# "Dual Nature of Radiation and Matter."

## Can you recall ?

1. What is electromagnetic radiation?

Ans: Radiations that are caused by the acceleration of charged particles and consist of electric and magnetic fields vibrating sinusoidally at right angle to each other and to the direction of propogation are called electromag-netic radiations.

2) What are the characteristics of wave?

Ans) All the vibrating particles of medium have same amplitude, period and frequency.

3) What do you mean by frequency and wave number associated with a wave?

Ans: Frequency:- The number of cycles or vibration of wave per unit time is called the frequency of the wave.

Wave Number:- Wave number is defined as number of waves per unit distance.

4) What are the characteristic properties of particles of matter?

Ans: a) The particles of matter are very, very small.
b) The particles of matter have space between them.
c) The particles of matter are constantly ~~d)~~ moving.

d) The particles of matter attract each other.

v) What are the different types of energies that a particle of matter can possess?

Ans According to the Kinetic theory, particles of matter are in constant motion, and the energy possess by the particles during this motion is Kinetic energy.

vi) How do we define momentum of a particle?

Ans Momentum is the product of mass and velocity of an object.

★ Introduction:-

- The wave nature of electromagnetic radiation in regions X-Rays, Y-rays, infrared, ultraviolet radiation, microwaves and visible light.

- Electromagnetic radiation consists of mutually perpendicular oscillating electric and magnetic fields, both being perpendicular to the direction in which the wave and energy are travelling.
- Planck assumed that atoms behave like tiny oscillations that emit ~~em~~ electromagnetic radiation only in discrete packets ($E = nh\nu$).
- where $\nu$ is the frequency of oscillator.
- The emissions occur only when the oscillator makes a jump from one quantized level of energy to another of lower energy.

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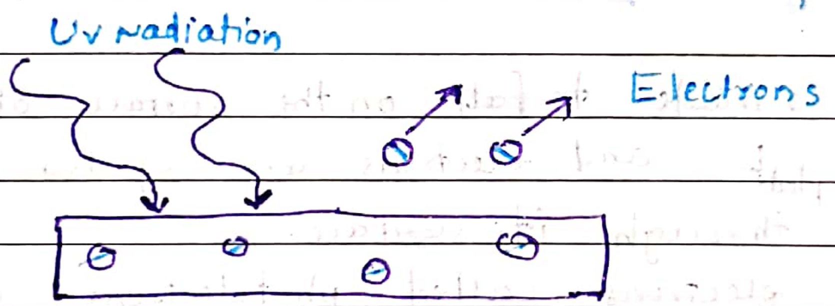
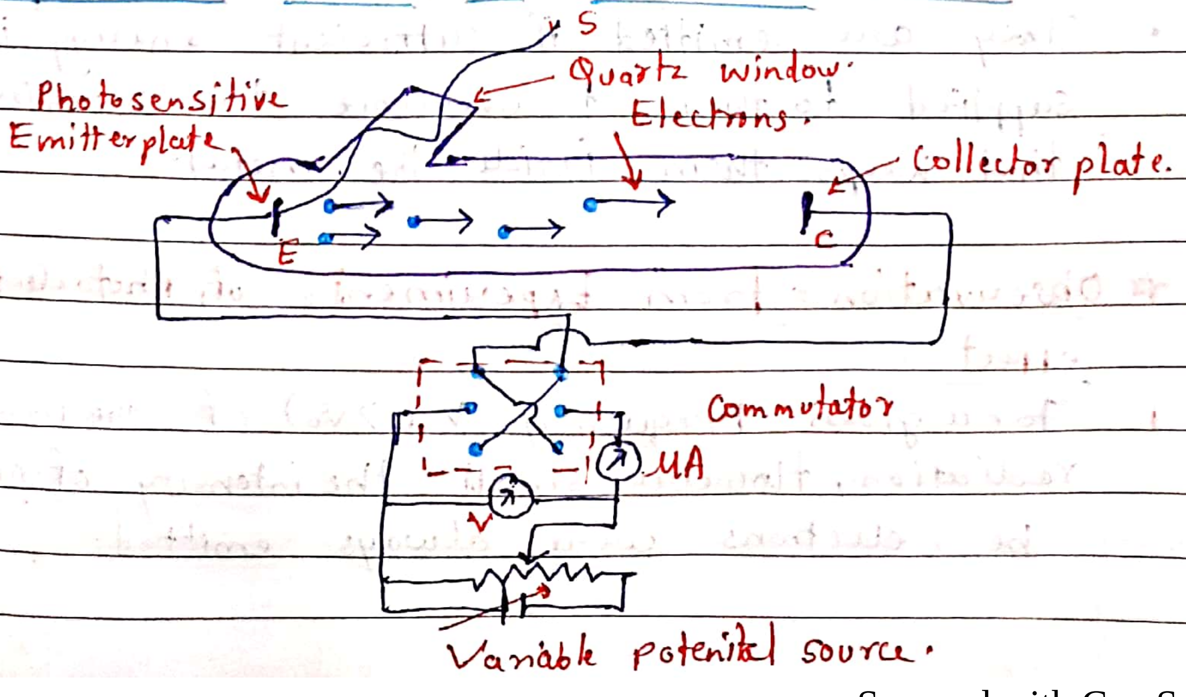
## The photoelectric effect →

