Optimal Feedback Control of Gaze Shift

BME 580.491 Learning Theory

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Introduction

In biological control, a saccade is a biological phenomenon in which the eye makes a quick, simultaneous movement towards a target, fixating the object over an object of interest. Saccadic eye movements are so brief that sensory feedback appears to play no role in control, yet remarkably, saccades are able to make mid-flight corrections when perturbed by blinking, transcranial magnetic stimulation, target-jump conditions in the Superior colliculus, and gaze shifts. As a result, saccades are thought to be modeled by an internal feedback system, in which motor commands are made to maximize rewarding states and minimize expensive states towards the end goal of being fixated at the center of the target. Specifically, during a gaze shift, the eye first makes a saccade towards an object, with the head following afterwards. As the fovea moves over the target, the head rotates to recenter the eyes such that the end goal is reached with the least cumulative cost of movement. In this project, I used optimal feedback control theory to model gaze shift, and simulated this motion with different perturbations and disturbances.

Methods

First, I used a Kalman Filter to model the state uncertainty of our system as our eye and head makes movements. The implication of Kalman gains is that as when system is pushing a large mass, it will produce relatively large motor commands, and thus, we would have a larger uncertainty regarding the consequences of these commands. These Kalman gains are calculated forwards in time from feedback control gains, which are first initialized to be 0.

$$K^{(k)} = S_e^{(k|k-1)} H^T (H S_e^{(k|k-1)} H^T + Q_y)^{-1}$$

$$S_e^{(k+1|k)} = A(I - K^{(k)} H) S_e^{(k|k-1)} A^T + Q_x + \sum_i B C_i G^{(k)} S_e^{(k|k-1)} G^{(k)} C_i^T B^T$$

$$S_x^{(k+1|k)} = (A_B G^{(k)}) S_e^{(k|k-1)} (A - BG^{(k)})^T + (A - BG^{(k)}) S_x^{(k|k-1)} (AK^{(k)} H)^T$$

$$+ AK^{(k)} H S_{xe}^{(k|k-1)} (A - BG^{(k)})^T + AK^{(k)} H S_e^{(k|k-1)} A^T$$

$$S_{xe}^{(k+1|k)} = (A - BG^{(k)}) S_{xe}^{(k|k-1)} (I - K^{(k)} H)^T A^T$$

Though the feedback control gains are first initialized to be 0, after the Kalman gains are obtained in the first pass, the feedback control gains are re calculated backwards in time. After ten iterations, the Kalman and feedback control gains converge, which becomes our optimal control policy. There iz zero signal-dependent sensory noise.

$$G^{(k-1)} = (L + C_x^{(k)} + C_e^{(k)} + B^T W_x^{(k)} B)^{-1} B^T W_x^{(k)} A$$

$$W_e^{(k)} = (A - AK^{(k)}H)^T W_e^{(k+1)} (A - AK^{(k)}H) + G^{(k)T} B^T W_x^{(k+1)} A$$

$$W_x^{(k)} = T^{(k)} + A^T W_x^{(k+1)} A - G^{(k)T} B^T W_x^{(k+1)} A$$

$$C_x^{(k)} = \sum_i C_i^T B^T W_x^{(k+1)} B C_i$$

$$C_e^{(k)} = \sum_i C_i^T B^T W_e^{(k+1)} B C_i$$

Our policy, which is a time-dependent feedback gain, transforms our estimate of state into optimal motor commands. Our model penalizes not only the difference between the goal and the fovea, but also the eccentricity of the eye.

$$u^{(k)} = -G^{(k)} \hat{x}^{(k)}$$

$$\hat{x}^{(k)} = A\hat{x}^{(k-1)} + AK^{(k-1)}(y^{(k-1)} - H\hat{x}^{(k-1)}) + Bu^{(k-1)}$$

