HOMEWORK 4–CSC 320-SUMMER2016-SOLUTIONS

(1) We showed in class that $W = \{(i,j) \mid i,j\}$, natural languages is countable. Let $f(i,j)$ be the 1-1 mapping from (i,j) to the natural numbers. Then $f(f(i,j),k)$ is a 1-1 mapping of (i,j,k) to the natural numbers.

(2) Yes for each. Suppose we have TM's for languages A and B. We design a NTM. For concatentation, run the program which nondeterministically chooses a prefix of the input to be on tape 1 and the rest of the input to be on tape 2, runs the program for A on tape 1 and the program for B on tape 2. If they both accept, accept. Else reject. Since they are both decidable, they both always halt.

For intersection, copy the input onto tape 2, run program A on tape 1 and program B on tape 2. If they both accept, accept. Else reject.
For complement: For the deterministic TM which decides A, change its accept states to reject and reject states to accept

(3) Using the encoding technique in class, give an encoding of a two state TM M such that $L(M) = \{0,1\}^*$

Note that for every TM, state q_1 is the start state and state q_2 in the final (accepting) state. (See Lecture Notes 14).
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This encodes $\delta(q_1, x) = (q_2, 0, L)$, for $x = 0, 1, B$.

(4) Let f map $< M, w >$ to $< M' >$ where M' given ϵ as input, writes w on its tape and then simulates M on w. If M accepts, M' writes \$ on its tape. f is computable. It is not hard to see that $< M, w > \in A_{TM}$ iff M' writes \$ on its tape when started on the blank tape (ϵ input) iff $< M' > \in L$.

(5) Simulate M running on a blank tape, until it writes a nonblank symbol somewhere on the tape, in which case accept or it runs for $m+1$ states, in which case reject– it is reading a blank in each of its states and is in some state twice when reading a blank. Note that there is no way for the TM to distinguish hits the leftmost side of the tape and bounces back so all positions look the same. Therefore it will continue in the same loop.

(6) To create an NTM N, N guess a string w. If $< M > \in \bar{E_{TM}}$ then there is a w for which M accepts. Therefore N recognizes the language.

(7) We showed in class E_{TM} is not decidable. We just showed $\bar{E_{TM}}$ is recognizable. Therefore E_{TM} must not be recognizable. Reduce E_{TM} to this language L as follows. Let f map $< M >$ to $< M_1, M >$ where M_1 accepts every string. We

show f is a reduction. If $< M > \in E_{TM}$ then $L(M_1) \cap L(M) = \emptyset$ so $< M_1, M > \in L$. If $< M > \notin E_{TM}$ then $< M_1, M > \neq \emptyset$ and so is not in L.

(8) Give a reduction from $HALT_{TM} = \{< M, w > \ |M$ is a TM which halts on $w\}$ to $L = \{< M, j > \ \mid M$ halts on all inputs with less than j $1's\}$. Prove it's a reduction.

Answer: Given $< M, w >$, f creates $< M_1, 1 >$ where M_1 checks if its input contains a 1. If so, it goes into a loop. Else if its input has no 1's, M_1 simulates M on w. f is computable since it's easy to write a program that check the number of 1's in a string and go into a loop, and easy to simulate M on w given $< M, w >$. Proof that f is a reduction: if M halts on w then $L(M_1)$ halts exactly on strings which contain no 1's, and so $< M_1, 1 > \in L$. If M doesn't halt on w, M_1 doesn't halt on any strings, so $< M_1 > \notin L$.

(9) Which of these problems are not recursive, by Rice's Theorem? (circle each one that is correct and put an X through each one that is incorrect.)
 (a) $\{M \mid M$ accepts an infinite number of strings$\}$
 (b) $\{M \mid M$ halts on at least one string$\}$
 (c) $\{M \mid M$ visits all its states on some input $\}$
 (d) $\{(M, w) \mid M$ accepts $w\}$

The first one only. (b), (c) and (d) are not about the property of a TM's language given its encoding.