Lecture 3

Image Enhancement

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Slides modified from materials provided by Dr. Tien D Bui

Image Enhancement: improving the quality of images (make it better) or modifying image to bring out hidden features. Two kinds: *spatial domain* and *frequency domain*.

Image Restoration: correcting images subjected to noise, blurs, distortions, atmospheric effects, etc

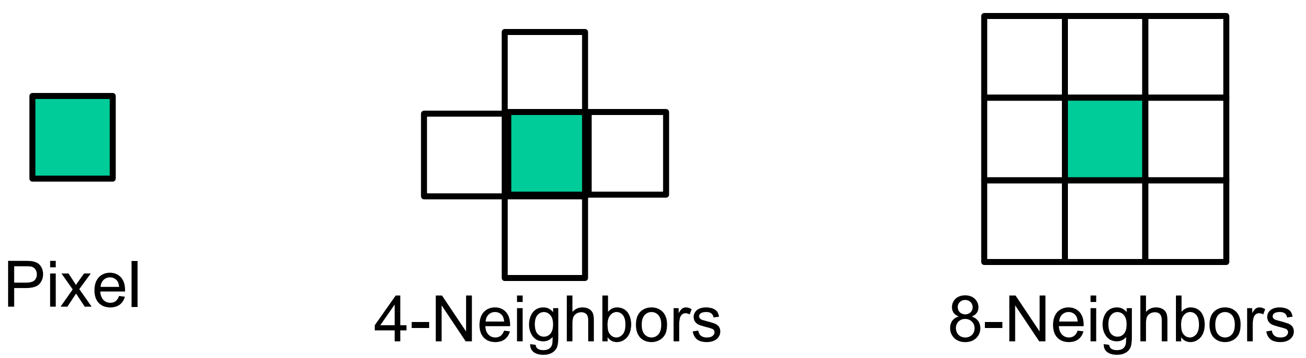
Image Enhancement I: Pixel-wise Operations

Image Enhancement: Operating on the Pixels

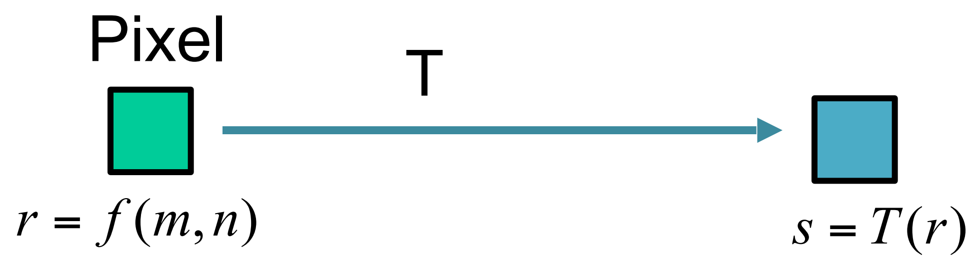
- Make image "better" for a specific application
  - The idea of "better" is somewhat subjective

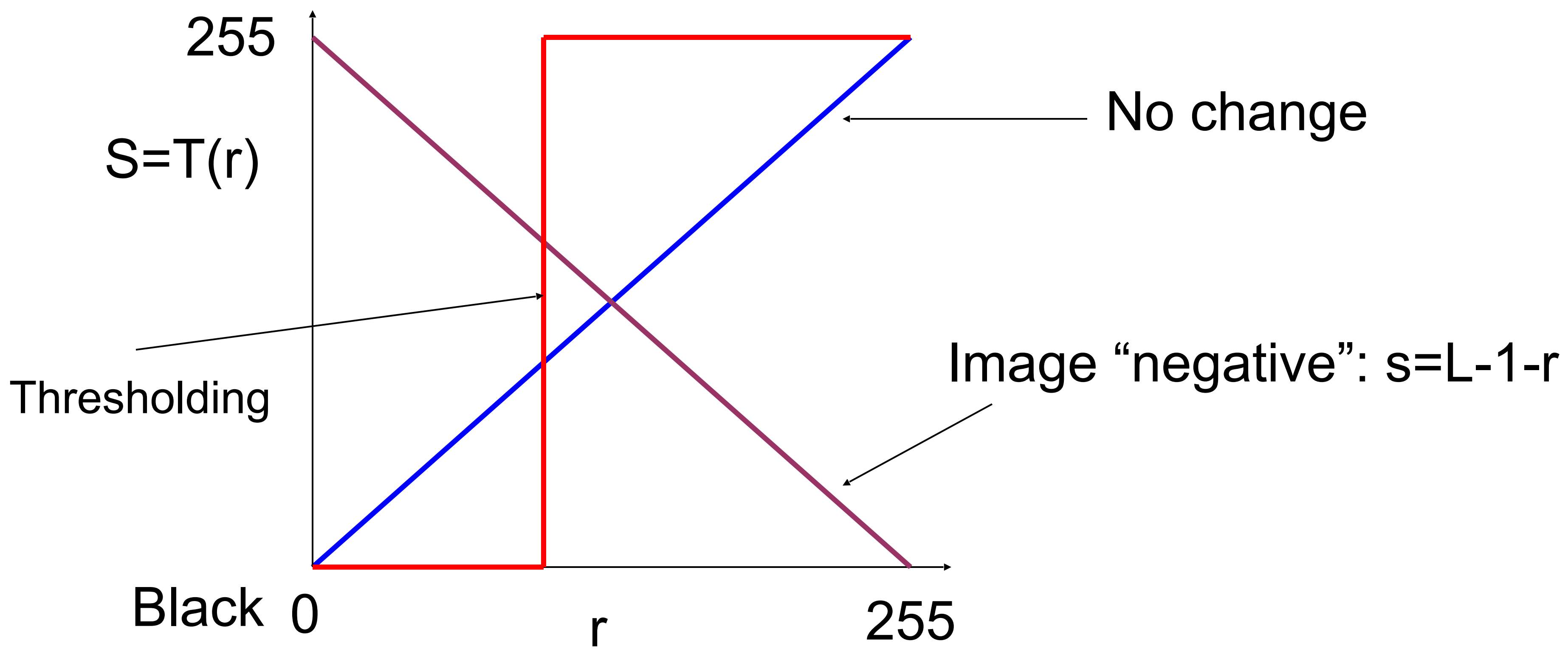
- We distinguish two domains:
  - Spatial or Pixel domain: $f(x, y)$ or $f(m, n)$
  - Frequency Domain: $F(w_x, w_y)$ or $F(u, v)$

- For this section: Pixel Domain
  - Operations on single pixel at a time
  - Operations on groups of pixels (neighborhoods)

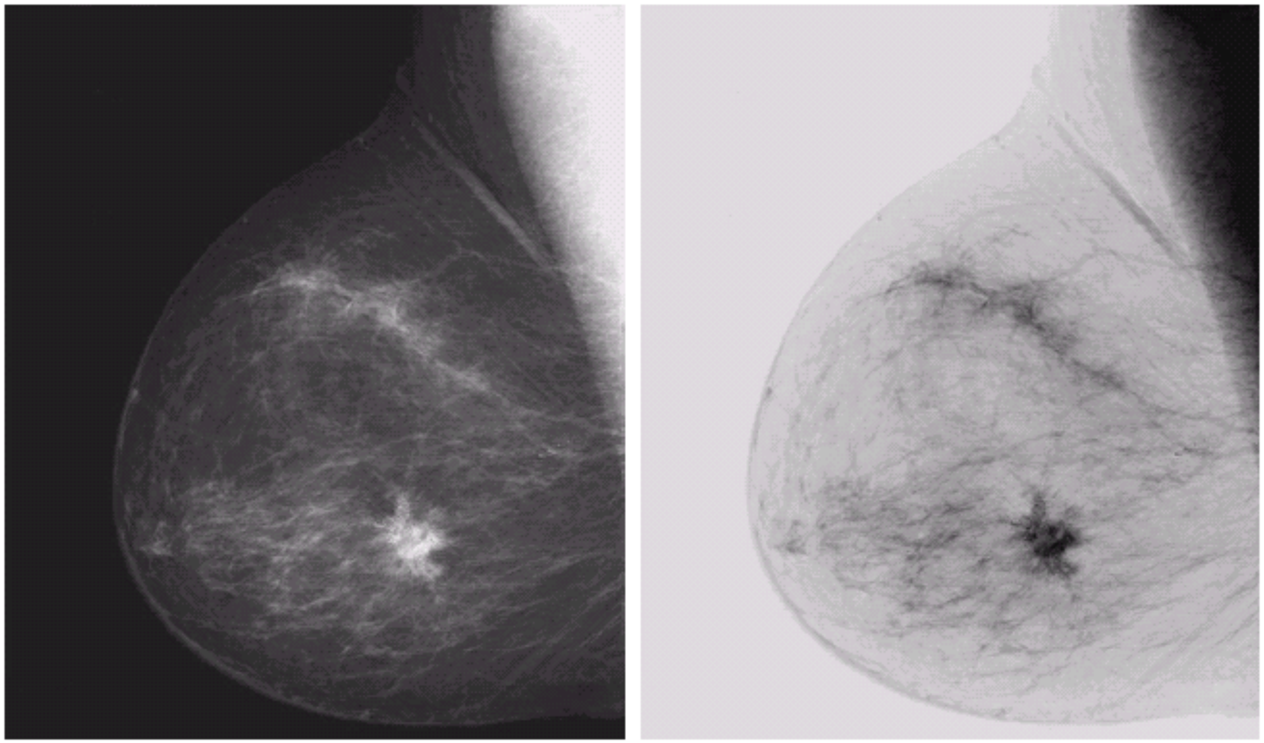


Simplest form of processing: Point Processing





Negative of an image

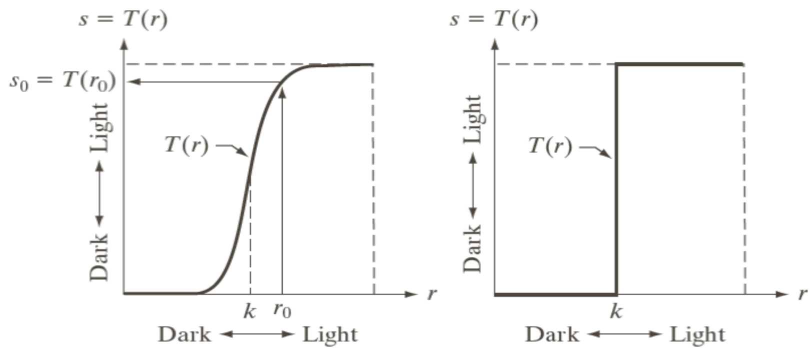


Original Negative

a b

FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

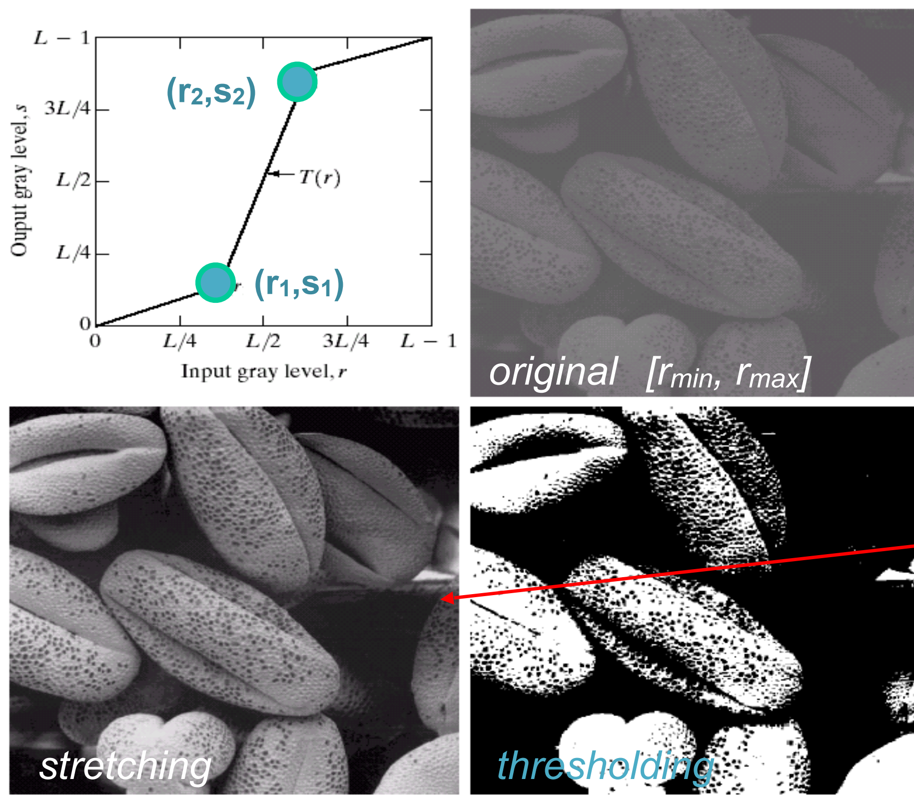
Intensity Transformation: Contrast Stretching



a b

FIGURE 3.2 Intensity transformation functions. (a) Contrast-stretching function. (b) Thresholding function.

