Protective Quarantine Model of Hand-foot-mouth Disease

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Abstract. In this paper, we construct the protective quarantine model of hand-foot-mouth disease (HFMD), which provides us a new strategy for controlling the spread of HFMD. Then, we discuss the properties of disease-free equilibrium and endemic equilibrium in the protective quarantine model. The application of protective quarantine model is shown in the simulation.

1. Introduction

Using ordinary differential equations to study the spread of infectious disease has always been a popular topic in order to learn how the infectious disease spreads. Many models and methods have been proposed and applied. It has been proved in many papers that the ordinary differential models work well to explain the development of infectious disease. [1] provided an overlook for how to use differential equation models to describe and explain the infectious disease. Many theorems and lemmas related to modelling for infectious disease have been stated in that paper. [2] also summarized the mathematical models for infectious disease. Besides, one may refer to [3] and [4] for many useful theorems about ordinary differential equations. [5] and [6] are good examples for how to construct differential equations to analyze the spread of infectious disease.

The hand-foot-mouth disease (HFMD) is a kind of disease spreading mainly around young children. The most significant feature of the HFMD is that people older than 10 years of age almost have lifelong immunization (Centers for Disease Control and Prevention, 2017). Considering the property of hand-foot-mouth disease (HFMD), it is reasonable to consider different differential equation models to perform the spread of HFMD. [6,7,8,9] constructed models for hand-foot-mouth disease. [10] considered SIRC model for HFMD, which was innovative for the analysis of HFMD. Many strategies for controlling the spread of HFMD have also been discussed. And in this paper, we will consider the protective quarantine model for HFMD. Considering that the special property of HFMD mentioned above, protective quarantine model means the susceptible and the exposed are the ones to be protected. Since the susceptible and the exposed are mainly children under 10 years old who are lack of public activities, the protective quarantine model is reasonable when applying in reality.

This paper is organized as follows. In the first part, the assumptions and notations are stated. The construction of the protective quarantine model is the second part. Then, we consider the basic reproduction number, disease-free equilibrium and endemic equilibrium. Meanwhile, the properties of disease-free equilibrium and endemic equilibrium are discussed. In the fourth part, we discuss the results of the protective quarantine model. And in the last part, the simulation for the protective quarantine model is provided.

2. Assumptions and notations of the modelling

In this section, we will provide some basic assumptions and notations used in the modeling.

2.1. Assumptions

In the modelling, the following assumptions are always made:

- The population includes five groups: susceptible (S), exposed(E), quarantine(Q), infected (I), and removed (R). The removed population group comprises the people immune to the HFMD.
- If we use N as the total number of people, and N is equal to the sum of S, E, Q, I, and R population groups. All the variables are the functions of time.
- The people who are older than 5 years old are assumed to have lifelong immunization. And all the new-born children are assumed to be susceptible to the HFMD.
- The people who recover from the HFMD are assumed to have lifelong immunization.
- We do not consider the people who move in and out city. The population is growing at a low speed.
- The protection isolation measures will be applied to two kinds of groups. The first group is susceptible people that contact with the infected, and the second is exposed ones. If people in the first group do not turn infected after the incubation period, the isolation measure will come to an end. Or, the patient will be cured.

2.2. Notations

The following notations will be used in the modeling.

- s: ratio of S to N; e: ratio of E to N; i: ratio of I to N; q: ratio of Q to N; r: ratio of R to N.
- b: Natural birth rate; d: natural mortality rate; β: contact coefficient between the S and I; σ: ratio of people who get lifelong immunization at an age above 5 (Centers for Disease Control and Prevention, 2017); α: mortality rate for illness; β_0: incubation ratio of S; ε: transferred coefficient from E to I; λ: incubation ratio of E; γ: recovery coefficient of I; ω: exclusive coefficient of Q; m_1: incubation controlling coefficient of S; m_2: incubation controlling coefficient of E.
- We apply the standard contact number between the susceptible and infected population, $\beta SI / N$.

