1. $P(A)=0.4$ $P(B)=0.3$ $P(A \vee B)=0.5$

The agent is rational if it chooses the action that has the heightest expected utility and has possible outcomes of the action.

All possibilities are from 0 to 1 for any proportion a that $0 \le P(a) \le 1$

$P(true)=1$

$P(false)=0$

	B	¬B
A	a	b
A	c	d

$P(A)=0.4=a+b$

$P(B)=0.3=a+c$ $P(A \cap B)=a$

then $P(A \cup B)=P(A)+P(B)-P(A\cap B)$

$= a+b+a+c-a$

$= a+b+c$

given $P(A \cup B)=0.5$

$\therefore$ $a+b+c=0.5$

$\because$ $a+b=0.4$, $a+c=0.3$

$b-c=0.1$

$b=0.1+c$

$a+0.1+c=0.4$ $b=0.4-a$

$a+c=0.3$

$c=0.3-a$

$\because$ $a+b+c=0.5$

$a+0.4-a+0.3-a=0.5$

$-a+0.7=0.5$

$\underline{a=0.2}$

$a+b=0.4 \rightarrow 0.2+b=0.4 \rightarrow \underline{b=0.2}$

$a+c=0.3 \rightarrow 0.2+c=0.3 \rightarrow \underline{c=0.1}$

$P(\neg A)=1-P(A)=1-0.4=0.6$

$0.6=c+d \rightarrow d=\underline{0.5}$

	B	$\neg B$
A	0.2	0.2
$\neg A$	0.1	0.5

$P(A\cap B)=P(A)+P(B)-P(A\cup B)$

$=0.2$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=0.7$

$a+b+c=0.7$

$a+b=0.4$

$a+c=0.3$

$\therefore b=0.4-a \quad c=0.3-a$

$a+b+c=0.7$

$a+0.4-a+0.3-a=0.7$

$\underline{a=0}$

$a+b=0.4 \rightarrow b=\underline{0.4}$

$a+c=0.3 \rightarrow c=0.3$

$d = 0.3$

	B	$\neg B$
A	0	0.4
$\neg A$	0.3	0.3

$P(A \cap B) = a = 0$ means A and B will never happen together

Agent 1		Agent 2		outcome of agent 1			
Proposition	belief	bet	stakes	$A \cap B$	$A \cap \neg B$	$\neg A \cap B$	$\neg A \cap \neg B$
A	0.4	A	4 to 6	-6	4	4	4
B	0.3	B	3 to 7	3	-7	-7	3
$A \cup B$	0.7	$\neg(A \cup B)$	3 to 7	5	5	5	-5
				2	2	2	2

if $P(A \cup B) = 0.7$

Agent 1		Agent 2		outcome of agent 1			
Proposition	belief	bet	stakes	$A \cap B$	$A \cap \neg B$	$\neg A \cap B$	$\neg A \cap \neg B$
A	0.4	A	4 to 6	-6	-6	4	4
B	0.3	B	3 to 7	-7	3	-7	3
$A \cup B$	0.7	$\neg(A \cup B)$	3 to 7	3	3	3	-7
					0	0	0

2. (a) $P(toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2$

(b) $P(cavity) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2$

(c) $P(toothache \mid cavity) = \frac{P(t \cap c)}{P(c)} = \frac{0.108}{0.2} = 0.54$

(d) $P(c \mid t \vee c) = \frac{P(c \cap (t \vee c))P(c)}{P(t \vee c)} = \frac{0.108}{\ } = 0.871$

$P(\cdots)$ $= 0.108 + 0.016$

3. $P(\text{disease}) = 0.0001$

$P(\neg \text{disease}) = 0.9999$

$P(\text{positive} | \text{disease}) = 0.99$

$P(\text{negative} | \text{disease}) = 1 - 0.99 = 0.01$

$P(\text{negative} | \neg \text{disease}) = 0.99$

$P(\text{positive} | \neg \text{disease}) = 0.1$

$P(A|B) = P(A \cap B) / P(B)$

$= 0.0001 \times 0.99 / (0.0001 \times 0.99 + 0.9999 \times 0.01$

$= 0.0098$

4 ① Payback = coins return × P(coin return)

$19 \times \frac{1}{64} + 14 \times \frac{1}{64} + 4 \times \frac{1}{64} + 2 \times \frac{1}{64} + 1 \times \frac{1}{16} + 0 \times \frac{1}{4}$

$= \frac{19 + 14 + 4 + 2 + 4}{64} = \frac{43}{64}$

② $\frac{1}{64} + \frac{1}{64} + \frac{1}{64} + \frac{1}{16} + \frac{1}{4} = \frac{24}{64}$

16
4
2.

4 16

20