8-bit image, low contrast



a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

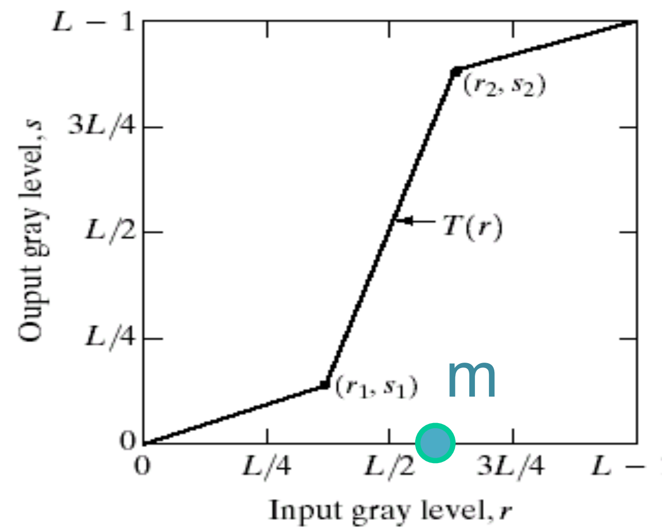
$(r_1, s_1) = (r_{min}, 0)$
$(r_2, s_2) = (r_{max}, L-1)$
r_{min} = r_{min} of original image

$(r_1, s_1) = (m, 0)$
$(r_2, s_2) = (m, L-1)$

The locations of (r_1, s_1) and (r_2, s_2) control the shape of the transformation function.

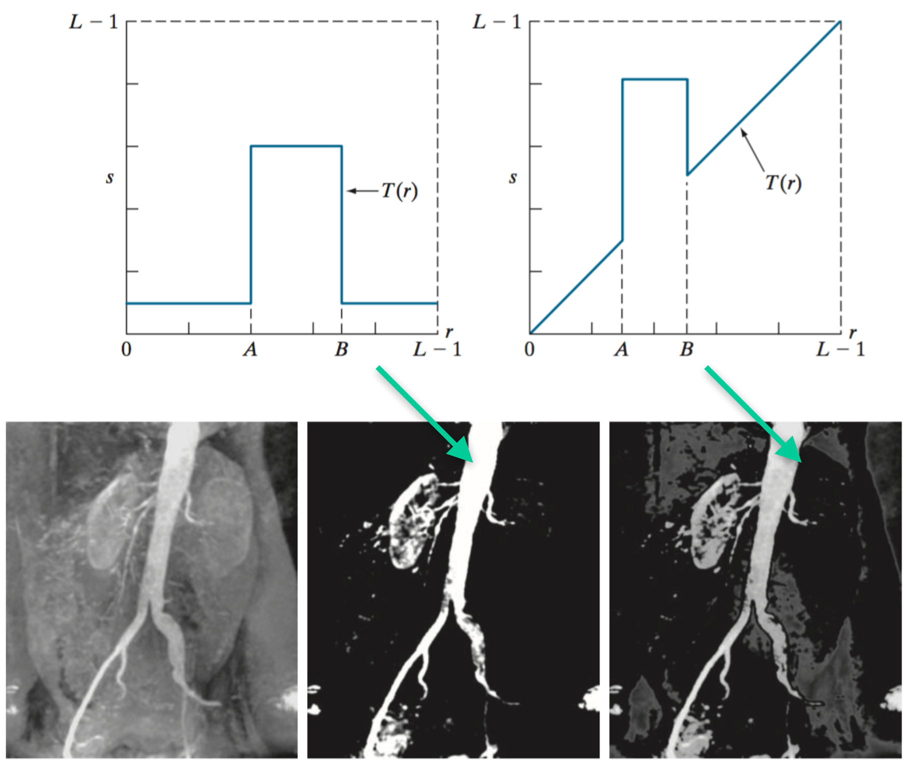
a) If $r_1 = s_1$ and $r_2 = s_2$ the transformation is a linear function: no change in intensity.

b) If $r_1 = r_2 =$ m, $s_1 = 0$ and $s_2 = L-1$ the transformation is a thresholding function: a binary image.



Intensity Transformation: Intensity Slicing

The aim is to highlight specific range of intensity of interest.

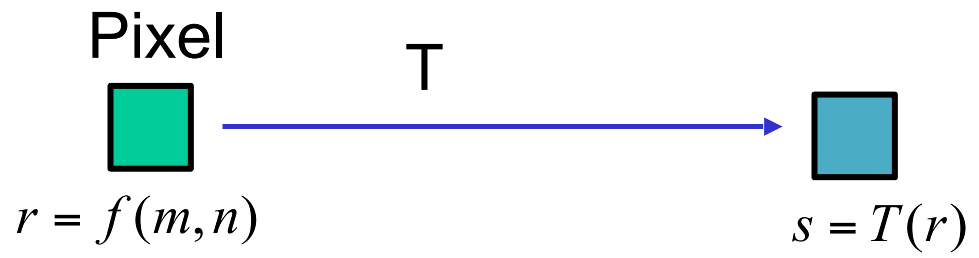


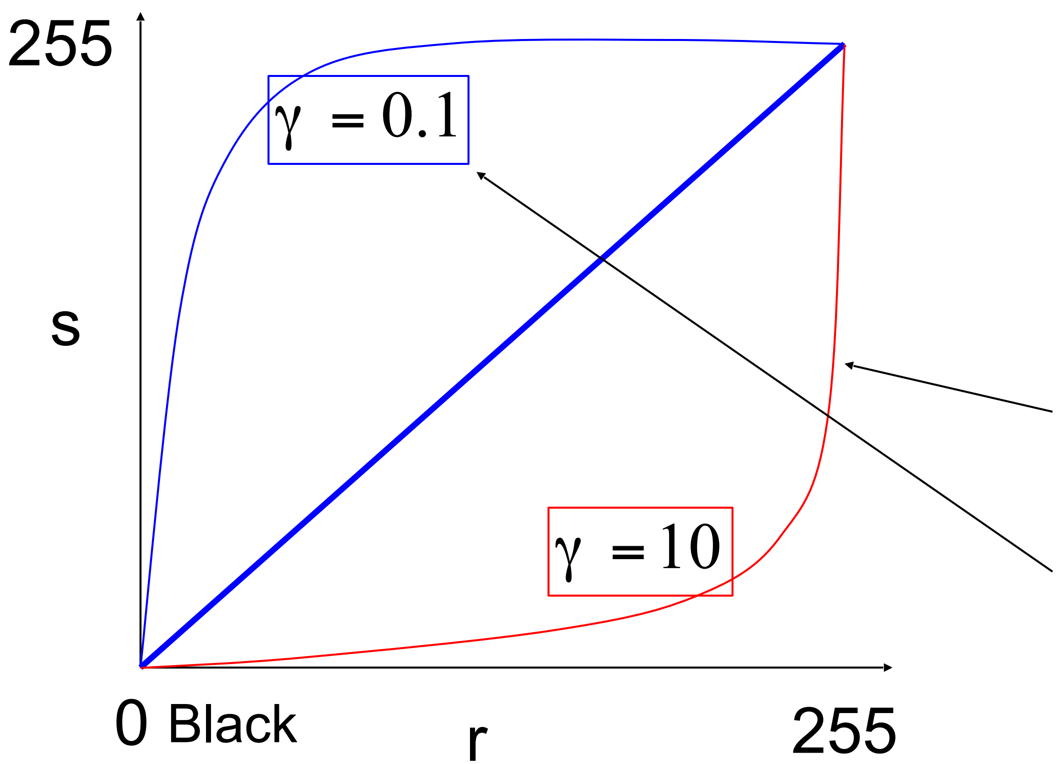
a b c

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected range set near black, so that the grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

See also Fig. 3.12 (p. 117 of the 3rd Ed., or p. 145 of the 4th Ed.)

Simplest form of processing: Point Processing





Common Examples:

Gamma Correction

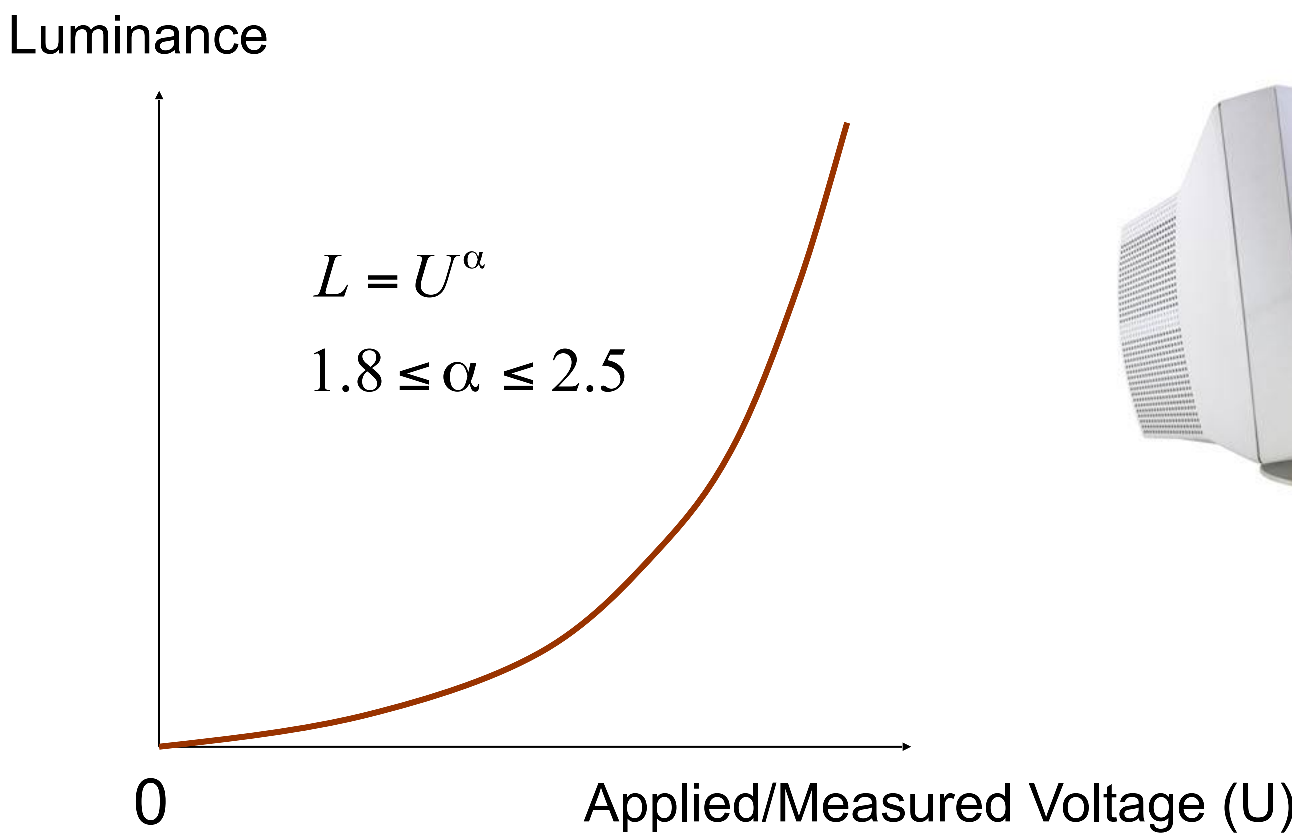
$$T(r) = cr^{\gamma}$$

Narrow range of “dark” gets mapped to broad range of “gray”

