Model Predictive Control for HVAC Systems - A Review

Roger Kwadzogah, Mengchu Zhou, *Fellow, IEEE*, and Sisi Li, *Student Member, IEEE*

Abstract— **The world faces an energy problem. Oil supply is gradually running out. Its use is polluting the planet with greenhouse gas. Most alternative energy sources also pose some environmental problems. Hence the efficient use of energy must be an integral part of any solution to them. Whenever possible, one must use advanced control engines and technologies to achieve the highest energy efficiency. In Heating Ventilation and Air Conditioning (HVAC) industry, much research has identified a number of energy-saving control methods. Derived from optimal control theory, Model Predictive Control (MPC) is one of them. This paper presents a comprehensive review of its applications to HVAC systems.**

I. INTRODUCTION

Heating, ventilation and air-conditioning (HVAC) systems in buildings are responsible for maintaining good indoor air quality through adequate ventilation and provide thermal comforts for their occupants via managing air and water, in collaboration with multiple system components: pumps, fans, heat exchangers, chillers, and boilers for the purpose of indoor space healing and cooling of multiple building zones [1]. The simplest HVAC control uses thermostats to provide temperature feedback for what is known as bang-bang control. In cold weather, whenever they show temperature below a set-point, a heating unit is turned on until it returns to the set-point. In hot weather, whenever they show it above a set-point, a cooling unit is turned on until temperature returns to the set-point. While this is a very simple and inexpensive control method, it is not only ineffective in tracking accurately the set-point temperature due to constant temperature overshoots but also because it is very wasteful of energy. The temperature tracking error and overshooting problems are overcome by replacing the bang-bang controller with so-called proportional, integral and derivative (PID) controllers. A standalone PID controller, though simple and relatively inexpensive to implement, is very tedious to tune and does not solve the energy-efficiency problem [2]. For example, an improperly tuned PID controller is not only energy wasting, but also can destabilize the system. A standalone PID controller is usually not designed based on the minimum energy requirement.

In HVAC industry, rule-based control (RBC) is commonly used to compensate for the standalone PID's incapability to deal with the large systems with real-time constraints and multivariable-coupling nonlinear dynamics. Designers set up a sequence of rules to control each HVAC component separately for the purpose of decoupling. The set-points of each PID loop of HVAC components are adjusted based on the predefined control rules. The sequence of rules is often based on the experience of designers. For example, a typical HVAC system with variable air volume (VAV) terminal boxes equipped with each zone may have the following rules. If a zone's temperature is higher than its set-point through the deadband, the set-point of the VAV damper is calculated and the damper is turned on to inject the air flow into a zone; If it is lower, the VAV damper is closed to its minimum setting, and turn on the reheat valve. These set-point values are set according to designers' experience.

To exploit the energy-saving potential in HVAC system operations, an approach based on optimal control theory is desired. Model predictive control (MPC) has shown great potential in comparison with RBC [3]. Generally speaking, it solves an online optimization problem to obtain the control action at each time instance, by approximating infinite optimization to finite one for the objective functions of interest, with the prediction of future evolution of the process, and subject to some constraints on control inputs and states. MPC is not a single strategy. It is a vast class of control methods that use a model of the process explicitly to obtain control signal by minimizing cost function subject to some constraints. The minimization process is posed over a finite future time window and then solved to acquire N step prediction of the optimal control signal subject to certain constraints. This implementation has no robustness and no stability guarantee. However, they can be overcome if the implementation is carried out in a manner called a receding horizon method. In such a method, only the first control signal in the sequence is applied during each step [3]. The mathematics to design and implement MPC dictates that the design model of a process be described using the state space paradigm. The system states that are not measurable have to be estimated. The system model usually includes disturbance models that describe internal and external heat loads. For accurate performance of MPC the disturbance must be measured or estimated. Fig. 1 depicts the general schematic of MPC for HVAC systems.

A cost function could be the discrepancies between the outputs and their set-points. It can also be any arbitrary quantity of interest, such as the energy consumption of a system, which renders the possibility to achieve energy-efficient operations. MPC is the only control methodology that takes the prediction of future system behavior into consideration while satisfying the system constraints.

