Supplementary Material of "A Sparse Large-scale Multi-objective Evolutionary Optimization based on Bi-level Interactive Grouping"

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1. Computational Efficiency

The computational efficiency of BLIGEA was assessed in comparison to five other MOEAs. Fig. S1 illustrates the runtime (in seconds) for benchmark SMOPs with decision variables ranging from 500 to 5000, along with five real-world application problems (FS, CN, NN, PO, and SR). The comparison reveals that BLIGEA outperforms PM-MOEA in terms of efficiency and is competitive with SparseEA and MSKEA. This is attributed to the fact that the evolutionary patterns formed by small populations in PM-MOEA lead to excessive runtime. Additionally, DSGEA requires sorting of the *Dec* vectors for each individual based on the grouping method, which results in longer runtime, while BLIGEA sorts the *Dec* vectors based on the distribution of *Dec* in the population. In summary, BLIGEA achieves satisfactory results with lower computational costs.

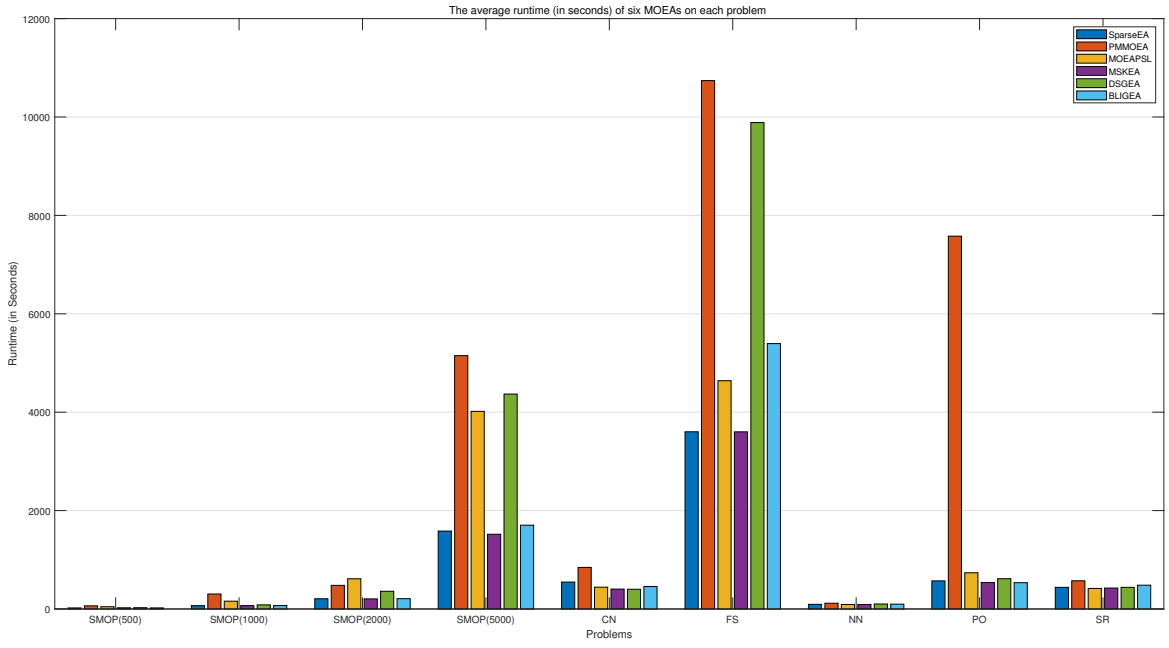

Figure S1: The runtime (in seconds) of six MOEAs on four benchmark SMOPs with 500, 1000, 2000 and 5000 decision variables and five real-world problems (FS, CN, NN, PO, and SR).

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2. Ablation Experiments

Table S1: THE MEAN AND STANDARD DEVIATION OF IGD VALUE OBTAINED BY INDEPENDENTLY RUNNING BLIGEA AND ITS FOUR VARIANTS ON THE BENCHMARK SMOPS WITH 1000 DECISION VARIABLES FOR 30 TIMES. (THE CONTENTS IN PARENTHESES IN THE TABLE REPRESENT THE STANDARD DEVIATION)

