Lab lecture 4

Learning objectives

- Applying the Nyquist theorem (without filtering)

Choosing the sampling frequency: Nyquist Sampling theorem

- Nyquist theorem: An analog signal should be sampled at a frequency that is twice the bandwidth of the signal

Choosing the sampling frequency: Nyquist Sampling theorem

- Nyquist theorem: An analog signal should be sampled at a frequency that is twice the bandwidth of the signal
- What is the bandwidth of a signal?
- To find out the bandwidth, obtain the Fourier transform of the signal …

Choosing the sampling frequency: Nyquist Sampling theorem

- Nyquist theorem: An analog signal should be sampled at a frequency that is twice the bandwidth of the signal
- What is the bandwidth of a signal?
- To find out the bandwidth, obtain the Fourier transform of the signal …
- … but the Fourier transform requires the entire signal to be available,…

Choosing the sampling frequency: Nyquist Sampling theorem

- Nyquist theorem: An analog signal should be sampled at a frequency that is twice the bandwidth of the signal
- What is the bandwidth of a signal?
- To find out the bandwidth, obtain the Fourier transform of the signal …
- … but the Fourier transform requires the entire signal to be available,…
- Which is not possible in practice!

Choosing the sampling frequency: Nyquist Sampling theorem

- Nyquist theorem: An analog signal should be sampled at a frequency that is twice the bandwidth of the signal
- What is the bandwidth of a signal?
- To find out the bandwidth, obtain the Fourier transform of the signal …
- … but the Fourier transform requires the entire signal to be available,…
- Which is not possible in practice!
- If we know the functional form of a signal and need to sample it at the Nyquist rate, we could get its Fourier transform analytically…

Choosing the sampling frequency: Nyquist Sampling theorem

- Nyquist theorem: An analog signal should be sampled at a frequency that is twice the bandwidth of the signal
- What is the bandwidth of a signal?
- To find out the bandwidth, obtain the Fourier transform of the signal …
- … but the Fourier transform requires the entire signal to be available,…
- Which is not possible in practice!
- If we know the functional form of a signal and need to sample it at the Nyquist rate, we could get its Fourier transform analytically…
- … But this is seldom possible and one needs to resort to a numerical calculation …

Choosing the sampling frequency: Nyquist Sampling theorem

- Nyquist theorem: An analog signal should be sampled at a frequency that is twice the bandwidth of the signal
- What is the bandwidth of a signal?
- To find out the bandwidth, obtain the Fourier transform of the signal …
- … but the Fourier transform requires the entire signal to be available,…
- Which is not possible in practice!
- If we know the functional form of a signal and need to sample it at the Nyquist rate, we could get its Fourier transform analytically…
- … But this is seldom possible and one needs to resort to a numerical calculation …
- … But any numerical calculation requires sampling, which brings us back to the question of what should be the sampling frequency!

Choosing the sampling frequency: Nyquist Sampling theorem

- Nyquist theorem: An analog signal should be sampled at a frequency that is twice the bandwidth of the signal
- What is the bandwidth of a signal?
- To find out the bandwidth, obtain the Fourier transform of the signal …
- … but the Fourier transform requires the entire signal to be available,…
- Which is not possible in practice!
- If we know the functional form of a signal and need to sample it at the Nyquist rate, we could get its Fourier transform analytically…
- … But this is seldom possible and one needs to resort to a numerical calculation
- … But any numerical calculation requires sampling, which brings us back to the question of what should be the sampling frequency!
- In practical signal processing applications, the Nyquist rate is enforced using an anti-aliasing filter, not by guessing its value

Choosing the sampling frequency: Nyquist Sampling theorem

- In this Lab: Where the Fourier transform cannot be obtained analytically, we will use a best guess approach to get the Nyquist rate

Choosing the sampling frequency: Nyquist Sampling theorem

- In this Lab: Where the Fourier transform cannot be obtained analytically, we will use a best guess approach to get the Nyquist rate
  - A good starting guess for bandwidth: highest **instantaneous frequency** $(f(t))$ in the signal

$$s(t) = a(t)\cos\big(2\pi\phi(t)\big)$$

$$f(t) = \frac{d\phi}{dt}$$

  - Best guess Nyquist rate: $2\times f_{max}$ where

$$f_{max} = \max_{t\in[T_1,T_2]} f(t)$$

Choosing the sampling frequency: Nyquist Sampling theorem

- In this Lab: Where the Fourier transform cannot be obtained analytically, we will use a best guess approach to get the Nyquist rate
  - A good starting guess for bandwidth: highest **instantaneous frequency** $(f(t))$ in the signal

$$s(t) = a(t)\cos\big(2\pi\phi(t)\big)$$

$$f(t) = \frac{d\phi}{dt}$$

  - Best guess Nyquist rate: $2\times f_{max}$ where

$$f_{max} = \max_{t\in[T_1,T_2]} f(t)$$

Note: Instantaneous frequency is not the same as Fourier frequency!

