Opacus: User-Friendly Differential Privacy Library in PyTorch

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Abstract

We introduce Opacus, a free, open-source PyTorch library for training deep learning models with differential privacy (hosted at opacus.ai). Opacus is designed for simplicity, flexibility, and speed. It provides a simple and user-friendly API, and enables machine learning practitioners to make a training pipeline private by adding as little as two lines to their code. It supports a wide variety of layers, including multi-head attention, convolution, LSTM, and embedding, right out of the box, and it also provides the means for supporting other user-defined layers. Opacus computes batched per-sample gradients, providing better efficiency compared to the traditional "micro batch" approach. In this paper we present Opacus, detail the principles that drove its implementation and unique features, and compare its performance against other frameworks for differential privacy in ML.

1 Background and Introduction

Differential privacy (DP) [5] has emerged as the leading notion of privacy for statistical analyses. It allows performing complex computations over large datasets while limiting disclosure of information about individual data points. Roughly stated, an algorithm that satisfies DP ensures that no individual sample in a database can have a significant impact on the output of the algorithm, quantified by the privacy parameters ϵ and δ .

Formally, a randomized mechanism $M\colon \mathcal{D}\to\mathcal{R}$ is (ϵ,δ) -differentially private for $\epsilon>0$ and $\delta\in[0,1)$ if for any two neighbouring datasets $D,D'\in\mathcal{D}$ (i.e., datasets that differ in at most one sample) and for *any* subset of outputs $R\subseteq\mathcal{R}$ it holds that

$$\mathbb{P}(M(D) \in R) \le \exp(\epsilon) \, \mathbb{P}(M(D') \in R) + \delta.$$

Differentially Private Stochastic Gradient Descent (DP-SGD) due to Abadi et al. [1], building on Song et al. [16] and Bassily et al. [2], is a modification of SGD that ensures differential privacy on every model parameters update. Instead of computing the average of gradients over a batch of samples, a DP-SGD implementation computes per-sample gradients, clips their ℓ_2 norm, aggregates them into a batch gradient, and adds Gaussian noise. (See Fig. 1 for an illustration.) However, mainly for efficiency reasons, deep learning frameworks such as PyTorch or TensorFlow do not expose intermediate computations, including per-sample gradients; users only have access to the gradients averaged over a batch.

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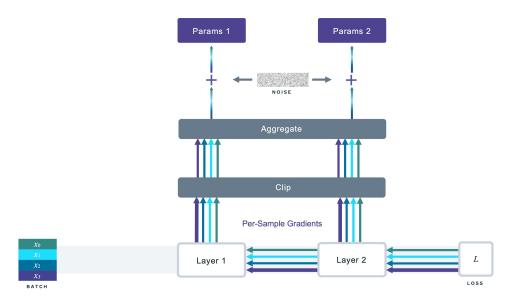


Figure 1: Pictorial representation of the DP-SGD algorithm. The single-colored lines represent per-sample gradients, the width of the lines represent their respective norms, and the multi-colored lines represent the aggregated gradients.

A naïve way to implement DP-SGD is thus to separate each batch into micro-batches of size one, compute the gradients on these micro-batches, clip, and add noise (sample code to obtain the persample gradients using this approach is provided in Appendix A). While this procedure (called the "micro-batch method" or "micro-batching") does indeed yield correct per-sample gradients, it can be very slow in practice due to underutilization of hardware accelerators (GPUs and TPUs) that are optimized for batched, data-parallel computations.

Opacus implements performance-improving vectorized computation instead of micro-batching. In addition to speed, Opacus is designed to offer simplicity and flexibility. In this paper, we discuss these design principles, highlight some unique features of Opacus, and evaluate its performance in comparison with other DP-SGD frameworks.

2 Design Principles and Features

Opacus is designed with the following three principles in mind:

- *Simplicity*: Opacus exposes a compact API that is easy to use out of the box for researchers and engineers. Users need not know the details of DP-SGD in order to train their models with differential privacy.
- *Flexibility*: Opacus supports rapid prototyping by users proficient in PyTorch and Python, thanks to its rich set of features (described below).
- *Speed*: Opacus seeks to minimize performance overhead of DP-SGD by supporting vectorized computation.

We explain throughout the paper how these principles manifest themselves in the Opacus API.

