To whom it may concern,

I wrote this research report as a research analyst at the Summer Undergraduate Research Program at the Chinese University of Hong Kong. It mainly discusses how I investigate the causality between American interest rates and the price of Bitcoin using the VAR model and other data science techniques.

Thank you for reading, and have a great day.

Regards,

Yunyao (Richard) Xue

The Causal Relationship Between the American Interest Rate and the Price of Bitcoin in the Past 4 Years

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Introduction

Interest rate is a critical index in macroeconomics. It is indispensable for every individual within society and the development of the whole economy. At the individual level, for instance, it influences the cost of borrowing. So those who need money to start a business must wait for the right time to negotiate with banks or credit unions. Also, the interest rate influences the return on savings. Hoarding the money in your bank account or taking it out for an active investment is closely connected with the interest rate level.

In the past, we usually connected the interest rate with traditional currencies- US dollars, Japanese yen, and Chinese Yuan- and classic economic and financial activities, such as mortgage applications, saving and borrowing, stock and bond investment, etc. However, in recent years, the popularity of cryptocurrencies has announced a new stage of financial investment and economic progress. Suppose we want to research more and understand the impact and influence of cryptocurrencies from different perspectives. In that case, an appropriate method is to link them to an essential economic index, such as the American interest rate. Meanwhile, investigating the relationship between the price of cryptocurrencies and the American interest rate can also help us figure out more potential variables that can impact the American interest rate. The results of this paper provide a lot of new possible perspectives for us to understand the behaviors of interest rates and cryptocurrencies in today's world.

This research paper aims to discuss the causal relationship between the American interest rate and the Bitcoin price, the most renowned cryptocurrency currently. We focus on the influence the American interest rate, the independent variable, can bring on the Bitcoin price, the dependent variable.

Data

If we want to understand the causal relationship between two variables, we need to collect a lot of data, implement the data into statistical models, and analyze the results. In terms of data, the daily open Bitcoin price in the workdays of the past four years will be contained. Also, I include two types of interest rates as independent variables: the daily treasury real long-term rate (the long-term interest rate in the following paragraphs) and the daily treasure par yield curve rates (the short-term interest rate in the following section). All data is on a day-to-day level, from the

beginning of 2018 to the end of 2021, but only on workdays (because the two independent variables are only calculated on workdays).

These two interest rates are from the United States Department of the Treasury. According to the Department of the Treasury (n.d.), the Long-Term Real Rate Average is "the unweighted average of bid real yields on all outstanding Treasury Inflation-Protected Securities with remaining maturities of more than ten years and is intended as a proxy for long-term real rates." The daily treasure par yield curve rate is commonly regarded as the constant maturity treasury rate. This daily-based index relates the yield on a security to its time to maturity. It is "based on the closing market bid prices on the most recently auctioned Treasury securities in the over-the-counter market" (US Department of the Treasury, n.d.).

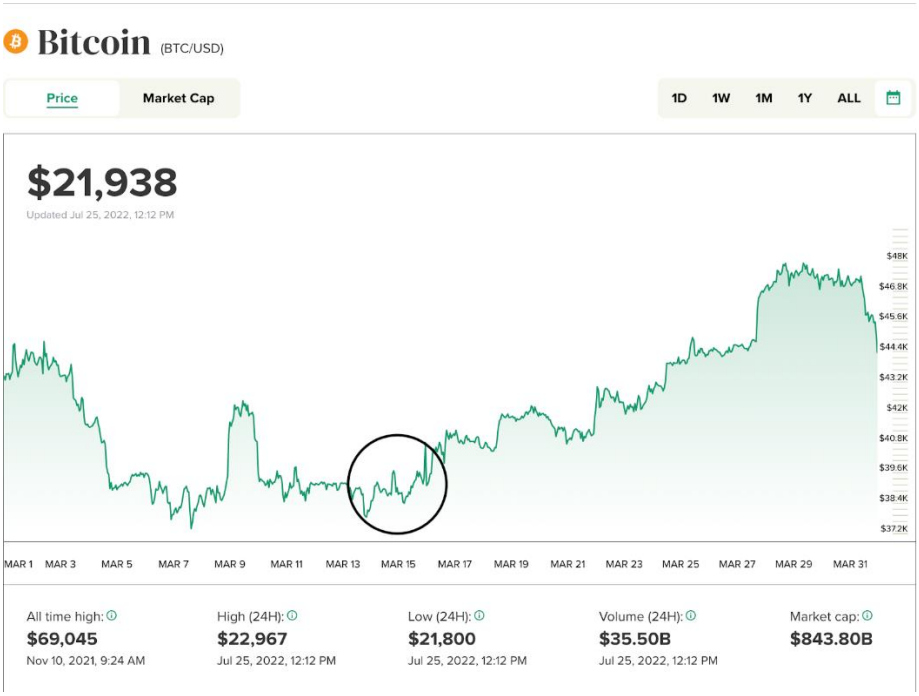
I have both a short-term interest rate and a long-term interest rate to have a more comprehensive understanding of the topic. Many interest rate types have different units, functions, and interpretations. To contain both a long-term interest rate and a short-term interest rate is to have a more general look and answer the question from different perspectives.

A potential problem that can cause biases is that the Bitcoin price is a global variable. The American interest rate is not the only one that influences it. Other countries' interest rates are also more or less related to the price of Bitcoin. Even if we include both the short-term interest rates and the long-term interest rates, without more global variables beyond the range of the U.S., our conclusion is not general and objective. So, I also include some global variables, just like the Bitcoin price, such as the daily open price of gold, the open index of the NASDAQ Composite, and the US dollar to Japanese Yen exchange rate. Like the previous variables, all data is workday-only and daily based from the beginning of 2018 to the end of 2021. We have six variables and more than 1000 observations in total.