QUADRATIC CHIRP SIGNAL

$$f(t) = A\sin(2\pi\Phi(t))$$

Instantaneous phase:

$$\Phi(t) = a_1 t + a_2 t^2 + a_3 t^3$$

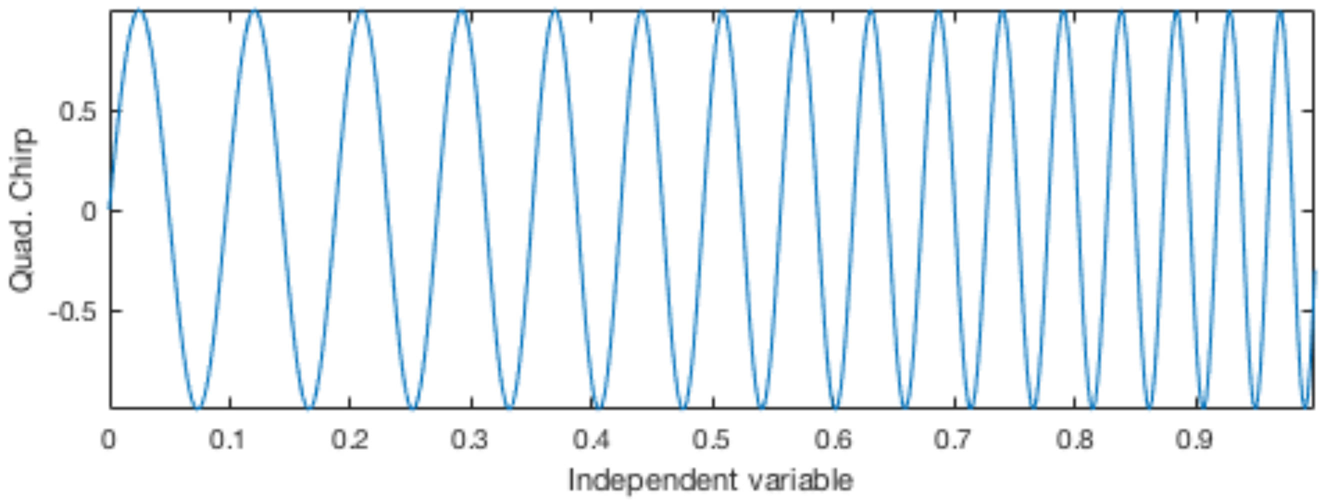
Parameters of the signal:

$$\mathrm{A}, a_1, a_2, a_3$$

Instantaneous frequency:

$$f(t) = \frac{d\Phi}{dt}$$
$$= a_1 + 2a_2 t + 3a_3 t^2$$

$f(t)$ increases with t
$1/f(t)$ (Instantaneous period) decreases with t



Example taken from textbook ("Swarm intelligence methods for Statistical Regression", Chapter 1)

QUADRATIC CHIRP SIGNAL

Instantaneous frequency:

$$f(t) = \frac{d\Phi}{dt}$$
$$= a_1 + 2a_2 t + 3a_3 t^2$$

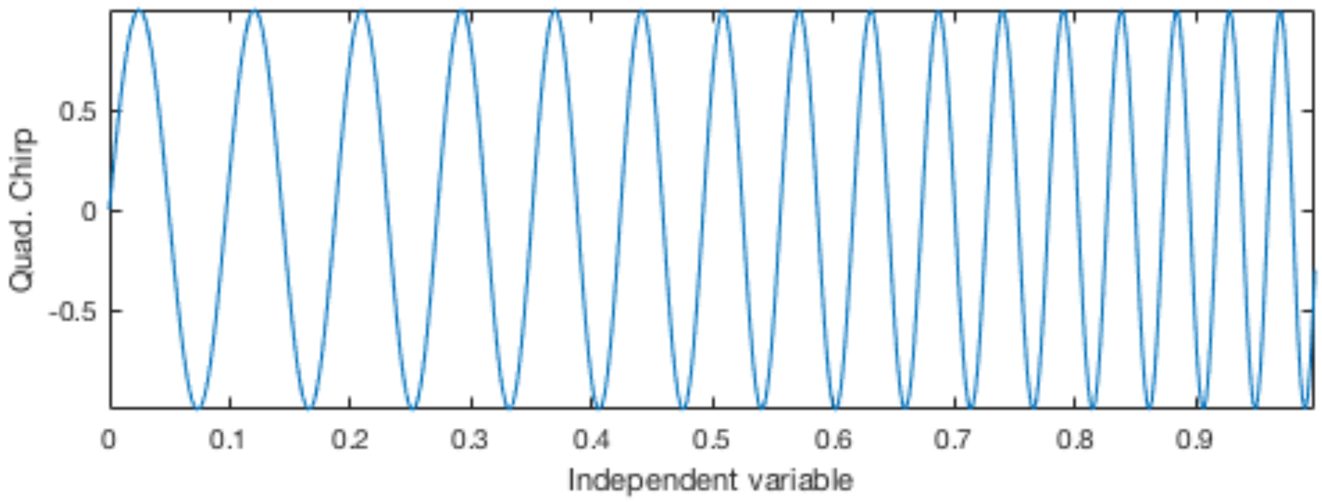
If a_2, a_3 are positive, $f(t)$ increases monotonically

