Week 3 Problem Set
Analysis of Algorithms

1. (Counting primitive operations)

   The following algorithm

   - takes a sorted array A[1..*n*] of characters
   - and outputs, in reverse order, all 2-letter words vω such that v≤ω.

```
for all i=n down to 1 do
   for all j=n down to i do
      print "A[i]A[j]"
   end for
end for
```

   Count the number of primitive operations (evaluating an expression, indexing into an array). What is the time complexity of this algorithm in big-Oh notation?

2. (Big-Oh Notation)

   a. Show that $\sum_{i=1}^{n} i^2 \in O(n^3)$

   b. Show that $\sum_{i=1}^{n} \log i \in O(n \log n)$

   c. Show that $\sum_{i=1}^{n} \frac{i}{2^i} \in O(1)$

3. (Algorithms and complexity)

   Develop an algorithm to determine if a character array of length *n* encodes a *palindrome*, that is, which reads the same forward and backward. For example, "racecar" is a palindrome.

   a. Write the algorithm in pseudocode.

   b. Analyse the time complexity of your algorithm.

   c. Implement your algorithm in C. Your program should prompt the user to inpout a stringb and check whether it is a palindrome. Examples of the program executing are

```
prompt$ ./palindrome
Enter a word: racecar
yes
prompt$ ./palindrome
Enter a word: reviewer
no
```

   Hint: You may use the standard library function `strlen(char[])`, defined in `<string.h>`, which computes the length of a string (without counting its terminating `'\0'`-character).

We have created a script that can automatically test your program. To run this test you can execute the `dryrun` *program that corresponds to this exercise. It expects to find a program named* `palindrome.c` *in the current directory. You can use autotest as follows:*

```
prompt$ 9024 dryrun palindrome
```

4. (Algorithms and complexity)

   Let $p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ be a polynomial of degree n. Design an O(n)-time algorithm for computing the function $p(x)$.

   Hint: Assume that the coefficients a_i are stored in an array A[0..n].

5. (Algorithms and complexity)

   A vector V is called *sparse* if most of its elements are 0. In order to store sparse vectors efficiently, we can use an array L to store only its non-zero elements. Specifically, for each non-zero element *V[i]*, we store an index-value pair *(i,V[i])* in L.

   For example, the 8-dimensional vector V=(2.3,0,0,0,-5.61,0,0,1.8) can be stored in an array L of size 3, namely L[0]=(0,2.3), L[1]=(4,-5.61) and L[2]=(7,1.8). We call L the *compact form* of V.

   Describe an efficient algorithm for adding two sparse vectors V_1 and V_2 of equal dimension but given in compact form. The result should be in compact form too, of course. What is the time complexity of your algorithm depending on the sizes m and n of the compact forms of V_1 and V_2, respectively?

   Hint: The sum of two vectors V_1 and V_2 is defined as usual, e.g. (2.3,-0.1,0,0,1.7,0,0,0) + (0,3.14,0,0,-1.7,0,0,-1.8) = (2.3,3.04,0,0,0,0,0,-1.8).

6. (Ordered linked lists)

   A particularly useful kind of linked list is one that is sorted. Give an algorithm for inserting an element at the right place into a linked list whose elements are sorted in ascending order.

   Example: Given the linked list

   ```
   L = 17 26 54 77 93
   ```

   the function `insertOrderedLL(L,31)` should return the list

   ```
   L = 17 26 31 54 77 93
   ```

   Hint: Develop the algorithm with the help of a diagram that illustrates how to use pointers in order to
   - find the right place for the new element and
   - link the new element to its predecessor and its successor.

7. (Advanced linked list processing)

   Describe an algorithm to split a linked list in two halves and output the result. If the list has an odd number of elements, then the first list should contain one more element than the second.

   Note that:
   - your algorithm should be 'in-place' (so you are not permitted to create a second linked list or use some other data structure such as an array);
   - you should not traverse the list more than once (e.g. to count the number of elements and then restart from the beginning).

   An example of the result of the algorihtm could be

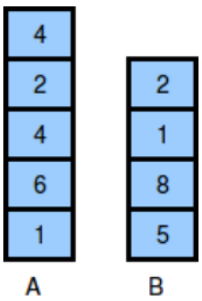
   ```
   Linked list: 17 26 31 54 77 93 98
   First half:  17 26 31 54
   ```

```
Second half: 77 93 98
```

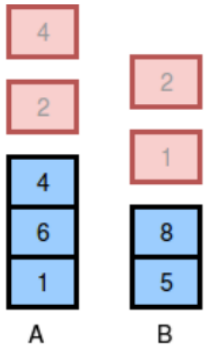
8. **Challenge Exercise**

   Suppose that you are given two stacks of non-negeative integers A and B and a target threshold $k \geq 0$. Your task is to determine the maximum number of elements that you can pop from A and B so that the sum of these elements does not exceed k.

   Example:

   Maximum number of elements that can be popped without exceeding $k = 10$ is 4:

   If k = 7, then the answer would be 3 (the top element of A and the top two elements of B).

   a. Write an algorithm (in pseudocode) to determine this maximum for any given stacks A and B and threshold k. As usual, the only operations you can perform on the stacks are pop() and push(). You *are* permitted to use a third "helper" stack but no other aggregate data structure.

   b. Determine the time complexity of your algorithm depending on the sizes m and n of input stacks A and B.

   Hints:
   - A so-called greedy algorithm would simply take the smaller of the two elements currently on top of the stacks and continue to do so as long as you haven't exceeded the threshold. This won't work in general for this problem.
   - Your algorithm only needs to determine the number of elements that can maximally be popped without exceeding the given k. You do not have to return the numbers themselves nor their sum. Also you do not need to restore the contents of the two stacks; they can be left in any state you wish.

Assessment

After you have solved the exercises, go to COMP9024 19T3 Quiz Week 3 to answer 5 quiz questions on this week's problem set (Exercises 1-7 only) and lecture.

The quiz is worth 2 marks.

There is no time limit on the quiz once you have started it, but the deadline for submitting your quiz answers is **Tuesday, 8 October 11:00:00am**. (Note the public holiday on Monday, 7 October!)

Please be mindful of the following **quiz rules**:

Do …

- use your own best judgement to understand & solve a question
- discuss quizzes on the forum only **after** the deadline on Tuesday

Do not …

- post specific questions about the quiz **before** the Tuesday deadline
- agonise too much about a question that you find too difficult