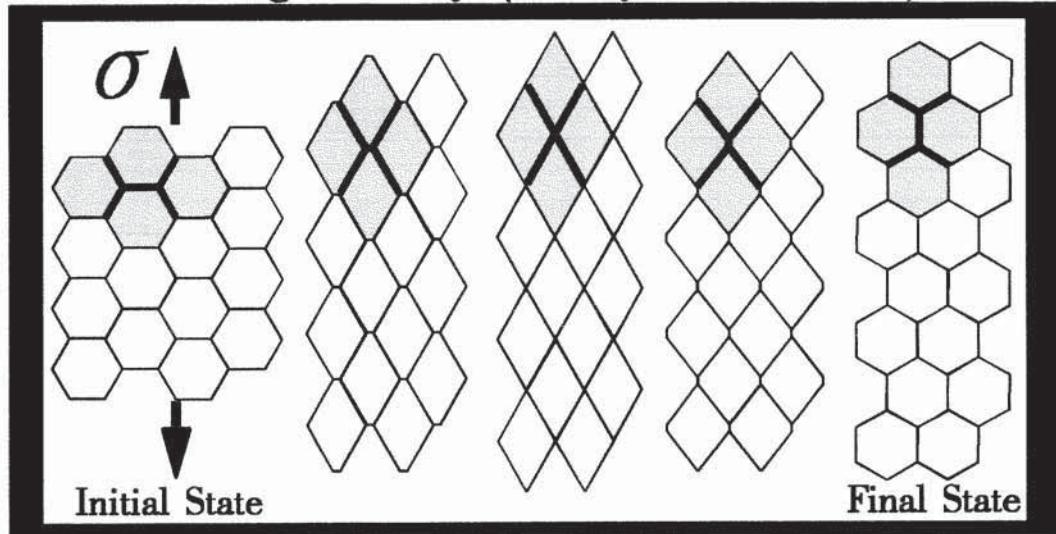
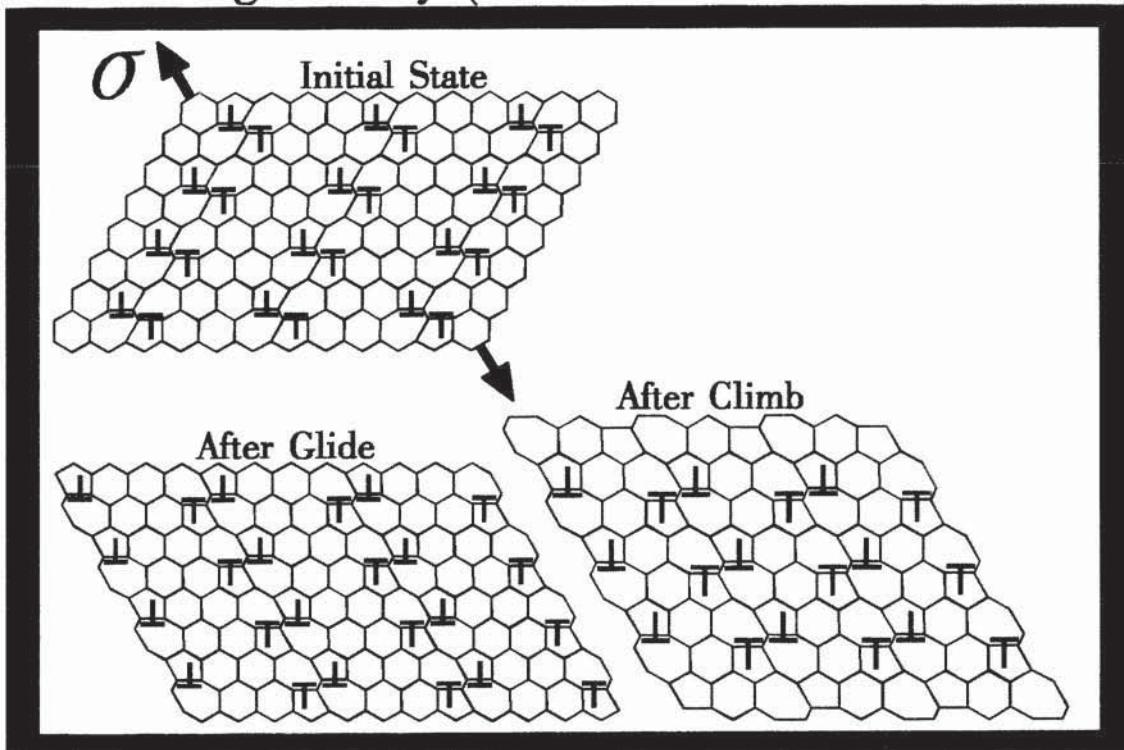


§5. Moving Grains—Superplasticity

Homogeneously (Ashby *et al.* 1978)



Less Homogeneously (Sherwood and Hamilton 1994)



MEANING OF FREQUENTLY USED SYMBOLS

<u>Symbol</u>	<u>Meaning</u>
<i>A</i>	constant in constitutive relationship
<i>b</i>	Burgers vector for cellular dislocations, and width of hexagon with diameter <i>d</i> : $\mathbf{b} = \sqrt{3} \mathbf{d}/2$
Γ	grain boundary surface tension
γ	shear strain
D_B	atomic diffusivity in grain boundary
D_V	atomic diffusivity in bulk material
<i>d</i>	average grain size, or cell diameter
d_0	initial grain size: $\mathbf{d}_0 \equiv \mathbf{d}(t = 0)$; e.g. $\mathbf{d}_0 =$ $9.2 \mu\text{m}$ for 7475 Aluminum
d_c	upper limit to grain size from second phase particles: e.g. $\mathbf{d}_c = 2(4r/3f)$
\dot{d}	grain growth rate
δ	grain boundary "width"
ε	strain: $\varepsilon = \ln[l_f/l_i]$, l_f and l_i denoting final and initial lengths, respectively
ε_o	strain required to switch four identical cells simultaneously, $\varepsilon_o = 0.55$

MEANING OF FREQUENTLY USED SYMBOLS

(Continued)

<u>Symbol</u>	<u>Meaning</u>
$\dot{\varepsilon}$	imposed or total strain rate
$\dot{\varepsilon}_{\text{GBS}}$	strain rate from grain boundary sliding/migration
η	material viscosity: <i>e.g.</i> $\eta = [6.93AX_{\text{d}}M_{\text{B}}\delta^2/d^3]^{-1}$
η_{o}	crystal viscosity: <i>e.g.</i>
	$\eta_{\text{o}} \equiv kT d^2/[40D_{\text{V}}\Omega(1 + \pi\delta D_{\text{B}}/dD_{\text{V}})]$
	for diffusional creep
f	volume fraction of second phase particles, <i>e.g.</i>
	$f \approx 0.015$ for 7475 Aluminum
K	constant of proportionality for production rate of "excess" cellular dislocations
k	Boltzmann's constant: 1.38054×10^{-23} J/K-atom
κ	ratio of the strain rate from grain boundary sliding/grain boundary migration to the imposed strain rate: $\kappa \equiv \dot{\varepsilon}_{\text{GBS}}/\dot{\varepsilon}$
M_{B}	grain boundary mobility: <i>e.g.</i> $M_{\text{B}} = (\Omega/\delta)[D_{\text{B}}/kT]$
M	cellular dislocation mobility
m	strain rate sensitivity: $m \equiv \partial(\ln \sigma)/\partial(\ln \dot{\varepsilon})$; m is determined from tensile testing

MEANING OF FREQUENTLY USED SYMBOLS

(Continued)

<u>Symbol</u>	<u>Meaning</u>
m	crystal strain rate sensitivity predicted from theory of plastic deformation mechanism
m_B	atomic mobility in grain boundary: $m_B \equiv D_B/kT$
N_v	number of grains (cells) per unit volume
Ω	atomic volume
Q	activation energy
R	the gas constant, 8.314 J/K-mole
r	average radii of second phase particles in quasi-single phase materials, <i>e.g.</i> $r \approx 0.1 \mu\text{m}$ for 7475 Aluminum
σ	flow stress or tensile stress
T	temperature of the system
t	time
τ	relaxation time period for cellular dislocation annihilation
τ	shear or resolved stress, <i>i.e.</i> Schmid Factor \times σ : $\tau = s\sigma$, s denoting the Schmid factor