3. Model Construction

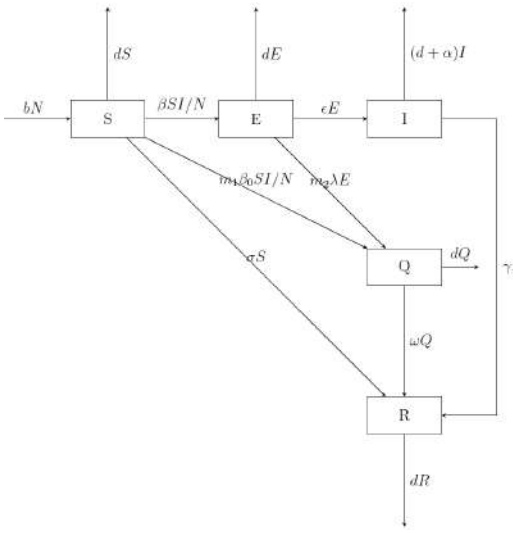
The model can be constructed as Figure 1.

where bN represents the new-born children and dS, dE, dQ, dR represents the natural death of the susceptible people, the exposed ones, the inoculated ones and the removed ones separately. $(d+\alpha)I$ represents the sum of natural death and death due to illness. $\beta SI / N$ denotes the susceptible people that are exposed to the pathogeny and become the exposed and εE denotes the exposed ones that move into the infected. σS represents those susceptible ones who get lifelong immunization. γI denotes the infected ones that recover. ωQ represents those removed from inoculation. Meanwhile, $m_1\beta_0 SI / N$ and $m_2\lambda E$ denotes the number of inoculation for the susceptible people.

Based on the flow chart in Figure 1, we can get the ordinary differential equation system (1).

Moreover, from (1) the change of N can be obtained. $N' = (b-d)N - \alpha I$.

Applying the notations of ratios, we can normalize the equation system and obtain the following system (2), where there is no relation between the first three equations and the last two. Thus, we consider the subsystem (3), where $\{(s,e,i) | 0 \le s+e+i \le 1\}$.



$$\begin{cases} S' = bN - (d+\sigma)S - \beta SI/N - m_1\beta_0 SI/N \\ E' = \beta SI/N - (\epsilon + dE + m_2)\lambda E \\ I' = \epsilon E - (\gamma + \alpha + d)I \\ Q' = m_2\lambda E + m_1\beta_0 SI/N - (d+\omega)Q \\ R' = \sigma S + \omega Q + \gamma I - dR \end{cases} \quad (1)$$

Figure 1. Model Construction.

$$\begin{cases} s' = b + (\alpha - \beta - m_1\beta_0)si - (b+\sigma)s \\ e' = \beta si - (b + \epsilon + m_2\lambda)e + \alpha ei \\ i' = \epsilon e - (\alpha + b + \gamma)i + \alpha i^2 \\ q' = m_2\lambda e + m_1\beta_0 si - (b+\omega)q + \alpha qi \\ r' = \sigma s + \omega q + \gamma i - br + \alpha ri \end{cases} \quad (2)$$

$$\begin{cases} s' = b + (\alpha - \beta - m_1\beta_0)si - (b+\sigma)s \\ e' = \beta si - (b + \epsilon + m_2\lambda)e + \alpha ei \\ i' = \epsilon e - (\alpha + b + \gamma)i + \alpha i^2 \end{cases} \quad (3)$$

4. Basic reproduction number

According to [2] and [6], basic reproduction number is used to describe the infectious system. Here, in the paper, considering that the form in the differential system is proportion to the total number, we use basic reproduction number in [2] and denote it as R_0.

Let e and i in equation system (3) equal zero, we obtain the disease-free equilibrium $U_0(b/(b+\sigma),0,0)$. And the average disease cycle with natural death and illness death should be adjusted from $1/\gamma$ to $1/(\alpha+b+\gamma)$.

The coefficient for disease propagation is β, and the adjusted proportion from the exposed to the infected is $\epsilon/(b+\epsilon+m_2\lambda)$. Thus, the effective propagation coefficient is $\beta\epsilon/(b+\epsilon+m_2\lambda)$.