Example Usage. The main entry point to Opacus is the PrivacyEngine class. It keeps track of "privacy budget" spent so far and is responsible for wrapping regular PyTorch training objects with DP-related code. The key method provided by PrivacyEngine is make_private(). It takes the three PyTorch training objects—model, optimizer and data loader— along with the privacy parameters (noise multiplier and maximum norm of the gradients) and outputs differentially private analogues of these objects:

• the model wrapped with GradSampleModule, which adds the ability to compute per-sample gradients;

- the optimizer wrapped with an additional code for clipping gradients and adding noise;
- the data loader transformed into one using Poisson sampling as required by DP-SGD.

This design provides a good balance between *simplicity* and *flexibility*. On the one hand, most users are able to switch to DP training by calling a single method. On the other hand, advanced users can modify the details of the private components' behavior as long as their interface is unchanged.

Attaching Opacus to an existing script can be done with changing as few as two lines of code, e.g., the lines containing privacy_engine in the following example:

```
dataset = Dataset()
model = Net()
optimizer = SGD(model.parameters(), lr)
data_loader = torch.utils.data.DataLoader(dataset, batch_size)
privacy_engine = PrivacyEngine()
model, optimizer, data_loader = privacy_engine.make_private(
    module=model,
    optimizer=optimizer,
    data_loader=data_loader,
    noise_multiplier=noise_multiplier,
    max_grad_norm=max_grad_norm,
)
# Now it's business as usual
```

Main features of Opacus. We highlight some of the key features Opacus provides.

Privacy accounting. Opacus provides out of the box privacy tracking with an accountant based on Rényi Differential Privacy [11, 12]. The PrivacyEngine object keeps track of how much privacy budget has been spent at any given point in time, enabling early stopping and real-time monitoring. Opacus also allows a user to directly instantiate a DP training targeting an (ϵ, δ) budget. In this instance, the engine computes a noise level σ that yields an overall privacy budget of (ϵ, δ) . Opacus also exposes an interface to write custom privacy accountants.

Model validation. Before training, Opacus validates that the model is compatible with DP-SGD. For example, certain layers (e.g., BatchNorm or GroupNorm modules in some configurations) mix information across samples of a batch, making it impossible to define a per-sample gradient; Opacus disallows those modules. It also ensures that no additional statistics without DP guarantees are tracked by the model (See Appendix C).

Poisson sampling. Opacus also supports uniform sampling of batches (also called Poisson sampling): each data point is independently added to the batch with probability equal to the sampling rate. Poisson sampling is necessary in some analyses of DP-SGD [12].

Efficiency. Opacus makes efficient use of hardware accelerators (See Appendix B). Opacus also supports distributed training via PyTorch's DistributedDataParallel.

Virtual steps. As Opacus is highly optimized for batched per-sample gradient computation, it faces an inevitable speed/memory trade-off. In particular, when computing per-sample gradients for the entire batch, the size of the gradient tensor it needs to store is increased by the factor of batch_size. To support a wider range of batch sizes, Opacus provides an option to decouple physical batch sizes, limited by the amount of memory available, and logical batch sizes, whose selection is driven by considerations of model convergence and privacy analysis.

Predefined and custom layers. Opacus comes with several predefined layer types, including convolution, multi-head attention, LSTM, GRU (and generic RNN), normalization, and embedding layers. Moreover, it allows users to add their own custom layers. When adding a custom layer, users can provide a method to calculate per-sample gradients for that layer and register it with a simple decorator provided by Opacus. The details can be found in opacus.ai/tutorials.

Secure random number generation. Opacus offers a cryptographically safe (but slower) pseudorandom number generator (CSPRNG) for security-critical code. This can be enabled by the option secure_mode, which enables CSPRNG for noise generation and random batch composition.

Noise scheduler and variable batch size. Similar to learning rate scheduler in deep learning, the noise scheduler in Opacus adjusts the noise multiplier during training by evolving it according to some

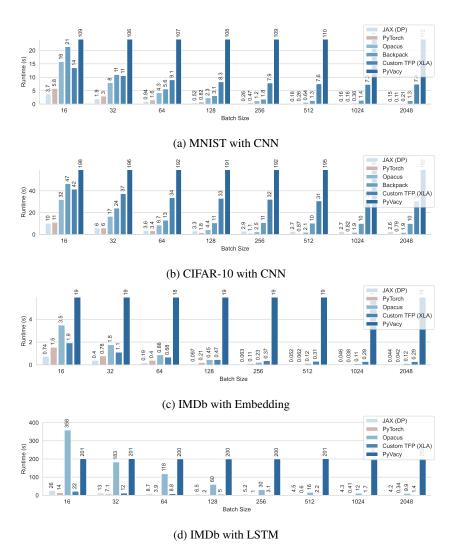


Figure 2: Median runtime (per epoch, computed over 20 epochs) of different frameworks for various batch sizes. Note that BackPACK does not support LSTM and embedding, hence theses data are omitted.

predefined schedule, such as exponential, step, and custom function. Opacus also supports varying batch sizes throughout training.

