Weekly progress report

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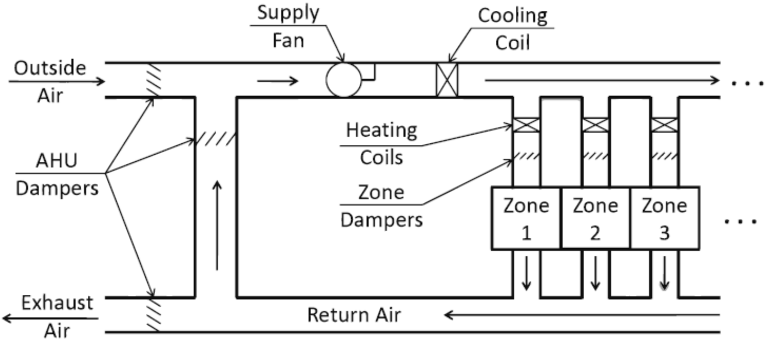
Outline

- HVAC system diagram
- Dynamic models
- Full nonlinear MPC
- Bounding of $\dot{m}_s^i$ for CO_2

HVAC system diagram



Temperature dynamics

The following two-mass dynamics are considered [Ma et al 2011]

$$C_1^j \dot{T}_1^j = c_p \dot{m}_s^j (T_s^j - T_1^j) + \frac{T_s^j - T_1^j}{R^j} + \frac{T_a - T_1^j}{R_a^j} + P_d^j + \sum_{i \in \mathcal{N}^j} \frac{T_1^i - T_1^j}{R_{ij}}$$

$$C_2^j T_2^j = \frac{T_1^j - T_2^j}{R_{12}^j}$$

$$T_s^j = \delta \frac{\sum_{i \in \mathcal{R}} \dot{m}_s^i T_1^i}{\sum_{i \in \mathcal{R}} \dot{m}_s^i} + (1 - \delta) T_a - \Delta T_c + \Delta T_h^j$$

States: T_1^j has fast dynamics (air temperature) and T_2^j has slow dynamics (temperature of furniture, floor, etc)

Inputs: $\dot{m}_s^j$ (air mass flow), δ (recirculation rate), ΔT_c (cooling temperature in chiller), ΔT_h^j (heating in each VAV box)

Disturbances: T_a (ambient temperature), P_d^j (internal gains: people, electronics, etc)

Other variables: T_s^j (supply temperature to room j)

CO_2 concentration dynamics

The following dynamics are considered [Parisio et al 2013]

$$\rho V^j \dot{C}^j_{CO_2} = \dot{m}^j_s (C_{CO_2,s} - C^j_{CO_2}) + g_{CO_2} N_{\text{people}}$$
$$C_{CO_2,s} = \delta \frac{\sum_{i \in \mathcal{R}} \dot{m}^i_s C^i_{CO_2}}{\sum_{i \in \mathcal{R}} \dot{m}^i_s} + (1-\delta) C_{CO_2,a}$$

States: $C^j_{CO_2}$ is the CO_2 concentration in room j

Inputs: $\dot{m}^j_s$, δ

Disturbances: $C_{CO_2,a}$ is the ambient CO_2 concentration

Other variables: ρ is the air density, V^j is the air volume in room j, g_{CO_2} is the rate of generation of CO_2 per person, $C_{CO_2,s}$ is the supply CO_2 concentration (i.e. from right after the chiller until being supplied to the room)

Humidity dynamics

The following dynamics are considered [Sun et al 2013][Wu et al 2015]

$$
\begin{aligned}
\rho V^j \dot{H}^j &= \dot{m}_s^j (H_s - H^j) + g_H N_{\text{people}} \\
H_m &= \delta \frac{\sum_{i \in \mathcal{R}} \dot{m}_s^i H^i}{\sum_{i \in \mathcal{R}} \dot{m}_s^i} + (1-\delta) H_a \\
H_s &= \min(H_m, H_{sat,T_s})
\end{aligned}
$$

