COMP478 Final Exam Review Guide

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Fall 2022

key concepts

- **Point processing**
  - ✓ Contrast stretching
  - ✓ Gamma correction
  - ✓ Bit-plane slicing
- **Histogram processing**
  - ✓ Histogram equalization (global & local)
  - ✓ Histogram matching

- **Linear averaging filters**
- **Order-statistics filters**
- **Derivative-type of filters - image sharpening**
- **Unsharp mask and Sobel gradient**

- **Aliasing: reasons and impacts**
- **Discrete Fourier Transform (DFT)**
- **Convolution and properties**
- **Properties of FT: phase & magnitude**
- **Intuitive understanding of images vs. FT**

- **Image smooth and sharpening (ringing artefacts)**
- **Different filter designs: ideal, butterworth, and Gaussian Correspondence b/w spatial and frequency domains**
- **Homomorphic filtering & Notch filtering**

- **Different types of noise and denoising solutions**
- **Inverse filter (dealing with noise) & Wiener filter**
- **Backprojection, sinogram and Radon transformation**
- **The projection slice theorem**

- **Point and line detection with 2nd derivative filters**
- **1st and 2nd derivative edge detection**
- **Edge detection (1st and 2nd derivative filters)**
- **Marr-Hildreth/LoG and Canny algorithms**

- **Edge linking: fixing broken edges**
- **Hough transform**
- **Image threshold: optimal global & Otsu's method**
- **k-means & graph cuts segmentation**

- **Set operations, structuring elements, objects**
- **Dilation, erosion -> closing, opening**
- **Hit-or-miss transform, boundary extraction, hole filling, thinning, thickening, skeletonization**
- **Simple grayscale morphology: flat SE**

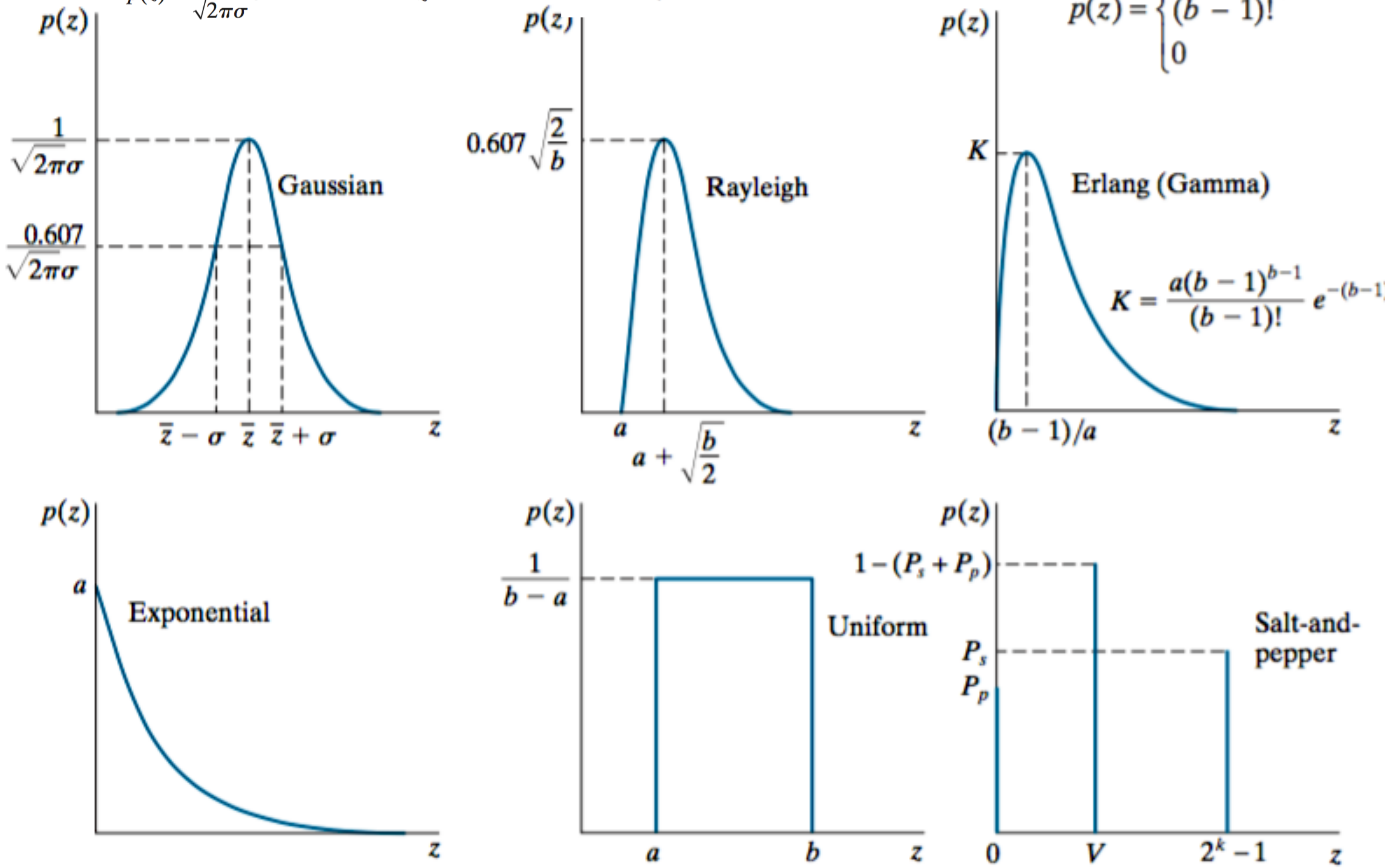
- **Definition and benefits of wavelets**
- **Image Pyramid**
- **Filter banks: decomposition & synthesis**
- **Scaling and wavelet functions & properties**
- **Scaling and wavelet function space**
- **Different wavelet families (Haar, Daubechies)**

Noise models

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\bar{z})^2}{2\sigma^2}} \qquad -\infty < z < \infty$$

