Assignment/Problem Set 12

Heinrich Dinkel
ID: 1140339107
E-mail: heinrich.dinkel@sjtu.edu.cn

1 Exercise 6

We need to determinate whether $f(x)$ is a quadratic spline. Therefore we must verify whether $f \in C^2(R)$ or $f \in C^3(R)$.

$$f'(x) = \begin{cases} 1 & x \in (-\infty, 1] \\ 2 - x & x \in [1, 2] \\ 0 & x \in [2, \infty) \end{cases}, f''(x) = \begin{cases} 0 & x \in (-\infty, 1] \\ -1 & x \in [1, 2] \\ 0 & x \in [2, \infty) \end{cases}$$

Now we use the limits between two of the spline cases to calculate whether the functions are quadratic and so on.

$$\lim_{x \to 1^-} f(x) = 1 = \lim_{x \to 1^+} f(x), \lim_{x \to 2^-} f(x) = \frac{3}{2} = \lim_{x \to 2^+} f(x)$$

hence $f \in C^1(R)$

$$\lim_{x \to 1^-} f'(x) = 1 = \lim_{x \to 1^+} f'(x), \lim_{x \to 2^-} f'(x) = 0 = \lim_{x \to 2^+} f'(x)$$

hence $f \in C^2(R)$.

$$\lim_{x \to 1^-} f''(x) = 0, \lim_{x \to 1^+} f''(x) = -1, \lim_{x \to 2^-} f''(x) = -1, \lim_{x \to 2^+} f''(x) = 0$$

Hence $f \notin C^3(R)$, so we can see that f is a quadratic, but not a cubic spline.

2 Exercise 7

As already seen in paragraph 6, it is not a cubic spline.

3 Exercise 8

The given functions and its derivatives are:

$$f(x) = \begin{cases} a(x-2)^2 + b(x-1)^3 & x \in (-\infty, 1] \\ c(x-2)^2 & x \in [1, 3] \\ d(x-2)^2 + e(x-3)^3 & x \in [3, \infty) \end{cases}$$

$$f(x)' = \begin{cases} 2a(x-2) + 3b(x-1)^2 & x \in (-\infty, 1] \\ 2c & x \in [1, 3] \\ 2d(x-2) + 3e(x-3)^2 & x \in [3, \infty) \end{cases}$$

$$f(x)'' = \begin{cases} 2a + 6b(x-1) & x \in (-\infty, 1] \\ 2c & x \in [1, 3] \\ 2d + 6e(x-3) & x \in [3, \infty) \end{cases}$$

To determinate the spline we first use the knot conditions:

$$\begin{aligned}
S_{i-1}(t_i) &= S_i(t_i) \\
S_0(1) == S_1(1) &= \\
a(x-2)^2 &= c(x-2)^2 \\
&= a = c
\end{aligned}$$

Moreover we get:

$$\begin{aligned}
S_1(3) &= S_2(3) \\
&= -c = -d \\
c = d &\Rightarrow a = c = d
\end{aligned}$$

Interestingly, we didn't use the derivatives as conditions, since they actually give us exactly the same results for the knots. $f(x)$ is a cubic spline, if and only if $a = c = d$, the parameters b, e are arbitrary chosen. Now we calculate the parameters, using $f(0) = 26$ in equation S_0:

$$4a - b = 26$$

Using $f(1) = 7$ in S_0:

$$a + 0 = 7 \Rightarrow a = 7 = c = d \Rightarrow b = 4a - 26 = 2$$

Lastly using $f(4) = 25$, in S_2.

$$\begin{aligned}
4d + e &= 25 \\
28 + e &= 25 \\
e &= -3
\end{aligned}$$

So our result is : $a = c = d = 7$, $b = 2$, $e = -3$.