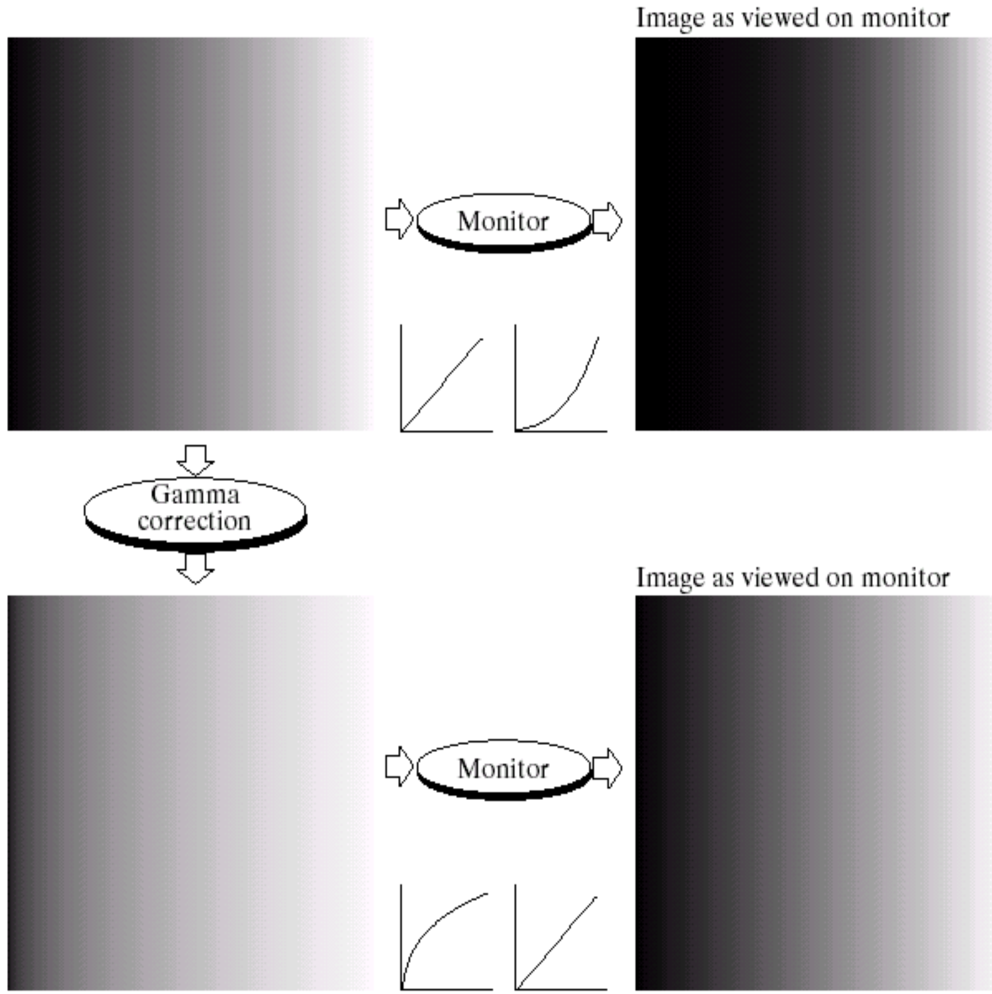
Origins of gamma correction:

- Nonlinear response of CRT's and imagers

- To correct for this in image display, the images or commands to the CRT are "pre-distorted"



Gamma Correction:



a b
c d

FIGURE 3.7 (a) Intensity ramp image. (b) Image as viewed on a simulated monitor with a gamma of 2.5. (c) Gamma-corrected image. (d) Corrected image as viewed on the same monitor. Compare (d) and (a).

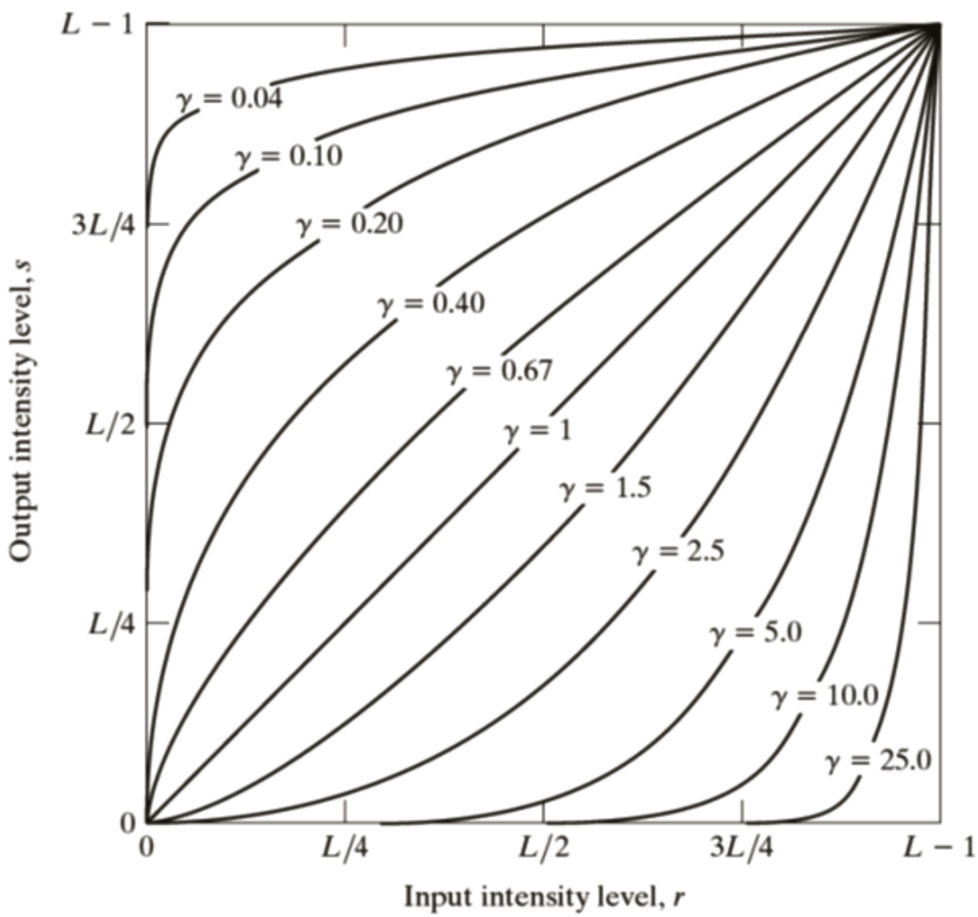
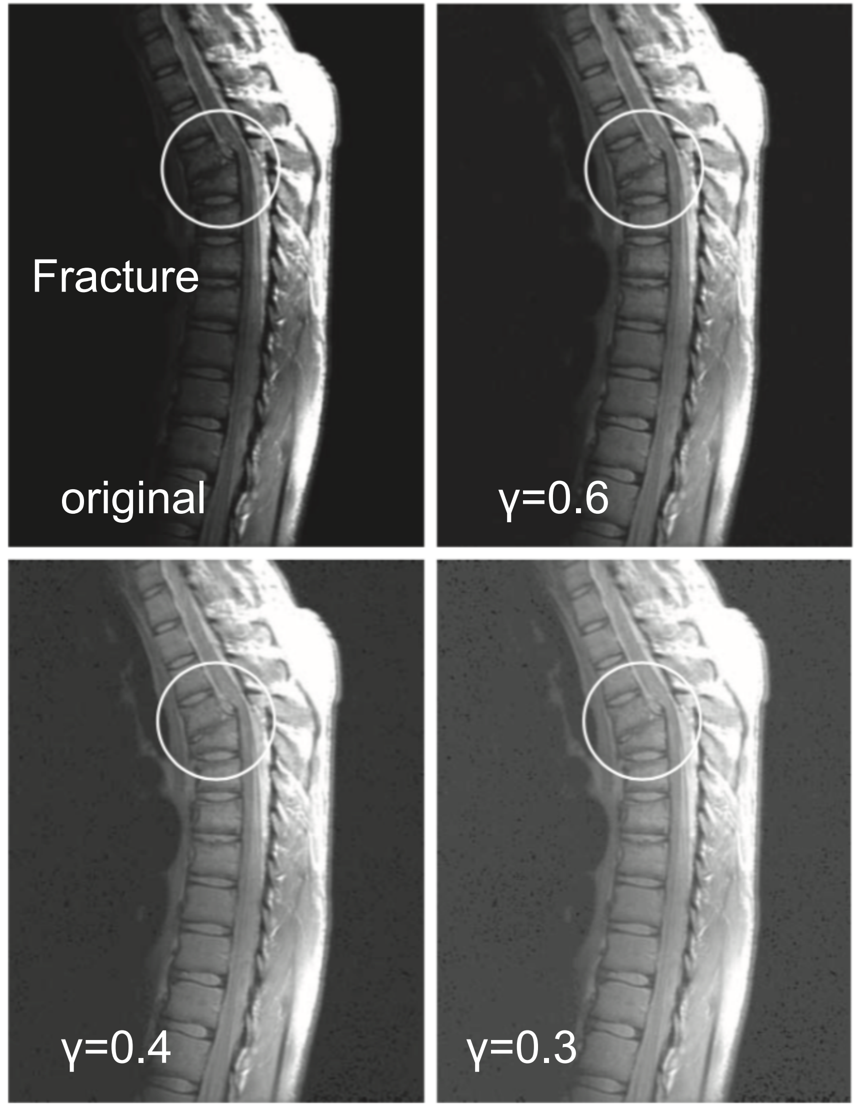
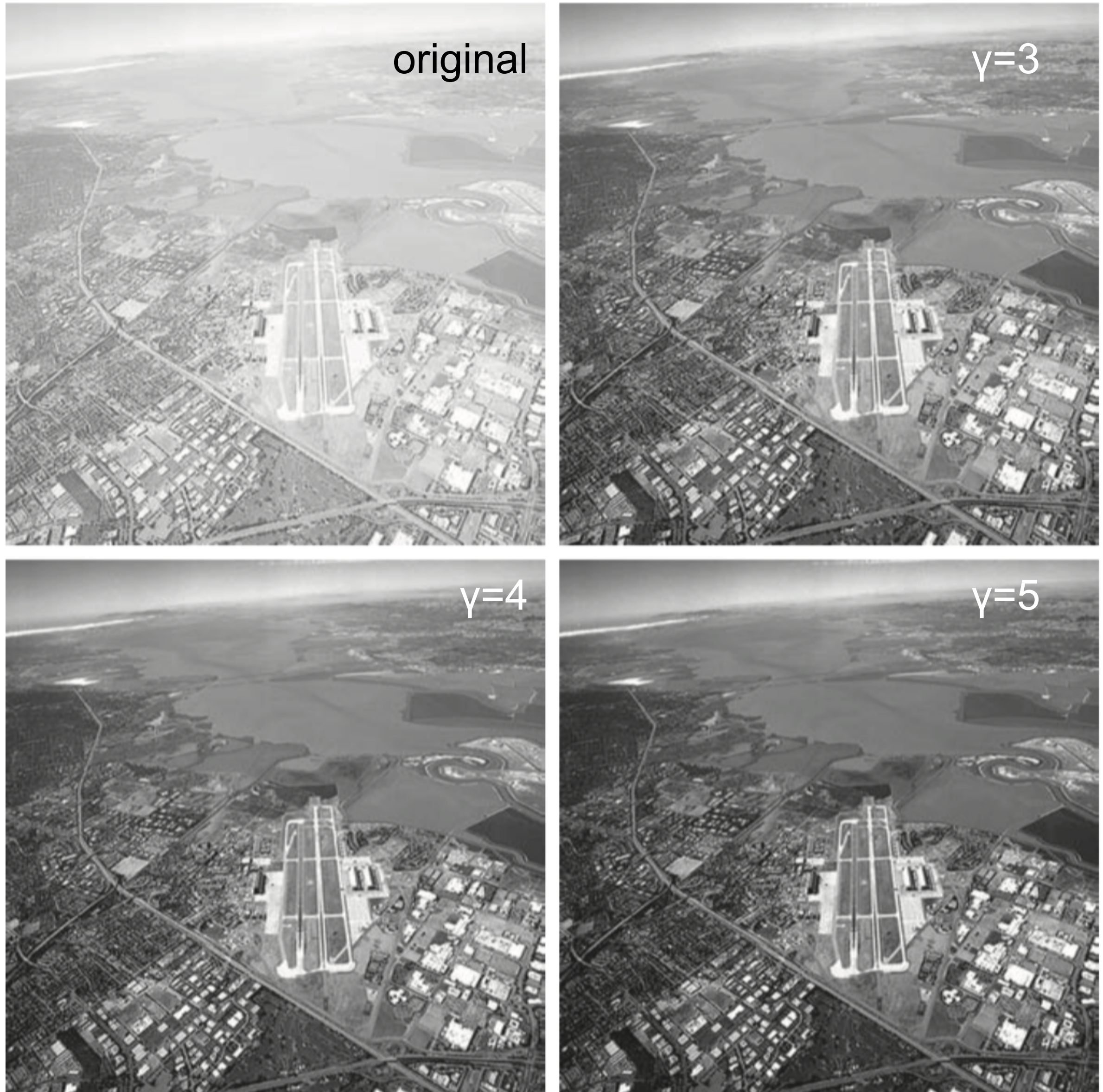


