# Design Project

## MECH 323 Machine Design (Winter 2021)

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|---|---|
| **Team Number** | **23** |

| | |
|---|---|
| **Phase Number** | **4** |

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|---|---|---|---|
| **Team Members** | | **Student Number (Required)** | **Student Name (Optional)** |
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## Executive Summary

Phase 4 consisted of 7 tasks where Lectures 18, 19, and 20 were carefully reviewed to iterate the preliminary features of the gears and shafts. This was done to create a finalized design with a corresponding housing to be 3D printed.

The 7 tasks included the following defined requirements, CAD design, and decisions. Task 1 focused on the input and output shafts where the three designed shafts shown in the Drawing Package were updated to have a d-shaft section of at least 0.021 m long. Task 2 focused on the spline connection, where the preliminary shafts with keyways were updated to have splines. This would allow the gears and shafts to be coupled.

Task 3 consisted of bushing selection following the procedure in Lecture 19 [1]. The bushings that were chosen to support the designed shafts were five 6391K403 ($0.70/each) bearings and one 6391K126 ($0.51/each) bearing for a total of 6 bearings and a total cost of USD 4.01 [2]. Task 4 consisted of creating the housing design shown in the drawing package below. The housing design was created following all the defined constraints to ensure it would function properly. A detailed description of the completed housing design can be seen in Section 3.1.

Task 5 was completed to ensure the final design of the gearbox was prepared for 3D printing. All parts were exported to Catalyst and the final printing time was determined to be 6 hours and 21 minutes.

Finally, the 5 defined tasks prepared the final project report and drawing package to be completed in Task 6 and Task 7. The final report below consists of a two-page description of the overall gearbox design, where the housing design was highlighted and the key performance metrics were stated and justified to ensure the competitive advantage of the designed gearbox. A description of the evolution of the gearbox design was then written to explain how the final design of the gearbox was generated and iterated. A parts list, tools required, and assembly procedure were then stated and explained to ensure that the drawing package completed in Task 7 was understood and justified accordingly.

# Contents

## 1.0 Introduction and Problem Statement

The team was tasked with designing a gearbox for a toy vehicle to compete in race events. The first event is a hill climb. The goal of the event was to climb as high as possible up an arced ramp with a radius of 1.05 m. A depiction of the event can be seen in Figure 1. [1]

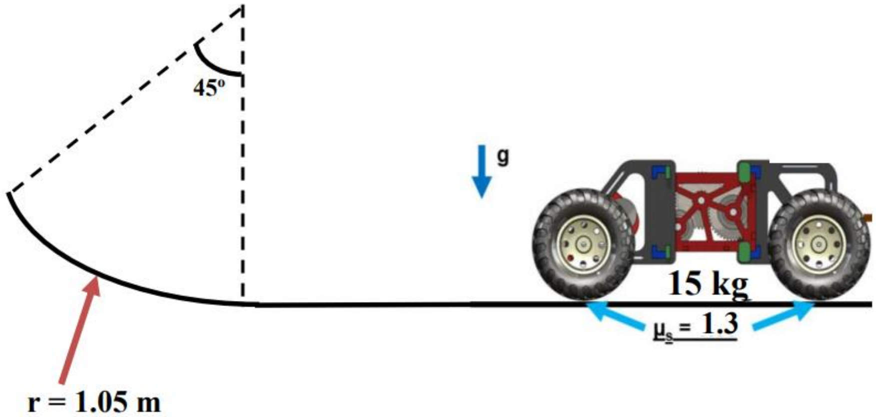


*Figure 1 - Depiction of the hill climb event.*

The second event was a speed event, where the vehicle needed to reach the highest average speed within a 2 m zone having an acceleration distance of 4 m. A depiction of this event can be seen in Figure 2. [1]

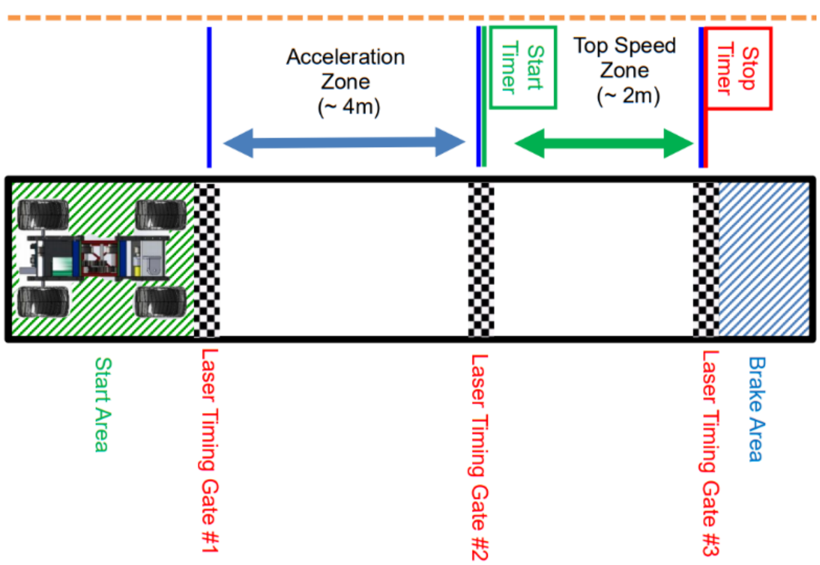


*Figure 2 - Speed event racetrack.*

Along with competing in these events, the vehicle also had to withstand an endurance test. The gears and shafts needed to be able to act without failure over five years with a safety factor greater than 1. Each of these events was used to design the components of the vehicle for desired performance. [1]

Based on the above information, the problem at hand was to design a vehicle gearbox 0.08 m wide and high with a length between 0.05 and 0.13 m long, which would optimize its performance in the hill climb and speed event. It would also need to have safety factors above 1 for the shaft and gears for performance in a five-year endurance test.

## 2.0 Overall Gearbox Design

Section 3 covers a two-page description of the overall gearbox design. The section highlights why the housing and performance metrics were determined to generate an advantage over others.

### 2.1 Housing Design

The final design of the housing can be seen in Figure 3. The housing design was generated by creating the Housing Side Right part to ensure that none of the timing chain sprockets would interfere with the mounting notches. The shafts were positioned to ensure that none of the gears exceeded the top of the gearbox and that the gears meshed accordingly. The shafts were not increased by the recommended 10-20% at the locations of the smallest safety factor, as there was risk of gear rim failure. Additionally, the lowest safety factor on the shafts was 2.04, which is double the limit of 1, indicating no need for increasing shaft diameters. The total length of the housing was then selected to ensure all the parts fit seemingly, while focusing on reducing the length of the gearbox. The holes were cut to ensure that a shoulder was left on the outside so that the required constraints were met. An additional beam was also added on Shaft A and Shaft B to better resist the expected loading. The dovetails were then added at the correct locations to ensure the timing chains could pass through the housing. Dovetails were chosen to increase strength and allow the housing to maintain its structure before being put in the vehicle.

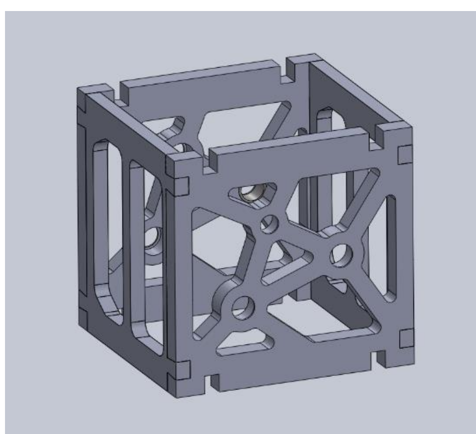

*Figure 3 – Final housing design.*

The detailed housing design creates a competitive advantage by ensuring a short length of the vehicle and reducing the weight of the vehicle. This was accomplished with the completed skeletonization while keeping a reasonable thickness to account for the expected applied loads. A short-length vehicle will reduce weight, drag, and printing time. This was shown when the estimated printing time was determined to be 6 hours and 21 minutes, much lower than the limit of 9 hours.

## 2.2 Key Performance metrics

The key performance metrics of the gearbox design and their significance are as follows. The gearbox was designed to have a shifting gear system using Shaft A, Shaft B, and Shaft C shown in the Drawing Package below. This was determined to be the preferable system and would generate a key advantage compared to others. The top speed and hill climb event demand different characteristics and thus require a shifting gear system to reduce and increase the torque needed for its corresponding event. A summary of the key performance metrics for the designed gearbox can be seen in Table 1.

Table 1 defines the calculated gear ratios for both the hill climb and top speed events. The required torque equal to 4.24 Nm was found using Figure 33 shown in Appendix A. The required torque was used to calculate the overall gear ratio, stage 1 gear ratio, and stage 2 gear ratio, highlighted in Table 1. The optimal gear ratio was found to be 3.029 for the hill climb event. The optimal gear ratio allowed the designed car to travel 0.503 m using the expected angle of equilibrium to be 27.5 degrees. The determined gear ratio and the distance covered for the hill climb event were calculated using Equation 1 to Equation 4 in Appendix D.

The expected time and speed for the top speed event were calculated using a numerical solution of the differential equation of motion. The expected velocity versus distance of various gear ratios can be seen in Figure 31, calculated using a linear-fit version of the torque-speed curve in Figure 32. The expected 2 m time was calculated to be 2.02 s with an expected speed of 0.99 m/s.

The key performance metrics and justifications shown in Table 1 were calculated to ensure the pre-built race vehicle could achieve success in all three events. In conclusion, the calculations were created to ensure the designed gearbox would produce the maximized angular speed of the motor and output torque. The calculations were used to ensure the designed gearbox would produce the greatest angular speed and output torque of the motor. Other key parameters including gear ratios, number of weights added, number of gear teeth, module and pitch diameter are summarized in the Phase 4 Summary at the end of this document and in Figure 18 to Figure 29.

*Table 1 – Summary of key performance metrics of the gearbox.*

| Hill Climb Event | |
|---|---|
| Gear ratio ($m_g$) | 3 |
| Stage 1 gear ratio ($m_{g1}$) | 1.732 |
| Stage 2 gear ratio ($m_{g2}$) | 1.732 |
| Angle of reached equilibrium ($\theta$) | 27.5 degrees |
| Distance covered | 0.503 m |
| **Top Speed Event** | |
| Gear ratio ($m_g$) | 1 |
| Stage 1 gear ratio ($m_{g1}$) | 1.732 |
| Stage 2 gear ratio ($m_{g2}$) | 0.577 |
| Expected speed | 0.99 m/s |
| Expected 2 m time | 2.02 s |
| Expected torque | 0.625 Nm |
| Angular velocity at top speed | 151.280 RPM |

## 3.0 Evolution of the Gearbox Design

The gearbox started its evolution in Phase 1 after careful calculations on the theoretically optimal gear ratios and feasibility analysis. From the compromises required, the actual gear ratios were used in the initial design of the system, featuring a shifting central shaft. In the second phase, fatigue analysis of the gear under both bending and surface stress revealed a probable failure. A redesign occurred, requiring changes in the gear parameters and ultimately, in the shifting mechanism. Instead of the shaft moving to perform the shifting, the mechanism used shifting gears. No major design changes occurred in Phase 3, as the design passed the shaft fatigue criterion. In Phase 4, the bearings were chosen, the housing was created, and several adaptations were performed on the gears and shafts, yielding the final design for prototyping. A flowchart of the evolution can be seen in Figure 4.

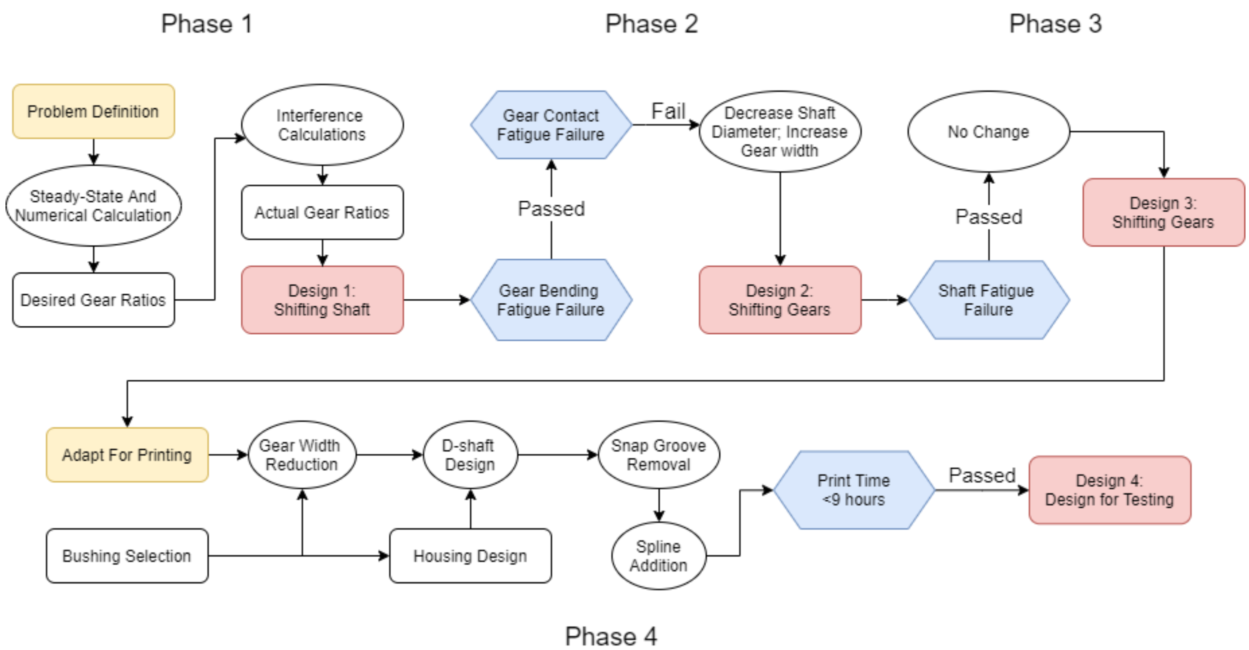


*Figure 4 - A flow chart of the design evolution. Rectangles represent design stages, where yellow represents input from the problem statements, red represents critical design points. Circles represent changes to the system and blue hexagons represent evaluation points where the system had to overcome new criteria.*

The problem definition in Phase 1 involved first breaking down the two events. The hill climb event was decomposed into several limitations and was treated as a steady-state problem. Four limitations were examined: flipping (which was assessed to be unimportant), friction (which was determined to be the limiting factor), the torque, and the time constraint. To accommodate this, a gear ratio of 3.029:1 was chosen such that the torque would reach the frictional limit. With an analysis of the system's minimal speed at the operating torque, the design was validated against the time constraints. The transient forces of the speed event were analyzed with a computational approach. A numerical solution and subsequent optimization of the differential equations of motion were used to generate a gear ratio of 0.55:1 for the speed event.