MEANING OF FREQUENTLY USED SYMBOLS

(Continued)

<u>Symbol</u>	<u>Meaning</u>
X_d	fractional concentration of non-hexagonal cells, or "defects," here assumed to be in 5-7 combinations in an array otherwise comprised of hexagons
X_d^0	defect level during annealing
X_d^{ss}	steady state ($\partial X_d / \partial t \equiv 0$) defect level during superplastic deformation: $X_d^{ss} \equiv X_d^0 + \delta X_d^{ss}$
δX_d	incremental defect level from superplastic deformation
δX_d^{ss}	steady state defect increment: $\delta X_d^{ss} \equiv K\tau\kappa\varepsilon$
ρ	concentration of mobile cellular dislocations: $\rho = 0.77X_d/d^2$; here, X_d is the fractional concentration of non-hexagonal cells moved by the imposed strain rate
v	cellular dislocation velocity: $v = \mathbf{M}\mathbf{b}\boldsymbol{\tau}$; the velocity with which cellular dislocations must move in order to make the material flow with the imposed strain rate

The grains comprising a polycrystalline material are the appropriate subdivision of matter for describing superplastic behavior. The polycrystalline microstructure is modelled as either a cellular array or its "dual" lattice. Defects (dislocations and disclinations) in the cellular array provide a simplistic description for the grain size distribution. Cellular dislocation glide and climb model microstructural flow and the latter also models deformation-enhanced grain growth. Cellular dislocation motion, like the crystalline counterpart, is an inhomogeneous process. After the introductory remarks, much of this section is from Sherwood and Hamilton (1994).

For the case of diffusional creep in a polycrystalline material, Ashby (1972) relates the engineering strain rate, $\dot{\gamma}$, to the shear stress, τ , by $\dot{\gamma} = \tau/\eta_o$, where η_o is the *crystal viscosity*: $\eta_o \equiv kT\bar{d}^2/[40D_v\Omega(1 + \pi\delta D_b/\bar{d}D_v)]$. Here, \bar{d} denotes the average grain size, D_v the lattice diffusivity and D_b the grain boundary diffusivity, Ω the atomic volume, δ the grain boundary width, and kT the usual. Since $\dot{\gamma} \propto \tau$, diffusional creep is an example of Newtonian flow; the material becomes more viscous as \bar{d} increases. Diffusional creep is *accommodated* by grain boundary sliding (Langdon 1970, Stevens 1972); however, sliding "makes no direct contribution to the total strain" (Langdon 1970). McLean (1952-53) observed grain boundary migration during creep of "super pure" Al at 200 °C and noted that it became more prevalent as \bar{d} decreased.

Rachinger (1952-53) deformed 99.95% Al with an initial grain size of 99 μm over a temperature range of 20 - 350 °C at a strain rate of 0.1%/hr ($2.8 \times 10^{-7}/\text{s}$) to strains of about 0.5. For $T \leq 200$ °C the grains were *plastic*, exhibiting about the same level of strain as the tensile specimens. For $T \geq 250$ °C *grain strain* was an order of magnitude lower

than the specimen strain. Rachinger suggests for these higher temperatures that deformation occurs by *grain boundary rearrangements*, a mechanism(s) that produces bulk strain without grain strain, for example, the neighbor switching mechanism. Also, at $T = 350^{\circ}\text{C}$ he observed that d increased from $99\ \mu\text{m}$ to $150\ \mu\text{m}$: *i.e.* *grain growth* had occurred. When deformed at 350°C with a much higher strain rate of $10\%/\text{min}$ ($1.7 \times 10^{-3}/\text{s}$), no grain growth was detected and the grain strain was nearly equal to the specimen strain. Grain growth during diffusional creep might produce non-Newtonian behavior (Nix 1982); *i.e.* $\eta_o \rightarrow \eta_o = \eta_o(\dot{\gamma})$.

Superplastic materials exhibit a high *strain rate sensitivity*, m , of the flow stress σ :

$$m \equiv \partial(\ln \sigma)/\partial(\ln \dot{\varepsilon}),$$

where $\dot{\varepsilon}$ is the *imposed tensile strain rate*: m is greater than about 0.3 (Holt and Backofen 1966). This is the strain rate sensitivity determined from repeated tensile testing of a material over some range of imposed strain rates. Relationships such as the following are, on the other hand, predicted by assuming that a particular deformation mechanism is "rate limiting":

$$\sigma = \eta_o(d, T)\dot{\varepsilon}^m,$$

where $m \leq 1$ is the *crystal strain rate sensitivity*. If $m = 1$ and $\eta_o(d, T) \neq \eta_o(\dot{\varepsilon}, T)$, then *Newtonian behavior*, $\sigma \propto \dot{\varepsilon}$ (*i.e.* $m = 1$), is predicted. High m values produce large, local

increases in strain rate in regions of a tensile specimen that are tending to neck down, resulting in local hardening and suppression of further neck growth; as $m \rightarrow 1$ specimen elongation can exceed 1000% (Ghosh and Hamilton 1982). The strain rate sensitivity of a polycrystalline material changes with strain rate, $m = m(\dot{\varepsilon})$, because $\eta_o = \eta_o(\dot{\varepsilon})$ even for a *constant* temperature: the load can be taken up by the grains via atomic diffusion, dislocation movement, grain boundary sliding, and combinations of these mechanisms (Langdon 1970); see also Sherby and Wadsworth (1989). Since some of these processes are associated with diffusional creep as well, Rai and Grant (1983) conclude that superplastic deformation is similar, the difference being that the fine grain structure typical of superplastic materials ($d \leq 10 \mu\text{m}$) promotes "highly mobile" grain boundaries.

Both diffusional creep and superplastic deformation are highly *heterogeneous processes*: "Local strains" between some grains can be large while only small "local strains" are imparted to other grains (Rai and Grant 1983). Also, some grains can rotate substantially while other grains do not rotate at all (Beeré 1976). Grains have furthermore been observed to slide both in clumps of many grains and individually (Edington *et al.* 1976), *e.g.* the *cooperative grain boundary sliding* of Astanin *et al.* (1994) and Zelin *et al.* (1994).

Unlike diffusional creep however, superplastic behavior is probably not associated with a unique mechanism, *i.e.* $m \neq \text{constant}$. For very low strain rates these materials can exhibit diffusional creep with $m \approx 1$; but at higher strain rates where superplasticity is generally studied, m follows a sigmoidal relationship with the strain rate (Arieli and Mukherjee 1982, Langdon 1982): At very low strain rates diffusional creep results and $m \approx 1$; for somewhat higher strain rates *region I* superplasticity is obtained with $m \approx 0.3$; as

the strain rate increases further *region II* superplasticity with $m \geq 0.5$ results; and the largest strain rates considered are those of *region III* superplasticity with $m \approx 0.2$. See Figure 1. Potential mechanisms for superplastic flow are considered in the recent reviews of Kashyap and Mukherjee (1985) and Sherby and Wadsworth (1989).

The crystal strain rate sensitivity is, furthermore, likely to be a local property of the grains in a polycrystalline material: Some grains may deform by one type of mechanism while others deform from an entirely different one (Ghosh and Raj 1986); but nearly all theories of superplastic flow propose a behavior for either a single grain or a small group of grains and extrapolate this behavior to the entire grain boundary array.

Ashby and Verrall (1973) reproduce the sigmoidal strain rate behavior typifying superplastic flow by superposing a *diffusionally-accommodated neighbor switching* mechanism ($m = 1$) with dislocation creep ($m \approx 1/6$, $\eta_o = \eta_o(T)$). Grain boundary rearrangements modelled by neighbor switching reactions likely play a major role in superplastic flow (Kashyap *et al.* 1985). Neighbor switching results from grain boundary sliding/migration and produces strain, atomic diffusion accommodates sliding/migration. This relationship between grain boundary sliding/migration and atomic diffusion is inverse to that for diffusional creep; see Cannon (1972), and Cannon and Nix (1973) for a full account.