- Heinrich Hertz discovered photo-electric emission in 1887.
- The phenomenon of emission of electrons from a metal surface, when radiation of appropriate frequency is incident on it, is known as photoelectric effect.
- For metals like zinc, cadmium, magnesium etc, ultraviolet radiation ~~et~~ is necessary while for alkali metals even visible radiation is sufficient.
- Electrical energy can be obtained from light (i) photo emissive effect and (ii) photo-voltaic effect, used in a solar cell.




## Experimental Set-up of Photoelectric effect →



- An expirimental set-up consists of an evacuated glass tube with a quartz window containing emitter E and collector C.
- The emitter and collector are connected to a voltage source whose voltage can be changed to an ammeter to measure the current in the circuit.
- A potential difference of V, as measured by the voltmeter, is maintained between the emitter E (+ve) and collector C (-ve).
- A source S of monochromatic light of sufficiently high frequency is used.

## Working of Experiment →

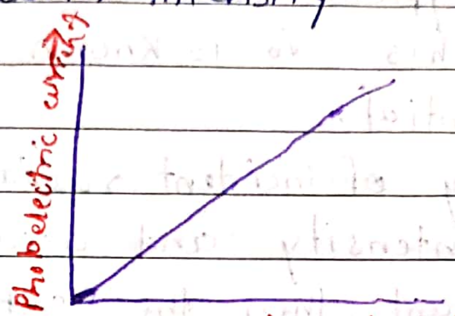
- Light is made to fall on the surface of the metal plate E and electrons are ejected from the metal through its surface.
- These electrons, called photelectrons!, are collected at the collector C.
- Free electrons are available in a metal plate.
- They are emitted if sufficient energy is supplied to them to overcome the barrier that keeps them inside the metal.

## Observations from Experiments of Photoelectric effect →

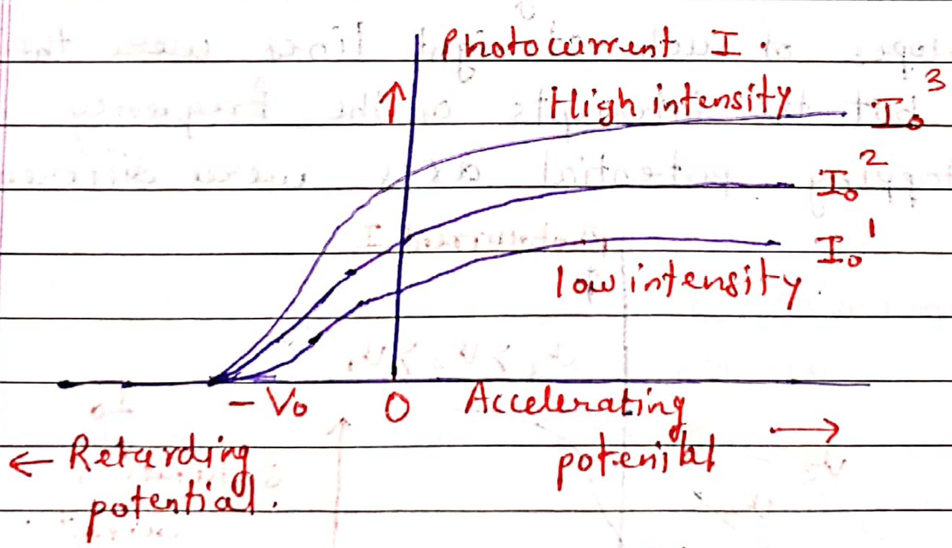
1. For a given frequency $\nu$ ($>\nu_0$) of the incident radiation, however small the intensity of radiation be, electrons were always emitted.

2. There were no time lag between the incidence of light and emission of electrons i.e. instantaneous (within $10^{-9}$ s).

3. Keeping the frequency $\nu$ of the incident radiation and accelerating potential V fixed. if the intensity was increased, the photo current increased linearly with intensity.




4. Keeping intensity and frequency fixed, Initially the current increased with Voltage then it remained constant. saturation current $I_o$.




5. Keeping the accelerating voltage and incident frequency fixed; if the intensity of incident radiation increased, the value of saturation current also increased proportionately.

