(1) $P(tg) = 0.9$ tg : taxi gree

$P(tb) = 1 - P(tg) = 0.1$ tb : taxi blue

$P(yg \mid tg) = 0.75$ yg : you saw green

$P(yb \mid tg) = 1 - P(yg \mid tg) = 0.25$ yb : you saw blue

$P(yb \mid tb = 0.75$

$P(yg \mid tb) = 1 - P(yb \mid tb) = 0.25$

$P(tb \mid yb)$

$= P(tb \text{ AND } yb) / P(yb)$

$= P(tb \text{ AND } yb) / P(tb \text{ AND } yb) + P(tg \text{ AND } yb))$

$= P(yb \mid tb) P(tb) / P(yb \mid tb) P(tb) + P(yb \mid tg) P(tg))$

$= 0.75 \cdot 0.1 / (0.75 \cdot 0.01 + 0.25 \cdot 0.9) = 0.25$

$P(tg \mid yb) = 1 - P(tb \mid yb) = 0.75$

The taxi was most likely green.

2. (a) The model consists of the prior probability P(category) and the conditional probabilites P(world$_i$ | category), where Word$_i$ is true iff the document in question contains the ith word in dictionary. For each category c, P(category=c) is estimated as fraction of all documents that are of category c. Similarly, P(Word$_i$=true | category=c) is estimated as the fraction of category c that contains word i.

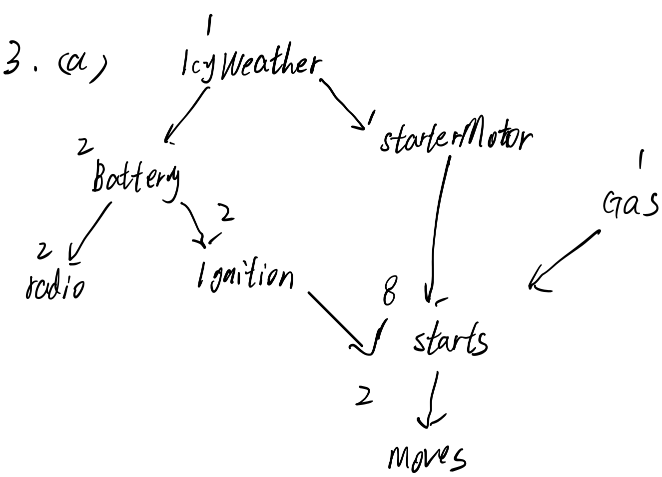
(b) $P(\text{category} \mid \text{word}_i \ldots \text{word}_n)$

$= \alpha P(\text{category}, \text{word}_1 \ldots \text{word}_n)$

$= \alpha P(\text{category}) \prod_i P(\text{word}_i \mid \text{category})$

(c) No, For example, a phrase like "computer science" occurs more frequenty than the probabilities "computer" x "science" in document category. So the true probability of any set of words occuring higher than the model suggests. So the relative category probabilieties of documents of different lengths tend to be very unreliable

3. (a)



(b) Suppose IcyWeather = 0.05

$P(\text{Battery} \mid \text{Icy}) = 0.95$, $P(\text{Battery} \mid \neg\text{Icy}) = 0.997$

$P(\text{StarterMotor} \mid \text{Icy}) = 0.98$, $P(\text{Battery} \mid \neg\text{Icy}) = 0.999$

$P(\text{Radio} \mid \text{Battery}) = 0.9999$, $P(\text{Radio} \mid \neg\text{Battery}) = 0.05$

$P(\text{Ignition} \mid \text{Battery}) = 0.998$. $P(\text{Ignition} \mid \neg\text{Battery}) = 0.01$

$P(\text{Gas}) = 0.995$

$P(\text{starts} \mid \text{Ignition}, \text{StarterMotor}, \text{Gas}) = 0.9999$, other entries 0.0

$P(\text{Moves} \mid \text{Starts}) = 0.998$

(c) $2^8 - 1 = 255$

(d) $1+2+2+2+2+1+8+2 = 20$