If t is in units of seconds, then

$$f(1) = a_1 + 2a_2 + 3a_3$$

is the highest instantaneous frequency in the signal for $t \leq 1$

$f(t)$ increases with t
$1/f(t)$ (Instantaneous period) decreases with t



Example taken from textbook ("Swarm intelligence methods for Statistical Regression", Chapter 1)

QUADRATIC CHIRP SIGNAL

- Look at DATASCIENCE_COURSE/SIGNALS/testcrcbgenqcsig.m
- To generate the sampled QC signal, we have to fix the sampling times
- To fix the sampling times, we have to fix the sampling interval (reciprocal of sampling frequency)

```
%% Plot the quadratic chirp signal
% Signal parameters
a1=10;
a2=3;
a3=3;
A = 10;
% Instantaneous frequency after 1 sec is
maxFreq = a1+2*a2+3*a3;
samplFreq = 5*maxFreq;
samplIntrvl = 1/samplFreq;

% Time samples
timeVec = 0:samplIntrvl:1.0;
```

QUADRATIC CHIRP SIGNAL

1. We have fixed the time interval over which we want the signal to be $[0,1]$ sec

```
%% Plot the quadratic chirp signal
% Signal parameters
a1=10;
a2=3;
a3=3;
A = 10;
% Instantaneous frequency after 1 sec is
maxFreq = a1+2*a2+3*a3;
samplFreq = 5*maxFreq;
samplIntrvl = 1/samplFreq;

% Time samples
timeVec = 0:samplIntrvl:1.0;
```

QUADRATIC CHIRP SIGNAL

1. We have fixed the time interval over which we want the signal to be $[0,1]$ sec
2. We obtain the highest instantaneous frequency over this interval (time derivative of instantaneous phase)

```
%% Plot the quadratic chirp signal
% Signal parameters
a1=10;
a2=3;
a3=3;
A = 10;
% Instantaneous frequency after 1 sec is
maxFreq = a1+2*a2+3*a3;
samplFreq = 5*maxFreq;
samplIntrvl = 1/samplFreq;

% Time samples
timeVec = 0:samplIntrvl:1.0;
```

QUADRATIC CHIRP SIGNAL

1. We have fixed the time interval over which we want the signal to be $[0,1]$ sec
2. We obtain the highest instantaneous frequency over this interval (time derivative of instantaneous phase)
3. We set the sampling frequency to be (at least) twice this value

```
%% Plot the quadratic chirp signal
% Signal parameters
a1=10;
a2=3;
a3=3;
A = 10;
% Instantaneous frequency after 1 sec is
maxFreq = a1+2*a2+3*a3;
samplFreq = 5*maxFreq;
samplIntrvl = 1/samplFreq;

% Time samples
timeVec = 0:samplIntrvl:1.0;
```

QUADRATIC CHIRP SIGNAL

1. We have fixed the time interval over which we want the signal to be $[0,1]$ sec
2. We obtain the highest instantaneous frequency over this interval (time derivative of instantaneous phase)
3. We set the sampling frequency to be (at least) twice this value
4. The sampling interval is obtained from the sampling frequency

```
%% Plot the quadratic chirp signal
% Signal parameters
a1=10;
a2=3;
a3=3;
A = 10;
% Instantaneous frequency after 1 sec is
maxFreq = a1+2*a2+3*a3;
samplFreq = 5*maxFreq;
samplIntrvl = 1/samplFreq;

% Time samples
timeVec = 0:samplIntrvl:1.0;
```

Tasks

- For your assigned signal …

1. Task: Guess the Nyquist sampling frequency following the example shown earlier
    1. Identify the instantaneous phase function: $s(t) = a(t)\sin(2\pi\Phi(t))$
        1. If $a(t)$ is also oscillatory, then it will be the one that oscillates more slowly than $\sin(2\pi\Phi(t))$
    2. Get the instantaneous frequency $f(t) = \frac{d\Phi(t)}{dt}$, and find out its maximum value in the time interval over which the signal samples are to be computed
2. Task: If you have not done so, modify the `test<funcname>.m` script that you wrote to compute the sampling frequency based on the above guess (see DATASCIENCE_COURSE/SIGNALS/`testcrcbgenqcsig.m`)

- Task: Plot the signal for a sampling frequency that is:
    1. 5 times the Nyquist sampling frequency
    2. ½ of the Nyquist sampling frequency