

Measure Contribution of Participants in Federated Learning

Guan Wang
Digital and Smart Analytics
 Swiss Re
 Hong Kong, China
 guan_wang@swissre.com

Charlie Xiaoqian Dang
Digital and Smart Analytics
 Swiss Re
 Beijing, China
 charlie_dang@swissre.com

Ziye Zhou
Research Center
 Ping An P&C Insurance Co., Ltd.
 Shenzhen, China
 zhouziye659@pingan.com.cn

Abstract—Federated Machine Learning (FML) creates an ecosystem for multiple parties to collaborate on building models while protecting data privacy for the participants. A measure of the contribution for each party in FML enables fair credits allocation. In this paper we develop simple but powerful techniques to fairly calculate the contributions of multiple parties in FML, in the context of both horizontal FML and vertical FML. For Horizontal FML we use deletion method to calculate the grouped instance influence. For Vertical FML we use Shapley Values to calculate the grouped feature importance. Our methods open the door for research in model contribution and credit allocation in the context of federated machine learning.

Index Terms—federated learning, machine learning, deletion, shapley values

I. INTRODUCTION

Federated Learning or Federated Machine Learning (FML) [1] is introduced to solve privacy issues in machine learning using data from multiple parties. Instead of transferring data directly into a centralized data warehouse for building machine learning models, Federated Learning allows each party to own the data in its own place and still enables all parties to build a machine learning model together. This is achieved either by building a meta-model from the sub-models each party builds so that only model parameters are transferred, or by using encryption techniques to allow safe communications in between different parties [2].

Federated Learning opens new opportunities for many industry applications. Companies have been having big concerns on the protection of their own data and are unwilling to share with other entities. With Federated Learning, companies can build models together without disclosing their data and share the benefit of machine learning. An example of Federated Learning use case is in insurance industry. Primary insurers, reinsurers and third-party companies like online retailers can all work together to build machine learning models for insurance applications. Number of training instances is increased by different insurers and reinsurers, and feature space for insurance users is extended by third-party companies. With the help of Federated Learning, machine learning can cover more business cases and perform better.

For the the ecosystem of Federated Learning to work, we need to encourage different parties to contribute their data and participate in the collaboration federation. A credit allocation

and rewarding mechanism is crucial for the incentives current and potential participants of Federated Learning. A fair measure of the contribution for each party in Federated Learning enables fair credits allocation. Data quantity alone is certainly not enough, as one party may contribute lots of data that doesn't help much on building the model. We need a way to fairly measure the data quality overall and hence decide the contribution.

In this paper we develop simple but powerful techniques to fairly calculate the contributions of multiple parties in FML, in the context of both horizontal FML and vertical FML. For Horizontal FML, each party contributes part of the training instances. We use deletion method to calculate the grouped instance influence. Each time we delete the instances provided from one certain party and retrain the model, and calculate the difference of the prediction results between the new model and the original one, and use this measure of difference to decide the contribution of this certain party. For Vertical FML, each party owns part of the feature space. We use Shapley Values [3] to calculate the grouped feature importance, and use this measure of importance to decide the contribution of each party. The method we propose in our knowledge is the first attempt of research on model contribution and credit allocation in the context of federated machine learning.

In the next chapters of this paper, we first briefly introduce Federated Learning. We then cover the Federated Deletion method and Federated Shap method we propose on measuring contributions of multiple parties for horizontal and vertical FML models, followed by some experiments. We conclude the paper with some discussions in the last chapter.

II. FEDERATED LEARNING

Federated Learning originated from some academic papers like [1], [4] and a follow-up blog from Google in 2017. The idea is that Google wants to train its own input method on its Android phones called "Gboard" but does not want to upload the sensitive keyboard data from their users to Google's own servers. Rather than uploading user's data and training models in the cloud, Google lets users train a separate model on their own smartphones (thanks to the neural engines from several chip manufacturers) and upload those black-box model parameters from each of their users to the cloud and merge

the models, update the official centralized model, and push the model back to Google users. This not only avoids the transmission and storage of user's sensitive personal data, but also utilizes the computational power on the smartphones (a.k.a the concept of Edge Computing) and reduce the computation pressure from their centralized servers.