Morral and Ashby (1974) suggest an analogy between neighbor switching and the slip of lattice planes by dislocation glide. Lattice slip from dislocation motion is, of course, a heterogeneous process, and this analogy therefore addresses the experimental observation that superplastic flow is heterogeneous. Their analogy follows by associating neighbor switching with the glide of cellular dislocations in an array of non-uniform cells. Morral and Ashby

note that if the cellular dislocation density is a function of the strain rate, that the strain rate sensitivity (m) for the cellular array is predicted to vary with strain rate as observed experimentally (m decreasing with increasing strain rate), even for the case where neighbor switching results from a mechanism with $m = 1$, such as atomic diffusion. This hypothesis is examined here (Sherwood and Hamilton 1994). See Arieli and Mukherjee (1982) for full discussion of the variation of m with the strain rate. Ovid'ko (1989) discusses the plastic deformation of metallic glasses with dislocations and disclinations analogous to the cellular defects utilized here.

Grain growth, or deformation-enhanced grain growth (Clark and Alden 1973) often accompanies superplastic flow (Kashyap *et al.* 1985), and this may lead to general strain hardening of the material, perhaps also in a reduction of m (Ghosh and Hamilton 1982), but perhaps not (Langdon 1982). Grain growth from time at the test temperature ("static" grain growth) and deformation-enhanced grain growth may be either deleterious to superplastic forming or beneficial, depending on the grain growth rate and the strain rate (Hamilton 1989). Correctly accounting for grain growth during superplastic flow can therefore be used to optimize the strain rate path during forming. Models for deformation-enhanced grain growth based on cellular dislocation climb, *i.e.* Hillert (1965), are presented and analyzed here (Sherwood and Hamilton 1994). Holm, Embury and Purdy (1976) were the first to attribute deformation-enhanced grain growth to neighbor switching. See Kaibyshev, Valiev and Astanin (1976), Wilkinson and Cáceres (1984), Nes (1985), Senkov and Myshlyaev (1986), Campenni and Cáceres (1988), and Perevezentsev, Rybin and Chuvil'deev (1992) for other treatments.

5.1. Grain Rearrangements

Grains rotate until the most viscous boundaries are perpendicular to the applied stress, then $\dot{\omega} = 0$ and there is very little sliding on these boundaries. Such stability is "disturbed" by "grain rearrangements." Grain rearrangements, or neighbor switching, produce grain boundary rotations. Grains rotate to reduce the work performed by the applied stress in achieving unit strain, and grain boundaries rotate to minimize the free energy associated with the boundaries; these processes are therefore coupled during diffusional creep and superplastic deformation (Beeré 1978). *Grain rearrangements* occur during annealing (*i.e.* recrystallization and grain growth), diffusional creep, and superplastic deformation, to maintain equilibrium angles at grain corners (dihedral angles are $\frac{2}{3}\pi = 120^\circ$ at triple points, or junctions, in two dimensions). Relative movement of grain "centers" ("elastic" deformation of "dual" lattice), *i.e.* grain stretching, requires grain boundary migration to maintain equilibrium angles at grain junctions, Figure 32a. Grain rearrangements are not required at small strains to maintain surface tension equilibrium, *e.g.* for uniaxial deformation as in Figure 32b, at larger strains the free energy is lowered if they do occur, Figure 32c, since boundary area (length in two dimensions) is reduced, Figure 32d.

Stress-driven cell boundary rotations* \equiv *transilatory cell movements* \equiv *cellular dislocation motion; that is, Morral and Ashby's (1974) model for the flow of a cellular array. ***Stress-driven cell rotations* \equiv *cellular disclination motion***. Transilatory cell movement (neighbor switching) occurs preferentially at cellular dislocations because cells associated with these defects have some edges that are smaller than average; deformation mechanisms for creep and superplastic flow predict faster strain rates for smaller grain sizes.

Let N_L and N_{LT} denote the average number of grain boundaries per unit length in the longitudinal and long-transverse directions, respectively, for the rolling plane in a sheet product, such as 7475 Al; and let N_{ST} denote the same quantity for the short-transverse direction in the plane perpendicular to the rolling direction. Then d_L , d_{LT} , and d_{ST} are average grain dimensions. Let d denote the average of these grain dimensions. A dislocated BCC lattice that models this microstructure has a primitive unit cell with volume $\Omega = g^{1/2} \approx d_L d_{LT} d_{ST} \approx d^3$. "Dual" to this lattice is the cellular array where cells are tetrakaidecahedrons, except for irregularly shaped cells at cellular dislocation cores (Morral and Ashby 1974). Now consider only the rolling direction, after Rachinger (1952-53); N_T and d_T replace N_{LT} and d_{LT} , and N_{ST} and d_{ST} are ignored. A dislocated lattice with *sixfold* symmetry that models this microstructure has a primitive unit cell with volume $\Omega = g^{1/2} \approx d_L d_T \times 1$. "Dual" to this lattice is a hexagonal array containing cellular dislocations. This cellular array is used throughout this section to illustrate the considerations.

After Rachinger (1952-53):^[57] For a tensile load applied in the longitudinal direction that produces a total strain $\bar{\epsilon} \equiv \ln(l_f/l_i)$, l_f and l_i denoting the initial and final gauge lengths of the tensile specimen, respectively, the final configuration of the grains in a tensile specimen arises from three separate processes:^[58]

[57]: See also Ishida, Mullendore and Grant (1965), Cannon (1972), and Cannon and Nix (1973),

[58]: Like Rachinger, I neglect complicating factors such as "directional grain growth," grain agglomeration, and dynamic recrystallization.

- (1) A "non-directional" growth of the grains by a "linear" factor, say C .
- (2) A longitudinal tensile strain of the grains (grain strain) of amount ε_G .
- (3) Relative translatory grain movements accounting for the difference between the specimen elongation $\tilde{\varepsilon}$ and the grain strain ε_G .

The first corresponds to cellular dislocation climb. The second results from "conventional" creep mechanisms (*e.g.* diffusional creep for lower strain rates and dislocation creep for higher strain rates) and corresponds to an elastic deformation of the "dual" lattice. The third corresponds to cellular dislocation glide and climb resulting from stress-driven neighbor switching reactions. Cellular dislocation climb and glide are illustrated for the two dimensional case by Figure 33.

Rachinger's "Compatibility" Argument: A "*superposition principle*" of sorts is invoked with the suggestion that the three processes above can be added (Rachinger 1952-53), as follows. In the initial condition an equiaxed microstructure is characterized by $N_L^0 = N_T^0 = N^0$ and $d = d_0 = 1/N^0$. (1) After non-directional grain growth in an equiaxed microstructure $N_L = N_T = N^0/C$ and then $d = d_L = d_T = Cd_0$. (2) The tensile specimen is pulled in the direction corresponding to the longitudinal direction of the microstructure. Each grain suffers a longitudinal tensile strain ε_G which increases d_L by the factor $(1 + \varepsilon_G)$. Since plastic deformation preserves volume (or area in two dimensions), d_T decreases by the factor $(1 + \varepsilon_G)^{-1/2}$. Including the grain growth:

$$d_L = C(1 + \varepsilon_G) \mathbf{d}_0 = 1/N_L \text{ and, } d_T = C(1 + \varepsilon_G)^{-1/2} \mathbf{d}_0 = 1/N_T; \quad (12a)$$

and

$$\varepsilon_G = (d_L/d_T)^{2/3} - 1 \text{ and, } C = (d_L d_T^2)^{1/3}/\mathbf{d}_0. \quad (12b)$$

(3) Relative translatory movements (neighbor switching or cellular dislocation glide) now occur to increase the plastic strain in the specimen to its final value $\tilde{\varepsilon}$:

$$\tilde{\varepsilon} = \varepsilon_G + \varepsilon_{GBS}. \quad (13)$$

Here, ε_{GBS} , for grain boundary sliding, could just as well have been labeled as ε_{NS} for neighbor switching (cellular dislocation glide). For the "dual" lattice ε_G corresponds to an "elastic" strain, or stretching of the lattice, and ε_{GBS} corresponds to plastic strain, or relative motion (slip) of the lattice. Thus, grain rearrangements, grain stretching, and grain growth are coupled so that space always remains occupied. Fracture is not considered here.

