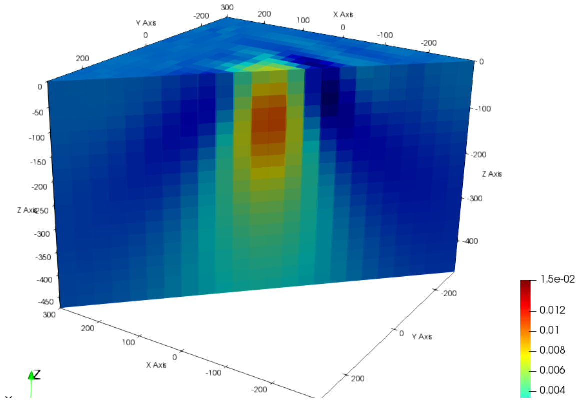
True model



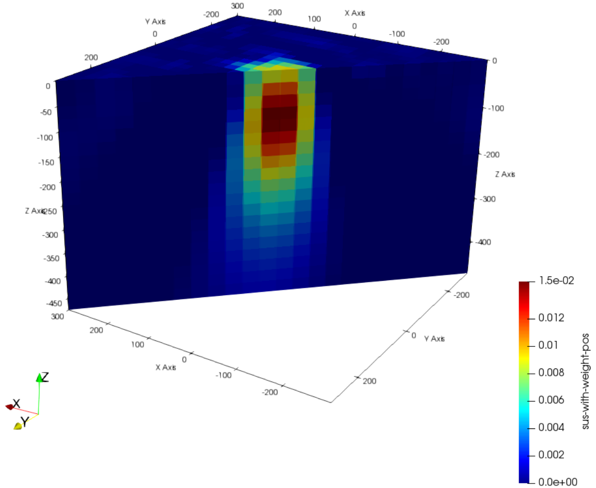
No weight



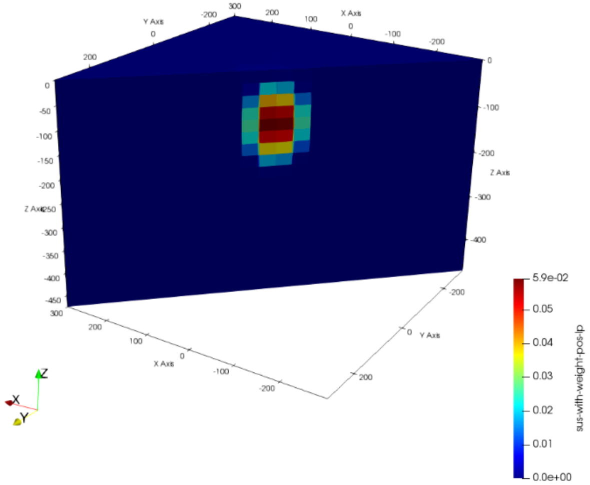
Sensitivity weight



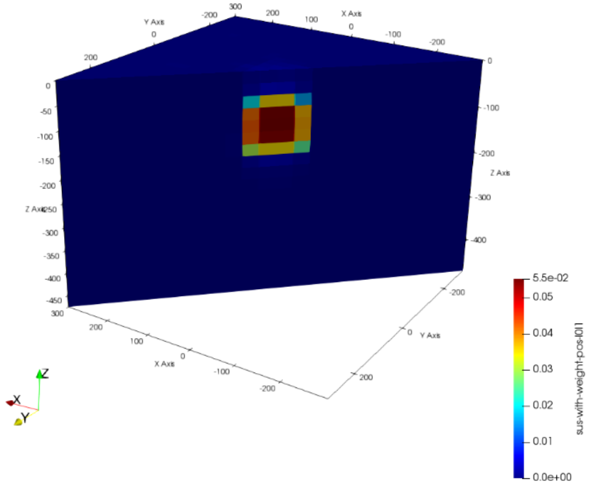
$p_s=p_x=p_y=p_z=2$



$p_s=0,\ p_x=p_y=p_z=2$

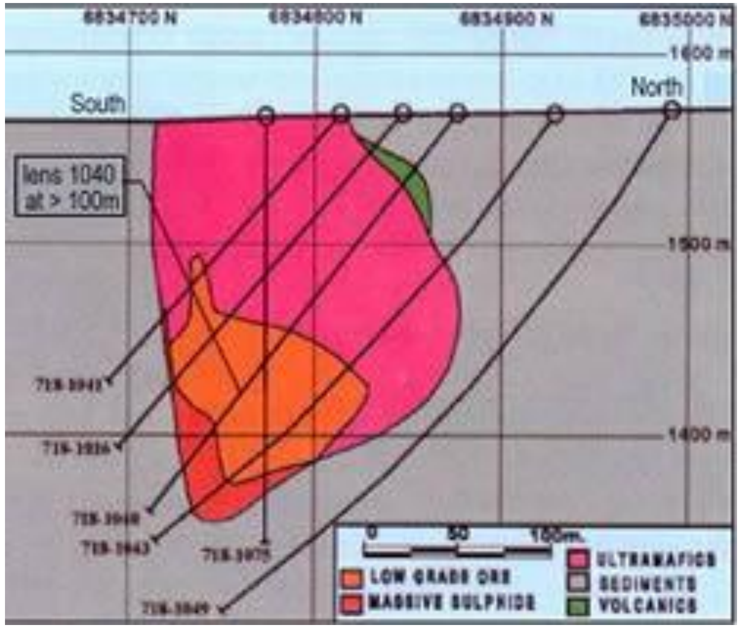