When the concept of Federated Learning was published, Google's focus was on the transmission of models as the upload bandwidth of mobile phones is usually very limited. One possible reason is that similar engineering ideas have been discussed intensively in distributed machine learning. The focus of Federated Learning was thus more on the "engineering work" with no rigorous distributed computing environment, limited upload bandwidth and slave nodes as massive number of users.

Data privacy is becoming an important issue, and lots of relating regulations and laws have been taken into action by authorities and governments [5], [6]. The companies that have been accumulating tons of data and have just started to make value of it now have their hands tightened. On the other hand, all companies value a lot their own data and feel reluctant from sharing data with others. Information islands kill the possibility of cooperation and mutual benefit. People are looking for a way to break such prisoner dilemma while complying with all the regulations. Federated Learning was soon recognized as a great solution for encouraging collaboration while respecting data privacy.

[2] describes Federated Learning in three categories: Horizontal Federated Learning, Vertical Federated Learning and Federated Transfer Learning. Such categorization extends the concept of Federated Learning and clarify the specific solutions under different use cases.

Horizontal Federated Learning applies to circumstances where we have a lot of overlap on features but only a few on instances. This refers to the Google Gboard use case and models can be ensembled directly from the edge models.

Vertical Federated Learning refers to where we have many overlapped instances but few overlapped features. An example is between insurers and online retailers. They both have lots of overlapped users, but each owns their own feature space and labels. Vertical Federated Learning merges the features together to create a larger feature space for machine learning tasks and uses homomorphic encryption to provide protection on data privacy for involved parties.

Federated Transfer Learning uses Transfer Learning [7] to improve model performance when we have neither much overlap on features nor on instances.

An example in insurance industry to illustrate the idea is the following. Horizontal FML corresponds to primary insurers working with a reinsurer. For the same product primary insurers share the similar features. Vertical FML corresponds to reinsurer working with another third-party data provider like online retailer. An online retailer will have more features for a certain policyholder that can increase the prediction power for models built for insurance.

For a detailed introduction of Federated Learning and their respective technology that is used, please refer to [2].

III. DELETION METHOD FOR HORIZONTAL FML

Most model interpretation methods can be applied, with some minor modifications, to contribution measure for Horizontal Federated Learning as all parties have data for the full feature space. There is no special issue for interpreting prediction results on both training data and new data, for both specific single predictions as granular check or for batch predictions as holistic check.

Approaches to identifying influential instances, such as deletion diagnostics [9] and influence functions [8], can be used to measure the importance of individuals to a machine learning model. Here we propose a method based on deletion diagnostics to measure contributions of different parties for horizontal FML.

Deletion diagnostics is intuitive. With the deletion approach, we retrain the model each time an instance is omitted from training dataset and measure how much the prediction of re-trained model changes. Supposing we are evaluating the effect of the i th instance on the model predictions, the influence measure can be formulated as follows,

$$\text{Influence}^{-i} = \frac{1}{n} \sum_{j=1}^n |\hat{y}_j - \hat{y}_j^{-i}|, \quad (1)$$

where n is the size of dataset, \hat{y}_j is the prediction on j th instance made by the model trained on all data, and \hat{y}_j^{-i} is the prediction on j th instance made by model trained with the i th instance omitted.

For one party in horizontal FML with a subset of instances D , we define the contribution as the total influence of all instances it possesses in the following form,

$$\text{Influence}^{-D} = \sum_{i \in D} \text{Influence}^{-i}. \quad (2)$$

We propose an approximation algorithm to implement the above influence measure, considering a batch of instances as a whole during each deletion, which is shown in Algorithm 1.

Algorithm 1 Approximating influence estimation for each party in horizontal FML

Input

number of parties K , model f
instance subsets D_1, \dots, D_K

Output

Influence measure Influence^{-D_k} for $k = 1, \dots, K$

for all $k=1, \dots, K$ **do**

delete D_k from training dataset

retrain model f'

compute $\text{Influence}^{-D_k} = \frac{1}{n} \sum_j |\hat{y}_j - \hat{y}_j^{-D_k}|$

end for

return Influence^{-D_k} for $k = 1, \dots, K$

IV. SHAPLEY VALUES FOR VERTICAL FML

In this section we focus on the contribution measure of different parties in vertical Federated Machine Learning. **In the vertical mode a party contributes to FML model by sharing its features with other parties, which means the contribution of the party can be represented by the combined contributions of its shared features.** Therefore, we first introduce how to distribute the contributions among individual features and then show the extension to measuring contribution of grouped features.

