

# Investigative Physics

## Activity Units for Physics 125

Emory F. Bunn,<sup>1</sup> Mirela Fetea,<sup>1</sup> Gerard P. Gilfoyle,<sup>1</sup> Henry Nebel,<sup>1</sup>  
Matthew Trawick,<sup>1</sup> Philip D. Rubin,<sup>2</sup> and Michael F. Vineyard<sup>3</sup>

<sup>1</sup>Department of Physics, University of Richmond, VA

<sup>2</sup>Department of Physics, George Mason University, Fairfax, VA

<sup>3</sup>Department of Physics, Union College, Schenectady, NY

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### Abstract

The exercises in this manual have been developed to support an investigative physics course that emphasizes active learning. Some of these units have been taken from the Workshop Physics project at Dickinson College and the Tools for Scientific Thinking project at Tufts University and modified for use at the University of Richmond. Others have been developed locally.

The units are made up of activities designed to guide your investigations in the laboratory. The written work will consist primarily of documenting your class activities by filling in the entries in the spaces provided in the units. The entries consist of observations, derivations, calculations, and answers to questions. Although you may use the same data and graphs as your partner(s) and discuss concepts with your classmates, all entries should reflect your own understanding of the concepts and the meaning of the data and graphs you are presenting. Thus, each entry should be written in your own words. Indeed, it is very important to your success in this course that your entries reflect a sound understanding of the phenomena you are observing and analyzing.

We wish to acknowledge the support we have received for this project from the University of Richmond and the Instrumentation and Laboratory Improvement program of the National Science Foundation. Also, we would like to thank our laboratory directors for their invaluable technical assistance.

## Contents

1	Walking Speed	3
2	Measurement and Uncertainty	4
3	Measurement of Length, Mass, Volume, and Density	7
4	Measurement Uncertainty and Variation	9
5	Position vs. Time Graphs	13
6	Velocity vs. Time Graphs	21
7	Relating Position and Velocity Graphs	29
8	Changing Motion	34
9	Slowing Down, Speeding Up, and Turning	47
10	Gravity and Free Fall	60
11	Projectile Motion	64
12	Force and Motion I	67
13	Force and Motion II	79
14	Newton's 3rd Law, Tension, and Normal Forces	87
15	The Electrical and Gravitational Forces	94
16	Centripetal Force	99
17	Work and Power	107
18	Conservation of Mechanical Energy	113
19	Momentum and Momentum Change	116
20	Introduction to Rotation	122
21	Conservation of Angular Momentum	129
22	Hooke's Law	132

<b>23 Periodic Motion</b>	<b>134</b>
<b>24 The Period of a Pendulum</b>	<b>141</b>
<b>25 Resonance in Tubes</b>	<b>143</b>
<b>26 Heat, Temperature, and Internal Energy</b>	<b>145</b>
<b>27 Calorimetry</b>	<b>147</b>
<b>28 Boyle's Law</b>	<b>152</b>
<b>29 Charles' Law</b>	<b>156</b>
<b>30 The P-T Relationship of a Gas</b>	<b>159</b>
<b>31 Electrostatics</b>	<b>161</b>
<b>32 Electric Fields and Equipotential Lines</b>	<b>165</b>
<b>33 Ohm's Law</b>	<b>168</b>
<b>34 Magnetism I</b>	<b>173</b>
<b>35 Magnetism II</b>	<b>176</b>
<b>36 Refraction of Light</b>	<b>179</b>
<b>37 Refraction at Spherical Surfaces: Thin Lenses</b>	<b>182</b>
<b>38 The Diffraction Grating</b>	<b>185</b>
<b>39 The Optical Spectrum of Hydrogen</b>	<b>186</b>
<b>40 The Twins Paradox</b>	<b>188</b>
<b>A Treatment of Experimental Data</b>	<b>191</b>
<b>B Introduction to DataStudio</b>	<b>193</b>
<b>C Introduction to Excel</b>	<b>195</b>
C.1 Data and formulae . . . . .	195
C.2 Graphs . . . . .	195
C.3 Making Histograms . . . . .	196
<b>D Video Analysis</b>	<b>198</b>
<b>E Instrumentation</b>	<b>200</b>

# 1 Walking Speed

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

## Objectives

- Introduction to measuring length and time.
- Introduction to metric units of length.

## Apparatus

- stop watch
- 2-meter stick

## Activity

(a) Use the stop watch and meter stick to determine your walking speed in m/s.

walking speed \_\_\_\_\_

(b) Describe the technique you used to perform the measurement and show the calculation of walking speed.

(c) Use the stop watch and meter stick to determine your partner's walking speed in m/s.

walking speed \_\_\_\_\_ How does this compare with his/her result?

## Question

What sort of unit is speed (fundamental or derived)?

## 2 Measurement and Uncertainty<sup>1</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

- To learn to measure length using a meter stick and a vernier caliper.
- To learn to express the results of measurements with the appropriate number of significant figures.
- To learn how to compensate for systematic error in measurements so that accuracy can be improved.

### Measuring Lengths and Significant Figures

We are interested in determining the number of significant figures in length measurements you might make. How is the number of significant figures determined? Suppose God could tell us that the “true” width of a certain car key in centimeters was:

2.435789345646754456540123544332975774281245623... etc.

If we were to measure the key width with a ruler that is lying around the lab, the precision of our measurement would be limited by the fact that the ruler only has lines marked on it every 0.1 cm. We could estimate to the nearest 1/100th of a centimeter how far the key edge is from the last mark. Thus, we might agree that the best estimate for the width of the key is 2.44 cm. This means we have estimated the key width to three significant figures.

If God announces that the width of a pair of sun glasses is 13.27655457787654267787... cm, then upon direct measurement we might estimate the width to be 13.28 or 13.27 or 13.26 cm. In this case the estimated width is four significant figures. Obviously, there is uncertainty about the “true” value of the right-most digit.

The number of significant figures in a measurement is given by the number of digits from the most certain digit on the left of the number up to and including the first uncertain digit on the right. In reporting a number, all digits except the significant digits should be dropped. (See the discussion of significant figures in Appendix A.)

Let’s do some length measurements to find out what factors might influence the number of significant figures in a measurement.

### Apparatus

- A meter stick
- A vernier caliper
- A rectangular board

### Activity 1: Length Measurements with the Meter Stick

(a) What factors might make a determination of the “true” length of an object measured with the meter stick uncertain?

(b) Measure the width of the board with the meter stick at least seven times and create a table in the space below to list the measurements. For best results, you should use different regions of the meter stick so that, when an average of these measurements is made, non-uniformities in the scale will tend to cancel. Also, you

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should avoid using the end of the meter stick which might be worn and, therefore, would not be a true zero. The apparent change in the reading on the scale due to the position of the eye is called parallax, an effect which can introduce error into the reading. To reduce the uncertainty due to parallax, you should place the meter stick on edge so that the scale is close to the object being measured. (See Appendix E)

(c) In general, when a series of measurements is made, the best estimate is the average of those measurements. In the space below list the minimum measurement, the maximum measurement, and the best estimate for the width of your board.