These two gear ratios were the basis for the intended preliminary design. The gearbox would implement a two-stage gear shift to accommodate the two requirements. Using gear interference equations and appropriate values for the pressure angle, module, and tooth type, the necessary number of teeth were found for each gear and event. Due to interference issues, the actual gear ratio in the design had to be adapted from the desired values, changing to 3:1, and 1:1, for the hill and speed events, respectively. The first iteration was based on all these considerations.

This preliminary design would use a shifting central shaft with all gears fixed relative to their respective shafts. In the hill climb event, two consecutive increases in pitch diameter would deliver an increased torque. The central shaft would then shift, moving the hill climb gears on the last two shafts out of alignment and setting a gear reduction for the speed event. The front two shafts would mate identically in either configuration, using a doubly wide gear to accommodate the translation of the central shaft. Additional features such as shoulders and grooves for snap rings were added as well. An image of the first design can be seen in Figure 4.

However, the first design faced certain issues in Phase 2.  The concerns of this phase's task involved safeguarding the gears against fatigue failure in an endurance race where the vehicle would undergo continuous operation for five years. Two safety factors were considered: the safety factor against bending fatigue and the safety factor against surface contact fatigue.

To determine these values, a variety of correction factors had to be determined, involving temperature, geometry, gear width, shaft diameter, surface condition, hardness, loading, and cycle life. It was at this point that a mistake became apparent: the gears would be protected from bending, but the surface contact fatigue would cause failure. Careful evaluation of the variables revealed that most of the factors were fixed with the operating conditions: the gear width would have to increase, and the shaft diameters would have to decrease. With the original design nearly impinging on the space constraints, a major redesign would have to occur to meet these two requirements.

Further analysis determined that these two mistakes were primarily caused by the shifting shaft mechanism: the gears had to be thin, and the shoulders were numerous, increasing the shaft diameters.  Thus, the team had to shift gears. A choice was made to transition to a design where the gears would shift on the output shaft instead, and all shafts were stationary relative to the housing. The thick gear on the input shaft could be halved, permitting the width of the other gears to increase without surpassing the constraints. Furthermore, a stationary gear requires a shoulder on one side whereas a sliding gear can employ a snap ring on both sides. The overall reduction in the number of shoulders, along with a decrease in the diameter of the shoulders, further improved the safety factors. What emerged was the second design, which resisted gear fatigue failure and is displayed in Figure 6.

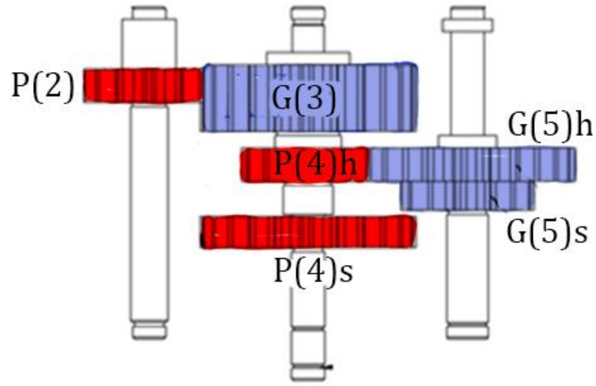


*Figure 5 - A diagram of the layout of the first design. Red indicates pinions, blue indicates gears.*

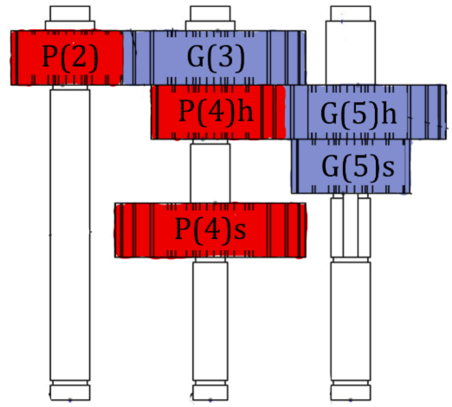


*Figure 6 -A diagram of the layout of the design at the second and third stages. Red indicates pinions, blue indicates gears.*

As mentioned previously, no major design changes occurred in Phase 3 of the project. Like Phase 2, the goal was to ensure the gearbox would perform with a high degree of confidence for the duration of the super endurance event. It required an in-depth analysis of the shafts to determine the safety factor of all critical points using the DE-Gerber criterion. In the event a safety factor was lower than 1, a redesign was required. To determine this, the following calculations were completed. The corrected endurance limit had to be found. Next, all reaction forces and torques acting on the shaft were determined. The locations of all critical points were established and the fatigue stress concentration factor of each was calculated. Lastly, the midrange torque and resultant amplitude moment were determined at the critical points. Using these values, the required safety factors were calculated, with a minimum of 2.04. The main equations used in these calculations can be seen in Appendix D. Since the safety factor requirements were met, it was decided that the current design was sufficient. Therefore, no redesign was required, and no changes were made to the Phase 2 design. [3]

The basis of Phase 4 was adapting the design for 3D printing. Until this point, it was assumed all aspects of the gearbox were made of AISI 4130 steel. First, the following changes had to be made to the shafts and gears. The shaft to gear couplings could no longer use a keyway and instead required a splined connection. There was no longer a need for the retaining ring features as the spline connection relies on a frictional fit. The input and output shafts had to be modified to have a certain length of D-shaft to be compatible with the testing rig. The D-shaft section had to have a lofted surface on either side to minimize stress concentrations. It was also crucial that any major changes made to the shafts and gears did not drop their respective safety factors below 1.

Once these changes were completed, the bearings had to be selected. The price of the bearings used could not surpass $5 and from the selections, the shafts and housing had to be made compatible. While selecting these, the team realized that there would not be enough room on the output shaft to fit the required 0.021 m D-shaft section for the sprocket attachment. To counter this, the gear face widths were reduced by 0.001 m. Calculations to justify this design change are shown in Appendix D. It was determined that this reduction caused the minimum safety to be reduced to 1.09. This would not cause the safety factors to fall below 1 and proved that there was no concern of failure.

The gearbox housing was then designed based on the changes made to the shafts and gears. It had to restrain all forces from the shafts, properly locate the bearings, and meet all dimensional requirements set in Phase 1. The design aimed to be as compact as possible, use minimal material, and decrease the length of the car. Once this was completed, the question remained as to whether all parts of the gearbox could be printed within the required 9-hour limit. With the use of the Catalyst software and the settings shown in Table 5, the print time was found to be 6 hours and 21 minutes, which is well within the limit. Since all constraints had been met, the team arrived at the final gearbox design presented in Section 2.0.

# 4.0 Technical Specification

The following section covers the assembly instructions of the final gearbox design. Included is a parts list, as shown in Table 2, and a technical specification summary shown in Table 3.

## 4.1 Assembly Instructions

A full parts list of all gearbox components can be found in Table 4 of the Drawing Package. Parts include housing components, shafts, sprockets, and bearings. Vehicle components are excluded from this list. Full drawings of all components with dimensions and other manufacturing instructions can be found in Figure 18 to Figure 29. The printing time and layout of the print table for all components are seen in Figure 30.

A complete list of tools and materials needed for the gearbox is shown in Table 2. Included are tools for making the components as well as assembling the parts.

*Table 2 - List of tools and materials required for assembly.*

| **Tool** | **Use** |
|---|---|
| 3D printer | Prints the components of the gearbox aside from the bearings. |
| Safety Glasses and Safety Shoes | Worn for safe assembling in the workshop. (Safety first). |
| Arbor press | Will be used to insert the bearings in the housing to ensure a press fit. |
| Clamp | Used to hold the housing parts to allow safe insertion of bearings when using the arbor press. |
| Sandpaper (multiple grits) | Sanding down parts to finish surfaces. Allows for slip fit of shafts in bearings and press fit of gears on splines. |
| Alan key | Used to connect the timing chain sprocket to the D-Spline by a set screw. |
| **Materials** | **Use** |
| ABS Thermoplastic | The material used in the 3D printer to make all components. |
| Soluble Printing Material | The material used in the 3D printer for support material. |

To assemble the components, the following procedure outlined below should be completed.

1. Use the sandpaper to finish all surfaces that will fit together. The wall surfaces that connect to each other and the bearings as well as gear and shaft surfaces should be finished for a press fit. The gears and shaft splines that are part of the gear shift mechanism (Gear 5h and Gear 5s) should be finished for a slip fit. Surfaces of the shaft that are inserted into the bearings should be finished for a slip fit.

2. Line up the Housing Side Right part on the arbor press so that bearing holes are beneath the press head. Clamp the Housing Side Right part to the base of the arbor press and align the large bearings. Press bearings into the part as shown in Figure 7.

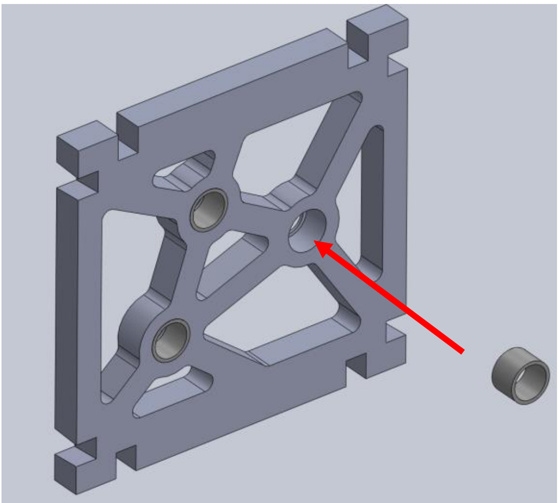

*Figure 7 - Inserting the bearings into the walls.*

3. Repeat step 2 with Housing Side Left, two large bearings, and one small bearing. The location of the small bearing is circled in red in Figure 8.

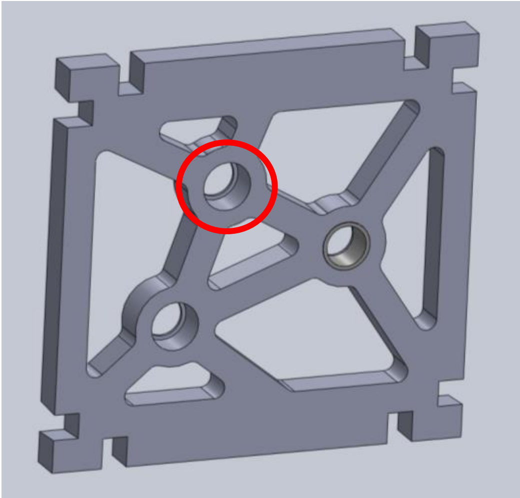

*Figure 8- Location of small bearing.*

4. Snap panels onto Housing Side Right using dovetail cutouts as shown in Figure 9. Note that these will need to be removed in step 8 to assemble the Housing Left Side, but they are temporarily put together in this step for ease of assembly.

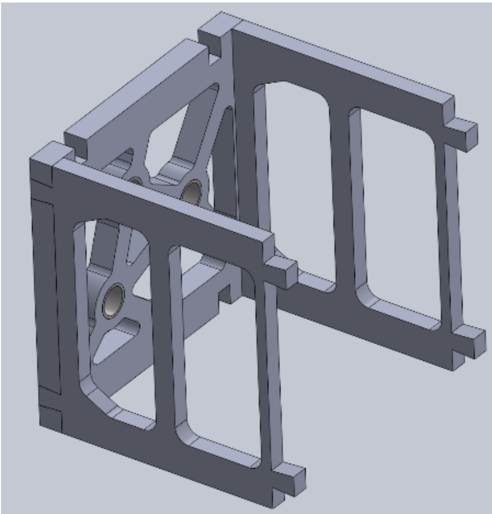

*Figure 9 - snapping together Panels.*

5. Put Shaft A, B, and C into the bearings on the Housing Right side as shown in Figure 10. Shaft A should be placed on the far left of the housing near the input, Shaft B in the center slot, and Shaft C at the far-right side near the output.