A. Shapley Values for Individual Feature

Generally, we are interested in how a particular feature value influences the model prediction. For an additive model like linear regression

$$f(x) = \beta_0 + \sum_{i=1}^n \beta_i x_i, \quad (3)$$

where β_i is the model coefficient and x_i the feature value, we can measure the influence of $X_i = x_i$ according to the situational importance [11]

$$\phi_i(x) = \beta_i x_i - \beta_i \mathbb{E}[X_i]. \quad (4)$$

The situational importance is the difference between what a feature contributes when its value is x_i and what it is expected to contribute. For a more general model which we treat as a black box, the feature influence can be computed in the following way similar to the situational importance:

$$\phi_i(x) = f(x_1, \dots, x_n) - \mathbb{E}[f(x_1, \dots, x_i, \dots, x_n)], \quad (5)$$

which is the difference between a prediction for an instance and the expected prediction for the same instance if the i th feature had not been known.

The Shapley value [12], which is originated from coalitional game theory with proven theoretical properties, provides an effective approach to distribute contributions among features in a fair way by assigning to each feature a number which denotes its influence [13] [14] [15] [16]. In a coalitional game, it is assumed that a grand coalition formed by n players has a certain worth and each smaller coalition has its own worth. The goal is to ensure that each player receives his fair share, taking into account all sub-coalitions. In our case, the Shapley value is defined as

$$\phi_i(x) = \sum_{Q \subseteq S \setminus \{i\}} \frac{|Q|!(|S| - |Q| - 1)!}{|S|!} (\Delta_{Q \cup \{i\}}(x) - \Delta_Q(x)), \quad (6)$$

where S is a feature index set, $Q \subseteq S = \{1, 2, \dots, n\}$ is a subset of features, x is the vector of feature values of the instance in consideration and $|\cdot|$ is the size of a feature set. **$\Delta_Q(x)$ denotes the influence of a subset of feature values, which generalizes (5), in the following form**

$$\Delta_Q(x) = \mathbb{E}[f|X_i = x_i, \forall i \in Q] - \mathbb{E}[f]. \quad (7)$$

The Shapley value $\phi_i(x)$ gives a strong solution to the problem of measuring individual feature contribution. However, computing (6) has an exponential time complexity, making the

method infeasible for practical scenarios. An approximation algorithm with Monte-Carlo sampling is proposed in [13] to reduce the computational complexity:

$$\phi_i(x) = \frac{1}{M} \sum_{m=1}^M (f(x_{+i}^m) - f(x_{-i}^m)), \quad (8)$$

where M is the number of iterations. $f(x_{+i}^m)$ is the prediction for instance x , with a random number of feature values replaced by feature values from a randomly selected instance z , except for the respective value of feature i . The vector x_{-i}^m is almost identical to x_{+i}^m , except that the value x_i^m is taken from the sampled z . The approximation algorithm is summarized in Algorithm 2.

Algorithm 2 Approximating Shapley estimation for individual feature value

Input

number of iterations M , instance feature vector x , model f , feature space \mathcal{X} , and feature index i

Output

Shapley value for the value of the i th feature $\phi_i(x)$

for all $m = 1, \dots, M$ do

select a random instance $z \in \mathcal{X}$

select a random permutation of the feature values

construct two new instances:

$$x_{+i} = (x_1, \dots, x_{i-1}, x_i, z_{i+1}, \dots, z_n)$$

$$x_{-i} = (x_1, \dots, x_{i-1}, z_i, z_{i+1}, \dots, z_n)$$

compute marginal contribution $\phi_i^m = f(x_{+i}) - f(x_{-i})$

end for

compute Shapley value $\phi_i(x) = \frac{1}{M} \sum_{m=1}^M \phi_i^m$

B. Shapley Values for Grouped Features

Vertical Federated Learning raises new issues for measuring contributions of multiple parties where the feature space is divided into different parts. Directly using methods like Shapley values for each prediction will very likely reveal the protected feature value from the other parties and cause privacy issues. Thus it is not trivial to develop a safe mechanism for vertical Federated Learning and find a balance between contribution measurement and data privacy.