Problem	M	D	BLIGEA′	BLIGEA″	BLIGEA‴	BLIGEA⁗	BLIGEA
SMOP1	2	1000	4.2080e-3 (2.43e-4) =	1.7664e-2 (3.36e-3) -	8.0972e-3 (2.90e-3) -	1.1863e-2 (2.49e-3) -	4.0792e-3 (1.96e-4)
SMOP2	2	1000	1.0181e-2 (6.21e-4) -	5.9560e-2 (8.30e-4) -	2.9623e-2 (4.53e-3) -	3.8542e-2 (4.07e-3) -	7.9225e-3 (1.98e-3)
SMOP3	2	1000	3.8570e-3 (6.09e-5) =	2.6243e-2 (9.92e-4) -	4.7653e-3 (6.04e-4) -	2.7254e-2 (3.12e-3) -	3.7747e-3 (3.86e-5)
SMOP4	2	1000	4.1758e-3 (4.45e-5) -	4.1456e-3 (4.08e-5) -	4.1111e-3 (4.69e-5) =	4.6959e-3 (2.31e-4) -	4.0899e-3 (1.59e-5)
SMOP5	2	1000	5.5520e-3 (1.97e-4) -	7.0442e-3 (2.34e-4) -	4.4780e-3 (8.35e-5) -	6.0131e-3 (4.21e-4) -	4.2315e-3 (6.99e-5)
SMOP6	2	1000	5.1664e-3 (2.50e-4) -	9.1650e-3 (3.63e-4) -	4.9447e-3 (2.78e-4) =	6.9821e-3 (4.03e-4) -	4.5666e-3 (1.66e-4)
SMOP7	2	1000	2.1016e-2 (8.84e-3) =	7.5807e-2 (1.04e-2) -	3.9638e-2 (2.84e-3) -	8.0102e-2 (7.03e-3) -	1.3399e-2 (1.05e-3)
SMOP8	2	1000	1.4386e-1 (7.08e-3) =	2.5078e-1 (1.21e-2) -	1.6217e-1 (9.96e-3) -	2.5027e-1 (1.23e-2) -	1.3666e-1 (1.17e-2)
+/-/≈			0/4/4	0/8/0	0/6/2	0/8/0	

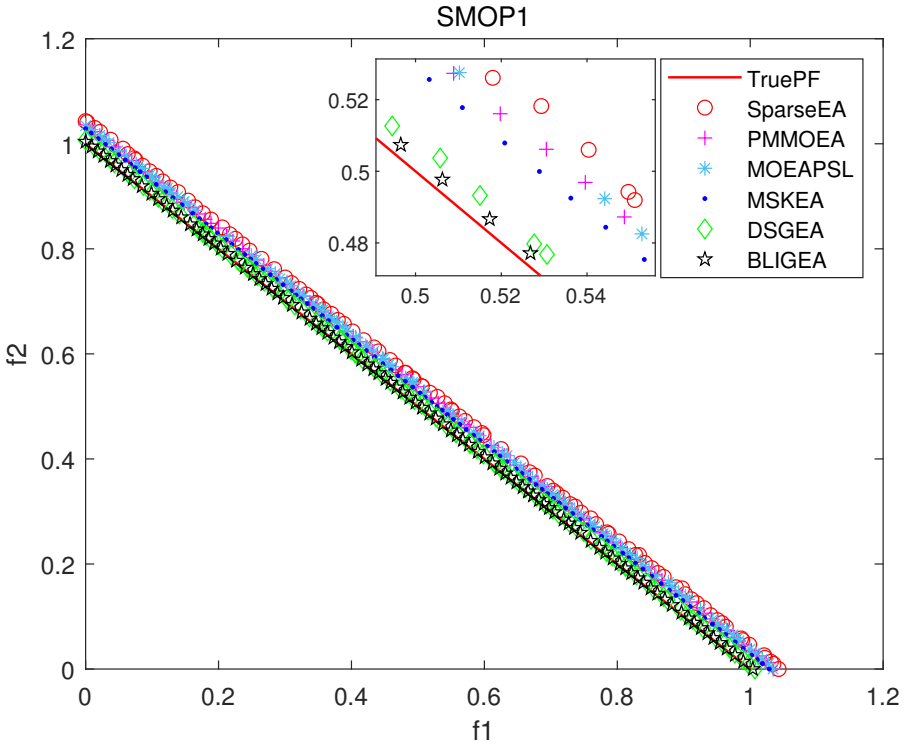

Figure S2: The Pareto front of six MOEAs on SMOP1 with 2000 decision variables.