(d) Based on these measurements, write the width of your board as a value plus or minus an “uncertainty”.

(e) How many significant figures should you report in your best estimate? Why?

(f) For your board, what limits the number of significant figures most - variation in the actual width of the board or limitations in the accuracy of the meter stick? How do you know?

### **Activity 2: Length Measurements with the Vernier Caliper**

(a) Measure the thickness of the board at least seven times and record the results in a table in the space below. Make the measurements at different places along each of the two edges. If you have questions about how to read the vernier, see Appendix E. If you still have questions, consult your instructor.

(b) Record the minimum, maximum, and average of your measurements below.

(c) Based on these measurements, write the thickness of your board as a value plus or minus an uncertainty.

- (d) How many significant figures should you report in your average? Why?

**Activity 3: Calculation of Cross-Sectional Area**

- (a) Calculate the cross-sectional area of the board in the space below.

- (b) Calculate the uncertainty in the area,  $\Delta A$ , as follows:

$$\frac{\Delta A}{A} = \sqrt{\left(\frac{\Delta w}{w}\right)^2 + \left(\frac{\Delta t}{t}\right)^2}$$

- (c) Write the cross-sectional area as a value plus or minus an uncertainty, rounding off as appropriate.

- (d) How many significant figures should you report in your result? Why?

**The Inevitability of Uncertainty**

In common terminology there are three kinds of “errors”: (1) mistakes or human errors, (2) systematic errors due to measurement or equipment problems and (3) inherent uncertainties.

**Activity 4: Error Types**

- (a) Give an example of how a person could make a “mistake” or “human error” while taking a length measurement.

- (b) Give an example of how a systematic error could occur because of the condition of the meter stick when a set of length measurements are being made.

- (c) What might cause inherent uncertainties in a length measurement?

### 3 Measurement of Length, Mass, Volume, and Density

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

**Objectives:**

- Learn to measure length with the vernier caliper and mass with the platform balance
- Apply knowledge of units and significant figures
- Understand the dependence of mass and density on dimensions

**Apparatus:**

- vernier caliper
- platform balance
- set of wooden disks

**Activity:**

1. Find the dimensions in centimeters of each of the disks using the vernier caliper (see Appendix E). Use the averages of three trials for each dimension (diameter,  $D$ ; width,  $W$ ) in the calculations of the volumes ( $V = \pi r^2 W$ ).
2. Find the mass,  $M$ , of each disk using the laboratory balance.
3. Calculate the density,  $\rho$ .

disk	$D_1$ (cm)	$D_2$ (cm)	$D_3$ (cm)	$W_1$ (cm)	$W_2$ (cm)	$W_3$ (cm)	$D$ (cm)	$W$ (cm)	$V$ (cc)	$M$ (g)	$\rho$ (g/cc)
1											
2											
3											
4											
5											

4. Graph mass versus radius and mass versus radius squared.



**Questions:**

1. How does the density depend on the size of a disk?
2. What is the nature of the relationship between mass and radius? What is the dependency?
3. What is the smallest part of a centimeter that can be read or estimated with a meter stick? With a vernier caliper? Which reading is more reliable? Explain.
4. When determining the volume of a disk, which dimension, diameter or width, should be measured more carefully? Explain.
5. What is the volume of the largest disk in cubic millimeters? In liters? What is its mass in kilograms?

## 4 Measurement Uncertainty and Variation<sup>2</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

- To explore the mathematical meaning of the standard deviation and standard error associated with a set of measurements.
- To investigate random and systematic variations associated with a set of measurements.

### Statistics - The Inevitability of Uncertainty

With care and attention, it is commonly believed that both mistakes and systematic errors can be eliminated completely. However, inherent uncertainties do not result from mistakes or errors. Instead, they can be attributed in part to the impossibility of building measuring equipment that is precise to an infinite number of significant figures. The ruler provides us with an example of this. It can be made better and better, but it always has an ultimate limit of precision.

Another cause of inherent uncertainties is the large number of random variations affecting any phenomenon being studied. For instance, if you repeatedly drop a baseball from the level of the lab table and measure the time of each fall, the measurements will most probably not all be the same. Even if the stop watch was gated electronically so as to be as precise as possible, there would be small fluctuations in the flow of currents through the circuits as a result of random thermal motion of atoms and molecules that make up the wires and circuit elements. This could change the stop watch reading from measurement to measurement. The sweaty palm of the experimenter could cause the ball to stick to the hand for an extra fraction of a second, slight air currents in the room could change the ball's time of fall, vibrations could cause the floor to oscillate up and down an imperceptible distance, and so on.

### Repeated Time-of-Fall Data

In the first two activities, you and your partners will take repeated data on the time of fall of a ball and study how the data varies from some average value for the time-of-fall.

### Apparatus

- A ball
- A stop watch
- A 2-meter stick

### Activity 1: Timing a Falling Ball

(a) Drop the ball so it falls through a height of exactly 2.0 m at least 20 times in rapid succession and measure the time of fall. Be as exact as possible about the height from which you drop the ball. Record the data in a table below.

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(b) Enter your data as a single column in Excel and find the average (mean) of the data. See Appendix C for instructions. Report the mean value in the space below.

### The Standard Deviation as a Measure of Uncertainty

How certain are we that the average fall-time determined in the previous activity is accurate? The average of a number of measurements does not tell the whole story. If all the times you measured were the same, the average would seem to be very precise. If each of the measurements varied from the others by a large amount, we would be less certain of the meaning of the average time. We need criteria for determining the certainty of our data. Statisticians often use a quantity called the standard deviation as a measure of the level of uncertainty in data. The standard deviation is usually represented by the Greek letter  $\sigma$  (sigma). A customary way of expressing an experimentally determined value is: Mean  $\pm \sigma$ . The formal mathematical definition of  $\sigma$  can be found in Appendix A.

In the next activity you will use Excel (see Appendix C) to calculate the value of the standard deviation for the repeated fall-time data you just obtained and explore how the standard deviation is related to variation in your data. In particular, you will try to answer this question: What percentage of your data lies within one standard deviation of the average you calculated?

### Activity 2: Standard Deviation

(a) Report the value for the standard deviation of your data in the space below.

(b) Calculate the average plus the standard deviation,  $\langle t \rangle + \sigma$ , and the average minus the standard deviation,  $\langle t \rangle - \sigma$ , and record the results in the space below.

(c) Determine the number of your data points that lie within  $\pm\sigma$  of the average and write the result in the space below. Also, calculate the percentage of data points lying within a standard deviation of the average and report that result.

(d) Combine your results with those obtained by the other groups in the class and create a table in the space below with the following column headings: Lab Station,  $\langle t \rangle$  (s),  $\sigma$  (s), %Data  $\pm\sigma$ .

(e) Study the last column, which represents the percentage of data points lying within one standard deviation of the average. What does the standard deviation,  $\sigma$ , tell you about the approximate probability that another measurement will lie within  $\pm\sigma$  of the average?