Methodology & Result

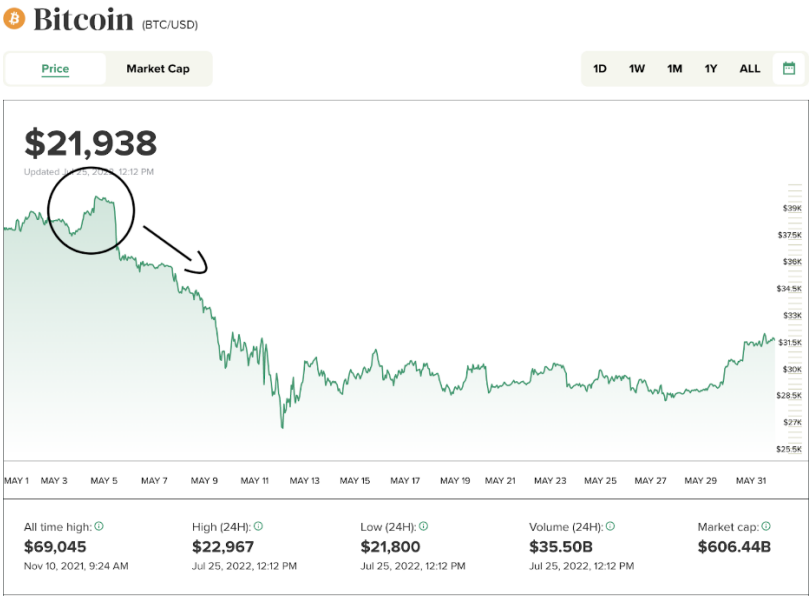
Before we get into the part of model selection and data cleaning, I also read some papers and online passages that discuss the relationship between the Bitcoin price and interest rates to get some empirical insights.

One passage talks about the relationship between American interest rates and the Bitcoin price and uses two specific examples to answer this question (Alex Gailey, 2022). Let's look at them.



As the first graph above shows, on March 13th, Bitcoin's price first dropped (then gradually climbed up to a higher level), while the Federal Reserve decided the interest rate: "the Fed approved a 0.25% rate hike, which was the first increase since 2018" (Alex Gailey, 2022). So, from this simple example, the higher the interest rate, the lower the Bitcoin price.

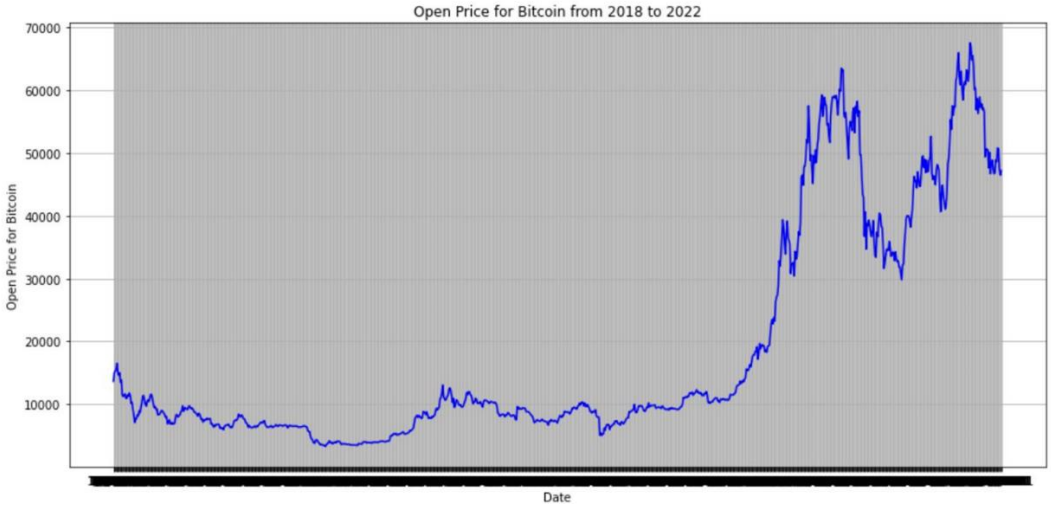
However, the second graph below also doesn't provide a similar result. Bitcoin's price spiked immediately after May 3rd. On the same day, the Federal Reserve had a meeting lasting for two days. During the meeting, the Fed approved a half-percentage point hike in interest rates, which was a sharp increase. Now, we got a different conclusion. From this example, the higher the interest rate, the higher the Bitcoin price.



Since we got a contradictive conclusion based on the article's data, we can't easily reach a guaranteed answer. So, we still have to implement the data into some models and analyze the result by modifying models.

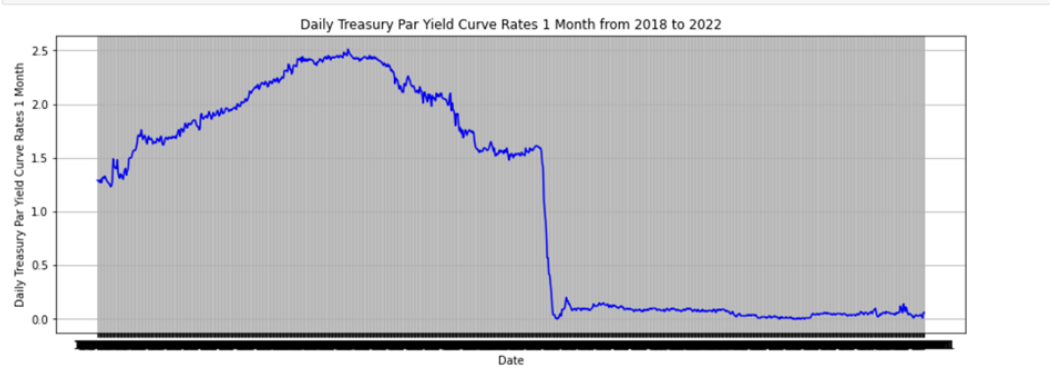
The model will be the vector autoregressive model, also known as the VAR model. As a multivariate time series model, it connects current observations of a variable with past observations of itself and past observations of other variables. It's a powerful tool for focusing on the causal relationship between multiple independent and dependent variables.