3. Extra Experiment

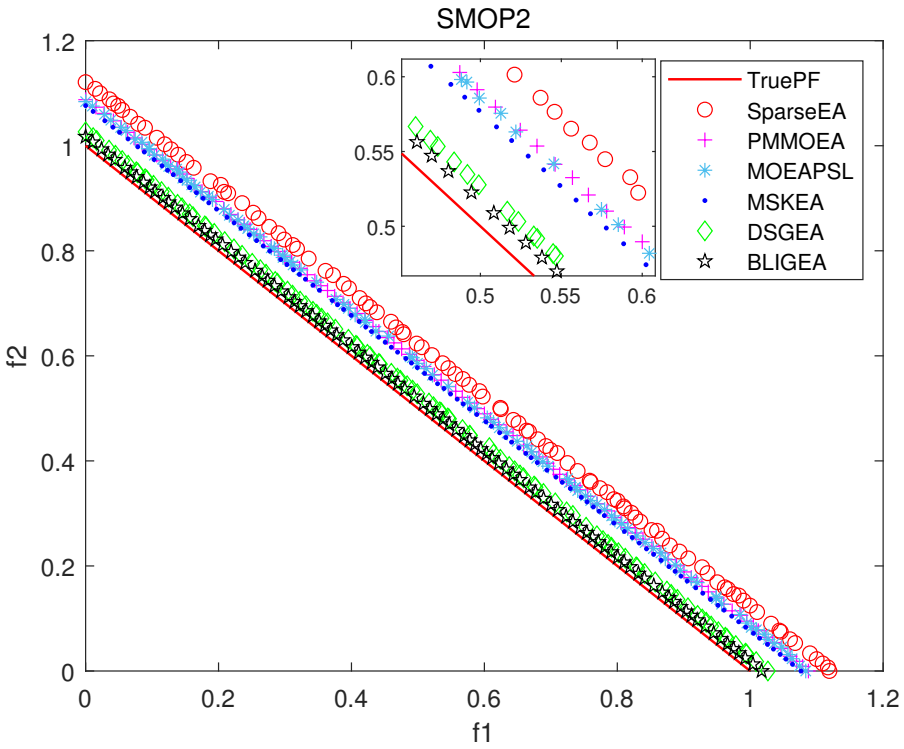

Figure S3: The Pareto front of six MOEAs on SMOP2 with 2000 decision variables.

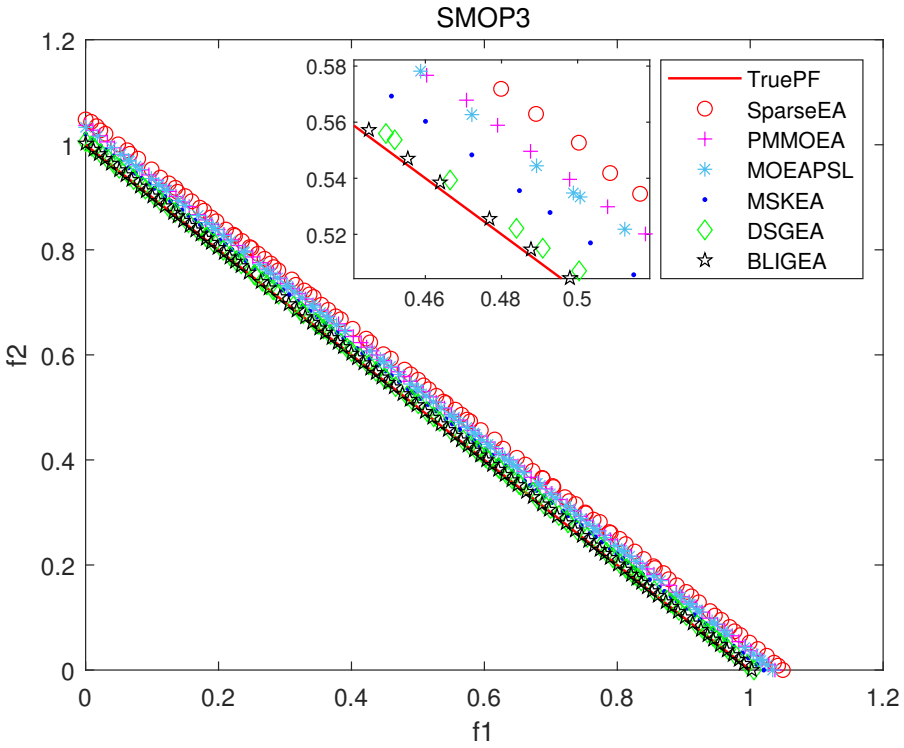

Figure S4: The Pareto front of six MOEAs on SMOP3 with 2000 decision variables.