We propose a variant version of the approach proposed in [14] to use Shapley value for measuring contributions of different parties in vertical FML. Here we take the dual-party vertical Federated Learning as an example, while the idea can be extended to multiple parties. For the k th instance, the label is y_k and one party owns part of the features $x^{h,k}$ and the other party owns the rest part of the features $x^{g,k}$, where $k = 1, \dots, K$ as we suppose both parties have K overlapped instances with the same IDs. By using vertical FML, the two parties collaborate to develop a machine learning model for predicting labels Y . We first give some definitions and assumptions in this problem and then propose an approximation algorithm to compute the Shapley group value for measuring the contributions of different parties.

Definition 1. (United Federated Feature). For the vertical FML with a set of parties G and a set of features S , the united federated feature x^{fed} is a combination feature of the features $x^g \in X^g \subset S$ for party $g \in G$.

We treat a united federated feature as a single feature since individual features of each party are private and not visible to other parties.

Definition 2. (Shapley Group Value). The Shapley group value is the group value that sums the individual Shapley values for all elements in the group. Formally, the Shapley group value for a subset $P \subset S$ is given by

$$\phi_P(x) = \sum_{i \in P} \phi_i(x). \quad (9)$$

The Shapley group value denotes the contribution of a subset of features.

Definition 3. (Shapley Group Interaction Index). The Shapley group interaction index is the additional combined feature effect of group $P \subset G$ given by

$$\phi_P(x) = \sum_{Q \subseteq S \setminus P} \frac{|Q|!(|S| - |Q| - 1)!}{|S|!} \delta_P(x), \quad (10)$$

where

$$\delta_P(x) = \Delta_{Q \cup P}(x) - \sum_{i \in P} \Delta_{Q \cup \{i\}}(x) + (|P| - 1) \Delta_Q(x). \quad (11)$$

The Shapley group interaction index is a variant of the Shapley interaction index [17] which extends the definition of the combined feature effect from two features to a group of features.

Assumption 1. The Shapley group interaction index for feature set $X^g \subset S$ of any party in vertical FML is zero, i.e., $\phi_{X^g}(x) = 0, \forall g \in G$.

Assumption 2. All features in the feature set X^g of party g are dummy features with $\Delta_{Q \cup \{j\}}(x) = \Delta_Q(x) + \Delta_{\{j\}}(x), \forall j \in X^g, \forall g \in G$ and $\forall Q \subset S$.

Proposition 4. If either of Assumption 1 and 2 holds, then the Shapley group value for a party $g \in G$ with feature set X^g is given by

$$\phi_{X^g} = \sum_{Q \subseteq S \setminus \{j^{fed}\}} \frac{|Q|!(|S| - |Q| - 1)!}{|S|!} (\Delta_{Q \cup \{j^{fed}\}}(x) - \Delta_Q(x)), \quad (12)$$

where j^{fed} is the index of the united federated feature x^{fed} .

Proof. We consider the vertical FML scenario where the other parties act collaboratively as a whole and reach an agreement on a protocol of sharing and permutation among all their features when computing the Shapley group value for one party. Actually this reduces to the dual-party FML case. If Assumption 1 holds, then

$$\sum_{i \in P} (\Delta_{Q \cup \{i\}}(x) - \Delta_Q(x)) = \Delta_{Q \cup P}(x) - \Delta_Q(x).$$

According to the definition of federated feature, we can treat $\{j^{fed}\}$ as X^g . Thus putting the above equation into (6) and (9) gives (12). If Assumption 2 holds, then

$$\begin{aligned} \Delta_{Q \cup \{j_1^g, \dots, j_k^g\}}(x) &= \Delta_{Q \cup \{j_1^g, \dots, j_{k-1}^g\}}(x) + \Delta_{\{j_k^g\}}(x) \\ &= \dots \\ &= \Delta_Q(x) + \sum_{j \in X^g} \Delta_{\{j\}}(x) \end{aligned}$$

The above equation together with the dummy property makes (11) equal to zero. Thus Assumption 1 holds. \square

Proposition 4 indicates that we can measure the importance of a feature subset without revealing the details of any private feature of a party in the vertical FML. Suppose we want to measure the contribution of one party to the prediction of an instance by looking at the Shapley group value of the feature set shared by the party. Instead of giving out individual Shapley values for all features in its feature space, we combine the private features as one united federated feature, and compute the Shapley value for this federated feature together with the features of all the other parties.