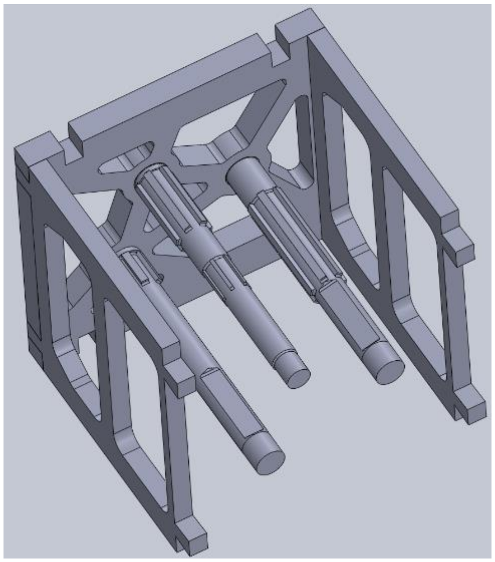

*Figure 10 - Inserting the shafts into the bearings.*

6. Slide the gears onto the correct shafts along splines as shown in Figure 11. Gears are labelled in the image. Ensure that teeth are aligned so that gears mesh. The configuration shows set up for the Hill Climb event. For set up of the Speed event, shift both Gear 5s away from Housing Side Right until they align with Pinion 4s.

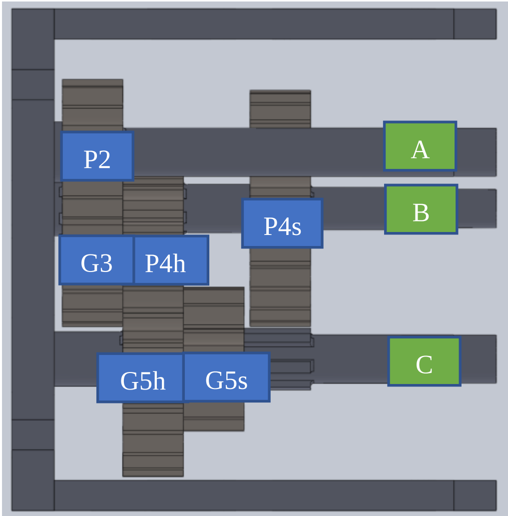


*Figure 11- Gear placement on shafts shown using the bottom view.*

7. Referring to Figure 12, put timing chain sprockets on the D-spline highlighted in green of Shaft A for the input timing chain sprocket and Shaft C for the output timing chain sprocket. Use the Alan key to fasten the set screw into the timing chain sprocket. Add the chain to sprockets and feed chain through openings highlighted in blue.

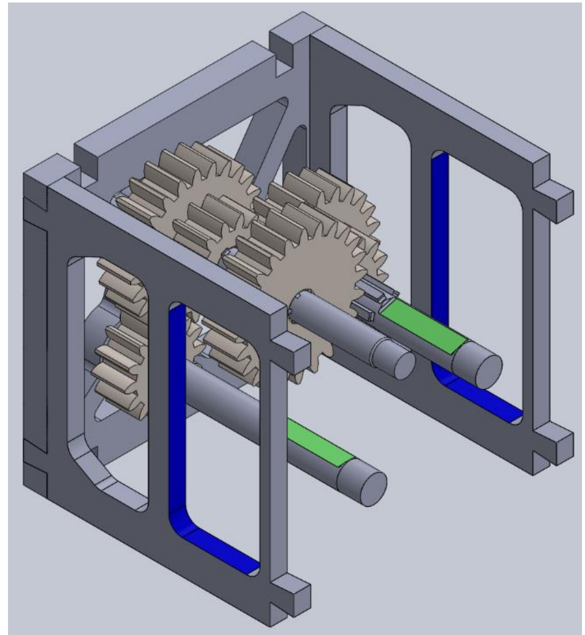

*Figure 12 - Placement of timing chain sprockets and chains.*

8. Remove the 2 panels that were placed in step 4. Align Housing Side Right with the shafts so that they insert into the bearings. Finally, put the panels back into place by snapping the panels into the dovetail grooves as shown in Figure 13.

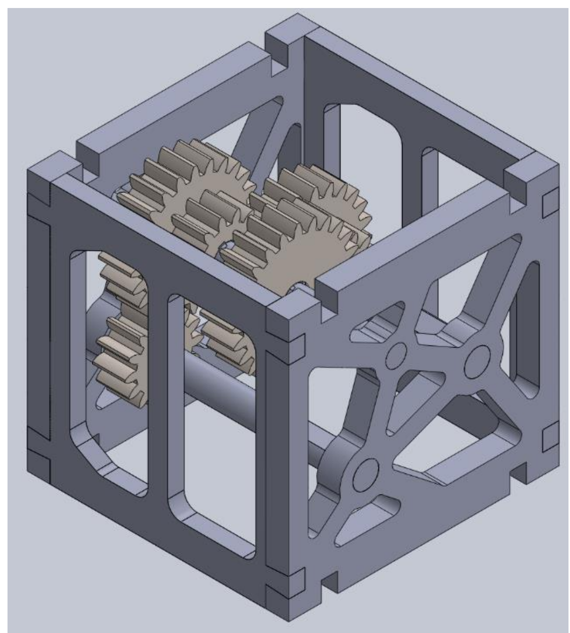

*Figure 13 - Snapping housing together after aligning shafts with bearings.*

9. Place gearbox into vehicle housing by snapping vehicle parts into grooves circled in red in Figure 14. Ensure that Shaft A is connected to the input of the vehicle and Shaft C connects to the output of the vehicle.

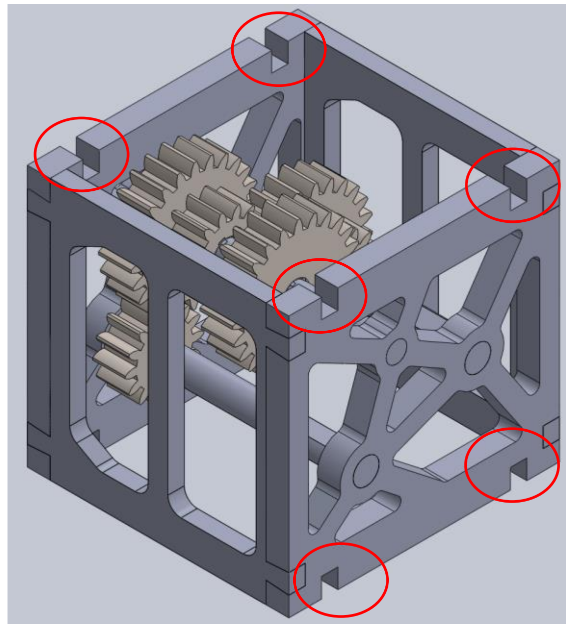

*Figure 14 - Placement of snapping onto vehicle housing.*

## 4.2 Technical Specification Summary

The technical specifications of the project including performance specifications, safety factors and overall dimensions are shown in Table 3. Specifications are based on the use of AISI 4130 steel for the gears and pinions and the three competitive events as described in the introduction.

*Table 3 - Summary of technical Specifications of the gearbox performance.*

| Gearbox Technical Specification | Value | Unit |
|---|---|---|
| Gearbox Type | Two-Stage Shifter | - |
| Gearbox Design Lifetime | 43,680 | Hours |
| Motor Operating Torque – Climb | 1.4 | Nm |
| Motor Operating Torque – Speed | 0.625 | Nm |
| Pinion Lowest Safety Factor | 1.17 | - |
| Gear Lowest Safety Factor | 1.14 | - |
| Gear Ratio – Climb | 3:1 | - |
| Gear Ratio – Speed | 1:1 | |
| Print Time | 6:21 | Hours |
| Gearbox Footprint (L x W x H) | 82.76 x 80 x 80 | mm |

Table 4 of the Drawing Package highlights each component of the gearbox along with their quantity and location within the housing as a parts list. Also shown is an exploded view of the assembly in Figure 17.

## 5.0 Future Recommendations

The most important improvement of the final design would be to properly implement the desired speed event gear ratio of 0.55:1. To have implemented this theoretically optimal gear ratio would have meant an increase in the 2 m time of 0.445 s or an increase in the end zone speed of 0.28 m/s. Such a bonus could have drastically improved the probability of success in both the speed event and the super endurance race. However, implementing the desired gear ratio would mean researching gear interference models for ratios less than one. Additionally, it would have increased the number of cycles on the system, requiring reanalysis of fatigue failure.

Other considerations if the gearbox were to be manufactured would be alternative modes of torque transfer. In a competitive environment, a CVT (Continuously Variable Transmission) would be worth exploring. CVT's can deliver efficient torque at a wide range of vehicle speeds, making the design more optimized for both events. [4]

There are additional improvements in the design process that would be beneficial for perfecting the design. There is a lack of testing and computer validation of the design. Many key parameters of the system were assumed, such as the resistance of the vehicle and the friction at the wheels. Experimenting with prototypes prior to final manufacturing would help find accurate parameters for gear ratio optimization. Testing would also validate the system under loading. Finite element analysis could be used in this regard as well.

## 6.0 Conclusion

In summary, the housing design was created to ensure all requirements and constraints were met. This was concluded and confirmed through completing the required Phase 4 checklist shown in Appendix C. The checklist confirms that the conducted iterations completed and highlighted in the evolution of the gearbox design are accurate and ready to be 3D printed with a confirmed printing time of 6 hours and 21 minutes.

It can be concluded that the defined gearbox creates an extreme competitive advantage over others with the lightweight and short-length shifting gearbox system. Since the defined specifications ensure minimal drag and print time, and they allow for the torque to be changed to succeed in both the top speed and hill climb event.

# References

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[4] J. Gorzelany, "Pros And Cons Of A CVT (Continuously Variable Transmission)," CARFAX, 27 November 2019. [Online]. Available: https://www.carfax.com/blog/cvt-pros-and-cons.

[5] Machine Design - An Intergrated Approach 4th Edition, Norton, Prentice Hall.

[6] "SolidWorks," SolidWorks, [Online]. Available: http://help.solidworks.com/. [Accessed April 2021].