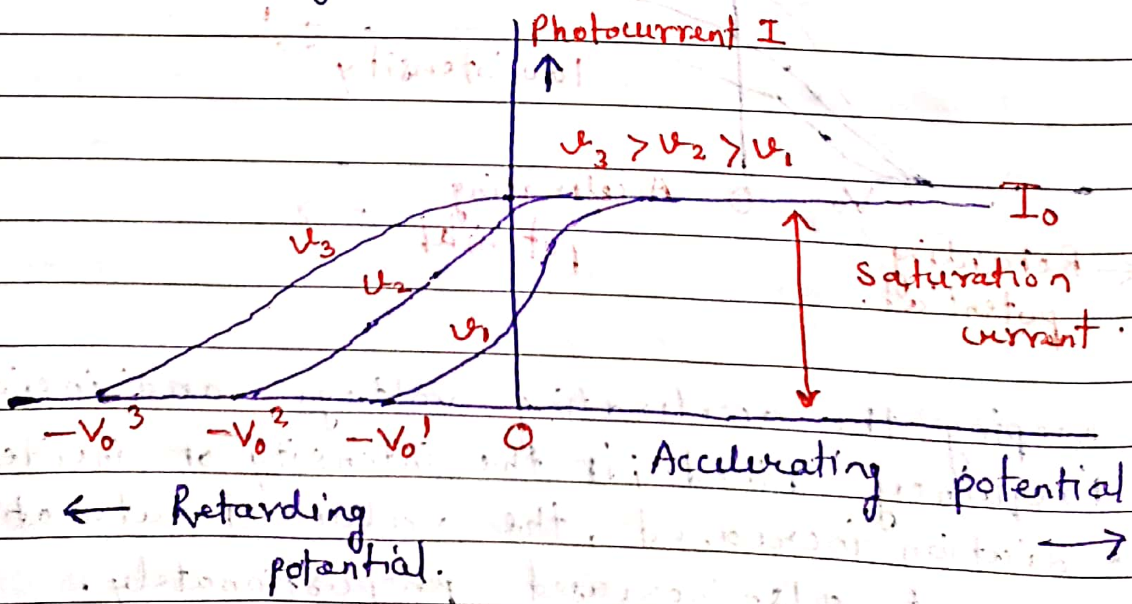
6. The maximum Kinetic energy K.Emax of the

depended on the potential V for a given metal plate and frequency of the incident radiation. it did not depend on the intensity of the incident radiation.

7. If increasingly negative potentials were applied on the collector, the photocurrent decreased and for some typical value $-V_0$, photo current became zero this $V_0$ is known as cut off or stopping potential.

8. If the frequency of incident radiation changed keeping the intensity and accelerating potential V constant, then the saturation current remained the same but the stopping potential $V_0$ changed.

9. The stopping potential $V_0$ varied linearly with $\nu$ as shown in fig. for different metals, the slopes of such straight lines were the same but the intercepts on the frequency and stopping potential axes were different.

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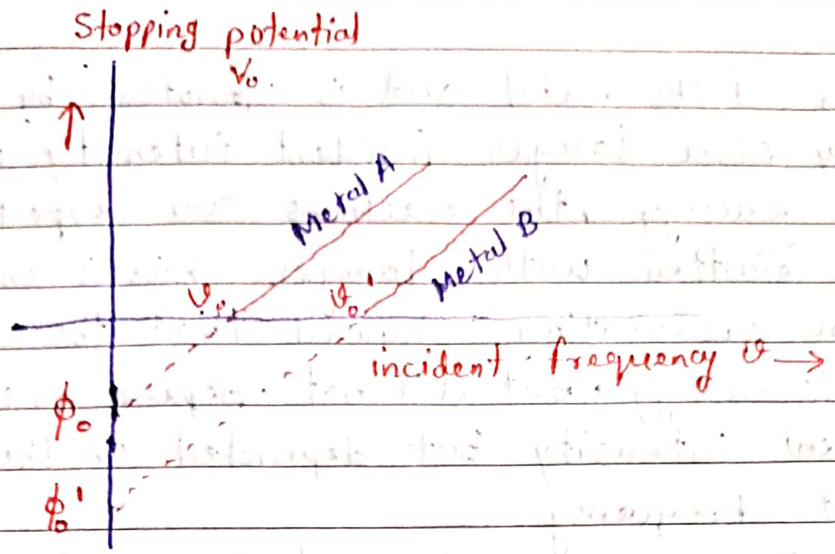


9. The photocurrent and hence the number of electrons depended on the intensity but not on the frequency of incident radiation, as long as the incident frequency was larger than the threshold frequency $\nu_0$ and the potential of anode was higher than that of cathode.

★ Failure of wave Theory to Explain the observations from Experiments of Photoelectric effect.

