SPE-178303-MS

Empirical Wellhead Pressure-Production Rate Correlations for Niger Delta Oil Wells

Anietie N. Okon, Department of Chemical & Petroleum Engineering, University of Uyo, Nigeria; Francis D. Udoh, Department of Chemical & Petroleum Engineering, University of Uyo, Nigeria, Department of Chemical & Petroleum Engineering, Afe Babalola University, Nigeria; Dulu Appah, Department of Gas Engineering, University of Port Harcourt, Nigeria

This paper was prepared for presentation at the Nigeria Annual International Conference and Exhibition held in Lagos, Nigeria, 4–6 August 2015.

Abstract

Several correlations have been developed to predict wellhead pressure–production rate relationship in the Niger Delta region. Regrettably, most of these correlations were developed from field data that are not from the Niger Delta region and with limited field test data ranges, so their predictions are lower than expected field values when applied to the Niger Delta. Additionally, some developed wellhead pressure–production rate correlations based on Niger Delta field data are made using in-house equations by the operating companies in the Niger Delta region. To ameliorate this anomaly, sixty four (64) field test data: choke size (S), production rate (q), gas-liquid ratio (GLR), flowing wellhead pressure (P_{wh}), flowing temperature (FTHP) and basic sediment and water (BS&W) were collected from oil producing wells in the Niger Delta region to develop wellhead pressure–production rate correlations based on Gilbert correlation and modified Gilbert equations. The developed correlations using Niger Delta field test data were compared with several authors' correlations. The results obtained indicate that the developed correlations resulted in better predictions than earlier correlations. In addition, the statistical analysis of the developed correlations used to ascertain the extent of their predicted values differ from the field test data and resulted in average error, absolute error and standard deviation of −0.1477, 0.4430 and 0.9582 for Gilbert formula and −0.2515, 0.4737 and 1.0997 for modified Gilbert formula, respectively. Furthermore, the developed correlations are comparable with an average correlation coefficient of 0.9869. Therefore, the developed correlations can be used as a quick tool to estimate the wellhead pressure–production rate relationship in Niger Delta oil fields.

Key Words: Wellhead pressure-production rate correlation, Gilbert formula, Modified Gilbert formula, Niger Delta region

Introduction

In the petroleum industry, wellhead chokes are used to limit production rates for regulations, protect surface equipment from pressure fluctuations, avoid sand problem due to high drawdown and control flow rate to avoid water or gas coning (Guo *et al*., 2007). Therefore, installing a choke at the wellhead implies fixing the wellhead pressure and, thus, the flowing bottom-hole pressure and production rate. On the other hand, wellhead pressure is predicted in order to determine the choke size to be installed on a particular well for optimal oil production. As standard oil field practice, wellhead performance is evaluated under critical or sonic flow (Mesallati *et al*., 2000). The sonic flow is used to provide stable well production and separation operation conditions (Guo *et al*., 2007). Consequently, the existence of sonic flow in the choke depends on a downstream-to-upstream pressure ratio. Clegg (2007) maintains that, for this condition to occur, the downstream pressure must be approximately 0.55 or less of the upstream pressure. Sadiq (2012) opined that under critical flow conditions, the flow rate is a function of the upstream pressure. The theoretical framework for gas-liquid two-phase flow through restrictions was laid by Tangren *et al*. in 1949 (Guo *et al*., 2007). They presented an analysis of the behaviour of an expanding gas-liquid system and established that when gas bubbles are added to an incompressible fluid, above a critical flow velocity, the medium becomes incapable of transmitting pressure change upstream against the flow. In this connection, several empirical equations have been developed to estimate the relationship between production rate and wellhead pressure for two-phase critical flow. The pioneer work was provided by Gilbert in 1954. He suggested an empirical correlation for critical flow through choke that predicts liquid flow rates as a function of flow wellhead pressure, gas-liquid ratio and surface wellhead choke size. Additionally, Ghareeb and Shedid (2007) maintain that Gilbert's correlation is valid for critical flow occurring when the upstream pressure of the choke is at least 70% higher than the downstream pressure or when the ratio of downstream-upstream pressure is equal to 0.588. Other researchers have proposed various correlations rather than Gilbert's equation in the literature. Baxendell in 1957developed a revised correlation to Gilbert (1954) equation based on incremental data to update the correlation exponents. Ros (1960) and Achong (1961) presented another Gilbert's correlation form with modification on the constant and exponents using regression parameters based on data from different oil fields. Similarly, Pilehvari (1981) and Beiranvand *et al*. (2012) revised the Gilbert equation with new constant and exponents. Furthermore, Beiranvand *et al*. (2012) and Khorzoughi *et al*. (2013) introduced additional parameters: basic sediments and water (BS&W) and temperature (T) to modify the Gilbert (1954) equation form to predict wellhead pressure-flow rate relationship. Thus, several correlations have been published to describe critical two-phase flow through wellhead chokes, but most of these correlations were based on limited ranges of flow variables (Ghassan and Maha, 1991). In other words, the validity of these developed correlations is limited by the quantity and scope of the data upon which they are based. In most literature, emphasis is laid on the flowing wellhead pressure, gas-liquid ratio and chokes size to develop the empirical equation to estimate the relationship between production rate and wellhead pressure for two-phase critical flow. This approach, however, has resulted in several correlations currently available in the literature; but the concern is the variability of the production rate at the wellhead. Therefore, in this paper, emphasis is laid on the production rate, gas-liquid ratio and chokes size to develop empirical correlations to predict wellhead pressure–production rate relationship in Niger Delta oil fields.

Materials and Methods

Data Acquisition and Correlation Formulation

For the development of the wellhead pressure–production rate correlations for Niger Delta oil wells, sixty four (64) production test data were collected from different oil wells in Niger Delta fields. The data collected as presented in Table 1 include: gas-liquid ratio (GLR), choke size (S), flowing wellhead pressure (P_{wh}), production rate (q), basic sediments and water (BS&W), and flowing temperature (FTHP). The ranges of these production test data used in the correlations formulation are also included in Table 1. The developed wellhead pressure (P_{wh}) – production rate (q) correlations were based on Gilbert (1954) formula and modified Gilbert formula presented by Beiranvand *et al*. (2012) and Khorzoughi *et al*. (2013). These equations are expanded as;

Table 1—Data Range

Parameters	Range
Flowing Wellhead Pressure (P_{wh}), psig	36 – 2320
Choke size (S), 1/64 inch	16 – 76
Production rate (q), stb/day	263 – 5313
Basic sediment & water (BS&W), fraction	0 – 0.884
Gas-liquid ratio (GLR), scf/stb	93 – 4134
Temperature (T), °F	100 – 150

Gilbert formula:

$$P_{wh} = \frac{C(GLR)^m q}{S^n} \tag{1}$$

Modified Gilbert formula:

$$P_{wh} = \left[\frac{1}{A}\frac{GLR^C q}{S^B\left(1-\frac{BS\&W}{100}\right)^D \left({}^{T}/_{T_{sc}}\right)^E}\right]^{1/F} \tag{2}$$

Several authors have presented different correlations by determining the constant (C) and the exponents *m* and *n* for Gilbert equation as presented in Table 2. The modified Gilbert equation constant and exponents established by Beiranvand *et al*. (2012) and Khorzoughi *et al*. (2013) are presented in Table 3. To develop the wellhead pressure correlations for Niger Delta oil fields, a multivariate regression analysis approach was used to determine and optimize the constants and exponents in the Gilbert and modified Gilbert equations in order to minimize the difference between the field data and predicted values. There are several methods to perform multivariable regression analysis; these include Gauss-Newton, the Marquardt-Levenberg, the Nelder-Mead, the steepest descent, among others. Interestingly, a robust and reliable SOLVER in Microsoft Excel which uses another iteration protocol known as General Reduced Gradient (GRG) was used to determine the constants and exponents in the developed wellhead pressure correlations for Niger Delta oil wells. The determined constants and exponents for Gilbert and modified Gilbert equations are also depicted in Tables 2 and 3 respectively.