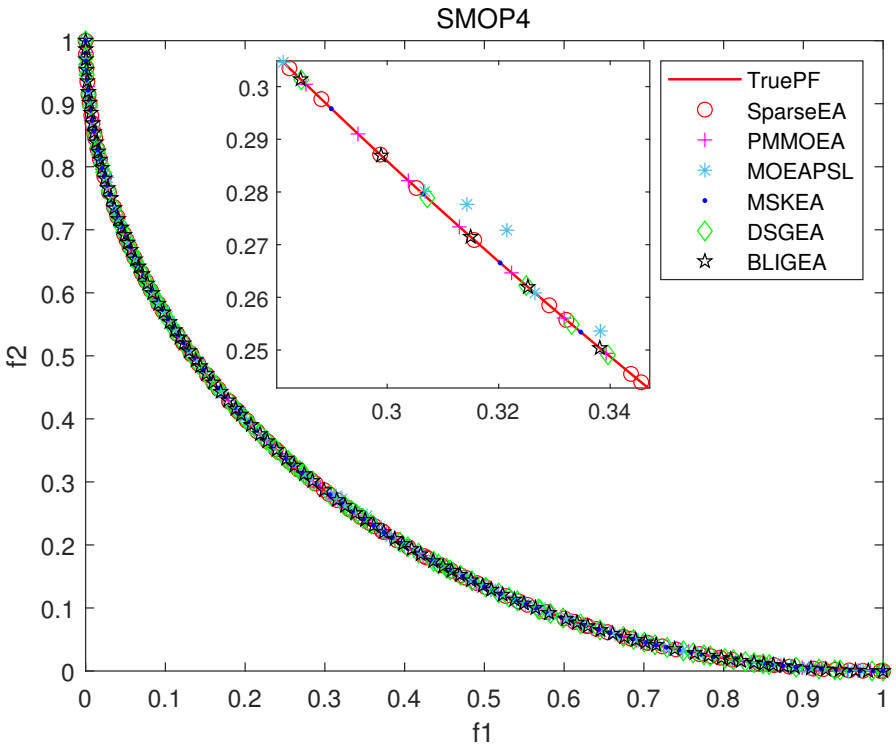

Figure S5: The Pareto front of six MOEAs on SMOP4 with 2000 decision variables.

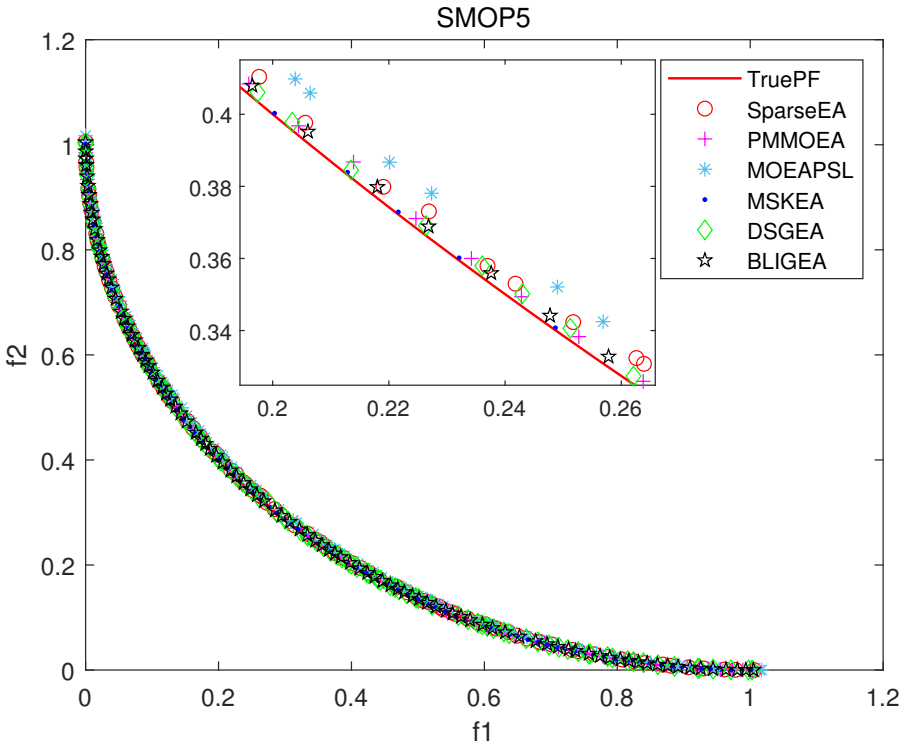

Figure S6: The Pareto front of six MOEAs on SMOP5 with 2000 decision variables.