5.2. Superplastic Deformation

The tensile specimen is polycrystalline and the grain size is very small, less than about $10 \mu\text{m}$. The specimen is heated and slowly pulled. Consider the case of a superplastic metal that undergoes a superplastic strain of several thousand percent with no significant microstructural change. Clearly any grain strain is transitory (not cumulative) and the plastic strain results completely from neighbor switching. This is the idealization of superplastic flow considered here. It is described like laminar flow in a crystal lattice from dislocation slip on many finely spaced slip bands.

Outline of the Sub-Section: After reviewing the neighbor switching mechanisms proposed as models of superplastic deformation it will be shown which mechanism is analogous to dislocation slip. The constitutive relationship is developed from the Orowan equation by using a relationship for the cellular dislocation mobility \mathbf{M} extracted from Ashby and Verrall's (1973) constitutive relationship. Two models for deformation-enhanced grain growth are developed. Results are illustrated with analyses of superplastic 7475 Al and Sn-1% Bi (Clark and Alden) alloys (Sherwood and Hamilton 1994).

Ashby and Verrall (1973) develop the following constitutive relationship for superplastic deformation from an analysis of Figure 34:

$$\dot{\varepsilon} = A \frac{\Omega}{kT d^2} (\sigma - 0.72\Gamma/d) D_v \left(1 + \frac{3.3\delta}{d} \frac{D_b}{D_v}\right). \quad (14a)$$

Here, Γ denotes the grain boundary surface tension, $A = 100$, d is the diameter of each hexagonal grain and also the grain thickness, and $\sigma = \text{constant}$ is the steady state flow stress.

The strain rate is entirely from neighbor switching, *i.e.* it is plastic from either the viewpoint of the cellular array or its "dual" lattice. According to Ashby and Verrall (1973), over the range of temperatures and grain sizes for which superplastic flow is generally observed, grain boundary diffusion is the dominant mode of atomic transport, so that $3.3\delta D_b/dD_v \gg 1$, and therefore to a good approximation

$$\dot{\varepsilon} = A \left(\frac{D_b \delta \Omega}{k T d^3} \right) [\sigma - 0.72 \Gamma / d], \quad (14b)$$

where $A = 330$. Aside from the *threshold stress*,

$$\sigma_o \equiv 0.72 \Gamma / d,$$

the Ashby-Verrall relationship is similar to the Coble creep equation; but the strain rate predicted by equation (14a) can be almost an order of magnitude larger than a corresponding relationship for diffusional creep, since A is larger.^[59] This is because: (1) there is less volume of material transported per unit strain, (2) diffusion distances are shorter, and (3) the number of diffusion paths per grain is greater. According to Ashby and Verrall, if there is deformation-enhanced grain growth, then an expression for the grain growth kinetics

[59]: When $3.3\delta D_b/dD_v \gg 1$, there is not too much difference between the strain rate predicted by the Ashby-Verrall theory and the Coble creep equation, equation (5). Ashby and Verrall (1973) provide a relationship which gives $A = 44$ for Coble creep, whereas equation (5) gives $A = 148$.

can be substituted into the constitutive relationship. Sherwood and Hamilton (1994), however, note that in this case the flow stress increases, and then $\sigma \neq \text{constant}$. This complication is ignored here.

Figure 34 is the ***neighbor switching mechanism of superplastic deformation***, originally suggested as a creep mechanism by Rachinger (1952-53), and as a mechanism for superplastic flow by Lee (1970). When cells elongate by a critical amount ($\varepsilon = 0.275$ in Figure 34b), boundaries reorient themselves to preserve an equiaxed microstructure. Cells in the group of four finish switching neighbors when the strain is $\varepsilon_0 = 0.55$: cells one and four are initially separated while cells two and three are neighbors, following the reaction depicted by Figure 34, cells one and four become neighbors while cells two and three are separated. Neighbor switching begins when the cells have elongated such that one boundary disappears; if the boundaries are able to migrate, they will do so until dihedral angles at the triple points again make angles of 120° , which concludes the reaction (Beeré 1976). Strains of several hundred (or even thousand) percent develop during superplastic flow and adjacent grains become separated by many grain diameters (Beeré 1976). The neighbor switching mechanism therefore models an important characteristic of superplastic behavior.

When the intermediate state, Figure 34b, occurs at $\varepsilon = 0.275$, the cell boundary area is increased by $0.26d^2$, thereby storing free energy in the array since the cells have surface tension (energy per unit area) (Ashby and Verrall 1973). Spingarn and Nix (1978) criticized the sliding/migration sequence and diffusive atomic movements of the Ashby-Verrall neighbor switching mechanism, and in particular the fact that cell boundary area is increased without the applied stress doing any work. They consequently analyze Lee's (1970) version

of the neighbor switching reaction to develop an expression for the strain rate. Beeré (1976), however, had already suggested Lee's "reaction path" for the sliding/migration required to produce the intermediate state in the Ashby-Verrall mechanism, Figure 35. Here, the strain at the "critical" intermediate state (iii of Figure 35a) is $\varepsilon_o = 0.55$; further boundary migration producing configuration iv from iii in Figure 35a does not produce more strain, only a reduction in boundary length in the ratio 1:1.15, which lowers the free energy of the cells (Beeré 1976). Also, cells deform to achieve the intermediate state, so the applied stress does work. Of further significance to Beeré's (1976) interpretation of the neighbor switching mechanism is that he leaves out the 30° rotation of the cell group considered by both Lee (1970) and Ashby and Verrall (1973): in Figure 35a cells one and two move to the right by half the width of a hexagon and cells three and four move left by the same amount. Beeré (1976) refers to this as *grain rolling*, Figure 35d. The final state of Beeré's (1976) neighbor switching reaction is a mirror transformation of the initial state, Figure 35e; this is identical to the unit slip considered by Figure 29 for a lattice with *sixfold* symmetry ("dual" to the hexagonal array).

The neighbor switching mechanisms of superplastic deformation are summarized in Figure 36a. Figure 36b and 36c indicate that Beeré's (1976) initial and final states along with the Ashby-Verrall (1973) intermediate state is "slip-like;" see also Ashby *et al.* (1978). To complete the analogy between neighbor switching and crystal slip, cellular dislocation glide is shown, after Sato *et al.* (1990), by Figure 36d.

The Superplastic Strain Rate

Spingarn and Nix (1978) conclude that neighbor switching does not significantly affect the creep rate. So how can the strain rate for superplastic flow be any greater than that for conventional creep (Edington *et al.* 1976)? Some theories propose unique deformation mechanisms to accommodate grain rearrangements, such as Arieli and Mukherjee (1980) and Gifkins (1978). And of course, mechanisms unique to superplastic flow cannot be compared with conventional creep.

For example, Gifkins' (1978) theory is a reexamination of Ashby and Verrall's (1973) model: he considers neighbor switching as a grain rearrangement in three dimensions which results from grain boundary sliding accommodated by grain boundary dislocation climb and glide; dislocation motion results in material flow in a narrow "mantle" of material adjacent to the grain boundaries. This mantle has width $(d/2)[1 - \cos 30^\circ] = 0.07d$, practically the same as from the Ashby-Verrall mechanism. Of course, dislocation motion transports matter much more rapidly than diffusive processes. Gifkins obtains the following constitutive relationship for superplastic flow:

$$\dot{\varepsilon} = A \left(\frac{D_b \Omega}{G k T d^2} \right) \sigma^2,$$

where $A = 32$ and G denotes the shear modulus of the material. Note that the stress exponent is 2, indicating that $\alpha = 1/2$, and therefore $m = 1/2$ is predicted.

Morral and Ashby (1974) and Ashby and Verrall (1973) observed that neighbor switching is in fact associated with the non-hexagonal cells in an array, as in Figure 36d, and furthermore that deformation is inhomogeneous. Morral and Ashby (1973) suggest that superplastic deformation should be modelled with neighbor switching reactions that involve only non-hexagonal cells. The combination of 5- and 7-sided cells in a hexagonal array is an edge dislocation with Burgers vector $\mathbf{b} = \sqrt{3} \mathbf{d}/2$ (Cahn and Padawer 1965), where \mathbf{b} is the width of a hexagon with diameter \mathbf{d} . In a polycrystalline material the grain size is distributed and grains switch neighbors *sequentially* (McLean 1970) as in Figure 37, instead of *simultaneously* as in Figures 34e and 35d.

