

Name: \_\_\_\_\_

**Instruction:** You are encouraged to work on this assignment collaboratively with your peers in class. You can also ask me for hints if you are stuck. Yet you should write up your own work and submit it on Canvas in **ONE** pdf file. Show all of your work for full credit, and your work should be clearly written and organized. This homework covers some important concepts in §5.4, §5.5, §5.6, and §5.7.

**Problem 1.** Evaluate the following definite integrals using the FTC part I.

$$(a) \int_0^{\frac{\pi}{3}} \frac{\sin(\theta) + \sin(\theta) \tan^2(\theta)}{\sec^2(\theta)} d\theta$$

$$(b) \int_1^2 \frac{(x-1)^3}{x^2} dx$$

**Problem 2.** If  $f(x) = \int_0^{\sin(x)} \sqrt{1+t^2} dt$  and  $g(y) = \int_3^y f(x) dx$ , find  $g''\left(\frac{\pi}{6}\right)$ .

**Hint:** Find the relationship between these two functions  $f$  and  $g$  first. Is  $f$  the derivative or antiderivative of  $g$ ?

**Problem 3.** A particle moves in a straight line with the velocity

$$v(t) = 36 - 24t + 3t^2 \quad (\text{in m/s})$$

Find the *displacement* and the *total distance traveled* over the interval  $[0, 10]$ .

**Problem 4.** Wind engineers have found that wind speed  $v$  (in m/s) at a given location follows a **Rayleigh distribution** of the type

$$W(v) = \frac{1}{32}ve^{-v^2/64}$$

This means that at a given moment in time, the probability that  $v$  lies between  $a$  and  $b$  is equal to the shaded area in the figure below.

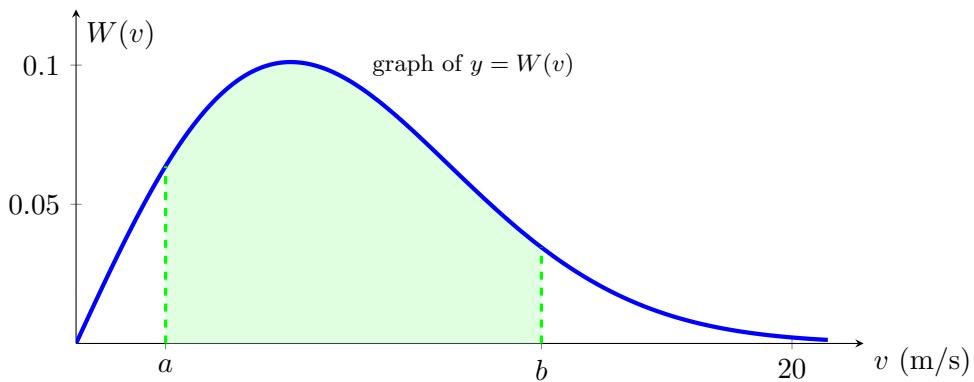


Figure 1: the shaded area is the probability that  $v$  lies between  $a$  and  $b$

- (a) Show that the probability that  $v$  lies between 0 and  $b$  is  $1 - e^{-b^2/64}$ .

- (b) Calculate the probability that  $v$  lies between 2 and 5.