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.



a b
c d

FIGURE 3.8
(a) Magnetic resonance image (MRI) of a fractured human spine (the region of the fracture is enclosed by the circle).
(b)–(d) Results of applying the transformation in Eq. (3-5) with $c = 1$ and $\gamma = 0.6, 0.4$, and 0.3, respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)



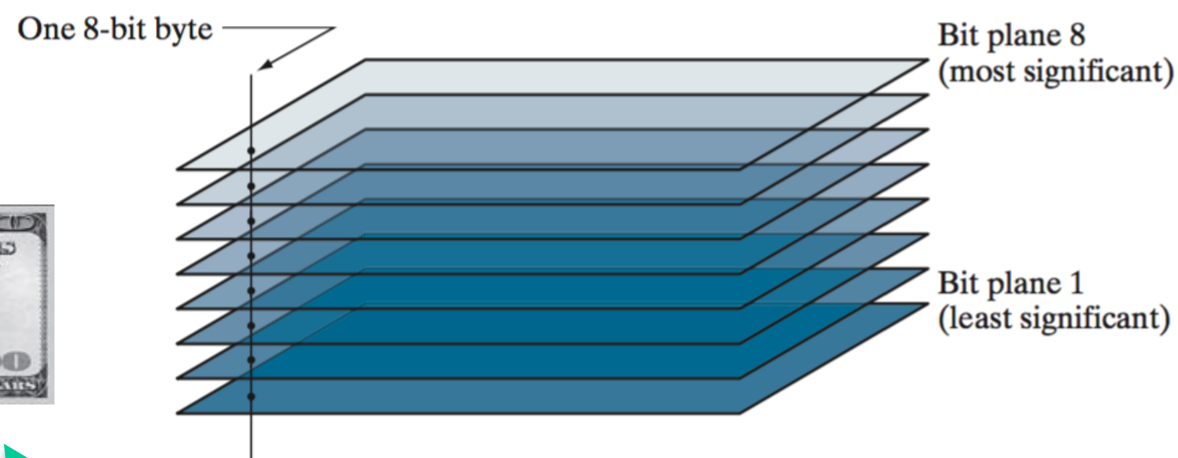
a b
c d

FIGURE 3.9
(a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3-5) with $\gamma = 3.0, 4.0$, and 5.0, respectively. ($c = 1$ in all cases.) (Original image courtesy of NASA.)

Intensity Transformation: Bit-plane Slicing

Ex. In 8-bit image, grey level 194 is (1 1 0 0 0 0 1 0).
Grey levels (0-127) have the 8th bit = 0, and levels (128-255) have the 8th bit = 1 (counting bits from right to left)

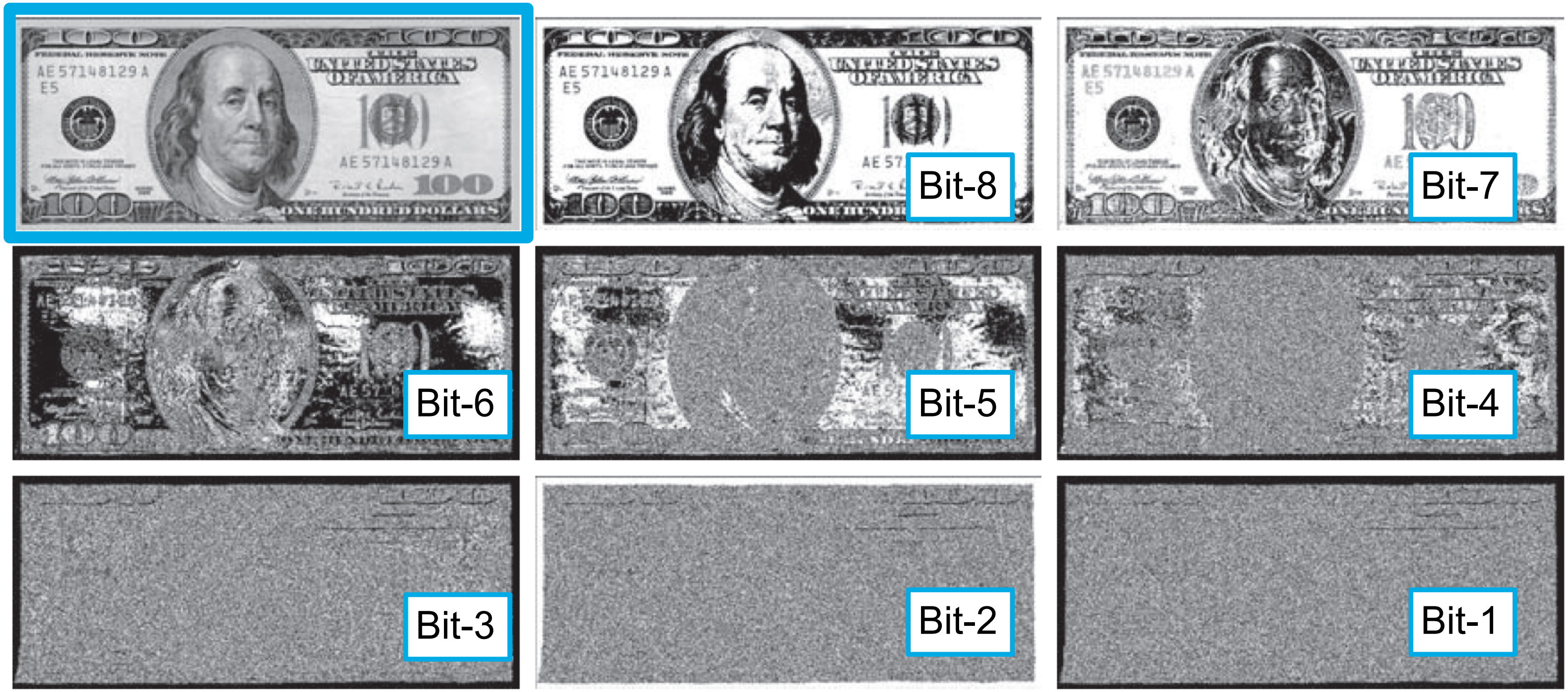
FIGURE 3.13
Bit-planes of an 8-bit image.



1
1
0
0
0
0
1
0

194

Intensity Transformation: Bit-plane Slicing

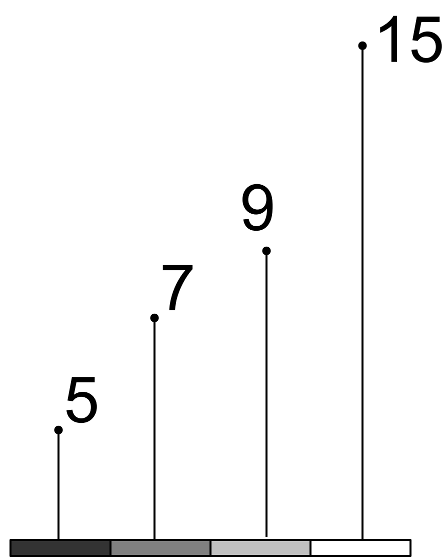


Take a break!

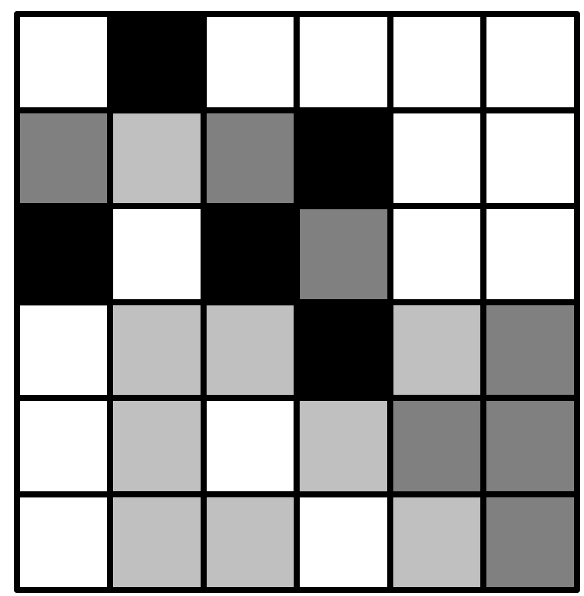
Histogram Processing:

- Distribution of gray-levels can be judged by measuring a Histogram

Histogram:



Graylevel



Histogram Processing

FIGURE 3.16 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.