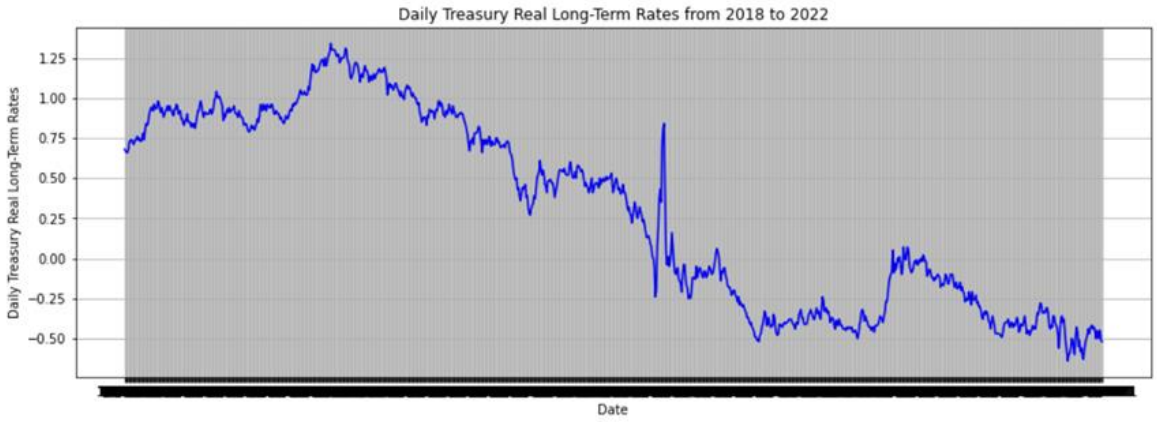
After we prepare all the essential data and tools, we do the data cleaning. Since the VAR model is only accessible to stationary data, it's necessary to check whether our data is stationary. If not, we still have to take the first difference of each variable and get the stationary data. A quick way to check if data is stationary is to look at the diagram that shows how the data goes throughout the period.



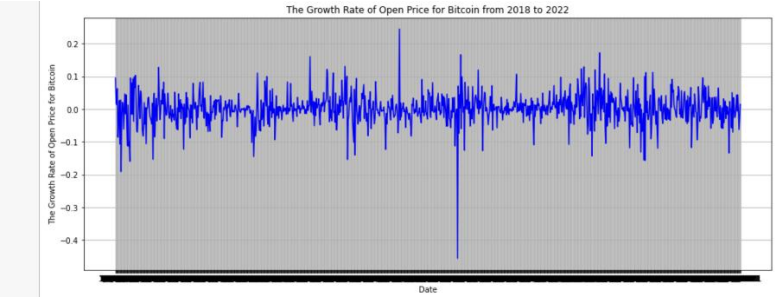
As we can see, the data of the open price for Bitcoin is not stationary: it keeps being stable for almost 2/3 of the whole time, suddenly hikes to a higher level, and fluctuates for the remaining 1/3 of the time.

The same thing applies to all four other variables: they are all not stationary. They don't fluctuate about a constant mean value. The following graphs are for short-term interest rates and long-term interest rates.

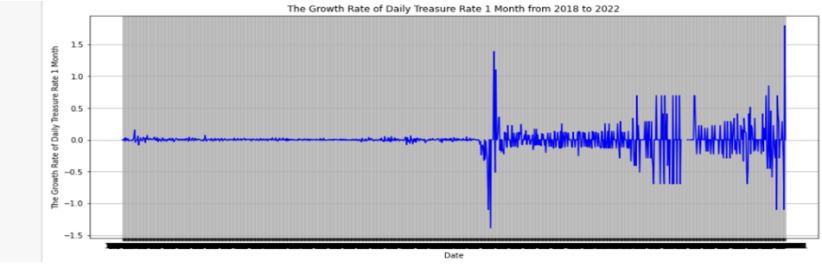


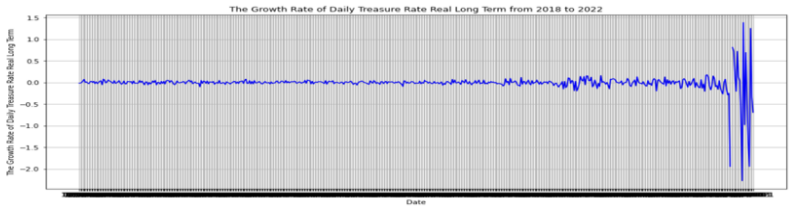


The same applies to the gold price and the exchange rate, but we skip the graphs for now. Now, we successfully take the first difference of all data by using the python codes that take the natural log of data and drop the meaningless numbers (infinity or naN). Let's redraw their graphs and see how things go.



The open price for Bitcoin is stationary now: it fluctuates about a constant mean value, which is a good thing. The same result happens in other kinds of data, too.





After the data cleaning, we should get to the part of applying data to the model. In terms of the VAR model, there are two kinds of them in this research paper: simple VAR and advanced VAR.

Simple VAR stands for the VAR models with limited variables: only two to three variables. By contrast, advanced VAR represents the models that contain four to five variables, which is the maximum number of variables we can have here. I divide them into two parts to make the research progress more logical and steadier. We first record some direct results and make some inferences based on our observations of simple models that don't have a lot of variables. When we have enough understanding and knowledge, we get to models with four or five variables and do more advanced research steps with them, such as variance decomposition, impulse responses, and sensitivity analysis.

Even though I have a lot of results of different combinations of variables, I won't show all of them due to limited space. I will only show some important models with content worth analyzing and discussing. I will first lay out my tried combinations and observe the results. So, this part is not only about Methodology but also about the Results of experiments.

For the two-variable-test, the first one is the VAR (2) test between the short-term interest rate and the Bitcoin price. The result is below.