Table 2—Gilbert Equation Authors' Constant

	Correlation constants		
Authors	C	m	n
Gilbert (1954)	10.0	0.546	1.89
Baxendell (1957)	9.56	0.546	1.93
Ros (1960)	17.40	0.50	2.00
Achong (1961)	3.82	0.65	1.88
Pilehvari (1980)	46.67	0.313	2.11
Beiranvand *et al.* (2012)	30.49	0.589	2.275
Developed Correlation	5.1474	0.5048	1.7093

Table 3—Modified Gilbert Equation Authors' Constants

	Correlation constants					
Authors	A	B	C	D	E	F
Beiranvand*et al*. (2012)	0.0382	2.151	0.5154	0.5297	0	1.0
Khorzoughi*et al*.(2013)	1.0	1.50	0.10	1.0	−0.8	0.5
Developed correlation (2014)	0.0509	1.8134	0.6749	0.2235	0.000029	1.321

Developed Correlations and Comparison

The developed correlation in both Gilbert and modified Gilbert equations were used to predict the flowing wellhead pressure (P_{wh}) of the Niger Delta fields. The obtained results were compared with the actual wellhead pressure from the production test data of the Niger Delta fields. Furthermore, the flowing wellhead pressure-production rate plot of field test data and predicted values from both Gilbert and modified Gilbert equations were compared. These results are presented in Figures 1 through 5. In addition to this, other authors' correlation presented in Tables 2 and 3 were used to predict the flowing wellhead pressure of the Niger Delta oil fileds. Their respective results are presented in Figures 7 through 16 for Gilbert equation and Figures 17 through 20 for modified Gilbert equation. Aside from the prediction and comparison of the developed correlation results with field test data and other authors' correlation, the reliability of the developed correlations was validated using three statistical approaches of evaluating the extent of the forecasted or predicted values' deviation from the test (measured) data. These statistical methods used are the average error, absolute error and standard deviation; thus, their respective mathematical equation is expanded as:

$$E_a = \frac{1}{N}\sum_{i=1}^{N}\frac{A_t - F_t}{A_t} \tag{3}$$

$$E_{ab} = \frac{1}{N}\sum_{i=1}^{N}\frac{|A_t - F_t|}{A_t} \tag{4}$$

$$S_D = \frac{1}{N}\sqrt{N\sum_{i=1}^{N}\left(\frac{A_t - F_t}{A_t}\right)^2 - \sum_{i=1}^{N}\left(\frac{A_t - F_t}{A_t}\right)^2} \tag{5}$$

Where:
A_t = *Actual value*
F_t = *Forecast value*
N = *Sample number*
E_a = *Average error*
E_{ab} = *Absolute error*
S_D = *Standard deviation*

The results obtained from these statistical estimations for the developed correlations and other authors' correlations for both Gilbert and modified Gilbert equations are presented in Tables 4 and 5. In addition to the statistical validation of the developed correlations with the field test data, the developed correlations were compared with each other to ascertain the extent of their correlation and are presented in Figure 6.

Table 4—Gilbert equation statistical analysis

Authors	Average Error	Absolute Error	Standard Deviation
Gilbert (1954)	−0.5604	0.6734	1.4645
Baxendell (1957)	−0.3000	0.4960	1.1790
Ros (1960)	−0.3830	0.5409	1.2854
Achong (1961)	−0.2125	0.4675	1.0936
Pilehvari (1980)	0.2288	0.4909	0.7332
Beiranvand *et al.* (2012)	−0.6890	0.7940	1.7845
Developed Correlation	−0.1477	0.4430	0.9582

Table 5—Modified Gilbert equation statistical analysis

Authors	Average Error	Absolute Error	Standard Deviation
Beiranvand *et al.* (2012)	−0.3745	0.5391	1.3291
Khorzoughi *et al.*(2013)	−0.6711	0.9922	1.5708
Developed correlation	−0.2515	0.4737	1.0997

Results and Discussion

From the results of the multivariate regression analysis performed on the field test data using Gilbert and modified Gilbert equations, the determined constants and exponents of these equations as presented in Tables 2 and 3 result in the following proposed correlations for Niger Delta oil field wellhead pressure as expanded in Equations 6 and 7.

Authors' Correlation based on Gilbert formula:

$$P_{wh} = \frac{5.1474\left(GLR^{0.5048}\right)q}{S^{1.7098}} \tag{6}$$

Authors' Correlation based on Modified Gilbert formula:

$$P_{wh} = \left[\frac{1}{0.0509} * \frac{GLR^{0.6749}q}{S^{1.8133}\left(1-\frac{BS\&W}{100}\right)^{0.2235}\left(\frac{T}{T_{sc}}\right)^{0.000029}}\right]^{1/1.321} \tag{7}$$

Alternatively, equation 7 can be expressed as;

$$P_{wh} = \left[\frac{19.65\left(GLR^{0.6749}\right)q}{S^{1.8133}\left(1-\frac{BS\&W}{100}\right)^{0.2235}\left(\frac{T}{T_{sc}}\right)^{0.000029}}\right]^{0.757} \tag{8}$$

Where;

P_{wh} = *Wellhead pressure (psig)*
GLR = *Gas-Liquid Ratio (scf/stb)*
q = *Production Rate (stb/day)*
S = *Choke Size (1/64 inch)*
T = *Flowing Temperature* (^oF)
BS&W = *Basic Sediment & Water (% volume)*
T_{sc} = *Standard surface Temperature* (60 ^oF)

Figures 1 and 2 depict the predicted wellhead pressure obtained from developed correlation based on Gilbert formula presented in Equation 6. The predicted wellhead pressure was compared with the field test wellhead pressure as shown in Figure 1. The figure indicates that the predicted wellhead pressure correlates with the field test wellhead pressure with correlation coefficient (R^2) of 0.6941. This correlation coefficient is in the range of other authors' correlation predictions as presented in Figures 7 through 12. Furthermore, Figure 2 presents the wellhead pressure–production rate plot of the field test data and the developed correlation predictions. The figure (Figure 2) indicates that there is a close match between the field test data and the predicted values as observed, when compared to other authors' predictions presented in Figures 13 through 16. Also, Figures 3 and 4 present the comparative results obtained using the developed correlation based on modified Gilbert equation expanded in Equation 8. Figure 3 indicates that a correlation coefficient (R^2) of 0.7185 was obtained between the field wellhead pressure and the predicted wellhead pressure. Interestingly, the obtained correlation coefficient was higher than Beiranvand *et al*. (2012) and Khorzoughi *et al*. (2013) correlation predictions as presented in 17 and 18, respectively. Figure 4 presents the modified Gilbert equation wellhead pressure–production rate comparison with field test data. The figure indicates that there was a better match between field test data and predicted values of the developed correlation compared to other authors' correlation as presented in Figures 19 and 20. Additionally, Figure 5 presents the wellhead pressure–production rate plot of the developed correlations (i.e., Gilbert and modified Gilbert equations) compared with field test data. As observed in Figure 5, the two developed correlations closely predict the field wellhead pressure. In this connection, the developed correlation wellhead pressure predictions resulted in correlation coefficient (R^2) of 0.9869 as presented in Figure 6. This result indicates that there is a strong and close relationship between the two developed wellhead pressure correlations based on Gilbert and modified Gilbert equations. As presented in Tables 4 and 5, the statistical analysis of the developed correlations with field test data resulted in average error, absolute error and standard deviation of −0.1477, 0.4430 and 0.9582 respectively from Gilbert equation and −0.2515, 0.4737 and 1.0997 from modified Gilbert equation. From the tables, it can be observed that other authors' correlations resulted in slightly higher average error, absolute error and standard deviation values when compared with the developed correlations. This means that the developed correlations will better predict the wellhead pressure in Niger Delta oil fields than earlier predicted correlations.