**Systematic Error - How About the Accuracy of Your Timing Device and Timing Methods?**

As the result of problems with your measuring instrument or the procedures you are using, each of your measurements may tend to be consistently too high or too low. If this is the case, you probably have a source of systematic error. There are several types of systematic error.

Most of us have set a watch or clock only to see it gain or lose a certain amount of time each day or week. In ordinary language we would say that such a time keeping device is inaccurate. In scientific terms, we would say that it is subject to systematic error. In the case of a stopwatch or digital timer that doesn't run continuously like a clock, we have to ask an additional set of questions. Does it start up immediately? Does it stop exactly when the event is over? Is there some delay in the start and stop time? A delay in starting or stopping a timer could also cause systematic error.

Finally, systematic error can be present as a result of the methods you and your partner are using for making the measurement. For example, are you starting the timer exactly at the beginning of the event being measured and stopping it exactly at the end? Are you dropping the ball from a little above the exact starting point each time? A little below?

It is possible to correct for systematic error if you can quantify it. Suppose that God, who is a theoretical physicist, said that the distance in meters,  $y$ , that a ball falls after a time of  $t$  seconds near the earth's surface in most places is given by the equation

$$y = \frac{1}{2}gt^2$$

where  $g$  is the gravitational constant [equal to 9.8 (m/s)/s]. (In this idealized equation the effects of air resistance have been neglected.)

In the activity that follows, you should compare your average time-of-fall with that expected by theory to determine if a systematic error exists.

**Activity 3: Is There Systematic Error in the Data?**

(a) Calculate the theoretical, God given, time-of-fall in the space below.

(b) Does the theoretical value lie in the range of your own average value with its associated uncertainty? If not, you probably have a source of systematic error.

(c) If you seem to have systematic error, explain whether the measured times tend to be too short or too long and list some of the possible causes of it in the space below.

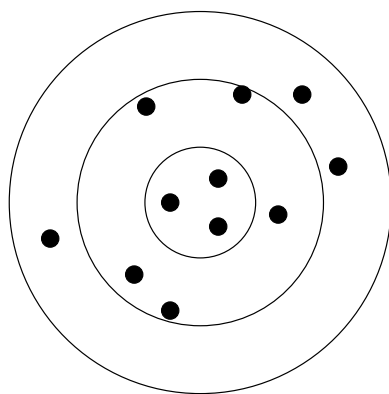
**Homework**

1. Suppose you made the following five length measurements of the width of a piece of  $8\frac{1}{2}'' \times 11''$  paper which has been cut carefully by a manufacturer using an unfamiliar centimeter rule: 21.33 cm, 21.52 cm, 21.47 cm, 21.21 cm, 21.45 cm. (a) Find the mean and standard deviation of the measurements. The formal mathematical definition of standard deviation is given in Appendix A.

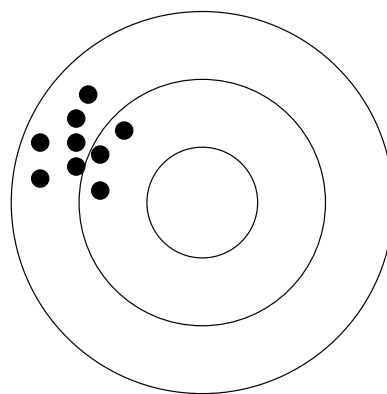
(b) Is there any evidence of uncertainty in the measurements or are they precise? Explain.

(c) Is there any evidence of systematic error in the measurements? If so, what might cause this? Explain.

2. Suppose Ashley and Ryan each throw darts at targets as shown below. Each of them is trying very hard to hit the bulls eye each time. Discuss in essay form which of the two students has the least amount of random error associated with his or her throws and is thus more precise. Is one of the students less accurate in the sense of having a systematic error associated with his or her throws? What factors like eyesight and coordination might cause one to be more precise and another more accurate?



Ashley



Ryan

## 5 Position vs. Time Graphs<sup>3</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

- To learn about two of the ways that physicists can describe motion in one dimension-words and graphs.
- To learn how to relate graphs of position vs. time to the motions they represent.

### Introduction

The focus of this unit on kinematics is to be able to describe your position as a function of time using words and graphs. You will use a motion detector attached to a computer in the laboratory to learn to describe one-dimensional motion.

The ultrasonic motion detector sends out a series of sound pulses that are of too high a frequency to hear. These pulses reflect from objects in the vicinity of the motion detector and some of the sound energy returns to the detector. The computer is able to record the time it takes for reflected sound waves to return to the detector and then, by knowing the speed of sound in air, figure out how far away the reflecting object is. There are several things to watch out for when using a motion detector. (1) Do not get closer than 0.15 meters from the detector because it cannot record reflected pulses which come back too soon. (2) The ultrasonic waves come out in a cone of about 15°. It will see the closest object. Be sure there is a clear path between the object whose motion you want to track and the motion detector. (3) The motion detector is very sensitive and will detect slight motions. You can try to glide smoothly along the floor, but don't be surprised to see small bumps in velocity graphs. (4) Some objects like bulky sweaters are good sound absorbers and may not be "seen" well by a motion detector. You may want to hold a book or a board in front of you if you have loose clothing on.

### Apparatus

- *Science Workshop 750 Interface*
- Ultrasonic motion detector
- *DataStudio* software (Position Graphs application)
- Wooden board
- Masking tape for marking distances

### Position vs. Time Graphs of Your Motion

The purpose of this unit is to learn how to relate graphs of position as a function of time to the motions they represent. How does a position vs. time graph look when you move slowly? Quickly? What happens when you move toward the motion detector? Away? After completing the next few activities, you should be able to look at a position vs. time graph and describe the motion of the object. You should also be able to look at the motion of an object and sketch a graph representing that motion.

Note that the motion detector measures the distance of an object from the detector, and that the motion detector is located at the origin of each graph. It is common to refer to the distance of an object from some origin as the position of the object. Therefore, it is better to refer to these graphs as position vs. time graphs than distance vs. time graphs.

You will use the *DataStudio* software to do the following activities. Launch the **Position Graphs** application by going to **Start** → **Programs** → **Physics Applications** → **131 Workshop** → **Position Graphs**. To start a data run, click the **Start** button. To stop a data run, click the **Stop** button. After a data run, the graph

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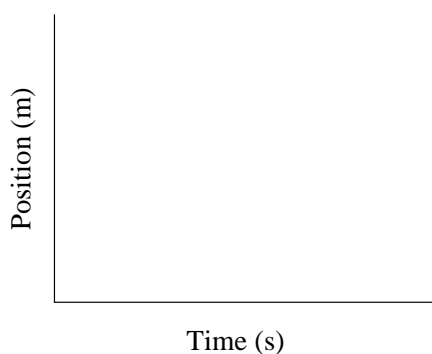
can be expanded by clicking on the **Scale to fit** button in the upper left corner of the Graph window. Multiple data sets can be displayed on the same graph. Data can be removed from the graph by selecting **Delete Last Data Run** or **Delete All Data Runs** from the **Experiment** menu. When you are finished with the activities, choose **Quit** from the **File** menu and do not save this activity.