Modular. Opacus integrates well with PyTorch Lightning, a high-level abstraction framework for PyTorch, which reduces boilerplate and simplifies coding complex networks.

3 Numerical Evaluation

We experimentally evaluate the runtime performance of Opacus and other frameworks for training models with DP-SGD on end-to-end benchmarks. We also run experiments to quantify the runtime and memory overhead of enabling DP with Opacus compared to using PyTorch (without DP) for each layer that Opacus currently supports.

3.1 End-to-end Benchmarks

We benchmark the runtime performance of Opacus on end-to-end model training and compare Opacus's performance to a JAX implementation of DP-SGD, a custom implementation of TensorFlow Privacy, BackPACK, PyVacy, and PyTorch (without DP). The experiments are based on

the Fast-DPSGD benchmarks [17], but executed with the most current releases of the respective frameworks as of December 8th, 2021 (see Appendix E.3 for details). JAX is a general-purpose framework for high-performance numerical computing that uses just-in-time (JIT) compilation and static graph optimization, and we choose the custom implementation for DP-SGD found in [17]. For TensorFlow Privacy, we choose the custom implementation that uses vectorization and XLA-driven JIT compilation, which outperforms both the custom TensorFlow implementation without XLA and TensorFlow Privacy in [17]. To be consistent with with [17], we annotate it as Custom TFP (XLA) in our graphs. Results are shown in Fig. 2.

Following [17], we train a CNN with 26,010 parameters on MNIST, a handwritten digit recognition dataset, and train a CNN with 605,226 parameters on CIFAR-10, a dataset of small color images. We also train an embedding network and an LSTM network with 160,098 and 1,081,002 parameters respectively on IMDb dataset [10], a movie review sentiment classification dataset.

All experiment results were obtained by running each framework on a VM with an Intel(R) Xeon(R) CPU @ 2.20GHz, NVIDIA A100 SXM4 (40GB VRAM), and 83GB of RAM. Compared to the testbench of [17] (NVIDIA GTX Titan V GPU (12GB VRAM)), our setup contains GPUs with more VRAM, which enables us to include benchmarks for larger batch sizes (512, 1024, 2048) for a more informative comparison. For some benchmarks (notably, LSTM and Embeddings) we are also able to significantly increase maximum batch size for Opacus due to library performance improvements.

All approaches implement the same model and algorithm for a given privacy budget; we do not compare accuracy, but rather focus on runtime. We report the median per-epoch runtime from a total of 20 epochs for each framework in Fig. 2.

JAX consistently performs very well in these experiments, though JAX's first epoch is often very slow (up to $100 \times$ the runtime of the subsequent epochs), which is not captured in Fig. 2 as it computes the median over 20 epochs. See Fig. 4 for a cumulative runtime comparison over 20 epochs. We can see that for small batch sizes, the custom DP-SGD implementation in JAX consistently achieves the fastest performance compared to other frameworks for DP-SGD and even pure PyTorch on MNIST and the embedding network. However, at larger batch sizes PyTorch (without DP) outperforms JAX, and Opacus outperforms JAX at large batch sizes on CIFAR-10.

While all frameworks except PyVacy achieve higher performance on larger batch sizes, the performance increase is most evident in Opacus: By increasing the batch size, we can reduce runtime in Opacus by a factor ranging between 17 (for CIFAR-10) and 75 (for MNIST), depending on the model and dataset. Opacus becomes more efficient as the batch size increase. This is largely due to its implementation being based on PyTorch: We compute the mean runtime reduction over all datasets and models for all frameworks: 12.8 (JAX), 37.8 (PyTorch), 40 (Opacus), 10.5 (Backpack), 6.3 (TensorFlow Privacy), and 1 (PyVacy).