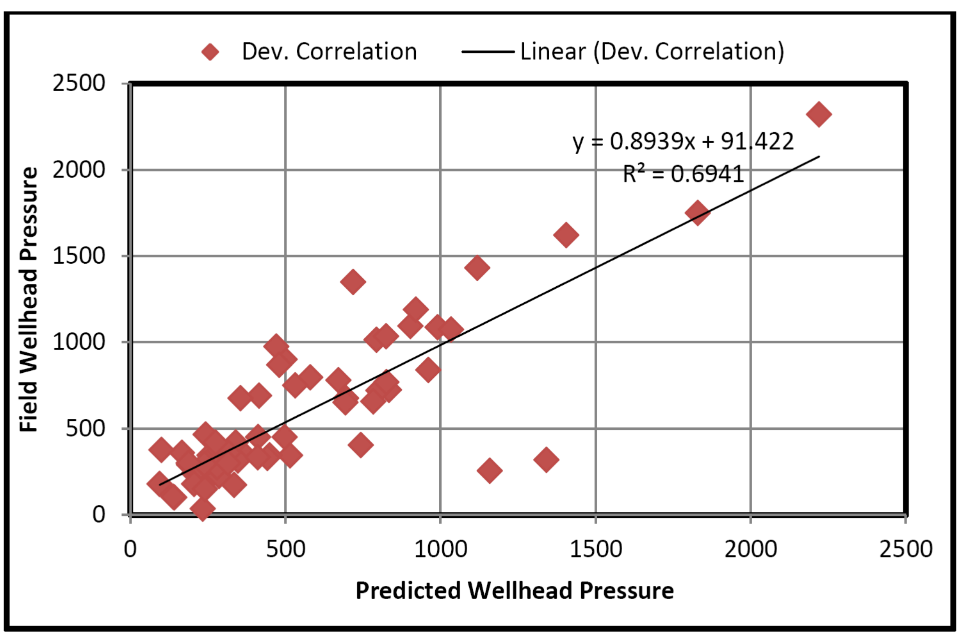

Figure 1—Comparison of Field wellhead pressure with developed correlation prediction (Gilbert correlation)

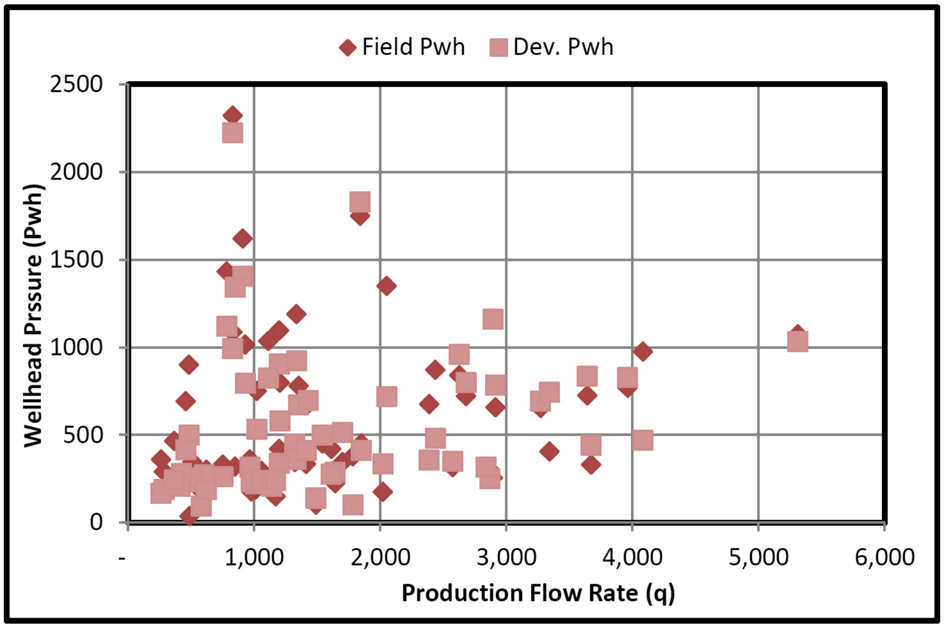

Figure 2—Comparison of Field data with developed correlation (Gilbert correlation)

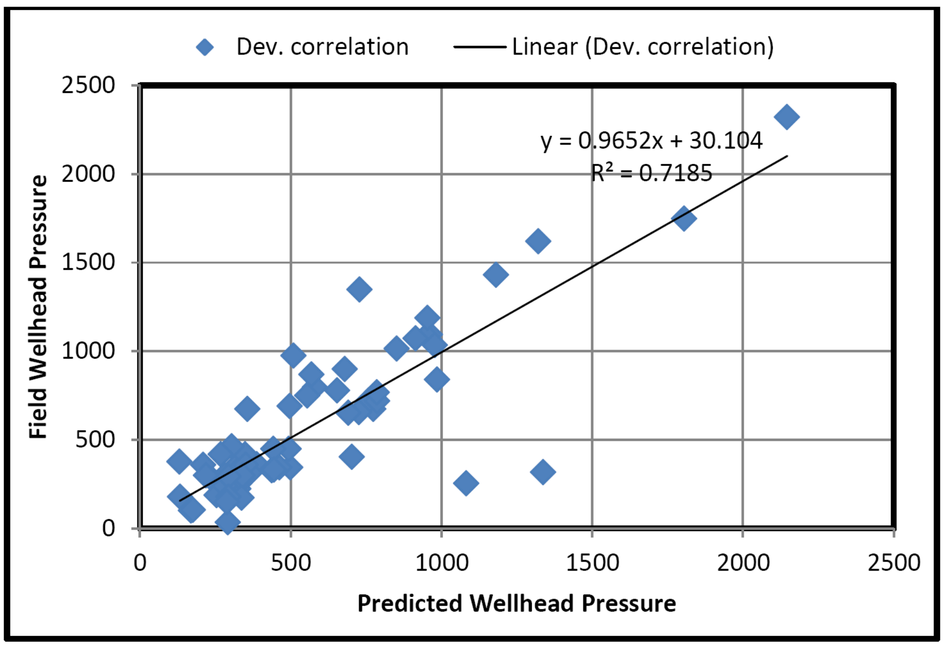

Figure 3—Comparison of Field wellhead pressure with developed correlation prediction (Modified Gilbert correlation)