Even though many research efforts have been devoted

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R. Kwadzogah, M. Zhou and S. Li are with the Department of Electrical and Computer Engineering, New Jersey Institute of Technology, Newark, NJ 07013, USA. M. Zhou is also with The Key Laboratory of Embedded System and Service Computing, Ministry of Education, Tongji University, Shanghai 201804, China {rkk2, zhou, sl98}at njit.edu

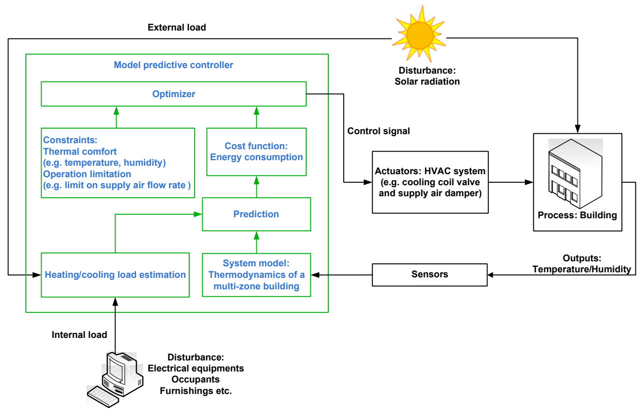

Fig. 1. General schematic of MPC for HVAC systems.

into the optimization of set-points of each components in an HVAC system [4], MPC is greatly advocated since it can systematically handle the system constraints and multivariable-coupling systems while realizing both objectives in HVAC industry: 1) to minimize the energy-consumption during HVAC operations by coordinating actuators and 2) to track each set-point. It is shown to outperform other controllers in saving energy while maintaining acceptable occupants' comforts in the buildings [5]–[7]. Due to the rich computational resources requested by MPC, it is inclined to be applied in the supervisory level of control. Its drawback is that it heavily depends on the accuracy of a model. Hence, the simplification of building models and unpredicted external disturbance such as solar radiation make its practical deployment difficult. To address this issue, three alternative approaches are under study. The first one is adaptive MPC that uses the updated time-variant models at each time intervals. It obtains a better model, but loses the robustness as a trade-off [8]. Robust MPC adopts the worse-case scenario to guarantee the stability by bounding the uncertainties, but tends to be too conservative [9]. Stochastic MPC considers the uncertainties and explicitly formulates it into an optimization problem, but the resulting problem is difficult to solve [3], [15].

In Section II, description and operation of a generic HVAC system is outlined. Section III presents an MPC design for it. Section IV is a survey of current research on applications of MPC to HVAC systems. The conclusions are presented in Section V.

II. GENERIC HVAC SYSTEM

A. Function of an HVAC System

HVAC is to deliver the air to the indoor space of a building to maintain its occupants' comforts. In cold weather, comfortable indoor temperatures can only be maintained by heating devices that provide heat to the space at the same rate as the space is losing heat. Similarly, in hot weather, heat is removed from the space at the same rate that it is gaining heat. The rate at which heat is gained or lost is a function of the difference between the inside and outside air temperatures. Therefore, in order to maintain a stable thermal comfort, the heat balance that determines the indoor temperature needs to be properly controlled by heating and cooling devices [10]. Indoor thermal comfort is determined not only by temperature but also by relative humidity.