Before you begin the activities, you should mark a position scale on the floor. To do this, position one person at approximately 1 meter in front of the motion detector and take data for 1 second. The computer will display a horizontal line showing the position measured by the detector. The person standing in front of the detector should then adjust his/her position and the procedure repeated until the 1 meter position is established. Mark the 1 meter position on the floor with a piece of masking tape and then mark the 2, 3 and 4 meter positions using the 2 meter stick.

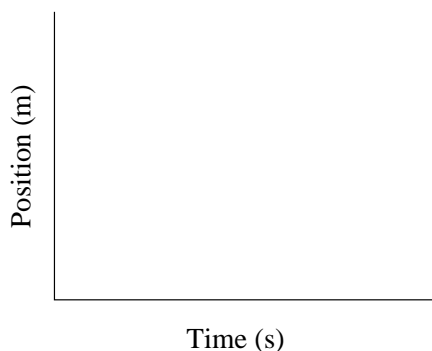
### Activity 1: Making Position vs. Time Graphs

Make position-time graphs for the following motions and sketch the graph you observe in each case:

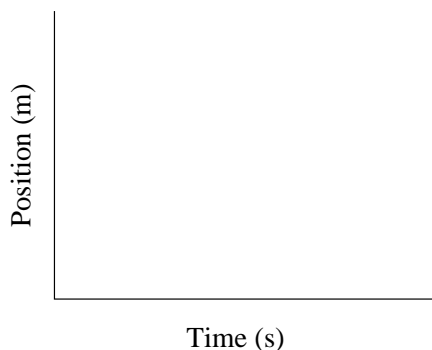
- (a) Starting at 0.5 m, walk away from the origin (i.e., the detector) slowly and steadily.



- (b) Walk away from the origin medium-fast and steadily.



- (c) Walk toward the detector (origin) slowly and steadily.

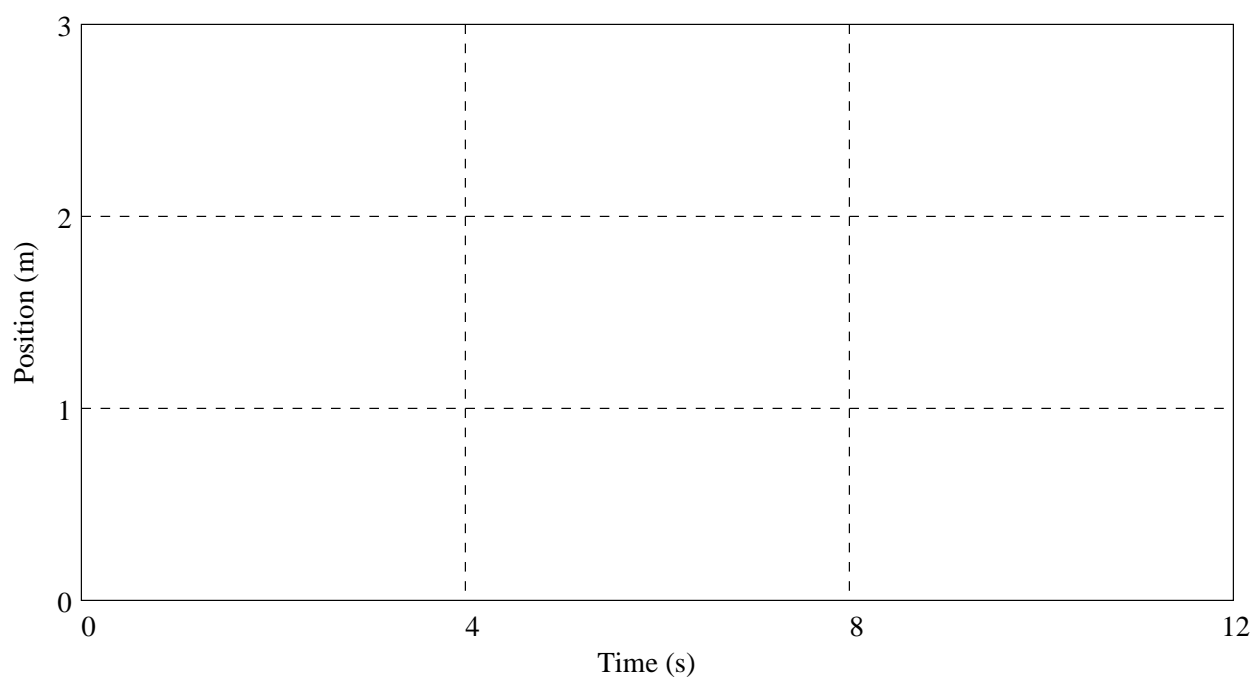


(d) Describe the difference between the graph you made by walking away slowly and the one made by walking away more quickly.

(e) Describe the difference between the graph made by walking toward and the one made walking away from the motion detector.

### Activity 2: Predicting a Position vs. Time Graph

(a) Suppose you were to start 1.0 m in front of the detector and walk away slowly and steadily for 4 seconds, stop for 4 seconds, and then walk toward the detector quickly. Sketch your prediction on the axes below using a dashed line.

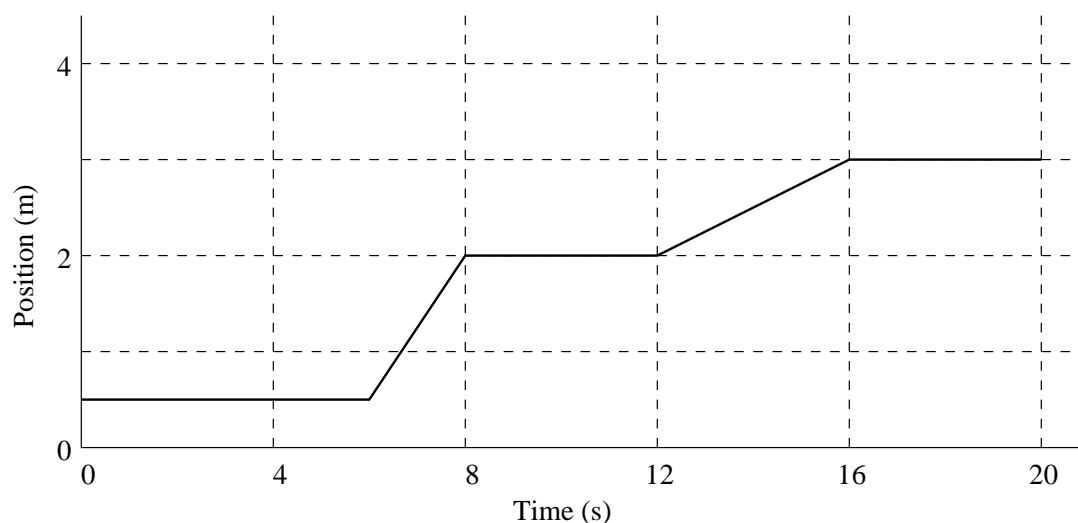


(b) Test your prediction by moving in the way described and making a graph of your motion with the motion detector. Sketch the trace of your actual motion on the above graph with a solid line.