- Most of these observations could not be explained by the wave theory of electromagnetic radiation.
- first observation the instantaneous emission of electrons on incidence of light.
- for small incident intensity, the energy incident on unit area in unit time will be small and will take longer to knock off the electrons. These arguments were contradictory to observations.
- The minimum amount of energy required to be provided to an electron to pull it out of the metal from the surface is called the work

function of the metal and is denoted by $\phi_0$.

- Secondly since larger incident intensity implies larger energy, the electrons are expected to be emitted with larger kinetic energy. But the observation showed that the maximum kinetic energy. But did not depend on the incident intensity but depended on the incident frequency.
- Wave theory expected photoelectrons to be emitted for any frequency, if the intensity of radiation was large enough. But observations indicated that for a given metal surface, some characteristics cut-off frequency $\nu_0$ existed below which no photoelectrons were emitted however intense the incident radiation was and photoelectrons were always emitted if incident frequency $\nu$ was greater than $\nu_0$ even if the intensity was low.

## ★ Einstein's Postulate of Quantization of Energy and the ~~Quantization of Energy~~ Photoelectric Equation :→

- Einstein proposed light behaves as a particle and its energy is released or absorbed in bundles or quanta with energy $E = h\nu$;
- where $\nu$ is the frequency of light and $h$ is planck constant $= 6.626 \times 10^{-34}$ Js.
- The explanation using Einstein's postulate of quantization of energy for the observations

of Photoelectric Effect

1. A photon of ultraviolet radiation arrives at the metal surface and collides with an electron then electrons will be emitted if and only if the energy gained by the electrons is more than or equal to the work function i.e. $h\nu \geq \phi_0$, Thus, a minimum or threshold frequency $\nu_0 \; (= \phi_0 / h)$ is required to eject electrons from the metal surface.

2. The exchange of energy between the photon and electron is instantaneous. Hence there is no time lag between the incidence of light and emmission of electrons.

3. If the intensity of incident radiation is increased, there will be an increase in the number of energy quanta (Photons). Therefore larger intensity radiation will knock off more number of electrons from the surface and hence the current will be larger (if $\nu > \nu_0$).

4. for intensity of incident radiation. Higher is the accelerating potential, more will be number of electrons reaching the collector, hence the photocurrent $I$ increases up to saturation current $I_0$.

5. Increasing the incident intensity will increase the number of incident photons and eventually the saturation current.

6. If $\nu > \nu_0$, then the energy $\phi_0$ is used by the electron to escape from the metal surface and remaining energy is kinetic energy

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of the electron is known as Einstein's photoelectric equation. K.E max depends on the material and varies linearly with the incident frequency $\nu$ and independent of the intensity of the incident radiation.

7. If KE max is the energy of the most energetic electron at the emitter surface and $-V_0$ is the stopping potential will fail to reach the collector.

- If K.E max $< eV_0$, where e is the electron charge and $eV_0$ is the energy needed for the electron to overcome the retarding potential $V_0$ if the electron just fails to reach the collector, i.e., the photo current becomes zero.
- Equation K.E max $= h\nu - \phi_0$ explains that stopping potential $V_0$ depends on the incident frequency and the material of the emitter and does not depend on the incident intensity.

8. If the ejected electrons have kinetic energy more than $eV_0$, electrons can reach the collector, hence current flows, when the kinetic energy of the electron is less than or equal to $eV_0$, no current will flow. Photo current will become zero when K.E max $= eV_0$, Using K.E max $= eV_0$, we can write eq. K.Emax $= h\nu - \phi_0$ as:

$$eV_0 = h\nu - \phi_0$$

or

$$V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi_0}{e}$$

Above equation tell us that $V_0$ varies linearly with incident frequency $\nu$, and the slope of the straight line depends on constants h and e while the intercept of the line depends on the material through $\phi_0$.

9. All the above requirements thus bring out the fact that the magnitude of photocurrent depends on the incident intensity through the number of emitted photoelectrons and the potential V of the collector but not on the incident frequency $\nu$ as long as $\nu > \nu_0$.

## Wave - Particle Duality of Electromagnetic Radiation:

1. Particle nature was confirmed by compton in 1924 in experiment on scattering of X - Rays due to electrons of matter.
2. Some phenomena like interference and differaction can be explained by considering as a wave.
3. On the other hand some other observations like Photoelectric effect and black body radiation can be explained only if we consider electromagnetic radiation as consisting of photons with definite quantum of energy.
4. Also there are some phenomena which can be explained by both the theories.
5. It is therefore essential to consider that both the characters or behaviours hold good, one dominates in some situations and the other works in rest of the situations.

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6. There is thus a need to hypothesize the dual character not only for light but for the whole electromagnetic spectrum.
7. This phenomenon is termed as wave particle duality of electromagnetic radiation.