Since this method requires to turn on and off certain features for calculating the Shapley value, for the federated feature we will need a specific ID to inform the party in consideration to return its part of the prediction with all its features turned off. For models that takes in NA values, this mean that the features will be set to NA. For models that cannot handle missing values, we follow the practice of [14] to set the feature values to be the median value of all the instances as the reference value.

Although the two assumptions are quite strong, the experiment results show that the approximation algorithm works well in real scenarios. Also, as discussed above the approximation algorithm for the dual-party case can be extended to measuring contributions of multiple parties as long as an agreement on a protocol of feature sharing and permutation is reached. In summary, the process of computing Shapley group value for one party is described in Algorithm 3. Repeating the estimation algorithm for all parties gives their corresponding contribution measure.

V. EXPERIMENT

We developed the algorithms for calculating multi-party contribution as derived in Section 3 and 4. In this section, we test our algorithm by training a machine learning model on the Cervical cancer (Risk Factors) Data Set [10] and experimenting by calculating the participant contributions on both horizontal and vertical FML setups. The Cervical cancer dataset is used to predict whether an individual female will get cervical cancer given indicators and risk factors as features of the dataset, which is showed in Table I. We normalized the data and used Scikit-learn to train a SVM (Support Vector Machine) model for the cervical cancer classification task.

The contribution of the FML participants can be of two ways, namely the horizontal FML setup where we use deletion method for indicating group instance importance, and the

Algorithm 3 Approximating Shapley estimation for federated feature for one party in vertical FML

Input

number of iterations M , instance feature vector x ,
model f , the federated feature index j^{fed} ,
the index set of other features I^h , party g

Output

Shapley group value $\phi_{j^{fed}}$ for j^{fed}

for all $m = 1, \dots, M$ **do**

select a subset $Q \in I^h \cup \{j^{fed}\}$

construct new instance x' :

Set $x'_k = x_k$ for $k \in Q$

Set x'_k to reference value for $k \notin Q$

if $j^{fed} \in Q$ **then**

Send encrypted ID of x to party g

Set $x'_{j^{fed}} = x_{j^{fed}}$

else

Send special ID to party g

Set $x'_{j^{fed}}$ to reference value

end if

Run federated model prediction for x'

Save prediction result of Q

end for

compute $\phi_{j^{fed}}$ using Algorithm 2

return Shapley group value $\phi_{j^{fed}}$

TABLE I
CERVICAL CANCER (RISK FACTORS) DATA SET ATTRIBUTE
INFORMATION

Attribute	Type
Age	int
Number of sexual partners	int
First sexual intercourse (age)	int
Num of pregnancies	int
Smokes	bool
Smokes (years)	int
Hormonal Contraceptives	bool
Hormonal Contraceptives (years)	int
IUD	bool
IUD (years)	int
STDs	bool
STDs (number)	int
STDs: Number of diagnosis	int
STDs: Time since first diagnosis	int
STDs: Time since last diagnosis	int
Biopsy ^a	bool

^aTarget Variable

vertical FML setup where Shapley value is used for evaluating importance of features from different parties. For purpose of illustration, we separately considered the experiments in order to illustrate that for both horizontal FML and vertical FML our proposed methods can give a reasonable explanation for the contribution of multiple participants.

A. Deletion Method (Horizontal FML)

As we explained in Section 3, the deletion method can be used to evaluate importance of a single instance and then

generalized to the scenario where different groups of instances are from different parties. The experiments are performed on the Cervical dataset with “Biopsy” as the target value. For simplicity, we only considered binary classification problem, where the Biopsy takes the value Health and Cancer. Since the deletion method is defined for the training process, without losing the generality we don’t split the Cervical dataset in the experiment, so the training set has the number of instances the same as the entire dataset. We use SVM as the classification algorithm, where the output is set to ‘probability’ and the kernel is RBF (Radial Basis Function) with the coefficient as $1/(\text{number of features})$. In order to simulate the participant contribution, we evenly split the dataset with a given number of instances, so that in our experiment we conceptually build up a Horizontal Federated Learning ecosystem with five players. We used deletion method to calculate the contribution of those five participants and the results are shown in Fig. 1.