(c) Is your prediction the same as the final result? If not, describe how you would move to make a graph that looks like your prediction.

### Activity 3: Matching Position vs. Time Graphs



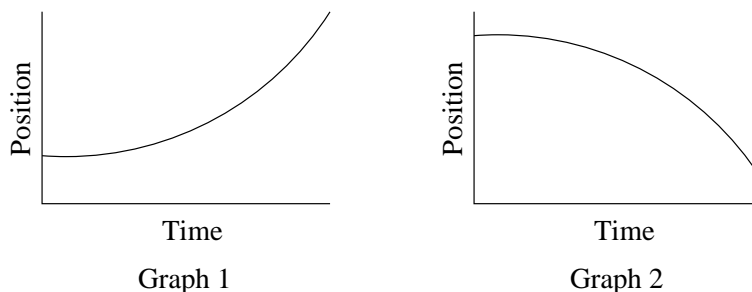


(a) Describe in your own words how you would move in order to match the graph shown above.

(b) Move to match the above graph on the computer screen. You may try a number of times. It helps to work as a team. Get the times right. Get the positions right. Do this for yourself. (Each person in your group should do his or her own match.) You will not learn very much by just watching!

(c) What was the difference in the way you moved to produce the two differently sloped parts of the graph you just matched?

(d) Make curved position vs. time graphs like those shown below.



(e) Describe how you must move to produce a position vs. time graph with each of the shapes shown.

Graph 1 answer:

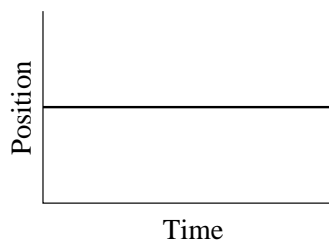
Graph 2 answer:

(f) What is the general difference between motions which result in a straight-line position vs. time graph and those that result in a curved-line position vs. time graph?

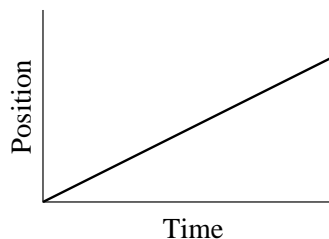
### Homework

Answer the following questions in the spaces provided.

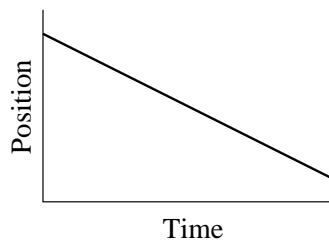
1. What do you do to create a horizontal line on a position-time graph?



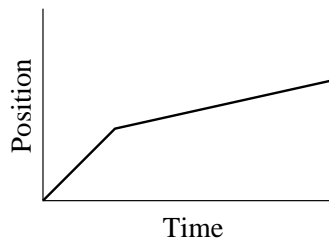
2. How do you walk to create a straight line that slopes up?



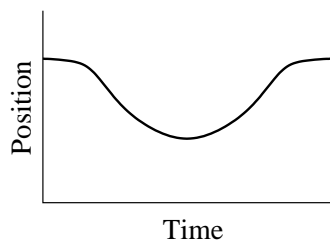
3. How do you walk to create a straight line that slopes down?



4. How do you move so the graph goes up steeply at first, then continues up gradually?

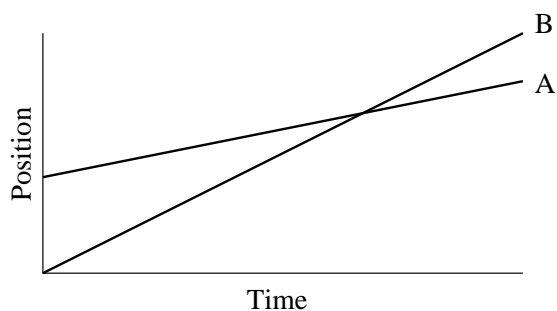


5. How do you walk to create a U-shaped graph?

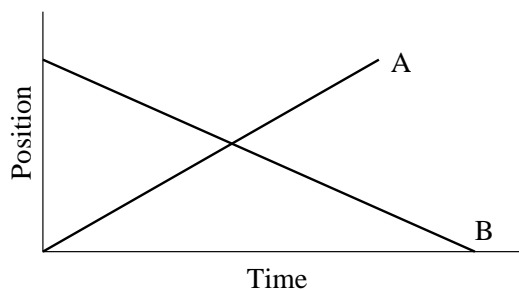


Answer the following about the two objects, A and B, whose motion produced the following position-time graphs.

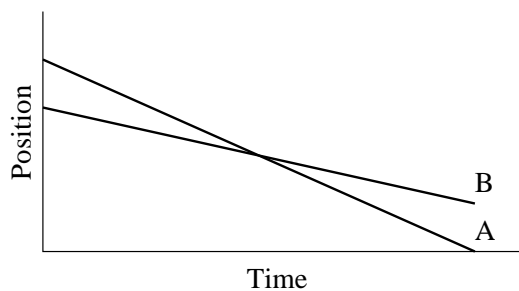
6. (a) Which object is moving faster? (b) Which starts ahead? Define what you mean by "ahead."  
 (c) What does the intersection mean?



7. (a) Which object is moving faster? (b) Which object has a negative velocity according to the convention we have established?



8. (a) Which object is moving faster? (b) Which starts ahead? Define what you mean by "ahead."



Sketch the position-time graph corresponding to each of the following descriptions of the motion of an object.

9. The object moves with a steady (constant) velocity away from the origin.



10. The object is standing still.



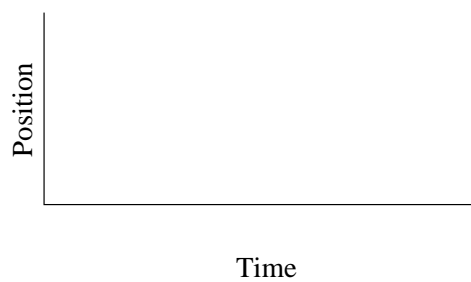
11. The object moves with a steady (constant) velocity toward the origin for 5 seconds and stands still for 5 seconds.



12. The object moves with a steady velocity away from the origin for 5 seconds, then reverses direction and moves at the same speed toward the origin for 5 seconds.



13. The object moves away from the origin, starting slowly and speeding up.



## 6 Velocity vs. Time Graphs<sup>4</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

- To acquire an intuitive understanding of speed and velocity in one dimension.
- To learn how to relate graphs of velocity vs. time to the motions they represent.