Following Morral and Ashby (1974): The *cellular dislocation velocity* is $v = \frac{1}{2} \mathbf{M} \sigma \mathbf{b}$, where \mathbf{M} denotes the *dislocation mobility* for glide, $\frac{1}{2} \sigma = \tau$ is the *resolved shear stress*. The strain rate of the cellular array, $\dot{\varepsilon}$, is related to this velocity by the *Orowan equation*:

$$\dot{\varepsilon} = \rho \mathbf{b} v = \frac{1}{2} \rho \mathbf{M} \sigma \mathbf{b}^2 = \frac{3}{8} \rho \mathbf{M} \sigma \mathbf{d}^2 \equiv \sigma / \eta, \quad (15)$$

ρ denoting the *cellular dislocation density* and η the *material viscosity*,

$$\eta = [\frac{3}{8} \rho \mathbf{M} \mathbf{d}^2]^{-1}. \quad (16)$$

The *number of columnar cells per unit volume*, N_v , is

$$N_v = 1/[3\sqrt{3} (\mathbf{d}/2)^2/2 \times 1] = 1.54/\mathbf{d}^2. \quad (17)$$

If X_d is the fraction of non-hexagonal cells (e.g. 5- and 7-sided) in a two-dimensional cellular array, then

$$\rho = [1/2X_d \times 1]N_v = 0.77X_d/d^2. \quad (18)$$

The strain rate of equation (15) arises from cellular dislocations passing entirely through the array, that is, it is *superplastic*, as opposed to the *plastic* strain rate from grain stretching in conventional creep. With respect to the "dual" lattice: equation (15) gives a plastic strain rate from dislocation slip whereas conventional creep mechanisms give an "elastic" strain rate.

Neglecting the threshold stress, equations (14b) and (15) are equivalent when

$$M = 6.93A \left(\frac{D_B \delta \Omega}{kT d^3} \right) = 6.93A[M_B \delta^2 / d^3], \quad (19)$$

provided that $X_d = 1/2$. Here, equations (17) and (18) are used, and the second expression results from the assumption that $M_B \equiv (\Omega/\delta)m_B = (\Omega/\delta)D_B/kT$, where $m_B \equiv D_B/kT$ is the *atomic mobility* in a grain boundary and M_B is the *grain boundary "mobility"*. The following *constitutive relationship for superplastic deformation* results (Sherwood and Hamilton 1994):

$$\dot{\varepsilon} = 2AX_d \left(\frac{D_B \delta \Omega}{kT d^3} \right) \sigma = [2AX_d(\dot{\varepsilon})M_B \delta^2 / d^3] \sigma \equiv \eta^{-1} \sigma, \quad (20)$$

where a strain rate dependency for X_d has been postulated in the second expression. The

viscosity of the cellular array could be strain rate dependent if there is grain growth or if X_d is strain rate dependent:

$$\eta = \{2AX_d(\dot{\varepsilon})M_B\delta^2/|\mathbf{d}(\varepsilon, \dot{\varepsilon})|^3\}^{-1}, \quad (21a)$$

the functional dependency of the grain size being emphasized. Note for $X_d = 1/2$ that equation (20) is the same as equation (14b), but since $X_d = X_d(\dot{\varepsilon})$ is possible, non-Newtonian viscosity is predicted by equation (20) even if $\mathbf{d} = \text{constant}$. For example, if $X_d \propto \dot{\varepsilon}^{1/2}$ and $\mathbf{d} = \text{constant}$, then equation (20) gives $m = 1/2$. Here, X_d refers to the fraction of defects responding to the applied stress by neighbor switching. For $X_d \ll 1$ (e.g. Figure 37), superplastic flow must be slow and inhomogeneous. Figure 38, from Sherwood and Hamilton (1994), shows a two dimensional cellular array with $X_d = 1/2$ which undergoes superplastic flow from cellular dislocation glide. In this case superplastic flow is relatively rapid and homogeneous.

I will treat two types of cellular dislocation climb: thermally-activated cellular dislocation climb and stress-driven cellular dislocation climb.

Thermally-Activated Cellular Dislocation Climb (Cahn and Padawer 1965, Morral and Ashby 1974) corresponds to Hillert's (1965) *grain growth "reaction"*, Figure 39. This process models either grain growth during annealing or deformation-enhanced grain growth (Sherwood and Hamilton 1994). The grain growth rate for annealing a single phase material is

$$\dot{\mathbf{d}} = \frac{1}{3}M_B(2\Gamma/\mathbf{d})X_d, \quad (22a)$$

where d is the average grain size, the grain boundary "mobility" could be $M_B = (\Omega/\delta)m_B = (\Omega/\delta)(D_B/kT)$ again, although several other relationships are possible (Christian 1965, Gleiter and Chalmers 1972, Smith *et al.* 1980), and X_d is the fraction of defects that participate in the grain growth "reaction."^[60] Equation (22a) integrates to give the *grain growth "law"* (Atkinson 1988)

$$d(t) = [d_0^2 + (4/3)X_d M_B \Gamma t]^{1/2}, \quad (23)$$

only if $X_d = \text{constant}$ (Morral and Ashby 1974). Hillert (1965) obtains $X_d = X_d^0 = 3/4$ for *normal grain growth*.

Grain growth is inhibited, or even prevented in alloys with a fine dispersion of second phase particles (Zener 1948, Ashby, Harper and Lewis 1969, Nes, Ryum and Hunderi 1985, Elst, Van Humbeeck and Delaey 1988, Gore *et al.* 1989, Sherwood and Hamilton 1993, Rios 1994). Such *quasi-single phase materials* are frequently employed for superplastic forming (Kashyap *et al.* 1985); see Figure 40 for an illustration of the microstructures employed for superplastic forming. For these materials, Hillert (1965) offers the following, approximate

[60]: Hillert's defects are the non-hexagonal cells in a honeycomb array; note that all three elementary reactions in Figure 39 involve such cells. His theory is consequently focussed on grain boundaries that are most likely to move, *i.e.* on the microstructure, instead of on, for example, atoms or grain boundary dislocations. This theory of grain growth is therefore consistent with the level of description being used here. See Chen (1987) for a theory of grain growth based on a somewhat different level of description, grain boundary dislocation climb.

grain growth rate:

$$\dot{d} \approx \frac{1}{3} M_B (2\Gamma/d) X_d [1 - d/d_c]^2. \quad (22b)$$

Here,

$$d_c \equiv 2(4r/3f),$$

with r denoting the average second phase particle size and f their volume fraction (Zener 1948); when $d = d_c$, grain growth ceases. For example, Ghosh (1985) reports for 7475 Al that Cr₂Al₃ particles with $r = 0.1 \mu\text{m}$ and $f = 0.015$ inhibit grain boundary migration, giving $d_c = 18 \mu\text{m}$. For $X_d = \text{constant} \equiv X_d^0$, the grain growth kinetics from equation (22b) are (Sherwood and Hamilton 1993)^[61]

$$\frac{d_c(d - d_0)}{(d_c - d)(d_c - d_0)} + \ln\left(\frac{d_c - d}{d_c - d_0}\right) = \frac{2}{3} X_d^0 [\Gamma M_B / d_c^2] t. \quad (24)$$

Since Hillert's grain growth rate is proportional to the fraction of defects in the cellular array, Sherwood and Hamilton (1991, 1994) attribute deformation-enhanced grain growth to Morral and Ashby's (1974) suggestion that cellular dislocations are generated during superplastic deformation, as shown, for example, by Figure 21. These defects should also be produced during annealing, Figure 41, and during superplastic deformation by other mechanisms such as those shown by Figures 42 and 43. Such "excess" cellular dislocations provide more "sites" for the grain growth "reaction," and consequently increase the grain

[61]: Correcting an error involving a factor of two on the left-hand side of the expression.