Overall, PyVacy has the highest running time among all frameworks. Both BackPACK and TensorFlow Privacy have comparable performance on CNNs with small batches, though BackPACK outperforms Tensorflow Privacy at larger batch sizes. Since BackPACK does not support LSTM and embedding, the corresponding data are omitted. Opacus outperforms BackPACK, Tensorflow Privacy, and PyVacy on MNIST, CIFAR-10, and IMDb with embedding network at large batch sizes, while Tensorflow Privacy outperforms Opacus on LSTM.

To summarize, Opacus is a library for training deep learning models with differential privacy via a simple and flexible API. Opacus implements custom privacy functionalities (e.g., Poisson sampling and virtual steps; see Section 2) and is based on PyTorch, which has a large community and numerous libraries. For large-scale ML tasks, the compute- and memory-intensive tasks of matrix multiplication and other tensor operations ultimately form the real bottleneck for any framework. We believe that the picture will look different when a functionality similar to vmap in JAX is implemented in PyTorch and Opacus is able to leverage it.

3.2 Micro-benchmarks

We report our micro-benchmark results for each layer that Opacus currently supports. In particular, we benchmark the average runtime and peak allocated memory of 2,000 forward and backward passes on 100 different inputs through each layer wrapped in GradSampleModule with DP enabled.

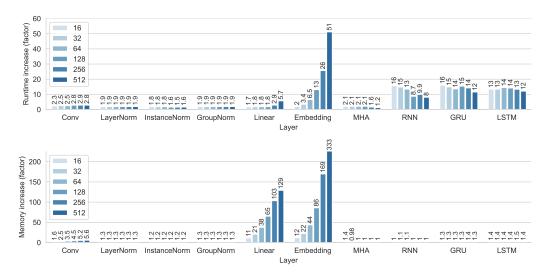


Figure 3: Runtime and memory overhead of the DP-enabled layer relative to its non-private baseline for different modules under different batch sizes. Top: Runtime increase factor. Bottom: Memory increase factor. The runtime increase factor is calculated as the ratio of the mean runtime for one forward and one backward pass of the DP-enabled module divided by the mean runtime for one forward and one backward pass of the corresponding (non-private) torch.nn module. The increase in peak allocated memory is calculated similarly.

We benchmark the same for the corresponding torch.nn layer without DP. In Fig. 3, we report the runtime and memory overhead of the DP-enabled layer relative to the non-private baseline.

These results are obtained on a cloud instance running Ubuntu 18.04.5 LTS with an Intel(R) Xeon(R) Platinum 8275CL CPU @ 3.00GHz, NVIDIA A100 SXM4 (40GB VRAM), and 1.1TB of RAM.

We see that for convolutional layers, normalization layers, and multi-head attention, enabling DP with Opacus's GradSampleModule results in a $\sim 2\times$ slowdown and some additional memory consumption. For linear and embedding layers, we see that the runtime and memory overhead increase with the batch size. In our experiments, we found that the PyTorch embedding layer and linear layer (without DP) have the lowest and fourth lowest runtime respectively out of all layers we benchmarked regardless of batch size.

For RNN-based models, there is a consistently large (up to 16×) runtime cost for enabling DP across different batch sizes. Opacus has custom implementations for RNN, GRU, and LSTM layers that allows for the computation of per-sample gradients once wrapped in GradSampleModule. For the exact breakdown of the overhead added by Opacus's custom implementation of each layer compared to the cost of enabling DP via GradSampleModule, see Fig. 5 in the appendix. However, note that when running with large batch sizes in practice, Opacus's DPLSTM performs competitively with other DP frameworks, for example coming within a factor of 2.5 of the custom JAX DP-SGD implementation without incurring the JIT compilation overhead (see Fig. 2), Fig. 4.

4 Related Work

Gradient Clipping. At the heart of implementing DP-SGD is the need to compute clipped gradients, for which there are several different approaches. A first option, as implemented in Opacus, is to directly compute and clip the per-sample gradients. A second option is to compute only the *norm* of each sample's gradient (in an exact or approximate form), and form the weighted average loss over the batch by weighting samples according to their norm. Typically, each sample's weight is $C/\max(N_i,C)$, where C is the clipping threshold and N_i is the norm of the sample's gradient. The gradient of this loss with respect to the parameters yields the average of clipped gradients. This option was proposed by Goodfellow [6] with exact norms, and was considered more recently along with Johnson-Lindenstrauss projections [3] to compute approximate gradient norms.

