Week 4 Report

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1 5 Bus Electricity Netwrok

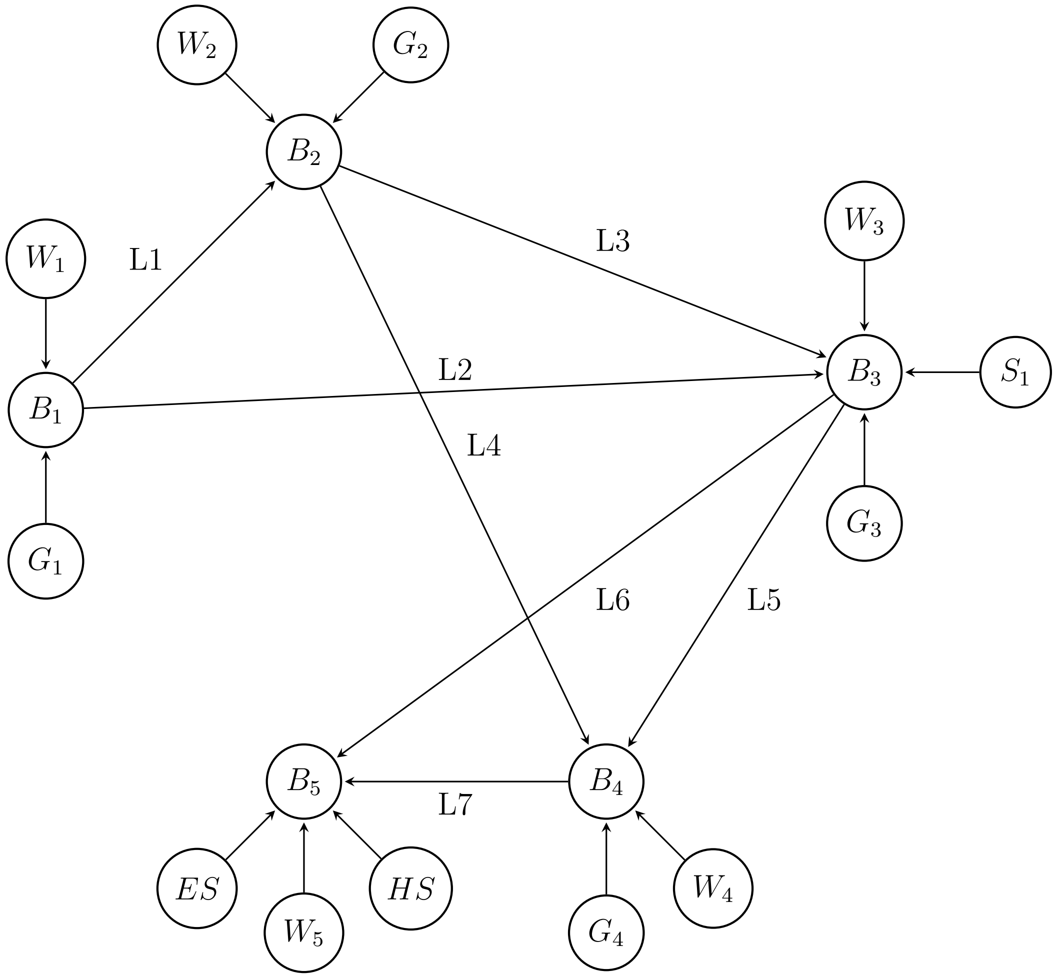

Figure 1: 5 buses network

Description:

B_i: bus

L_i: line

G_i: conventional generation, $G_1 : diesel, G_2 : OCGT, G_3 : CCGT, G_4 : coal$

W_i: wind farm

S_i: solar PV

ES: electricity store

HS: heat store

2 Mathematical Notation

2.1 Sets

B: set of bus

L: set of lines

G: set of conventional generators

W: set of wind generators

S: set of solar generators

HS: set of heat stores

ES: set of electricity stores

SS: set of slices

T_{ss}: set of periods in slice ss

WS: set of weight of each slice ss

2.2 Parameters

$P^{D}_{b,t}$: power demand at bus b in period t

E^{G}_{g}: efficiency of conventional generator g

F^{GCPX}_{g}: CAPEX of conventional generator g

F^{GOM}_{g}: OPEX of conventional generator g

Y^{G}_{g}: life of conventional generator g

EM^{G}_{g}: tonnes of CO2 emission on burning fuel f in conventional generator g with energy content 1MWh