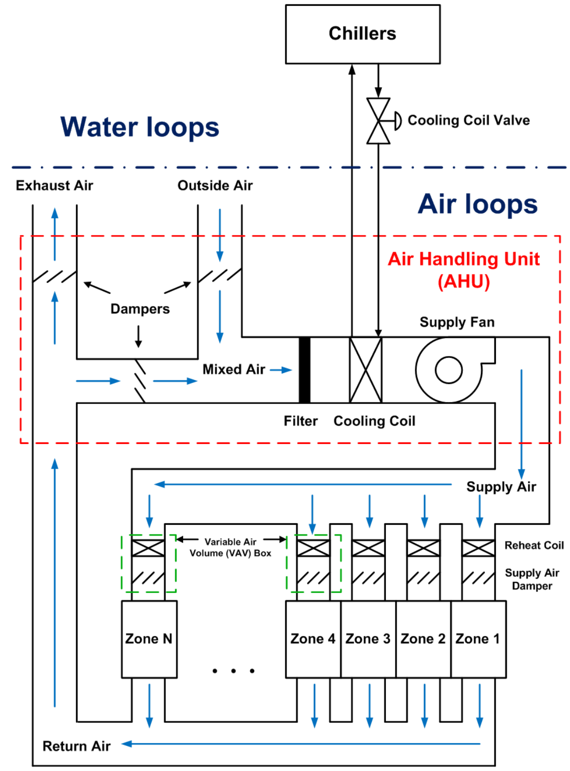

Fig. 2. A typical multi-zone HVAC system.

B. Functional Model of an HVAC System

HVAC is divided into two sub-systems that fulfill their tasks in their individual loops: 1) the water system that handles the water cooling, storage and distribution, and 2) the air system that handles the cooling air conversion and distribution to each zone. As depicted in Fig. 2, a typical multi-zone HVAC system consists of a central air handling unit (AHU). Within AHU are a cooling unit and a supply fan. AHU is connected to the indoor space by tubes. An AHU is to force the supply air to track its set-point. The return air and outside air are mixed in a mixing chamber, and then the mixed air traverses air filters and cooling coils in which the heat carried by the mixed air is removed by heat exchanging with the chilled water. The supply fan drives the supply air at a constant temperature to each zone to remove the heat generated by solar radiation, occupants, and electrical equipment, etc.. A part of the return air exits the indoor space back into the mixing chamber. Thus an AHU provides air at a controlled temperature for use in heating or cooling the indoor space. The temperature of the supply air is controlled by regulating the rate at which cold water flows through a cooling coil. The flow rate of the discharged air is regulated to maintain a predetermined static air pressure within the temperature-controlled indoor space. Each zone is equipped with its own VAV terminal box, which includes a reheat coil for reheating the supply air and a supply damper to manipulate the air flow rate. After going through the VAV box, the supply air is discharged to each zone. The VAV unit supplies variable air volume required to match changes in the thermal load of the indoor space in response to changes in the thermal load.

HVAC is a complex and nonlinear multiple-input multiple-output (MIMO) system due to all the components within the air and water control loops and the thermodynamics of the indoor space. Extensive literatures exist on thermal dynamic model of the space. The building thermodynamics can be simplified as an undirected graph. The conduction cross surfaces (e.g., walls, windows, ceilings and floors) are assumed to be a resistance-capacitor (RC)-network with current and voltage being analogs of heat flow and temperature [11], [12]. Gouda *et al.* show that a second-order RC-network model with three resistors and two capacitors is sufficient to capture thermodynamics between two spaces separated by a surface [13].

III. IMPLEMENTING MPC IN HVAC SYSTEMS

MPC is usually implemented as a supervisory controller at a higher hierarchical level in HVAC. At lower one, controlling individual thermal zones are usually via PID [14]. The main objective to use MPC in HVAC is to increase energy efficiency and satisfy thermal comfort requirements simultaneously.

Each HVAC control project begins with model design and description. Central to any model is the thermodynamics of the thermal space or rooms within the target building. Other elements are the dynamics of the heating and cooling components, actuators and sensors. During the model definition, assumptions are usually made on what to include in the dynamics of sub-components to reduce overall system complexity. Further simplifications are usually made about disturbance models. Once a complete model is written down, the next step is to estimate parameters of the model from available data or through identification experiments. In the end the overall model obtained is usually nonlinear in the system states and inputs. Then a performance cost function is formulated in terms of control input, system states, and their prediction over the prediction horizon. Typically constraints on control actuators, available power, allowable error in states, and thermal comfort must be met when the formulated cost function is optimized. The MPC implementation can proceed in simulation or finally in real experiment. At any point in time, the desired thermal comfort and its measured value are input into the cost function. The resulting function is an equation in control input that is then minimized and solved iteratively backwards from a selected terminal future time until the present one. Once a solution sequence of optimal control inputs from the present time into the future is obtained, only the control input corresponding to the present time is applied to advance the process to the next time step where the whole process is repeated. The difficulty in solving the optimization problem depends on the model of the system and the nature of the specified constraints. A real-time optimization solver is required for implantation in a real experiment. Model accuracy is a very important requirement in MPC design and implementation. Model mismatch means inaccurate prediction, leading to degraded performance.