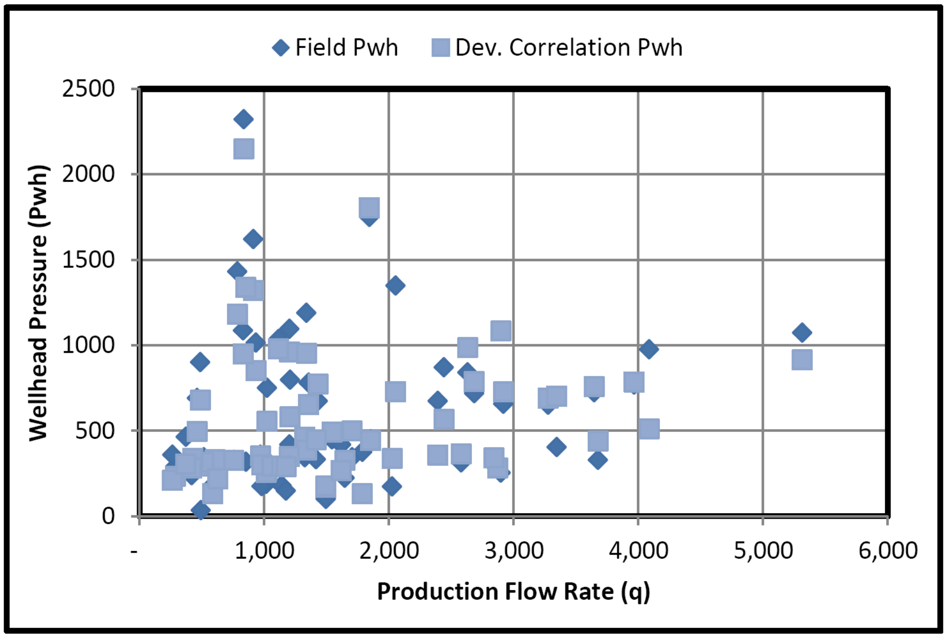

Figure 4—Comparison of field data with developed correlation (modified Gilbert correlation)

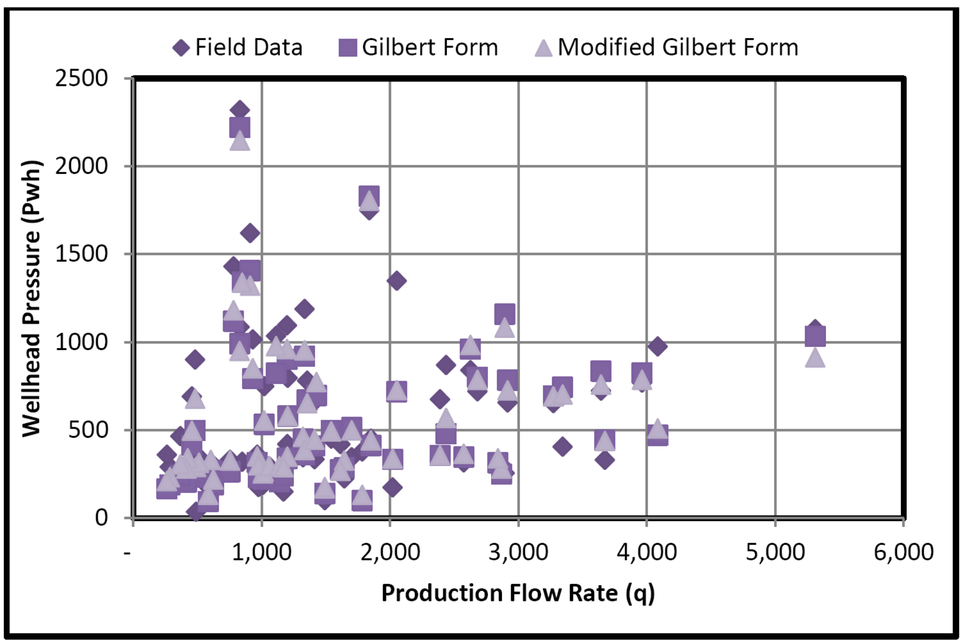

Figure 5—Comparison of the developed correlations with field test data

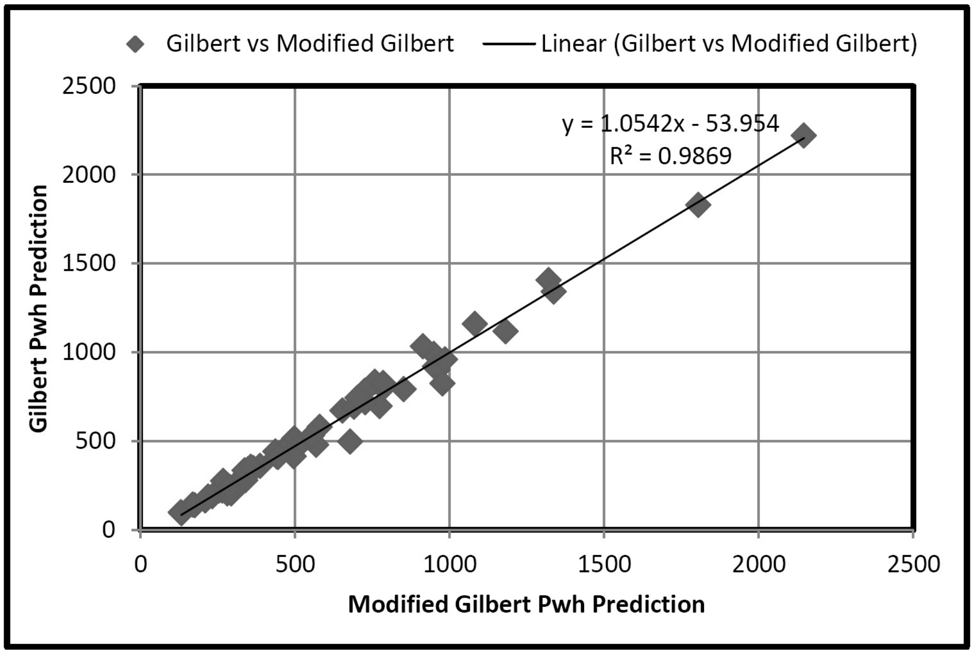

Figure 6—Comparison of the developed correlation coefficient

- Comparison of Field wellhead pressure with other Researchers' correlation prediction based on Gilbert's Equation:

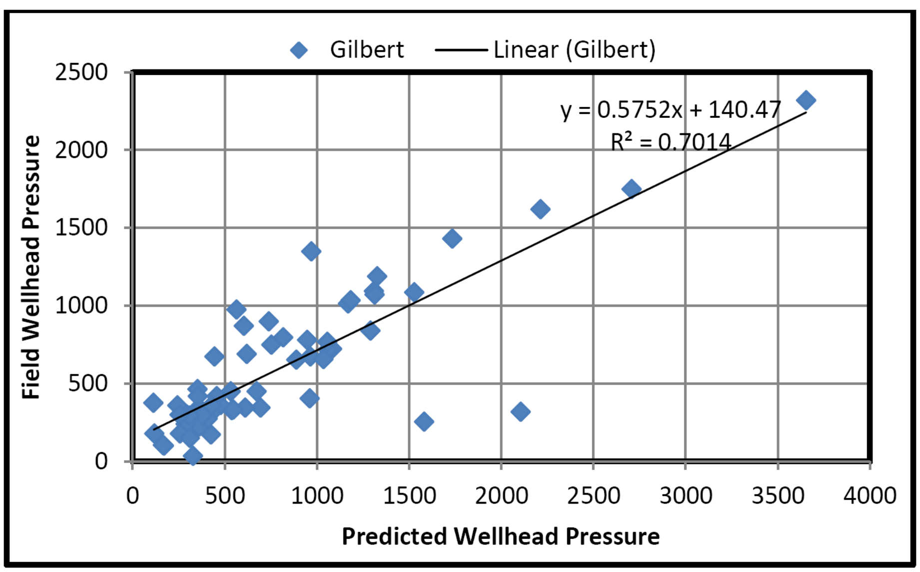

Figure 7—Gilbert's correlation prediction

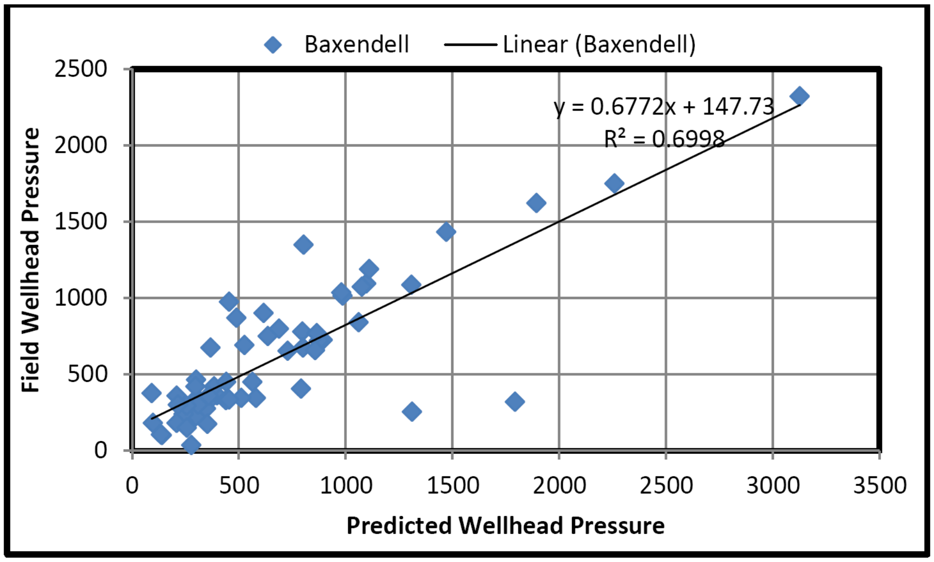

Figure 8—Baxendell's correlation prediction

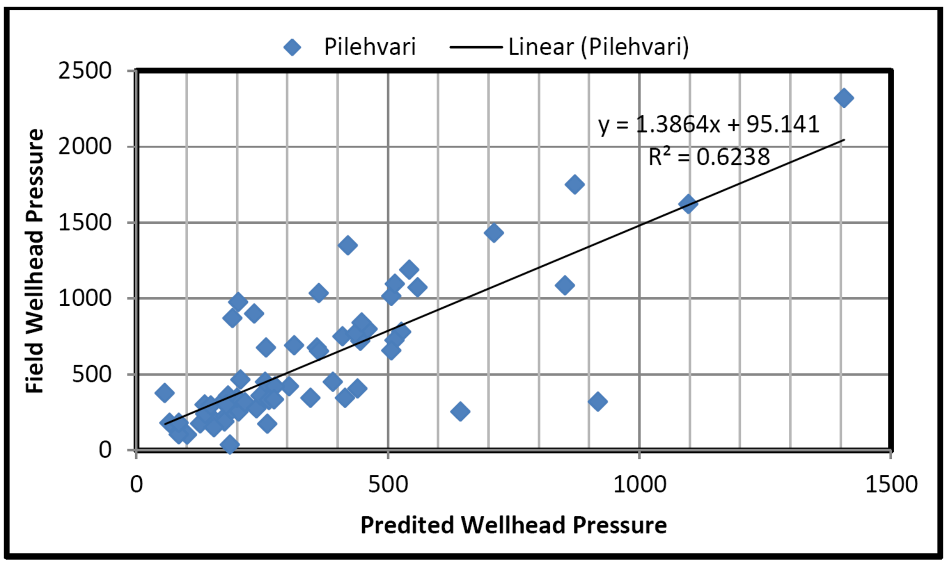

Figure 9—Pilehvari's correlation prediction

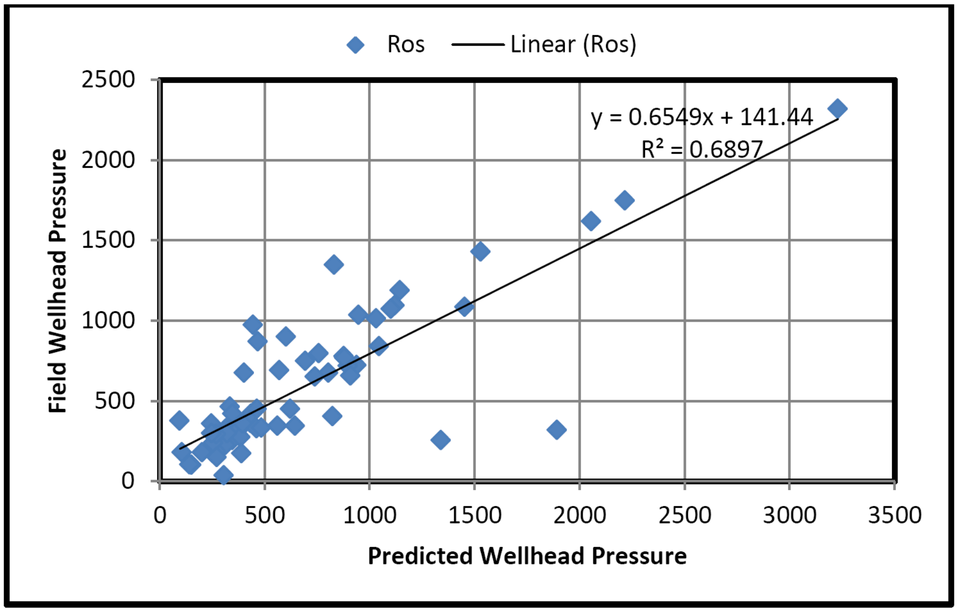

Figure 10—Ros' correlation prediction

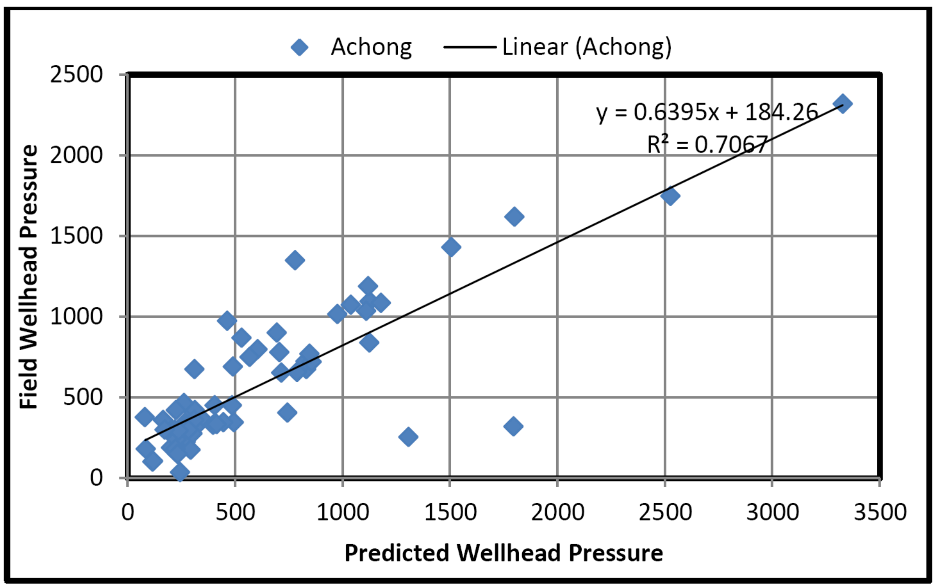

Figure 11—Achong's correlation prediction

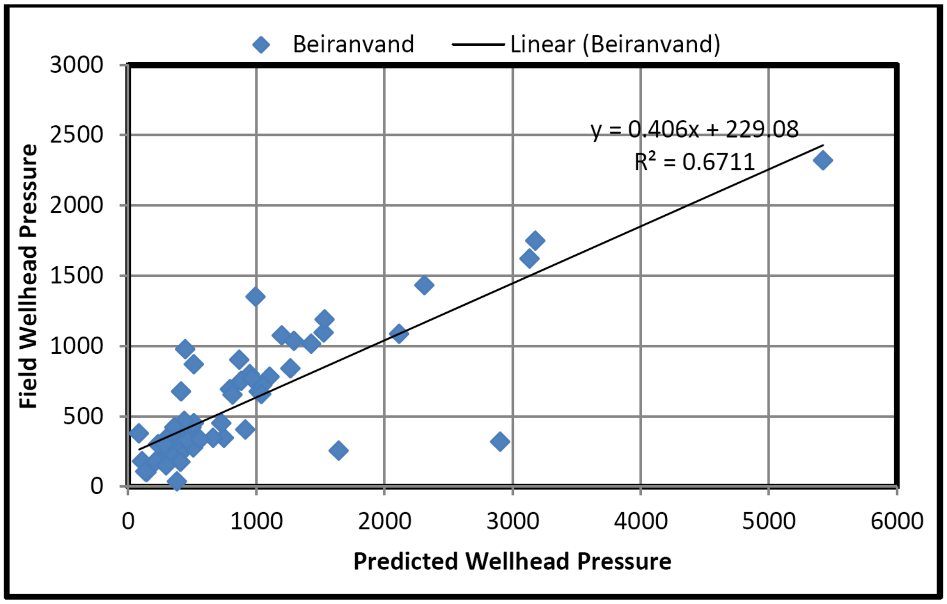

Figure 12—Beiranvand's correlation prediction

- Comparison of Field wellhead pressure–production rate with other Researchers' correlation prediction based on Gilbert's Equation:

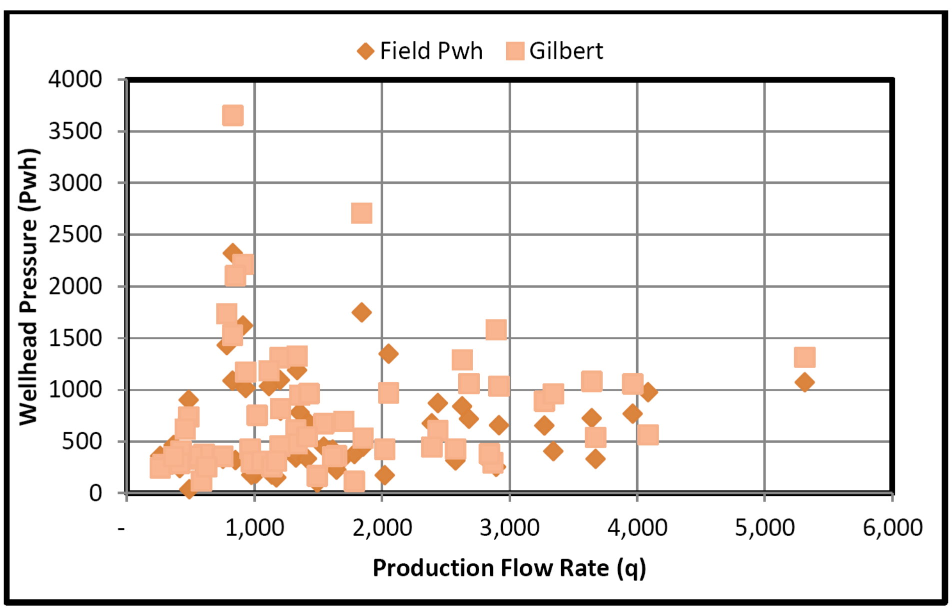

Figure 13—Comparison of Field test with Gilbert's prediction

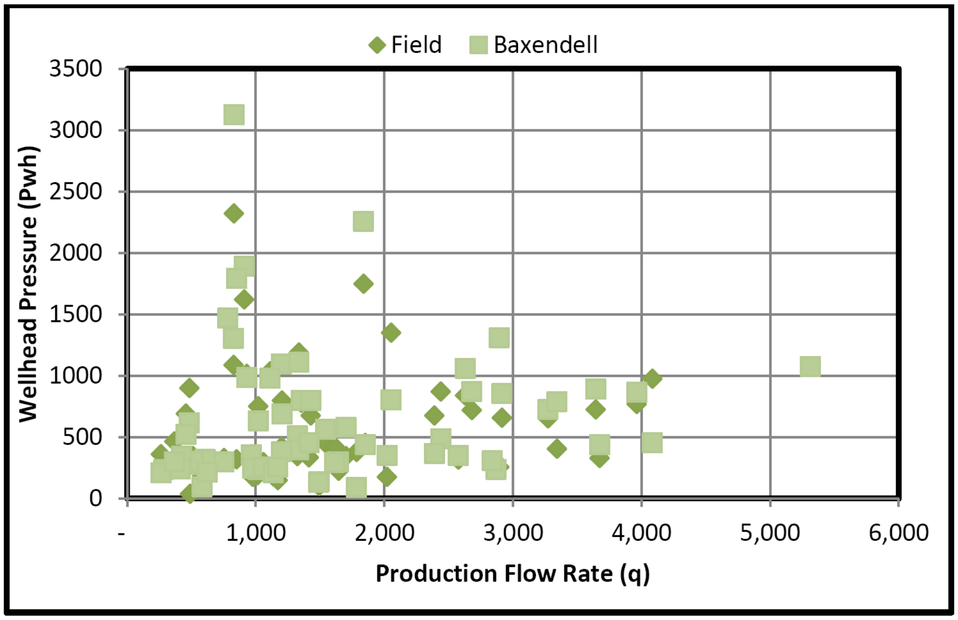

Figure 14—Comparison of Field test with Baxendell's prediction

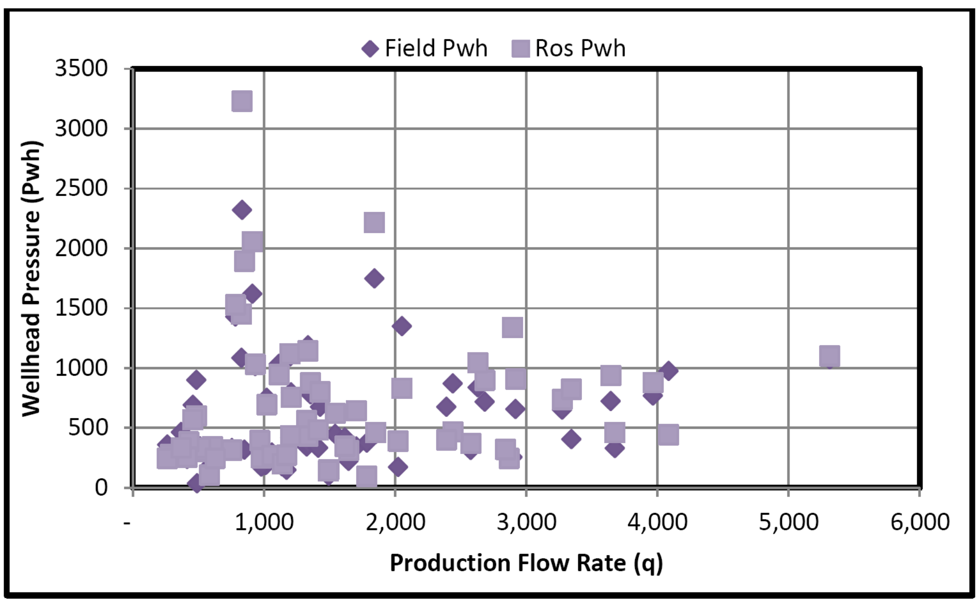

Figure 15—Comparison of Field test with Ros' prediction

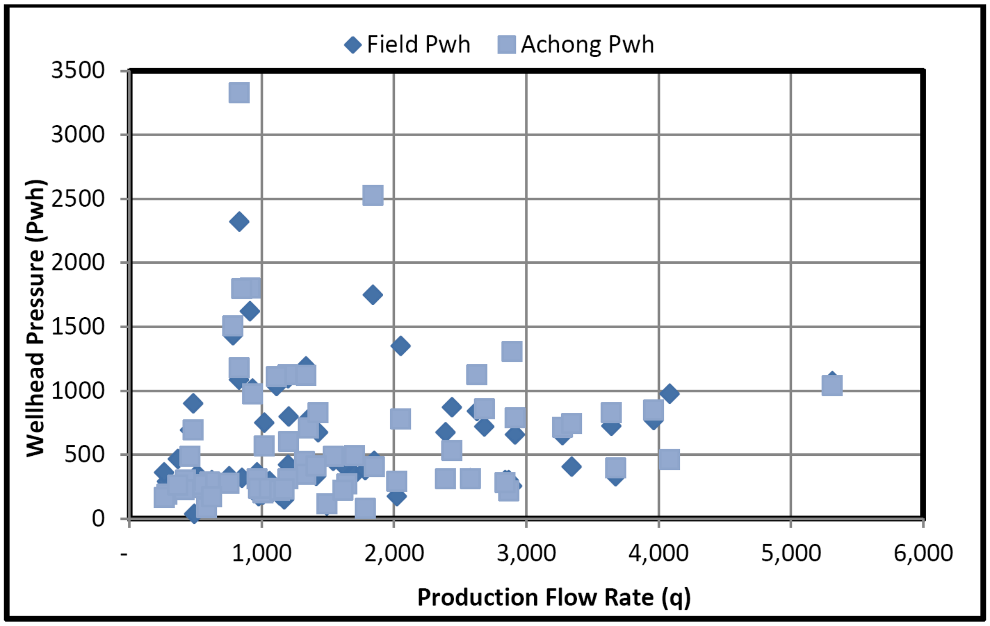

Figure 16—Comparison of Field test with Achong's prediction