growth rate. These defects are assumed to be increased from the initial level, $X_d(t = 0) \equiv X_d^0$, to $X_d(t) = X_d^0 + \delta X_d$ during superplastic deformation. The quantity $X_d(t)$ is assumed to be higher than X_d^0 by an amount $\delta X_d \equiv X_d - X_d^0$, which represents the level of "excess" cellular dislocations. "Excess" defects are assumed to be produced at a net rate

$$\dot{X}_d = K\kappa\dot{\varepsilon} - \delta X_d/\tau, \quad (25)$$

where K is a parameter, κ is the fraction of the imposed strain rate that causes transilatory cell movement (boundary sliding instead of cell stretching), and τ is the *relaxation time* of "excess" defects. The first and second terms on the right hand side of equation (25) are *production, p_{τ_1}* , and *annihilation, a_{τ_1} , rates*, respectively, of "excess" defects:

$$p_{\tau_1} \equiv K\kappa\dot{\varepsilon}, \text{ and } a_{\tau_1} \equiv \delta X_d/\tau.$$

At a *steady state* $p_{\tau_1} \equiv a_{\tau_1}$ and, if $\dot{\varepsilon} = constant$, then $X_d \rightarrow X_d^{ss}$, where X_d^{ss} is the *steady state defect level*:

$$X_d^{ss} = X_d^0 + K\tau\kappa\dot{\varepsilon} \equiv X_d^0 + \delta X_d^{ss},$$

Of course, $X_d^{ss} \leq 1$, and again, X_d is the fraction of defects that participate in the grain growth "reaction," Figure 39, as opposed to those engaged by the applied stress for stress-driven cellular dislocation climb and glide, or which remain undisturbed. Cellular dislocation

annihilation is suggested by Hillert (1965) and Cahn and Padawer (1965), and is illustrated by Figure 44.

Integration of equation (25) gives the temporal dependency of δX_d (Sherwood and Hamilton 1992):

$$\delta X_d = \exp(-t/\tau) \int_0^t \exp(t/\tau) K \kappa \dot{\varepsilon} dt \approx K \kappa \dot{\varepsilon} \tau [1 - \exp(-t/\tau)], \quad (26)$$

where the approximation holds for $\kappa \dot{\varepsilon} = constant$, if K and τ are constants. Deformation-enhanced grain growth is predicted by equation (22) when $X_d = X_d^0 + \delta X_d$ is substituted therein: for single phase materials

$$d^2 - d_0^2 = \frac{4}{3} M_B \Gamma [(X_d^0 + K\tau \kappa \dot{\varepsilon})t - K\tau^2 \kappa \dot{\varepsilon} \{1 - \exp(-t/\tau)\}], \quad (27)$$

and for quasi-single phase materials

$$\frac{d_c(d - d_0)}{(d_c - d)(d_c - d_0)} + \ln\left(\frac{d_c - d}{d_c - d_0}\right) = \frac{2}{3} \frac{M_B \Gamma}{d_c^2} [(X_d^0 + K\tau \kappa \dot{\varepsilon})t - K\tau^2 \kappa \dot{\varepsilon} \{1 - \exp(-t/\tau)\}]. \quad (28)$$

The grain growth kinetics from this theory are essentially the same as those of Clark and Alden's "excess vacancy" model of deformation-enhanced grain growth (Sherwood and

Hamilton 1991, 1994).

Stress-Driven Cellular Dislocation Climb is suggested as a model for both superplastic flow and deformation-enhanced grain growth by Sato *et al.* (1990). The resulting grain growth kinetics are very similar to those proposed by Wilkinson and Cáceres (1984). Wilkinson and Cáceres suggest that "lattice damage" at triple points from *impeded grain boundary sliding* (Holt and Backofen 1966) provides a driving force for grain boundary migration such that small grains shrink. Figure 45 illustrates this suggestion with "reaction paths" for neighbor switching and cell disappearance proposed for conventional creep by Ishida *et al.* (1965). The grain growth kinetics are approximately

$$\dot{d}_\sigma = \lambda d\dot{\varepsilon} \quad (29)$$

for the deformation-enhanced component of the grain growth rate, \dot{d}_σ , where λ is a constant. Sherwood and Hamilton (1992) show that $\lambda = Bc\varkappa X_d$, where c is a constant, $B = 1/2$, and $\varkappa = \text{strain rate from grain boundary sliding/imposed strain rate}$ again. Here, X_d is the fraction of grains engaged by the stress for the neighbor switching mechanism of superplastic flow. If $\lambda = \text{constant}$, and if \dot{d}_σ is much larger than the grain growth rate from the time at the test temperature, then equation (29) integrates to

$$d = d_0 \exp[\lambda\varepsilon]. \quad (30)$$

The parameter $\lambda = \frac{1}{2}c\kappa X_d$ is estimated next. Figure 46 (Sherwood and Hamilton 1994) shows an array with an initial state like Figure 38 (24 dislocations in 96 cells, $X_d = 1/2$) that deforms by cellular dislocation climb: each dislocation climbs once, resulting in a final state with only 72 cells and $X_d = 2/3$, 24 cells being eliminated by climb. In two dimensions, $N\mathbf{d}^2 = constant$, where N denotes the number of cells in the array (Hillert 1965); thus, $\mathbf{d}/\mathbf{d}_0 = 1.15$, with $\mathbf{d}_0 \equiv \mathbf{d}(\varepsilon = 0)$. Figure 46c indicates that each unit cell is strained by $\varepsilon_c = \ln[l_f/l_i] = 0.7$, l_f and l_i denoting the diagonal lengths of the "unit cell" before and after deformation, respectively. The entire array is therefore strained by $\varepsilon = (N_f/N_i)\varepsilon_c = (N_f/N_i)\ln[l_f/l_i] = 0.5$, where $N_i = 96$ and $N_f = 72$ are the initial and final number of cells in the array, respectively. Since $\lambda = \frac{1}{2}c\kappa X_d$, if $X_d = 0.5$ and $\kappa = 1$ as in Figure 46, then $\lambda = 0.25c$. From equation (30), $\lambda\varepsilon = \ln[\mathbf{d}/\mathbf{d}_0]$; so for $\varepsilon = 0.5$ and $\mathbf{d}/\mathbf{d}_0 = 1.15$, $\lambda \approx 0.3$, and therefore $c \approx 1$.

5.3. Illustrations of Results

Data from Clark and Alden's (1973) Sn-1% Bi alloy and a 7475 Al alloy are used to illustrate the prior considerations. Grain growth data for the 7475 Al alloy (Sherwood and Hamilton 1993,1994) are given in the Appendix.

5.3.1. Thermally-Activated Cellular Dislocation Climb

"Excess" defects δX_d for the grain growth "reaction" per equation (26) are assumed to be generated during superplastic deformation. Here, $X_d \equiv X_d^0 + \delta X_d$ refers to the fraction of grains that participate in Hillert's (1965) grain growth "reaction," Figure 39. The grain growth kinetics are given by equation (27) for single phase materials like Sn-1% Bi and equation (28) for quasi-single phase materials like 7475 Al (Ghosh and Hamilton 1982, Hamilton, Bampton and Paton 1982, Paton *et al.* 1982, Mahoney, Hamilton and Ghosh 1983, Ghosh 1985).^[62]

Figure 47 illustrates equation (27) with data from Clark and Alden's (1973) Sn-1% Bi alloy (Sherwood and Hamilton 1994).^[63] The Sn-1% Bi grain sizes were determined from Clark and Alden's Fig. 13, and these are shown against curves generated from equation (27) with the following information:

[62]: See also Rao and Mukherjee (1986,1987), Lee and McNelley (1987), Matsuki *et al.* (1988), Shin *et al.* (1990), and Xinggang, Jianzhong and Longxiang (1993) for descriptions of more recent variations on superplasticity in 7475 Al alloys.

[63]: Clark and Alden (1973) did not test the Sn-1% Bi alloy with a constant strain rate apparatus; this complication is not addressed here.

Sn-1% Bi

$K = 6.8 \times 10^{-6}$, $\tau = 1$ min, $\varkappa \equiv 1$, and $(4/3)M_B\Gamma X_d^0 = 4.49 \times 10^{-3} \mu\text{m}^2/\text{min}$.

Note for $\varkappa\dot{\varepsilon} \equiv 1/\text{s}$ that $\delta X_d^{\text{ss}} = K\tau = 4.08 \times 10^{-4}$, a very small perturbation on $X_d^0 = 3/4$.

Thus, there is very little deformation-enhanced grain growth in this material; the grain growth exhibited is largely associated with time at the test temperature, or "*static*" grain growth.