☆ Compton showed that →

- Photon has an associated momentum along with the energy it carries.
- All photons of electromagnetic radiation of a particular frequency have the same energy and momentum.
- Photons are electrically neutral and are not deflected by electric or magnetic fields.
- Photons can have particle-like collisions with other particles such as electrons.
- In photon-particle collision, energy and momentum of the system are conserved but the number of photons is not conserved.
- Photons can be absorbed or new photons can be created.
- Photons can transfer their energy and momentum during collisions with particles and disappear.
- When we turn on light, they are created. Photon always moves with the speed of light. it is never at rest.
- Mass of a photon is not defined as we do for a particle in Newtonian mechanics. its rest mass is zero (in all frame of reference.)

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## ★ De-Broglie Hypothesis (1924) →

- Radiation has dual nature, matter may also possess dual nature.
- De Broglie used the properties frequency $\nu$ and wavelength $\lambda$, of a wave and to connect these with the particles properties energy, E and momentum p.
- The momentum p carried by a photon of energy E is given by the relation $p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$.
- Thus frequency and wavelength of a matter wave associated with a material particle, of mass m moving with a velocity v are given as $\nu = \frac{E}{h}$ and $\lambda = \frac{h}{p} = \frac{h}{mv}$
- For a particle of mass m moving with a velocity v, the kinetic energy $E_K = \frac{1}{2}mv^2$, or $v = \sqrt{\frac{2E_K}{m}}$

- $$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2E_K}{m}}} = \frac{h}{m}\sqrt{\frac{m}{2E_K}}$$

- $$\lambda = \frac{h}{\sqrt{2mE_K}}$$

- For a charged particle of charge q accelerated from rest through a potential difference V, the work done is qV. This provides the kinetic energy. Thus $E_K = qV$.
- $$\lambda = \frac{h}{\sqrt{2mqV}}$$
- For an electron moving through a potential difference of V.

- $\lambda = \frac{h}{\sqrt{2m_e eV}}$
- $\lambda = \frac{1.228 \times 10^{-8}}{\sqrt{V}} m$
- Experimentally found that electrons and subatomic particles like protons and neutrons also exhibit wave properties.
- The wave property of electron was confirmed experimentally in 1927 by Davisson and Germer in America in 1928 by George P.

## * Wave-Particle Duality of matter.

~~Marti~~

- Material particles show wave-like nature under certain circumstances. This phenomenon is known as wave-particle duality of matter.
- frequency $\omega$ and wave number $k$ are used to describe waves in classical theories while mass $m$ and momentum $p$ are used to describe particles.
- Wave-particle duality implies that all moving particles have an associated frequency and an associated wave number and all waves have an associated energy and an associated momentum.
- The wavelengths associated with macroscopic particles do not play any significant role in our every day life and we need not consider their wave nature. Also the

wavelengths for macroscopic particles are so small that they cannot be measured.

- On the other hand if we try to estimate the associated de Broglie wavelength of a moving electron passing through a small aperture of size $10^{-10}$ m or an oxygen molecule in air is comparable to the size of the hole through which the electron is passing.
- In conclusion, for both electromagnetic radiation and atomic and sub-atomic particles, particle nature is dominant during their interaction with matter.
- On the other hand, while travelling through space, particularly when their confinement is of same order of magnitude as their associated wavelength, the wave nature is dominant.

# ★ Dual Nature of Radiation and Matter

## Numericals :→

7] What will be the energy of each photon in monochromatic light of frequency $5 \times 10^{14}$ Hz?

Ans Given $\nu = 5 \times 10^{14}$ Hz

$\because E = h\nu$

$E = 6.63 \times 10^{-34} \times 5 \times 10^{14}$

$E = 3.31 \times 10^{-19}$ J

$E = \frac{3.31 \times 10^{-19}}{1.6 \times 10^{-19}}$ eV

$\boxed{E = 2.068 \text{ eV}}$

8) Observations from an experiment on photoelectric effect for the stopping potential by varying the incident frequency were plotted. The slope of the linear curve was found to be approximately $4.1 \times 10^{-15}$ V s. Given that the charge of an electron is $1.6 \times 10^{-19}$ C. find the value of the Planck's constant h.

Solution :- Stopping potential for a material is given by the formula, $V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi_0}{e}$

comparing this with equation of straight line, $y = mx + c$

slope of a graph $V_0$ v/s $\nu$ will be

$m = \frac{h}{e}$

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$\therefore 4.1 \times 10^{-15} = \frac{h}{1.6 \times 10^{-19}}$

$h = 4.1 \times 10^{-15} \times 1.6 \times 10^{-19}$

$\boxed{h = 6565 \times 10^{-34} \text{ Js}}$

9. The threshold wavelength of tungsten is $2.76 \times 10^{-5}$ cm.

i. Explain why no photoelectrons are emitted when the wavelength is more than $2.76 \times 10^{-5}$ cm.

ii. What will be the maximum kinetic energy of electrons ejected in each of the following cases.

a. if ultraviolet radiation of wavelength $\lambda = 1.80 \times 10^{-5}$ cm and.

b. Radiation of frequency $4 \times 10^{15}$ Hz is made incident on the tungsten surface.

