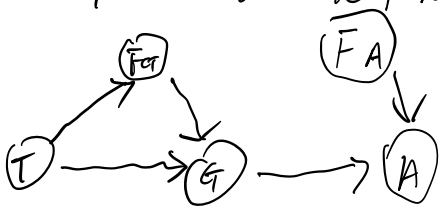
1 (a) A - alarm sounds

F_A - Alarm is faulty

F_G - Gauge is faulty

G - Gauge reading

T - Actual Temperature



(b) No, a poly tree is a directed acyclic graph for which there is almost one undirected path between any two nodes in the network. And this graph contains a cycle.

(c)

	T=normal		T=high	
	F_G	$\neg F_G$	F_G	$\neg F_G$
G=normal	1-y	1-x	y	x
G=high	y	x	1-y	1-x

(d) conditional probability table of A depends on F_A and G

G	F_A	P(A=sounds | G, F_A)	P(A=¬sounds | G, F_A)
High	T	0	1
High	F	1	0
	T	0	1
	F	0	1

2(a) $P(B|j,m) = \alpha P(B) \sum_e P(e) \sum_a P(a|b,e) P(j|a) P(m|a)$

$= \alpha P(B) \sum_e P(e) \left[0.9 \times 0.7 \times \begin{pmatrix} 0.95 & 0.29 \\ 0.94 & 0.001 \end{pmatrix} + 0.05 \times 0.01 \times \begin{pmatrix} 0.05 & 0.71 \\ 0.06 & 0.999 \end{pmatrix} \right)$

$= \alpha P(B) \sum_e P(e) \left[\begin{pmatrix} 0.5985 & 0.1827 \\ 0.5922 & 0.00063 \end{pmatrix} + \begin{pmatrix} 0.000025 & 0.000355 \\ 0.00003 & 0.0004995 \end{pmatrix} \right)$

$= \alpha P(B) \sum_e P(e) \left[\begin{pmatrix} 0.598525 & 0.183055 \\ 0.59223 & 0.0011295 \end{pmatrix} \right)$

$= \alpha P(B) \left[0.002 \times \begin{pmatrix} 0.598525 \\ 0.183055 \end{pmatrix} + 0.998 \times \begin{pmatrix} 0.59223 \\ 0.0011295 \end{pmatrix} \right)$

$= \alpha P(B) \left[\begin{pmatrix} 0.001197 \\ 0.000366 \end{pmatrix} + \begin{pmatrix} 0.591045 \\ 0.00127 \end{pmatrix} \right)$

$= \alpha P(B) \times \begin{pmatrix} 0.592242 \\ 0.001492 \end{pmatrix}$

$= \alpha \begin{pmatrix} 0.001 \\ 0.999 \end{pmatrix} \times \begin{pmatrix} 0.592242 \\ 0.001492 \end{pmatrix}$

$= \alpha \begin{pmatrix} 0.000592242 \\ 0.00149058 \end{pmatrix}$

$\approx \langle 0.284, 0.716 \rangle$

α is a normalization constant

α is 480 (approx)

(b) number of addition = 7

number of $\times$ = 16

of $\div$ = 2

total # = 23

The number of arithmetic operations performed by enumeration are 25. There will be 2 more multiplications done in enumeration algorithm

(c) $P(x_1|x_n = true) = \alpha \sum_{x_{n-1}} P(x_n = true|x_{n-1}) \sum_{x_{n-2}} P(x_{n-1}|x_{n-2}) \cdots \sum_{x_2} P(x_3|x_2)$

$= \alpha \sum_{x_{n-1}} P(x_n = true|x_{n-1}) \sum_{x_{n-2}} P(x_{n-1}|x_{n-2}) \cdots \sum_{x_2} P(x_4|x_3) f_{x_2}(x_3)$

There are n-2 summation on each

running time O(n)

(d) Inductive hypothesis:

Assume any polytree with n nodes

Evaluate the n nodes to the size of polytree

Consider polytree with n+1 nodes

Eliminate some of the leaf nodes, propotional to the size of its conditional probability Table.

Since, the network is a polytree, there will be only independent subproblems, one for each parent.

Each subproblem takes total work propotional to the sum of its CPT sizes

Hence the total work for n+1 nodes is propotional to the sum of CPT sizes.