MPC implementation is the easiest when an HVAC model is linear. This is why in most projects the nonlinear model is first linearized at the desired operating point of the system. Without the linear system model, the optimization problem is non-convex which is very challenging to solve. The ability to specify additional constraints in MPC formulation and to have the optimization routine handle them directly is the key strength of the MPC approach [10].

The fundamental idea of MPC is to solve an open-loop optimization problem with constriants at each time instance. Consider J, a cost function in terms of system state x_t, control input u_t, disturbance d_t, and their prediction. It is to be minimized subject to the specified constraints that follow:

$$\min_{k=1}^{N-1} J = f(x_{t+k|t}, u_{t+k|t}, d_{t+k|t}) \tag{1}$$

where $x_{k+t|t}$ denotes the system state x_k at time $t+k$ predicted at time t, $u_{k+t|t}$ denotes the control input u_k at time $t+k$ predicted at time t, $d_{k+t|t}$ denotes the system state d_k at time $t+k$ predicted at time t, and N denotes the prediction horizon.

The main constraint is linear or nonlinear dynamics of the system given as follows: $x_{t+1} = f(x_t, u_t, d_t)$. This equality constraints is used as the predictor to acquire the future behavior of the system.

Other specific constraints can include thermal comfort, HVAC operation limits such as limits on air supply fan, water pump, heating and cooling coils power. They are the inequality constraints in the formulation of optimization problem [15]. The direct and generic algorithm for solving it is by dynamic programming e.g., Sequential Quadratic Programming (SQP).

IV. MPC EXAMPLES FOR HVAC SYSTEMS

The use of MPC as the controller in building energy and HVAC systems is relatively new. For this reason, the pertinent research literature is relatively scant. Most of the literature comes from researchers at two research teams: the Center for the Built Environment (CBE) at the University of California at Berkeley and the OptiControl organization of the Swiss Federal Institute of Technology.

In a simulation example [14], the target building is the Bancroft Library at University of California at Berkeley. Its HVAC model is bilinear in states and inputs. Included in the model are internal and external heat loads described by equations affine in carbon dioxide concentration and outside temperature, respectively. Weather forecast to estimate the outside temperature from and the building's occupancy schedule are used to estimate the carbon dioxide concentration. Using historical data and the building's parameters in simulation employing nonlinear regression technique, they are able to extract the bilinear model. The model is linearized

TABLE I

I. Major MPC implementations and their qualitative comparison (B-Building, S-Computer Simulation, E-Experiment, P-Prototype)

Ref.	Mode l	Cost Function	Solution Method	Optimal	Study	Features
[15]	Linearized bilinear	Total energy, peak airflow	YALMIP and MATLAB	Yes	B, S	Unmodeled dynamics estimated, and both peak and total energy reduced
[5]	Linearized bilinear	Utility, heat/cooling	SQP solver BPMPD	Yes	B, S	Real-time utility cost minimization
[16]	Linearized bilinear	Electric bill, COP	Moving Window Blocking	Yes	B, E	Weather and thermal load prediction considered
[18]	Linearized bilinear	Energy cost, comfort	SQP on Scilab	Yes	B, E	Weather and occupancy forecast, thermal capacities for storage
[19]	MIMO/LTI	Multi-step-ahead error	YALMIP and SeDuMi solver	No	B, S	MPC relevant identification
[25]	Single time constant	Quadratic unconstrained	Linear Quadratic Regulation	No	B, S	Exploiting thermal capacities and solar energy
[26]	Linearized/nonlinear	Electric, water flow rate	Optimal Set-point Synthesis	Yes	P, S	No supervisory level, and decoupled subsystems
[28]	Feedback linearized bilinear	Control and output	Linear Quadratic Regulation	No	P, S	Feedback linearization
[29]	Stochastic bilinear	Thermal comfort, input and state	Dynamic programming	Yes	B, S	Models with uncertainties, and easy tuning
[30]	Linear polytropic	Lyapunov	Required gains computed off-line	No	P, S	Reduced computational load