States: H^j is the humidity ratio in room j

Inputs: $\dot{m}_s^j$, δ

Disturbances: H_a is the ambient humidity ratio

Other variables: g_H is the rate of water vapor generation per person, H_m is the mixed humidity (recirculation and ambient), H_s is the supply humidity ratio and H_{T_c} is the saturation humidity right after the chiller

Nonlinear MPC

The prediction dynamics are

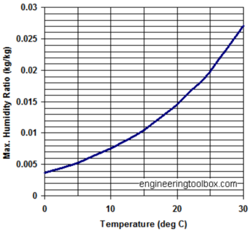
$$
\begin{aligned}
T_k^{i+} =& T_k^i + \frac{\Delta t c_p}{C_i} \dot{m}_{s,k}^i (T_s - T_{1,k}^i) + \sum_{j \in \mathcal{N}_i} \frac{\Delta t}{C_i R_{i,j}} (T_k^j - T_k^i) + \\
& \frac{\Delta t}{C_i R_{a,i}} (T_k^a - T_k^i) + \frac{\Delta t}{C_1} P_{d,i} \\
T_{s,k}^i =& \delta_k \frac{\sum \dot{m}_{s,k}^i T_k^i}{\sum \dot{m}_{s,k}^i} + (1-\delta_k) T_k^{amb} + \Delta T_k \\
C_{CO_2,k}^{i+} =& C_{CO_2,k}^i + \frac{\Delta t}{V_i \rho_{air}} (\dot{m}_{s,k}^i (C_{CO_2,k}^s - C_{CO_2,k}^i) + \frac{g_{CO_2,k}^i}{V_i}) \\
C_{CO_2,k}^s =& \delta_k \frac{\sum \dot{m}_{s,k}^i C_{CO_2,k}^i}{\sum \dot{m}_{s,k}^i} + (1-\delta_k) C_{CO_2,k}^{amb} \\
H_k^{i+} =& H_k^i + \frac{\Delta t}{V_i \rho_{air}} (\dot{m}_{s,k}^i (H_k^s - H_k^i) + \frac{g_{CO_2,k}^i}{\rho_{air} V_i}) \\
H_k^m =& \delta_k \frac{\sum \dot{m}_{s,k}^i C_{CO_2,k}^i}{\sum \dot{m}_{s,k}^i} + (1-\delta_k) H_k^{amb}, \quad H_k^s = \min(H_k^m, H_{sat,T_s})
\end{aligned}
$$

Nonlinear MPC

Note the term

$$H_k^s = \min(H_k^m, H_{sat,T_s})$$

The following approximation is used $H_{sat,T_s} = a_1 + b_1 T_s$



Also, in order to have a smooth optimization the approximation is considered

$$\min(x, y) \approx -\frac{log(exp(-kx) + exp(-ky))}{k}$$

for some large k.

Nonlinear MPC

We optimize the cost

$$J_{power} = -\sum_{k=0}^{N_1} \frac{c_p}{\eta_c} \dot{m}^i_{s,k} \Delta T_k + \kappa_f \left(\sum_i \dot{m}^i_{s,k} \right)^2$$

Impose the input constraints

$$-\Delta T_{max} \leq \Delta T_k \leq 0, \quad 0 \leq \dot{m}^i_{s,k} \leq 5[kg/s], \quad 0 \leq \delta \leq 1$$

and the state constraints

$$21 \leq T^i_k \leq 24, \quad C^i_{CO_2,k} \leq 1200[ppm], \quad 35\% \leq RH^i_k \leq 65\%$$

Where it is used that $RH = H/H_{sat,T}$. The last constraint can then be implemented as

$$0.35(a_2 + b_2 T_s) \leq H^i_k \leq 0.65(a_2 + b_2 T_s)$$

Bounding of $\dot{m}_s^i$ for CO_2

The Standard ASHRAE 62.1 provides rules for finding the minimum fresh-air mass flows sent to the rooms, which for our settings is given by

$$\dot{m}_{fa} = \frac{\sum_{i=1}^{q} R_p P_i + R_a A_i}{E_z}$$
$$\dot{m}_{fa}^i = \frac{R_p P_i + R_a A_i}{E_z}$$

where the distribution efficiency $E_z \in [0.5, 1]$ is a function of the type of room, R_p, R_a are the minimum outdoor airflow rate per person and per unit area, P_i is the number of occupants and A_i is the occupable area in region i.