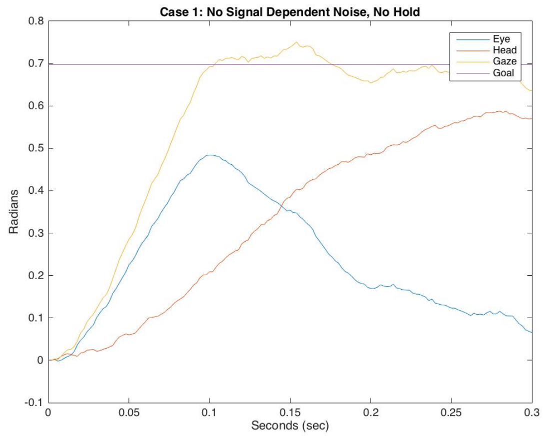
$$x^{(k+1)} = Ax^{(k)} + Bu^{(k)} + \epsilon_x^{(k)} + B\sum_i C_i u^{(k)} \phi_i^{(k)}$$

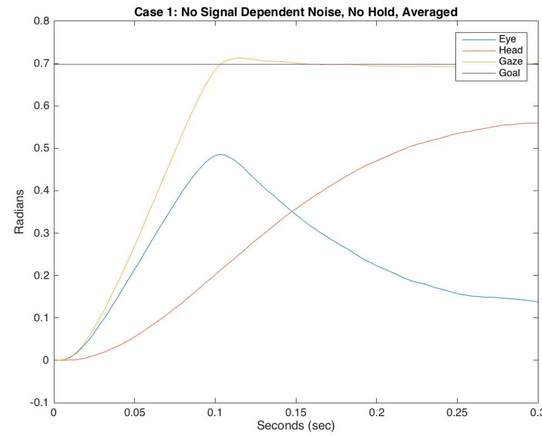
$$y^{(k+1)} = Hx^{(k)} + \epsilon_y^{(k)}$$

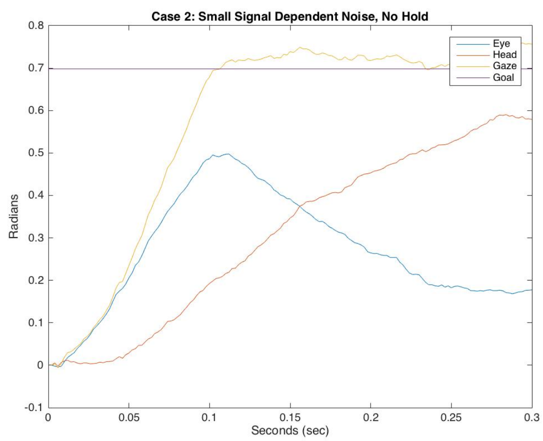
Results

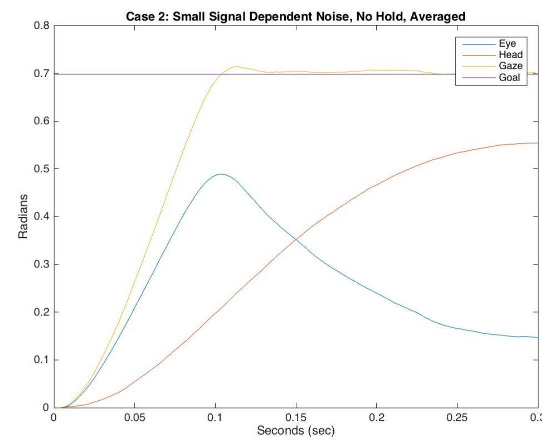
$$A = \begin{bmatrix} 0.9993 & 0.0018 & 0.0006 & 0 & 0 & 0 & 0 \\ -0.6337 & 0.8492 & 0.4953 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6065 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0.0020 & 0.0000 & 0 \\ 0 & 0 & 0 & -0.0489 & 0.9794 & 0.0443 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8187 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