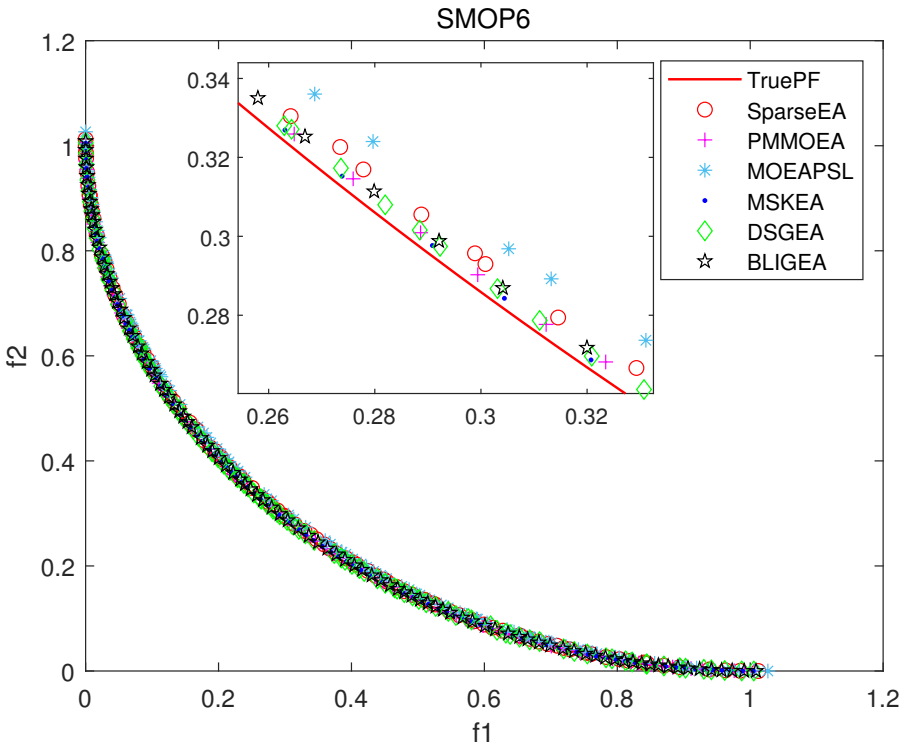

Figure S7: The Pareto front of six MOEAs on SMOP6 with 2000 decision variables.

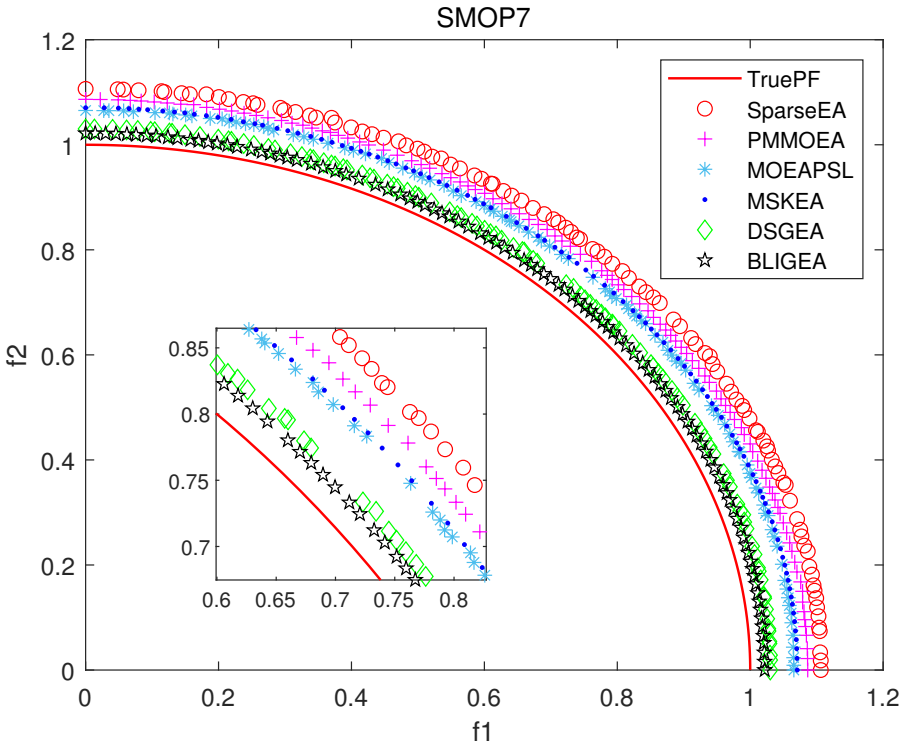

Figure S8: The Pareto front of six MOEAs on SMOP7 with 2000 decision variables.

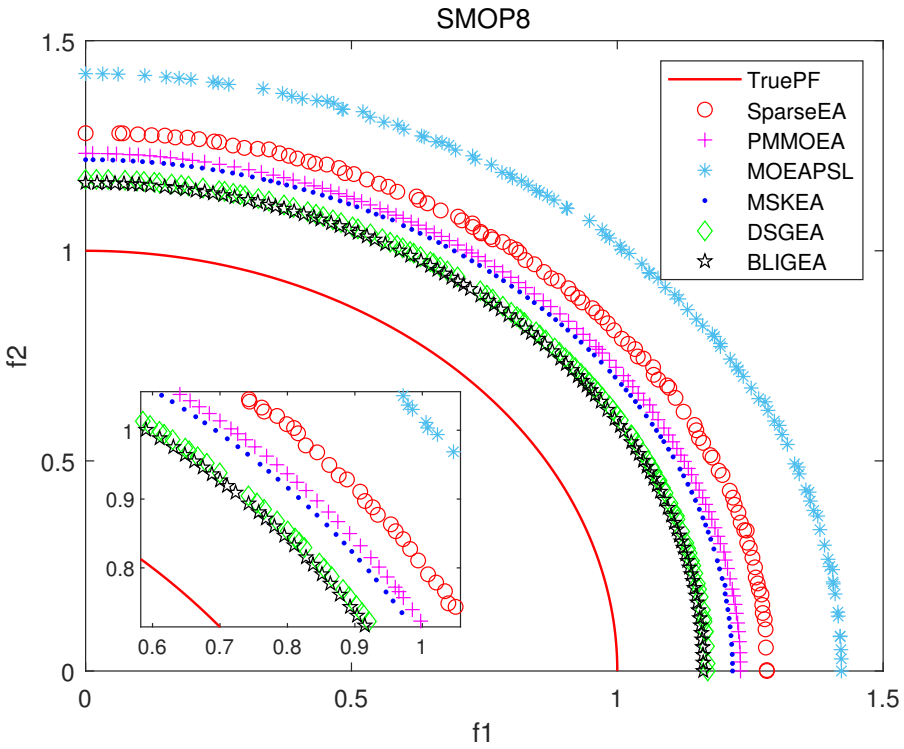

Figure S9: The Pareto front of six MOEAs on SMOP8 with 2000 decision variables.