Thus, the basic reproduction number in this paper is

$$R_0 = \frac{b}{b+\sigma}\frac{1}{\alpha+b+\gamma}\frac{\beta\epsilon}{b+\epsilon+m_2\lambda}. \quad (4)$$

5. Disease-free equilibrium

The disease-free equilibrium $U_0(b/(b+\sigma),0,0)$, has been obtained above. Then we will move to the properties of the equilibrium.

5.1. Local stability of disease-free equilibrium.

Theorem 5.1: When $R_0 < 1$, the disease-free equilibrium is locally asymptotically stable.

When $R_0 > 1$, the disease-free equilibrium is unstable.

Proof: As we have solved in the derivation of basic reproduction number, the disease-free equilibrium is $U_0(b/(b+\sigma),0,0)$. And the Jacobian matrix correlated with the disease-free equilibrium is

$$A = \begin{pmatrix} -(b+\sigma) & 0 & (\alpha-\beta-m_1\beta_0)s_0 \\ 0 & -(b+\epsilon+m_2\lambda) & \beta s_0 \\ 0 & \epsilon & -(\alpha+b+\gamma) \end{pmatrix}. \quad (5)$$

Assume the eigenvalue of matrix A is x_1, x_2, x_3.

$x_1 = -(b+\sigma) < 0$, $x_2 + x_3 = -(b+\epsilon+m2\lambda) - (\alpha+b+\gamma) < 0$,

$x_2 x_3 = (b+\epsilon+m_2\lambda)(\alpha+b+\gamma) - \epsilon\beta s_0 = (b+\epsilon+m_2\lambda)(\alpha+b+\gamma)(1-R_0)$.

When, $R_0 < 1$, we can obtain that $x_1, x_2, x_3 < 0$, and the disease-free equilibrium is locally asymptotically stable. When, $R_0 > 1$, to make problem simple, we can make $x_2 > 0$, which leads to the unstableness of the disease-free equilibrium.

5.2. Global asymptotic stability of disease-free equilibrium.

Theorem 5.2: $\hat{R}_0 = \dfrac{\beta\epsilon}{(\alpha+b+\gamma)(b+\epsilon+m_2\lambda)} \le 1$. When $\widehat{R_0} \le 1$, the disease-free equilibrium U_0 is globally asymptotically stable.

Proof: Construct Liapunov function as follows

$$L(t) = \epsilon e + (b+\epsilon+m_2\lambda)i \ge 0. \quad \frac{dL}{dt} = \epsilon\frac{de}{dt} + (b+\epsilon+m_2\lambda)\frac{di}{dt}$$

$$= \epsilon[\beta si - (b+\epsilon+m_2\lambda)e + \alpha ei] + (b+\epsilon+m_2\lambda)[\epsilon e - (\alpha+b+\gamma)i + \alpha i^2]$$

$$= i[\epsilon\beta s + \epsilon\alpha e + (b+\epsilon+m_2\lambda)\alpha i - (\alpha+b+\gamma)(b+\epsilon+m_2\lambda)]$$

$$\le i[max\{\epsilon\beta, \epsilon\alpha, (b+\epsilon+m_2\lambda)\alpha\} - (\alpha+b+\gamma)(b+\epsilon+m_2\lambda) \le 0. \quad (6)$$

Equality is established iff i = 0. According to LaSalle Invariant Set theory, the disease-free equilibrium is global asymptotically stable.

6. Endemic equilibrium

This section is organized as follows. The existence and uniqueness of the endemic equilibrium is proved. Then, the properties of the equilibrium will be discussed.

6.1. Existence and uniqueness of endemic equilibrium.

Theorem 6.1: When $R_0 > 1$, the endemic equilibrium is unstable.

Proof: Denote the endemic equilibrium as $P^* = (s^*, e^*, i^*)$, then P^* satisfies

$$\begin{cases} b + (\alpha-\beta-m_1\beta_0)s^*i^* - (b+\sigma)s^* = 0 \\ \beta s^*i^* - (b+\epsilon+m_2\lambda)e^* + \alpha e^*i^* = 0 \\ \epsilon e^* - (\alpha+b+\gamma)i^* + \alpha(i^*)^2 = 0 \end{cases} \quad (7)$$

From (7), the following can be obtained

$b + \alpha i(s+e+i) - b(s+e+i) - \sigma s - \alpha i - \gamma i - \lambda e - \beta_0 si = 0$

$\Leftrightarrow (b-\alpha i)(1-s-e-i) = \sigma s + \gamma i + \lambda e + \beta_0 si \ge 0$

where i satisfies $\{i \mid i \le max\{1, b/\alpha\}\}$. However, if $b > \alpha$, the constraint is useless.