at the equilibrium point nearest to the desired operating point. The energy consumption is a function of the heat and air supply inputs. A cost function expressed as a linear combination of the total heating power and peak airflow is then formulated. The constraints are bounds on both the control input and output temperature. The optimization is implemented using YALMIP and simulated in MATLAB. According to the report, a reduction of 67.2% is seen in total input airflow. The peak air flow rate is reduced by 33.3% and 73.2% in total energy consumption in comparison with the original controller.

In a simulation study [5], MPC control is designed and simulated on a same HVAC system [14]. It provides cooling using a variable speed supply fan and cooling coil located in a central AHU. HVAC in each room within this building is described with a bilinear model. Heat load due to the radiation among rooms is neglected such that the only thermal load considered is affine in internal temperature. Air supply fan speed determines the total air flow rate to all rooms. The work [5] ignores humidity effects and combines thermal capacity of the air, walls and furnishings of each room into a single lumped parameter to obtain a first order thermal dynamics model. They assume ambient outside temperature and thermal loads since the room occupants and equipment usage are known in advance. Energy consumption by the cooling and heating coils is determined based on the air-side thermal power that is calculated easily from the models. The bilinear model is then linearized to make the optimization problem convex. The cost function is formulated as a weighted sum of power consumed by heating circuit, power consumed by cooling circuit and air supply fan power. They employ a commercial SQP solver in their simulation to implement MPC. The simulation results demonstrate that MPC can produce the same results but in a coordinated manner.

In an experimental study [16], MPC is developed to control cooling in HVAC for a building at University of California at Merced. This building is equipped with water storage tank used to store cold water produced by a series of chillers. The energy-efficiency operation of HVAC is considered from the water-side. The work [16] aims to: 1) develop a simple switching nonlinear model for the storage tank identified and validated by historic data, 2) achieve the systematic integration of weather prediction in the MPC design to optimize the chiller operation and 3) design a low-complexity MPC scheme guaranteed to be robust against uncertain is considered load demands. The system model includes the chillers, cooling towers, thermal storage tanks and building itself. It also includes a thermal load component with solar and internal load predictors. The solar and internal load predictor uses time, date and cloud coverage as inputs to calculate inside and outside solar and internal loads. The outside solar load reflects the solar energy on outer surface of the building. Internal heat from occupants, lights and equipment are part of the internal load. The building thermal load predictor predicts the cooling load of buildings. The system and its load models are all calibrated using collected historical data. Their objective is to satisfy required cooling load, to minimize electricity bill and to maximize the coefficient of performance (COP). They employ a strategy known as moving window block to solve the optimization problem [17]. The results show about 19.1% improvement of the system COP over the baseline controller.

In a real experiment by Siroky *et al.* [18], MPC is implemented on HVAC inside a building at the Czech Technical University (CTU) in Praque. This building has two heating circuits and two reference rooms. Each room has one water heating circuit. The two rooms are each modeled as a RC-network. The overall model of the system includes stochastic output disturbance-based noise-corrupted sensor measurements. From the measured sensor data they employ the Maximum Likelihood algorithm to estimate the model parameters. Using the building plan according to material used and their tabular values they are able to also estimate the thermal resistance and capacitance parameters of the nodes in the model. The overall model is linear. The MPC they implement is at a higher level of a two level hierarchical controller. At the lower level, they also use PID controllers. The desired set-point temperature for each room is provided as a schedule. The cost function is then formulated as the sum of weighted 2-norm of the desired temperature tracking error and 1-norm of temperature difference at input and output of the water heater circuit. They use the Kalman filter to estimate the states during the implementation. The entire model of the system is implemented in RcWare. Even in the worst case, their experiment successfully shows 17% in energy savings over a conventional controller.