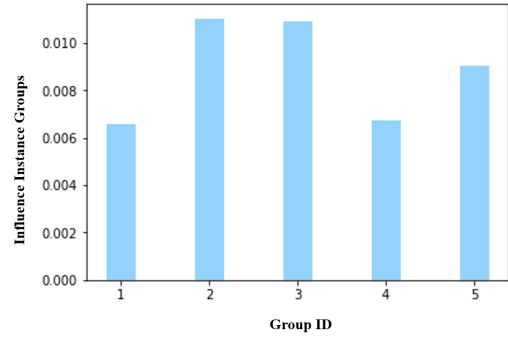


Fig. 1. The importance of instance groups of cervical data. We simulated five parties and each party has same number of training instances. The vertical axis shows the value of horizontal FML instance group importance value.

B. Shapley Value (Vertical FML)

We also did the experiments to calculate the Shapley value for feature importance, where we simulate the vertical FML ecosystem and each participant shares a certain part of the feature space. The experiments are performed on the same Cervical cancer dataset as explained in the previous session. We randomly shuffled the data and used 70% instances for training and 30% for testing. For testing data the accuracy reaches 95.42%. The algorithm’s setup is exactly the same as the experiment in the previous session. In order to avoid the inconsistency due to the algorithm’s hyperparameter choices, the random state for splitting and shuffling the dataset is set to the same random seed.

As the first demonstration, we pick one specific instance from the training data, run the prediction and test our Shapley Federated algorithm for feature importance. The result can be seen on Fig. 2.

Following the demonstration, we then considered the Vertical FML ecosystem framework. We first calculated the Shapley value for the whole feature space, as shown in Fig. 3, which directly reflects the importance for different features as normal Shapley value indicates.

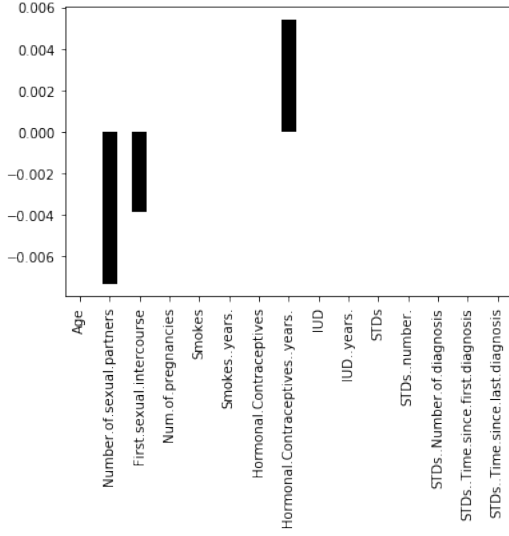


Fig. 2. The Shapley value for predicting one instance in Cervical cancer dataset. This is the demonstration that Shapley value can give a reasonable explanation for feature importance on each prediction.

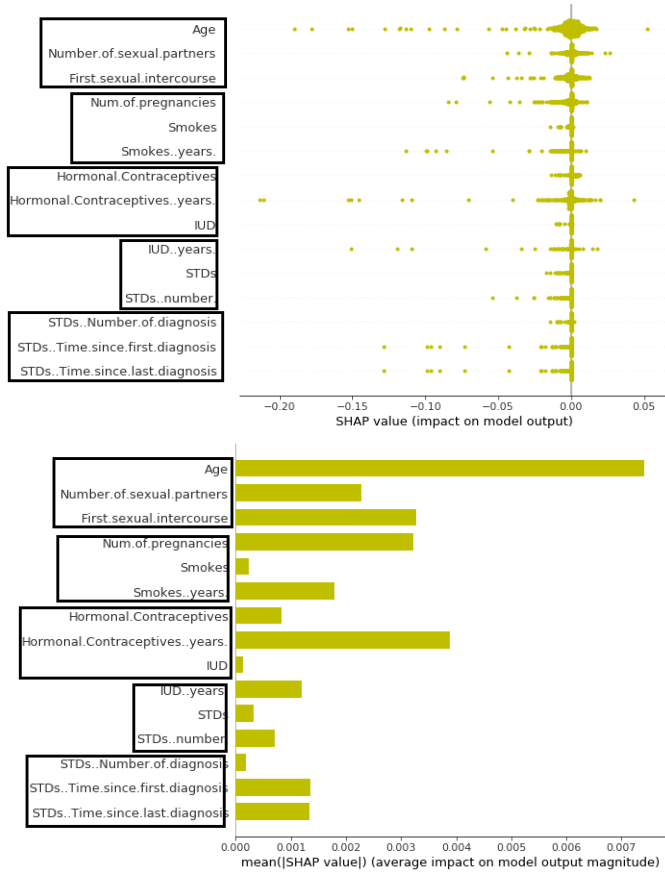


Fig. 3. Feature Importance (Shapley values) for 855 instances for the whole feature space. Above one is the scatter plot for each prediction on the data, below one is bar plot for each feature's total contribution from all the predictions.