Histogram Processing:

For B-bit image,

- Initialize 2^B counters with 0
- Loop over all pixels x,y
- When encountering gray level $f(x,y)=i$, increment the counter number i

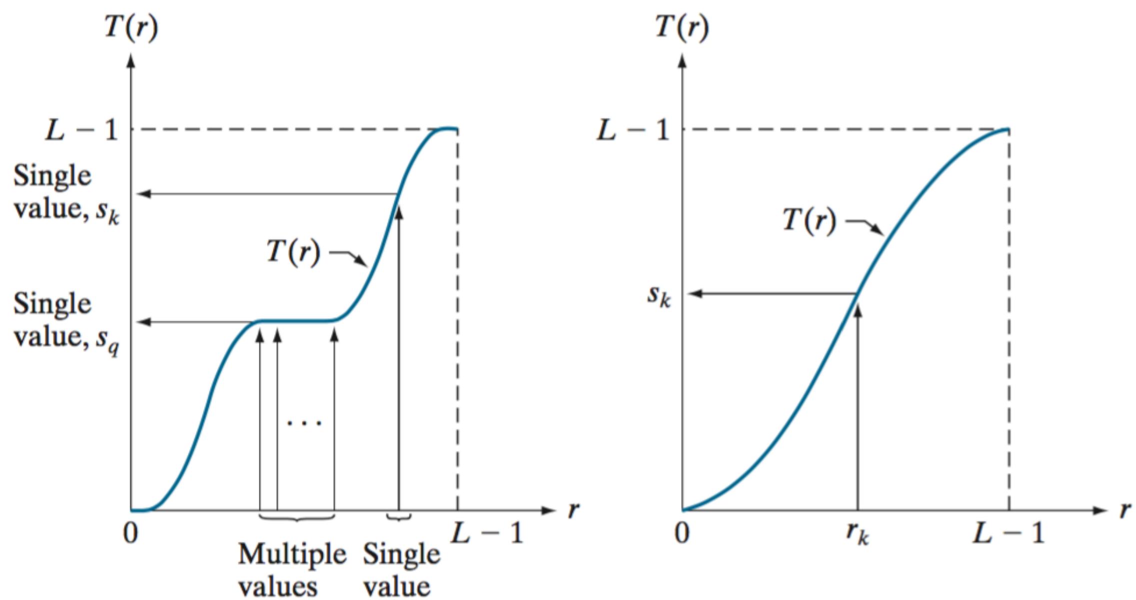
- With proper normalization, the histogram can be interpreted as an estimate of the probability density function (pdf) of the underlying random variable (the gray-level)

- You can also use fewer, larger bins to trade off amplitude

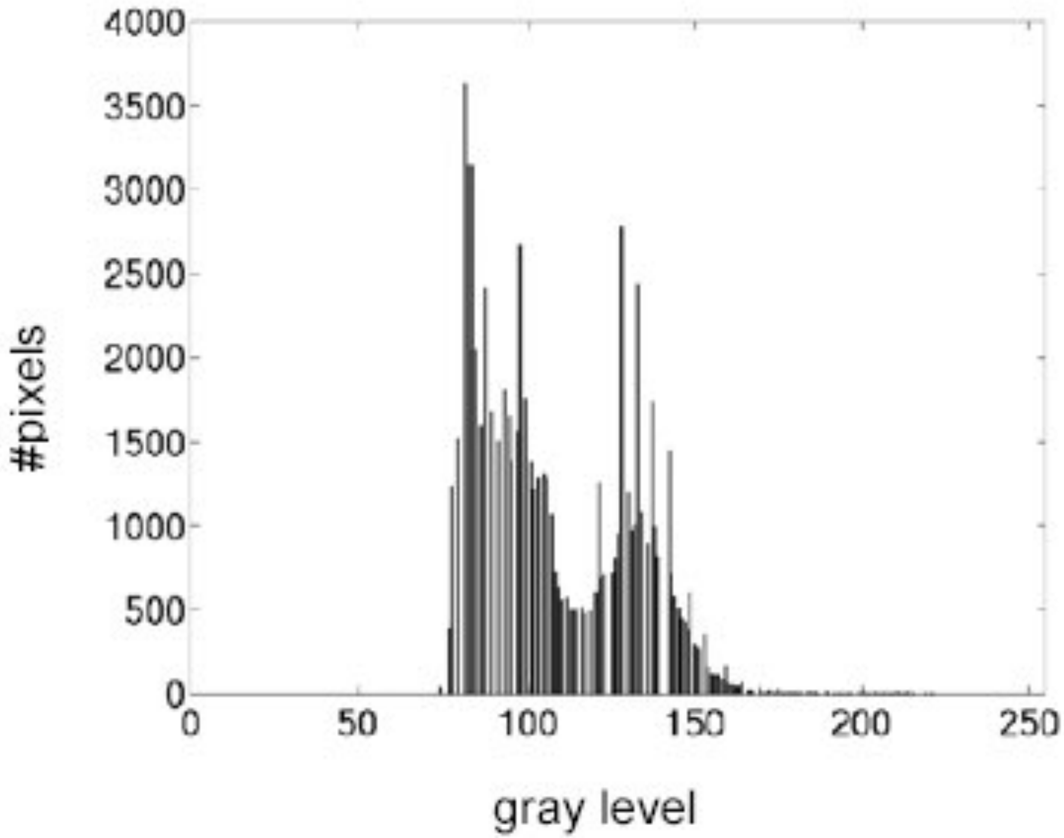
Histogram Processing

a b

FIGURE 3.17
(a) Monotonic increasing function, showing how multiple values can map to a single value. (b) Strictly monotonic increasing function. This is a one-to-one mapping, both ways.



Example:

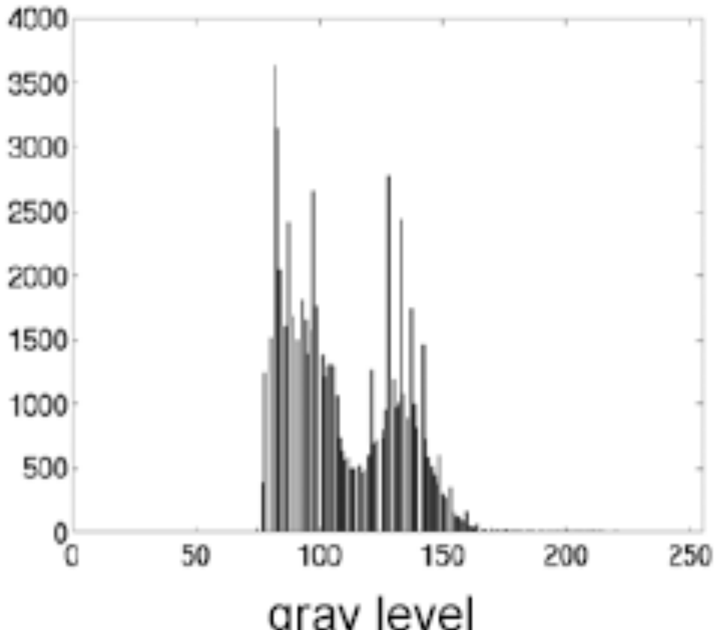


Pout image

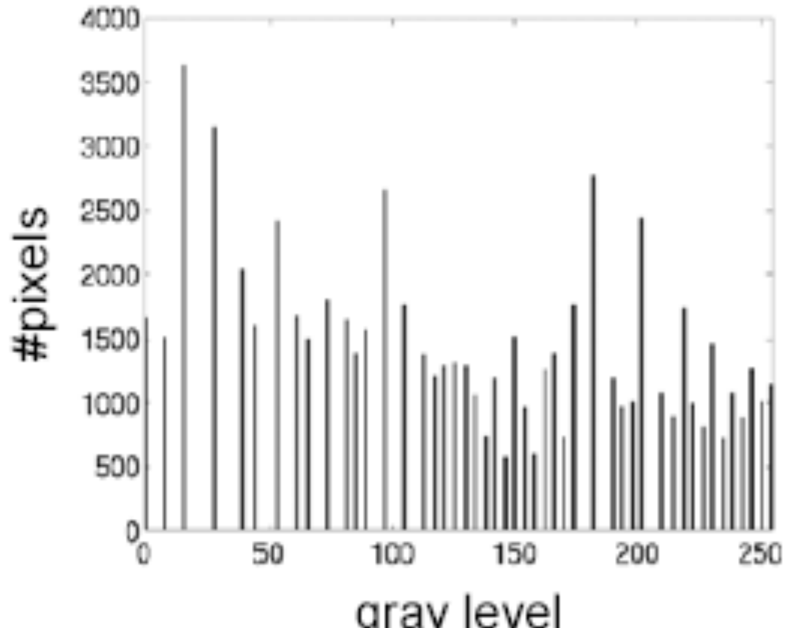
Original image *Pout*

Pout
after histogram equalization

Original image *Pout*



. . . after histogram equalization



Histogram Equalization

- The general idea: map the histogram of the given image to a flat histogram by performing a *nonlinear operation on the gray value at each pixel*.

- Nonlinear Transformation: $s = T(r)$
- Questions:
  - What is the right transformation?
  - How do we find it, given a particular image.

- Analysis for the continuous grayscale first

Histogram Equalization

- Consider the histogram of the given (continuous grayscale) image as a pdf ***p*(*r*)**, where *r* is in the interval [*0,L-1*]. eg L=256
  - Recall that as a pdf we have $p(r) \geq 0$ and $\int_0^{L-1} p(r)dr = 1$
  - Any pixel operation *T*(*r*) should map [*0,L-1*] to [*0,L-1*]

- Desired properties of *T*(*r*):
  - Keep the black/white order (*T*(*r*) should be monotonic increasing)
  - *T*(*r*) should be single valued (one-to-one), hence invertible.

- Question: Given image with histogram *p*(*r*), what does histogram of *s*=*T*(*r*) look like in general?

Histogram Equalization

- Consider continuous values of intensity
- *r* = gray level of image to be equalized in the range [*0, L-1*]

  0 : black

  L-1: white

Goal is to design a transformation

$$s = T(r) \ : \ \ 0 \le r \le L-1, \quad 0 \le s \le L-1$$

Assumption about *T(r)* :

(a) *T(r)* is single valued and monotonically increasing in the interval $0 \le r \le L-1$

(b) $0 \le T(r) \le L-1 \text{ for } 0 \le (r) \le L-1$

Condition *(a)* assures that $T^{-1}(s) = r$

- From Probability theory: $$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$ *(See Review Slides p. 34)*

- We consider Cumulative Distribution Function (CDF) as a Transformation

$$s = T(r) = (L-1)\int_0^r p_r(\omega)d\omega$$

(Not normalized. If normalized then L-1 = 1)

$$\frac{ds}{dr} = (L-1)\frac{d}{dr}\left\{\int_0^r p_r(\omega)d\omega\right\} = (L-1)p_r(r)$$

$$\text{Hence:}\quad p_s(s) = p_r(r)\left|\frac{1}{(L-1)p_r(r)}\right| = \frac{1}{L-1}\quad \text{for } 0 \le s \le L-1$$

- If $T(r)$ is just a CDF or the integral of the input pdf $(p_r(r))$ then applying $s = T(r)$ results in an image whose pdf is uniform.

(For PDF and CDF see pages 30-32 of the Review Slides)

Discrete Case

r_k = discrete intensity, $k = 0,\ldots,L-1$

$$p_r(r_k) = \frac{n_k}{n}$$

n_k = number of pixels that have intensity r_k

n = total number of pixels of the image

Then $s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = (L-1)\sum_{j=0}^{k} \frac{n_j}{n}$; $k = 0,\ldots,L-1$

new value

input value

max intensity

Example

r_k	n_k		$p_r(r_k) = n_k/MN$	
$r_0 = 0$	790	$s_0 = 1$	0.19	
$r_1 = 1$	1023	$s_1 = 3$	0.25	
$r_2 = 2$	850	$s_2 = 5$	0.21	
$r_3 = 3$	656	$s_3 = 6$	0.16	0.24
$r_4 = 4$	329	$s_4 = 6$	0.08	
$r_5 = 5$	245	$s_5 = 7$	0.06	
$r_6 = 6$	122	$s_6 = 7$	0.03	0.11
$r_7 = 7$	81	$s_7 = 7$	0.02	

TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64×64 digital image.

n = MxN

8 grey levels

Example: 3-bit image, L= 8, image size = n = 64x64 = **4096**, intensity levels [0,7].
Therefore, $p_0(r_0) = 790/4096 = 0.19$, etc…

$$\boxed{s_k = (L-1)\sum_{j=0}^{k}\left(\frac{n_j}{n}\right), \; k = 0,\ldots,L-1}$$ Therefore $s_0 = 7 \times 0.19 = 1.35;\; s_1 = 7 \times (0.19 + 0.25) = 3.1$

$$s_2 = 4.55,\; s_3 = 5.67,\; s_4 = 6.23,\; s_5 = 6.65,\; s_6 = 6.86,\; s_7 = 7.00$$

Note that the gray levels are integers so the values of the equalized histogram are: $s_0 = 1$, $s_1 = 3$, $s_2 = 5$, $s_3 = s_4 = 6$, $s_5 = s_6 = s_7 = 7$.