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & z \geq a \\ 0 & z < a \end{cases}$$

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$



$$p(z) = \begin{cases} ae^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

$$p(z) = \begin{cases} \frac{1}{b-a} & a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$p(z) = \begin{cases} P_s & \text{for } z = 2^k - 1 \\ P_p & \text{for } z = 0 \\ 1-(P_s+P_p) & \text{for } z = V \end{cases}$$

Spatial filters vs. FT

Shapes in frequency domain

Gaussian

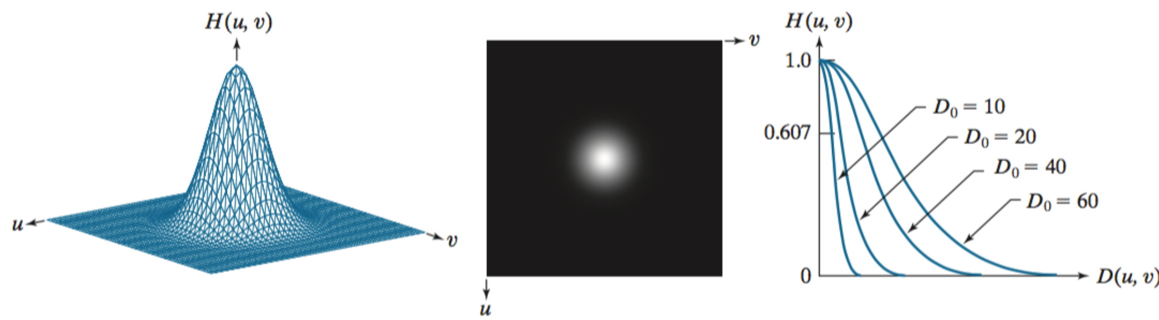


FIGURE 4.43 (a) Perspective plot of a GLPF transfer function. (b) Function displayed as an image. (c) Radial cross sections for various values of D_0.

Ideal

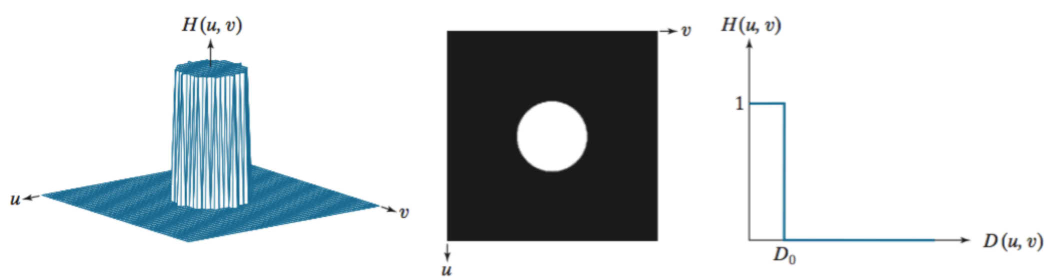


FIGURE 4.39 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Function displayed as an image. (c) Radial cross section.

Butter worth

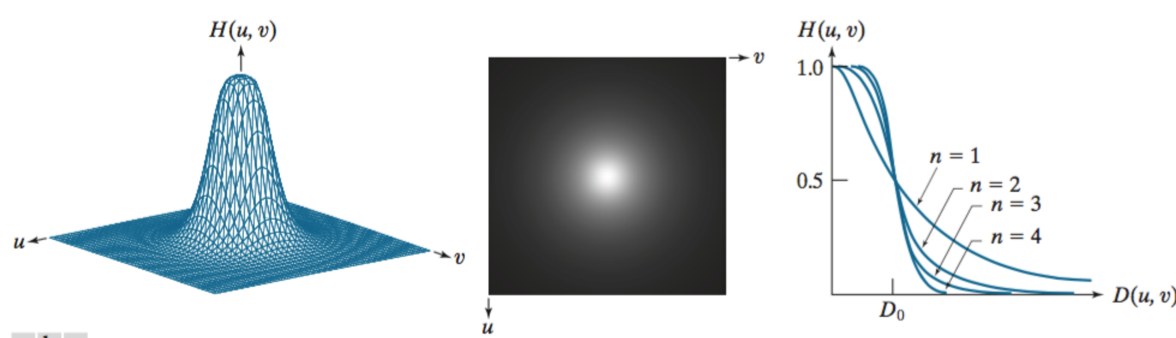
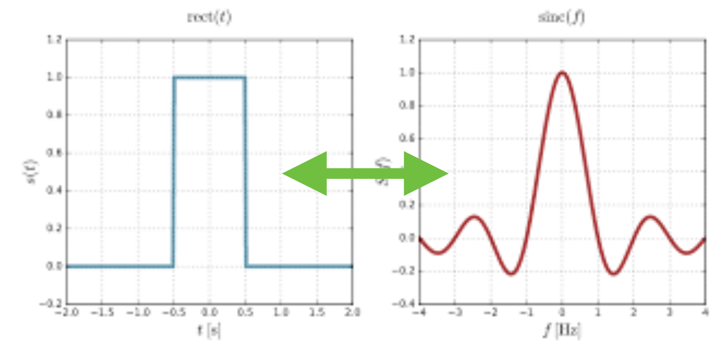
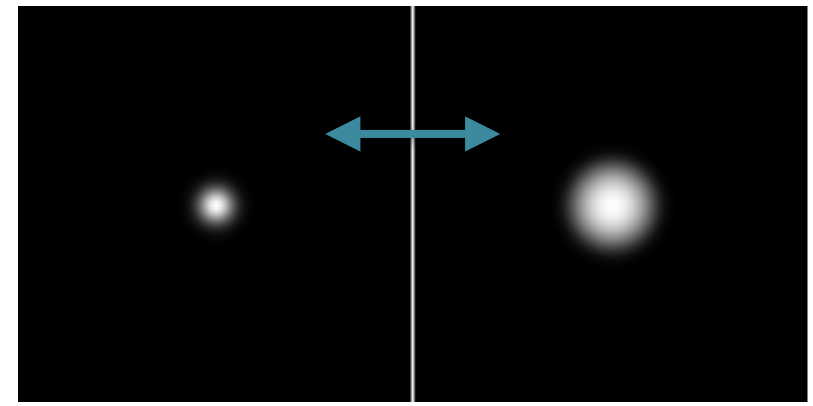


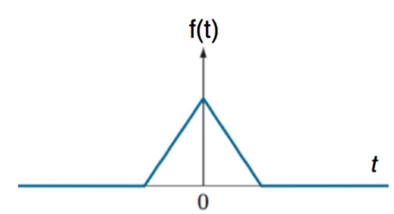
FIGURE 4.45 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Function displayed as an image. (c) Radial cross sections of BLPFs of orders 1 through 4.