### Introduction

You have already plotted your position as a function of time. Another way to represent your motion during an interval of time is with a graph which describes how fast and in what direction you are moving from moment to moment. How fast you move is known as your speed. It is the rate of change of position with respect to time. Velocity is a quantity which takes into account your speed and the direction you are moving. Thus, when you examine the motion of an object moving along a line, its velocity can be positive or negative depending on whether the object is moving in the positive or negative direction.

Graphs of velocity over time are more challenging to create and interpret than those for position. A good way to learn to interpret them is to create and examine velocity vs. time graphs of your own body motions, as you will do in the next few activities.

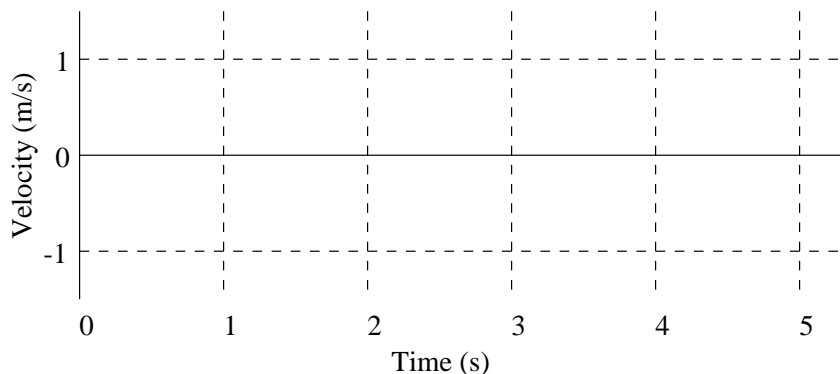
### Apparatus

- *Science Workshop 750 Interface*
- Ultrasonic motion detector
- *Data Studio* software (Velocity Graphs application)
- Wooden board

To make the graphs in the following activities, use the **Velocity Graphs** application by going to **Start** → **Programs** → **Physics Applications** → **131 Workshop** → **Velocity Graphs**. Click **Start** to begin, **Stop** to end a data run.

### Activity 1: Making Velocity vs. Time Graphs

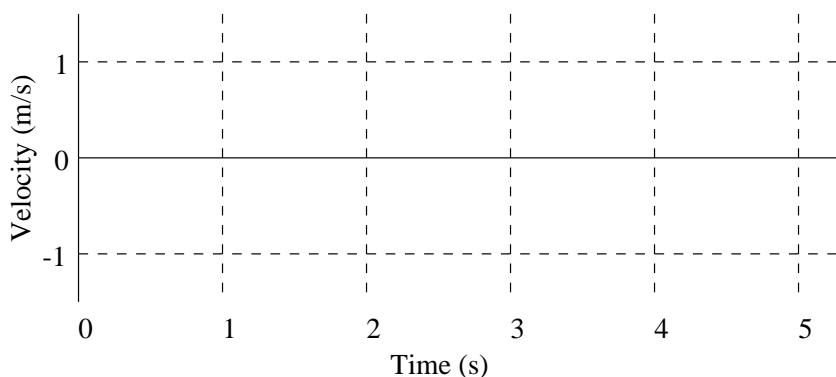
(a) Make a velocity graph by walking away from the detector slowly and steadily. Try again until you get a graph you're satisfied with and then sketch your result on the graph that follows. (We suggest you draw smooth patterns by ignoring smaller bumps that are mostly due to your steps.)



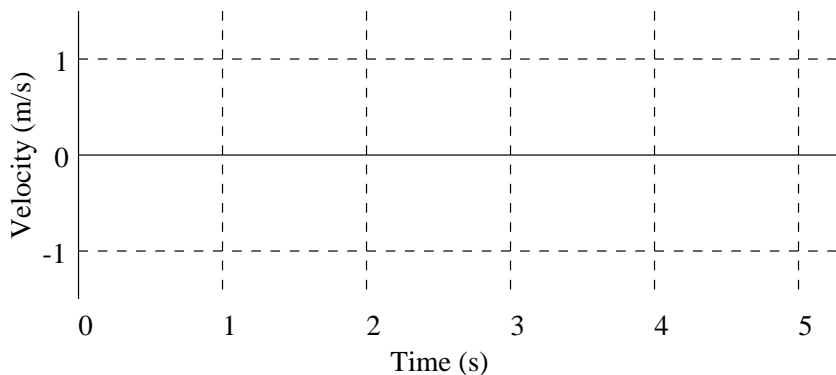
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<sup>4</sup>1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

(b) Make a velocity graph, walking away from the detector steadily at a medium speed. Sketch your graph below.



(c) Make a velocity graph, walking toward the detector slowly and steadily. Sketch your graph below.



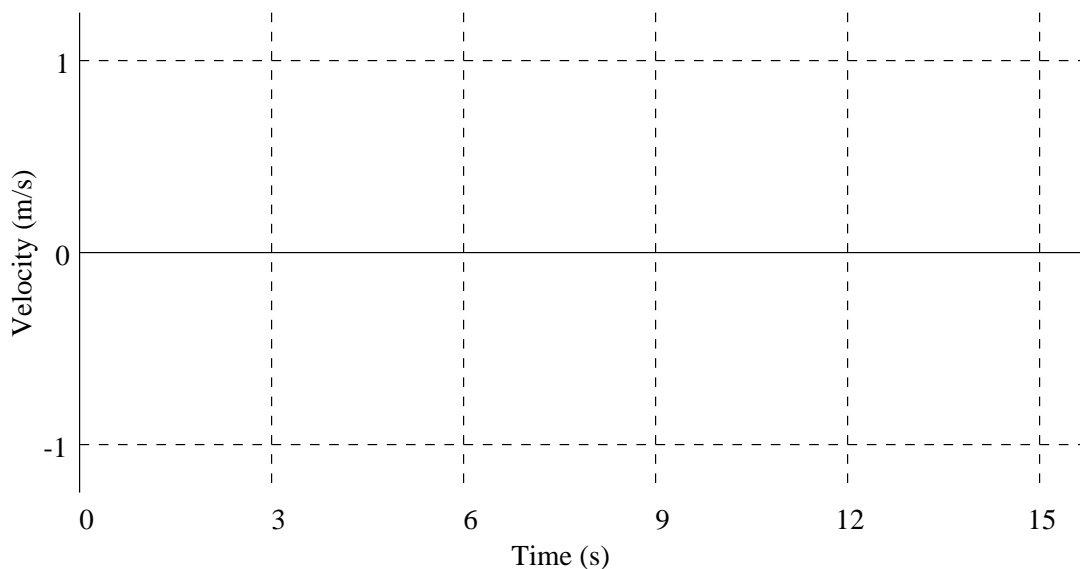
(d) What is the most important difference between the graph made by slowly walking away from the detector and the one made by walking away more quickly?

(e) How are the velocity vs. time graphs different for motion away and motion toward the detector?