The constraint is

$$(1 - \delta)\dot{m}_s^i \geq \dot{m}_{fa}^i$$

Bounding of $\dot{m}_s^i$ for CO_2

This lower bound could be conservative or too loose. It is proposed to use an adaptive procedure based on Stochastic Approximation

$$\alpha_{k+1} = \alpha_k + \mu_k Yk$$

where Y_k measures the constraint violations (could also use moving average or other filter), and α is a scaling so that the bound on air mass flow is

$$(1-\delta)\dot{m}_s^i \geq \alpha \dot{m}_{fa}^i$$

so that:

- If air quality is too bad (high CO_2) too often, then more fresh air is required
- If air quality is good all the time (low CO_2) then less fresh air can be allowed

Humidity control

A static control is considered for humidity.

From the dynamic equation

$$\rho V^i \dot{H}^i = \dot{m}_s^i (H_s - H^i) + g_H^i N_{\text{people}}^i$$
$$H_m = \delta \frac{\sum_{i \in \mathcal{R}} \dot{m}_s^i H^i}{\sum_{i \in \mathcal{R}} \dot{m}_s^i} + (1 - \delta) H_a$$
$$H_s = \min(H_r, H_{sat,T_s})$$

It follows that the static equilibrium is

$$H^i = H_s + \frac{g_H^i N_{\text{people}}^i}{\dot{m}_s^i}$$
$$H_r = H_s + \frac{\sum_i g_H^i N_{\text{people}}^i}{\sum_i \dot{m}_s^i}$$
$$H_m = (1 - \delta) H_{amb} + \delta H_r$$
$$H_s = \min(H_m, H_{sat,T_s})$$

Humidity control

The constraint $35\% \leq RH^i \leq 65\%$, where $RH = H^i / H_{sat,T^i}$, can be implemented as

$$0.35(a_2 + b_2 T^i) \leq H^i \leq 0.65(a_2 + b_2 T^i)$$

which using the static equilibrium described above can be cast as

$$0.35(a_2 + b_2 T^i)\dot{m}_s^i \leq H_s \dot{m}_s^i + g_H^i N_{\text{people}}^i \leq 0.65(a_2 + b_2 T^i)\dot{m}_s^i$$

Finally, in order to incorporate the adaptive stage, this is replaced by

$$0.35(a_2 + b_2 T^i)\dot{m}_s^i \underline{\alpha} \leq H_s \dot{m}_s^i + g_H^i N_{\text{people}}^i \leq \overline{\alpha} 0.65(a_2 + b_2 T^i)\dot{m}_s^i.$$

The adaptive stage is given by

$$\underline{\alpha}_{k+1} = \underline{\alpha}_k + \mu_k \underline{Y} k, \quad \overline{\alpha}_{k+1} = \overline{\alpha}_k + \mu_k \overline{Y} k$$

where $\underline{Y}_k$, $\overline{Y}_k$ measure the constraint violations (using moving average or other filter), and $\underline{\alpha}_k$, $\overline{\alpha}_k$ are the adaptive scaling scaling parameters

Increased actuation needed

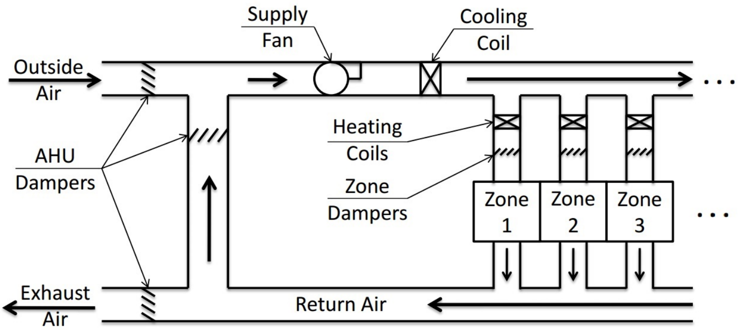
Previous simulations have shown that the system is under-actuated for joint temperature-CO2-humidity control.