## Drawing Package

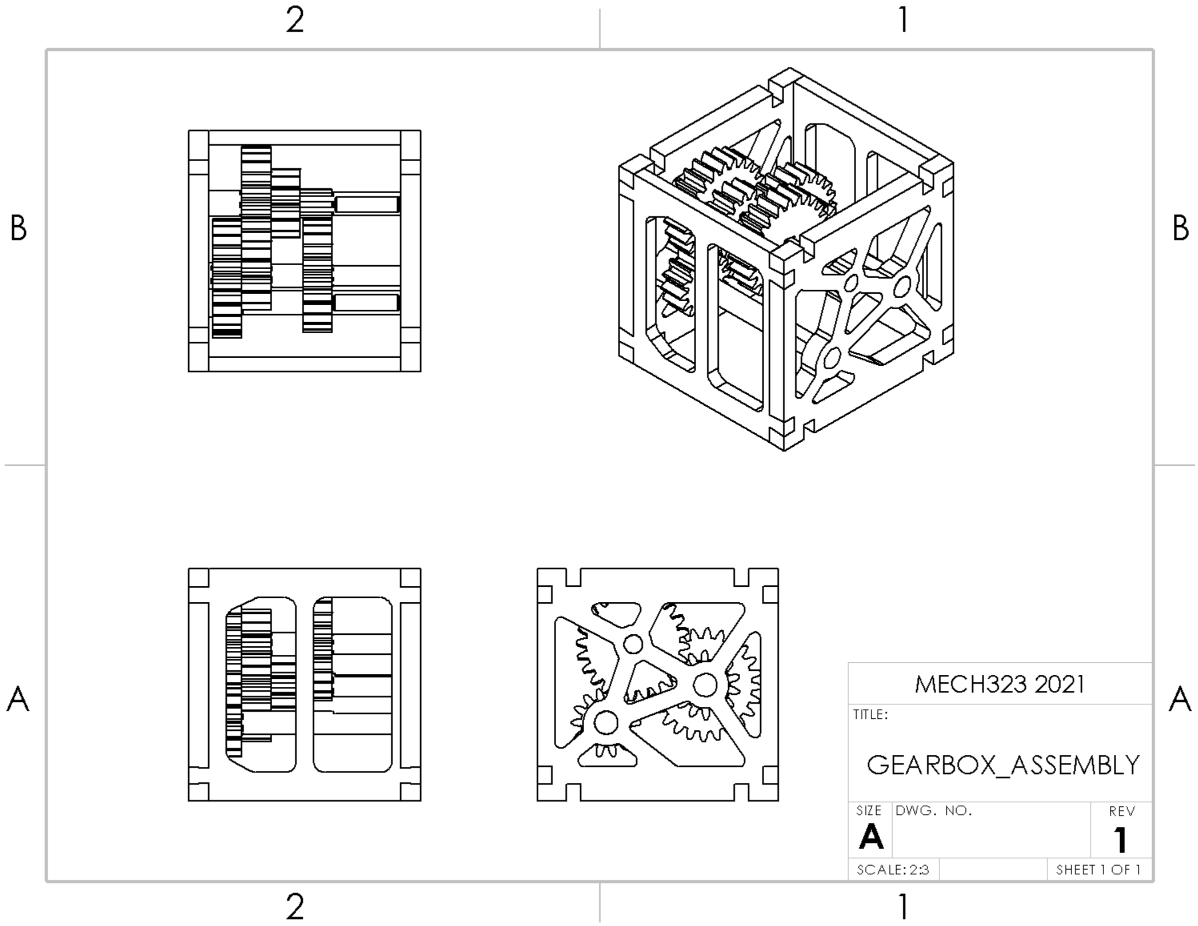



*Figure 15 - Full Assembly drawing showing all included components.*

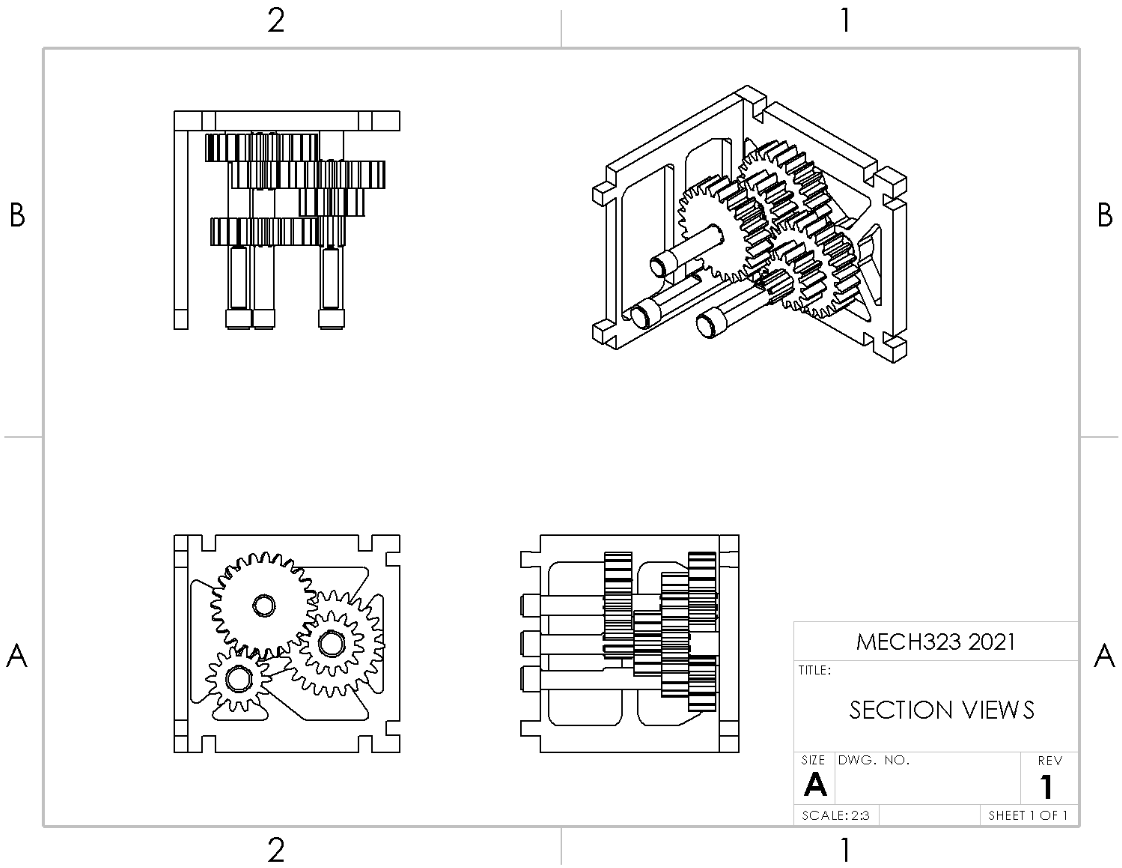



*Figure 16 - Section view of the assembly. Left side Housing and a Panel were removed to display all inner components and locations.*

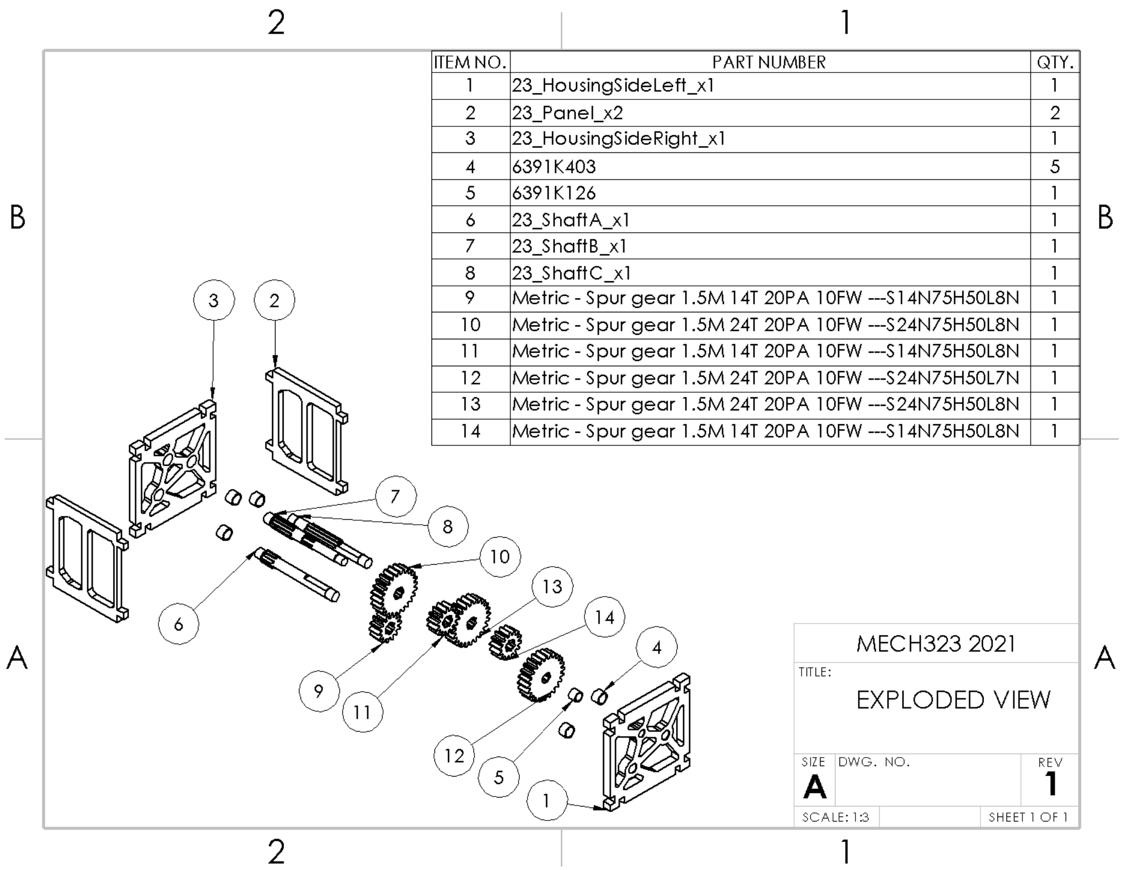
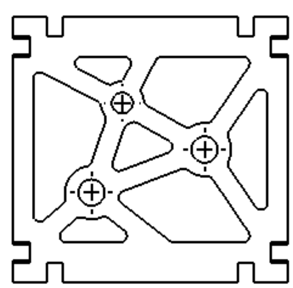
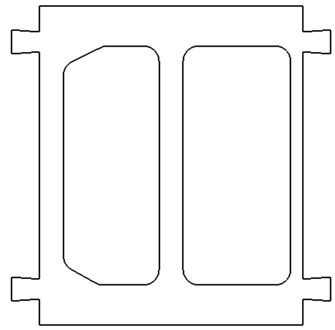
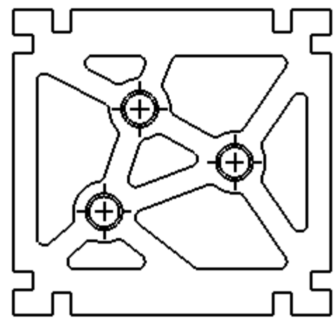
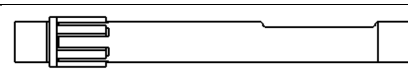
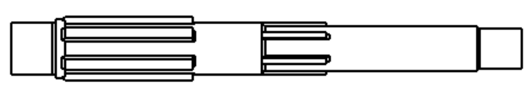
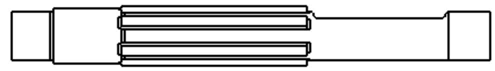
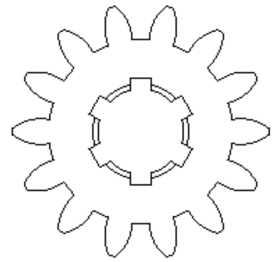
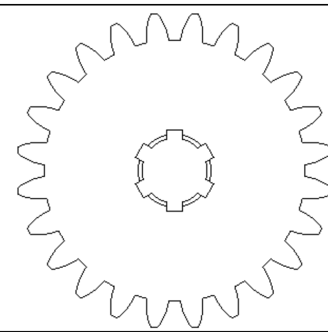
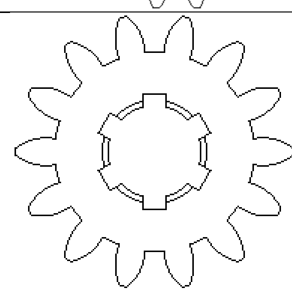
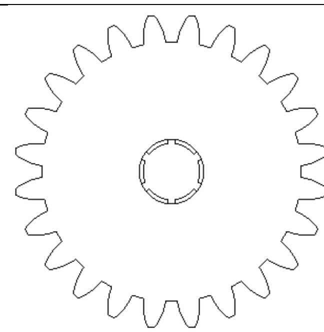


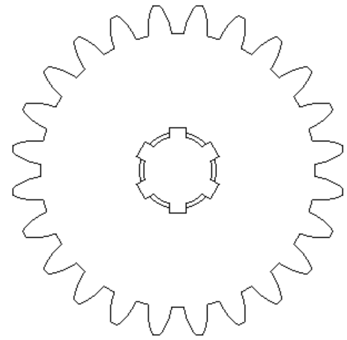
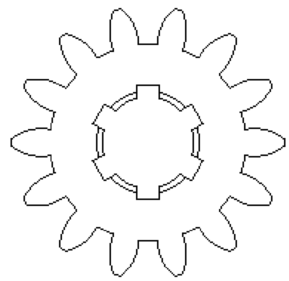

*Figure 17 - Full exploded view showing all components and assembly instructions. Included is a Bill of materials where the item numbers relate to the part numbers of the parts list.*

*Table 4 - Parts list of all components used in the gearbox assembly.*

| Part # | Name | Description | Quantity | Drawing |
|---|---|---|---|---|
| 1 | Housing Side Left | Wall for the housing which is closest to the timing chain sprockets. | 1 |  |

| | | | | |
|---|---|---|---|---|
| 2 | Panel | Panels that enclose the front and rear of the vehicle. | 2 |  |
| 3 | Housing Side Right | Wall for the housing which is farthest from the timing chain sprockets. | 1 |  |
| 4 | 6391K403 | Bearing of dimensions 0.0079375 m ID, 0.009525 m OD, 0.00635 m long. | 5 | |
| 5 | 6391K126 | Bearing of dimensions 0.00635 m ID, 0.0079375 m OD, 0.00635 m long. | 1 | |
| 6 | Phase 4 Shaft A | The input shaft of the gearbox, which connects to the input timing chain sprocket and holds Pinion 2. | 1 |  |
| 7 | Phase 4 Shaft B | Center shaft of the gearbox, which holds Gear 3, Pinion 4 Hill climb, and Pinion 4 Speed. | 1 |  |

| | | | | |
|---|---|---|---|---|
| 8 | Phase 4 Shaft C | The output shaft of the gearbox, which connects to the output timing chain sprocket and holds Gear 5 Hill Climb and Gear 5 Speed. | 1 |  |
| 9 | Pinion 2 | 14 teeth, pressure angle of 20°, face width of 0.01 m, pitch diameter of 0.021 m | 1 |  |
| 10 | Gear 3 | 24 teeth, pressure angle of 20°, face width of 0.01 m, pitch diameter of 0.036 m | 1 |  |
| 11 | Pinion 4 Hill Climb | 14 teeth, pressure angle of 20°, face width of 0.01 m, pitch diameter of 0.021 m | 1 |  |
| 12 | Pinion 4 Speed | 24 teeth, pressure angle of 20°, face width of 0.01 m, pitch diameter of 0.036 m | 1 |  |

| | | | | |
|---|---|---|---|---|
| 13 | Gear 5 Hill Climb | 24 teeth, pressure angle of 20°, face width of 0.01 m, pitch diameter of 0.036 m | 1 |  |
| 14 | Gear 5 Speed | 14 teeth, pressure angle of 20°, face width of 0.01 m, pitch diameter of 0.021 m | 1 |  |