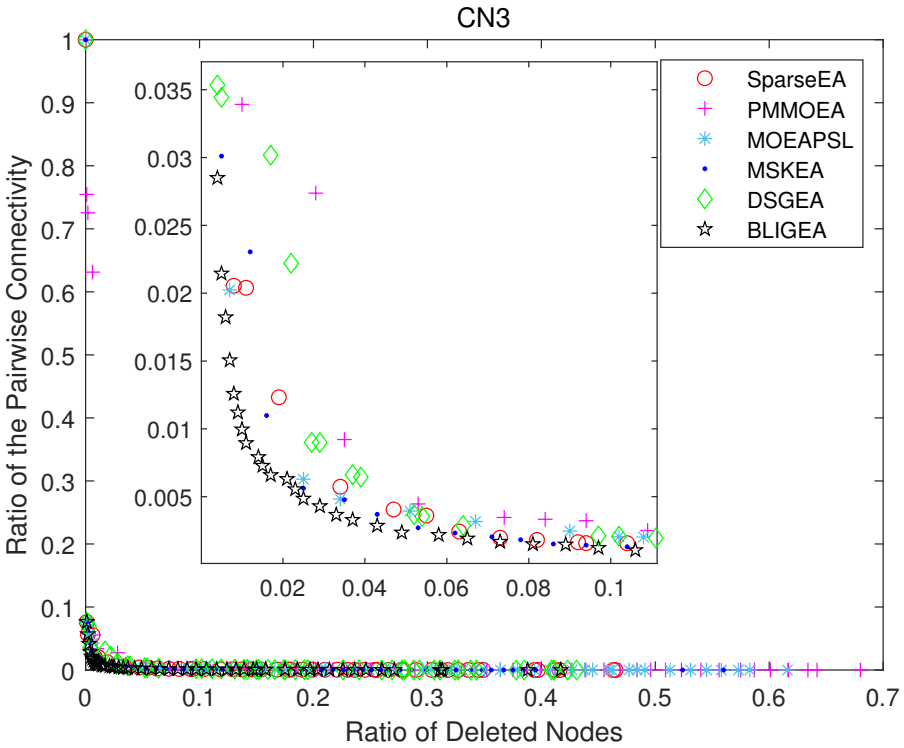

Figure S10: The Pareto front of six MOEAs on CN3.

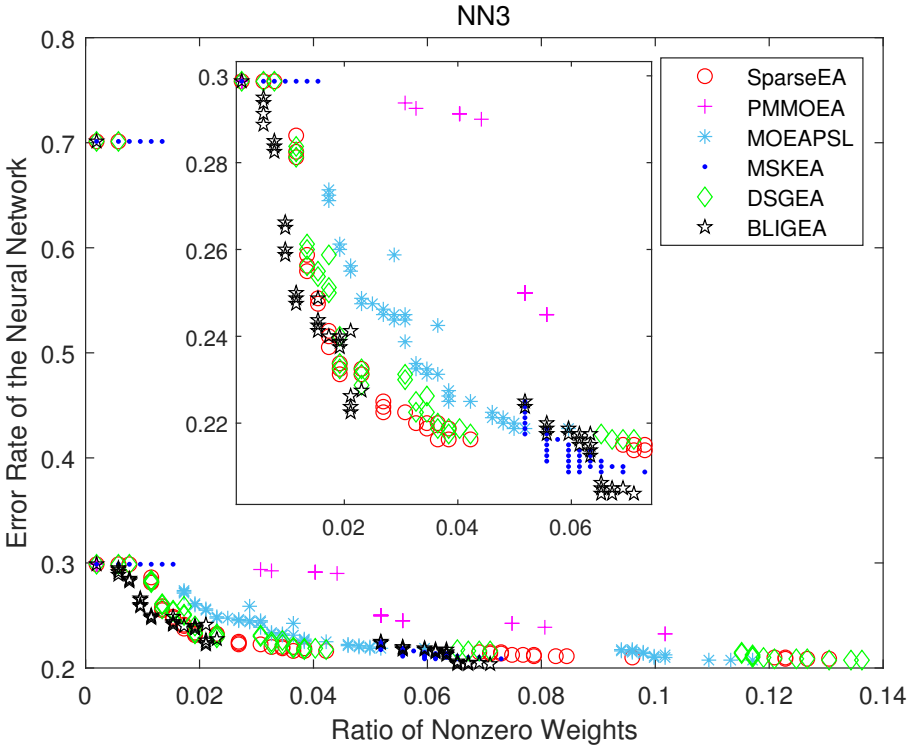

Figure S11: The Pareto front of six MOEAs on NN3.