$p_s=0,\ p_x=p_y=p_z=1$

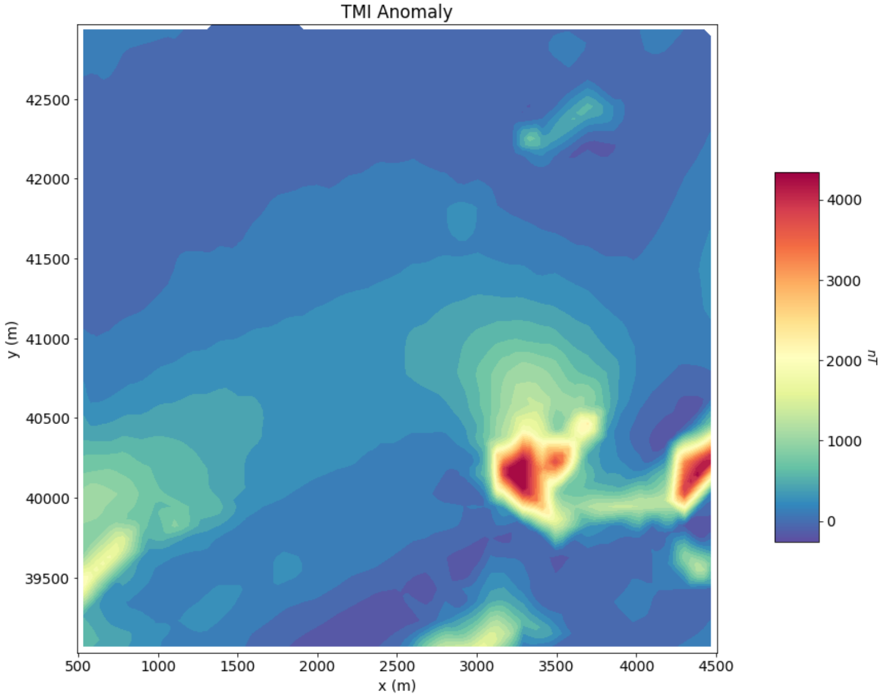


Revisit the case history

Geologic section



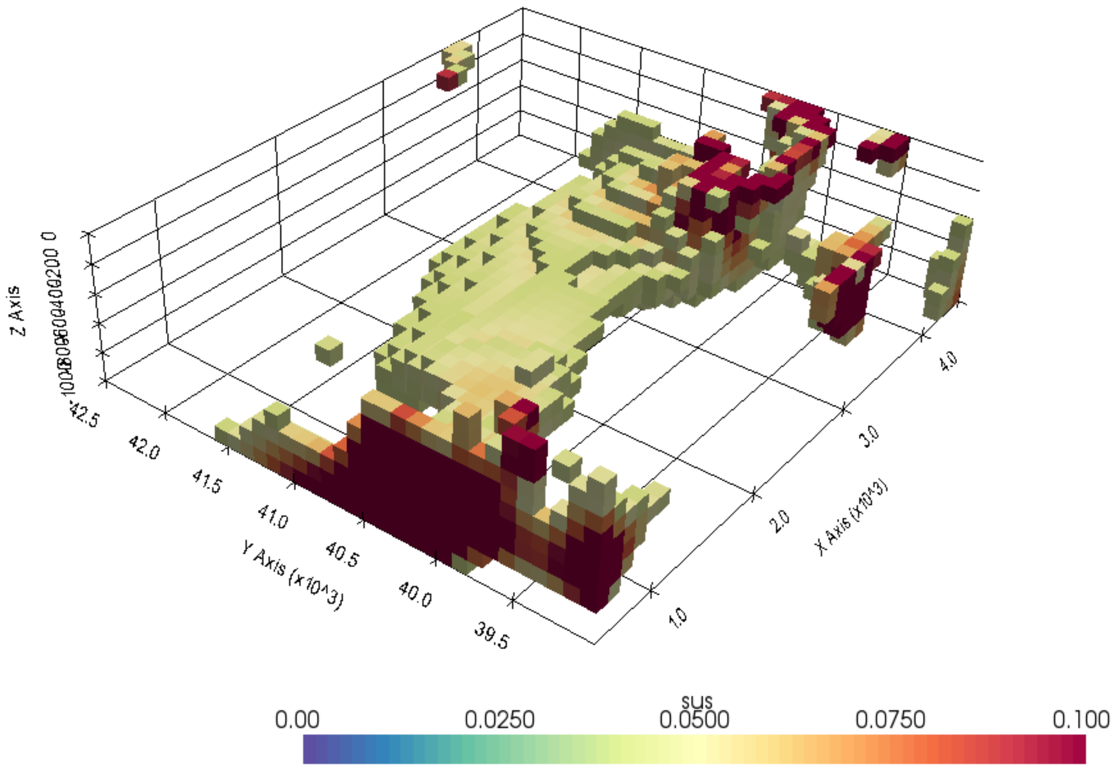
Magnetic data



Initial conceptual model: two ultramafic pipes
Can make impact on drilling location and mineral reserve