Box sinc



Gaussian Gaussian



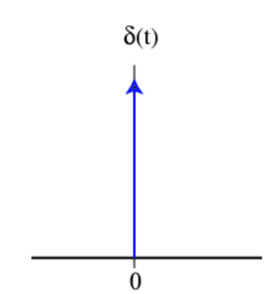
Duality

$$\boxed{f(x) * h(x) \Leftrightarrow H(u)F(u)}$$

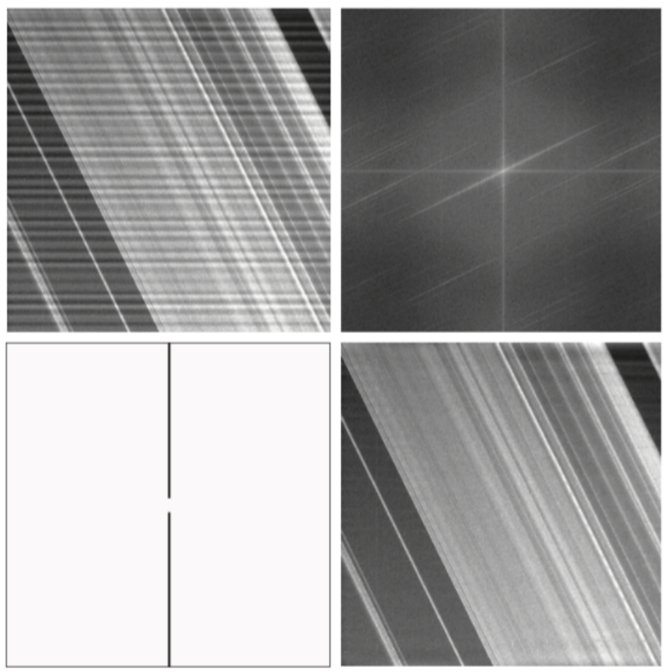
$$\boxed{f(x)g(x) \Leftrightarrow H(u) * F(u)}$$

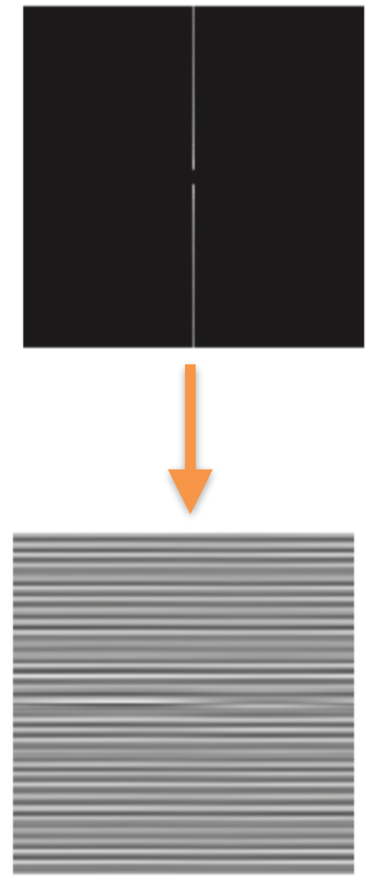
SIFTING Theorem

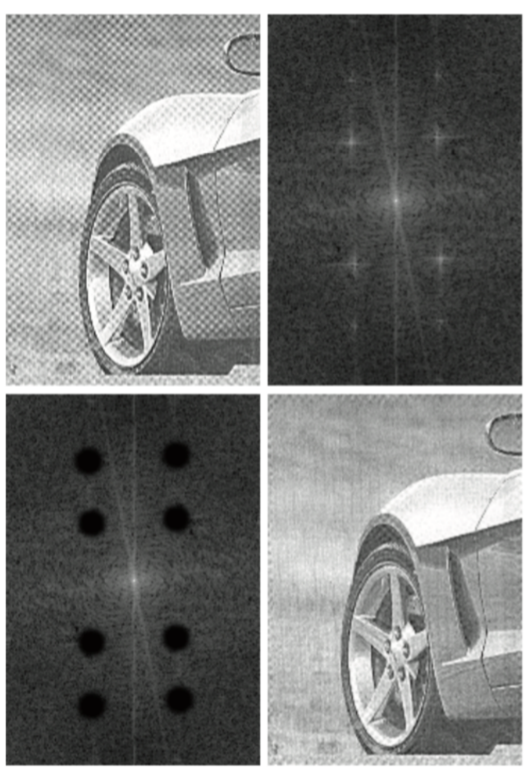
$$\boxed{\int_{-\infty}^{\infty} f(\alpha)\delta(x-\alpha)d\alpha = f(x).}$$