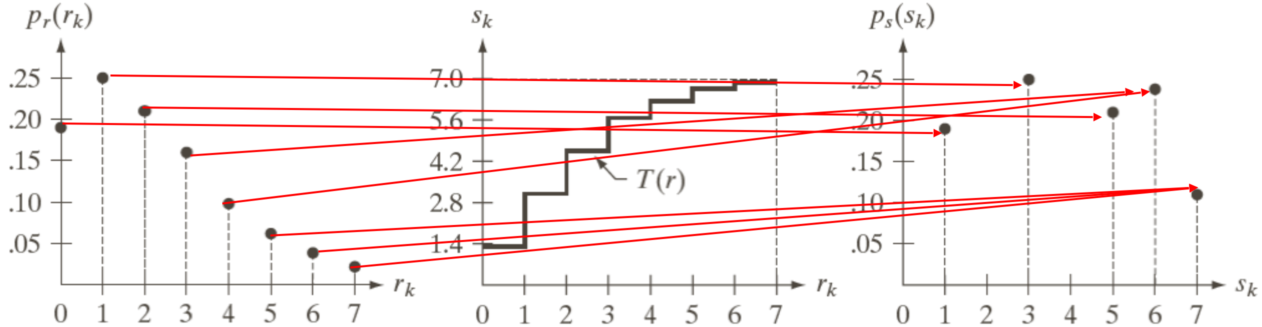
$r_0 = 0$ is mapped to $s_0 = 1 \Rightarrow n_{s_0} = n_{r_0} = 790 \Rightarrow p_s(s_0) = \dfrac{790}{4096} = 0.19$

$r_1 = 1$ is mapped to $s_1 = 3 \Rightarrow n_{s_1} = n_{r_1} = 1023 \Rightarrow p_s(s_1) = \dfrac{1023}{4096} = 0.25$

$r_2 = 2$ is mapped to $s_2 = 5 \Rightarrow n_{s_2} = n_{r_2} = 850 \Rightarrow p_s(s_2) = \dfrac{850}{4096} = 0.21$

But r_3 and r_4 both mapped to $s_3 = s_4 = 6 \Rightarrow n_{s_3} = n_{s_4} = 656 + 392 \Rightarrow p_s(s_3) = \dfrac{1048}{4096} = 0.26$

r_5, r_6 and r_7 all mapped to $s_5 = s_6 = s_7 = 7 \Rightarrow n_{s_5} = 245 + 122 + 81 \Rightarrow p_s(s_5) = \dfrac{448}{4096} = 0.11$



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Histogram Equalization (examples)

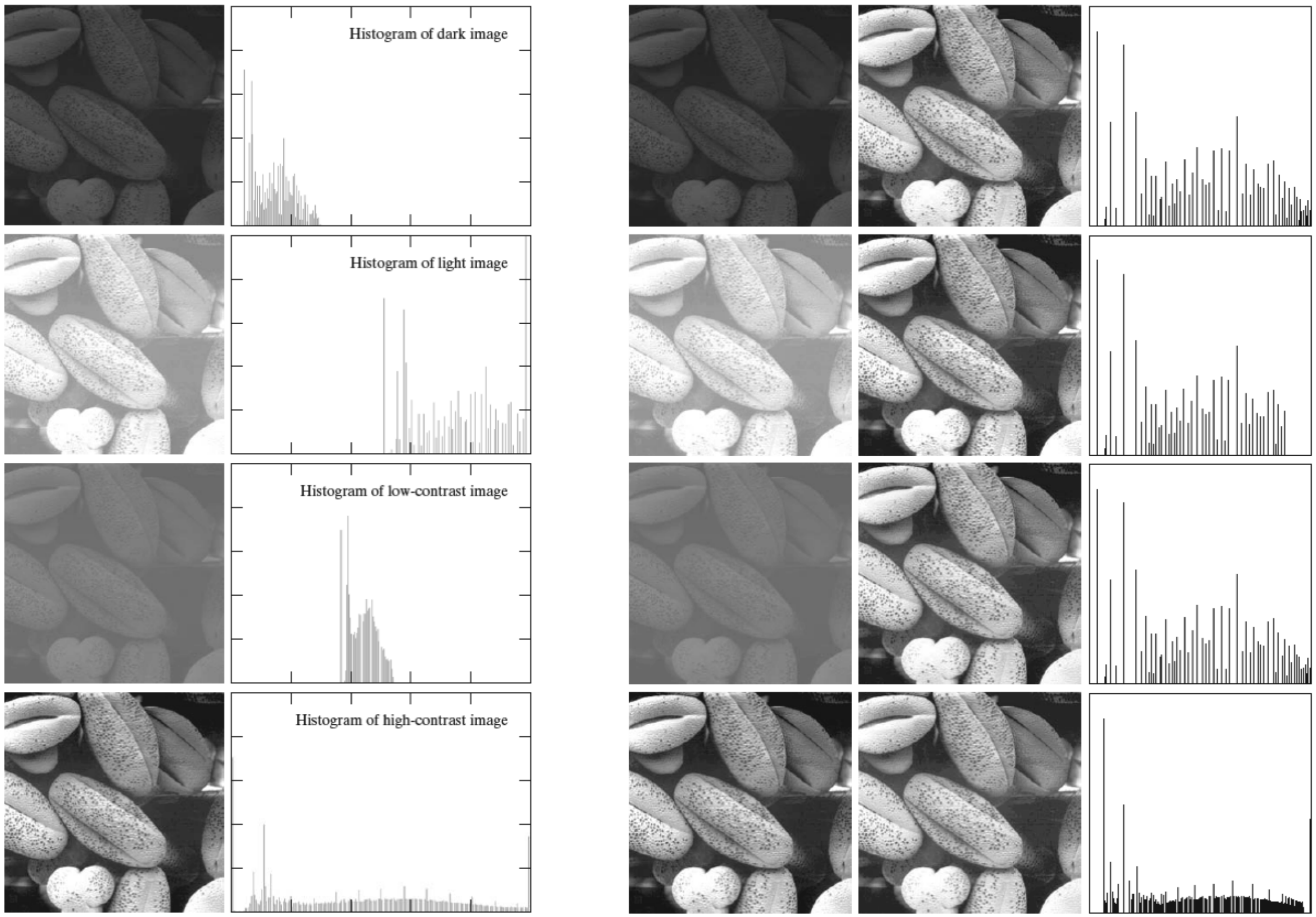
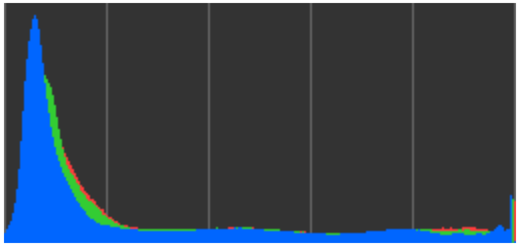
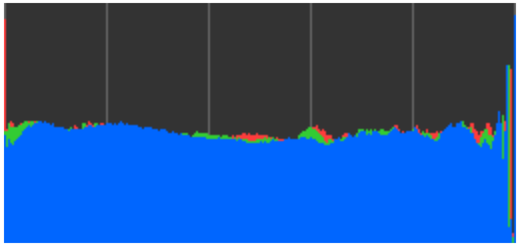


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.





Generalizations

- Adaptive Histogram Equalization
- Histogram Matching
  - Instead of flattening the histogram, we want to make the histogram of **A** look like the histogram of another image **B**.
  - Procedure (given two images):
    - Find transformation (T and U) that flatten images **A**, **B**
    - Required transformation is $U^{-1}(T(\mathbf{A}))$

Histogram Matching

- Rather than $p_s(s)$ uniform we want $p_s(s)$ to match as closely as possible to a desired, given *pdf*.
- r = pixel value before matching
- z = pixel value after matching
- We can compute $p_r(r)$ from the given image, and $p_z(z)$ is given. What is the Transformation from *r* to *z* ?
- In the continuous case:

$$s = T(r) = (L-1)\int_0^r p_r(\omega)\,d\omega$$

$$v = G(z) = (L-1)\int_0^z p_z(t)\,dt$$

- Can form Look-up table

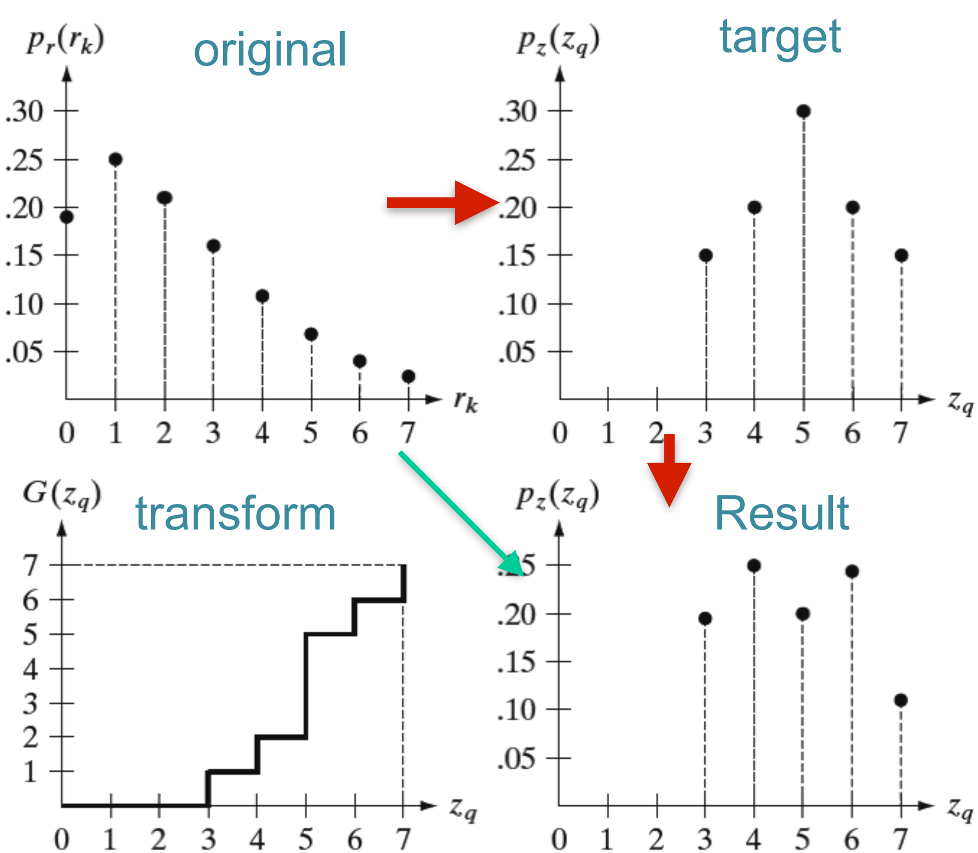
(Could be normalized to L-1 = 1)

Approach

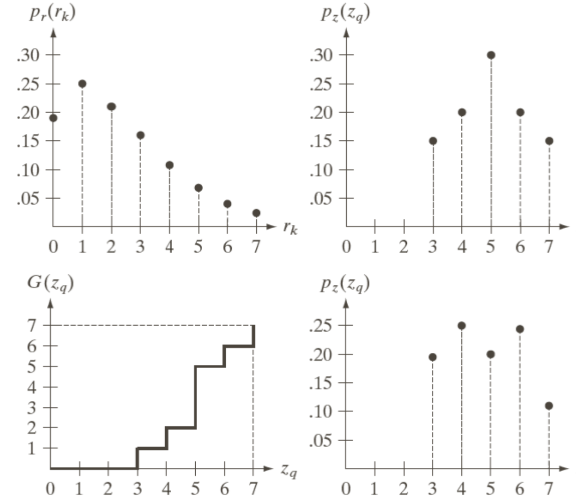
$$s = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = L-1\sum_{j=0}^{k}\frac{n_j}{n} \qquad (1)$$

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i) \qquad (2)$$

$$Matching: \quad G(z) = T(r) \longrightarrow z = G^{-1}\{T(r)\} = G^{-1}(s) \quad (3)$$



Approach



$$s = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = L-1\sum_{j=0}^{k}\frac{n_j}{n} \quad (1)$$

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i) \quad (2)$$

$$Matching: \ G(z) = T(r) \Longrightarrow z = G^{-1}\{T(r)\} = G^{-1}(s) \quad (3)$$

Method:

1. Compute $p_r(r)$ from input image and use (1) to find values of s. Round the resulting values to the integer range $[0, L\text{-}1]$.