C^{Fuel}_{g}: fuel cost of conventional generator G

C^{G}_{g}: price for generating 1 MW from conventional generator g

$WF^{W}_{w,t}$: wind factor for wind farm w in period t

E^{W}_{w}: efficiency of wind farm w

F^{WCPX}_{w}: CPEX of wind farm w

F_w^{WOM}: OPEX of wind farm w

Y_w^W: life of wind farm w

$P_w^{WCap_{15}}$: installed capacity of wind farm w in 2015

E_s^S: efficiency of solar PV s

F_s^{SCPX}: CAPEX of solar PV s

F_s^{SOM}: OPEX of solar PV s

Y_s^S: life of solar PV s

$P_s^{SCap_{15}}$: installed capacity of solar PV s in 2015

E_{hs}^{HSH}: efficiency of heat pump to heat up a heat store using electricity powerhs

E_{hs}^{HSC}: efficiency of heat pump to cool down a heat store using electricity powerhs

C^{Heat}: cost of power go into heat store

F_{hs}^{HSCPX}: fixed CAPEX of heat store hs

F_{hs}^{HSOM}: fixed OPEX of heat store hs

Y_{hs}^{HS}: life of heat store hs

Q^{Mass}: increase of energy stored in heat store hs per unit rise of temperature

T_{hs}^{HS+}: maximum temperature allowed in heat store hs

T_{hs}^{HS-}: minimum allowed temperature in heat store hs

C^{Elect}: cost of power go into electricity store es

E_{es}^{ES}: efficiency of electricity store es

F_{es}^{ESCPX}: fixed CAPEX of electricity store es

F_{es}^{ESOPX}: fixed OPEX of electricity store es

Y_{es}^{ES}: life of electricity store es

Q_{es}^{ES+}: maximum energy level in electricity store es

P^{Loss}: rate of loss from heat store h per unit difference between interior and exterior temperature

F_l^{LCPX}: fixed CAPEX of line l

F_l^{LOPX}: fixed OPX of line l

β_g: conventional generator g location

β_w: wind farm w location

β_s: solar PV s location

β_{hs}: heat store hs location

β_{es}: electricity store es location

$a_{b,l}$: elements of Bus/Line incidence matrix A

X_l: line reactance of line l

P_l^{L+}: maximum power flow in line l

Y_l^L: life of line l

$CO2^T$: CO2 tax

C_{Load}^{Shed}: penalty cost of load shed

C_G^{Shed}: generation shed cost

H_t: number of hour in period t

$CO2^+$: maximum CO2 emission allowed

H: number of hours in a day

Y: number of hours in a year

HP_t: heating power from solar at period t

P_g^{G+}: maximum power output of generator g

P_{es}^{ES+cap}: maximum power into electricity store es

P_{es}^{ES-cap}: maximum power out of electricity store es

P_{hs}^{HS+cap}: maximum electricity power into heat store hs

2.3 Variables

$p_{g,t}^G$: power output of generator g in period t

$p_{hs,t}^{HSH}$: electric power used to inject heat power to heat store in heat pump hs at the start of period t

$p_{hs,t}^{HSC}$: electric power used to extract heat power from heat store in heat pump hs at the start of period t