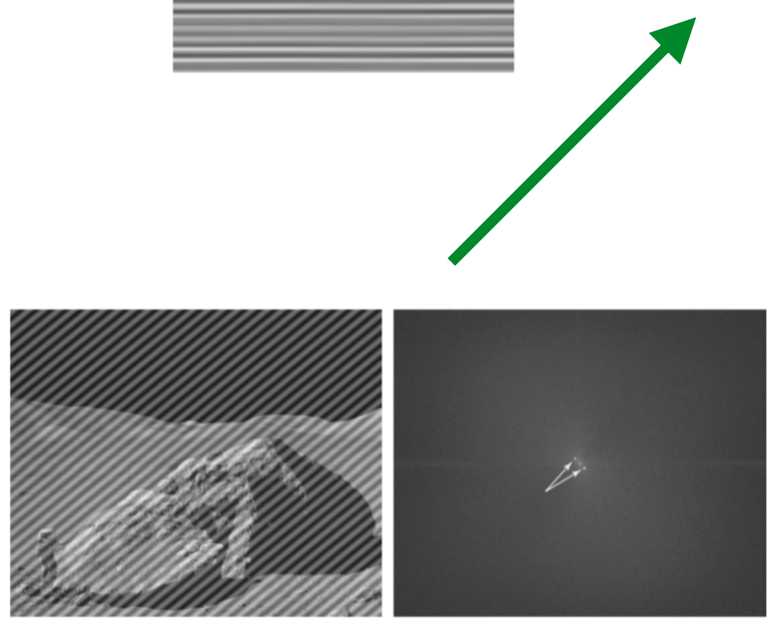


Notch filter









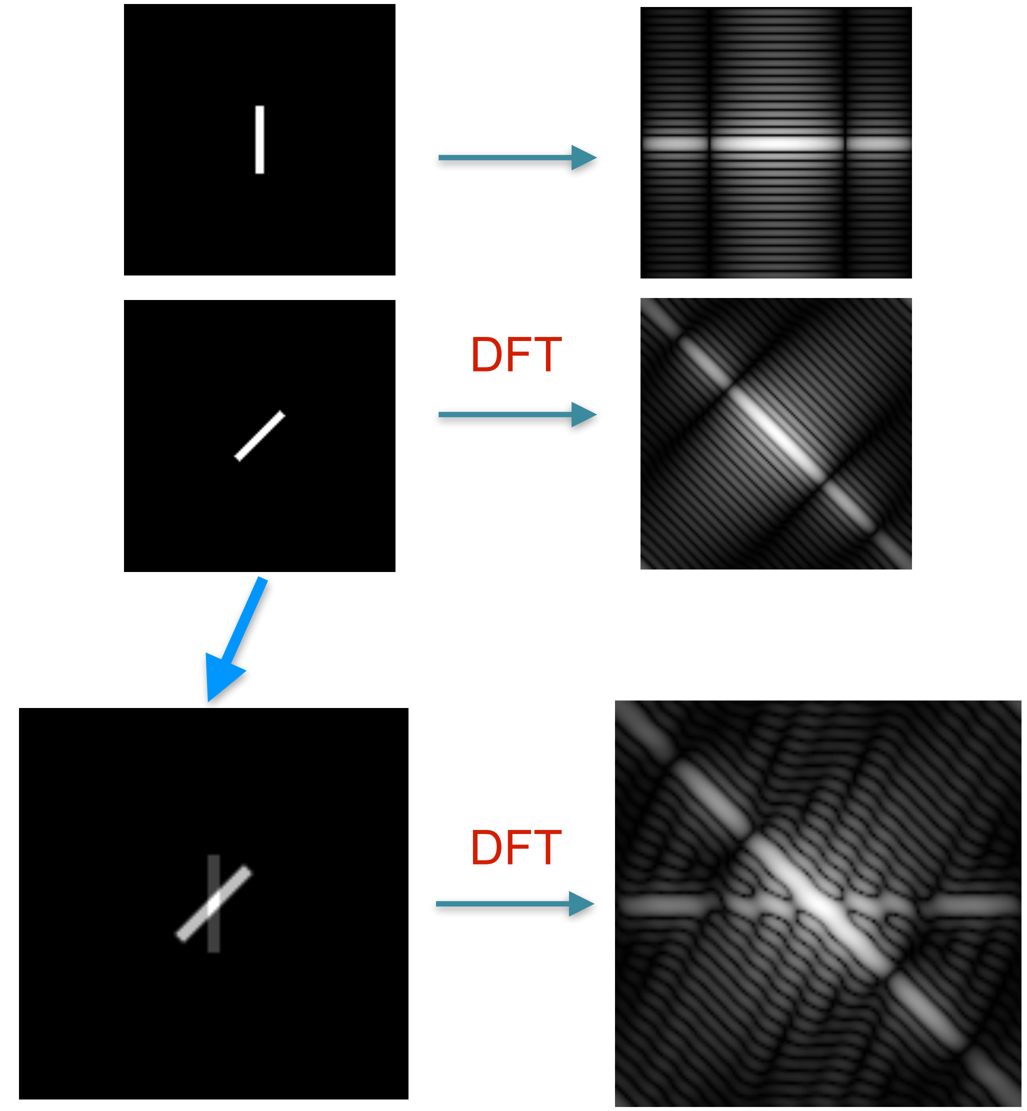
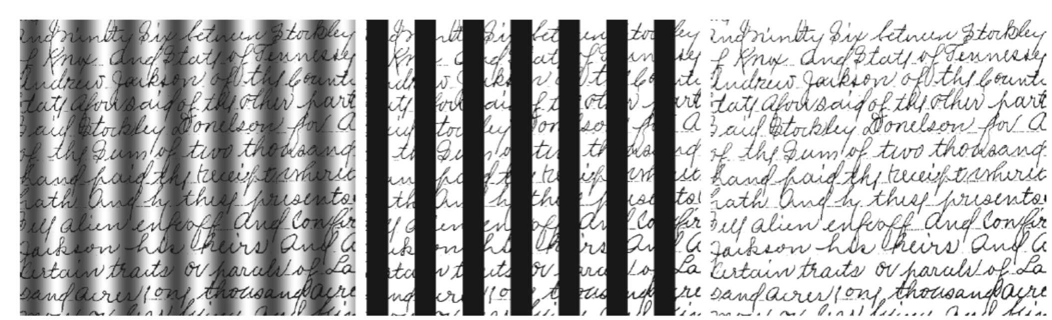