Magnetic inversion

The recovered model (SimPEG)



Model norm:

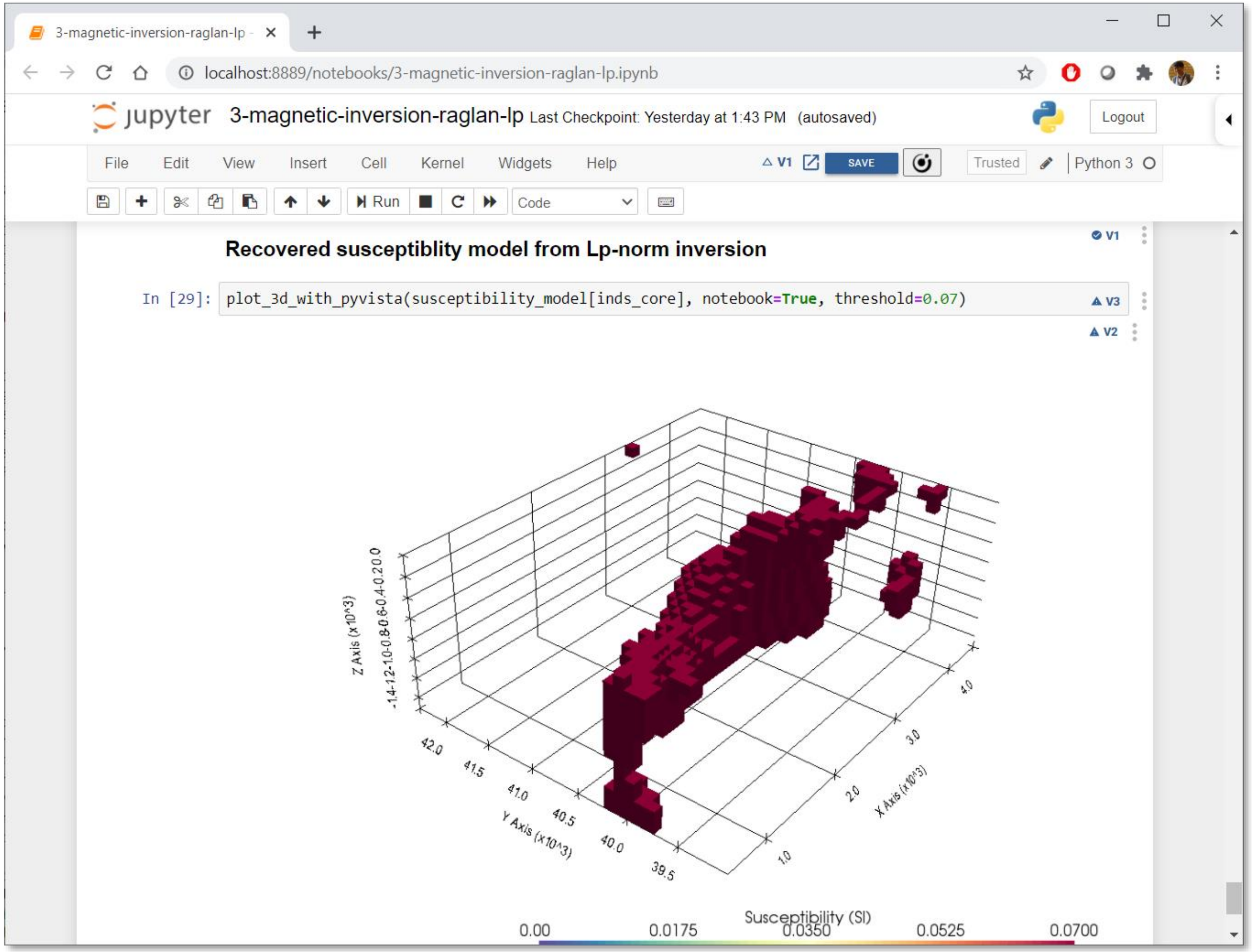
$$\phi_m = \alpha_s \int_v w_s \left(\kappa - \kappa_{\text{ref}}\right)^{\overset{p_s}{2}} dv +$$

$$\alpha_x \int_v w_x \left(\frac{d\kappa}{dx}\right)^2 dx + \alpha_y \int_v w_y \left(\frac{d\kappa}{dy}\right)^2 dy + \alpha_z \int_v w_z \left(\frac{d\kappa}{dz}\right)^2 dz$$

What if the target body is expected to be compact?