Ans: i) For any photosensitive material, part of incident energy equivalent to its work function is used by the electron to escape. the metal surface. Energy of a photon is given by,

$$E = \frac{hc}{\lambda}$$

$$\therefore E \propto \frac{1}{\lambda}$$

Thus increasing the wavelength beyond $\lambda_0$ decreases the energy of the photon and electron cannot eject from the metal surface.

ii) Given $\lambda_0 = 2.76 \times 10^{-5}$ cm $= 276$ nm

$\lambda_{UV} = 1.80 \times 10^{-5}$ cm $= 180$ nm.

$V = 4 \times 10^{15}$ Hz,

$\because \phi_0 = \frac{hc}{\lambda_0}$

Value of wavelength (nm) x energy (ev) can be taken numerically equal to 1250.
i.e. hc = 1250 eV - nm.
from formula.

$$\phi_0 = \frac{1250}{276} = 4.53 eV.$$

a) for UV radiations, using formula

$$K.E_{max} = h\nu - \phi_0 = \frac{hc}{\lambda} - \phi_0$$

$$K.E_{max} = \frac{1250}{180} - 4.53$$

$$= 6.94 - 4.53$$

$$K.E_{max} = 2.41 eV$$

b) For radiation of given frequency

$$K.E_{max} (eV) = \frac{6.63 \times 10^{-34} \times 10^{15} \times 4}{1.6 \times 10^{-19}} - 4.53$$

$$= \frac{26.52}{1.6} - 4.53$$

$$= 16.57 - 4.53$$

$$K.E_{max} (eV) = 12.04 \text{ eV}.$$

10) Photo current recorded in the micro ammeter in an experimental set up of photoelectric effect vanishes when the retarding potential is more than 0.8 V

if the wavelength of incident radiation is 4950 A°. If the source of incident radiation is changed, the stopping potential turns out to be 1.2V. find the work function of the cathode material and the wavelength of the second source.

Solution:-

Given $(V_o)_1 = 0.8V$, $\lambda_1 = 4950 A°$,

$(V_o)_2 = 1.2V$.

$\because eV_o = \frac{hc}{\lambda} - \phi_o$.

from formula.

$$e(V_o)_1 = \frac{hc}{\lambda_1} - \phi_o$$

$$(V_o)_1 = \frac{12500}{4950} - \phi_o \text{ (in eV)}.$$

$$0.8 = 2.52 - \phi_o$$

$\boxed{\phi_o = 1.72 eV.}$ Work function of metal.

Now, from formula $e(V_o)_2 = \frac{hc}{\lambda_2} - \phi_o$.

$$\therefore 1.2 = \frac{12500}{\lambda_2} - 1.72.$$

$$\therefore \lambda_2 = \frac{12500}{2.92}$$

$\boxed{\lambda_2 = 4280 A°.}$ Wavelength of second source.

11) Radiation of wavelength 4500 A° is incident on a metal having work function 2.0 eV. due to the presence of a magnetic field B. the most energetic photoelectrons emitted

in a direction perpendicular to the field move along a circular path of radius 20cm. What is the value of the magnetic field B?

Solution :-

Given $\lambda = 4500\ A^{\circ} = 4500 \times 10^{-10}\ m$.

$\phi_0 = 2.0\ eV = 2 \times 1.67 \times 10^{-19} J$.

$r = 20cm = 0.2 m$.

$B = ?$

$\because E = \frac{hc}{\lambda} - \phi_0$

$E = \frac{12500}{4500} - 2.0$

$E = (2.78 - 2.0)\ eV$.

$E = 0.78 \times 1.6 \times 10^{-19} J$.

$E = 1.248 \times 10^{-19} J$.

from formula $r = \sqrt{\frac{2mE}{qB}}$

$B = \sqrt{\frac{2mE}{qr}}$

$\therefore B = \sqrt{\frac{2 \times 1.248 \times 10^{-19} \times 9.11 \times 10^{-31}}{1.6 \times 10^{-19} \times 0.2}}$

$\therefore B = \sqrt{\frac{2.496 \times 10^{-50} \times 9.11}{3.2 \times 10^{-20}}}$

$\frac{1}{2} \times \begin{cases} \log 2.496 = 0.3973 \\ \log 9.11 = 0.9595 \end{cases}$

$\boxed{B = 1.490 \times 10^{-5}\ T.}$

$\log(3.2) = 0.5051$

$a\log(\bar{0}.1733) \times 10^{-5}$

12]. Given the following data for incident wavelength and the stopping potential obtained from an experiment on photoelectric effect, estimate the value of Planck's constant and the work function of the cathode material. what is the threshold frequency and corresponding wavelength? what is the most likely metal used for emitter?