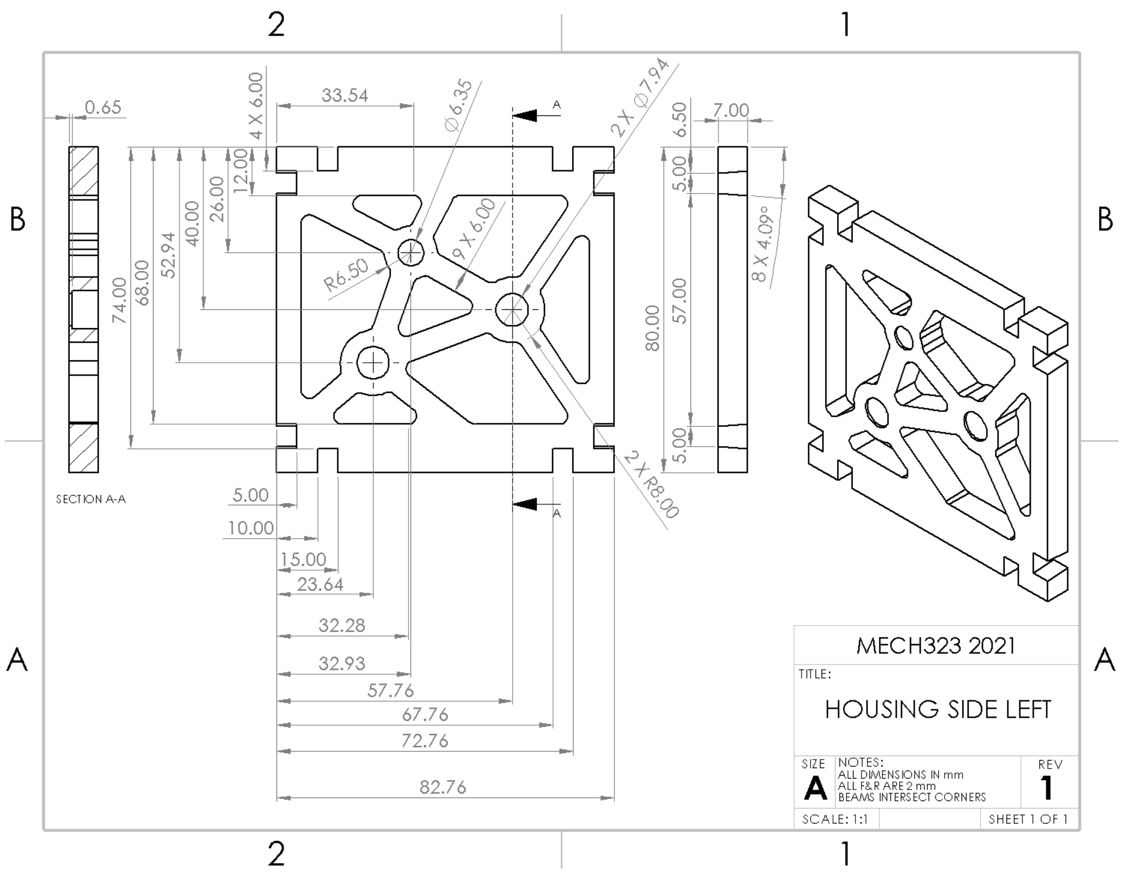



*Figure 18 - CAD drawing for the final design of the left side of the housing.*

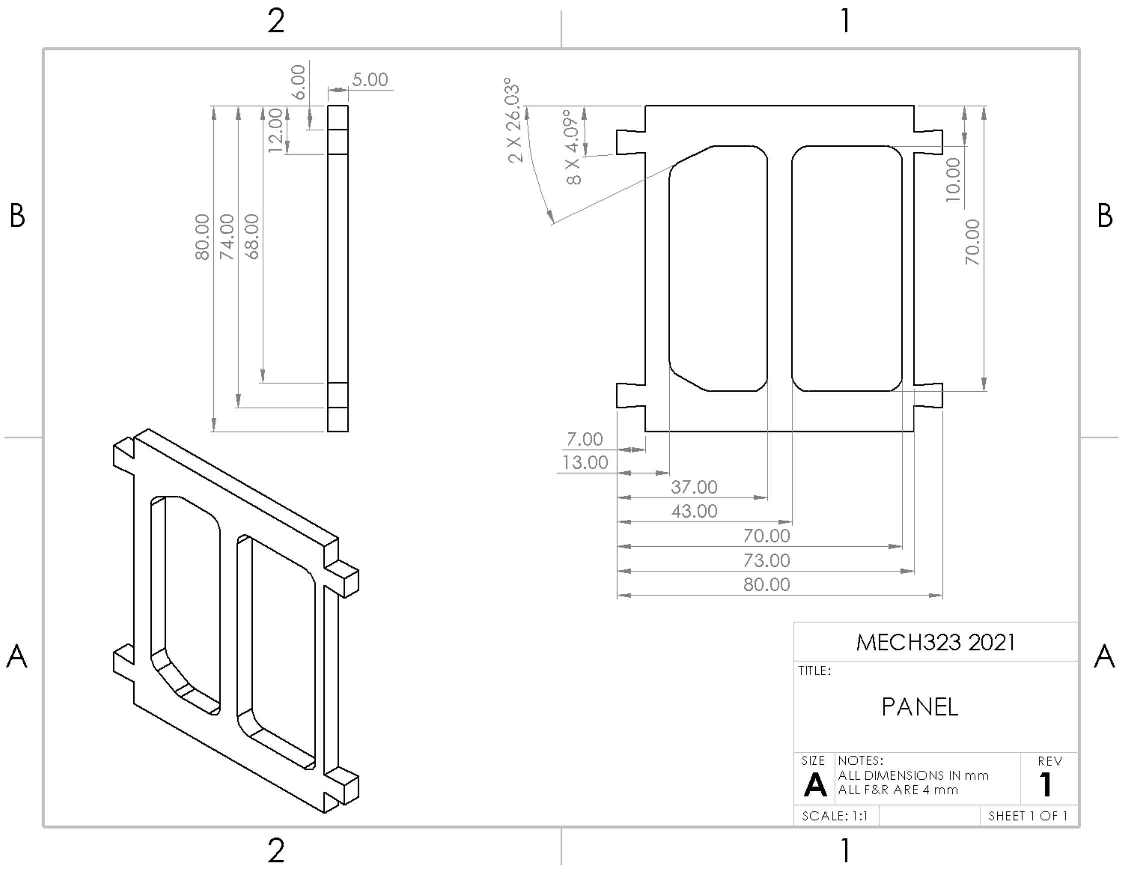



*Figure 19 - CAD drawing for the final design of the housing panels.*

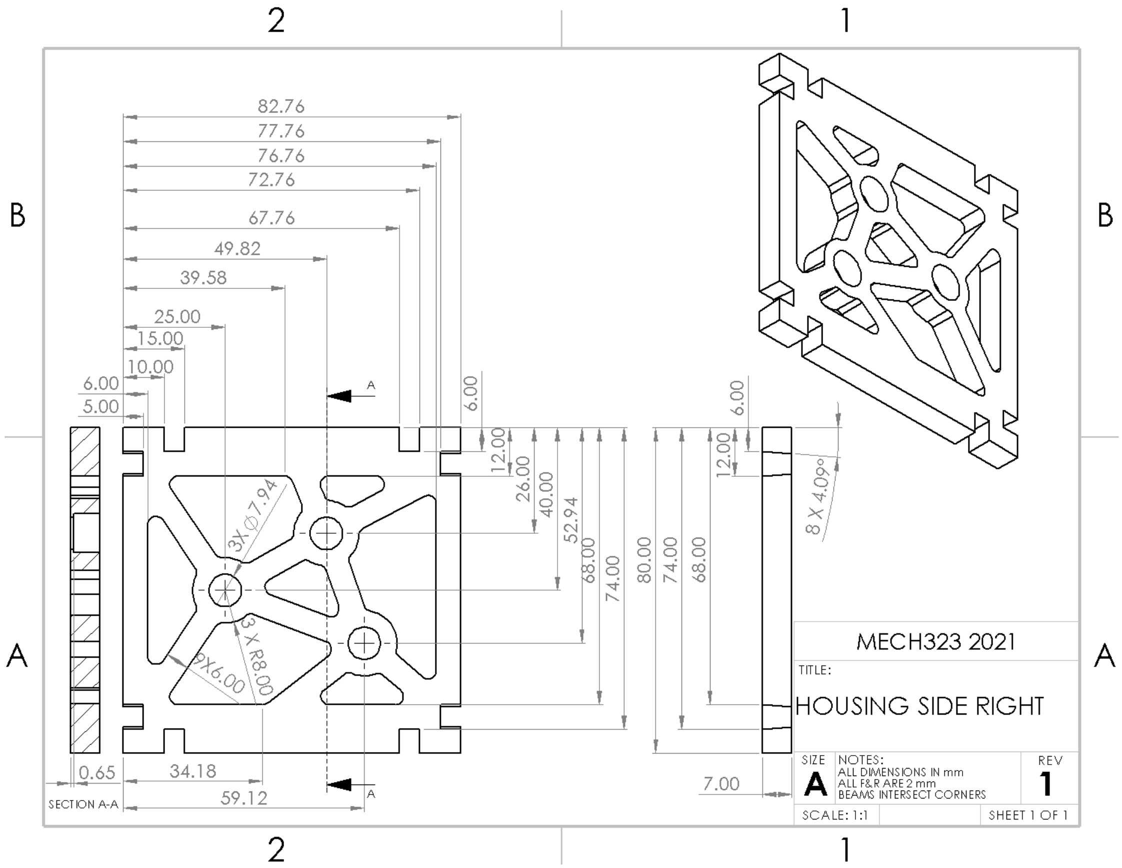



*Figure 20 - CAD drawing for the final design of the right side of the housing.*

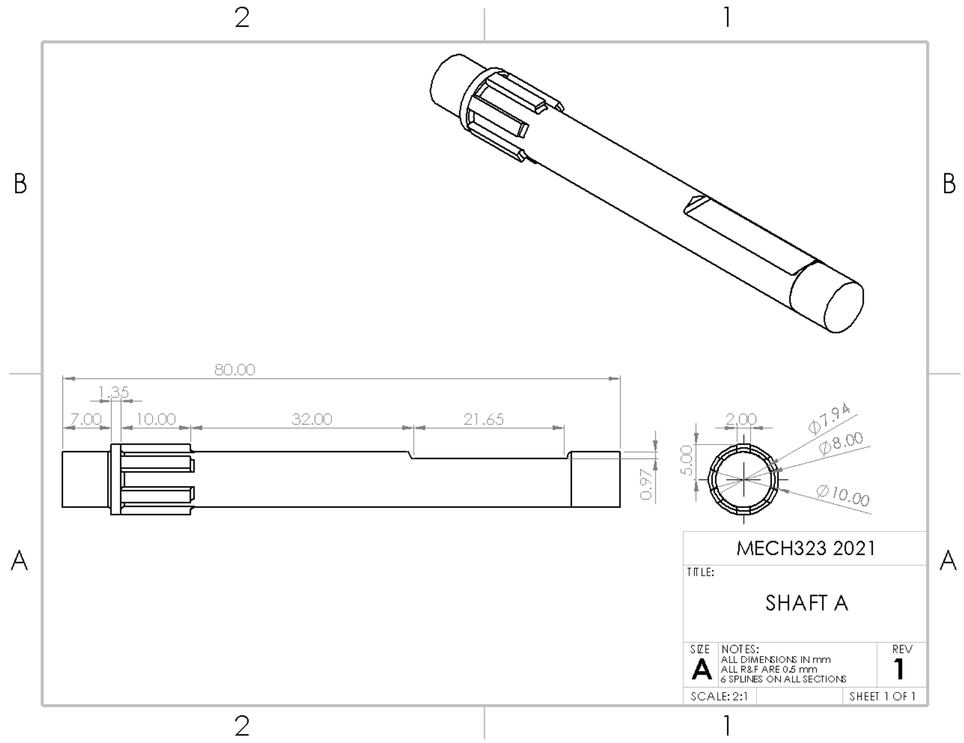



*Figure 21 - CAD drawing for the final design of Shaft A.*

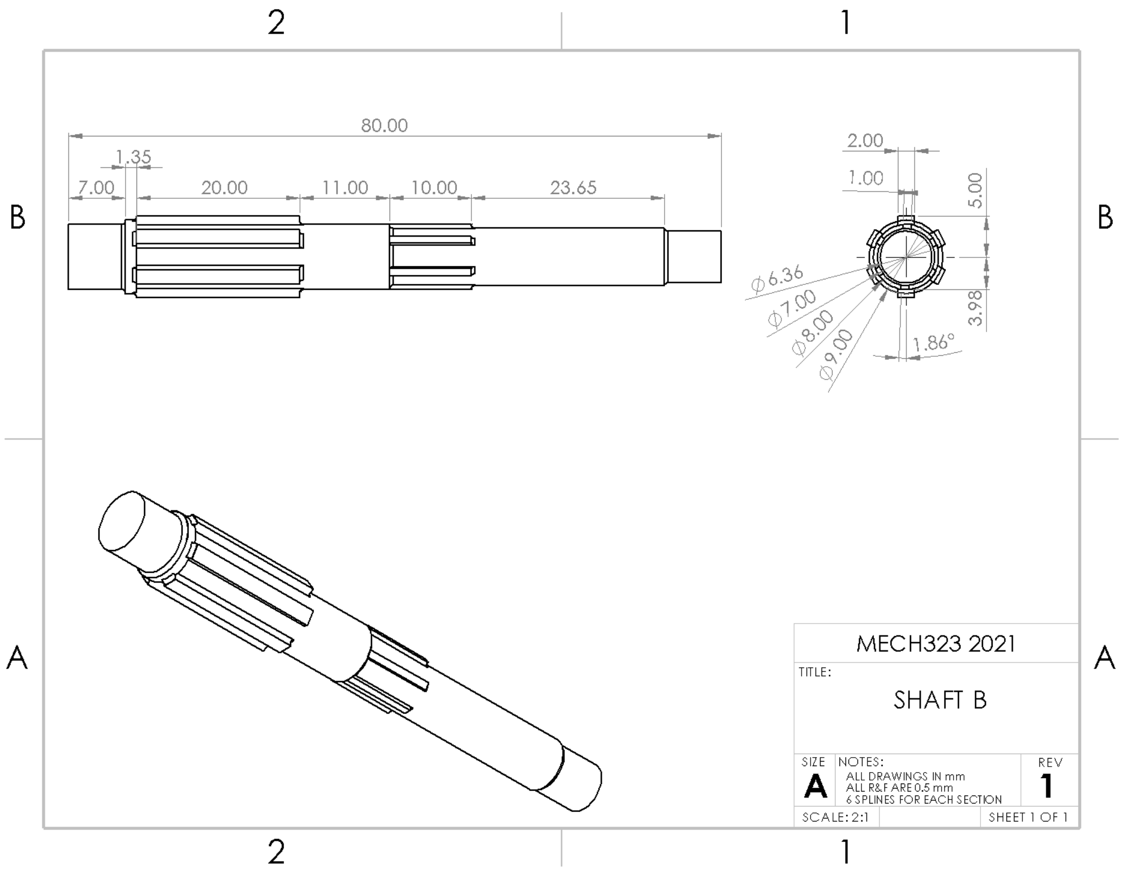



*Figure 22 - CAD drawing for the final design of Shaft B.*

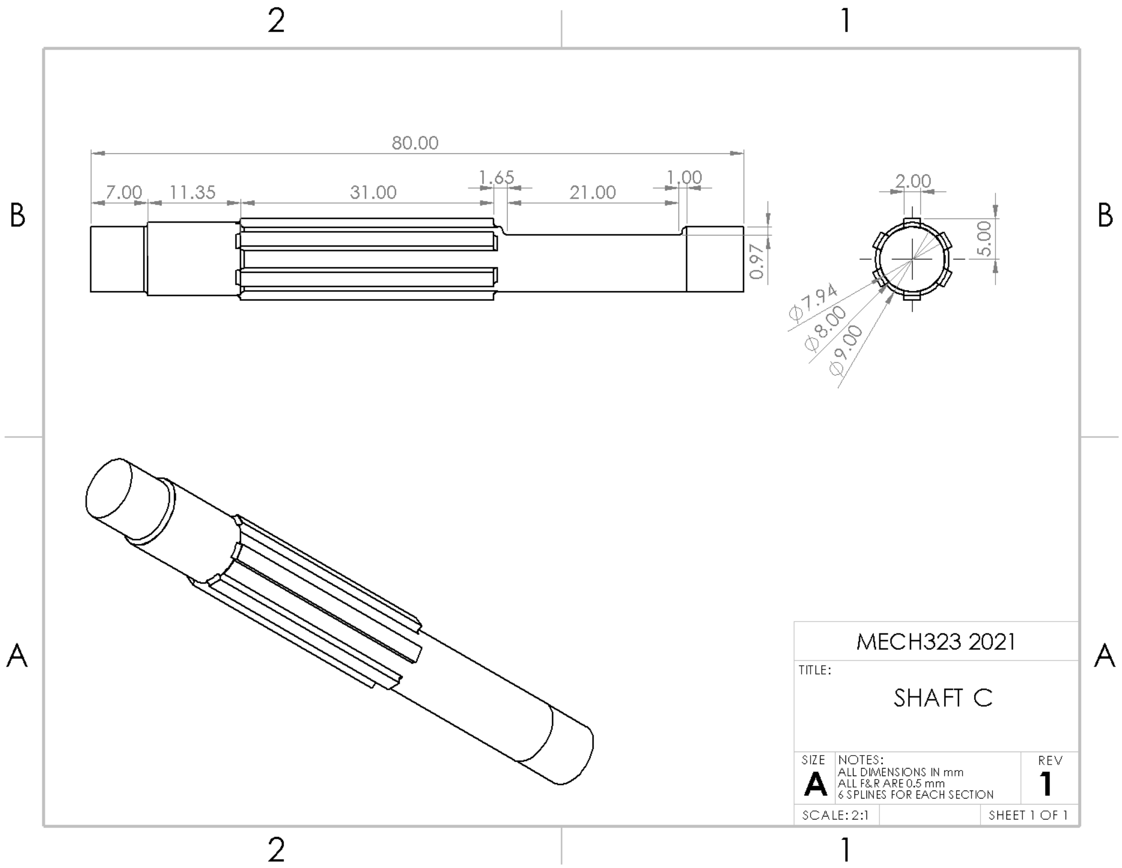



*Figure 23 - CAD drawing for the final design of Shaft C.*