Use a sparse norm (e.g., p_s=0)

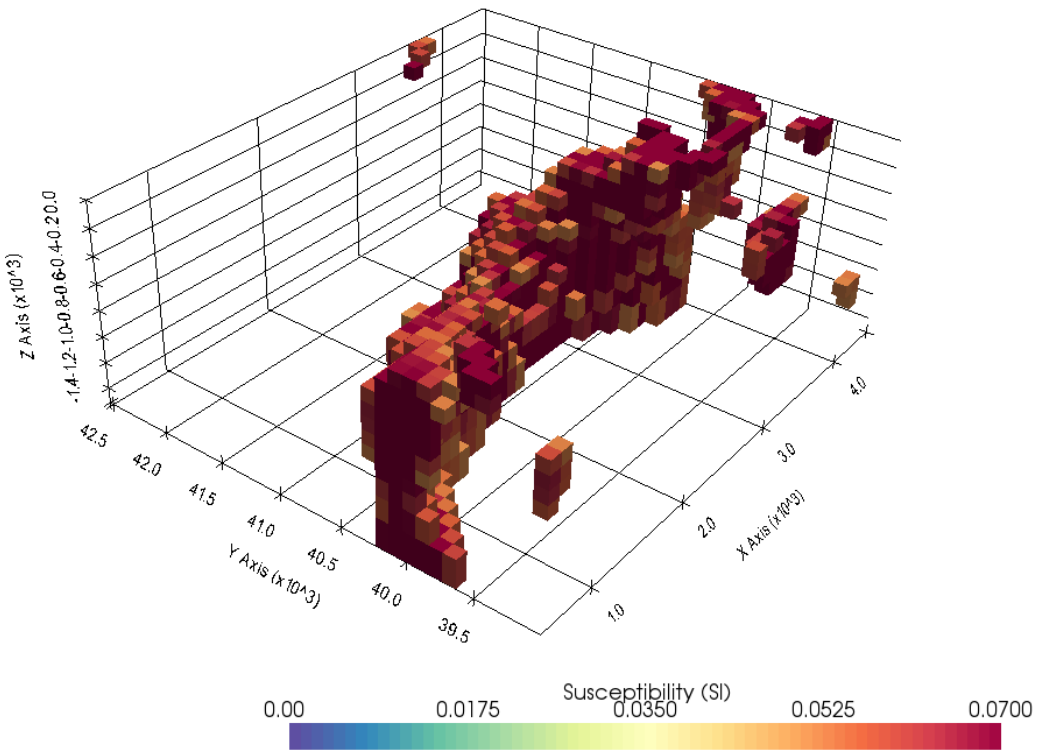
Lp-norm inversion of the Raglan magnetic data



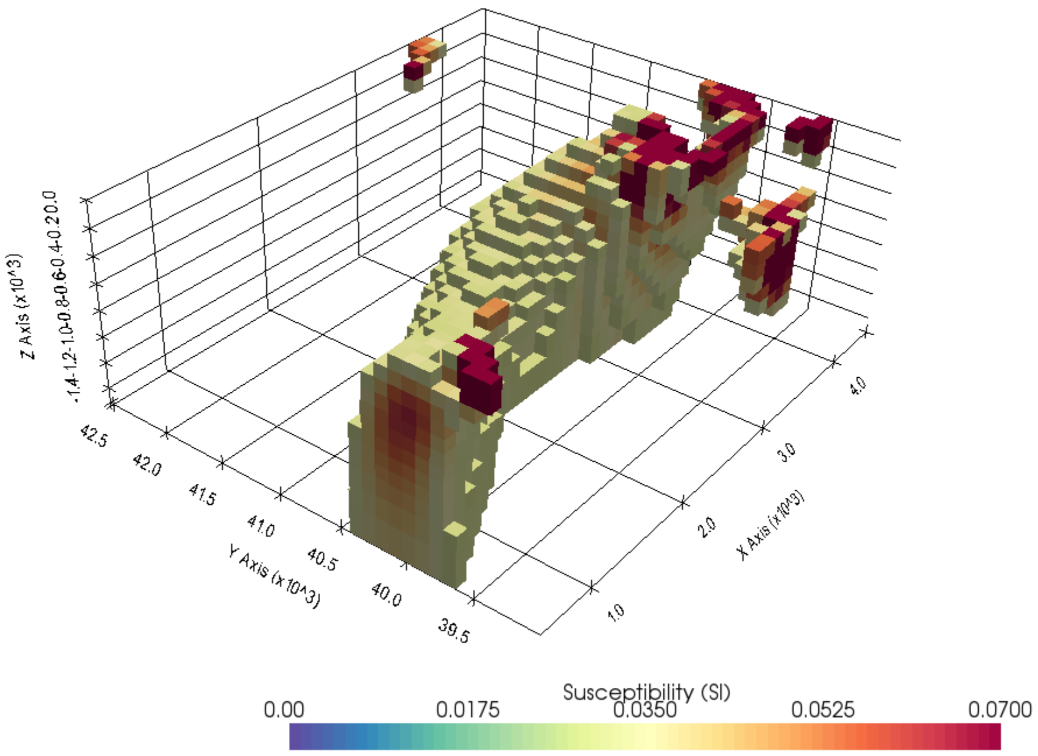
https://curvenote.com/@swung/inversion-for-geologists-transform-2021