We then simulated the vertical FML for multiple participants, where we evenly separate the 15 features into 5 groups and each group represents a single participants with 3 features. Each time we group the features from one party together as the federated feature, and run the Shapley value algorithms to calculate the feature importance for this single federated feature together with the other individual features from the other participants. The simulated results are showed in Fig. 4 and Fig. 5. Another way of doing this is to use federated features all together for all the participants and calculate the Shapley value at one go. We expect that this will give less accurate results. Our experiment indicates that in the multi-party Vertical FML setup, federated Shapley value is a good quantity to indicate the contribution for each participant.

VI. CONCLUSION

Fair calculating the contribution of each participants in Federated Machine Learning is crucial for credits and rewards allocation. In this paper, we proposed methods that can calculate participant contribution for both Horizontal FML and Vertical FML by using group instance deletion and group Shapley values. Our experiment results indicate that our method is effective and can give fair and reliable contribution measurements for FML participants without disclosing the data and breaking the initial intent of preserving data privacy.

Our work for contribution measurement for FML participants is model agnostic, meaning that this should work for almost any kind of machine learning algorithms, and become a general framework for this task. We expect our work can be built into FML tool sets like FATE and TFF and become the start of developing a standard model contribution measurement module for Federated Learning that is critical for industrial applications.

For future work, we expect some more advanced algorithms like influential functions [8] for horizontal FML and for some sampling version of calculating Shapley values for vertical FML. Those algorithms will help get an accurate and fair contribution measurement results with higher computational efficiency.