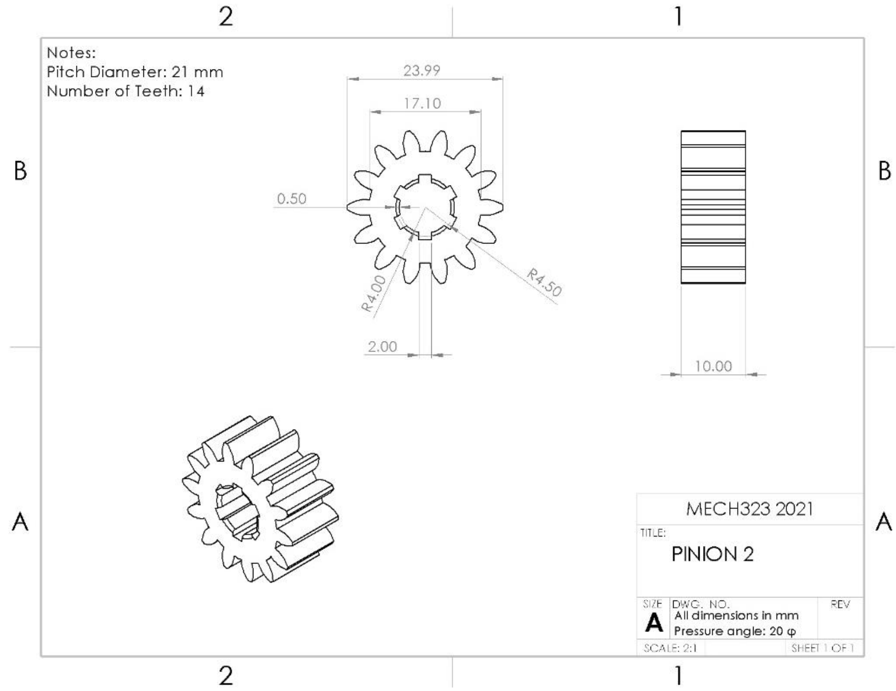



*Figure 24 – CAD drawing for the final design of Pinion 2.*

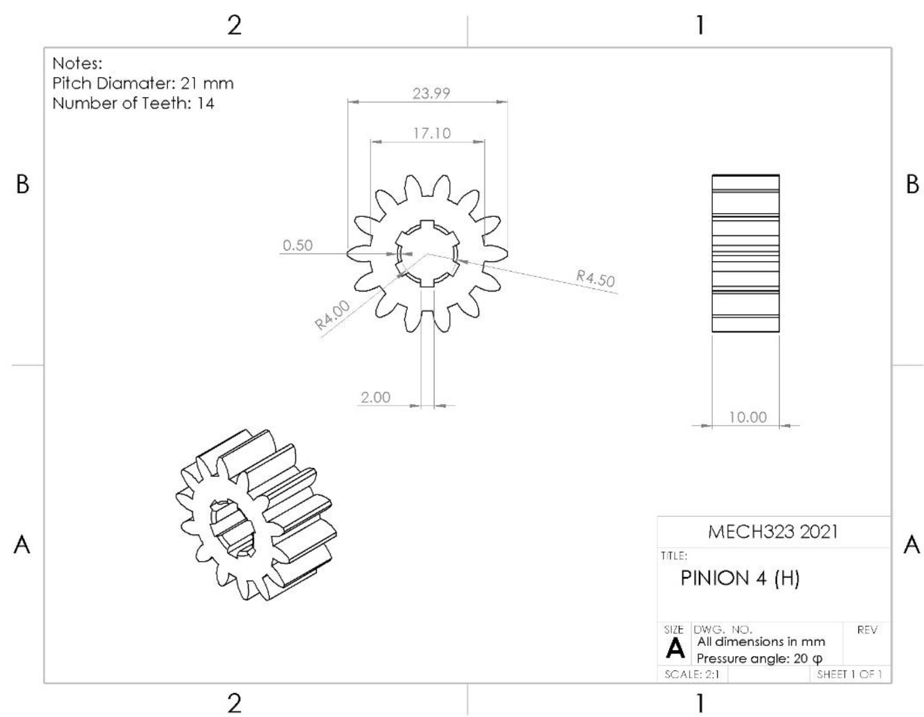



*Figure 25 – CAD drawing for the final design of Pinion 4 (h).*

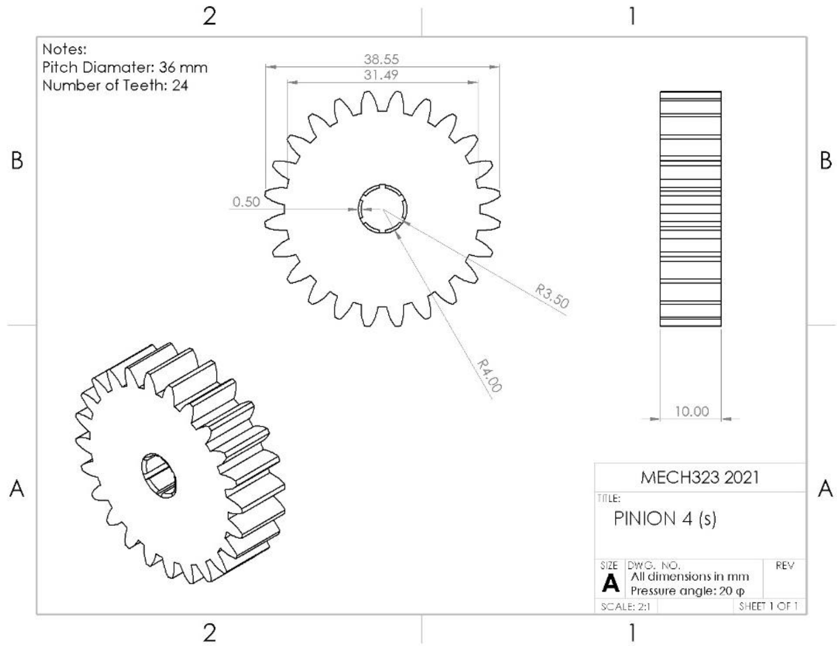



*Figure 26 – CAD drawing for the final design of Pinion 4 (s).*

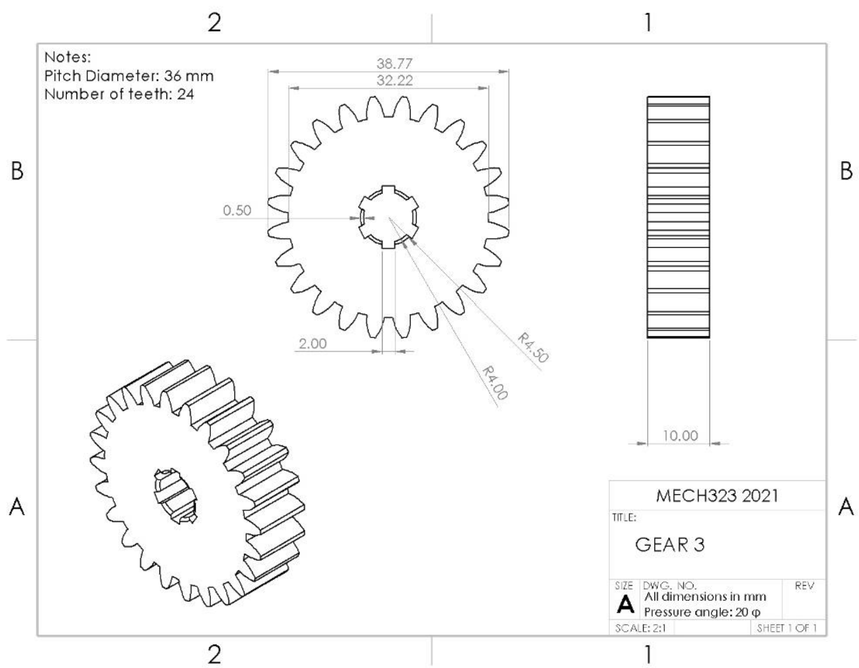



*Figure 27 – CAD drawing for the final design of Gear 3.*

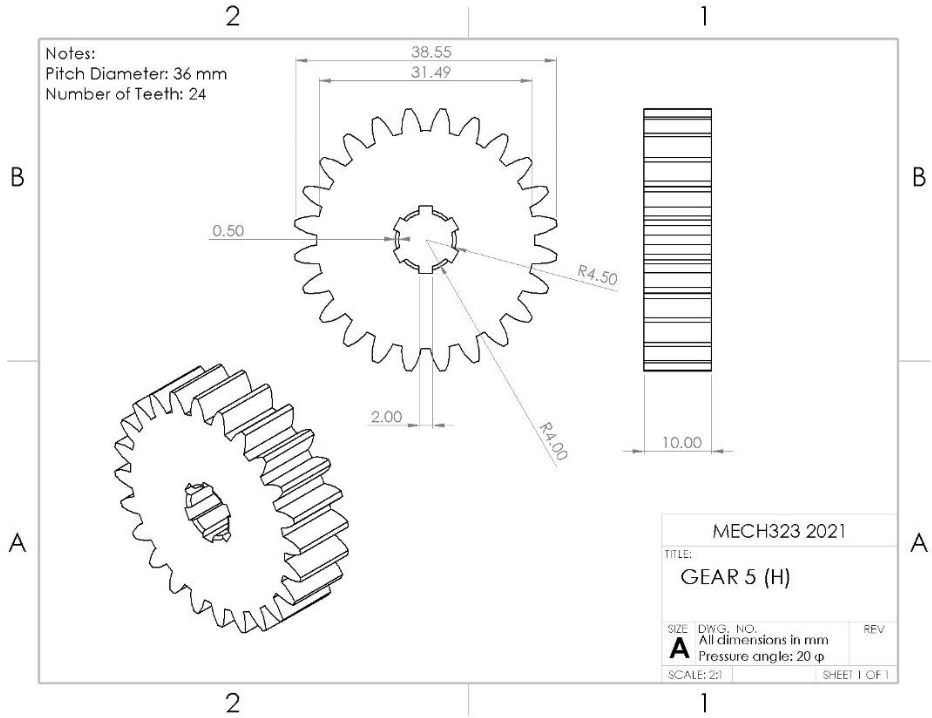



*Figure 28 - CAD drawing for the final design of Gear 5 (h).*

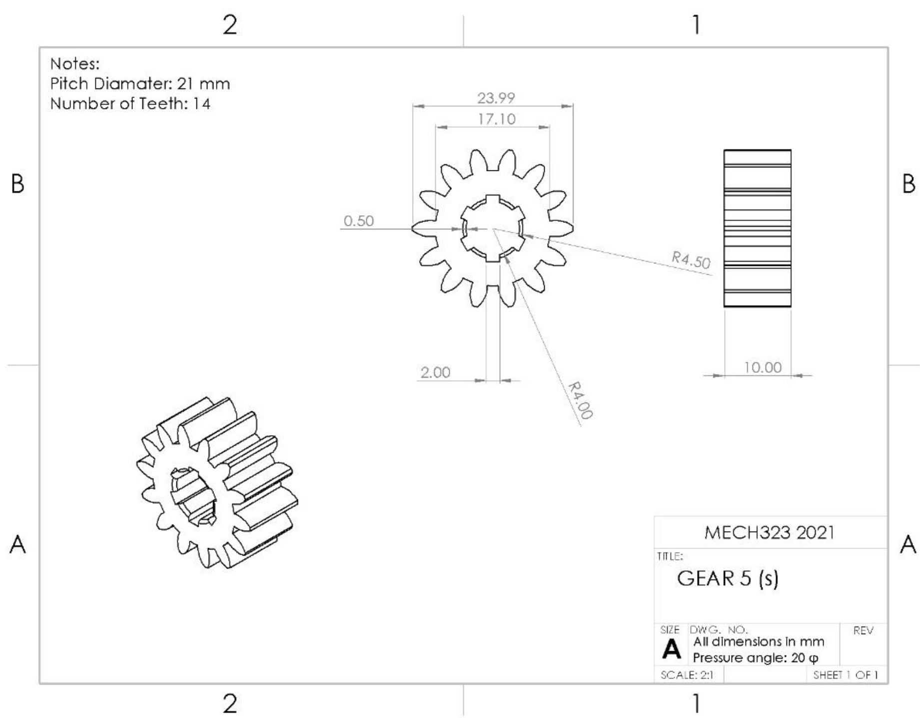



*Figure 29 – CAD drawing for the final design of Gear 5 (s).*