Meanwhile, we can obtain

$$s^* = \frac{b}{(b+\sigma)+(\beta+m_1\beta_0-\alpha)i} \quad (8) \qquad e^* = \frac{\beta si}{(b+\epsilon+m_2\lambda)-\alpha i} = \frac{(\alpha+b+\gamma)i-\alpha i^2}{\varepsilon} \quad (9)$$

$$[(b+\sigma)+(\beta+m_0\beta_0-\alpha)i^*][(b+\epsilon+\lambda)-\alpha i^*][(\alpha+b+\gamma)-\alpha i^*] - b\beta\epsilon = 0 \quad (10)$$

Denote the left side of (10) as $Y(i^*)$. Thus, the existence can be proved by the following two equations and existence theorem of zero points.

$Y(0)=(b+\sigma)(b+\epsilon+\lambda)(\alpha+b+\gamma)(1-R_0)<0,$

$Y(1)=[(b+\sigma)+(\beta+m_0\beta_0-\alpha)][(b+\epsilon+\lambda)-\alpha](b+\gamma)-b\beta\epsilon>0.$

Then it comes to the uniqueness.

$Y'(i^*)=3(i^*)^2[(\beta+m_0\beta_0-\alpha)\alpha^2]+2i^*[\alpha^2(b+\sigma)-\alpha(b+\epsilon+\lambda)(\beta+m_0\beta_0-\alpha)$
$-\alpha(\alpha+b+\gamma)(\beta+m_0\beta_0-\alpha)]+[(\beta+m_0\beta_0-\alpha)(b+\epsilon+\lambda)(\alpha+b+\gamma)$
$-\alpha(b+\sigma)(\alpha+b+\gamma)-\alpha(b+\sigma)(b+\epsilon+\lambda)],$

$\Delta=4\alpha^2(\beta+m_0\beta_0-\alpha)^2\{[\frac{\alpha(b+\sigma)}{(\beta+m_0\beta_0-\alpha)}-(b+\epsilon+\lambda)-(\alpha+b+\gamma)]^2$
$-3[(b+\epsilon+\lambda)(\alpha+b+\gamma)-\frac{\alpha(b+\sigma)(b+\epsilon+\lambda)}{(\beta+m_0\beta_0-\alpha)}-\frac{\alpha(b+\sigma)(\alpha+b+\gamma)}{(\beta+m_0\beta_0-\alpha)}]\}.$

Denote $\alpha(b+\sigma)/(\beta+m_0\beta_0-\alpha)=P$, $(b+\epsilon+\lambda)=Q$, $(\alpha+b+\gamma)=W$. Thus,

$\Delta=4\alpha^2(\beta+m_0\beta_0-\alpha)^2(P^2+Q^2+W^2+PQ+PW-WQ)>0.$

Thus, there are two solutions to $Y'(i^*)=0$. We consider the relative positions of the two solutions.

Because $\alpha+b+\gamma/\alpha>1$, then $Y(\alpha+b+\gamma/\alpha)=-b\beta\epsilon<0.$

Thus, at most there can exist one zero point in (0,1) for $Y'(i*)$.

6.2. Local stability of endemic equilibrium.

We add these assumptions to the following part of theorems, which is reasonable in the modelling of HFMD.

$b>3\alpha,\epsilon>3\alpha,m_2\lambda>\alpha,\gamma>2\alpha$

Theorem 6.2: When $R_0>1$, the endemic equilibrium is locally asymptotically stable.