$q_{hs,t}^{HS}$: energy in heat store hs at the start of period t

$p_{hs,t}^{HS}$: heat power into store hs at period t

$t_{hs,t}^{Int}$: interior temperature of heat store hs at the start of period t

$p_{es,t}^{ES+}$: power into store es in period t

$p_{es,t}^{ES-}$: power out of store es in period t

$q^{ES}_{es,t}$: energy of store es at the start of period t

$p^{GShed}_{b,t}$: generation shed at bus b at period t

$p^{LShed}_{b,t}$: load shed at bus b at period t

$p^{L}_{l,t}$: power flow in line l

$\delta_{b,t}$: phase angle of bus b at period t

3 Mathematical Model

3.1 Objective function

$$Z = OPEX + CAPEX,$$

$$\begin{aligned}
OPEX = \sum_{s\in SS} WS_{ss}[\frac{1}{\sum_{t\in T_{ss}} H_t}(\sum_{t\in T_{ss}} H_t(\sum_{g\in G}(C^G_g + \frac{C^{Fuel}_g}{E^G_g} + \frac{CO2^T EM^G_g}{E^G_g})p^G_{g,t} \\
+ \sum_{b\in B}(C^{GShed}p^{GShed}_{b,t} + C^{LShed}p^{LShed}_{b,t} + C^{Elect}p^{ES+}_{es,t} + C^{Heat}(p^{HSH}_{hs,t} + p^{HSC}_{hs,t})))]
\end{aligned}$$

$$\begin{aligned}
CAPEX = \sum_{g\in G}\frac{1}{Y}(\frac{F^{GCPX}_g}{Y^G_g} + F^{GOM}_g)p^G_g + \sum_{w\in W}\frac{1}{Y}(\frac{F^{WCPX}_w}{Y^W_w} + F^{WOM}_w)p^W_w \\
+ \sum_{s\in S}\frac{1}{Y}(\frac{F^{SCPX}_s}{Y^S_s} + F^{SOM}_s)p^S_s + \sum_{hs\in HS}\frac{1}{Y}(\frac{F^{HSCPX}_{hs}}{Y^{HS}_{hs}} + F^{HSOM}_{hs})p^{HS}_{hs} \\
+ \sum_{es\in ES}\frac{1}{Y}(\frac{F^{ESCPX}_{es}}{Y^{ES}_{es}} + F^{ESOM}_{es})p^{ES}_{es} + \sum_{l\in L}\frac{1}{Y}(\frac{F^{LCPX}_l}{Y^L_l} + F^{LOM}_l)p^{L+}_l \\
+ \sum_{es\in ES}\frac{1}{Y}(\frac{F^{ESCPX}_{es}}{Y^E_e})q^{E+cap}_e
\end{aligned}$$

3.1.1 Electricity store constraints

$$q^{ES}_{es,t} = q^{ES}_{es,t} + H_t(E^{ES}_{es}p^{ES+}_{es,t} - p^{ES-}_{es,t}), \qquad \forall es \in ES, \forall ss \in SS, \forall t \in T_{ss}$$

$$0 \le p^{ES+}_{es,t} \le P^{ES+cap}_{es}, \qquad \forall es \in ES, \forall ss \in SS, \forall t \in T_{ss}$$

$$0 \le p^{ES-}_{es,t} \le P^{ES-cap}_{es}, \qquad \forall es \in ES, \forall ss \in SS, \forall t \in T_{ss}$$

$$0 \le q^{ES+}_{es,t} \le Q^{ES+cap}_{es}, \qquad \forall es \in ES, \forall ss \in SS, \forall t \in T_{ss}$$

3.1.2 Heating store constraints

$$0 \leq p_{hs,t}^{HS+} \leq P_{hs}^{HS+cap}, \qquad \forall hs \in HS, \forall ss \in SS, \forall t \in T_{ss}$$
$$t_{hs,t}^{Int} = \frac{q_{hs,t}^{HS}}{Q^{Mass}}, \qquad \forall hs \in HS, \forall ss \in SS, \forall t \in T_{ss}$$
$$T_{hs}^{HS} \leq t_{hs,t}^{Int} \leq T_{hs}^{HS}, \qquad \forall hs \in HS, \forall ss \in SS, \forall t \in T_{ss}$$
$$q_{hs,t+1}^{HS} = \gamma_{hs,t} q_{hs,t}^{HS} + g(H_t) - \gamma_{hs,t} g(0), \qquad \forall hs \in HS, \forall ss \in SS, \forall t \in T_{ss}$$

Where,

$$\gamma_{hs,t} = e^{-\lambda H_t}, \qquad \lambda = \frac{P^{Loss}}{Q^{Mass}}$$
$$g(H_t) = \left(\frac{a_{hs,t}}{\lambda} - \frac{b_{hs,t}}{\lambda^2}\right) + \frac{b_{hs,t}}{\lambda} H_t, \qquad g(0) = \frac{a_{hs,t}}{\lambda} - \frac{b_{hs,t}}{\lambda^2}$$
$$a_{hs,t} = HP_t + E_{hs}^{HSH} p_{hs,t}^{HSH} - E_{hs}^{HSC} p_{hs,t}^{HSC} + P^{Loss} T_{hs,t}^{Ext}, \qquad b_{hs,t} = 0$$