- Comparison of Field wellhead pressure with other Researchers' correlation prediction based on modified Gilbert's Equation:

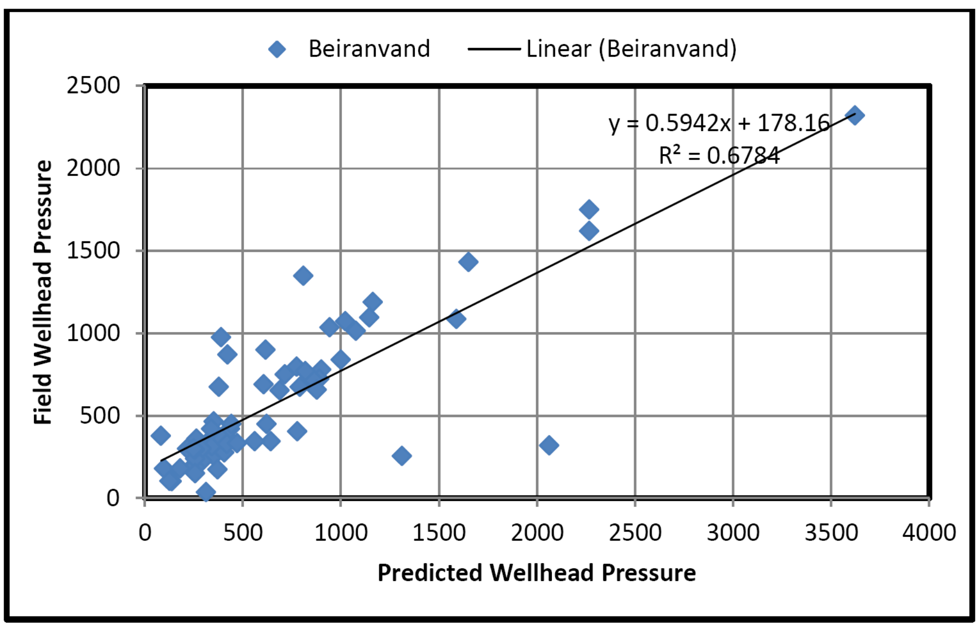

Figure 17—Beiranvand's correlation prediction

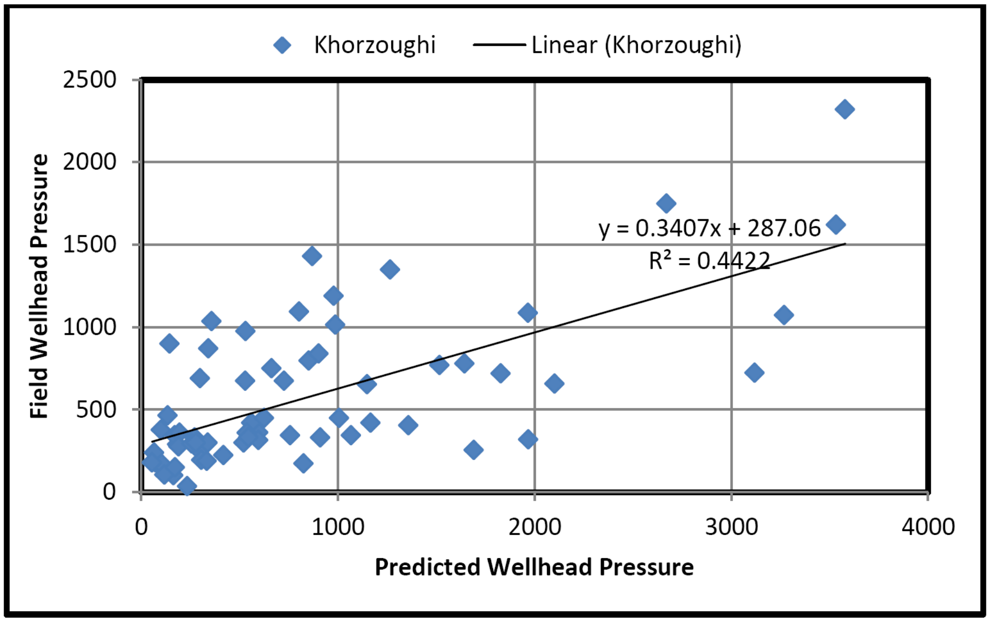

Figure 18—Khorzoughi's correlation prediction

- Comparison of Field wellhead pressure–production rate with other Researchers' correlation prediction based on modified Gilbert's Equation:

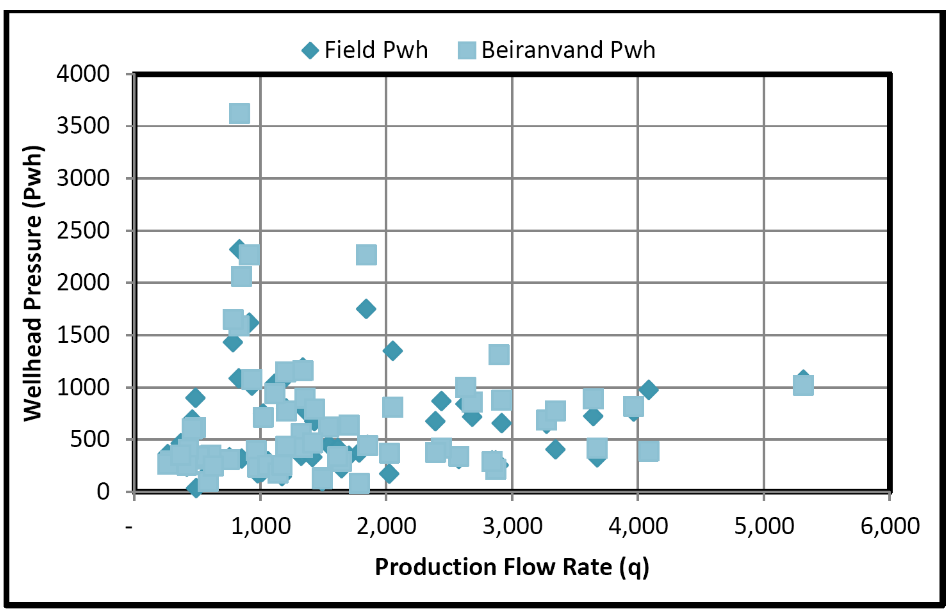

Figure 19—Comparison of Field test with Beiranvand's prediction

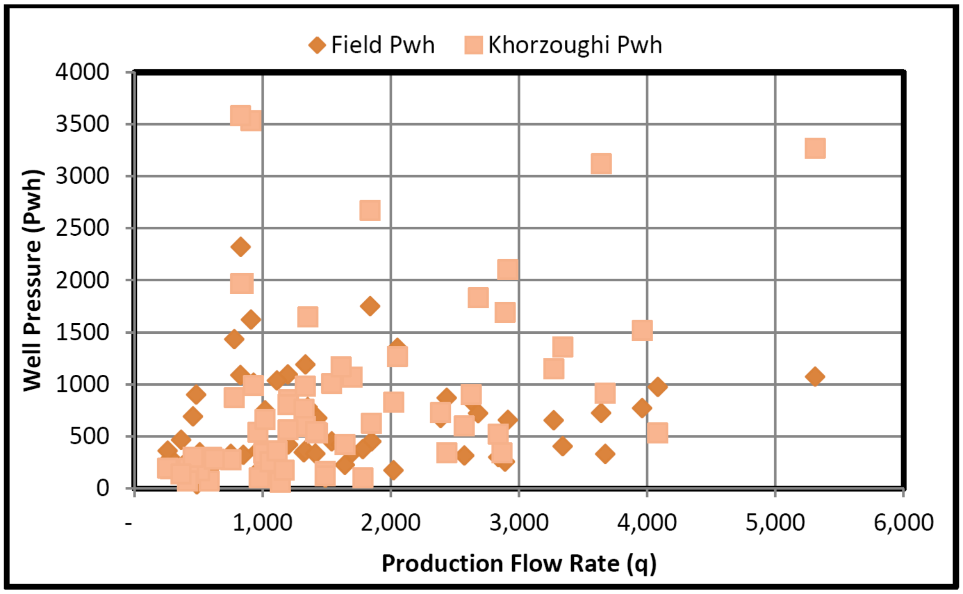

Figure 20—Comparison of Field test with Khorzoughi's prediction

Conclusion

The Niger Delta oil fields have been in existence for more than five decades. Empirical correlations used in predicting wellhead pressure–production rate relationship in the Niger Delta region are developed and controlled by the operating companies as proprieties for in-house estimation. Therefore, general empirical correlation that will establish the flowing wellhead pressure-production rate in the Niger Delta oil fields as a quick tool for field engineers in on-and-off shores of the region is a sine qua non. The developed correlations that can be used to predict Niger Delta wellhead pressure-production rate relationship based on field test data obtained from sixty four (64) oil wells in the Niger Delta region resulted in favourable prediction and high precision from its statistical analysis as compared to earlier correlations. Based on the results obtained, the following conclusions are drawn:

1. The developed correlations predict the field wellhead pressure with average error and absolute error of −0.1477 and 0.4430 for Gilbert equation and −0.2515 and 0.4737 for modified Gilbert equation.
2. The developed correlations: Gilbert and modified Gilbert formulae predictions are comparable with correlation coefficient of 0.9869.
3. The developed correlations standard deviation for Gilbert and modified Gilbert equations are 0.9582 and 1.0997 respectively.

References

1. Achong, I. (1961). Revised Bean Performance Formula for Lake Maracaibo Wells. Shell Internal Report, October 1961.
2. Baxendell, P. B. (1957). Bean Performance-Lake Wells. Shell Internal Report, October 1957.
3. Beiranvand, M. S., Mohammadmoradi, P., Aminshahidy, B., Fazelabdolabadi, B. and Aghahoseini, S. (2012). New Multiphase Choke Correlations for a High Flow Rate Iranian Oil Field. *Mechanical Science,* Vol. **3**. p. 43–47. doi: 10.5194/ms-3-43-2012.
4. Clegg, J. D. (2007). Petroleum Engineering Handbook – Production Operations Engineering. *SPE Textbook Series,* Volume **4,** Richardson, Texas.

5. Ghareeb, M. and Shedid, A. S. (2007).A New Correlation for Calculating Wellhead Production Considering Influences of Temperature, GOR and Water-cut for Artificially Lifted Wells. Prepared for presentation at the International Petroleum Technology Conference held in Dubai, U.A.E., December 4 - 6.
6. Ghassan, H. A. M. and Maha, R. A. A. (1991). Correlations Developed to Predict Two-Phase Flow through Wellhead Chokes. *Journal of Canadian Petroleum Technology,* Vol. **30,** No. 6, p. 1–10.
7. Gilbert, W. E. (1954). Flowing and Gas-Lift Well Performance. *API Drilling and Production Practice,* Vol. **20,** p. 126–157. Dallas, Texas.
8. Guo, B., Lyons, W. C. and Ghalambor, A. (2007).*Petroleum Production Engineering: A Computer-Assisted Approach*. Elsevier Science and Technology Books, p. 60.
9. Khorzoughi, M. B., Beiranvand, M. S. and Rasaei, M. R. (2013). Investigation of a New Multiphase Flow Choke Correlation by Linear and Non-linear Optimization Methods and Monte Carlo Sampling. *Journal of Petroleum Exploration, Production and Technology,* Vol. **3,** p. 279–285. doi: 10-1007/s 13202-103-0067-9.
10. Mesallati, A., Bizanti, M. and Mansouri, N. (2000). Multiphase-Flow Choke Correlations for Offshore Bouri Oil Field. International Gas Union 21st World Gas Conference, p. 49, Nice, France, June 6–9.
11. Pilehvari, A. A. (1981). Experimental Study of Critical Two-Phase Flow through Wellhead Chokes. University of Tulsa Fluid Flow Project Report, Tulsa, USA.
12. Ros, N. C. J. (1960). An Analysis of Critical Simultaneous Gas/Liquid Flow through a Restriction and Its Application to Flow Metering. *Applied Science Research,* Vol. **9,** p. 374–389.
13. Sadiq, D. J. (2012). Prediction of Oil Flow Rate through Choke at Critical Flow for Iraqi Oil Wells. Journal of Petroleum Research and Studies, Vol. **212,** No. 6, p. 53–79.