Thus we consider 2 options.

- Cool-reheat dehumidifier: air is cooled and dehumidified in the cooling coiled, and if needed it is reheated in reheating coils.
- Desiccant Wheel dehumidifier: air is dried chemically by absorbent material.

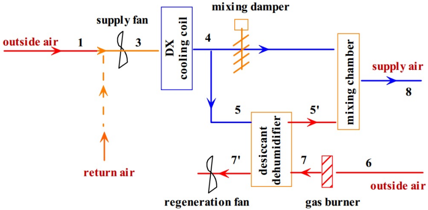
Cool-reheat dehumidifier

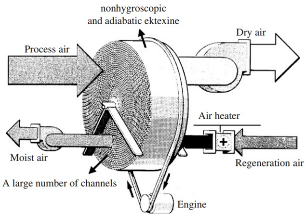
It does not require explicit/dedicated dehumidifier. It works as an appropriate configuration of cooling and heating elements.



Desiccant wheel dehumidifier

Desiccant wheel has absorbent material that absorbs moisture from input air. Absorbent material is regenerated with hot air.

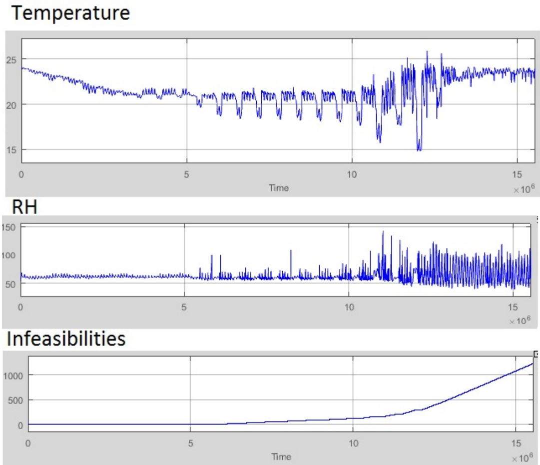




Simulation results A

For studying an increasedly actuated system we consider cool-reheat dehumification such that $0 \leq \Delta T_h \leq 8^o C$

We have a building of 4 rooms, and it is again observed that at some point to optimization problem stopped being feasible.



*temperature range is $[21, 24]$, α_k follows the adaptive law

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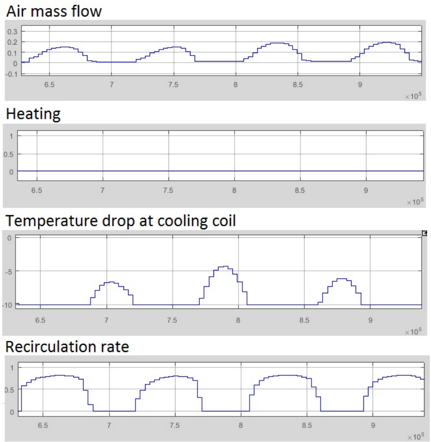

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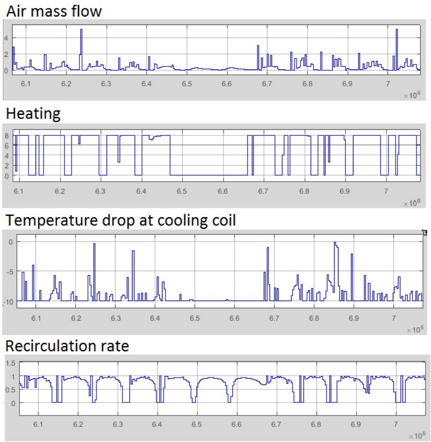