Proof: Do variable substitution to (10), $(\hat{s},\hat{e},\hat{i})=(s-s_0,e,i)=(s,e,i)-U_0.$

After the substitution, do linear approximation around the endemic equilibrium, we can get the matrix of the linear approximation system.

$$\begin{pmatrix} (\alpha-\beta-m_1\beta_0)i^*-(b+\sigma) & 0 & (\alpha-\beta-m_1\beta_0)s^* \\ \beta i^* & -(b+\epsilon+m_2\lambda)+\alpha i^* & \beta s^*+\alpha e^* \\ 0 & \varepsilon & -(\alpha+b+\gamma)+2\alpha i^* \end{pmatrix} \quad (11)$$

As defined in the Routh-Hurtwiz Criteria, we can obtain

$\Delta_1=(\alpha-\beta-m_1\beta_0)i^*-(b+\sigma)<0$, $\Delta_2=[-(b+\epsilon+m_2\lambda)+\alpha i^*]\Delta_1=-\frac{\beta s^* i^*}{e^*}\Delta_1,$

$\Delta_3=[-(\alpha+b+\gamma)+2\alpha i^*]\Delta_2-\epsilon[(\alpha-\beta-m_1\beta_0)i^*-(b+\sigma)](\beta s^*+\alpha e^*)+\epsilon(\alpha-\beta-m_1\beta_0)s^*\beta i^*$
$=[-(\alpha+b+\gamma)+2\alpha i^*][-(b+\epsilon+m_2\lambda)+\alpha i^*][(\alpha-\beta-m_1\beta_0)i^*-(b+\sigma)]$
$-\epsilon(\alpha-\beta-m_1\beta_0)i^*\alpha e^*+(b+\sigma)(b+\epsilon+m_2\lambda)[(\alpha+b+\gamma)-\alpha i^*]$
$=(\alpha-\beta-m_1\beta_0)i^*\{[-(\alpha+b+\gamma)+2\alpha i^*][-(b+\epsilon+m_2\lambda)+\alpha i^*]-\alpha i^*(\alpha+b+\gamma)+\alpha(i^*)^2\}$
$-(b+\sigma)\{[-(\alpha+b+\gamma)+2\alpha i^*][-(b+\epsilon+m_2\lambda)+\alpha i^*]-(b+\epsilon+m_2\lambda)[(\alpha+b+\gamma)-\alpha i^*]\}$
$=(\alpha-\beta-m_1\beta_0)i^*\{[(\alpha+b+\gamma)-2\alpha i^*][(b+\epsilon+m_2\lambda)-2\alpha i^*]-\alpha(i^*)^2\}$
$+\alpha i^*(b+\sigma)\{[(\alpha+b+\gamma)-2\alpha i^*]+(b+\epsilon+m_2\lambda)\}$
$=(\alpha-\beta-m_1\beta_0)i^*[(\alpha+b+\gamma)(b+\epsilon+m_2\lambda)-\alpha(i^*)^2]$
$+\alpha i^*[(b+\sigma)+2(\beta+m_1\beta_0-\alpha)]\{[(\alpha+b+\gamma)-2\alpha i^*]+(b+\epsilon+m_2\lambda)\}.$

According to $R_0>1$, $b\beta\epsilon>(b+\sigma)(\alpha+b+\gamma)(b+\epsilon+m_2\lambda)$

$\Delta_3\le-(\beta+m_1\beta_0-\alpha)i^*[(\alpha+b+\gamma)(b+\epsilon+m_2\lambda)-\alpha(i^*)^2]$

$$+\alpha i^*[\frac{b\beta\epsilon}{(\alpha+b+\gamma)(b+\epsilon+m_2\lambda)}+2(\beta+m_1\beta_0-\alpha)]\{[(\alpha+b+\gamma)-2\alpha i^*]+(b+\epsilon+m_2\lambda)\}$$
$$\le-(\beta+m_1\beta_0-\alpha)i^*[(\alpha+b+\gamma)(b+\epsilon+m_2\lambda)-\alpha(i^*)^2]$$
$$+\alpha i^*[\beta+2(\beta+m_1\beta_0-\alpha)]\{[(\alpha+b+\gamma)-2\alpha i^*]+(b+\epsilon+m_2\lambda)\}$$
$$\le-(\beta+m_1\beta_0-\alpha)i^*[(\alpha+b+\gamma)(b+\epsilon+m_2\lambda)-3\alpha(\alpha+b+\gamma)$$
$$-3\alpha(b+\epsilon+m_2\lambda)]+\alpha(\beta+m_1\beta_0-\alpha)(i^*)^2(i^*-5)$$

Thus, we only need to prove

$$(\alpha+b+\gamma)(b+\epsilon+m_2\lambda)-3\alpha(\alpha+b+\gamma)-3\alpha(b+\epsilon+m_2\lambda)>0$$
$$\frac{1}{3\alpha}-\frac{1}{(\alpha+b+\gamma)(b+\epsilon+m_2\lambda)}>0$$

which can be obtain under the newly added assumptions.