Image vs. FT



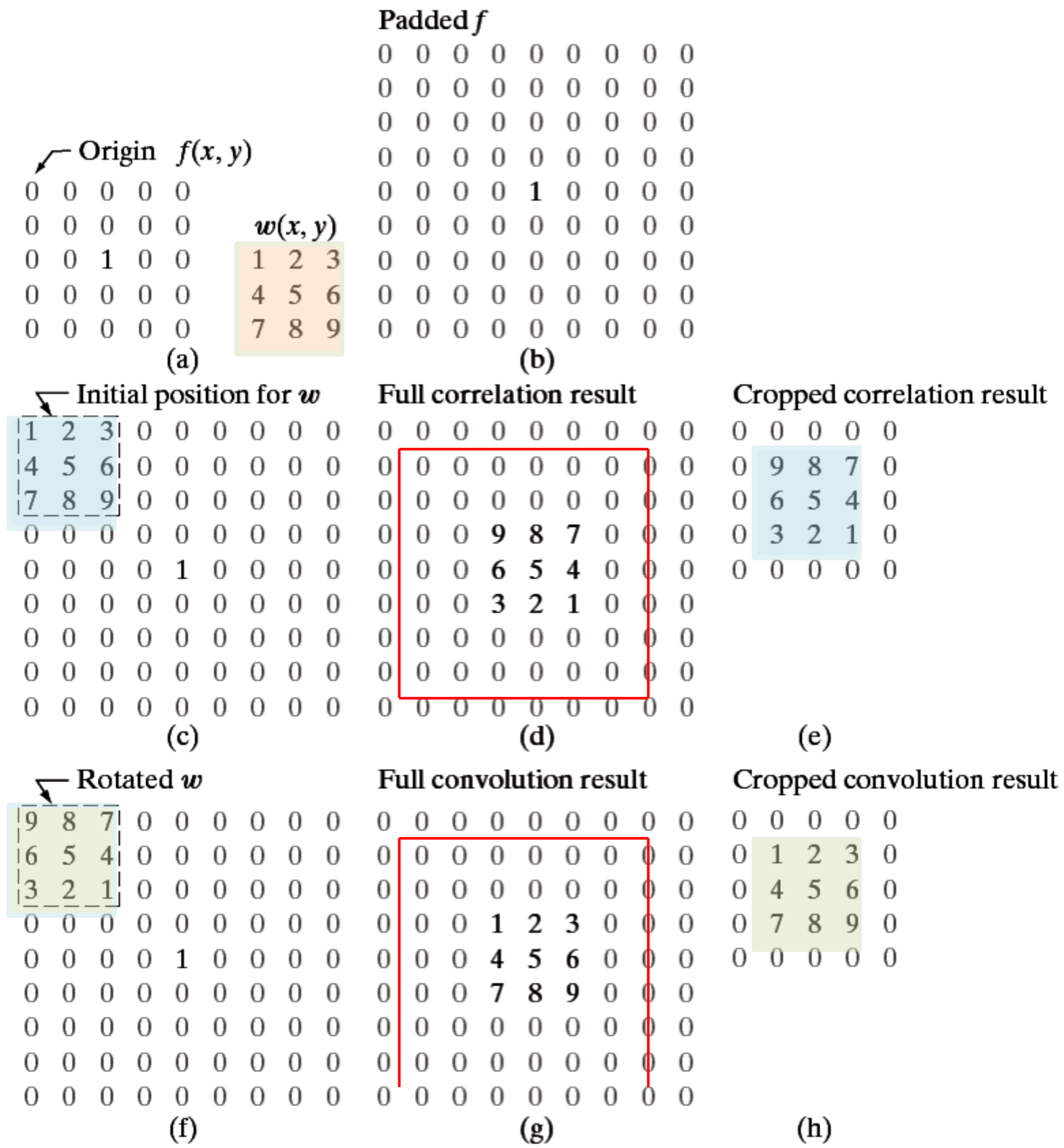
Variable thresholding

Masks, filters, and structuring elements

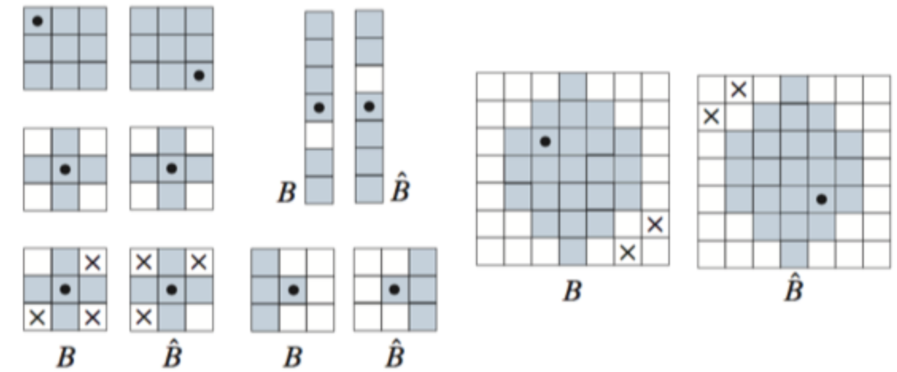
Filters and masks

Correlation

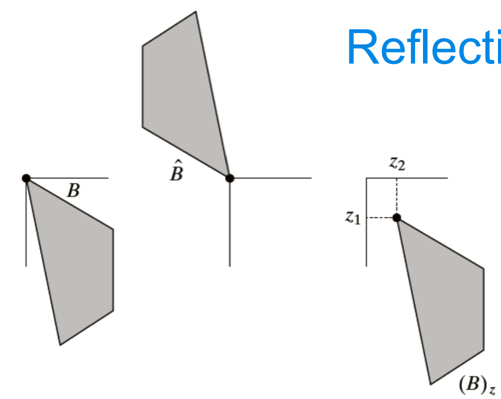
Convolution



Structuring elements

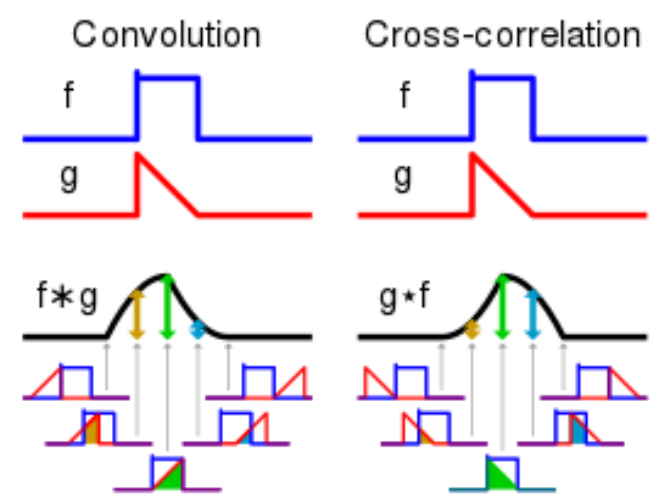


Reflection



Dilation $A \oplus B = \left\{ z \,\middle|\, (\hat{B})_z \cap A \neq \varnothing \right\}$