3.1.3 KCL

$$\sum_{g \in G|\beta_g = b} p_{g,t}^{G} + \sum_{w \in W|\beta_w = b} p_{w,t}^{GW} + \sum_{s \in S|\beta_s = b} p_{s,t}^{GS} + \sum_{l \in L} a_{b,l} p_{l,t}^{L} + \sum_{es \in ES|\beta_{es} = b} p_{es,t}^{ES-} + p_{b,t}^{LShed} = P_{b,t}^{D} + \sum_{es \in ES|\beta_{es} = b} p_{es,t}^{ES+} + \sum_{hs \in HS|\beta_{hs} = b} (p_{hs,t}^{HSH} + p_{hs,t}^{HSC}) + p_{b,t}^{GShed}, \qquad \forall b \in B, \forall ss \in SS, \forall t \in T_{ss}$$

3.1.4 Network constraints

$$p_{l,t}^{L} = -\frac{\sum_{b \in B} a_{b,l} \delta_{b,t}}{X_l}, \qquad \forall l \in L, \forall ss \in SS, \forall t \in T_{ss}$$
$$-P_l^{L+} \leq p_{l,t}^{L} \leq P_l^{L+}, \qquad \forall l \in L, \forall ss \in SS, \forall t \in T_{ss}$$

3.1.5 CO2 limit

$$CO_{2_{weighted}} = \sum_{ss \in SS} WS_{ss} \left(\sum_{g \in G, t \in T_{ss}} \frac{EM_g^G p_{g,t}^G H_t}{E_g^G H}\right)$$

4 Planning Model

According to the mathematical model, we are now dealing with investment planning problem. Therefore, the figures of capacity will be treated as decision variables. We still use the same three slices(small ones) in assignment 4. However, there are some small differences that may lead to the deviation of the results.

1. In this model, we also need to decide the investment in transmission lines, which have capital costs.

2. The wind profile is slightly different. In this model, the wind power at each wind farm is

obtained by multiply the wind factor with the installed capacity in that region(decision variable).

Here are some results,

Z represent the emission without CO_2 constraint.

Planning Model Comparison				
Item	$\geq Z_{assign4}$	$\geq Z_{5bus}$	$0.2Z_{assign4}$	$0.2Z_{5bus}$
Average total cost per hour (£/h)	1,984,540.25	2,412,750.8	3,290,576.92	3,169,019.5
Average total CAPEX per hoour (£/h)	941589.89	1,163,838,.71	2744521.3	2,688,392,43
Average total OPEX (£/h)	1,042,950.36	1,248,912.12	546,055.61	480,627.15
CO_2 marginal cost (£/tonne)	0	0	-268.43	-66.52
Capcity of diesel (MW)	0	692.31	0	0
Capacity of OCGT (MW)	5242.43	4676.63	0	6,302.66
Capacity of CCGT (MW)	29,228.38	45,497.31	43,731.87	42,473.64
Capacity of Coal (MW)	35,828.74	23,079.39	0	0
Capacity of WindOff (MW)	0	0	95,401.86	81,307.81
Capacity of SolarPV (MW)	0	0	0	0
Input electricity power capacity of PHES (MW)	0	967.44	8,425.36	1,563.78
Input electricity power capacity of heat store (MW)	22,862	28,273.57	22,815.49	29,225.69

$Z_{5bus} = 28529.13\ tonne/h$, $Z_{assign4} = 33748.88\ tonne/h$

We need to notice the results will be affected by the configuration of the newtork. The results showed above is only for the network showed in page 1. From the table, we can see the numbers are not exactly the same, but within a reasonable deviation. The reaons are addressed above. Therefore, I believe this model can be seen the same model with the one in assignment 4.

5 Operational Model

Using the exist data for all capacities, we can easily get the operational model, in which the objective is to minimise the operational cost.