Zacekova and Privara [19] present MPC simulation study on same building [18]. In their work the emphasis is placed on model accuracy since model mismatch leads to unacceptable MPC performance. They consider the use of multi-step-ahead-prediction model identification methods known as MPC Relevant Identification [20]–[24] to acquire a required model. The HVAC system has a ceiling radiant heating system. It is described with a linear MIMO model. Inputs to this model are solar radiation prediction, outside temperature prediction and heating water temperature. Specified constraints include bounds on the control input and a lower bound on the output temperature tracking error. They formulate a cost function with two slack variables to change inequality constraints into equality constraints. The implementation is then simulated with results showing successful estimation of the external solar radiation load and 27% increase in energy efficiency over a classical rule based controller.

Zucker *et al.* [25] present simulation analysis in which solar energy is used as the power source to drive centrifugal pump actuators of HVAC . In this work, the HVAC model has one time constant for cooling and a second one for heating. The heating and cooling are thus equivalent to first order differential equation. A linear model is thus designed. The work [25] demonstrates not only feasibility to use the solar energy source in HVAC control systems but adaptability of MPC. The MPC simulation result shows 5% energy savings over conventional PI controller.

Komareji *et al.* [26] present MPC design on HVAC consisting of air-to-air and water-to-air heat exchangers. They define set-point and cost functions and then linearize the model. The MPC task in this design is to track this set point while satisfying maximum possible exploitation of the air-to-air heat exchanger and also to ensure that at the steady state supply, water flow equals the tertiary water flow. The MPC used at the higher level determines the set-points for the lower level controllers whose jobs are direct regulation to obtain steady state performance. Simulation results of the implemented MPC shows that the system optimally tracks the given set point of inlet air temperature.

Kelman *et al.* [27] present two different configurations of an HVAC system, i.e. a dual-duct single fan and single-duct VAV with reheat systems . Each is described with a model bilinear in supply air temperature and mass flow rate. The resulting optimization problems for both systems are therefore non-convex. Their objective is to investigate local optima of the MPC optimization problem. They then use existing Nonlinear Programming solver Ipopt with a large number of initial guesses distributed throughout the optimization space to explore the set of locally optimal solutions. They then perform the analysis of the different types of local optima and their physical interpretation. According to their result, the local optimum of the dual-duct single fan system has notably different energy cost. Hence finding a global optimal control strategy has obvious significance. In the single-duct VAV with reheat systems, the local optima are very similar to the optimal one. Local optima with practically equivalent costs but significantly different control could have important consequences. For example, switching between two local optima at successive time steps could cause oscillatory control inputs, which is undesirable and detrimental to actuator reliability in practice.

Rehrl and Horn [28] propose an MPC control method that uses feedback linearization to transform the nonlinear model of HVAC to allow a linear feedback control . Because the nonlinear plant is feedback-linearized, an unconstrained MPC is designed for the resulting linear plant. A Practical Nonlinear Model Predictive Control strategy is implemented on a bioclimatic building in Spain. It is used to deal directly with the nonlinear optimization problem. The control objective is to maintain thermal comfort of the building occupants measured on the Predicted Mean Vote index scale. The main sources of thermal disturbance in this system are outdoor temperature, relative humidity, surface radiation from the room, and the room occupants.

Oldewurtel *et al.* [29] implement a stochastic Model Predictive Control (SMPC) for a building climate control to increase energy efficiency while respecting constraints resulting from desired occupant comfort. The HVAC model of the building is bilinear under stochastic uncertainty with probabilistic time varying constraints. The major challenge with the use of numerical weather predictions in MPC implementation is the uncertainties due to the stochastic nature of atmospheric processes. Thus there is prediction error in each predicted disturbance. The uncertainty can be modeled as an autoregressive model driven by Gaussian noise. This probabilistic model is tested and found to fit the error seen in the prediction data. The bilinear model of the system leads to a non-convex optimization, which is solved by Sequential Linear Programming. This involves iteratively linearizing the non-convex constraints around the current solution and then solving the optimization problem, till convergence. The simulation results show that SMPC outperforms traditional MPC.