Comparison: Lp-norm vs L2-norm

Lp-norm inversion



L2-norm inversion



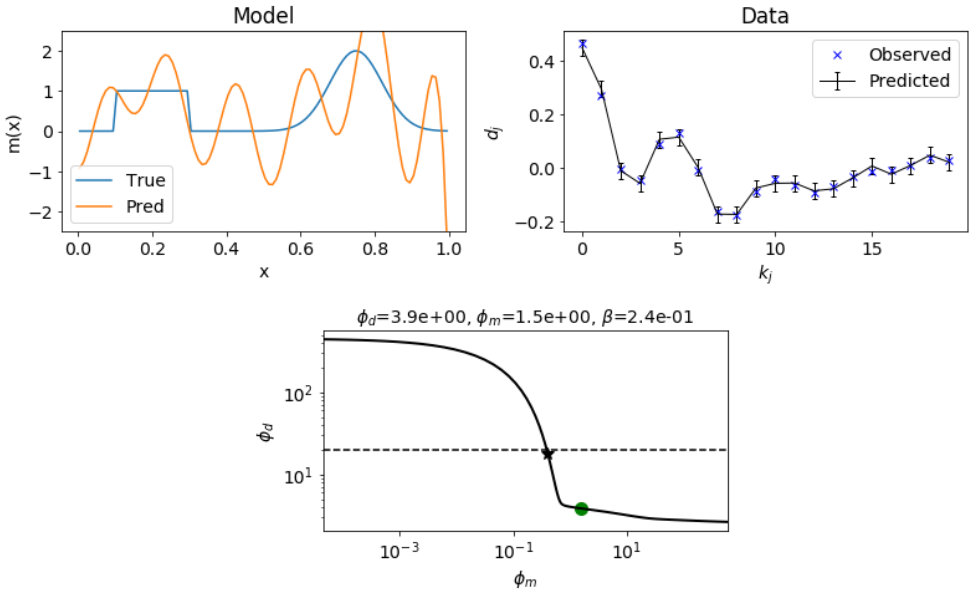
Beneficial for reserve estimate

Summary

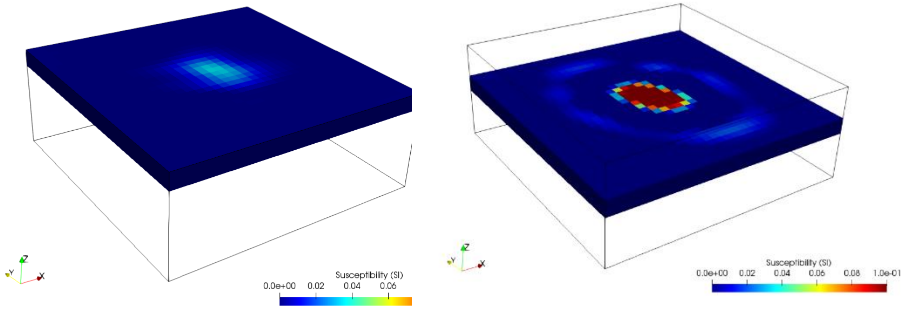
- Increasing complexity & volume of data
  - Increasing needs of data-driven approaches

- Fitting the observed data is not the enough condition!