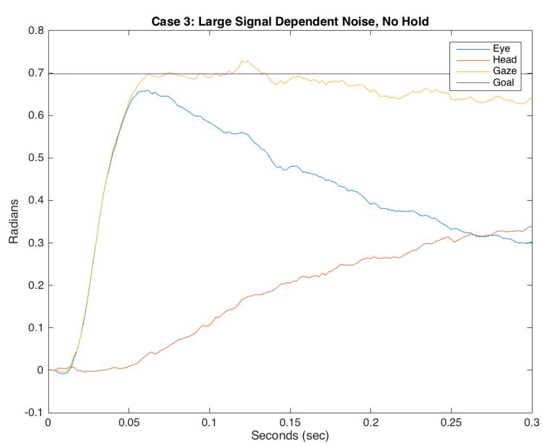
$$B = \begin{bmatrix} 9.73805813059788e-05 & 0 \\ 0.138390508200914 & 0 \\ 0.393469340287367 & 0 \\ 0 & 3.11758580250530e-06 \\ 0 & 0.00459250787534661 \\ 0 & 0.181269246922018 \\ 0 & 0 \end{bmatrix}$$

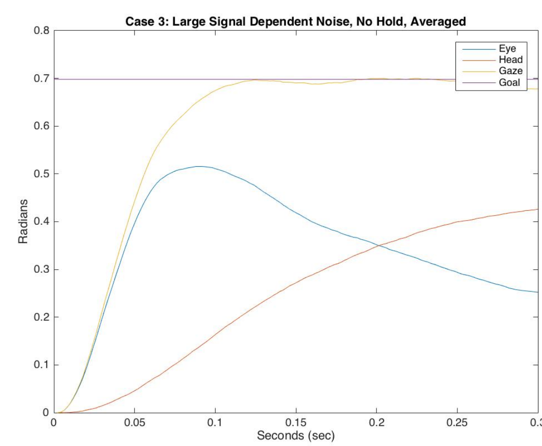

Case 1: No Signal Dependent Noise, No Hold
Radians
Seconds (sec)
Eye
Head
Gaze
Goal

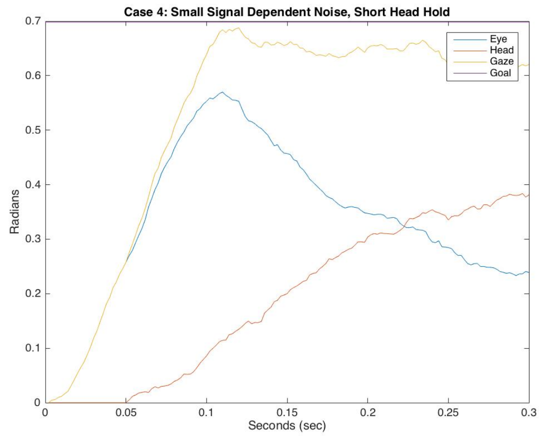

Case 1: No Signal Dependent Noise, No Hold, Averaged
Radians
Seconds (sec)
Eye
Head
Gaze
Goal

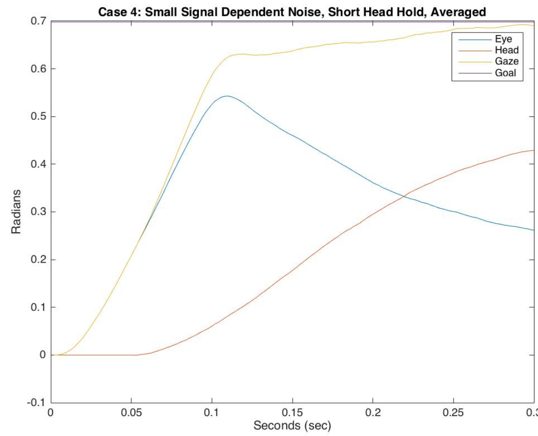

Case 2: Small Signal Dependent Noise, No Hold
Radians
Seconds (sec)
Eye
Head
Gaze
Goal

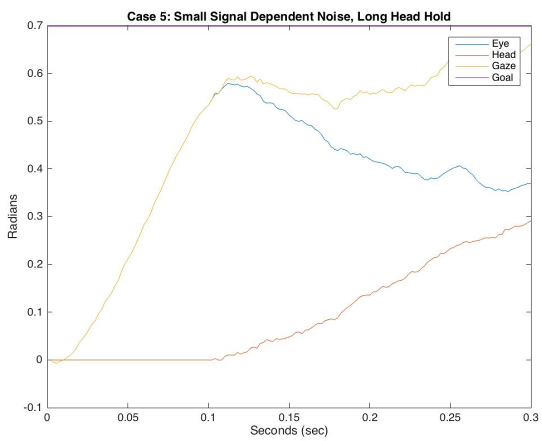

Case 2: Small Signal Dependent Noise, No Hold, Averaged
Radians
Seconds (sec)
Eye
Head
Gaze
Goal