### Activity 2: Predicting a Velocity vs. Time Graph

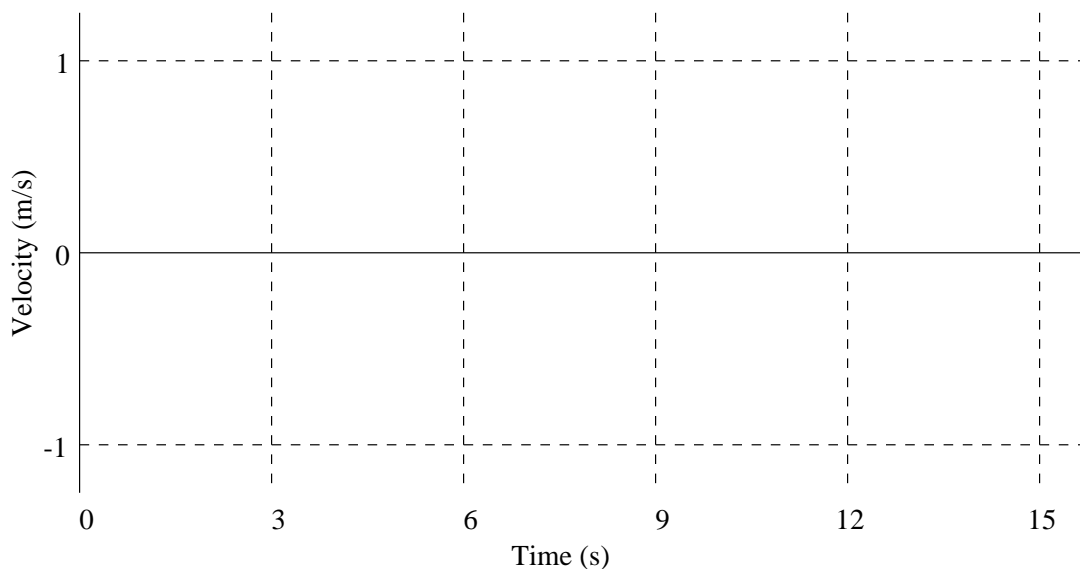
Suppose you were to undergo the following sequence of motions: (1) walk away from the detector slowly and steadily for 6 seconds, (2) stand still for 6 seconds, (3) walk toward the detector steadily about twice as fast as before.

(a) Use a dashed line in the graph that follows to record your prediction of the shape of the velocity graph that will result from the motion described above.



(b) Compare predictions with your partner(s) and see if you can all agree. Use a solid line to sketch your group prediction in the graph above.

(c) Adjust the sampling time to 15 s and then test your prediction. Repeat your motion until you are confident that it matches the description in words and then draw the actual graph on the axes below. Be sure the 6-second stop shows clearly.



(d) Did your prediction match your real motion? If not, what misunderstanding of what elements of the graph did you have?

### Velocity Vectors

The two ideas of speed and direction can be combined and represented by vectors. A velocity vector is represented by an arrow pointing in the direction of motion. The length of the arrow is drawn proportional to the speed;



the longer the arrow, the larger the speed. If you are moving toward the right, your velocity vector can be represented by the arrow shown below.



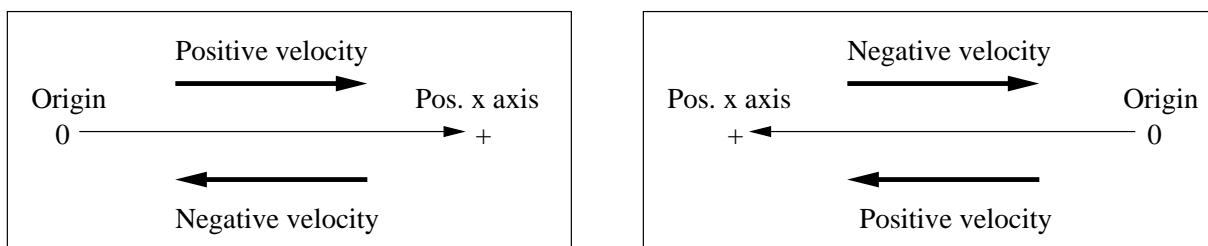
If you were moving twice as fast toward the right, the arrow representing your velocity vector would look like:



while moving twice as fast toward the left would be represented by the following arrow:



What is the relationship between a one-dimensional velocity vector and the sign of velocity? This depends on the way you choose to set the positive  $x$ -axis.



In both diagrams the top vectors represent velocity toward the right. In the left diagram, the  $x$ -axis has been drawn so that the positive  $x$ -direction is toward the right. Thus the top arrow represents positive velocity. However, in the right diagram, the positive  $x$ -direction is toward the left. Thus the top arrow represents negative velocity. Likewise, in both diagrams the bottom arrows represent velocity toward the left. In the left diagram this is negative velocity, and in the right diagram it is positive velocity.

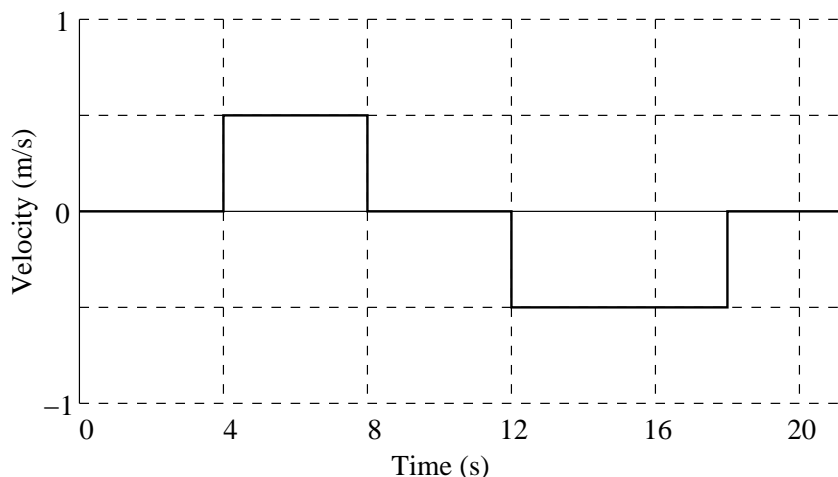
### Activity 3: Sketching Velocity Vectors

Sketch below velocity vectors representing the three parts of the motion described in the prediction you made in Activity 2.

- (a) Walking slowly away from the detector:
- (b) Standing still:
- (c) Walking rapidly toward the detector:

### Activity 4: Matching a Velocity Graph

- (a) Describe how you think you will have to move in order to match the velocity graph shown below.



(b) Move in such a way that you can reproduce the graph shown. You may have to practice a number of times to get the movements right. Work as a team and plan your movements. Get the times right. Get the velocities right. You and each person in your group should take a turn. Then sketch your group's best match on the above graph.

(c) Describe how you moved to match each part of the graph.

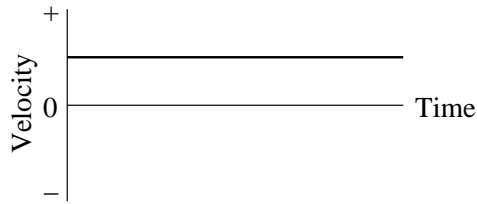
(d) Is it possible for an object to move so that it produces an absolutely vertical line on a velocity-time graph? Explain.