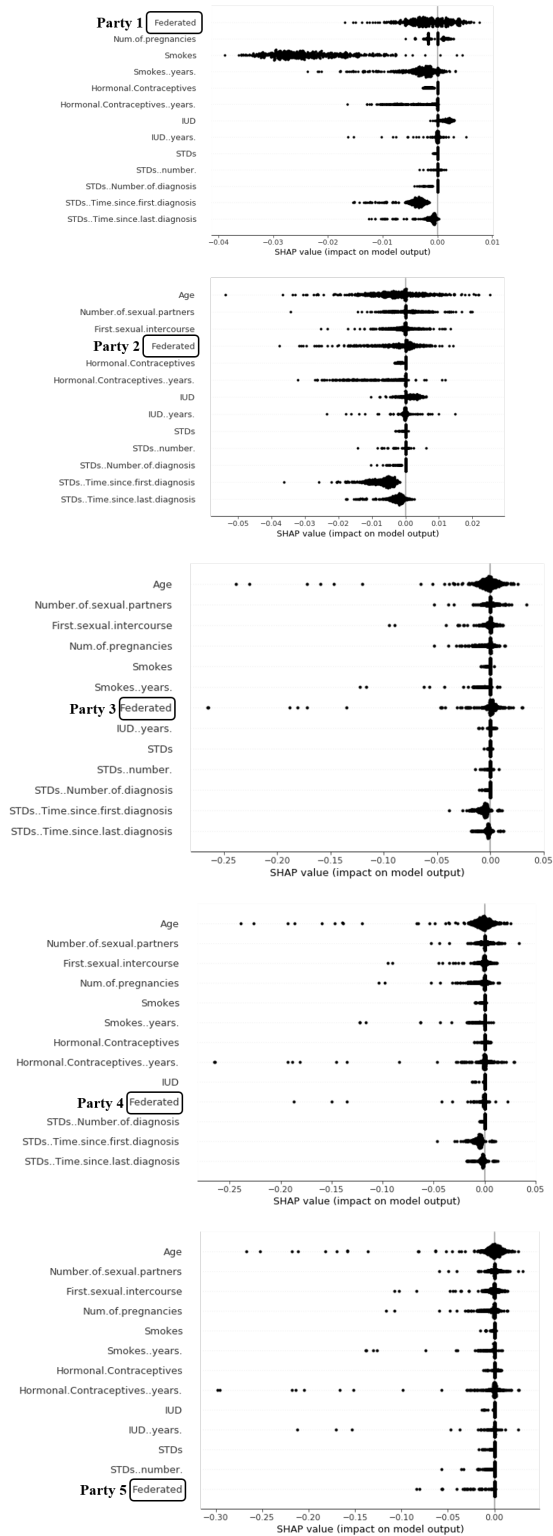


Fig. 4. Scatter plot for Feature Importance (Shapley values) for 855 instances. We considered different federated groups of different features. For combined feature has different impact on the feature importance. We evenly separated the 15 features into 5 groups, and each group has 3 features



Fig. 5. Bar plot for average Feature Importance (Shapley values) for 855 instances. We considered different federated groups of different features. For combined feature has different impact on the feature importance. We evenly separated the 15 features into 5 groups, and each group has 3 features

REFERENCES

- [1] Jakub Konen, H. Brendan McMahan, Felix X. Yu, Peter Richtarik, Ananda Theertha Suresh and Dave Bacon, 2016, "Federated Learning: Strategies for Improving Communication Efficiency", NIPS Workshop on Private Multi-Party Machine Learning.
- [2] Qiang Yang, Yang Liu, Tianjian Chen, and Yongxin Tong. 2019. Federated Machine Learning: Concept and Applications. ACM Trans. Intell. Syst.
- [3] Guan Wang, 2019, Interpret Federated Learning with Shapley Values. FML Workshop.
- [4] H. Brendan McMahan, Eider Moore, Daniel Ramage, and Blaise Agera y Arcas. 2016. Federated learning of deep networks using model averaging. CoRR abs/1602.05629 (2016). arXiv:1602.05629 <http://arxiv.org/abs/1602.05629>.
- [5] Albrecht, J. P. 2016. "How the gdpr will change the world." Eur. Data Prot. L. Rev. 2:287.
- [6] Goodman, B., and Flaxman, S. 2016. "European union regulations on algorithmic decision-making and a right to explanation". arXiv preprint arXiv:1606.08813.
- [7] Sinno Jialin Pan and Qiang Yang. 2010. A survey on transfer learning. IEEE Trans. Knowl. Data Eng. 22, 10 (Oct. 2010), 1345–1359. DOI:<https://doi.org/10.1109/TKDE.2009.191>
- [8] Koh, Pang Wei, and Percy Liang. "Understanding black-box predictions via influence functions." arXiv preprint arXiv:1703.04730 (2017).
- [9] Cook, R. Dennis. "Detection of influential observation in linear regression." Technometrics 19.1 (1977): 15-18.
- [10] Kelwin Fernandes, Jaime S. Cardoso, and Jessica Fernandes. 'Transfer Learning with Partial Observability Applied to Cervical Cancer Screening.' Iberian Conference on Pattern Recognition and Image Analysis. Springer International Publishing, 2017.
- [11] Achen, Christopher H. Interpreting and using regression. Vol. 29. Sage, 1982.
- [12] Shapley, Lloyd S. "A value for n-person games." Contributions to the Theory of Games 2.28 (1953): 307-317.
- [13] Erik Strumbelj, and Igor Kononenko. "Explaining prediction models and individual predictions with feature contributions." Knowledge and information systems 41.3 (2014): 647-665.
- [14] Lundberg, Scott M., and Su-In Lee. "A unified approach to interpreting model predictions." Advances in Neural Information Processing Systems. 2017.
- [15] Lundberg, Scott M., Gabriel G. Erion, and Su-In Lee. "Consistent individualized feature attribution for tree ensembles." arXiv preprint arXiv:1802.03888 (2018).
- [16] Staniak, Mateusz, and Przemyslaw Biecek. "Explanations of model predictions with live and breakDown packages." arXiv preprint arXiv:1804.01955 (2018).
- [17] Fujimoto, Katsushige, Ivan Kojadinovic, and Jean-Luc Marichal. "Axiomatic characterizations of probabilistic and cardinal-probabilistic interaction indices." Games and Economic Behavior 55.1 (2006): 72-99.