Erosion $A \ominus B = \{ z | (B)_z \cap A^c = \emptyset \}$

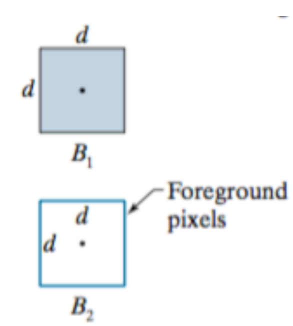


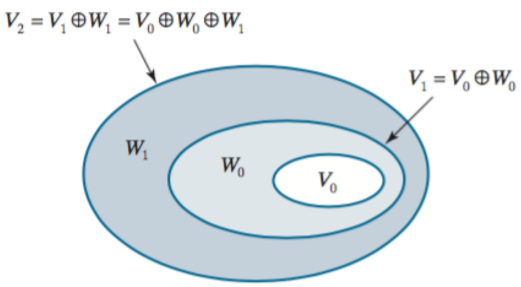
invert filter order

Cross-correlation vs. hit-or-miss transform

$$I \circledast B_{1,2} = \left\{ z \,\middle|\, (B_1)_z \subseteq A \text{ and } (B_2)_z \subseteq A^c \right\}$$

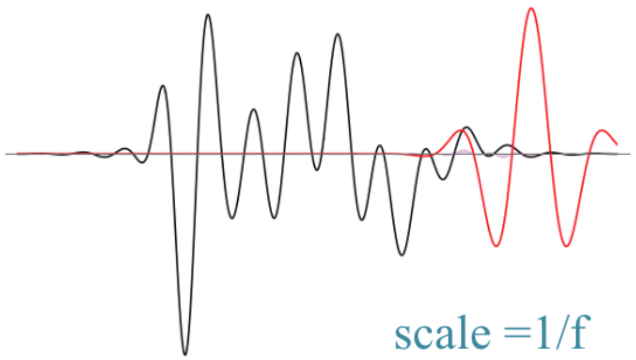
$$= (A \ominus B_1) \cap (A^c \ominus B_2)$$