# Appendix A: Supporting Figures

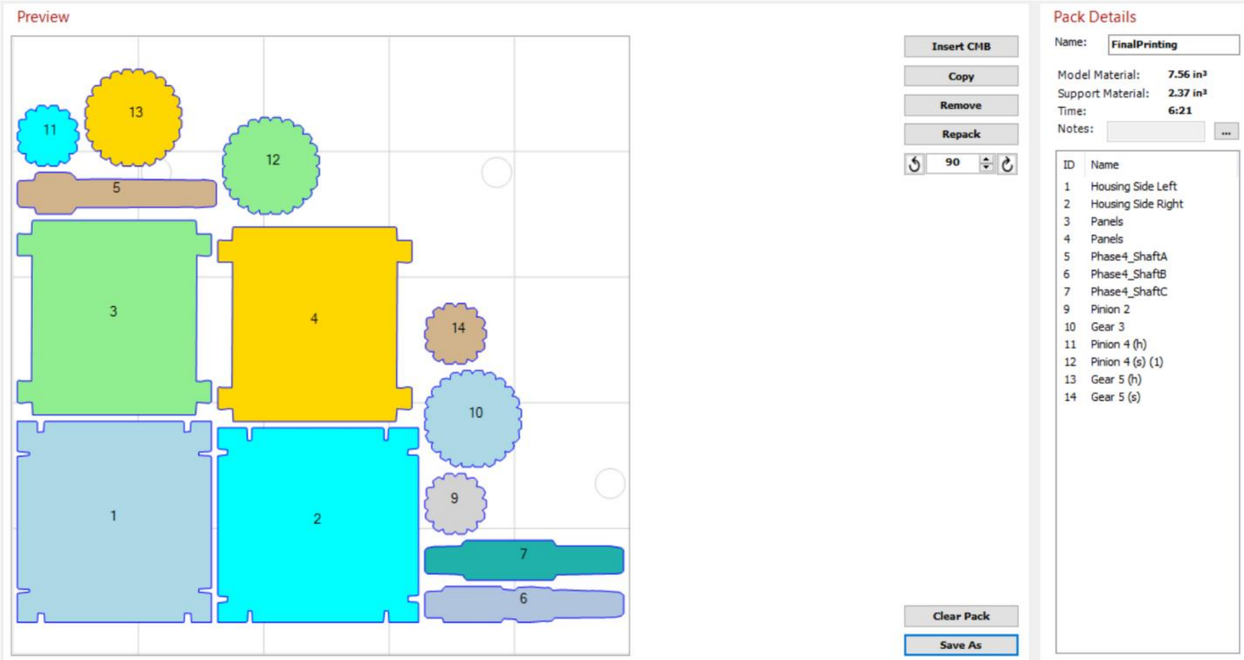



*Figure 30 – Print time for the defined gears, shafts, and housing. Time is 6 hours and 21 minutes.*

*Table 5 - Printing Parameters for Catalyst.*

| Part | Orientation | Model Interior | Support fill |
|---|---|---|---|
| Shafts | Horizontal axis | Solid | SMART |
| Gears | Vertical axis | Sparse: High Density | SMART |
| Housings and other parts | Auto Orient | Sparse: High Density | SMART |

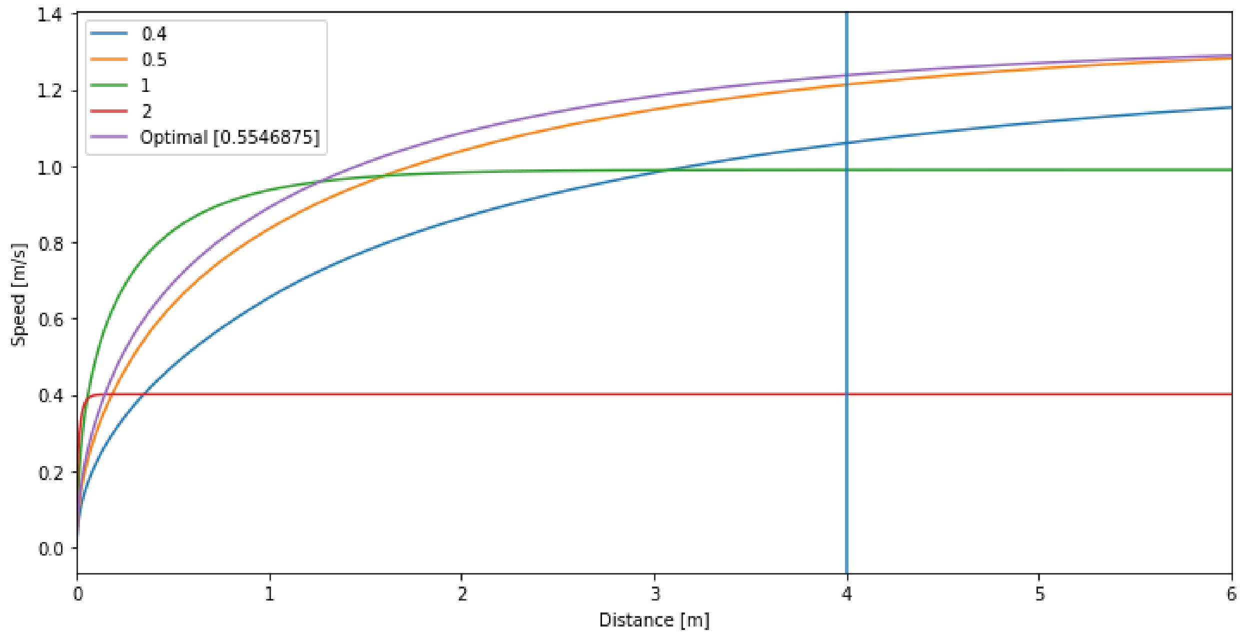



*Figure 31 - A plot of the expected velocity versus distance of a selection of gear ratios. The average velocity in the zone past the 4 m mark is the optimization objective function.*

