**CP4**

# Numerical Integration | Simple Pendulum

Nathan Ngqwebo

NGQLIN011

Department of Science, University of Cape Town

PHY2004W | Intermediate Physics

Dr Spencer Wheaton

10 April 2023

**Logo

Description automatically generated**

**CP4**

Numerical Integration | Simple Pendulum

This computational activity is an exercise in numerical integration techniques used in studying non-linear systems without analytical solutions. All code was produced using Python (Van Rossum & Drake, 2009) with the packages: NumPy (Harris et al., 2020), and Matplotlib (Hunter, 2007) used for data wrangling and plotting.

# Numerical Methods

## Derivative Approximation

We are tasked with deriving a first order approximation of the first derivative (1) using the Taylor expansion of while making note of the leading error term

Note, as thus making the truncated approximation (1) more accurate.

## Second Order Approximation

We also derived a second order approximation (4) for the first derivative using the Taylor expansions of and

# Amplitude dependence on Period

**Figure 1**

Scatter Plot of 50 Non-Linear Simple Pendulum Experiments Period vs Initial Angle Data with Horizontal Line Implying the Same Experiment under the Small Angle Assumption.

If we were to plot the negative initial angles as well, we would see that the period dependence on follows a parabolic relationship. As we expect, making the small angle approximation ensures that the system remains independent on the initial angle and is simply given by: .

# Euler Method Energy Non-Conservation

Here, we use the small angle approximation to show that energy is not conserved when using the Euler method for numerical integration:

|  |  |  |
| --- | --- | --- |
|  |  |  |

Note that always represents either a positive vertical displacement from equilibrium or zero displacement, exactly at equilibrium.

Also is always small compared to the potential energy whenever it is negative (near the peak of an oscillation). Therefore is always positive, and the total energy of the system will increase with each time step thus making the Euler method energy non-conservative.

# References

Harris, C. R., Millman, K. J., van der Walt, S. J., Gommers, R., Virtanen, P., Cournapeau, D., Wieser, E., Taylor, J., Berg, S., Smith, N. J., Kern, R., Picus, M., Hoyer, S., van Kerkwijk, M. H., Brett, M., Haldane, A., del Río, J. F., Wiebe, M., Peterson, P., … Oliphant, T. E. (2020). Array programming with NumPy. *Nature*, *585*(7825), 357–362. https://doi.org/10.1038/s41586-020-2649-2

Hunter, J. D. (2007). Matplotlib: A 2D graphics environment. *Computing in Science \& Engineering*, *9*(3), 90–95. https://doi.org/10.1109/MCSE.2007.55

Van Rossum, G., & Drake, F. L. (2009). *Python 3 Reference Manual*. CreateSpace.