COMP 6771 Image Processing: Assignment 2

Student name: YUNQI XU

Student id: 40130514

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1. Question 1

   (a) Based on the question, the mask is:

$$g(x,y) = \frac{1}{4}[f(x,y-1) + f(x,y+1) + f(x-1,y) + f(x+1,y)] \tag{1}$$

Also,

$$f(x-x_0, y-y_0) = F(u,v)e^{-j2\pi(ux_0/M+vy_0/N)} \tag{2}$$

Based on the Eq. 2, the Eq. 1 can be calculated like:

$$\begin{aligned} f(x,y-1) &= f(x-0, y-(1)) \\ &= f(u,v)e^-j2\pi(u(0)/M + v(1)/N) \\ &= F(u,v)e^{-j2\pi v/N} \end{aligned} \tag{3}$$

$$\begin{aligned} f(x,y+1) &= f(x-0, y-(-1)) \\ &= F(u,v)e^{-j2\pi(u(0)/M+v(-1)/N)} \\ &= F(u,v)e^{j2\pi u/M} \end{aligned} \tag{4}$$

$$\begin{aligned} f(x-1,y) &= f(x-(1), y-0) \\ &= F(u,v)e^{-j2\pi(u(1)/M+v(0)/N)} \\ &= F(u,v)e^{-j2\pi u/M} \end{aligned} \tag{5}$$

$$\begin{aligned} f(x-1,y) &= f(x-(1), y-0) \\ &= F(u,v)e^{-j2\pi(u(1)/M+v(0)/N)} \\ &= F(u,v)e^{-j2\pi u/M} \end{aligned} \tag{6}$$

$$\begin{aligned} f(x+1,y) &= f(x-(-1), y-0) \\ &= F(u,v)e^{-j2\pi(u(-1)/M+v(0)/N)} \\ &= F(u,v)e^{-j2\pi u/M} \end{aligned} \tag{7}$$

So, based on the Eq. 2 4 6 7,

$$G(u,v) = \frac{1}{4}F(u,v)[e^{-j2\pi v/N} + e^{j2\pi v/N} + e^{j2\pi u/M} + e^{j2\pi u/M}] \tag{8}$$

$$H(u,v) = \frac{1}{4}[e^{-j2\pi v/N} + e^{j2\pi v/N} + e^{j2\pi u/M} + e^{j2\pi u/M}] \tag{9}$$

Based on the Euler's Formula,

$$\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$$

,

$$\begin{aligned} H(i,v) &= \frac{1/4}{F}(u,v)[2\cos(\frac{2\pi v}{N} + 2\cos(\frac{2\pi u}{M}))] \\ &= \frac{1}{2}F(u,v)[\cos(\frac{2\pi v}{N} + \cos(\frac{2\pi u}{M}))] \end{aligned} \tag{10}$$

2. Question 2

   (a) If an equation is linear, which means that:

$$O(af_1(x,y)+bf_2(x,y)) = aO(f_1(x,y)) + bO(f_2(x,y)) \tag{11}$$

   In Eq. 11, the $O()$ is an operator. So in this queation:

$$\begin{aligned} O(af_1(x,y)+bf_2(x,y)) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(af_1(x,y)+bf_2(x,y))\delta(x\cos\theta + y\sin\theta - \rho)dxdy \\ &= a\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_1(x,y)\delta(x\cos\theta + y\sin\theta - \rho)dxdy + \\ & b\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_2(x,y)\delta(x\cos\theta + y\sin\theta - \rho)dxdy \\ &= aO(f1(x,y)) + bO(f2(x,y)) \end{aligned} \tag{12}$$

   So it is linear operator.

   (b) Based on the priciple of Integral by substitution:

$$\begin{aligned} u &= x - x_0 \\ v &= y - y_0 \end{aligned} \tag{13}$$

$$\begin{aligned} du &= dx \\ dv &= dy \end{aligned} \tag{14}$$

$$\begin{aligned} f(\rho,\theta) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x-x_0, y-y_0)\delta(x\cos\theta + y\sin\theta - \rho)dxdy \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)\delta[(u+x_0)\cos\theta + (v+y_0)\sin\theta - \rho)dudv \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)\delta(u\cos\theta + x_0\cos\theta + v\sin\theta + y_0\sin\theta - \rho)dudv \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)\delta(u\cos\theta + v\sin\theta - (\rho - x_0\cos\theta - y_0\sin\theta))dudv \\ &= g(\rho - x_0\cos\theta - y_0\sin\theta, \theta) \end{aligned} \tag{15}$$

3. Programming Question 1

   (a) The code is shown blow.

```
%read image
img_house = imread("house.tif");
img_jet = imread("jet.tiff");
img_house = img_house(:, :, 1);
img_jet = img_jet(:, :, 1);


% Fourier Transformer
img_house_f = fft2(double(img_house));
img_jet_f = fft2(double(img_jet));
```

```
%calculate the magnitude and phase of house
img_house_m = abs(img_house_f);
img_house_ph = angle(img_house_f);

%calculate the magnitude and phase of jet
img_jet_m = abs(img_jet_f);
img_jet_ph = angle(img_jet_f);

%reconstruct images
image_a=img_house_m.*cos(img_jet_ph)+img_house_m.*sin(img_jet_ph).*1i;
image_b=img_jet_m.*cos(img_house_ph)+img_jet_m.*sin(img_house_ph).*1i;
image_a=abs(ifft2(image_a));
image_b=abs(ifft2(image_b));
image_a=uint8(image_a);
image_b=uint8(image_b);


% plot images
subplot(2,2,1);imshow(img_house);title('House');
subplot(2,2,2);imshow(img_jet);title('Jet');
subplot(2,2,3);imshow(image_a);title('Magenitude of house with phase of
    Jet');
subplot(2,2,4);imshow(image_b);title('Magnitude of Jet with phase of House
    ');
```

The images are shown below:

Figure 1: Input House

Figure 2: Input Jet

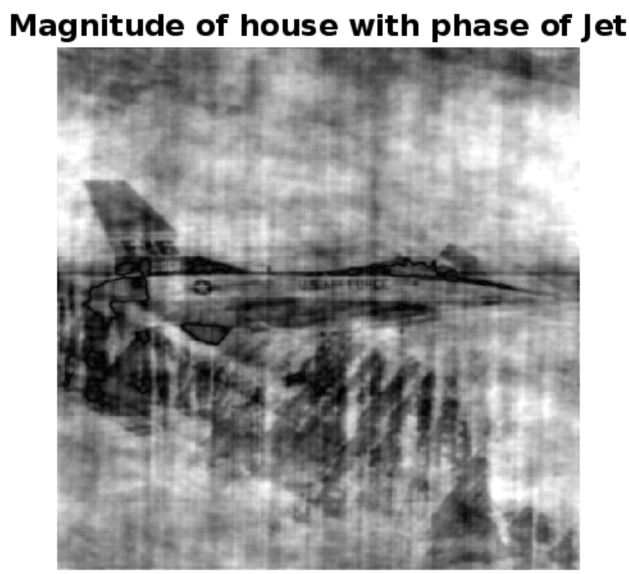

Figure 3: Magnitude of house + phase of Jet

Figure 4: Magnitude of Jet + phase of House

Suppoer the Fig. 1 is I_A, so the Fig. 4 have a better result to reconstruct the Fig. 1 compared with Fig. 3. On the other hand, the Fig. 3 reconstruct will for Fig. 2. Because the phase of the Fourier of an image keep more information compared with magnitude of the image. So it is clearly to find that in Fig. **??** the Jet keep more information, and in Fig. **??** there are more information present the house.