As we can see, a two-variable test is easy to observe due to the limited variable number, but it doesn't reflect many strong results. The p-value for all coefficients is not strong enough to provide robust results to our question. However, we can still record some key elements. For instance, the last period's price of Bitcoin brings a slightly negative influence on this period's Bitcoin price. Meanwhile, the last period's short-term interest rate slightly increases this period's Bitcoin price.

```
  Summary of Regression Results
==================================
Model:                         VAR
Method:                        OLS
Date:           Sun, 14, Aug, 2022
Time:                     18:18:03
----------------------------------------------------------------------
No. of Equations:         2.00000    BIC:                   -9.58442
Nobs:                     971.000    HQIC:                  -9.61553
Log likelihood:           1932.05    FPE:                6.54217e-05
AIC:                     -9.63466    Det(Omega_mle):     6.47531e-05
----------------------------------------------------------------------
Results for equation Bitcoin Price: Open (Only Weekdays)
=================================================================================================================
                                                  coefficient       std. error           t-stat            prob
-----------------------------------------------------------------------------------------------------------------
const                                                0.001513         0.001500            1.009           0.313
L1.Bitcoin Price: Open (Only Weekdays)              -0.005779         0.032043           -0.180           0.857
L1.Daily Treasury Par Yield Curve Rates 1 Month      0.014393         0.008645            1.665           0.096
L2.Bitcoin Price: Open (Only Weekdays)               0.074654         0.031985            2.334           0.020
L2.Daily Treasury Par Yield Curve Rates 1 Month     -0.014932         0.008668           -1.723           0.085
=================================================================================================================

Results for equation Daily Treasury Par Yield Curve Rates 1 Month
=================================================================================================================
                                                  coefficient       std. error           t-stat            prob
-----------------------------------------------------------------------------------------------------------------
const                                               -0.005390         0.005547           -0.972           0.331
L1.Bitcoin Price: Open (Only Weekdays)              -0.182116         0.118466           -1.537           0.124
L1.Daily Treasury Par Yield Curve Rates 1 Month     -0.074858         0.031961           -2.342           0.019
L2.Bitcoin Price: Open (Only Weekdays)               0.425730         0.118251            3.600           0.000
L2.Daily Treasury Par Yield Curve Rates 1 Month     -0.067046         0.032047           -2.092           0.036
=================================================================================================================

Correlation matrix of residuals
                                        Bitcoin Price: Open (Only Weekdays)  Daily Treasury Par Yield Curve Rates 1 Month
Bitcoin Price: Open (Only Weekdays)                                1.000000                                      0.010717
Daily Treasury Par Yield Curve Rates 1 Month                       0.010717                                      1.000000
```

What about the 2-variable test between the long-term interest rate and the Bitcoin price?

```
  Summary of Regression Results
==================================
Model:                         VAR
Method:                        OLS
Date:           Sun, 14, Aug, 2022
Time:                     18:18:03
----------------------------------------------------------------------
No. of Equations:         2.00000    BIC:                   -10.2484
Nobs:                     546.000    HQIC:                  -10.2963
Log likelihood:           1279.83    FPE:                3.27322e-05
AIC:                     -10.3272    Det(Omega_mle):     3.21408e-05
----------------------------------------------------------------------
Results for equation Daily Treasury Real Long-Term Rates
=================================================================================================================
                                                  coefficient       std. error           t-stat            prob
-----------------------------------------------------------------------------------------------------------------
const                                               -0.005595         0.005044           -1.109           0.267
L1.Daily Treasury Real Long-Term Rates              -0.028935         0.043993           -0.658           0.511
L1.Bitcoin Price: Open (Only Weekdays)               0.155966         0.104519            1.492           0.136
L2.Daily Treasury Real Long-Term Rates              -0.148188         0.043995           -3.368           0.001
L2.Bitcoin Price: Open (Only Weekdays)              -0.306724         0.104110           -2.946           0.003
=================================================================================================================

Results for equation Bitcoin Price: Open (Only Weekdays)
=================================================================================================================
                                                  coefficient       std. error           t-stat            prob
-----------------------------------------------------------------------------------------------------------------
const                                               -0.001571         0.002077           -0.756           0.449
L1.Daily Treasury Real Long-Term Rates              -0.036627         0.018114           -2.022           0.043
L1.Bitcoin Price: Open (Only Weekdays)               0.001891         0.043035            0.044           0.965
L2.Daily Treasury Real Long-Term Rates              -0.045583         0.018115           -2.516           0.012
L2.Bitcoin Price: Open (Only Weekdays)               0.100871         0.042867            2.353           0.019
=================================================================================================================
```

Compared to the model with the short-term interest rate involved, an interesting result here is the last two periods of the long-term interest rate have a strong negative correlation with the Bitcoin price. The p-value here is relatively convincing: it's lower than 0.05. The same thing applies to the last two periods of the Bitcoin price: it also has a convincing p-value, and the coefficient shows that it positively impacts the Bitcoin price.

Let’s apply independent interest rates together and get a 3-variable test. The result is below (with only the part reflecting the variables influencing the Bitcoin price).

```
  Summary of Regression Results
==================================
Model:                         VAR
Method:                        OLS
Date:           Sun, 14, Aug, 2022
Time:                     18:18:04
--------------------------------------------------------------------
No. of Equations:         3.00000    BIC:                   -16.0888
Nobs:                     546.000    HQIC:                  -16.1896
Log likelihood:           2134.21    FPE:                8.72649e-08
AIC:                     -16.2543    Det(Omega_mle):     8.39929e-08
--------------------------------------------------------------------
Results for equation Bitcoin Price: Open (Only Weekdays)
=================================================================================================================
                                                  coefficient       std. error           t-stat            prob
-----------------------------------------------------------------------------------------------------------------
const                                               -0.001883         0.002087           -0.903           0.367
L1.Bitcoin Price: Open (Only Weekdays)               0.000081         0.043144            0.002           0.999
L1.Daily Treasury Real Long-Term Rates              -0.041612         0.018389           -2.263           0.024
L1.Daily Treasury Par Yield Curve Rates 1 Month     -0.009634         0.038589           -0.250           0.803
L2.Bitcoin Price: Open (Only Weekdays)               0.105350         0.043009            2.449           0.014
L2.Daily Treasury Real Long-Term Rates              -0.048643         0.018219           -2.670           0.008
L2.Daily Treasury Par Yield Curve Rates 1 Month     -0.046451         0.038996           -1.191           0.234
=================================================================================================================
```

Still, most of the data are not significant. The p-value for most variables is not strong enough to show trustable correlations. However, a notable exception is the Long-term interest rate from the last two periods. It’s still negatively correlated with the bitcoin price, and the significance is lower than 0.05. Compared to it, the short-term interest rate doesn’t have a good enough statistical significance: its p-value is too high. So, from these three simple models, the long-term interest rate has a higher statistical significance than the short-term interest rate, and it always negatively influences the Bitcoin price. The short-term interest rate doesn’t have an apparent sign of correlation with the Bitcoin price.