Goodfellow's method is based on computing per-sample ℓ_2 -norms of the gradients and is restricted to fully-connected layers; more recently, Rochette et al. [15] extended it to CNNs. Lee and Kifer [8] propose computing the *norm* of the per-sample gradients directly, hence doing two passes of back-propagation: one pass for obtaining the norm, and one pass for using the norm as a weight. In Opacus per-sample gradients are obtained in a single back-propagation pass, without performance or accuracy penalties of alternative techniques.

Shortly after an initial version of this work was published, Li et al. [9] generalized the Goodfellow method to handle sequential inputs, making fine-tuning large Transformers under DP computationally efficient. The proposed method, *ghost clipping*, is memory-efficient and has good throughput. In their experiments on large language models, Opacus is as good in memory efficiency and better in throughput than JAX, thanks to some improvements in the implementation. We plan to incorporate such improvements to optimize performance of Opacus further.

Frameworks for differentially private learning. TensorFlow Privacy and PyVacy are two existing frameworks providing implementation of DP-SGD for TensorFlow and PyTorch, respectively. BackPACK [4], another framework for DP-SGD, exploits Jacobians for efficiency. BackPACK currently supports only fully connected or convolutional layers, and several activation layers (recurrent and residual layers are not yet supported). Objax [13] is a machine learning framework for JAX and also provides a DP-SGD implementation. The per-sample gradient clipping in Objax is based on a vmap method. It closely resembles approach implemented in [17] and we believe benchmark findings are applicable to both implementations. In Section 3 we compare the performance of these frameworks with Opacus.

Finally, we mention alternative approaches to ML training under different notions of privacy and security: using secure hardware to support oblivious training over encrypted data [14], or relying on secure multi-party computation techniques (SMPC) to train over data jointly held by several protocol participants [7].

5 Conclusions

Opacus is a PyTorch library for training deep learning models with differential privacy guarantees. The system design aims to provide simplicity, flexibility, and speed, for maximal compatibility with existing ML pipelines. We have outlined how these design principles have influenced the features of Opacus, and demonstrated that it achieves best-in-class performance on a battery of ML tasks compared to other DP training frameworks.

Opacus is actively maintained as an open source project, supported primarily by the privacy-preserving machine learning team at Meta AI. A number of extensions and upgrades are planned for Opacus in the future, including enhanced flexibility for custom components, further efficiency improvements, and improved integration with PyTorch ecosystem through projects like PyTorch Lightning.

Acknowledgement

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Appendix

A Micro-Batching

The following code snippet is a naïve way to yield the per-sample gradients through micro batching.

```
for batch in Dataloader(train_dataset, batch_size):
    all_per_sample_gradients = []
    for x,y in batch:
        y_hat = model(x)
        loss = criterion(y_hat, y)
        loss.backward()

        per_sample_grads = [p.grad.detach().clone() for p in model.parameters()]
        all_per_sample_gradients.append(per_sample_grads)
        model.zero_grad() # reset p.grad
```

B Vectorized Computation

In accordance with its speed objective, Opacus supports computing per-sample gradients efficiently, in a vectorized manner. This is achieved by deriving a per-sample gradient formula for every layer and transforming it into a form that can be implemented using a single application of the einsum operator. Due to space constraints, we discuss this approach only for the nn.Linear layer. The implementation details for other layers and other related tutorials can be found in opacus.ai/tutorials.

Consider one linear layer with weight matrix W. We omit the bias from the forward pass equation and denote the forward pass by Y = WX, where X is the input and Y is the output of the linear layer. X is a matrix of size $d \times B$, with B columns (B is the batch size), where each column is an input vector of dimension d. Similarly, the output matrix Y would be of size $r \times B$ where each column is the output vector corresponding to an element in the batch and r is the output dimension.

The forward pass can be written as follows:

$$Y_i^{(b)} = \sum_{j=1}^d W_{i,j} X_j^{(b)},$$

where $Y_i^{(b)}$ denotes the *i*'th coordinate of the *b*'th sample in the batch.