According to the Routh-Hurwitz Criteria, the endemic equilibrium is locally asymptotically stable.

6.3. Global asymptotic stability of endemic equilibrium.

According to [1] and [4], the global asymptotic stability of the endemic equilibrium can be proved via lemma 6.3. And the lemma is stated in [1] and partially stated in [4].

Lemma 6.3: Considering the open set $D\subset R^n$, $\forall x\in D, x\to f(x)\in R^n$ is C^1. As for the differential equation (12)

$x'=f(x)$ (12)

If the following four requirements are satisfied, then the unique equilibrium of system (12) is global asymptotic stability in D. (12) has a unique equilibrium in D and there exists a compact attractor $K\subset D$. The unique equilibrium is locally asymptotically stable. There is no equilibrium in any nonempty compact ω limit set. (This is the condition for Poincaré Bendixson property, Coddington et al. (1955)).

1. (12) has a unique equilibrium in D and there exists a compact attractor $K\subset D$.
2. The unique equilibrium is locally asymptotically stable.
3. There is no equilibrium in any nonempty compact ω limit set. (This is the condition for Poincaré Bendixson property, Coddington et al. (1955)).
4. For every periodic solution $x=p(t)$ to (12) when $P(0)\in D$, the second additive compound matrix is asymptotically stable.

For more details of lemma 6.3 and second additive compound matrix principal, one may refer to [1].

Theorem 6.3: When $R_0>1$, the endemic equilibrium is globally asymptotically stable.

Proof: Apply variable substitution to system (3), $x=(\hat{s},\hat{e},\hat{i})=(s-s^*,e-e^*,i-i^*)$. Denote the system after variable substitution as $x'=f(x)$ (13).

The Jacobian matrix for the system (13) is Df

$$\begin{pmatrix}(\alpha-\beta-m_1\beta_0)i^*-(b+\sigma) & 0 & (\alpha-\beta-m_1\beta_0)s^*\\ \beta i^* & -(b+\epsilon+m_2\lambda)+\alpha i^* & \beta s^*+\alpha e^*\\ 0 & \varepsilon & -(\alpha+b+\gamma)+2\alpha i^*\end{pmatrix}.$$

The feasible region for the system (13) is

$$D=\{(\hat{s},\hat{e},\hat{i})\,|-(s^*+e^*+i^*)\le\hat{s}+\hat{e}+\hat{i}\le1-(s^*+e^*+i^*)\}.$$

According to [1], the lemma 6.3 can be stated when the following three criteria are satisfied.

1. There is a compact attractor in D.
2. Poincaré-Bendixson property is satisfied.
3. The second additive compound matrix is asymptotically stable.

Proof of 1: For any starting point in D and system (3)

$$\frac{ds}{dt} \le b-(b+\sigma)s$$

Thus, when $t \to \infty$, $0 \le s \le b/b+\sigma$ and $0 \le e \le 1, 0 \le i \le 1$. Denote

$$K = \{(s,e,i) \mid 0 \le s \le \frac{b}{b+\sigma}, 0 \le e \le 1, 0 \le i \le 1\}.$$

K is compact subset and attract all the solutions to system (3). Thus, $K \subset D$ is a compact attractor.

Proof of 2: According to [1], we only need to prove the competitive system for Df is satisfied.

H = diag(- 1 , 1 , -1). HDfH=

$$\begin{pmatrix} (\alpha-\beta-m_1\beta_0)i^*-(b+\sigma) & 0 & (\alpha-\beta-m_1\beta_0)s^* \\ -\beta i^* & -(b+\epsilon+m_2\lambda)+\alpha i^* & -\beta s^*-\alpha e^* \\ 0 & -\epsilon & -(\alpha+b+\gamma)+2\alpha i^* \end{pmatrix}$$

satisfies competitive system. Thus, the Poincaré -Bendixson property is satisfied.