Because we only use the VAR (2) model for interpretation simplicity, we never try other VAR models with the same combination. I also try from VAR (1) to VAR (10) to see the best VAR model. In other words, I want to find the VAR model with the lowest AIC in three variables. And the result is VAR (6). It has the lowest AIC, around -17.6.

Let’s move to the advanced part with at least four variables to see more apparent results.

```
  Summary of Regression Results
==================================
Model:                         VAR
Method:                        OLS
Date:           Sun, 14, Aug, 2022
Time:                     18:18:04
----------------------------------------------------------------------
No. of Equations:         4.00000    BIC:                   -25.9052
Nobs:                     546.000    HQIC:                  -26.0780
Log likelihood:           4086.62    FPE:                4.22960e-12
AIC:                     -26.1889    Det(Omega_mle):     3.96185e-12
----------------------------------------------------------------------
Results for equation Bitcoin Price: Open (Only Weekdays)
=================================================================================================================
                                                  coefficient       std. error           t-stat            prob
-----------------------------------------------------------------------------------------------------------------
const                                               -0.001950         0.002085          -0.935           0.350
L1.Bitcoin Price: Open (Only Weekdays)              -0.002870         0.043469          -0.066           0.947
L1.Daily Treasury Real Long-Term Rates              -0.031345         0.019289          -1.625           0.104
L1.Daily Treasury Par Yield Curve Rates 1 Month     -0.012745         0.038714          -0.329           0.742
L1.Gold Price Open                                   0.512097         0.279660           1.831           0.067
L2.Bitcoin Price: Open (Only Weekdays)               0.097259         0.043442           2.239           0.025
L2.Daily Treasury Real Long-Term Rates              -0.044859         0.019311          -2.323           0.020
L2.Daily Treasury Par Yield Curve Rates 1 Month     -0.047037         0.039163          -1.201           0.230
L2.Gold Price Open                                  -0.106513         0.276242          -0.386           0.700
=================================================================================================================
```

This is the result panel of a four-variable test containing two kinds of interest rates, the gold price, and the Bitcoin price. The long-term interest rate still has the best significance. For now, the long-term interest rate seems more significant than the short-term interest rate. And the lowest AIC happens in VAR (6).

```
----------------------------------------------------------------------
Results for equation Bitcoin Price: Open (Only Weekdays)
=================================================================================================================
                                                  coefficient       std. error           t-stat            prob
-----------------------------------------------------------------------------------------------------------------
const                                               -0.001840         0.002095          -0.878           0.380
L1.Bitcoin Price: Open (Only Weekdays)              -0.000234         0.043326          -0.005           0.996
L1.Daily Treasury Real Long-Term Rates              -0.040359         0.018893          -2.136           0.033
L1.Daily Treasury Par Yield Curve Rates 1 Month     -0.011840         0.038844          -0.305           0.761
L1.USD to Yen Open                                  -0.419326         0.490120          -0.856           0.392
L2.Bitcoin Price: Open (Only Weekdays)               0.104168         0.043212           2.411           0.016
L2.Daily Treasury Real Long-Term Rates              -0.047775         0.018274          -2.614           0.009
L2.Daily Treasury Par Yield Curve Rates 1 Month     -0.050888         0.039412          -1.291           0.197
L2.USD to Yen Open                                  -0.159077         0.503117          -0.316           0.752
=================================================================================================================
```

Here we have the VAR (2) of the model with two interest rates, the bitcoin price, and the exchange rate between Dollars and Yen. The long-term interest rate still has the best performance. For this combination, the model with the best AIC is VAR (7).

Till now, we are still at the level of four different variables. We will enter the part with five variables, a slightly more advanced VAR model application. But we can get some simple conclusions now: First, the long-term interest rate always has a better significance than the short-term interest rate. Second, though the coefficient is not numerically huge (strong), the long-term interest rate's influence on the Bitcoin price always tends to be negative.

At this level, the increase in the long-term interest rate brings a 5 to 10% decrease in the price of Bitcoin.

Now, let's move to the 5-variable test. I only use VAR (2) for term interpretation, and mostly I use variables with lag 2 to interpret.

Results for equation Bitcoin Price: Open (Only Weekdays)

	coefficient	std. error	t-stat	prob
const	-0.002113	0.002067	-1.022	0.307
L1.Bitcoin Price: Open (Only Weekdays)	-0.005639	0.043476	-0.130	0.897
L1.Daily Treasury Real Long-Term Rates	-0.022196	0.019334	-1.148	0.251
L1.Daily Treasury Par Yield Curve Rates 1 Month	-0.021415	0.038634	-0.554	0.579
L1.Gold Price Open	0.548097	0.278011	1.971	0.049
L1.NASDAQ Composite Open	0.531010	0.155798	3.408	0.001
L2.Bitcoin Price: Open (Only Weekdays)	0.086701	0.043167	2.008	0.045
L2.Daily Treasury Real Long-Term Rates	-0.042222	0.019286	-2.189	0.029
L2.Daily Treasury Par Yield Curve Rates 1 Month	-0.054381	0.038909	-1.398	0.162
L2.Gold Price Open	-0.071439	0.274099	-0.261	0.794
L2.NASDAQ Composite Open	0.030060	0.158311	0.190	0.849

This is the 5-variable test with two interest rates, the Bitcoin price, the gold price, and the NASDAQ composite. With five variables, including two global variables, some results change significantly.