In an ML problem, we typically need the derivative of the loss with respect to weights. Correspondingly, in Opacus we need the "per-sample" version of that, which is the per-sample derivative of the loss with respect to the weights W:

$$\frac{\partial L}{\partial z} = \sum_{b=1}^{B} \sum_{i'=1}^{r} \frac{\partial L}{\partial Y_{i'}^{(b)}} \frac{\partial Y_{i'}^{(b)}}{\partial z}.$$

Applying the chain rule above, we can now replace variable z with $W_{i,j}$ and get

$$\frac{\partial L}{\partial W_{i,j}} = \sum_{b=1}^{B} \sum_{i'=1}^{r} \frac{\partial L}{\partial Y_{i'}^{(b)}} \frac{\partial Y_{i'}^{(b)}}{\partial W_{i,j}}.$$

We know from Y=WX that $\frac{\partial Y_{i'}^{(b)}}{\partial W_{i,j}}$ is $X_j^{(b)}$ when i=i', and is 0 otherwise. Continuing the above we have

$$\frac{\partial L}{\partial W_{i,j}} = \sum_{b=1}^{B} \frac{\partial L}{\partial Y_{i'}^{(b)}} X_{j}^{(b)}.$$

This equation corresponds to a matrix multiplication in PyTorch. In a regular backpropagation, the gradients of the loss function with respect to the weights (i.e., the gradients) are computed for the output of each layer and averaged over the batch. Since Opacus requires computing per-sample gradients, what we need is the following:

$$\frac{\partial L_{batch}}{\partial W_{i,j}} = \frac{\partial L}{\partial Y_{i'}^{(b)}} X_j^{(b)}.$$
 (1)

More generally, in a neural network with more layers, equation (1) can be written as

$$\frac{\partial L_{batch}}{\partial W_{i,j}^{(l)}} = \frac{\partial L}{\partial Z_i^{(l)(b)}} Z_j^{(l-1)(b)} \tag{2}$$

for every layer l, where $Z_i^{(l)(b)}$ is the activation of the hidden layer l for the b'th element of the batch of the neuron i. We refer to $\frac{\partial L}{\partial Z^{(l)(b)}}$ as the $highway\ gradient$.

We now explain how we compute the per-sample gradient equation (2) in Opacus efficiently. In order to remove the sum reduction to get to the equations (1) and (2), we need to replace the matrix multiplication with a batched outer product. In PyTorch, einsum allows us to do that in vectorized form. The function einsum is for evaluating the Einstein summation convention on the operands; it

allows computing many multi-dimensional linear algebraic array operations by representing them in a short-hand format based on the Einstein summation convention.

For instance, for computing the per-sample gradients for a linear layer, the einsum function can be written as torch.einsum("n...i,n...j->nij", B, A), where variables A and B refer to activations and backpropagations, respectively. In Opacus activations and backpropagations essentially contain what we need for equation (2): using module and tensor hooks in PyTorch, Opacus stores the activations $Z_j^{(l-1)(b)}$ in forward hooks and access the highway gradients $\frac{\partial L}{\partial Z_i^{(l)(b)}}$ through backward hooks. That is how the method torch.einsum("n...i,n...j->nij", B, A) implements equation (2) for computing per-sample gradients for a nn.Linear layer. To understand the einsum expression itself, it is useful to think of it as a generalized version of torch.bmm (batch matrix multiplication) for multi-dimensional inputs. For 2D matrices A and B, einsum is equivalent to torch.bmm(B.unsqueeze(2), A.unsqueeze(1)). For higher dimensional inputs the idea is the same, while we also sum over extra dimensions.

C Detection of DP Violations

In this section we explain how Opacus can detect whether an operation violates some DP guarantees by applying the following criteria. First, Opacus checks if all layers in the model are supported. Second, Opacus checks for violations that make a model incompatible with differentially private training, which is usually due to one of the two issues: 1) the model tracks some extra information not covered by DP guarantees, or 2) a module is known to do batch-level computations, thus rendering the computation of per-sample gradients impossible.

Some examples: 1) Opacus does not allow models to have batch normalization layers, as they share information across samples; 2) Opacus does not allow track_running_stats in instance-normalization layers, as they track statistics that are not covered by DP guarantees.

The above checks in Opacus for DP compatibility are not exhaustive. In particular, Opacus has no way of checking whether the model maintains the independence of the individual samples or tracks extraneous statistics. We plan to investigate ways to address this in the future releases of Opacus.