Proof of 3: The coefficient matrix of second additive compound of system (13) is

$$Df^{[2]} = \begin{pmatrix} \nu_1 & \beta s^*+\alpha e^* & -(\alpha-\beta-m_1\beta_0)s^* \\ \varepsilon & \nu_2 & 0 \\ 0 & \beta i^* & \nu_3 \end{pmatrix}$$

Then, we only need to prove the asymptotically stable of $Df^{[2]}$.

$$\nu_1 = (\alpha-\beta-m_1\beta_0)i^*-(b+\sigma)-(b+\epsilon+m_2\lambda)+\alpha i^*$$

$$\nu_2 = (\alpha-\beta-m_1\beta_0)i^*-(b+\sigma)-(\alpha+b+\gamma)+2\alpha i^*$$

$$\nu_3 = -(b+\epsilon+m_2\lambda)-(\alpha+b+\gamma)+3\alpha i^*$$

$$\beta s^*+\alpha e^* = (b+\epsilon+m_2\lambda)\frac{e^*}{i^*} = (b+\epsilon+m_2\lambda)[(\alpha+b+\gamma)-\alpha i^*]$$

$$\Delta_1 = \upsilon_1 = -(\beta+m_1\beta_0)i^*-(b+\sigma)-(b+\epsilon+m_2\lambda) < 0$$

$$\Delta_2 = \upsilon_1\upsilon_2-\epsilon(\beta s^*+\alpha e^*) = [(\beta+m_1\beta_0)i^*+(b+\sigma)+(\alpha+b+\gamma)-\alpha i^*]$$

$$[(\beta+m_1\beta_0)i^*+(b+\sigma)+(b+\epsilon+m_2\lambda)]-(b+\epsilon+m_2\lambda)[(\alpha+b+\gamma)-\alpha i^*]$$

$$= [(\beta+m_1\beta_0)i^*+(b+\sigma)]^2+[(\beta+m_1\beta_0)i^*+(b+\sigma)](b+\epsilon+m_2\lambda)$$

$$+[(\beta+m_1\beta_0)i^*+(b+\sigma)][(\alpha+b+\gamma)-\alpha i^*] > 0$$

$$\Delta_3 = \Delta_2\upsilon_3-\beta i^*\epsilon(\alpha-\beta-m_1\beta_0)s^* = \Delta_2\upsilon_3+\frac{b\beta\epsilon(\beta+m_1\beta_0-\alpha)i^*}{(b+\sigma)+(\beta+m_1\beta_0-\alpha)i^*}$$

$$< \Delta_2\upsilon_3+b\beta\epsilon = \Delta_2\upsilon_3+[(b+\sigma)+(\beta+m_1\beta_0-\alpha)i^*][(b+\epsilon+m_2\lambda)-\alpha i^*][(\alpha+b+\gamma)-\alpha i^*]$$

$$< \Delta_2\upsilon_3+[(b+\sigma)+(\beta+m_1\beta_0)i^*][(b+\epsilon+m_2\lambda)][(\alpha+b+\gamma)]$$

If we separate $[(\beta+m_1\beta_0)i^*+(b+\sigma)](\alpha+b+\gamma)$ from Δ_2 and separate $-(b+\epsilon+m_2\lambda)$ from ν_3, then we get $\Delta_3 < 0$.

According to Routh-Hurwitz criterion, system (13)' s second additive compound matrix is asymptotically stable, which means $Df^{[2]}$ is asymptotically stable.

In conclusion, the endemic equilibrium P^* is asymptotically stable.

7. Discussion

Considering the problem that is caused by the specific characteristic of the HFMD, the existing quarantine strategies for controlling the spread of infectious diseases may be not suitable for the HFMD. That is why we consider the protective quarantine model.

