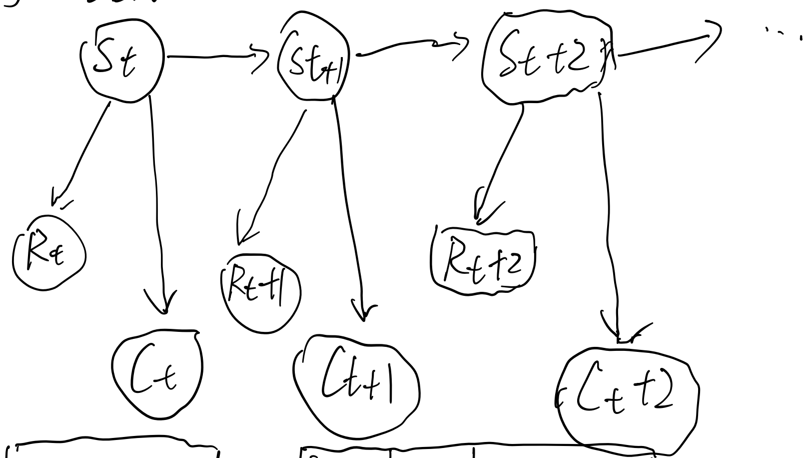
(1) DBN:



exam HW

s = e

c = s

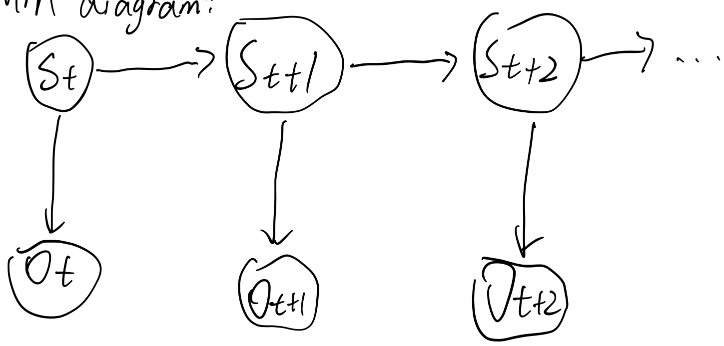
S_0	$P(S_0)$
s	0.7
$\neg s$	0.3

R_t	S_t	$P(R_t \mid S_t)$
r	s	0.2
$\neg r$	s	0.8
r	$\neg s$	0.7
$\neg r$	$\neg s$	0.3

S_{t+1}	S_t	$P(S_{t+1} \mid S_t)$
s_{t1}	s_t	0.8
$\neg s_{t1}$	s_t	0.2
s_{t1}	$\neg s_t$	0.3
$\neg s_{t1}$	$\neg s_t$	0.7

C_t	S_t	$P(C_t \mid S_t)$
c	s	0.1
$\neg c$	s	0.9
c	$\neg s$	0.3
$\neg c$	$\neg s$	0.7

HMM diagram:



S_0	$P(S_0)$
s	0.7
$\neg s$	0.3

S_{t+1}	S_t	$P(S_{t+1} \mid S_t)$
s_{t1}	s_t	0.8
$\neg s_{t1}$	s_t	0.2
s_{t1}	$\neg s_t$	0.3
$\neg s_{t1}$	$\neg s_t$	0.7

O_t	S_t	$P(O_t \mid S_t)$
r, c	s	0.02
$r, \neg c$	s	0.18
$\neg r, c$	s	0.08
$\neg r, \neg c$	s	0.12
r, c	$\neg s$	0.21
$r, \neg c$	$\neg s$	0.49
$\neg r, c$	$\neg s$	0.09
$\neg r, \neg c$	$\neg s$	0.21

2 (a) $P(E_{oughsleep,t} | e_{1:t})$ for each $t=1,2,3$

Apply Markov property:

$P(E_0) = \langle 0.7, 0.3 \rangle$

$P(E_1) = \sum_{E_0} P(E_1|E_0) P(E_0)$

$= (\langle 0.8, 0.2 \rangle 0.7 + \langle 0.3, 0.7 \rangle 0.3)$

$= \langle 0.65, 0.35 \rangle$

$P(E_1|e_1) = \alpha P(e_1|E_1) P(E_1)$

$= \alpha \langle 0.8 \times 0.9, 0.3 \times 0.7 \rangle \langle 0.65, 0.35 \rangle$

$= \alpha \langle 0.72, 0.21 \rangle \langle 0.65, 0.35 \rangle$

$= \langle 0.8643, 0.1357 \rangle$

$P(E_2|e_1) = \sum_{E_1} P(E_2|E_1) P(E_1|e_1)$

$= \langle 0.7321, 0.2679 \rangle$

$P(E_2|e_{1:2}) = \alpha P(e_2|E_2) P(E_2|e_1)$

$= \langle 0.501, 0.499 \rangle$

$P(E_3|e_{1:2}) = \sum_{E_3} P(E_3|E_2) P(E_2|e_{1:2})$

$= \langle 0.5505, 0.4495 \rangle$

$P(E_3|e_{1:3}) = \alpha P(e_3|E_3) P(E_3|e_{1:2})$

$= \langle 0.1045, 0.8955 \rangle$

(b) $P(e_3|E_3) = \langle 0.2 \times 0.1, 0.7 \times 0.3 \rangle$

$= \langle 0.02, 0.21 \rangle$

$P(e_3|E_2) = \sum_{E_3} P(e_3|E_3) P(E_3) P(E_3|E_2)$

$= \langle 0.02 \times 0.8 + 0.21 \times 0.2, 0.02 \times 0.3 + 0.21 \times 0.7 \rangle$

$= \langle 0.0588, 0.153 \rangle$

$P(e_{2:3}|E_1) = \sum_{E_2} P(e_2|E_2) P(e_3|E_2) P(E_2|E_1)$

$= \langle 0.0233, 0.0556 \rangle$

$$P(E_1|e_{1:3}) = \alpha P(E_1|e_1) P(e_{2:3}|E_1)$$

$$= \langle 0.7277, 0.2723 \rangle$$

$$P(E_2|e_{1:3}) = \alpha P(E_2|e_{1:2}) P(e_3|E_1)$$

$$= \langle 0.7757, 0.2243 \rangle$$

$$P(E_3|e_{1:3}) = \langle 0.1045, 0.8955 \rangle$$

(c) The probability at $t=1$ in state Estimation is computed as 0.8643

at $t=1$ in item Smoothening 0.7277

at $t=2$ in state Estimation 0.5010

at $t=2$ in item Smoothening 0.2757

3 We have 3 prior u_1, u_2, u_3 and we need to use DBN to get posterior. For each time we need three space. Repeat it n times need 3N