D Tracking Gradients

In this section we explain how Opacus makes it easy to keep track of gradient at different stages of DP training. In the following code snippet, we show how in Opacus we can access intermediate stages of gradient computation throughout training:

```
# model, optimizer and data_loader are initialized with make_private()

for data, labels in data_loader:
    output = model(data)
    loss = criterion(output, labels)
    loss.backward()

    print(p.grad) # normal gradients computed by PyTorch autograd
    print(p.grad_sample) # per sample gradients computed by Opacus (no clipping, no noise)

    optimizer.step()

    print(p.grad_sample) # same as before optimizer.step() - this field doesn't change
    print(p.grad_grad) # clipped and aggregated over a batch, but no noise
    print(p.grad) # final gradients (clipped, noise added, aggregated over a batch)

    optimizer.zero_grad() # all gradients are None now
```

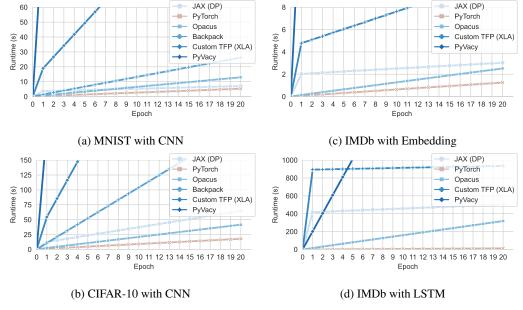


Figure 4: Cumulative runtime over 20 epochs with batch size 512. Using JIT compilation results in a slower first epoch.

E Additional Experimental Results

E.1 Additional End-to-end Benchmarks

Fig. 4 shows the cumulative runtime over 20 epochs of training with batch size 512 for each framework across the experiment datasets. Both JAX and Custom Tensorflow Privacy (XLA) incur a large runtime overhead in the first epoch and we can see that JIT compilation makes the first epoch substantially slower than all subsequent ones. If training only for a few epochs, disabling JIT compilation or using a framework that is optimized to run without JIT improves overall runtime.

E.2 Additional Micro-benchmarks

Fig. 5 compares torch.nn module, the corresponding Opacus layer (without enabling DP), and the corresponding Opacus layer wrapped in GradSampleModule with DP enabled for the multi-head attention, RNN, GRU, and LSTM layers. The majority of the runtime overhead of enabling DP with Opacus stems from Opacus's custom implementation of these layers, while the memory overhead (if present) is primarily due to wrapping the custom layer with GradSampleModule.

E.3 Experiment Setup

E.3.1 End-to-end Benchmarks

Hardware: Intel(R) Xeon(R) CPU @ 2.20GHz, NVIDIA A100 SXM4 (40GB VRAM), 83GB RAM

OS: Debian 10 (kernel 4.19.0)

Docker image source: nvidia/cuda:11.4.2-cudnn8-devel-ubuntu20.04

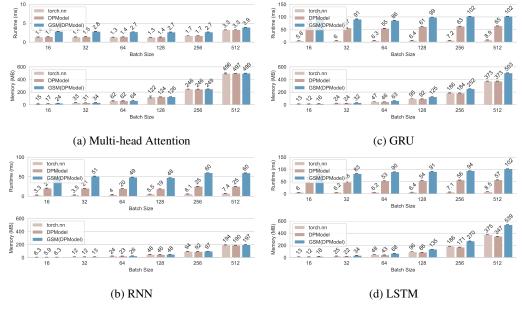


Figure 5: Microbenchmarks of the multi-head attention, RNN, GRU, and LSTM layers, comparing the torch.nn module, the corresponding Opacus layer without DP enabled, and the corresponding Opacus layer wrapped in GradSampleModule with DP enabled. Shows the mean and standard deviation of runtime (ms) and memory (MB).

Library	Version
python	3.8.10
dm-haiku	0.0.5
jax	0.2.25
jaxlib	0.1.73
pytorch	1.10.0
opacus	1.0.0
backpack	0.1
backpack-for-pytorch	1.4.0
tensorflow	2.7.0
tensorflow-privacy	0.7.3
pyvacy	0.0.1

For PyVacy, we used the most recent commit in their repository (commit hash: 2e0a9f4f71152990d60b154911070f5f60a493af).

E.3.2 Micro-benchmarks

Hardware: Intel(R) Xeon(R) Platinum 8275CL CPU @ 3.00GHz, NVIDIA A100 SXM4 (40GB VRAM), 1.1TB RAM

OS: Ubuntu 18.04.5 LTS (kernel 5.4.0)

Library	Version
opacus	1.0.0
python	3.9.7
pytorch	1.10.0