First, in terms of P-value, the gold price and the NASDAQ composite of the last period has outstanding statistical significance. Second, they all have a solid positive correlation with the Bitcoin Price. Third, the long-term interest rate with lag 2 is still negatively correlated with the Bitcoin price, and the significance is still substantial. The short-term interest rate doesn't have a very strong p-value. However, compared to previous cases, the gap of significance between the long-term interest rate and the short-term interest rate is decreasing.

I also notice that, in this case, the short-term interest rate negatively correlates with the Bitcoin price. Let's do two other possibilities to see if we can find a fixed sign of correlation of the short-term interest rate.

Results for equation Bitcoin Price: Open (Only Weekdays)

	coefficient	std. error	t-stat	prob
const	-0.001874	0.002093	-0.896	0.370
L1.Bitcoin Price: Open (Only Weekdays)	-0.000917	0.043677	-0.021	0.983
L1.Daily Treasury Real Long-Term Rates	-0.030556	0.019841	-1.540	0.124
L1.Daily Treasury Par Yield Curve Rates 1 Month	-0.016495	0.039071	-0.422	0.673
L1.Gold Price Open	0.466329	0.284764	1.638	0.102
L1.USD to Yen Open	-0.435867	0.517115	-0.843	0.399
L2.Bitcoin Price: Open (Only Weekdays)	0.098511	0.043577	2.261	0.024
L2.Daily Treasury Real Long-Term Rates	-0.047131	0.019428	-2.426	0.015
L2.Daily Treasury Par Yield Curve Rates 1 Month	-0.050384	0.039472	-1.276	0.202
L2.Gold Price Open	-0.244254	0.292752	-0.834	0.404
L2.USD to Yen Open	-0.268214	0.507874	-0.528	0.597

The diagram above is a 5-variable test with two interest rates, the Bitcoin price, the gold price, and the exchange rate from USD to Yen.

In terms of P-value, the gold price is still good: the sign of the coefficient is still positive, and it still has a strong relationship. The long-term interest rate's significance returns to a standard low level: though it's not as low as it is in the three-variable model, at least it's not as high as 0.8. The short-term interest rate also changes compared to the past. Its significance increases a lot. And it also has a negative relationship with the Bitcoin price, just like the usual tendency of the long-term interest rate.

	coefficient	std. error	t-stat	prob
const	-0.002085	0.002074	-1.006	0.315
L1.Bitcoin Price: Open (Only Weekdays)	-0.005836	0.043361	-0.135	0.893
L1.Daily Treasury Real Long-Term Rates	-0.030191	0.019005	-1.589	0.112
L1.Daily Treasury Par Yield Curve Rates 1 Month	-0.028478	0.038855	-0.733	0.464
L1.NASDAQ Composite Open	0.607328	0.161643	3.757	0.000
L1.USD to Yen Open	-0.937687	0.509938	-1.839	0.066
L2.Bitcoin Price: Open (Only Weekdays)	0.090729	0.042880	2.116	0.034
L2.Daily Treasury Real Long-Term Rates	-0.043759	0.018263	-2.396	0.017
L2.Daily Treasury Par Yield Curve Rates 1 Month	-0.061914	0.039189	-1.580	0.114
L2.NASDAQ Composite Open	0.146970	0.167021	0.880	0.379
L2.USD to Yen Open	-0.134360	0.515235	-0.261	0.794

Now, we have the final one. This is the model between two interest rates, the Bitcoin price, the NASDAQ Composite, and the exchange rate. What do we have here?

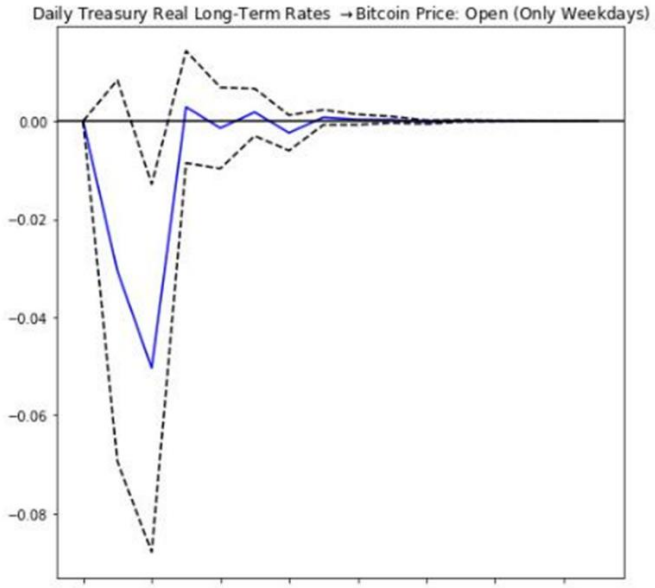
In terms of P-value, the NASDAQ Composite is extremely good: the sign of the coefficient is strongly positive. The Exchange rate is good enough, too. As usual, it has a robust negative coefficient.

Regardless of L1 or L2, both long-term and short-term interest rates have a negative sign of correlation with the Bitcoin price. But, compared to short-term interest rates, long-term interest rates always have better significance.

Moreover, the significance gap between the long-term and short-term interest rates is decreasing.

Since we get to the 5-variable case, some observation is worth emphasizing. First, regardless of short-term or long-term interest rates, interest rates always hurt the Bitcoin price. If the interest rate increases, then the Bitcoin price decreases.

I also find the best VAR model among all 5-variable cases. It's VAR (7) of the 5-variable case containing the gold price and the exchange rate. It has the lowest AIC. Let's use this model to do more advanced research.

