**Name:** ________________________________

This exam contains 5 pages (including this cover page) and 8 problems, including the bonus. Check to see if any pages are missing.

You are allowed one sheet of **handwritten** notes. Turn in your notesheet along with your response. You are also allowed to use a calculator on the exam. No other resources are allowed.

**Read the following instructions carefully:**

- Write your response on separate sheet of paper. **Start on a new sheet for each problem**.
- The exam has a time limit of **2 hours**. In addition, 15 minutes is added to the time limit for students to scan their work and submit it via Canvas. Make sure you include your notesheet in your submission.
- You can only use methods discussed in class so far. i.e., you may **NOT** use any method we haven't learned in class so far.
- **Show ALL your work!** You are being assessed on your work and explanations. Correct answers with little to no work or explanation will not receive full credit. I give partial credits so don't leave a blank if you have some ideas.
- **Organize your work.** Your work should be clearly written and organized. Please use correct mathematical notations when appropriate. For example, please pay attention to when you should use the integral sign (including the $dx$) and when to put the $+C$.
- By submitting this exam, you agree that all work represents your individual effort and understanding. Failure to comply with any of the above statements will be considered a violation of the Student Code of Conduct and will result in a 0 for this assessment and possible disciplinary action.

Adding the bonus, the total score won't exceed 100%.

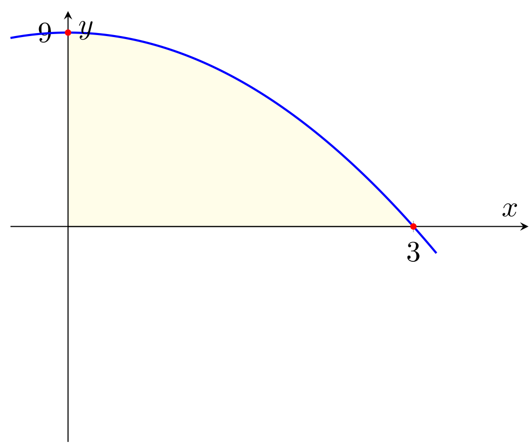
Please keep in mind that an assessment is just another means of communication between you and me to help me understand what you have learned so far. It is not a measure of your worth as a person nor your intelligence or aptitude as a student. Just give this your best try.

**Problem 1.** (15 points) Evaluate the following integrals. Make sure your notation is perfectly correct.

(a) $\displaystyle\int (\sec(x) + \tan(x))^2 \, dx$

(b) $\displaystyle\int_0^{1/2} \frac{x^3}{\sqrt{1-x^2}} \, dx$

**Problem 2.** (14 points) Let $\mathcal{R}$ be the region under $x^2 + y = 9$ in the first quadrant. The graph is shown below. Let $\mathcal{S}$ be the solid obtained by rotating $\mathcal{R}$ about the $x$-axis.




(a) Set up an expression to find the volume of $\mathcal{S}$ using the **Disk Method**. You do NOT need to evaluate this integral.

(b) Set up an expression to find the volume of $\mathcal{S}$ using the **Shell Method**. You do NOT need to evaluate this integral.

**Problem 3.** (14 points) Evaluate the following integrals. Also, check the box that best describes the solution.

(a) $\displaystyle\int_1^{\infty} \frac{\ln(x)}{x^3}\,dx$ ☐ the integral **converges**. ☐ the integral **diverges**.

(b) $\displaystyle\int_0^{3} \frac{5x}{(x+2)(x-3)}$ ☐ the integral **converges**. ☐ the integral **diverges**.

**Problem 4.** (15 points) Approximate the integral $\displaystyle\int_0^{2\pi} \frac{dx}{(5+3\sin(x))^2}$ using the following approximation methods:

(a) $M_4$ (b) $T_4$ (c) $S_4$

**Problem 5.** (14 points) Let $f(x) = xe^{-x}$ if $x \geq 0$ and $f(x) = 0$ if $x < 0$.

(a) *Verify* that $f$ is a probability density function.

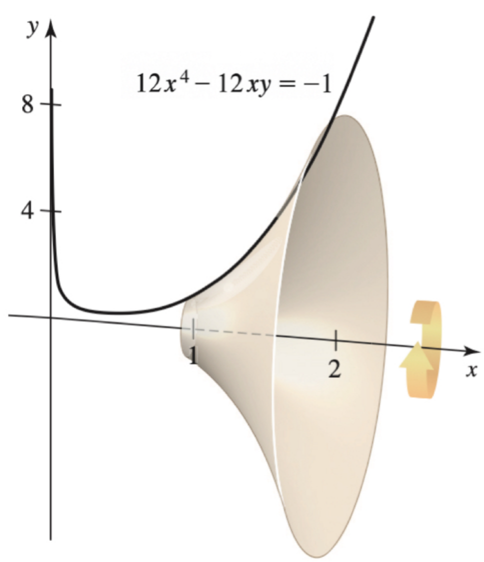
> **Hint:** Recall the two conditions for a function to be a probability density function based on the definition. You will need to argue why $f(x)$ satisfies BOTH conditions.

(b) Find $P(1 \leq X \leq 2)$. Round your answer to six decimal places.

**Problem 6.** (14 points) The curved surface of a funnel is generated by revolving the graph of

$$12x^4 - 12xy = -1$$

on the interval $[1, 2]$ about the $x$-axis (see the figure below). Set up an expression to find the **surface area** of the funnel. You do NOT need to evaluate this integral.



**Problem 7.** (14 points) Approximate the integral $\int_0^1 x\cos\left(x^3\right)\,dx$ using a Maclaurin polynomial in the following steps.

(a) Find the 4th-degree Maclaurin polynomial for $\cos(x)$ using the definition of Maclaurin polynomial.

(b) Find the 13th-degree Maclaurin polynomial for $x\cos\left(x^3\right)$ using the result you obtained in part (a).

(c) Approximate the integral using the Maclaurin polynomial you found in part (b). Round your answer to six decimal places.

**Bonus.** (10 points) Determine whether the following statements are true. If so, justify it; if not, provide a counterexample to show what breaks down (for example, what value for $m$ does not work).

(a) If $m$ is a positive integer, then $\int_0^\pi \sin^m(x)\,dx = 0$.

(b) If $m$ is a positive integer, then $\int_0^\pi \cos^{2m+1}(x)\,dx = 0$.

**Hint:** Notice that the power of $2m+1$ on cosine is always an odd number since $2m$ is always divisible by 2 and we added 1 to it. What do we do if we see an odd power on cosine?