CIS 419/519: Applied Machine Learning	Fall 2024

Homework 0

Handed Out: August 28 *Due: 7:59 pm September 4*

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1 Declaration

- **Person(s) discussed with:** *N/A*
- **Affiliation to the course: student, TA, prof etc.** *student*
- **Which question(s) in coding / written HW did you discuss?** ***N/A***
- **Briefly explain what was discussed.** *N/A*

2 Multiple Choice & Written Questions

1. (a) *C*
   (b) *A*
2. (a) *D*
   (b) *A*
3. (a) *A*
   (b) *A*
4. (a) *B*
   (b) Let X be a variable. The variance of X is defined as $\text{Var}(X) = E[(X - E[X])^2]$
   We want to prove $\text{Var}(X) = E[X^2] - (E[X])^2$:

$$E[(X - E[X])^2] = E[X^2 - 2X \cdot E[X] + (E[X])^2]$$

$$\text{Var}(X) = E[X^2 - 2X \cdot E[X] + (E[X])^2]$$

$$\text{Var}(X) = E[X^2] - 2 \cdot E[X \cdot E[X]] + E[(E[X])^2]$$

Because we notice that $E[X]$ is a constant, $2 \cdot E[X \cdot E[X]] = 2 \cdot E[X] \cdot E[X]$

Similarly, because we notice that $(E[X])^2$ is a constant, $E[(E[X])^2 = (E[X])^2$
Thus, the equation now becomes:

$$\mathrm{Var}(X) = E[X^2] - 2 \cdot E[X] \cdot E[X] + (E[X])^2$$

$$\mathrm{Var}(X) = E[X^2] - 2 \cdot (E[X])^2 + (E[X])^2$$

$$\mathrm{Var}(X) = E[X^2] - (E[X])^2$$

The proof is completed.

5. (a) C
   (b) D
   (c) A

6. (a) Let I be a $2 \cdot 2$ identity matrix. Let λ be the eigenvalue.
   We first need to calculate the determinant of the equation $A - \lambda I = 0$

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$\det\left(\begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} 4-\lambda & 2 \\ 1 & 5-\lambda \end{bmatrix}\right) = 0$$

$$(4-\lambda) \cdot (5-\lambda) - (2) \cdot (1) = 0$$

$$\lambda^2 - 9\lambda + 18 = 0$$

$$(\lambda - 3) \cdot (\lambda - 6) = 0$$

$$\lambda 1 = 3$$

$$\lambda 2 = 6$$

   The eigenvalues are 3 and 6, respectively.
   (b) C

3 Python Programming Questions

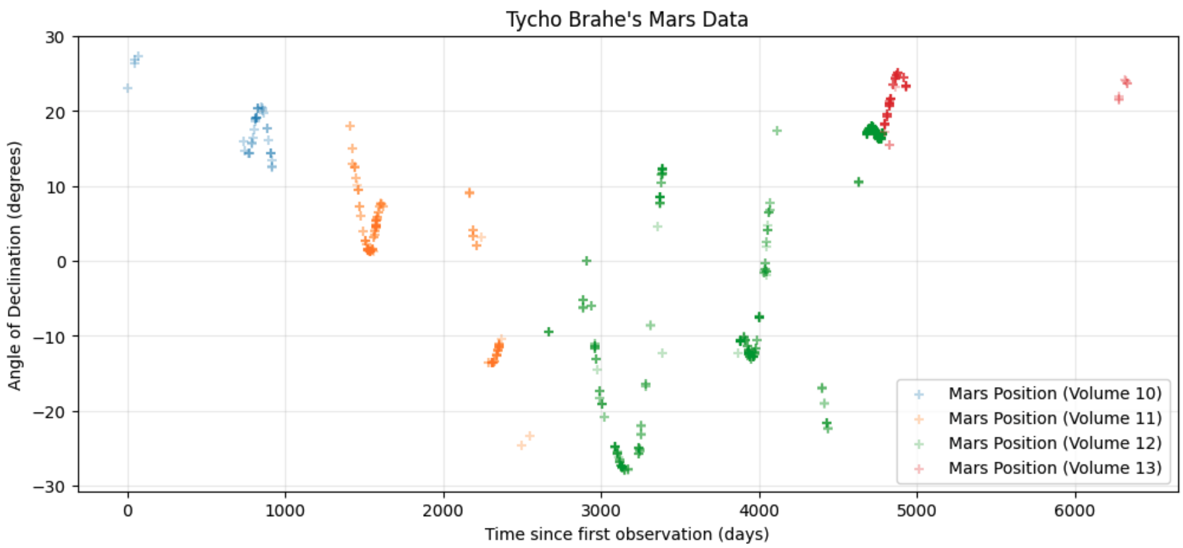

Figure 1: result