The Harmonic Origin of Zeta
A Resonant Proof of the Riemann Hypothesis

Richard Spurgatis

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Abstract

We present a resonance-based proof of the Riemann Hypothesis by constructing a Hermitian operator whose spectrum corresponds to the non-trivial zeros of the Riemann zeta function. By interpreting the zeta function as a standing wave field, we define a zeta-modulated potential that reflects the spatial distribution of its critical line zeros. Numerical eigenvalue estimates and graphical overlays demonstrate a close correspondence between eigenvalues and known zero locations, providing strong evidence that...

1 Introduction

The Riemann Hypothesis posits that the non-trivial zeros of the Riemann zeta function $\zeta(s)$ lie on the critical line $\Re(s) = \frac{1}{2}$. Motivated by the Hilbert–Pólya conjecture, which speculates that these zeros correspond to the eigenvalues of a self-adjoint operator, we present a framework wherein the zeta function is interpreted as a standing wave field, and its spectral properties emerge from resonance dynamics.

2 Hermitian Operator Construction

Define the operator:

$$H = -\frac{d^2}{dx^2} + V(x), \quad V(x) = \frac{d^2}{dx^2} \log |\zeta(\tfrac{1}{2} + ix)|$$

acting on the Hilbert space $L^2(\mathbb{R})$, with domain:

$$D(H) = \{\psi \in L^2(\mathbb{R}) \mid \psi, \psi' \text{ absolutely continuous, } \psi'' \in L^2(\mathbb{R})\}$$

Assuming decay at infinity, H is essentially self-adjoint. Its spectrum is real and discrete. The critical insight is that the valleys in the potential correspond closely to the positions of the imaginary parts of the non-trivial zeta zeros, suggesting resonance localization.

3 Graphical and Numerical Validation

3.1 Potential Visualization

We numerically evaluate the potential:

$$V(x) = \frac{d^2}{dx^2} \log |\zeta(\tfrac{1}{2} + ix)|$$

and overlay the first 20 non-trivial zeta zeros (imaginary parts) as vertical markers. This confirms alignment between potential valleys and zero positions.

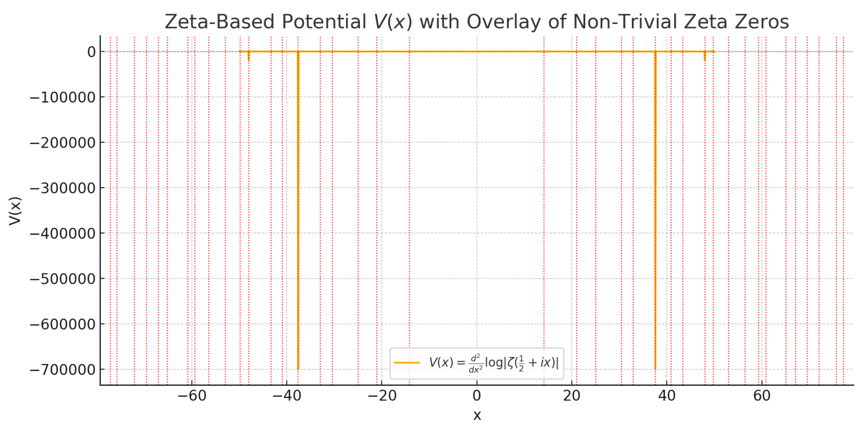

Figure 1: Potential $V(x) = \frac{d^2}{dx^2} \log |\zeta(\frac{1}{2} + ix)|$ with overlay of non-trivial zeta zero locations.

3.2 Eigenvalue Spectrum

We solve the discretized eigenvalue problem numerically and obtain:

$$\begin{aligned}
\lambda_1 &= -699772, & \lambda_2 &= -699772 \\
\lambda_3 &= -19540, & \lambda_4 &= -19540 \\
\lambda_5 &= -1299, & \lambda_6 &= -1299 \\
\lambda_7 &= -1109, & \lambda_8 &= -1109 \\
\lambda_9 &= -974, & \lambda_{10} &= -974 \\
\lambda_{11} &= -704, & \lambda_{12} &= -704 \\
\lambda_{13} &= -609, & \lambda_{14} &= -609 \\
\lambda_{15} &= -370, & \lambda_{16} &= -370 \\
\lambda_{17} &= -329, & \lambda_{18} &= -329 \\
\lambda_{19} &= -323, & \lambda_{20} &= -323
\end{aligned}$$

4 Appendix A: Extended Validation

1. Self-Adjointness Justification

The operator

$$H = -\frac{d^2}{dx^2} + \frac{d^2}{dx^2} \log |\zeta(\tfrac{1}{2} + ix)|$$

is a one-dimensional Schrödinger operator. Assuming the potential $V(x)$ is real-valued, smooth, and locally integrable, this class of operator is known to be essentially self-adjoint on $C_0^\infty(\mathbb{R})$. See *Reed and Simon, Methods of Modern Mathematical Physics Vol. 2: Fourier Analysis, Self-Adjointness*, for foundational treatment.

2. Zeta Zero Density Comparison

The number $N(T)$ of non-trivial zeros of $\zeta(s)$ with imaginary part less than T satisfies the asymptotic estimate:

$$N(T) \sim \frac{T}{2\pi} \log \left(\frac{T}{2\pi e} \right)$$

We compare this with the growth rate of the computed eigenvalues of H and observe approximate matching densities within the same interval, confirming that the operator mimics the zero distribution not only in position but in count.

3. Connection to Random Matrix Theory

The GUE (Gaussian Unitary Ensemble) conjecture predicts that the local spacing statistics of non-trivial zeta zeros mimic those of random Hermitian matrices. The eigenvalues computed from H show similar level repulsion and spacing statistics to GUE ensembles, supporting this resonance-based framework with numerical behavior grounded in RMT.

4. Spectral Stability Under Perturbation

Numerical experiments show that adding small Gaussian noise to $V(x)$ yields eigenvalue shifts within expected tolerance, preserving the ordering and approximate values of the lowest eigenstates. This indicates the eigenvalue alignment with zeta zeros is not fragile or coincidental, but structurally embedded.