Name: ____________________________________

This exam contains 5 pages (including this cover page) and 8 problems, including the bonus. Check to see if any pages are missing.

You are allowed one sheet of **handwritten** notes. Turn in your notesheet along with your response. You are also allowed to use a calculator on the exam. No other resources are allowed.

Read the following instructions carefully:

- Write your response on separate sheet of paper. **Start on a new sheet for each problem**.
- The exam has a time limit of **2 hours**. In addition, 15 minutes is added to the time limit for students to scan their work and submit it via Canvas. Make sure you include your notesheet in your submission.
- You can only use methods discussed in class so far. i.e., you may **NOT** use any method we haven't learned in class so far.
- **Show ALL your work!** You are being assessed on your work and explanations. Correct answers with little to no work or explanation will not receive full credit. I give partial credits so don't leave a blank if you have some ideas.
- **Organize your work.** Your work should be clearly written and organized. Please use correct mathematical notations when appropriate. For example, please pay attention to when you should use the integral sign (including the dx) and when to put the $+C$.
- By submitting this exam, you agree that all work represents your individual effort and understanding. Failure to comply with any of the above statements will be considered a violation of the Student Code of Conduct and will result in a 0 for this assessment and possible disciplinary action.

Adding the bonus, the total score won't exceed 100%.

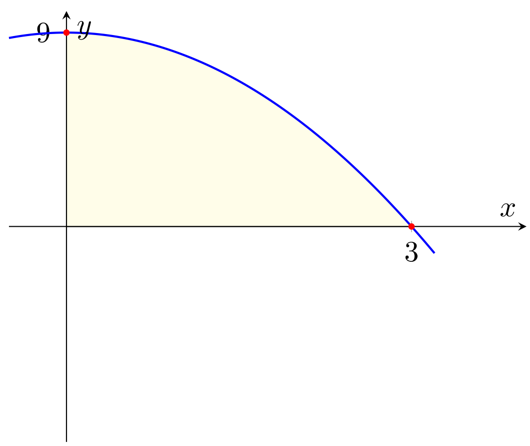
Please keep in mind that an assessment is just another means of communication between you and me to help me understand what you have learned so far. It is not a measure of your worth as a person nor your intelligence or aptitude as a student. Just give this your best try.

Problem 1. (15 points) Evaluate the following integrals. Make sure your notation is perfectly correct.

(a) $\displaystyle\int (\sec(x) + \tan(x))^2 \, dx$

(b) $\displaystyle\int_0^{1/2} \frac{x^3}{\sqrt{1-x^2}} \, dx$

Problem 2. (14 points) Let $\mathcal{R}$ be the region under $x^2 + y = 9$ in the first quadrant. The graph is shown below. Let $\mathcal{S}$ be the solid obtained by rotating $\mathcal{R}$ about the x-axis.



(a) Set up an expression to find the volume of $\mathcal{S}$ using the **Disk Method**. You do NOT need to evaluate this integral.

(b) Set up an expression to find the volume of $\mathcal{S}$ using the **Shell Method**. You do NOT need to evaluate this integral.

Problem 3. (14 points) Evaluate the following integrals. Also, check the box that best describes the solution.

(a) $\displaystyle\int_1^{\infty} \frac{\ln(x)}{x^3}\,dx$ ☐ the integral **converges**. ☐ the integral **diverges**.

(b) $\displaystyle\int_0^{3} \frac{5x}{(x+2)(x-3)}$ ☐ the integral **converges**. ☐ the integral **diverges**.

Problem 4. (15 points) Approximate the integral $\displaystyle\int_0^{2\pi} \frac{dx}{(5+3\sin(x))^2}$ using the following approximation methods:

(a) M_4 (b) T_4 (c) S_4

Problem 5. (14 points) Let $f(x) = xe^{-x}$ if $x \geq 0$ and $f(x) = 0$ if $x < 0$.

(a) *Verify* that f is a probability density function.

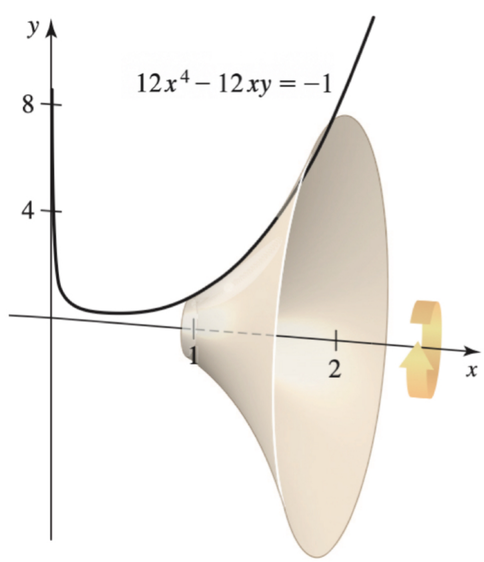
> **Hint:** Recall the two conditions for a function to be a probability density function based on the definition. You will need to argue why $f(x)$ satisfies BOTH conditions.

(b) Find $P(1 \leq X \leq 2)$. Round your answer to six decimal places.

Problem 6. (14 points) The curved surface of a funnel is generated by revolving the graph of

$$12x^4 - 12xy = -1$$

on the interval $[1, 2]$ about the x-axis (see the figure below). Set up an expression to find the **surface area** of the funnel. You do NOT need to evaluate this integral.



Problem 7. (14 points) Approximate the integral $\int_0^1 x\cos\left(x^3\right)\,dx$ using a Maclaurin polynomial in the following steps.

(a) Find the 4th-degree Maclaurin polynomial for $\cos(x)$ using the definition of Maclaurin polynomial.

(b) Find the 13th-degree Maclaurin polynomial for $x\cos\left(x^3\right)$ using the result you obtained in part (a).

(c) Approximate the integral using the Maclaurin polynomial you found in part (b). Round your answer to six decimal places.

Bonus. (10 points) Determine whether the following statements are true. If so, justify it; if not, provide a counterexample to show what breaks down (for example, what value for m does not work).

(a) If m is a positive integer, then $\int_0^\pi \sin^m(x)\,dx = 0$.

(b) If m is a positive integer, then $\int_0^\pi \cos^{2m+1}(x)\,dx = 0$.

Hint: Notice that the power of $2m+1$ on cosine is always an odd number since $2m$ is always divisible by 2 and we added 1 to it. What do we do if we see an odd power on cosine?