Grain growth kinetics of the quasi-single phase 7475 Al alloy are illustrated with equation (28) by Figure 48, as follows.

The 7475 Al alloy exhibits very little grain growth during annealing. For example, the grain size only doubles after one week at 516 °C (Sherwood and Hamilton 1993). Two key assumptions will be made to account for this behavior:

$$(1) X_d^0 \ll 1$$

and

$$(2) d_c \text{ being only slightly greater than } d_0 = 9.2 \mu\text{m}.$$

For the former

$$X_d^0(T) = 7.40 \times 10^4 \exp[-(117.1 \text{ kJ/K-mol})/RT]$$

will be assumed, where $R = 8.314 \text{ J/K-mol}$ is the gas constant. And for the latter, d_c increases slightly with temperature (Rios 1994):

$$d_c(450^\circ\text{C}) = 15 \mu\text{m}, d_c(475^\circ\text{C}) = 17 \mu\text{m}, \text{ and } d_c(516^\circ\text{C}) = 19 \mu\text{m}.$$

Brown and Ashby (1980) give the lattice diffusivity, $D_v = D_o \exp(-Q_v/RT)$, of Al as $D_o = 1.7 \times 10^{-4} \text{ m}^2/\text{s}$ and $Q_v = 142 \text{ kJ/K-mol}$, and the molar volume as $V_m = 9.998 \times 10^{-6} \text{ m}^3$. So I assume that for 7475 Al

$$D_B = (D_o/4) \exp[-(96.1 \text{ kJ/K-mol})/RT]$$

for the grain boundary diffusivity. The grain boundary width is $\delta \approx 5 \text{ \AA}$ (Peterson 1983). Thus, with $M_B = (\Omega/\delta)m_B = (\Omega/\delta)(D_B/kT) = (V_m/\delta)(D_B/RT)$, for 7475 Al

$$M_B = [(1.02 \times 10^{23})/T] \exp[-(96.1 \text{ kJ/K-mol})/RT] \mu\text{m}^4/\text{J-s}. \quad (31)$$

The grain boundary surface tension of Al is $\Gamma = 0.325 \text{ J/m}^2$ (Hirth and Lothe 1982).

The parameters K and τ are assumed to be given by the following relationships:

$$K = 6.84 \times 10^8 \exp[-(115.2 \text{ kJ/K-mol})/RT],$$

and

$$\tau = \{1.20 \times 10^8 \exp[-(144.1 \text{ kJ/K-mol})/RT]\}^{-1} \text{ s.}$$

Since κ decreases with increasing $\dot{\varepsilon}$, $\delta X_d^{\text{ss}} \equiv K\tau\kappa\dot{\varepsilon} < 1$, as shown by Figure 48 (Sherwood and Hamilton 1994). Here, δX_d^{ss} is taken as temperature independent, to reduce the number of

adjustable parameters for this illustration.

See Figure 49 for an interpretation of the meaning of a low value for X_d .

5.3.2. Stress-Driven Cellular Dislocation Climb

Grain growth kinetics predicted from this model are approximately $d = d_0 \exp[\lambda\varepsilon]$ where $\lambda \approx 0.6\kappa X_d$, with X_d denoting the fraction of cells engaged by the stress. This relationship is illustrated for the 7475 Al grain growth data shown previously by Figure 50 (Sherwood and Hamilton 1994). Here, I let $X_d = 1$ as an upper bound estimate for the case where $\kappa = 1$ so that $\lambda = 0.6$, and then assumed that κ decreases with increasing strain rate as shown, to fit the data.

5.3.3. The Strain Rate Sensitivity

An alternative expression for the viscosity is

$$\eta = \eta(\rho, d, T) = \{2.6A\rho M_b \delta^2 / d\}^{-1}, \quad (21b)$$

which is obtained from equation (21a) with equation (18), $X_d = 1.3d^2\rho$. This is the form of the viscosity used in the following analysis (Sherwood and Hamilton 1994).

Equation (20) predicts Newtonian-viscous behavior ($m = 1$) if X_d and d are *constants*, X_d denoting the fraction of defects moved by the stress. Morral and Ashby (1974) suggest that non-Newtonian viscosity results when ρ increases with the flow stress σ and, therefore with increasing strain rate. Since the cellular dislocation density is given by $\rho = 0.77X_d/d^2$,

ρ increases with increasing X_d and decreasing d . For a constant strain the average grain size d generally decreases with increasing strain rate, as indicated by the prior treatment of deformation-enhanced grain growth. For $d = \text{constant}$, X_d increases with the strain rate because: (1) defects are generated at a greater rate, or (2) the higher flow stress engages more extant defects. Figure 51 shows the strain rate as a function of the flow stress, $\dot{\varepsilon} = \dot{\varepsilon}(\sigma)$, for Sn-1% Bi from Clark and Alden (1973), and 7475 Al from Hamilton *et al.* (1982). These curves were generated with equation (20): For Sn-1% Bi: Values for $d(\varepsilon = 0.4)$ were taken from Clark and Alden's Fig.'s 4 and 5; $M_b = 4.48 \times 10^{-17} \text{ m}^4/\text{J}\cdot\text{s}$ was obtained from the grain growth kinetics by assuming $\Gamma = 1 \text{ J/m}^2$; $A = 1.75 \times 10^8$ was used to fit the data, along with the values for ρ shown by Figure 51b. For 7475 Al: $A = 2.5 \times 10^5$, M_b is given by equation (31), and values for ρ and d are given by Figures 51d and 51e, respectively.

5.3.4. Strain Hardening

Mahoney *et al.* (1983) report that deformation-enhanced grain growth in 7475 Al strain hardens the material. Figure 52 shows 7475 Al stress-strain curves from Ghosh (1985) against predictions obtained from equation (20) (Sherwood and Hamilton 1994). It appears that deformation-enhanced grain growth can account for strain hardening in 7475 Al at 516 °C, and for lower strains at 482 °C.

5.4. Discussion of Results

For the quasi-single phase 7475 Al $K\tau \approx 10^2$ s was assumed for deformation-enhanced grain growth from thermally-activated cellular dislocation climb. This is several orders of magnitude larger than the value $K\tau = 4.08 \times 10^{-4}$ s assumed for Clark and Alden's (1973) Sn-1% Bi single phase alloy. The Sn-1% Bi alloy exhibits much greater grain growth during annealing than the 7475 Al alloy because second phase particles impede grain boundary migration in the latter but not the former. For 7475 Al $X_d = X_d^0 \approx 10^{-3}$ was assumed for annealing (X_d denoting the fraction of cells participating in the grain growth "reaction") because so little grain growth is observed (Sherwood and Hamilton 1993), the grains are largely "static," Figure 49. The 7475 Al alloy exhibits substantially more grain growth during superplastic deformation, *i.e.* deformation-enhanced grain growth. Therefore, grains are "dynamic" during superplastic flow, and consequently $X_d \rightarrow X_d^0 + \delta X_d \approx \delta X_d$, since $\delta X_d \gg X_d^0$. This argument seems equivalent to a particularly simple explanation for deformation-enhanced grain growth from Beeré (1978):

"Grain boundaries are often mobile and migrate during deformation resulting in simultaneous grain growth."

The two models for deformation-enhanced grain growth either predict that the grain size is independent of the imposed strain rate (within the strain rate regime of superplasticity), equation (30), or slightly dependent on it, equations (27) and (28). With respect to the latter, varying the strain rate by orders of magnitude produces relatively minor

variations in observed grain sizes. The tenfold increase in the Sn-1% Bi grain size is rather large: it is the result of substantial grain growth from the time at the test temperature and relatively little deformation-enhanced grain growth, because δX_d is so small compared with X_d^0 . The grain growth kinetics of 7475 Al are, on the other hand, markedly affected by the strain rate, as shown in the Appendix, and $\delta X_d >> X_d^0$.