Ill-conditioning

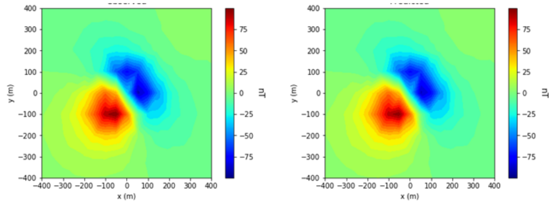


Non-uniqueness



observed

predicted

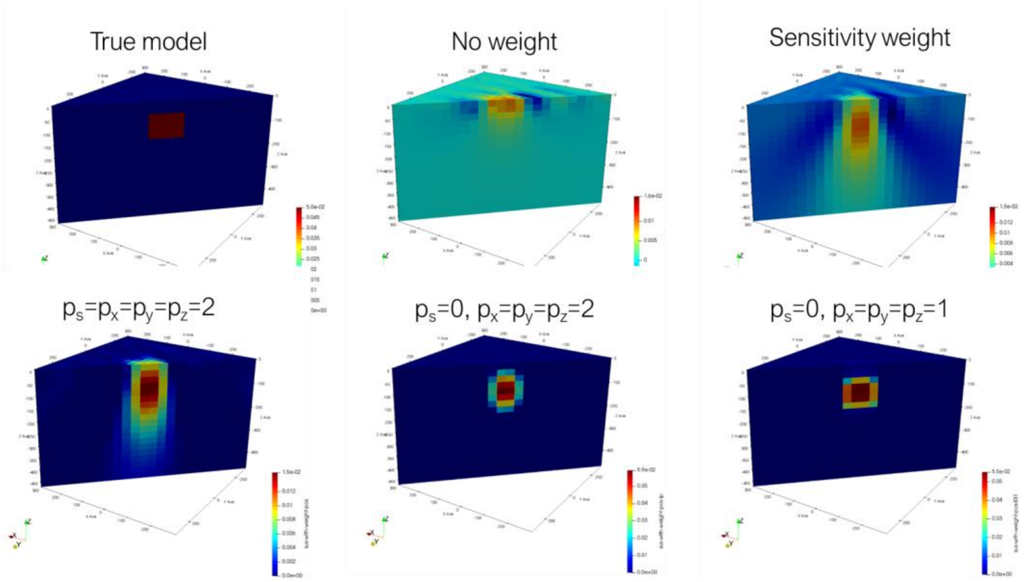


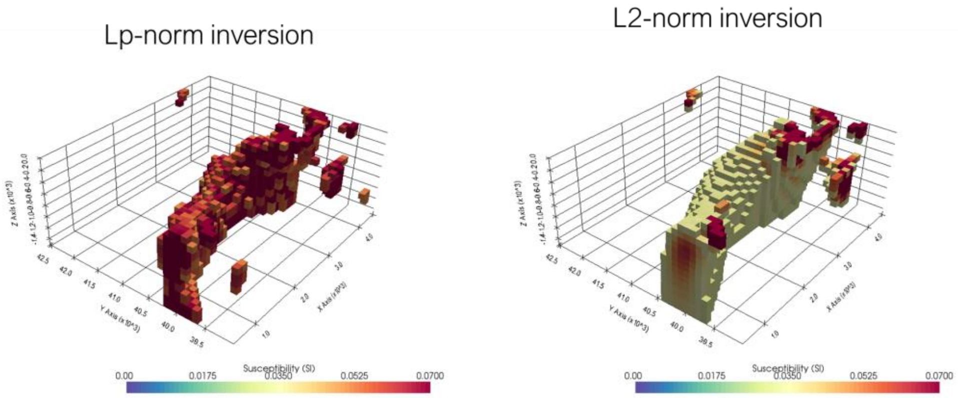
Summary

- An inversion framework can play an important role
  - Data-driven + *Prior knowledge*