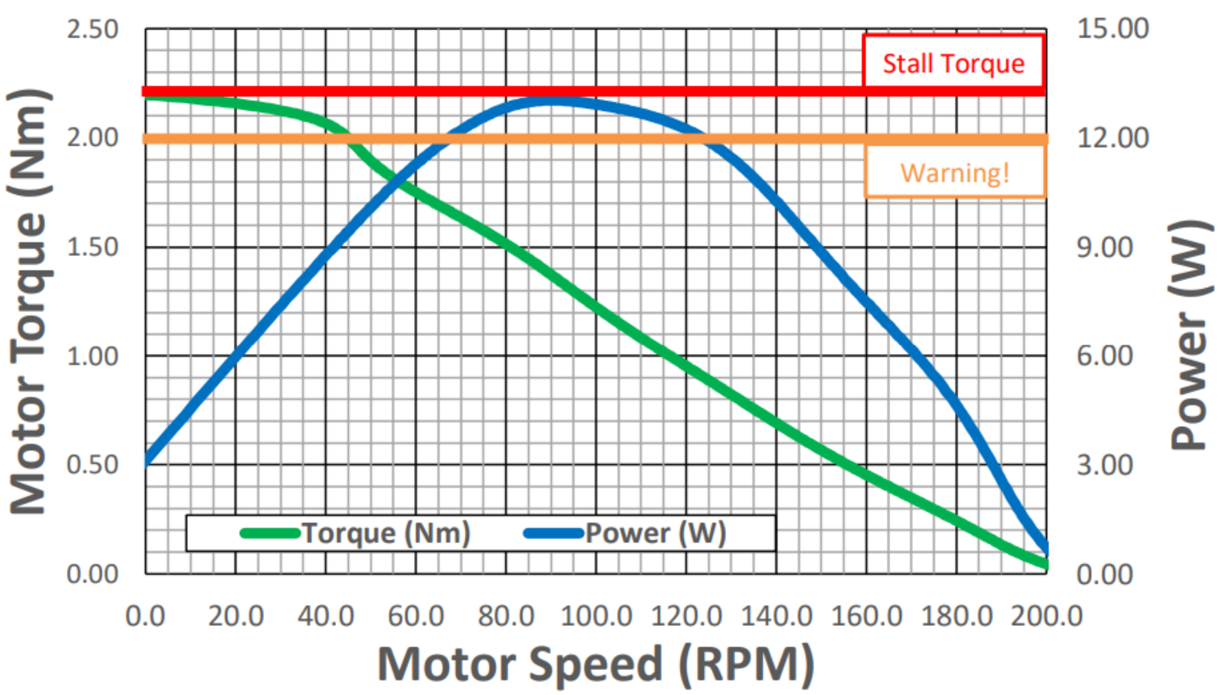



*Figure 32 - Motor Torque Curve for gearbox design.*

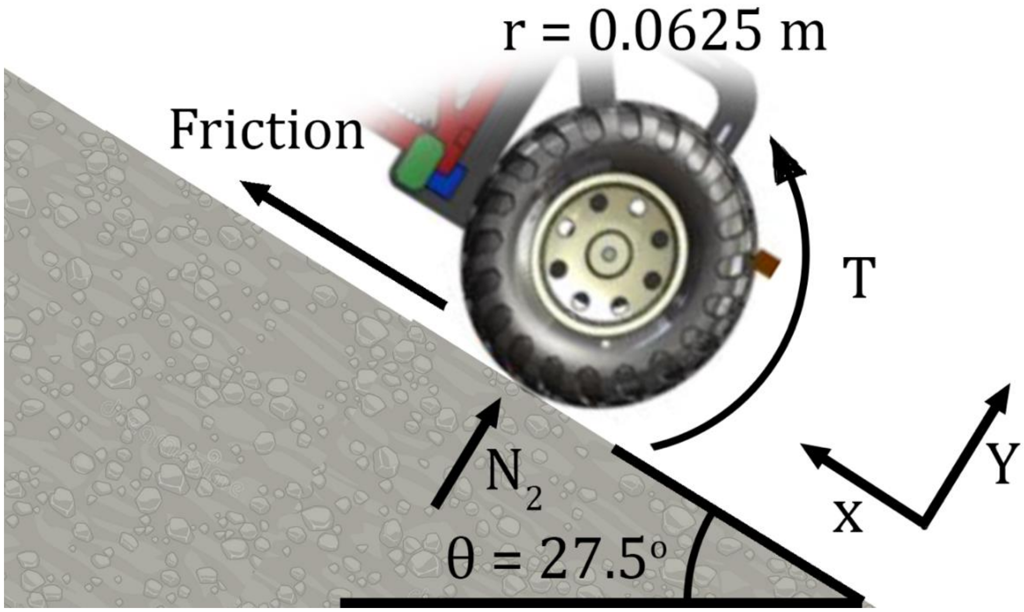



*Figure 33 - The moments acting on the rear wheel of the car.*

## Appendix B: McMaster-Carr Order Form

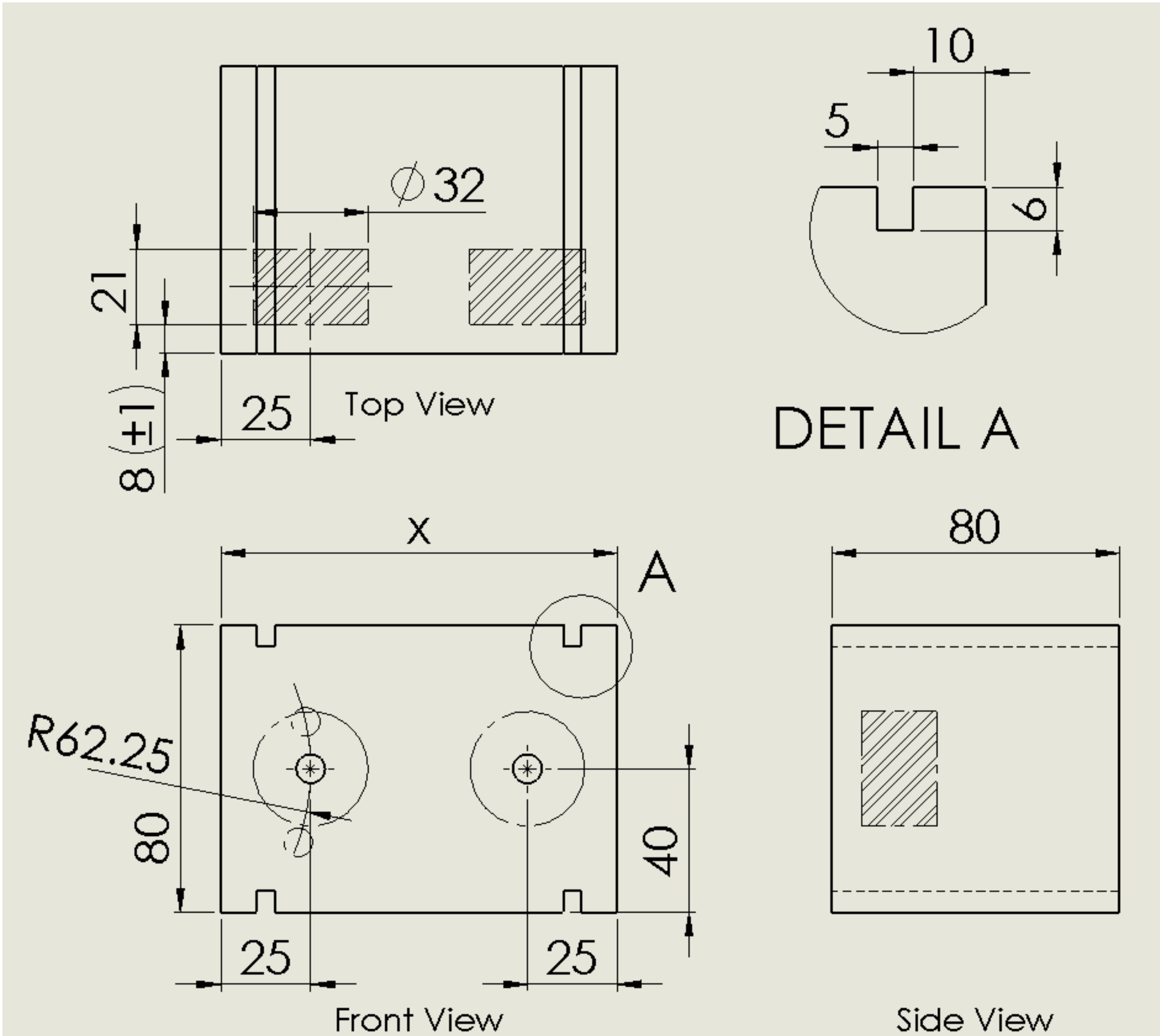
# McMaster-Carr Order Form

| Group Number | 23 |
|---|---|

| McMaster-Carr Part Number | ID (shaft diameter) | OD (outer diameter) | Length | Price/Unit | Quantity |
|---|---|---|---|---|---|
| 6391K403 | 5/16” | 3/8” | 1/4” | $0.70 | 5 |
| 6391K126 | 1/4" | 5/16” | 1/4" | $0.51 | 1 |
| | | | | | |
| | | | | | |
| | | | **Total Price:** | **$4.01** | |

## Appendix C: Phase 4 Checklist

| Dimensional Rules | Check |
|---|---|
| Distance from the base of the gearbox to the center of both the input and the output shafts should be exactly 40 mm<br>(Unless input shaft ONLY is along the arc specified in phase 1 instructions) | YES |
| Input and output shafts are 25 mm from the respective ends of the gearbox<br>(Unless input shaft ONLY is along the arc specified in phase 1 instructions) | YES |
| Gearbox outside width is exactly 80 mm | YES |
| Gearbox height is exactly 80 mm | YES |
| Four slots are present in the gearbox at each corner, each being 10 mm away from the respective ends of the gearbox, 6 mm deep, and 5 mm wide.<br>The slots are not obstructed by gears, sprockets or other objects in any way. | YES |
| Dimension X: The gearbox is over 50 mm in length, and under 130 mm in length. | YES |
| Gears do not protrude beyond the housing further than 10 mm and do not interfere with the slots in any way. | YES |
| Space has been left in the housing and on the input and output shafts for the sprockets, which are 21 mm in length, 32 mm in diameter, and must be located 8 ± 1 mm from the side of the gearbox<br>(sprockets must NOT interfere with one another or other components within the gearbox when assembled) | YES |
| Shafts mating to the sprockets have a D-profile with a 1 mm deep flat, and the diameter has been selected from the specified sizes:<br>8, 9, 10, 12 mm | YES |




| Shafts and Bushings | Check |
|---|---|
| Shafts have been sized to fit inside bushings | YES |
| Bushing part numbers have been provided in the McMaster-Carr Order From (below) and the appropriate bushings present in the CAD model | YES |
| Shafts provide adequate room for, and a method of transmitting torque to, gears | YES |
| Shaft design allows for assembly | YES |
| Input and output shafts have included the specified areas for the sprockets and have the required d-shaft profile | YES |
| Sprocket shaft diameters used (indicate actual sprocket shaft diameters) | YES / 8 mm |

| Housing | Check |
| --- | --- |
| Housing looks robust enough to not break. CAD model is well made and includes no broken features | YES |
| Housing provides some method of alignment between the two sides and constrains the shafts adequately | YES |
| No protrusions from the housing are smaller than 2-3mm (too small features will not be printed properly.) | YES |
| Housing is able to be assembled and maintain its form such that it can function without needing to be in the car and without falling apart. All parts of the housing are securely attached to each other. | YES |
| Housing can be easily assembled/disassembled multiple times | YES |
| Housing includes bearing (i.e. bushings in Phase 4) bores, and shoulders to keep bearings (i.e. bushings in Phase 4) from coming out | YES |
| Housing will fit into the car | YES |

| Printing Time | Check |
| --- | --- |
| Catalyst print time is under 9 hours | YES |
| Catalyst print time is clearly and explicitly indicated in “Group X Phase 4 Report.PDF” document. | YES |

# Appendix D: Supporting Calculations

## Phase 1: Distance covered and Average Velocity

Equation 1 and Equation 2 were used to calculate the optimal gear ratio used for the hill climb event.

$$\sum M = 0 = F_f * r - T \tag{1}$$

$$T = F_f = 0.4 * r\mu mg\cos(\theta)$$

$$T = 4.24\ Nm$$

$$m_g = \frac{T_{out}}{T_{in}} = \frac{4.24\ Nm}{1.4\ Nm} = 3.029 \tag{2}$$

Equation 3 and Equation 4 were used to calculate the distance the car travels along the track where the radius is equal to 0.0625 m and theta ($\theta$) was determined to be 27.5 degrees due to the highlighted frictional limitation found in Phase 1.

$$C = 2\pi r \tag{3}$$

$$Distance = C * \frac{\theta}{360°} \tag{4}$$

## Phase 3: Shaft Safety Factors Calculation

The following calculation was used to calculate the safety factors of the critical points along the shafts. For each factor, $S_{ut}$ was 1280 MPa, $S_e$ was the corrected endurance limit, and $d$ was the smaller diameter at the critical point.

$$n = \frac{1}{\frac{8A}{\pi d^3 S_e}\left\{1+\left[1+\left(\frac{2BS_e}{A\,S_{ut}}\right)^2\right]^{\frac{1}{2}}\right\}} \tag{5}$$

$A$ is a function of amplitude moment and torque at the critical points while B is a function of the midrange torque and moment at the critical points. They were calculated using the equations below.

$$A = \sqrt{4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2} \tag{6}$$

$$B = \sqrt{4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2} \quad (7)$$

The safety factors of all critical points were calculated, and all were above the minimum value of 1. The lowest safety factor found was 2.04, located at critical point 5 on Shaft B.

## Phase 4: Gear Width Adjustment Calculation

In the previous phases, it was unknown how long the bearings would be. Due to this, the previous designs did not allocate enough room for the final Phase 4 assembly. This can be seen in the calculations below, and Figure 34.

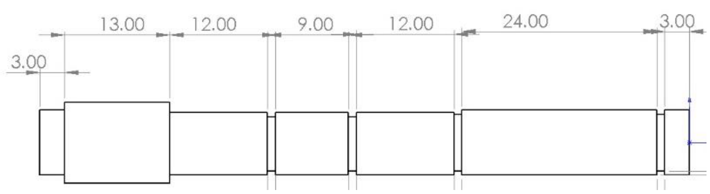



*Figure 34 - An image of the limiting shaft from the previous phase.*

Directly summing the previous design yields that it fits. However, with the new lengths needed for the bearings, this would not work.

$$6.35\ mm + 8\ mm + 21\ mm + 4(11\ mm) + 2\ mm = 81.35\ mm = 0.08135\ \mathrm{m}$$

This is over the limit and the only solution was to reduce the gear width.

$$6.35\ mm + 8\ mm + 21\ mm + 4(11\ mm) + 2\ mm = 77.35\ mm = 0.07735\ m$$

This is acceptable as it is below the limit and it allows some clearance between the parts. The shaft can also accommodate the lofted surface for D-shaft transitioning.

## Phase 4: Reduced Gear Face Width Safety Factor Calculation

When reducing the gear face width from 0.011 m to 0.01 m, the question of how it would affect the safety factors had to be answered. The gears were first widened from 0.008 m to 0.011 m to increase the surface contact fatigue safety factor of all the gears. To ensure this decrease still met the requirement, the surface contact fatigue safety factor was recalculated for all gears using a width of 0.01 m. This calculation was done using the equation seen below, where $\sigma_c$ is the corrected contact stress, and $S_c^{corrected}$ is the corrected surface fatigue strength. It should be noted that the calculation was completed in a Jupyter Notebook.

$$n_c = \frac{S_c^{corrected}}{\sigma_c} \tag{8}$$

The equations for the corrected contact stress and the corrected surface fatigue strength are provided below.

$$\sigma_c = Z_E \sqrt{W^t K_O K_v K_s \frac{K_H}{d_{w1} b} \frac{Z_R}{Z_I}} \tag{9}$$

$$S_c^{corrected} = S_c \frac{Z_N Z_W}{Y_\theta Y_Z} \tag{10}$$

The calculations returned surface contact fatigue safety factor values over 1 for all gears. The lowest safety factor calculated was 1.09 for gear 5. Therefore, the design change was justified as the safety factor requirements were met.

# Phase 4 Summary Page

| | |
|---|---|
| **Team Number** | **23** |

| | |
|---|---|
| **Phase Number** | **4** |

| Global Design Characteristics | | |
|---|---|---|
| **Gear Box Parameters** | **Number of Stages** | 2 |
| | **Speed Gear Ratio** | 1:1 |
| | **Hill Climb Gear Ratio** | 3:1 |
| **Vehicle Weights** | **Number of Weights Added** | 0 |

| Predicted Event Performance | | |
|---|---|---|
| **Speed Event** | **2m Time (s)** | 2.02 |
| | **Top Speed (m/s)** | 0.99 |
| | **Motor Operating Torque (Nm)** | 0.625 |
| | **Motor Operating Speed (rpm)** | 151.28 |
| **Hill Climb Event** | **Distance (°)** | 27.47 |