$s_0 = 1$, $s_1 = 3$, $s_2 = 5$, $s_3 = s_4 = 6$, $s_5 = s_6 = s_7 = 7$.

2. Use the given PDF $p_z(z_i)$ to obtain transformation $G(z)$ as in (2). Round the values of G as above. Create a table of G.

$G_0 = G_1 = G_2 = 0$, $G_3 = 1$ (round-off), $G_4 = 2$, $G_5 = 5$, $G_6 = 6$, $G_7 = 7$.

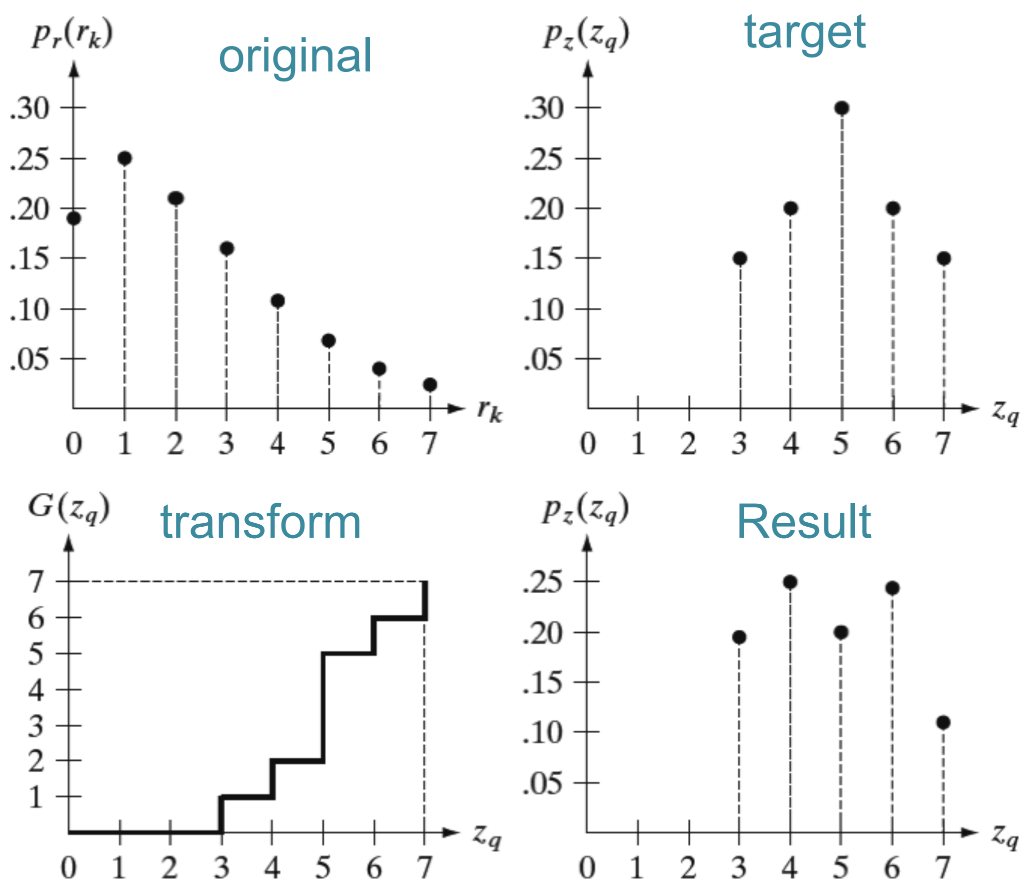
3. For every value of s_k use the stored values of G to find the corresponding of z_q so that $G(z_q) \approx s_k$ and store these mappings from s to z. When more than one z_q satisfy the given s_k use the smallest.

$s_0 = 1 \to G_3 = 1 \to z = 3$
$s_1 = 3 \to G_4 = 2 \to z = 4$
$s_2 = 5 \to G_5 = 5 \to z = 5$
$s_3 = s_4 = 6 \to G_6 = 6 \to z = 6$
$s_5 = s_6 = s_7 = 7 \to G_7 = 7 \to z = 7$

4. Obtain output image by first equalizing input image by using (1); pixel values in this image are s values. For each pixel with value s in the equalized image, perform inverse mapping (3) to obtain the corresponding pixel in output image.

r_k	s_k	z_k
0	1	3
1	3	4
2	5	5
3	6	6
4	6	6
5	7	7
6	7	7
7	7	7

Histogram Matching (example)



a b
c d

FIGURE 3.22 (a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).

r_k	s_k	z_k
0	1	3
1	3	4
2	5	5
3	6	6
4	6	6
5	7	7
6	7	7
7	7	7

Histogram Matching (example)

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

TABLE 3.2
Specified and actual histograms (the values in the third column are from the computations performed in the body of Example 3.8).

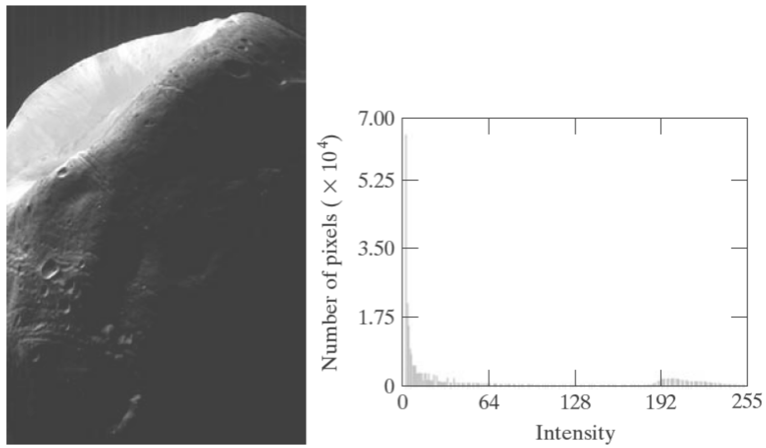
z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7

TABLE 3.3
All possible values of the transformation function G scaled, rounded, and ordered with respect to z.

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

TABLE 3.4
Mappings of all the values of s_k into corresponding values of z_q.

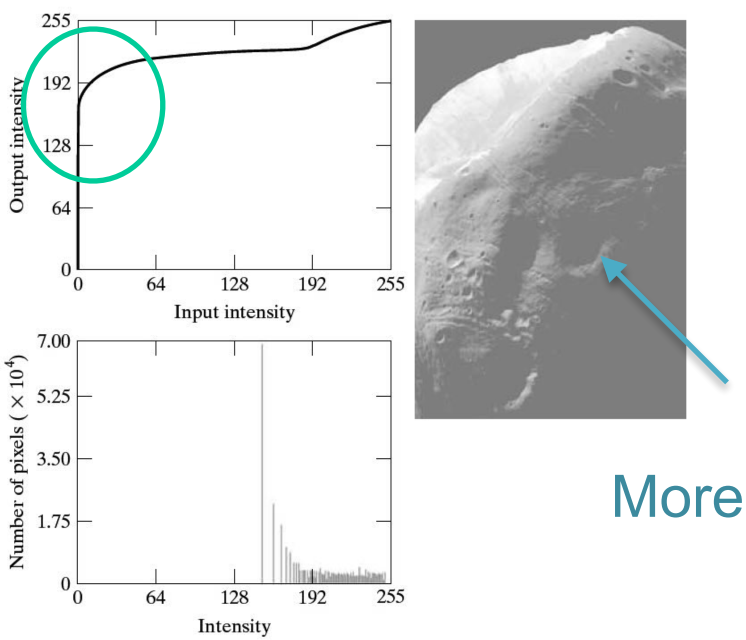
Comparison of Histogram Equalization and Histogram Matching



a b

FIGURE 3.23
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)

Histogram Equalization



More details

a b
c

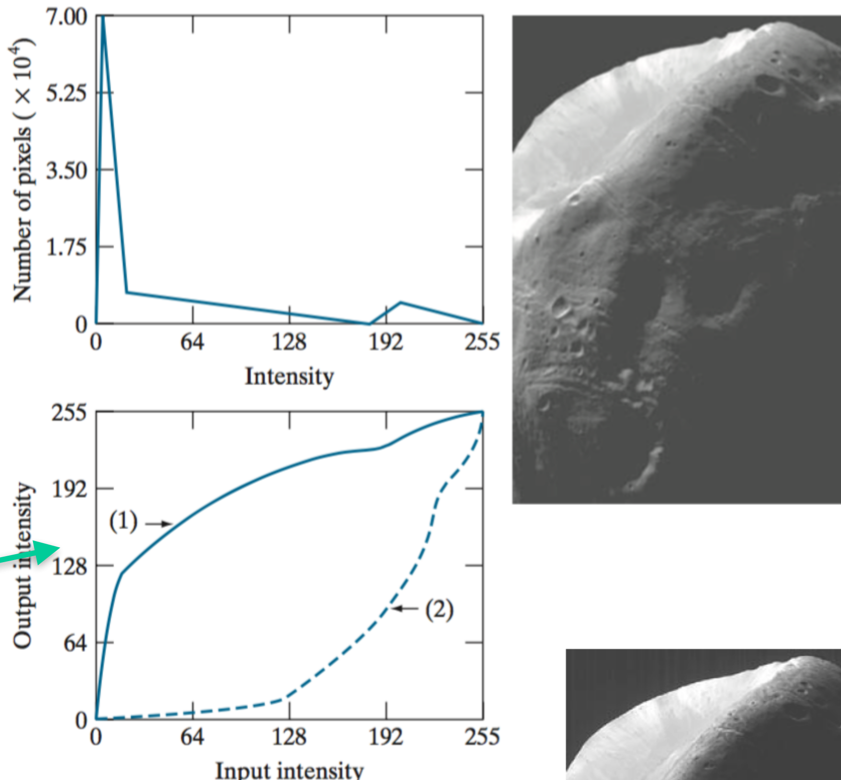
FIGURE 3.24
(a) Transformation function for histogram equalization. (b) Histogram-equalized image (note the washed-out appearance). (c) Histogram of (b).

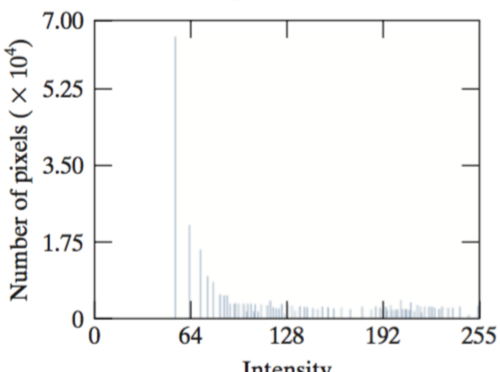
Histogram Matching (example)

target histogram

Transform

Result

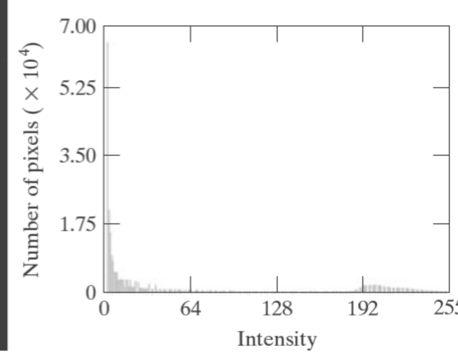




a c
b
d

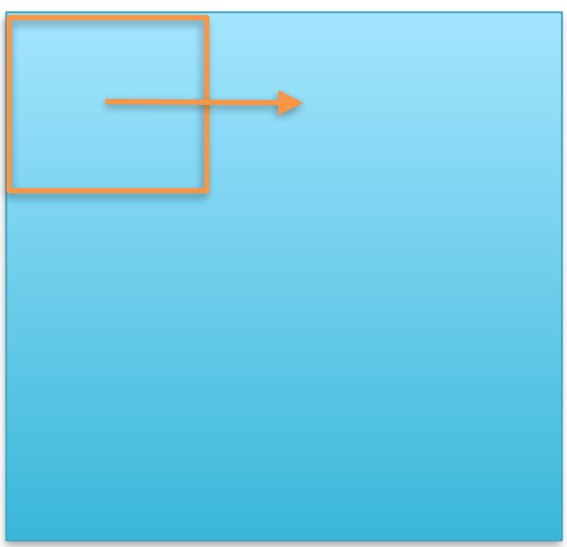
FIGURE 3.25 Histogram specification. (a) Specified histogram. (b) Transformation $G(z_q)$, labeled (1), and $G^{-1}(s_k)$, labeled (2). (c) Result of histogram specification. (d) Histogram of image (c).