From the assumptions of the protective quarantine model, it is easy to notice that the model is only suitable for hand-foot-mouth disease and its related diseases. When considering the social expense of protective quarantine, it will reduce the cost sharply when we the quarantine are kids under 5 years old because kids under this period of age always do not have school to go to and are mainly taken care of by their parents. As for the quarantine, in the model, there are mainly two kinds of quarantines. The first is the susceptible, and the second is the exposed. From the basic reproduction numbers (4), we can obtain the effect after the quarantine strategy is quite different when we make the susceptible to be the quarantine and make the exposed to be the quarantine. It seems more easy and effective to control the susceptible than the exposed. However, in reality, the amount of the exposed is largely less than the amount of the susceptible, it will be more applicable to make the exposed to be the quarantine.

From the results and properties of the protective quarantine model, the disease-free equilibrium and endemic equilibrium all have good properties, which makes them suitable for the reality. For example, when the system satisfies the assumptions of the protective quarantine and the condition of the disease-free equilibrium, the system will experience globally asymptotically stability wherever the starting point is.

In conclusion, the protective quarantine model is reasonable and applicable.

8. Empirical Simulations

In order to simulate the spread of hand-foot-mouth disease, we set the parameters in the following two tables, different conditions for the equilibriums of the protective quarantine model are satisfied.

Table 1. Parameter Setting 1.

Parameter	b d β ε α $m_1\beta_0$ $m_2\lambda$ σ ω γ
Number	0.013, 0.006, 1.7, 0.25, 0.003, 0.1, 0.1, 0.15, 0.2, 0.117

When we adopt the parameter setting in Table 1, $R_0 = 0.702079 < 1$, $\hat{R}_0 = 8.8029 > 1$. Thus, according to theorem 5.1 and 5.2, the disease-free equilibrium is locally asymptotically stable and globally asymptotically stable. Using the x-axis to represent time and the y-axis to represent proportion, we can get the Figure 2.

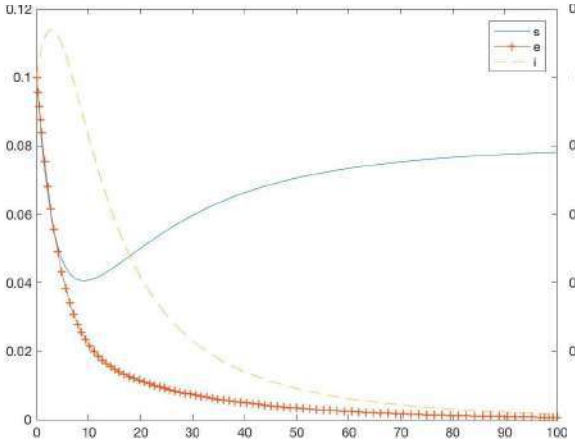


Figure 2. Simulation with disease-free equilibrium.

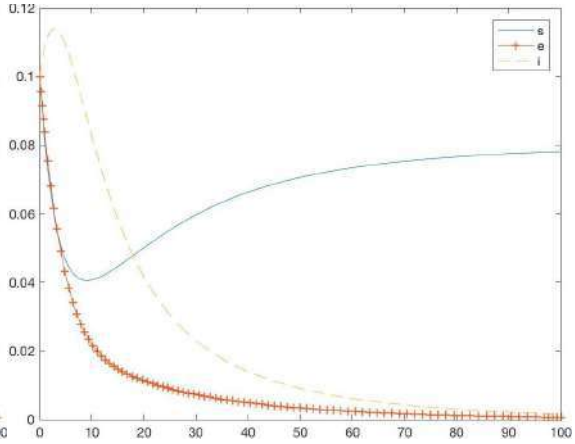


Figure 3. Simulation with endemic equilibrium.

When we adopt the parameter setting in Table 2, $R_0 = 1.026 > 1$. According to theorem 6.1, 6.2 and 6.3, since the assumptions in theorem 6.2 are satisfied, there exist an endemic equilibrium P = (0.07773, 0.001193, 0.002244). And the result is in Figure 3.

Table 2. Parameter Setting 2.

Parameter	b d β ε α $m_1\beta_0$ $m_2\lambda$ σ ω γ
Number	0.013, 0.006, 1.8, 0.25, 0.003, 0.1, 0, 0.15, 0.2, 0.117

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