Vesely *et al.* [30] develop a new variant of MPC in which the control horizon model is derived as a linear polytropic system with a cost function formulated as a Lyapunovs' function. Its advantage is that on-line computational load is significantly less than in standard MPC because the time demanding the computation of output feedback gain matrices is realized off-line. The actual value of the control variable is obtained through simple on-line computation of parameters and respective convex combination of already computed matrix gains.

Table I is a summary of the major MPC implementations. The column "Study" indicate if the corresponding method is applied to a real building (B), a prototype lab system (P), a real experiment (E), only computer-simulated building (S). Most use a bilinear model to describe an HVAC system but linearize it prior to MPC implementation. Different cost functions and solution methods are utilized. Most claim to achieve the optimal solutions and significant energy saving over conventional/baseline methods.

V. CONCLUSIONS

To achieve optimal energy efficiency in the operation of HVAC systems, MPC and its variants offer a good solution. As demonstrated by the research reviewed in this paper, MPC and it variants have outperformed all classical control methods in terms of energy efficiency due to its nearly unique ability of systematically handling the states/inputs constraints and nonlinear coupling MIMO systems. Thus in world of dwindling energy supply any means to increase efficiency of energy use is welcomed. However, the performance of MPC is highly sensitive to model mismatch. For applications in which model uncertainties cannot be neglected, the design of their MPC is a big challenge. Another common problem with MPC is the large computation required to implement it, which calls for distributed MPC variants.

REFERENCES

[1] *2003 ASHRAE HANDBOOK : Heating, Ventilating, and Air-Conditioning Applications : Inch-Pound Edition*. American Society of Heating, Refrigerating and Air-Conditioning Engineers, 2003.

[2] W. Guo and M. Zhou, "Technologies toward thermal comfort-based and energy-efficient hvac systems: A review," in *Proceedings of 2012 IEEE International Conference on Systems, Man and Cybernetics*, pp. 3883–3888.

[3] F. Oldewurtel, A. Parisio, C. Jones, M. Morari, D. Gyalistras, M. Gwerder, V. Stauch, B. Lehmann, and K. Wirth, "Energy efficient building climate control using stochastic model predictive control and weather predictions," in *Proceedings of 2010 American Control Conference*, pp. 5100–5105.

[4] M. A. Cascia, "Implementation of a near-optimal global set point control method in a ddc controller," *ASHRAE Transactions*, vol. 106, no. 1, pp. 249–263, 2000.

[5] A. Kelman, Y. Ma, and F. Borrelli, "Bilinear model predictive control of a hvac system using sequential quadratic programming," in *Proceedings of 18th IFAC World Congress*, 2011, pp. 9869–9874.

[6] M. Kummert, P. Andre, and J. Nicolas, "Optimised thermal zone controller for integration within a building energy management system," in *Proceedings of CLIMA 2000 conference*, 1997.

[7] B. Paris, J. Eynard, S. Grieu, T. Talbert, and M. Polit, "Heating control schemes for energy management in buildings," *Energy and Buildings*, vol. 42, no. 10, pp. 1908 – 1917, 2010.

[8] A. Aswani, N. Master, J. Taneja, A. Krioukov, D. Culler, and C. J. Tomlin, "Energy-efficient building hvac control using hybrid system lbmpc," in *Proceedings of 4th IFAC Nonlinear Model Predictive Control Conference*, 2012, pp. 496–501.

[9] R. Gondhalekar, F. Oldewurtel, and C. Jones, "Least-restrictive robust mpc of periodic affine systems with application to building climate control," in *Proceedings of 49th IEEE Conference on Decision and Control*, 2010, pp. 5257–5263.

[10] R. Yang and L. Wang, "Optimal control strategy for hvac system in building energy management," in *Proceedings of Transmission and Distribution Conference and Exposition*, 2012, pp. 1–8.

[11] K. Deng, P. Barooah, and P. Mehta, "Mean-field control for energy efficient buildings," in *Proceedings of 2012 American Control Conference*, pp. 3044–3049.

[12] B. Arguello-Serrano and M. Velez-Reyes, "Nonlinear control of a heating, ventilating, and air conditioning system with thermal load estimation," *IEEE Transactions on Control Systems Technology*, vol. 7, no. 1, pp. 56–63, 1999.