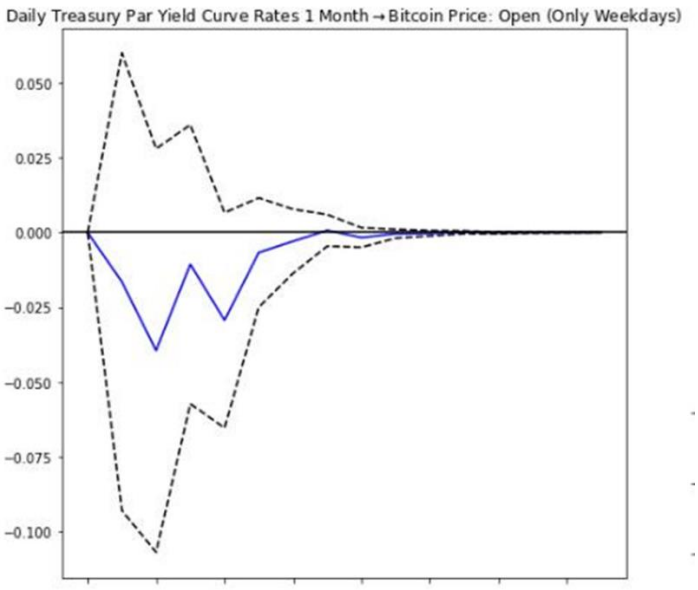


First, let's do the impulse response. The diagram above is an impulse response diagram about the long-term interest rates and the Bitcoin price. Why do we do impulse response tests? Because we are interested in the result of the Bitcoin price (dependent variable) when there is a shock to the two interest rates (independent variables). By using the impulse responses, we can see the reactions of the variables to shocks hitting the system and better understand how our dependent variable behaves.

The interpretation of this diagram is the following. A one standard deviation shock to the long-term interest rate causes significant decreases (the maximum is around 5 percent) in the open price of the Bitcoin price for two periods, after which the effect dissipates. And after decreasing

to the minimal value, it rises and becomes normal. At last, after the second period, it takes one period to go back to normal.

What about the short-term interest rate?



The interpretation is the following. A one standard deviation shock to the short-term interest rate causes decreases (the maximum is around 3 percent) in the open price of the Bitcoin price for one period, after which the effect dissipates.

After decreasing to the minimal value, it rises to a local minimum for another period. It starts to fall again in the second half of the second period and comes to a local minimum at the beginning of the third period. Then, it keeps rising (at a different rate) in the following periods, returning to normal.

Before we make some comprehensive conclusions, let's first see the result of variance decomposition. Below is the variance decomposition panel of our best model.

FEVD for Bitcoin Price: Open (Only Weekdays)

	Bitcoin Price: Open (Only Weekdays)	Daily Treasury Real Long-Term Rates	Daily Treasury Par Yield Curve Rates 1 Month	Gold Price Open	USD to Yen Open
0	1.000000	0.000000	0.000000	0.000000	0.000000
1	0.984994	0.007685	0.000094	0.005896	0.001331
2	0.972691	0.016431	0.001274	0.006324	0.003280
3	0.970637	0.016468	0.001290	0.006324	0.005282
4	0.968680	0.016536	0.002174	0.007310	0.005299
5	0.968601	0.016540	0.002217	0.007343	0.005299
6	0.968460	0.016540	0.002230	0.007466	0.005304
7	0.968455	0.016543	0.002230	0.007467	0.005305
8	0.968446	0.016544	0.002234	0.007470	0.005306
9	0.968444	0.016545	0.002234	0.007471	0.005306

The variance decomposition shows "how much of the variability in the dependent variable is explained by 'its own shocks' versus the 'shocks in another variable' in the model. As we can see, which fulfills our expectations: the variable other than the Bitcoin price with the highest shock is the long-term interest rate. It has risen since the second period and remained the most significant shock variable for a rate of 0.016. The short-term real interest rate is the next one, and then the gold price and the exchange rate. As expected, the long-term interest rate is still the one that matters the most.

At last, we also did a sensitivity analysis: to see which model has the lowest BIC.

A very convincing result to show that this model is the best is: the one with the lowest AIC and the lowest BIC is both the model between two interest rates, the Bitcoin price, the gold price, and the exchange rate. However, the one with the lowest AIC is VAR (7), and the one with the lowest BIC is VAR (6). But even if they are not the same, their coefficients are very similar.

We finish introducing all crucial combinations and show their results with comprehensive interpretations. Let's get to the conclusion and answer our question.

Conclusion

1) In short, the increase in American interest rates, combined with all kinds of global variables, negatively impacts the price of Bitcoin. In multiple variable cases (5-variable cases), whether long-term or short-term, the increase in interest rate always causes the Bitcoin price to drop.
2) Within all the models (2-variable-VAR, 3-variable-VAR, and 5-variable-VAR), most of the time, the long-term interest rate is more significant than the short-term interest rate. The long-term interest rate has a higher coefficient (absolute value) and a lower p-value. The coefficient in the 5-variable case tends to be -0.03% in L1 and -0.06% in L2.
3) If we take a careful look at the coefficients and the p-value of the best model (VAR (6) and VAR (7)), the long-term interest rate usually has the best significance in the last two or three periods. But the coefficient is usually the same: negatively influencing the Bitcoin price. The higher the long-term interest rate, the lower the Bitcoin price.

Results for equation Bitcoin Price: Open (Only Weekdays)

