HOMEWORK 1–CSC 320–SOLUTIONS PART 1

Homework is due in class on Friday Sept. 14. Text, Chapter 0 and Chapter 1 to 1.2 Exercises:

(1) Write a mathematical description of the set containing 0, 10, and 100. This is a little ambiguous. Here is a possible answer: $\{0, 10, 100\}$

(2) The natural numbers are the positive integers. Write a mathematical description of the set containing all natural numbers less than 5 $\{1, 2, 3, 4\}$.

(3) Write a mathematical description of the set containing the empty string. $\{\epsilon\}$.

(4) If a set contains c elements, how many elements are in its power set? 2^c

(5) Let X be the natural numbers. Let f be a mapping from X to itself. If $f(x) = x+1$, is f onto? No, no natural number is mapped to 1

Is f 1-1? Yes. If $f(x) = f(y)$ then $x = y$.
What is range of f? $\{2, 3, 4, ...\}$. What is its domain? The natural numbers.

(6) In a directed graph, the relation: node A can reach node B by a directed path consisting of one or more edges is
a) reflexive No, there is not necessarily a path of one or more edges from a node to itself
b) symmetric No, just because there is a path from A to B doesn't mean there is one from B to A
c) transitive Yes, if there's a path from A to B and B to C then there's a path from A to C
d) none of the above

Explain.

(7) Prove that every graph with two or more nodes have two nodes with equal degree. Hint: Use a proof by contradiction.
Proof
Proof by contradiction. Assume each node has a distinct degree. Then the degrees are $0, 1, 2, ..., n-1$. But this gives a contradiction since a node with degree $n-1$ has an edge to every other node which implies there is no node of degree 0.

(8) Write a formal description of the set containing strings with some number of 0's (possibly none) followed by at least as many 1's.
$\{0^n 1^m \mid 0 \leq n \leq m\}$.

(9) Assume $\Sigma = \{a, b\}$. Give a DFA for $\{w \mid w \text{ starts with an } a\}$ and another DFA for $\{w \mid w \text{ has at most one } b\}$. Now use the method described in class of directly constructing a DFA which recognizes the union.

Picture will be attached.

(10) Give a state diagram of a DFA recognizing the language over $\{0,1\}$:

$$\{w \mid w \text{ contains an even number of 0s or contains exactly two 1s}\}$$

.

Picture will be attached.

(11) Give the state diagram of an NFA with one state that recognizes 0^* .

It's a single state with a loop labeled 0, which is both the start state and the final state.

(12) Show by giving an example that if M is an NFA that recognizes language C, swapping the accept and nonaccept states in M doesn't necessarily yield a new NFA that recognizes the complement of C. Is the class of languages recognized by NFA's closed under complement? Explain.

Solution: This is a tricky question. For the first part, almost any NFA you come up with will have the property that if you switch the accept and nonaccept states the recognized language will not be the complement of the original language. However, the class of languages recognized by NFAs is the class of regular languages, since for any NFA there is an equivalent DFA, and the class of regular languages is closed under complement. To prove that just take a DFA, make the final states nonfinal states and the nonfinal states final states.

(13) For any string $w = w_1w_2...w_n$, the *reverse* of w written w^R is the string in reverse order, $w_nw_{n-1}...w_1$. For any language A, let $A^R = \{w^R \mid w \in A\}$. Show that if A is regular, so is A^R.

Solution: To show this, we need to show that we can build a DFA to accept the reverse of the strings in L. Given L is regular there is a machine $M = (Q, \Sigma, \delta, q_0, F)$ that accepts L. We can modify M to get the NDFA M' that accepts $\{w \mid w^R \in L\}$ by adding a new start state q_0' and reversing old transitions, as follows:

$$\begin{aligned} M' &= (Q', \Sigma, \delta', q_0', F) \\ Q' &= Q \cup \{q_0'\} \\ F' &= \{q_0\} \end{aligned}$$

Define δ' as follows: Put in an ϵ labelled edge from the new start state to each final state, i.e., $\delta'(q_0', \epsilon) = \{q \in F\}$ and reverse all other edges, that is, replace each $\delta(q, a) = R$ by a set of δ transitions from each state in R to q, i.e., $\{\delta'(r, a) = \{\{q\} \mid r \in R\}$.

(14) Let $B_n = \{a^k \mid \text{ where } k \text{ is a multiple of } n\}$. Show that for each $n \geq 1$, the language B_n is regular.

Solution: you can build a DFA to accept B_c, by just building a cycle of k states, with an a transition taking you from one state to the next, and the last state cycling back to the first state by an ϵ transition which is the only accepting state.