$$\text{minimize} \;\; \phi(m) = \phi_d(m) + \beta\phi_m(m)$$

ϕ_d: data misfit
ϕ_m: model norm
β: trade-off parameter

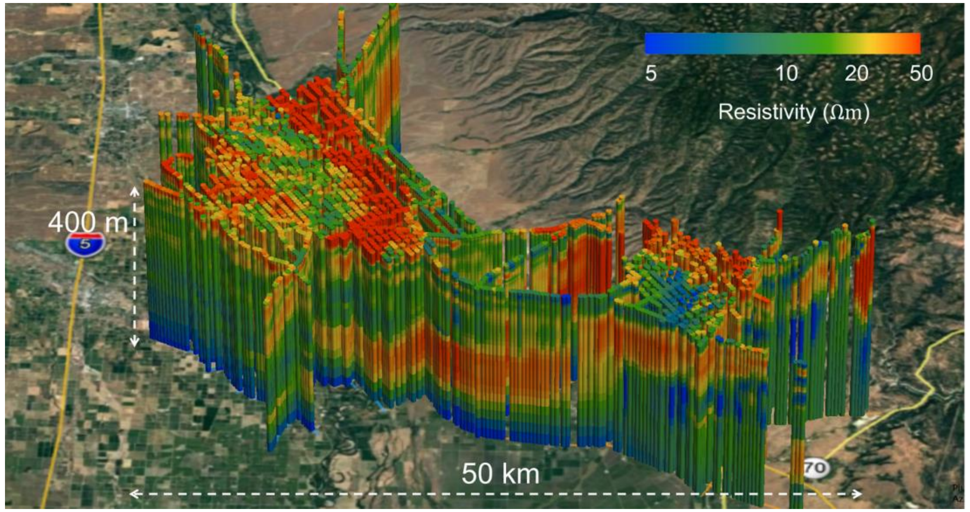




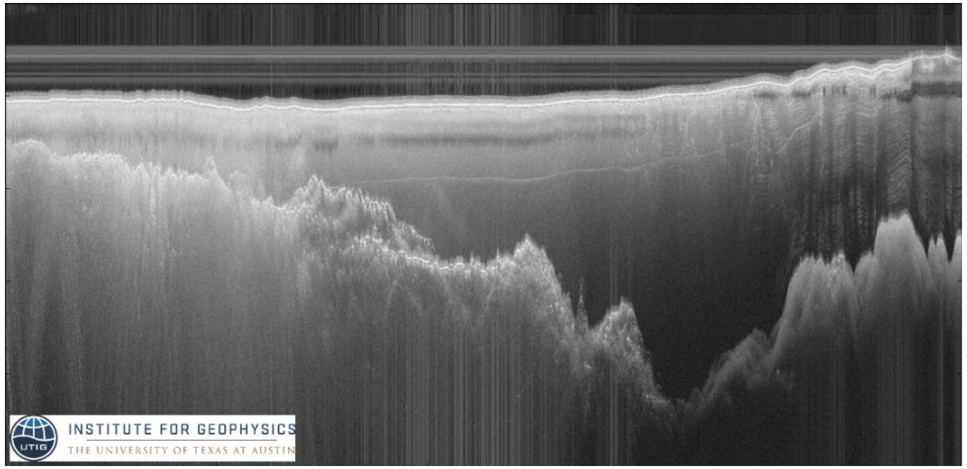
Multiple airborne geophysics

- Potential fields
  - *<u>Magnetics</u>*
  - Gravity

- Electromagnetics

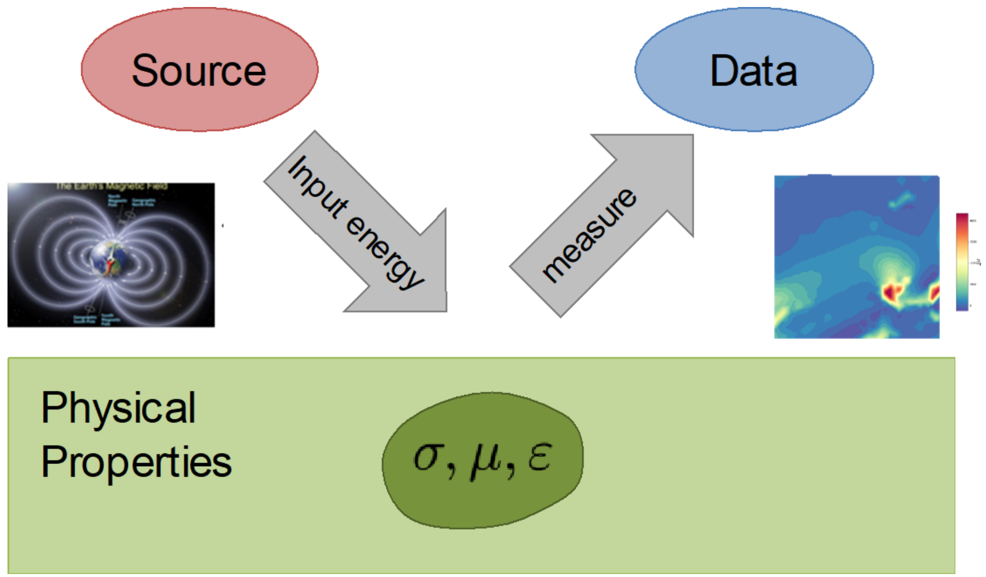


- Radar



Increasing Resolution

But in a generic level, they are very similar …

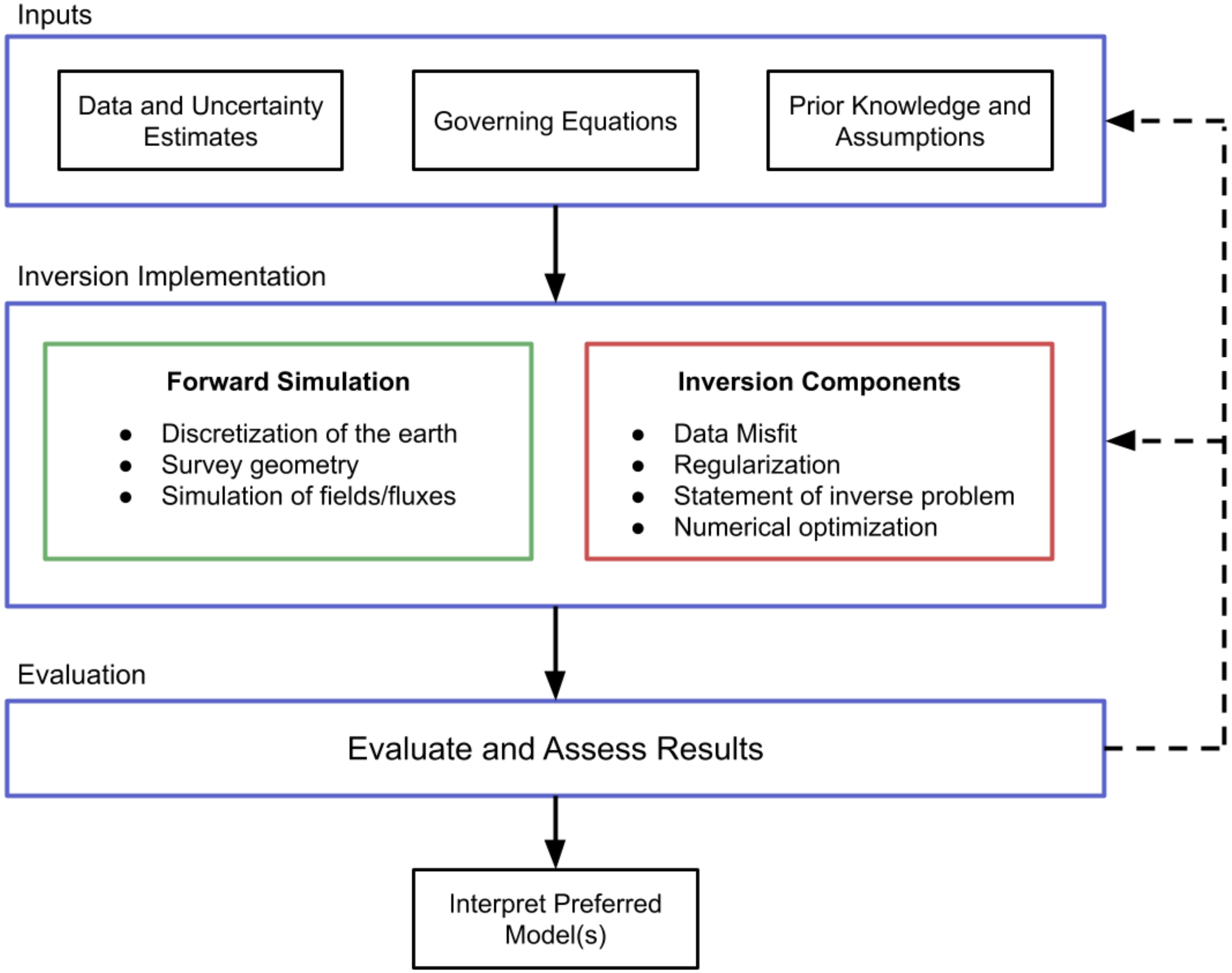


minimize $\phi(m) = \phi_d(m) + \beta\phi_m(m)$

ϕ_d: data misfit
ϕ_m: model norm
β: trade-off parameter

SimPEG provides ..

A common & modular inversion framework



- Gravity
- Magnetics
- Direct current resistivity
- Induced polarization
- Electromagnetics
- Fluid flow

There are many other geophysical software packages

- And they are growing!

pyGIMLi