	coefficient	std. error	t-stat	prob
const	-0.000769	0.001986	-0.387	0.698
L1.Bitcoin Price: Open (Only Weekdays)	0.066582	0.044279	1.504	0.133
L1.Daily Treasury Real Long-Term Rates	-0.013859	0.020580	-0.673	0.501
L1.Daily Treasury Par Yield Curve Rates 1 Month	0.016315	0.057860	0.282	0.778
L1.Gold Price Open	0.513030	0.281760	1.821	0.069
L1.USD to Yen Open	-0.498734	0.492051	-1.014	0.311
L2.Bitcoin Price: Open (Only Weekdays)	0.063215	0.044314	1.427	0.154
L2.Daily Treasury Real Long-Term Rates	-0.038763	0.020641	-1.878	0.060
L2.Daily Treasury Par Yield Curve Rates 1 Month	0.056177	0.068301	0.822	0.411
L2.Gold Price Open	-0.011204	0.287112	-0.039	0.970
L2.USD to Yen Open	-0.013701	0.513522	-0.027	0.979
L3.Bitcoin Price: Open (Only Weekdays)	-0.006705	0.043952	-0.153	0.879
L3.Daily Treasury Real Long-Term Rates	0.146221	0.020502	7.132	0.000
L3.Daily Treasury Par Yield Curve Rates 1 Month	0.005241	0.058245	0.059	0.953
L3.Gold Price Open	0.458003	0.311132	1.472	0.141
L3.USD to Yen Open	-0.140845	0.516370	-0.273	0.785
L4.Bitcoin Price: Open (Only Weekdays)	-0.024245	0.044402	-0.546	0.585
L4.Daily Treasury Real Long-Term Rates	-0.033505	0.023592	-1.420	0.156
L4.Daily Treasury Par Yield Curve Rates 1 Month	-0.128125	0.100842	-1.271	0.204
L4.Gold Price Open	-0.220604	0.319564	-0.690	0.490
L4.USD to Yen Open	0.916691	0.518849	1.767	0.077
L5.Bitcoin Price: Open (Only Weekdays)	-0.006903	0.044514	-0.155	0.877
L5.Daily Treasury Real Long-Term Rates	0.071208	0.023360	3.048	0.002
L5.Daily Treasury Par Yield Curve Rates 1 Month	-0.114115	0.101654	-1.123	0.262
L5.Gold Price Open	0.106215	0.319169	0.333	0.739
L5.USD to Yen Open	-0.045891	0.526338	-0.086	0.932
L6.Bitcoin Price: Open (Only Weekdays)	-0.004298	0.044394	-0.097	0.923
L6.Daily Treasury Real Long-Term Rates	0.033587	0.024886	1.350	0.177
L6.Daily Treasury Par Yield Curve Rates 1 Month	-0.103954	0.121244	-0.857	0.391
L6.Gold Price Open	0.221454	0.317713	0.697	0.486
L6.USD to Yen Open	0.437420	0.490102	0.893	0.372

Results for equation Bitcoin Price: Open (Only Weekdays)

	coefficient	std. error	t-stat	prob
const	-0.000753	0.001991	-0.378	0.705
L1.Bitcoin Price: Open (Only Weekdays)	0.063389	0.044359	1.429	0.153
L1.Daily Treasury Real Long-Term Rates	-0.013622	0.020939	-0.651	0.515
L1.Daily Treasury Par Yield Curve Rates 1 Month	-0.031278	0.077559	-0.403	0.687
L1.Gold Price Open	0.452934	0.287264	1.577	0.115
L1.USD to Yen Open	-0.546242	0.496152	-1.101	0.271
L2.Bitcoin Price: Open (Only Weekdays)	0.060189	0.044488	1.353	0.176
L2.Daily Treasury Real Long-Term Rates	-0.043294	0.021394	-2.024	0.043
L2.Daily Treasury Par Yield Curve Rates 1 Month	0.150097	0.079618	1.885	0.059
L2.Gold Price Open	-0.027582	0.297629	-0.093	0.926
L2.USD to Yen Open	0.050692	0.515610	0.098	0.922
L3.Bitcoin Price: Open (Only Weekdays)	-0.006357	0.044641	-0.142	0.887
L3.Daily Treasury Real Long-Term Rates	0.137522	0.021856	6.531	0.000
L3.Daily Treasury Par Yield Curve Rates 1 Month	0.017154	0.090184	0.190	0.849
L3.Gold Price Open	0.490009	0.312314	1.569	0.117
L3.USD to Yen Open	-0.097092	0.519446	-0.187	0.852
L4.Bitcoin Price: Open (Only Weekdays)	-0.030001	0.044593	-0.673	0.501
L4.Daily Treasury Real Long-Term Rates	-0.029766	0.023921	-1.244	0.213
L4.Daily Treasury Par Yield Curve Rates 1 Month	-0.167976	0.102471	-1.639	0.101
L4.Gold Price Open	-0.187717	0.321678	-0.584	0.560
L4.USD to Yen Open	0.873687	0.520988	1.677	0.094
L5.Bitcoin Price: Open (Only Weekdays)	-0.010663	0.044681	-0.239	0.811
L5.Daily Treasury Real Long-Term Rates	0.069644	0.024190	2.879	0.004
L5.Daily Treasury Par Yield Curve Rates 1 Month	-0.125919	0.103352	-1.218	0.223
L5.Gold Price Open	0.065409	0.320688	0.204	0.838
L5.USD to Yen Open	-0.095822	0.528241	-0.181	0.856
L6.Bitcoin Price: Open (Only Weekdays)	-0.012623	0.044813	-0.282	0.778
L6.Daily Treasury Real Long-Term Rates	0.038333	0.025704	1.491	0.136
L6.Daily Treasury Par Yield Curve Rates 1 Month	-0.114676	0.123202	-0.931	0.352
L6.Gold Price Open	0.227700	0.319786	0.712	0.476
L6.USD to Yen Open	0.617348	0.530733	1.163	0.245
L7.Bitcoin Price: Open (Only Weekdays)	0.018430	0.044787	0.412	0.681
L7.Daily Treasury Real Long-Term Rates	0.037968	0.034255	1.108	0.268
L7.Daily Treasury Par Yield Curve Rates 1 Month	-0.253117	0.122614	-2.064	0.039
L7.Gold Price Open	0.282190	0.326098	0.865	0.387
L7.USD to Yen Open	-0.052229	0.493813	-0.106	0.916

4) The impulse response and the variance decomposition results also consolidate the conclusion above. The negative change in the price of Bitcoin is more significant in the long-term interest rate shock than in the short-term interest rate shock.
5) The variance decomposition also tells us the same thing: the long-term real interest rate is always the most significant variable that can shock the Bitcoin Price.
6) There are no significant long-term relationships between the American interest rates and the Bitcoin price, which also fulfills our expectations. I try to do the error correction model, but it doesn't work.

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