Solution :-> Given $\lambda_1 = 2536$ A°, $(V_0)_1 = 1.95$ V,
$\lambda_2 = 3650$ A°, $(V_0)_2 = 0.5$ V.

i) $\because \frac{hc}{e}$ = slope of graph of $V_0$ v/s $(1/\lambda)$

Slope of graph $V_0$ vs $(1/\lambda)$ would be.

$$m = \frac{V_{02} - V_{01}}{\frac{1}{\lambda_2} - 1/\lambda_1} = \frac{0.5 - 1.95}{\frac{1}{3650 \times 10^{-10}} - \frac{1}{2536 \times 10^{-10}}}$$

$$= \frac{-1.45}{(2.74 \times 10^{6}) - (3.94 \times 10^{6})}$$

$$= \frac{1.45}{1.2 \times 10^{6}}$$

$$\frac{hc}{e} = 1.21 \times 10^{-6}$$

$$\therefore h = \frac{1.21 \times 10^{-6} \times 1.6 \times 10^{-19}}{3 \times 10^{8}}$$

$$\boxed{h = 6.45 \times 10^{-34} \text{ Js.}}$$

ii) $(V_0)_1 = \frac{hc}{e\lambda} - \frac{\phi_0}{e}$

$$1.95 \text{ (eV)} = \frac{12500}{2536} - \phi_0$$

$\therefore \phi_0 = 4.93 - 1.95 = 2.98\ eV$

As value of work function of calcium is $\phi_0 = 2.9$, the metal used for emitter is most likely calcium.

iii) $\phi_0 = h\nu_0 \quad c = \nu\lambda$

$\lambda_0 = \frac{c}{\nu} = \frac{3 \times 10^8}{7.191 \times 10^{14}} = 0.4172 \times 10^{-6}$

$\boxed{\lambda = 4172\ A^\circ}$ Threshold wavelength.

iv) $\phi_0 = h\nu_0$

$\nu_0 = \frac{\phi_0}{h} = \frac{2.98 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = \frac{4.768 \times 10^{-15}}{6.63}$

$\boxed{\nu_0 = 7.191 \times 10^{14}\ Hz}$ threshold frequency.

13) Calculate the wavelength associated with an electron, its momentum and speed.

i) when it is accelerated through a potential of 54 V,

ii) when it is moving with kinetic energy of 150 eV.

Ans :- $V = 54\ v$, $E = 150\ eV$.

$\therefore \lambda = \frac{1.228}{\sqrt{54}} = \frac{1.228}{7.35}$

$\therefore \lambda = 0.167\ nm$.

$\therefore p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{0.167 \times 10^{-9}}$

$\boxed{p = 39.70 \times 10^{-25}\ kgm/s.}$

$\log 6.63 = 0.8215$

$\log 0.167 = \bar{1}.2227$

$(1.5988) \times 10^{-25}$

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$\because c = \frac{P}{m_e} = \frac{39.70 \times 10^{-25}}{9.1 \times 10^{-31}}$

$\log 39.7 = 1.5988$
$\log 9.1 = 0.9590$
$\overline{a.\log (0.6398) \times 10^6}$

$c = \text{antilog } (0.6398) \times 10^6$

$\boxed{c = 4.363 \times 10^6 \ m/s.}$

$\because V = \frac{E \text{ in } J}{q} = \frac{q \times E \text{ (in eV)}}{q}$

$\boxed{V = 150 \ V.}$

from formula $\lambda = \frac{1.228}{\sqrt{150}} = \frac{1.228}{12.25} \simeq 0.100 nm.$

from formula $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{0.100 \times 10^{-9}}$

$\boxed{p = 6.63 \times 10^{-24} \ kg \ m/s.}$

from formula

$v = \frac{P}{m_e} = \frac{66.30 \times 10^{-25}}{9.1 \times 10^{-31}}$

$\log 663 = 1.8215$
$\log 9.1 = 0.9590$
$\overline{A.\log (0.8625) \times 10^6}$

$v = a.\log (0.8625) \times 10^6.$

$\boxed{V = 7.286 \times 10^6 \ m/s.}$

[4] The de-Broglie wavelengths associated with an electron and a proton are same, what will be ratio of i) their momenta?
ii) their kinetic energies?

Solution :- Given $\lambda_e = \lambda_p$.