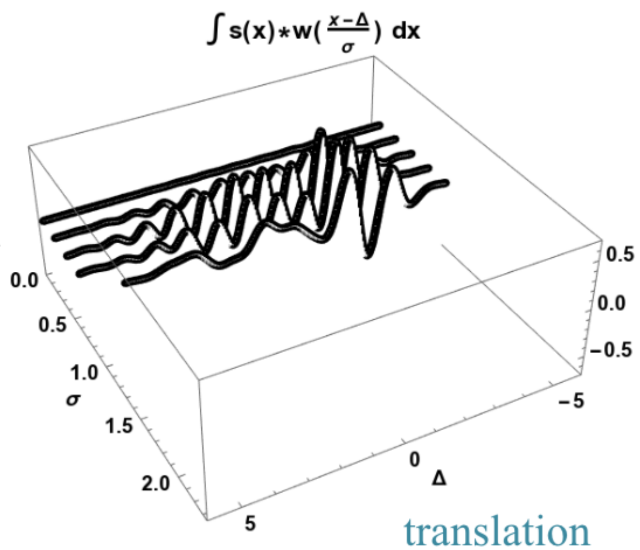


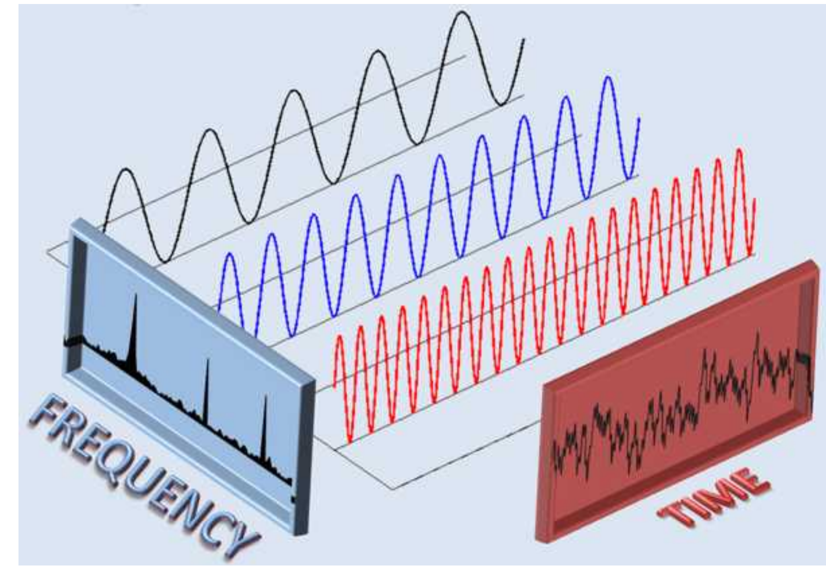


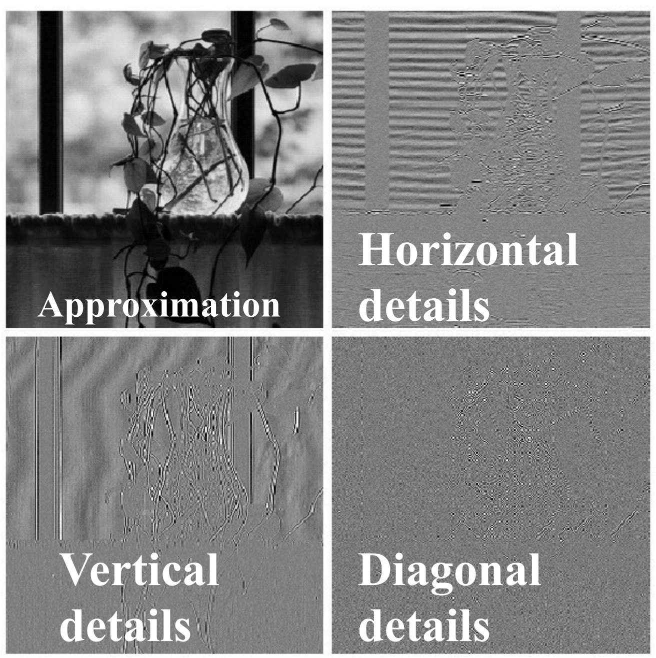
wavelet transform

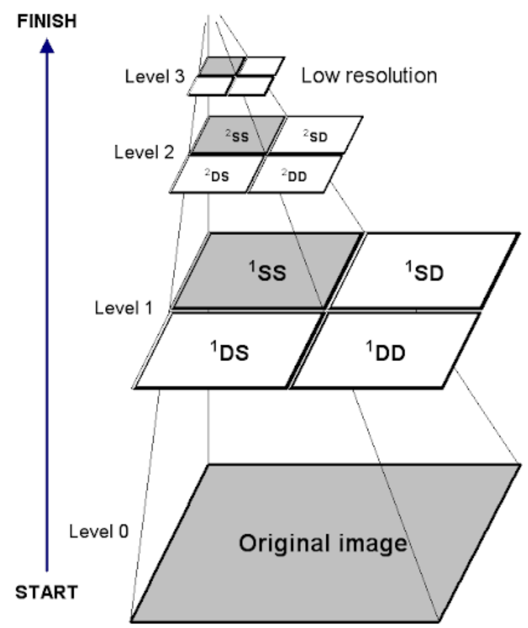
Fourier transform











Inverse filter

$$\hat{F}(\omega_1,\omega_2) = F(\omega_1,\omega_2) + \boxed{\frac{N(\omega_1,\omega_2)}{D(\omega_1,\omega_2)}}$$

Dealing with zeros in the inverse filter

Wiener filter

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right] G(u,v)$$

Choose K interactive to achieve the final restoration

ratio of the power spectrum of the noise and signal $S_\eta(u,v)/S_f(u,v)$

Solving a MMSE problem
Solve zero-value issues differently

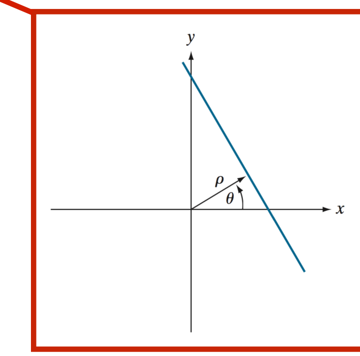
Scaling function vs. wavelet function

Haar wavelet

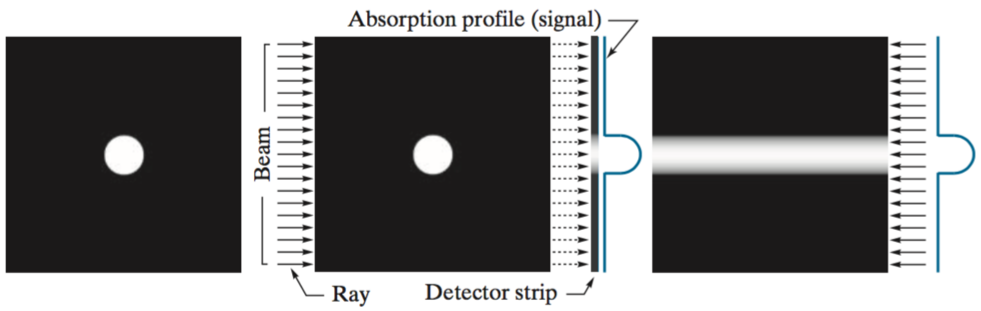
Radon transform

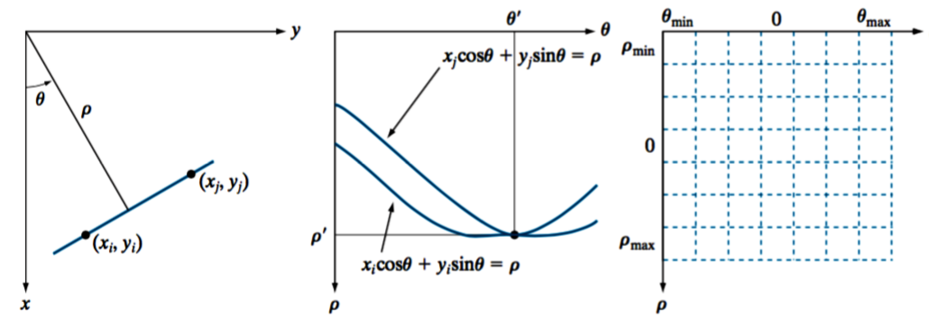
$$x\cos\theta + y\sin\theta = \rho$$

Hough transform



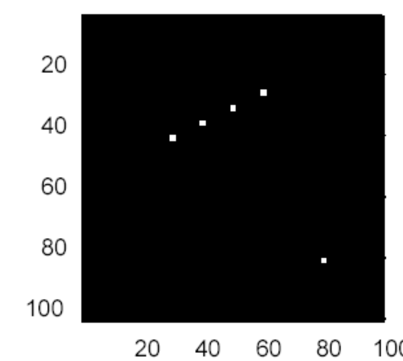
$$g(\rho,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta - \rho)\,dx\,dy$$

