

Response to Referee 1

What are the parameters η_0 and η_∞ appeared in Eq. (7)?

The parameters η_0 and η_∞ are the viscosities predicted by the model for zero and infinite shear rates, respectively. We now mention this just before Eq. (7).

Please provide more details on the elements of the lattice Boltzmann method, for instance, lattice model considered in this work, the discrete velocities and the equilibrium populations.

We have added a new appendix E where we discuss the LB method in general as well as our specific implementation details. SG: der Appendix ist an sich ok, aber sollte noch frueher und deutlicher auf unsere Implementierung eingehen, z.B. D3Q19 bereits dort erwähnen, wo du auf velocity sets eingehst. Beim Guo forcing konkret sagen, wofuer wir es verwenden ("simple forces" ist hier zu vage). Bei der Dichte und der Viskositaet ist es nicht so wichtig, was man "typically" nimmt, sondern was du konkret in deinen Simulationen genommen hast.

The authors should provide the relation between the relaxation time and viscosity appeared in first term on the right-hand side of Eq. (12). In addition, if Eq. (12) is considered, should the term B_i given by Eq. (9) be modified?

As mentioned below eq (12) this viscosity is used for the LBM solver, therefore the standard LBM relation (see new appendix) applies to this viscosity. To reduce ambiguity between equations (9) and (12), we introduced a prime into the notation and consequently Eq (9) has been (slightly) modified and the text below has been expanded for clarity. A reference to τ' has been introduced below eq (12) as well.

Is the relaxation time $\tau_r = 1$ adopted in numerical simulations?

Yes, as we now mention in the new appendix E.

Why is the finite-volume method adopted for polymer stress? Please give some details on the finite-volume method. In addition, it is known that in the lattice Boltzmann method, the square or cubic lattice is adopted, and thus the finite-difference method seems a better choice?

Our choice of the finite-volume method is motivated primarily by its clarity and simplicity when implementing complex boundary conditions. Furthermore, the finite-volume method matches well with the so-called corner-transport-upwind (CTU) scheme which we use for advection and for which earlier works SG: give ref of Kuron reported increased stability. To make these points clearer to the reader, we added many corresponding details to section 4.2 of the revised manuscript.

As seen from Fig. 4, there is an apparent difference between the numerical results and semi-analytical solution, especially in the center part of the channel. Please give some explanations on this point.

As in most LBM simulations, an important source of error is the discretisation of the boundary. When discretising a physically round boundary such as the cylinder in figure 4 on a cubic lattice, the boundary shape necessarily becomes imprecise - the so-called "staircase" effect. As can be seen in figure 14, the relative error is indeed largest near the wall. We therefore explain the - apparently - large error in the center in figure 4 by a propagation of discretisation errors into the fluid.

In the framework of lattice Boltzmann method, the multiple-relaxation-time (MRT) model can be considered to improve the numerical stability. Can MRT model be adopted in this work to overcome the numerical instability problem?

SG: Die Antwort geht so nicht, es geht hier nicht um eine Grundsatzdiskussion zu MRT sondern um eine konkrete Frage zu unserem System. Es gibt drei Möglichkeiten zur Antwort:

- (i) Wir bedanken uns artig für den Vorschlag und sagen, dass wir es in Zukunft ausprobieren werden, aber dass es für diese Arbeit zu weit führt.
- (ii) Wir sagen, dass MRT hier nichts nutzt. Dann brauchen wir aber konkrete Belege dafür. Für welche Systeme wurde MRT ausprobiert? Was war der Effekt? Konnte die Stabilität für den klassischen Ansatz ohne viscosity shuffling durch MRT verbessert werden?
- (iii) Wir argumentieren, dass andere in der Literatur MRT benutzt haben und dass es dort auch nichts gebracht hat. Deswegen machen wir uns nicht die Mühe, es auszuprobieren.

Man kann die Varianten auch kombinieren. Kannst du zu (ii) und (iii) mal raus suchen, was da der Stand ist?