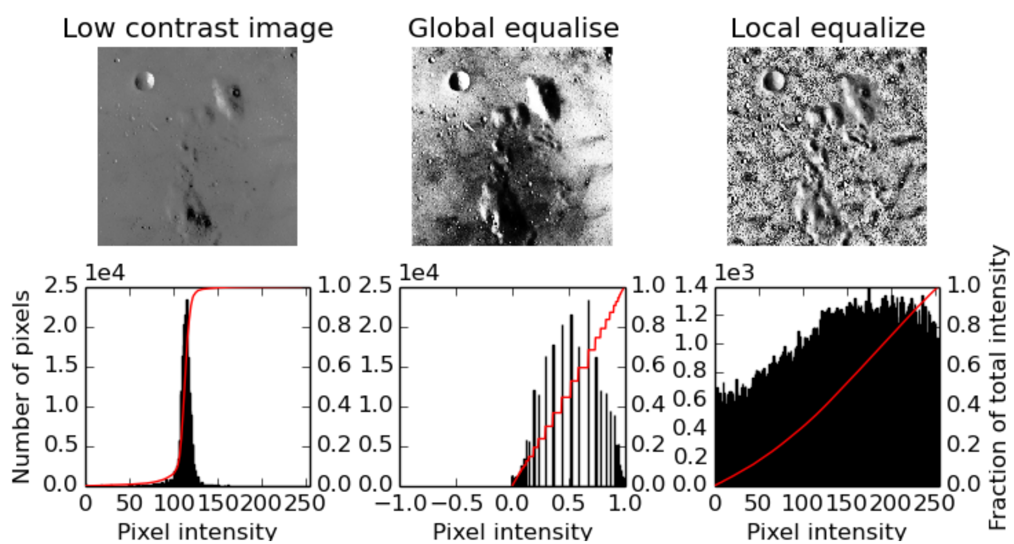
Original histogram



Local Enhancement using Local Histogram

- Define a "window" as a rectangle neighborhood.
- Move centre of windows across the image. At each location, compute histogram in the window and do histogram equalization.
- *Some constraints may be needed to ensure results*





scikit-image.org

Multi-Image "Averaging"

- It may be possible to obtain multiple noise-corrupted images of a scene and average them to obtain a less noisy result.
- It is important that there be no motion between the frames, or that the frames be aligned first.

Given: $g_k(x,y) = f(x,y) + n_k(x,y)$

Compute: $\hat{f}(x,y) = \sum_{k=1}^{N} \alpha_k g_k(x,y)$

$$\sum_{k=1}^{N} \alpha_k = 1$$

Simple Averaging Example:

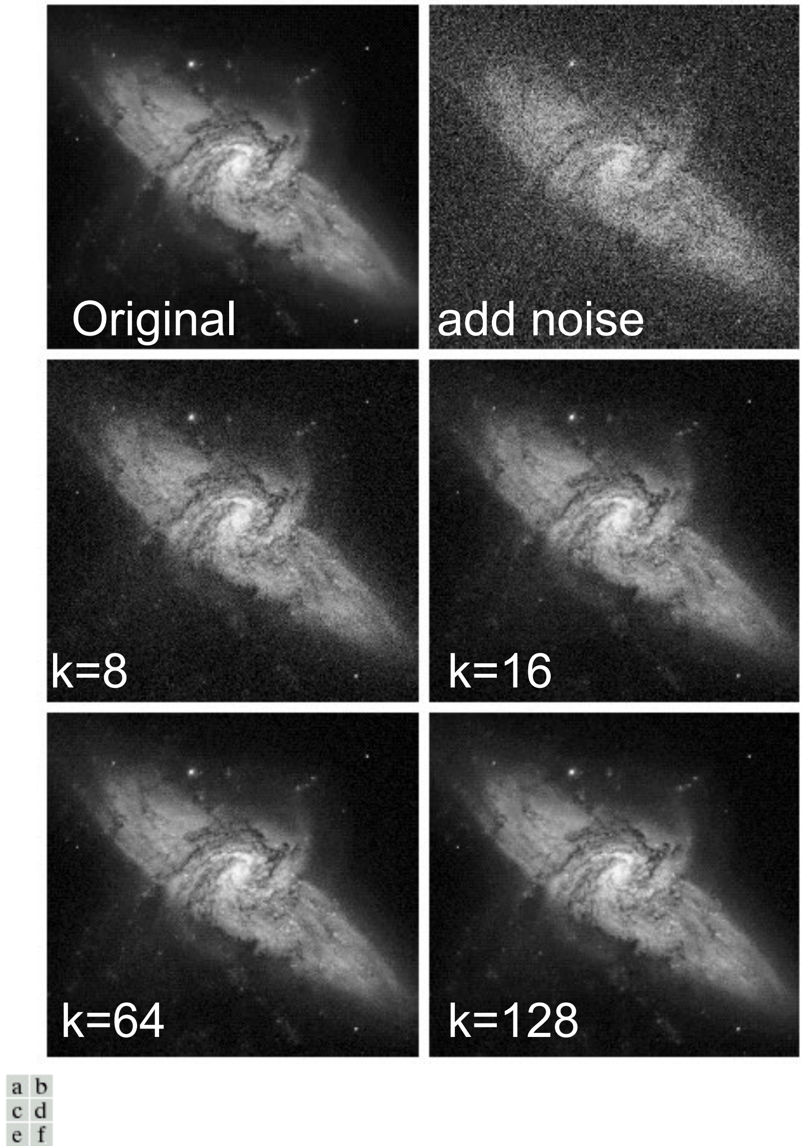


FIGURE 3.30 (a) Image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging $K = 8, 16, 64$, and 128 noisy images. (Original image courtesy of NASA.)

Additive noise = Gaussian with zero mean & standard deviation of 64 grey levels

Averaging k times

Optimal Multi-image "Averaging"

Given: $$g_k(x,y) = f(x,y) + n_k(x,y)$$

Compute: $$\hat{f}(x,y) = \sum_{k=1}^{N} \alpha_k g_k(x,y)$$

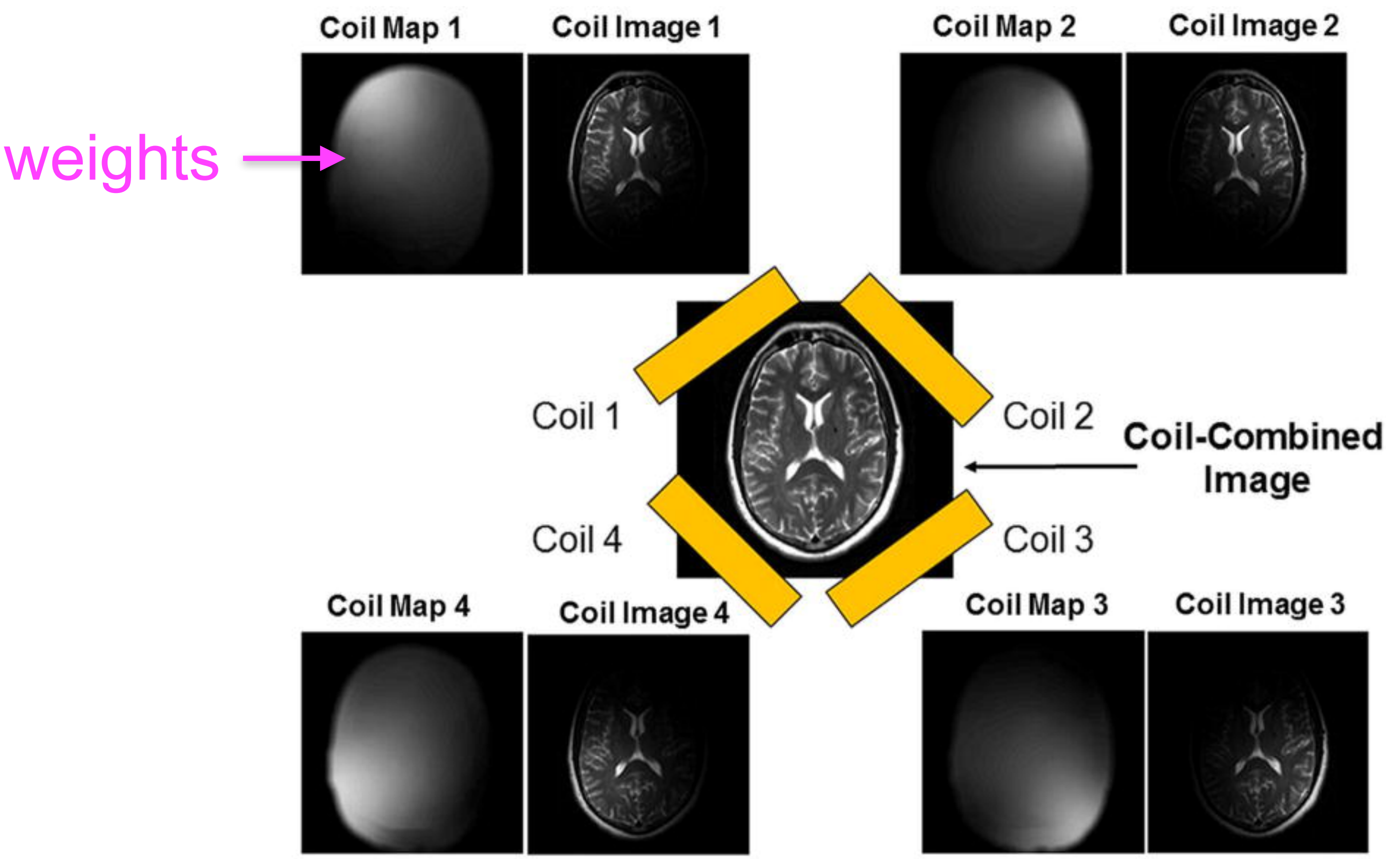
$$\boxed{\sum_{k=1}^{N} \alpha_k = 1}$$

Given the noise statistics of the measured images, what should these coefficients be to yield the "best" results?

- Inversely proportional to the variance of noise in each image

MRI scan reconstruction from multiple sensor data

Adaptive averaging for image reconstruction



(Hamilton et al. Europe PMC, 2017)

Summary

- **Point processing**
  - ✓ Contrast stretching
  - ✓ Intensity slicing
  - ✓ Gamma correction
  - ✓ Bit-plane slicing
- **Histogram processing**
  - ✓ Histogram equalization (global & local)
  - ✓ Histogram matching
- **Multi-image averaging**
  - ✓ Simple averaging
  - ✓ Optimal/adaptive averaging

Reading from textbook

Chapter 3: Page 120-153

Food for thought

- For the bit map, how will the image histogram change if we remove the lowest-order bit?
- When we preform histogram equalization, why it is very rare that a perfectly flat histogram can be obtained for the resulting image?
- Why histogram matching often won't result in the exact histogram as the template image?
- When using averaging of N images to reduce noise, if the standard deviation of the Gaussian noise is ***S***, what is the standard deviation of the noise in the noise-reduced image?

  https://www.youtube.com/watch?v=-RDYHXwJ1LY

- In what scenario is local histogram equalization beneficial?
- If we want to improve image contrast, will gamma correction be a good choice? compared to histogram equalization?