$\because \lambda = \frac{h}{p}$

$\frac{h}{P_e} = \frac{h}{P_p} \quad \therefore P_e = P_p$

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$$\therefore \frac{P_e}{P_p} = \frac{1}{1}$$

$$\because P = \sqrt{2mE}$$

substituting this result in formula

$$\sqrt{2m_eE_e} = \sqrt{2m_pE_p}$$

Squaring both the sides.

$$2m_e E_e = 2m_p E_p.$$

$$\frac{E_e}{E_p} = \frac{m_p}{m_e}$$

$$m_p = 1836 \times m_e$$

$$\boxed{\frac{E_e}{E_p} = 1836}$$

15] Two particles have the same de Broglie wavelength and one is moving four times as fast as the other. If the slower particle is an $\alpha$ particle, what are the possibilities for the other particle ?

Solution :- Given that $\lambda_1 = \lambda_2$ ; & $V_1 = 4V_2$

Also, slower particle is $\alpha$ particle, particle 2 is $\alpha$ particle.

Now, from de Broglie relation $\frac{h}{m_1V_1} = \frac{h}{m_2V_2}$

$$\therefore m_1V_1 = m_2V_2$$

$$\therefore 4m_1V_1 = m_2V_2$$

$$m_1 = \frac{1}{4} \times m_2 = \frac{1}{4} \times m_\alpha$$

$$\therefore m_1 = \frac{1}{4} \times 4U$$

$\boxed{m_1 = 1U}$, Thus the particle 1 could either be proton or neutron.

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16] What is the speed of a proton having de Broglie wavelength of 0.08 Å?
(Taken $m_p = 1.67 \times 10^{-27}$ kg)

Solution → Given $\lambda = 0.08\ Å = 8 \times 10^{-12}$ m

$m_p = 1.67 \times 10^{-27}$ kg

speed of proton (v) = ?

$$\because \lambda = \frac{h}{mv}$$

$$v = \frac{h}{mv}$$

$$\therefore v = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 8 \times 10^{-12}}$$

log 6.63 = 0.8215

log 13.36 = 1.1258

a.log $(\bar{1}.6957) \times 10^5$

$v = \text{antilog}(\bar{1}.6957) \times 10^5$

$$\boxed{v = 49.63 \times 10^3\ m/s}$$

17) In a nuclear reactors, neutrons travel with energies of $5 \times 10^{-21}$ J. find their speed and wavelength. (Taken $m_n = 1.67 \times 10^{-27}$ kg).

Solution → Given. $E = 5 \times 10^{-21}$ J, $m_n = 1.67 \times 10^{-27}$ kg

Speed of neutron (v) = ?

$$\because E = \frac{1}{2} m_n v^2$$

$$\therefore \sqrt{\frac{2E}{m_n}} = v$$

$$v = \sqrt{\frac{2 \times 5 \times 10^{-21}}{1.67 \times 10^{-27}}}$$

$$v = \frac{10^{-10}}{\sqrt{16.7 \times 10^{-14}}} = 0.2446 \times 10^4\ m/s$$

$$\boxed{v = 2.45 \times 10^3\ m/s}$$

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wavelength of neutron $(\lambda) = \frac{h}{m_n v}$.

$\therefore \lambda = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 2.45 \times 10^{3}}$

$\lambda = \frac{6.63}{1.67 \times 2.45} \times 10^{-10}$

$\lambda = 1.62 \times 10^{-10} m$

$\boxed{\lambda = 1.62 A^\circ}$

$\log 6.63 = 0.8215$
$\log 1.67 = 0.2227$
$\log 2.45 = 0.3892$
antilog $(0.2096) \times 10^{-10}$

18] Find the ratio of the de Broglie wavelength of an electron and a proton which both are moving with the
i) same speed, ii) same energy and
iii) same momentum?
iv) state which of the two will have longer wavelength in each case?

Ans → Ratio of de-Broglie wavelengths for electron and proton is,

$$\frac{\lambda_e}{\lambda_p} = \frac{h}{m_e v_e} \times \frac{m_p v_p}{h}$$

$$\frac{\lambda_e}{\lambda_p} = \frac{h}{\sqrt{2 m_e E_e}} \times \frac{\sqrt{2 m_p E_p}}{h}$$

i) For proton and electron moving with same speed.

$$\frac{\lambda_e}{\lambda_p} = \frac{m_p}{m_e} = 1836.$$

Thus de Broglie wavelength of electron is longer than that of proton.

ii) For proton and electron having same energy.

$$\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} = \sqrt{1836} = 42.85.$$

Thus, de Broglie wavelength of electron is longer than that of proton.

iii) For proton and electron having same momentum.

$$\boxed{\frac{\lambda_e}{\lambda_p} = \frac{P_p}{P_e} = 1.}$$

Thus both electron and proton will have equal de ~~Brg~~ Broglie wavelength.

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