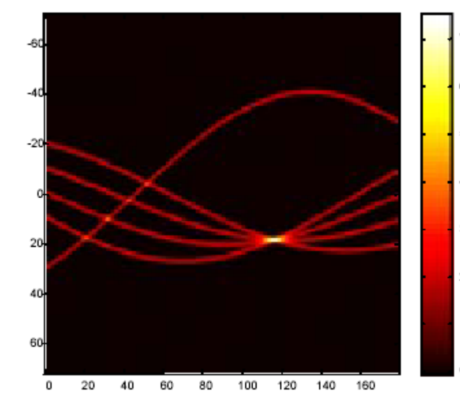




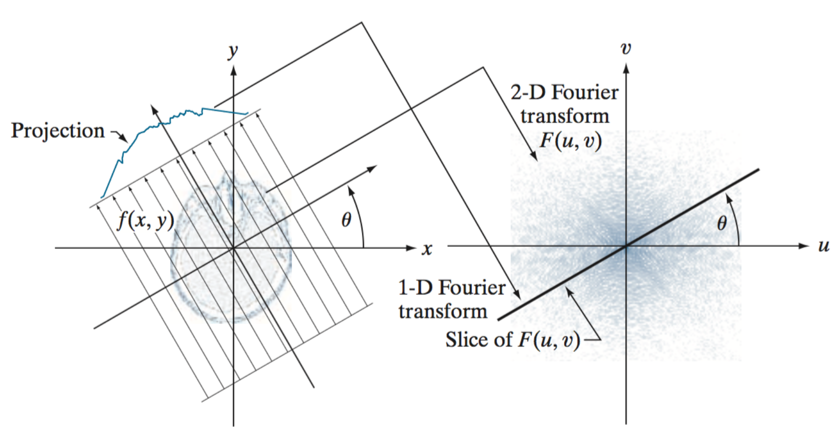
Sinogram

Original image

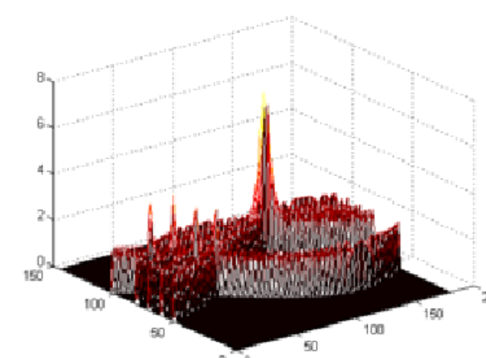




Projection slice theorem



Circles

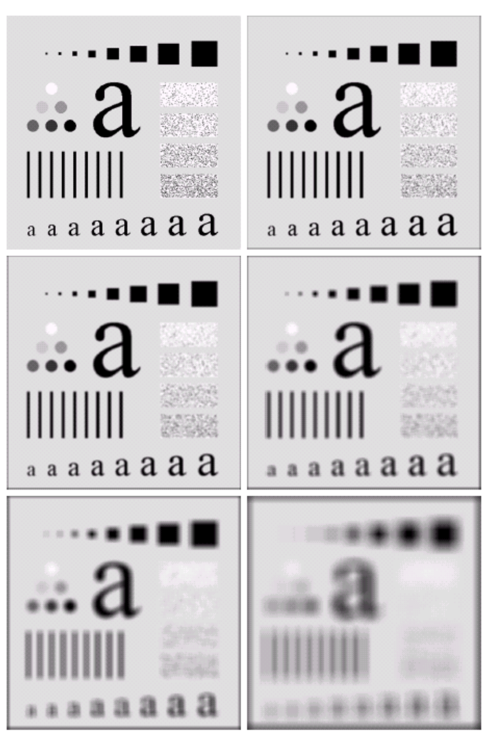


Averaging filters

$$\frac{1}{9}\times\begin{array}{|c|c|c|}\hline 1&1&1\\ \hline 1&1&1\\ \hline 1&1&1\\ \hline\end{array}\qquad \frac{1}{16}\times\begin{array}{|c|c|c|}\hline 1&2&1\\ \hline 2&4&2\\ \hline 1&2&1\\ \hline\end{array}$$

box filter *Weighted filter*

$$g(x,y)=\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\,f(x+s,y+t)$$



Derivative filters

1st derivative vs. 2nd derivative filters

Sobel filter $\begin{bmatrix}-1&-2&-1\\0&0&0\\1&2&1\end{bmatrix}\begin{bmatrix}-1&0&1\\-2&0&2\\-1&0&1\end{bmatrix}$

$$\text{Edge magnitude}: \sqrt{s_1^2+s_2^2}\quad \text{Edge direction}: \tan^{-1}\left[\frac{s_1}{s_2}\right]$$

LoG/Marr-Hildreth edge detector

Canny edge detector (what filter, two thresholds)

Edge linking

Zero-crossings
with 2nd derivative filters

$$\begin{array}{|c|c|c|}\hline 0&1&0\\ \hline 1&-4&1\\ \hline 0&1&0\\ \hline\end{array}\quad\begin{array}{|c|c|c|}\hline 1&1&1\\ \hline 1&-8&1\\ \hline 1&1&1\\ \hline\end{array}$$

$$\begin{array}{|c|c|c|}\hline 0&-1&0\\ \hline -1&4&-1\\ \hline 0&-1&0\\ \hline\end{array}\quad\begin{array}{|c|c|c|}\hline -1&-1&-1\\ \hline -1&8&-1\\ \hline -1&-1&-1\\ \hline\end{array}$$