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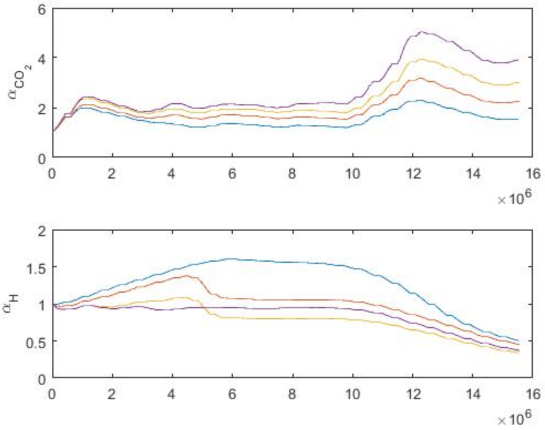

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Simulation results A

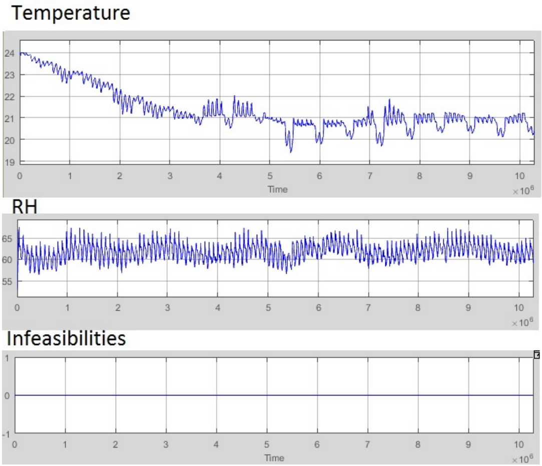
Infeasibilities happen because of the adaptation of α_k:

- this makes the static constraint for humidity stronger, which makes the temperature go down in time

and the limits in ΔT_h.

Simulation results B

Now the same simulation but with $0 \leq \Delta T_h \leq 30^o C$



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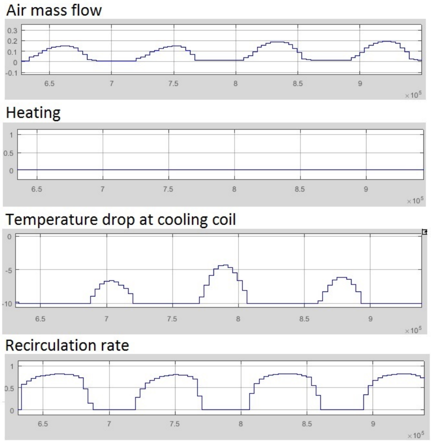

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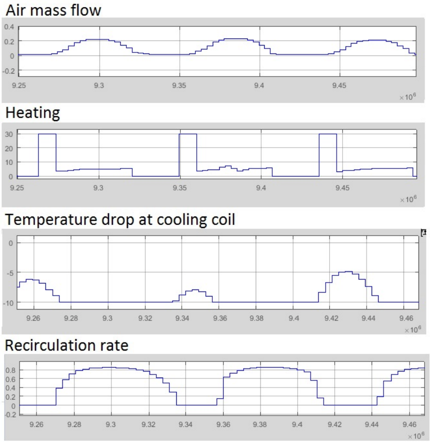

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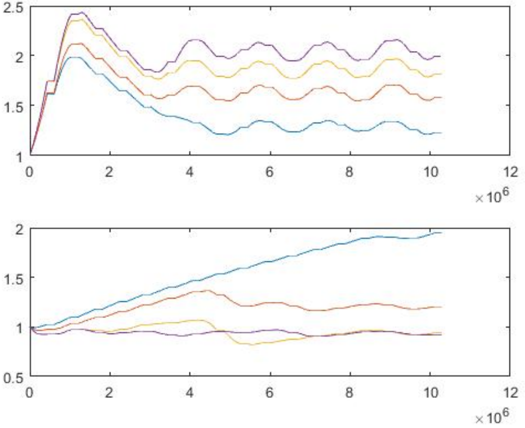

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Simulation B

There are no infeasibilities now (at least in the simulation range).

But system seems to be highly inefficient:

- Large re-heating is necessary. Lots of energy consumption.
- Is it really necessary for temperature to drop in time for humidity constraints to be satisfied? No ... this seems to be an artifact of the static constraints

Alternative type of constraint coupled with increased actuation is necessary.