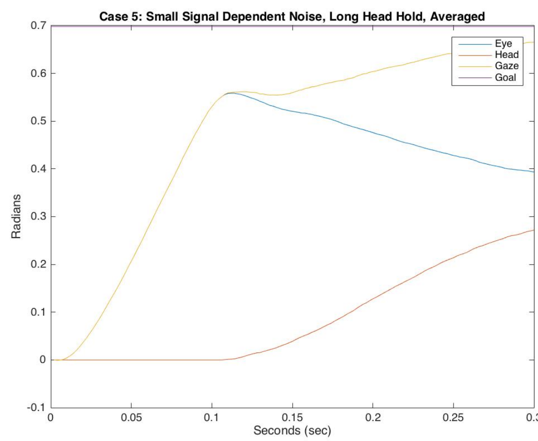

Case 3: Large Signal Dependent Noise, No Hold
Radians
Seconds (sec)
Eye
Head
Gaze
Goal


Case 3: Large Signal Dependent Noise, No Hold, Averaged
Radians
Seconds (sec)
Eye
Head
Gaze
Goal









Discussion

Case 1: No Signal Dependent Noise, No Hold: In the case no signal dependent noise, we see that the eye and head both move towards the goal, but the eye turns faster such that at the k_1 time step (100 ms), the fovea is centered and fixated over the target at around a $25.7831°(0.45$ radians) eccentricity. The gaze shift begins with motion of the eye because the dynamics of the eye present a lighter system to move, and therefore it costs less in terms of effort, and thus, smaller motor commands. Upon saccade completion, while keeping the fovea at the center (gaze is now at $40°$), we see that the eye rolls back as the head continues to move towards the goal. This was a result of applying state cost matrix T to penalize for the eye state for not being at the center of the head. We would expect that if we extended the simulation, the eye would be at an eccentricity of $0\ °$, while the head would be at $40\ °$, facing towards the goal.

Cases 2 and 3: Small and Large Signal Dependent Noises, No Hold In the case of signal dependent noise, because there is more uncertainty in state and measurement estimation, the eye turns much faster than the head, and is thus at a greater eccentricity compared to Case 1, which had no signal dependent noise. Specifically, in case 1, the eye made a 25.7831 °saccade, and in cases 2 and 3, the eye made 28.07493 °and 29.7938 °saccades respectively (approximately 0.49 radians and 0.52 radians). In comparing case 2 and 3, we see that though the goal is reached at 100 ms, the simulation with the larger signal dependent noise turns much faster than the head, such that peek eye eccentricity at 80 ms. Thus, we see that in this simulation, a saccade was not completed towards the goal, and it was only after the head finished turning at 100 ms did the fovea manage to reach the target.

Cases 4 and 5: Small Signal Dependent Noise, Short and Long Head Hold In these simulations, when the eye made a saccade towards the goal, because the head was held stationary, the eyes did not rotate back towards the center of the head. I observed that the saccade made by the eye in these two cases was lengthened such that the cumulative eccentricity of the gaze was as close to the goal as possible in order to minimize any costs. In addition, I observed that the amplitude of the saccade was much larger than that of cases 1-3, which can be explained by the fact that since the head was not allowed to move for 50 and 100 ms, the eye must make larger movements to minimize costs of being far away from the goal. It was only was only after the "brakes" were released was the head allowed to move. In comparing between cases 4 and 5, since case 5 was held stationary for an extra 50 ms, the duration and amplitude of the saccade was overall greater. In both scenarios, at 100 ms, the state cost matrix T penalized the eccentricity of the eye to return to center. Together, cases 4 and 5 demonstrate how the motor commands of the eyes do not follow an "open-loop" model, and depend on the state of the head.