[13] M. Gouda, S. Danaher, and C. Underwood, "Building thermal model reduction using nonlinear constrained optimization," *Building and Environment*, vol. 37, no. 12, pp. 1255 – 1265, 2002.

[14] M. Maasoumy and A. Sangiovanni-Vincentelli, "Total and peak energy consumption minimization of building hvac systems using model predictive control," *Design Test of Computers, IEEE*, vol. 29, no. 4, pp. 26–35, 2012.

[15] Y. Ma and F. Borrelli, "Fast stochastic predictive control for building temperature regulation," in *Proceedings of 2012 American Control Conference*, pp. 3075–3080.

[16] Y. Ma, F. Borrelli, B. Hencey, B. Coffey, S. Bengea, and P. Haves, "Model predictive control for the operation of building cooling systems," *IEEE Transactions on Control Systems Technology*, vol. 20, no. 3, pp. 796–803, 2012.

[17] R. Cagienard, P. Grieder, E. Kerrigan, and M. Morari, "Move blocking strategies in receding horizon control," *Journal of Process Control*, vol. 17, no. 6, pp. 563 – 570, 2007.

[18] J. Siroky, F. Oldewurtel, J. Cigler, and S. Privara, "Experimental analysis of model predictive control for an energy efficient building heating system," *Applied Energy*, vol. 88, no. 9, pp. 3079 – 3087, 2011.

[19] E. Zacekova and S. Privara, "Control relevant identification and predictive control of a building," in *Proceedings of 24th Chinese Control and Decision Conference*, 2012, pp. 246–251.

[20] R. Gopaluni, R. Patwardhan, and S. Shah, "Mpc relevant identification-tuning the noise model," *Journal of Process Control*, vol. 14, no. 6, pp. 699 – 714, 2004.

[21] R. B. Gopaluni, R. Bhushan, G. Rohit, R. S. Patwardhan, and S. L. Shah, "Bias distribution in mpc relevant identification," 2002.

[22] D. Shook, C. Mohtadi, and S. Shah, "Identification for long-range predictive control," *Control Theory and Applications, IEEE Proceedings D*, vol. 138, no. 1, pp. 75–84, 1991.

[23] D. Shook, C. Mohtadi and S. Shah, "A control-relevant identification strategy for gpc," *IEEE Transactions on Automatic Control*, vol. 37, no. 7, pp. 975–980, 1992.

[24] D. Lauri, M. Martinez, J. Salcedo, and J. Sanchis, "Pls-based model predictive control relevant identification: Pls-ph algorithm," *Chemometrics and Intelligent Laboratory Systems*, vol. 100, no. 2, pp. 118 – 126, 2010.

[25] G. Zucker, T. Ferhatbegovic, and D. Bruckner, "Building automation for increased energy efficiency in buildings," in *Proceedings of 2012 IEEE International Symposium on Industrial Electronics*, pp. 1191–1196.

[26] M. Komareji, J. Stoustrup, H. Rasmussen, N. Bidstrup, P. Svendsen, and F. Nielsen, "Optimal model-based control in hvac systems," in *Proceedings of 2008 American Control Conference*, pp. 1443–1448.

[27] A. Kelman, Y. Ma, and F. Borrelli, "Analysis of local optima in predictive control for energy efficient buildings," in *Proceedings of 50th Decision and Control and European Control Conference*, 2011, pp. 5125–5130.

[28] J. Rehrl and M. Horn, "Temperature control for hvac systems based on exact linearization and model predictive control," in *Proceedings of 2011 IEEE International Conference on Control Applications*, pp. 1119–1124.

[29] F. Oldewurtel, C. Jones, and M. Morari, "A tractable approximation of chance constrained stochastic mpc based on affine disturbance feedback," in *Proceedings of 47th IEEE Conference on Decision and Control*, 2008, pp. 4731–4736.

[30] V. Vesely, D. Rosinova, and M. Foltin, "Robust model predictive control design with input constraints," *ISA Transactions*, vol. 49, no. 1, pp. 114 – 120, 2010.