The procedure of lowering the ratio $\kappa \equiv \dot{\varepsilon}_{GBS}/\dot{\varepsilon}$ as the strain rate $\dot{\varepsilon}$ increases is required to reproduce the strain rate dependency of deformation-enhanced grain growth in 7475 Al with the cellular dislocation climb models. Wilkinson and Cáceres' (1984) were the first to incorporate κ into a relationship for deformation-enhanced grain growth; it is an important parameter reflecting the nature of superplastic flow mechanisms. For example, Ghosh (1985) observed significant grain elongation (plasticity) and some grain refinement in 7475 Al deformed at 516 °C to strain rates greater than $10^{-2}/\text{s}$; the latter is indicative of dynamic recrystallization and the former of dislocation creep. Because superplastic materials have very fine grain sizes, grain boundaries continue to play an important role even at high strain rates, e.g. $0.2 \leq \kappa \leq 0.5$ for region III superplasticity (Langdon 1982).

The thermally-activated cellular dislocation climb model of deformation-enhanced grain growth requires the generation of a large number of "excess" cellular dislocations at high strain rates in order to represent 7475 Al data. At high strain rates I also assumed that κ decreases because grain boundary rearrangements give way to atomic dislocation processes. To be consistent therefore, **the thermally-activated cellular dislocation climb model is applicable to 7475 Al only at lower strain rates.** For low strain rates the stress-driven cellular dislocation climb model of deformation-enhanced grain growth requires that X_d (the

fraction of defects engaged by the applied stress) be very large. For the analysis of the strain rate sensitivity of this material, however, I assumed that X_d is small at low strain rates. Consequently, to be consistent, **the stress-driven cellular dislocation climb model is applicable to 7475 Al at higher strain rates.** I am therefore suggesting that **the grain growth kinetics are strain rate dependent:** *i.e.* described by one relationship in some strain rate regime and by another relationship in a different strain rate regime. Ghosh and Raj (1986) go one step further by suggesting that at a given strain rate larger grains may deform by dislocation creep and recrystallize while smaller grains may deform by diffusional creep and coarsen. **It is possible that distinct grain growth kinetics are operating simultaneously in the microstructure and that their relative contributions change with the strain rate.** The only requirement that must be met is compatibility, or topology: *e.g.* the grains must fill space and satisfy surface tension equilibrium (Smith 1952).

Inspection of the 7475 Al deformation-enhanced grain growth curves suggests that the prior explanation is plausible. The strain rate changes by two orders of magnitude for these tests. About three hours elapse when a strain of one is attained at a strain rate of $10^{-4}/\text{s}$ while only about two minutes is required to produce this strain at a strain rate of $10^{-2}/\text{s}$. It seems reasonable to presume that in order for the grain size to change after only two minutes that the grain boundary migration for grain growth must occur very "efficiently" so as not to waste what little time is available; it must be "driven" by the applied stress, *i.e.* the stress-driven cellular dislocation climb model. Much more time becomes available for thermally-activated processes as the strain rate decreases, and the thermally-activated cellular dislocation climb model of deformation-enhanced grain growth is therefore applicable.

When cellular defects are plentiful and mobile the cellular array will flow in response to the imposed strain rate as these defects move throughout the array; "slip planes" will be finely spaced. On the other hand, if there are not many mobile defects in the array, then in order for it to flow the imposed strain rate must be diminished; the "slip planes" will then be more coarsely spaced; if the strain rate is not reduced enough, then the stress will be large and many cells will stretch as opposed to switching neighbors. In order to maintain a flow stress low enough to prevent substantial cell stretching during deformation, the microstructure must provide an increasing concentration of mobile defects as either the strain rate, or average cell size increases. If the imposed strain rate is too high, then defects in the array will not have enough time to move, the load will consequently be taken up by cell stretching, instead of defect motion.

An increased strain rate requires an increase in the cellular dislocation density for every $\dot{\varepsilon} = \dot{\varepsilon}(\rho)$ curve on Figure 51, excepting 7475 Al at 516 °C, where $\dot{\varepsilon}$ increases with ρ = constant until $\dot{\varepsilon} \approx 10^{-3}/\text{s}$. The 7475 Al is therefore exhibiting Newtonian viscosity at 516 °C for $\dot{\varepsilon} \leq 10^{-3}/\text{s}$; otherwise, the material exhibits non-Newtonian behavior. Note for the 7475 Al $\dot{\varepsilon} = \dot{\varepsilon}(\rho)$ curves that for $\dot{\varepsilon} \leq 2 \times 10^{-4}/\text{s}$, ρ is larger for higher temperatures at $\dot{\varepsilon} = \text{constant}$, and therefore for lower σ , which is the opposite trend suggested by Morral and

Ashby (1974). The explanation for this, seemingly anomalous behavior, is that 7475 Al tends to become Newtonian viscous, $\rho = \text{constant}$, as the strain rate decreases and the temperature increases. Evidently Region I superplasticity is not exhibited by the 7475 Al alloy; this often turns out to be the case (Arieli and Mukherjee 1982).

Strain hardening of 7475 Al also results from atomic dislocations accumulating in subcells (Rao and Mukherjee 1986 and Mahoney *et al.* 1983) and interacting with second phase particles (Rao and Mukherjee 1987). Such processes are neglected here to fully illustrate the ramifications of diffusional accommodation implicit in the development of Morral and Ashby's (1974) theory. Accommodation mechanisms for neighbor switching based on atomic dislocation processes (Arieli and Mukherjee 1980, Nix 1985) could be utilized with this procedure to obtain a relationship for M different from that presented here, *i.e.* it could be stress-dependent. For example, following Langdon's (1970) procedure^[64] for summing the strain rates, $\dot{\varepsilon}_i$, from $k = \sum i$ creep mechanisms to obtain the *total strain rate* $\dot{\varepsilon} = \sum_{i=1}^{i=k} \dot{\varepsilon}_i$:

$$\dot{\varepsilon} = \sum_{i=1}^k X_d^i \sigma^{1/m_i} / \eta_o^i(d, T). \quad (32)$$

Here, X_d^i is the fraction of cells that deform from mechanism i , and m_i and η_o^i are the crystal strain rate sensitivity and crystal viscosity for this mechanism, respectively. Of course,

[64]: An alternative to adapting Langdon's procedure with this treatment of the grain size distribution (X_d^i) is that suggested by Ghosh and Raj (1981), to whom the reader is referred.

$\sum_{i=1}^{i=k} X_d^i \leq 1$; and also, $X_d^i = X_d^i(\dot{\varepsilon}, \mathbf{d})$. With respect to the strain rate dependency of X_d^i for the dislocation creep mechanism, as an example: $X_d^{\text{disl.}} \rightarrow 0$ as the strain rate decreases through the sequence $\dot{\varepsilon}_{\text{III}} \rightarrow \dot{\varepsilon}_{\text{II}} \rightarrow \dot{\varepsilon}_{\text{I}} \rightarrow \dot{\varepsilon}_{\text{diff.}}$, where $\dot{\varepsilon}_{\text{I}}$, $\dot{\varepsilon}_{\text{II}}$ and $\dot{\varepsilon}_{\text{III}}$ denote "limiting" strain rates for Regions I, II and III of superplastic flow, respectively, and $\dot{\varepsilon}_{\text{diff.}}$ is the "limiting" strain rate for diffusional creep. With respect to the grain size dependency of X_d^i , using the neighbor switching mechanism of superplastic flow as an example: X_d^{GBS} decreases with increasing \mathbf{d} . Augmenting equation (32) are expressions for the grain growth kinetics:

$$\dot{\mathbf{d}} = \dot{\mathbf{d}}(\dot{\varepsilon}; \mathbf{d}, X_d).$$

In Closing: The description of superplasticity here is "geometrical." Initial and final states are emphasized, after Ashby and Verrall (1973), and the paths between these states are idealized as along "slip directions." It is suggested that the "geometry" of deformation, *i.e.* "slip band" spacing, or mobile cellular dislocation density (Morral and Ashby 1974), affects the observed strain rate sensitivity m . Thus, m is influenced by two factors: (1) the accommodation mechanism(s) (ω) and (2) the "geometry"; they likely "interact." "Geometry" may be determined by, for example, the distribution of grain sizes and shapes, grain boundary viscosity, and texture. Deformation-enhanced grain growth is "geometrical;" both the stress-driven and thermally-activated cellular dislocation climb models attribute this phenomenon to only the translatory grain motions that produce superplastic flow, as opposed to a mechanism, such as "excess" vacancies (Clark and Alden 1973).