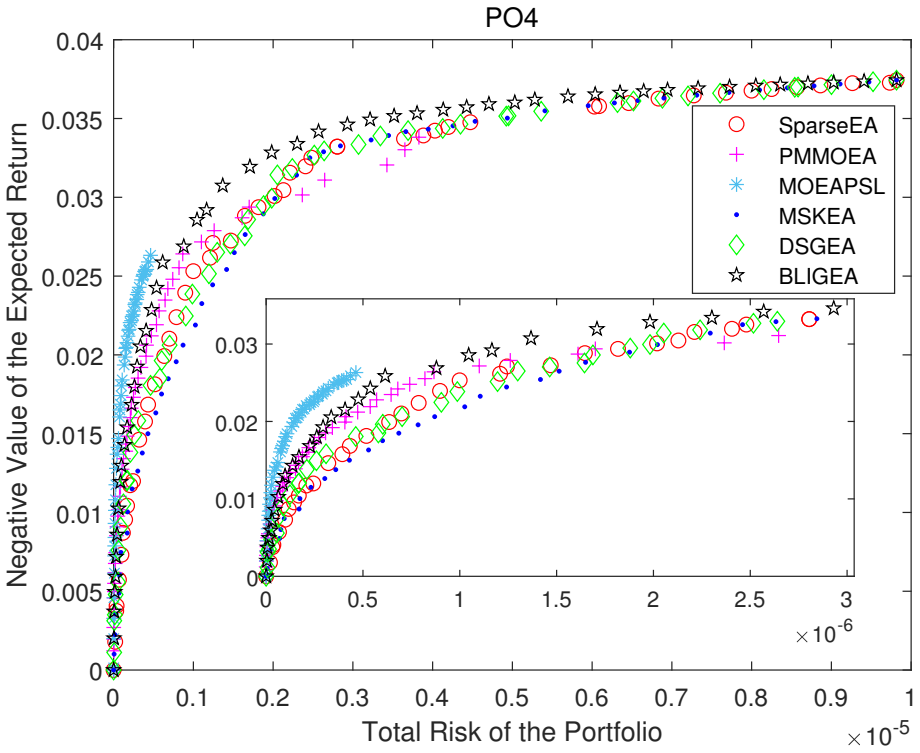

Figure S12: The Pareto front of six MOEAs on PO4.

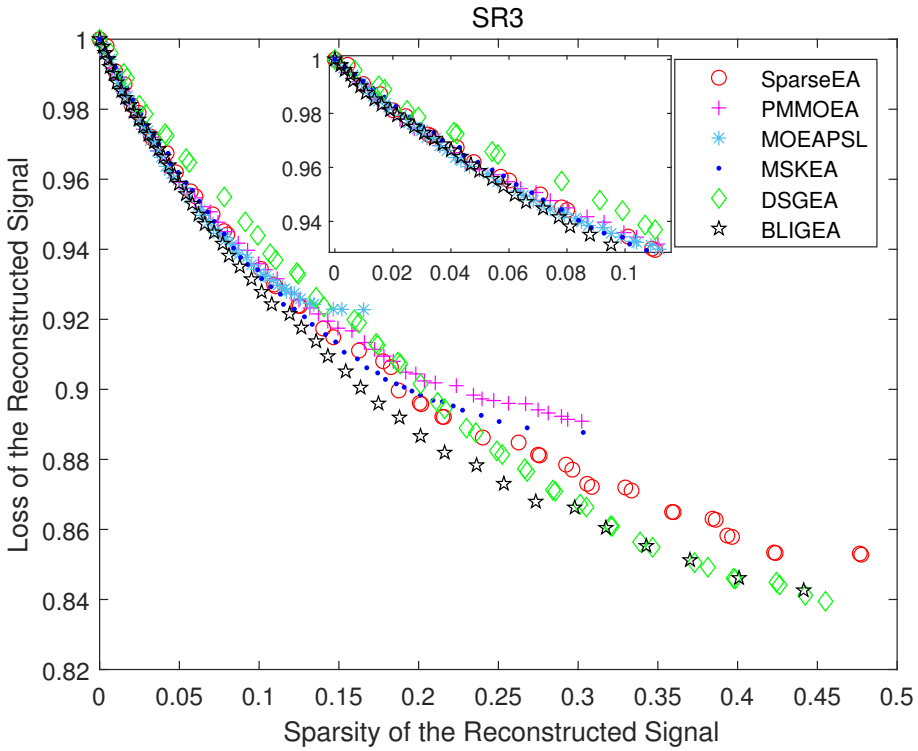

Figure S13: The Pareto front of six MOEAs on SR3.