Geophysical Inversion & Modelling Library

emg3d

PyLops

.....

harmonica

MARE2DEM

Open data

?

Challenging geoscience problems

Open data

?

Inversion framework can provide a "data-driven" approach

Challenging geoscience problems

Resources

Slides: http://bit.ly/transform-2021-slides

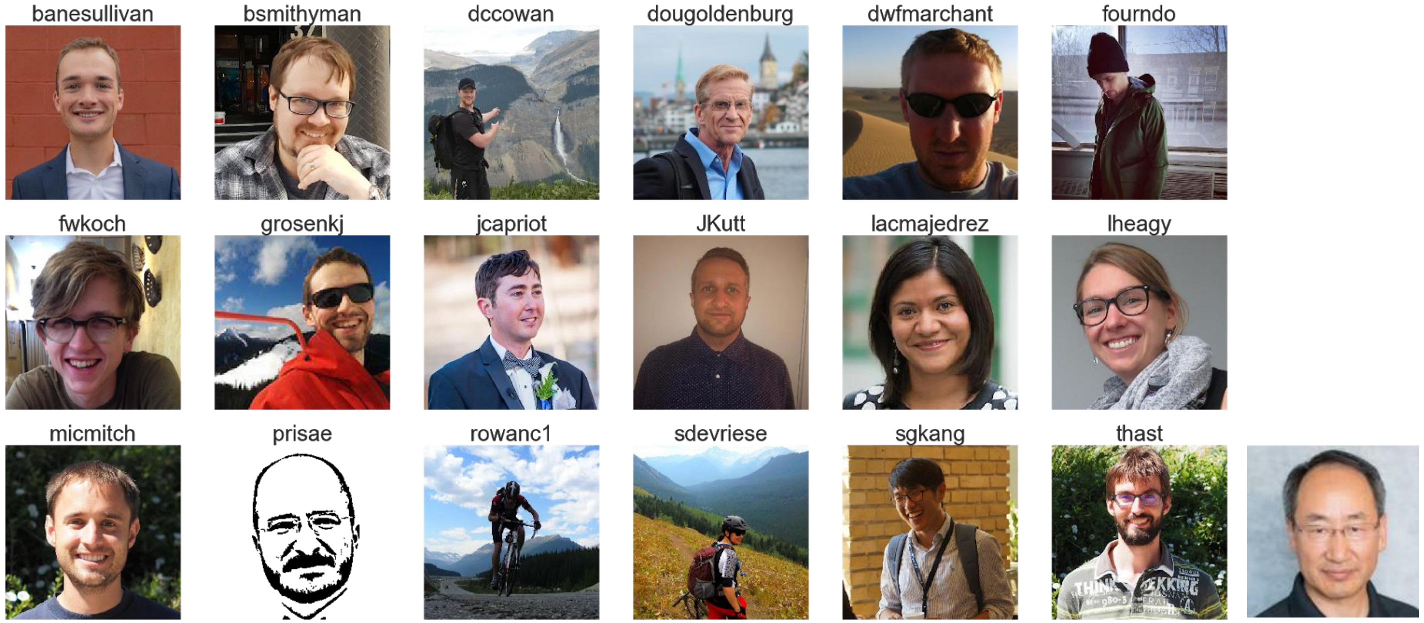
Notebooks: https://curvenote.com/@swung/inversion-for-geologists-transform-2021

Magnetics: https://gpg.geosci.xyz/content/magnetics/index.html

Github: https://github.com/simpeg/transform-2021-simpeg

SimPEG: https://www.simpeg.xyz

Thank you!



TRANSFORM 2021 organizers

The livestream has ended.

softwareunderground.org presents

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Virtual Conference on the Digital Subsurface, 16–23 April

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