Table S2: STATISTICS OF IGD RESULTS ACHIEVED BY THREE COMPARED MOEAS ON BENCHMARK SMOP1-8.

Problem	D	DKCA	MOEA/CKF	BLIGEA
SMOP1	500	7.1298e-3 (1.34e-3) -	6.2803e-3 (1.55e-3) -	4.0523e-3 (5.12e-4)
	1000	1.1318e-2 (1.97e-3) -	9.8863e-3 (2.83e-3) -	4.0108e-3 (1.96e-4)
	2000	1.5664e-2 (2.03e-3) -	9.1845e-3 (1.83e-3) -	4.9189e-3 (2.82e-4)
	5000	2.3911e-2 (7.51e-4) -	1.4137e-2 (1.93e-3) -	8.0952e-3 (8.39e-4)
SMOP2	500	2.0421e-2 (5.10e-3) -	1.4876e-2 (5.37e-3) -	7.2412e-3 (2.37e-3)
	1000	3.4586e-2 (7.25e-3) -	2.7309e-2 (6.34e-3) -	9.2684e-3 (2.58e-3)
	2000	5.0161e-2 (3.24e-3) -	4.0513e-2 (4.30e-3) -	1.2904e-2 (1.51e-3)
	5000	8.7173e-2 (1.61e-3) -	7.1470e-2 (2.62e-3) -	2.3465e-2 (2.94e-3)
SMOP3	500	7.2635e-3 (9.07e-4) -	6.2872e-3 (1.43e-3) -	3.7246e-3 (2.94e-5)
	1000	1.0324e-2 (1.16e-3) -	1.2873e-2 (1.73e-3) -	3.8781e-3 (1.04e-4)
	2000	1.5899e-2 (2.09e-3) -	1.9941e-2 (2.61e-3) -	4.2889e-3 (2.06e-4)
	5000	2.7319e-2 (1.97e-3) -	2.6513e-2 (2.25e-3) -	5.5688e-3 (2.27e-4)
SMOP4	500	5.0593e-3 (2.86e-4) -	4.1728e-3 (9.26e-5) =	4.1655e-3 (6.89e-5)
	1000	4.9727e-3 (1.43e-4) -	4.2127e-3 (5.24e-5) =	4.0901e-3 (2.19e-5)
	2000	4.9961e-3 (2.42e-4) -	4.2123e-3 (1.01e-4) -	4.0938e-3 (3.46e-5)
	5000	4.6437e-3 (2.84e-4) -	3.9910e-3 (7.52e-5) +	4.1672e-3 (5.05e-5)
SMOP5	500	7.9182e-3 (5.82e-4) -	7.1208e-3 (8.58e-4) -	4.3198e-3 (1.71e-4)
	1000	7.7726e-3 (3.51e-4) -	6.1295e-3 (2.05e-4) -	4.2563e-3 (7.15e-5)
	2000	7.6188e-3 (3.08e-4) -	5.3370e-3 (1.24e-4) -	4.5769e-3 (1.13e-4)
	5000	7.4478e-3 (6.15e-5) -	4.7034e-3 (2.76e-5) +	5.8260e-3 (2.58e-4)
SMOP6	500	9.1057e-3 (6.04e-4) -	5.9022e-3 (5.26e-4) -	4.3449e-3 (1.27e-4)
	1000	8.9003e-3 (6.12e-4) -	4.7691e-3 (1.52e-4) =	4.5417e-3 (1.56e-4)
	2000	8.9869e-3 (4.52e-4) -	4.2531e-3 (9.27e-5) +	5.4926e-3 (1.85e-4)
	5000	9.2600e-3 (6.89e-4) =	3.9357e-3 (8.87e-4) +	7.4427e-3 (3.92e-4)
SMOP7	500	2.4859e-2 (6.19e-3) -	1.8580e-2 (7.44e-3) -	7.7899e-3 (4.30e-3)
	1000	4.6710e-2 (1.01e-2) -	3.8750e-2 (7.79e-3) -	1.4132e-2 (2.55e-3)
	2000	9.2965e-2 (5.85e-2) -	7.1839e-2 (8.66e-2) -	2.2562e-2 (2.21e-3)
	5000	1.1195e-1 (3.55e-3) -	8.1147e-2 (5.53e-3) -	3.3556e-2 (2.59e-3)
SMOP8	500	1.3543e-1 (2.21e-2) =	1.2791e-1 (2.61e-2) =	1.2039e-1 (9.45e-3)
	1000	1.7103e-1 (1.81e-2) -	1.7324e-1 (1.93e-2) -	1.3873e-1 (9.64e-3)
	2000	1.8993e-1 (2.78e-3) =	1.7132e-1 (1.28e-2) =	1.5935e-1 (3.78e-3)
	5000	2.5992e-1 (1.66e-3) -	2.2072e-1 (4.96e-3) -	1.8579e-1 (2.46e-4)
+/-/=		0/29/3	4/23/5	