(e) Did you run into the motion detector on your return trip? If so, why did this happen? How did you solve the problem? Does a velocity graph tell you where to start? Explain.

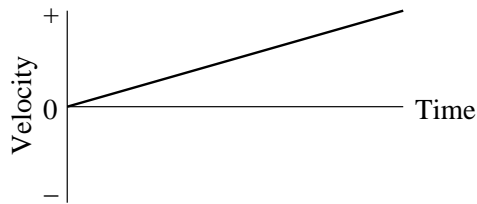
### Homework

Answer the following questions in the spaces provided.

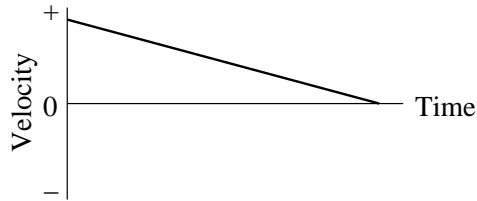
1. How do you move to create a horizontal line in the positive part of a velocity-time graph, as shown below?



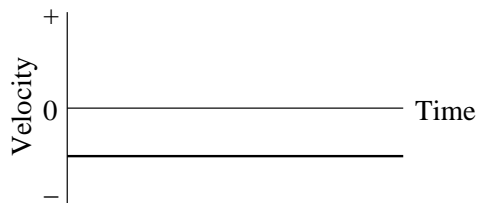
2. How do you move to create a straight-line velocity-time graph that slopes up from zero, as shown below?



3. How do you move to create a straight-line velocity-time graph that slopes down, as shown below?

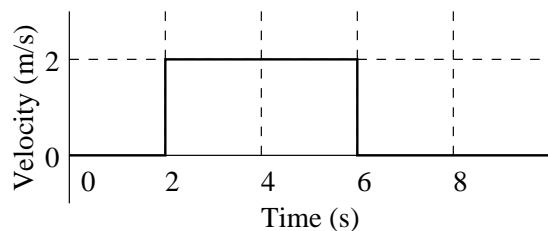


4. How do you move to make a horizontal line in the negative part of a velocity-time graph, as shown below?

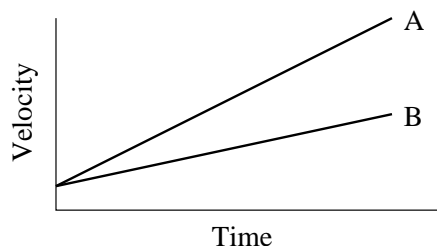


5. The velocity-time graph of an object is shown below. Figure out the total change in position (displacement) of the object. Show your work.

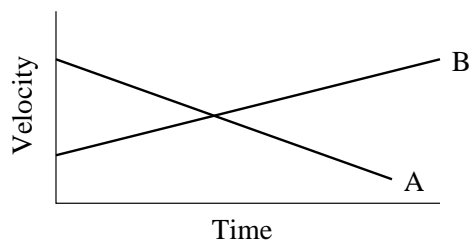
Displacement = \_\_\_\_\_ meters.



6. The velocity graph below shows the motion of two objects, A and B. Answer the following questions separately. Explain your answers when necessary. (a) Is one faster than the other? If so, which one is faster? (A or B) (b) What does the intersection mean? (c) Can one tell which object is “ahead”? (define “ahead”) (d) Does either object A or B reverse direction? Explain.



7. The velocity graph below shows the motion of two objects, A and B. Answer the following questions separately. Explain your answers when necessary. (a) Is one faster than the other? If so, which one is faster? (A or B) (b) What does the intersection mean? (c) Can one tell which object is “ahead”? (define “ahead”) (d) Does either object A or B reverse direction? Explain.



Sketch the velocity-time graph corresponding to each of the following descriptions of the motion of an object.

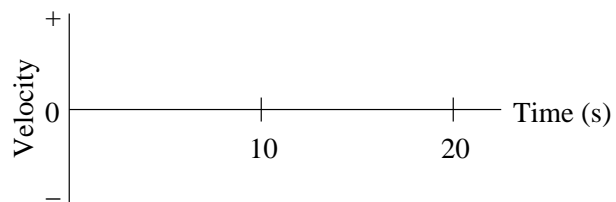
8. The object is moving away from the origin at a constant velocity.



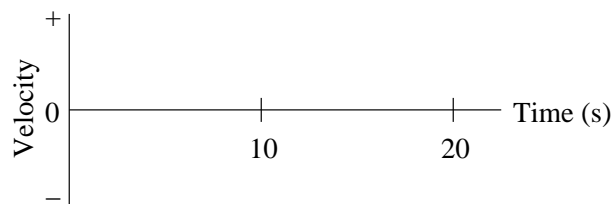
9. The object is standing still.



10. The object moves toward the origin at a steady (constant) velocity for 10 s and then stands still for 10 s.



11. The object moves away from the origin at a steady (constant) velocity for 10 s, reverses direction and moves back toward the origin at the same speed for 10 s.



## 7 Relating Position and Velocity Graphs<sup>5</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

To understand the relationship between position vs. time and velocity vs. time graphs.

### Introduction

You have looked at position and velocity vs. time graphs separately. Since position vs. time and velocity vs. time graphs are different ways to represent the same motion, it ought to be possible to figure out the velocity at which someone is moving by examining her/his position vs. time graph. Conversely, you ought to be able to figure out how far someone has traveled (change in position) from a velocity vs. time graph.

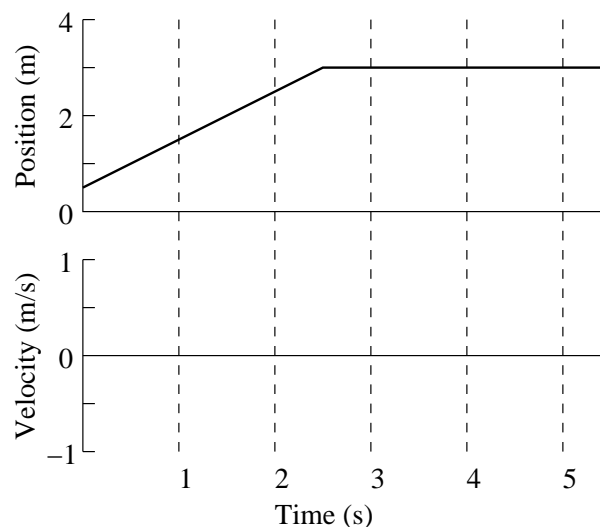
To explore how position vs. time and velocity vs. time graphs are related, go to **Start** → **Programs** → **Physics Applications** → **131 Workshop** → **Position & Velocity Graphs**. For some of the runs it may be necessary to adjust the time axis for one of the graphs so that the time scales are the same for the position and velocity graphs.

### Apparatus

- *Science Workshop 750 Interface*
- Ultrasonic motion detector
- *DataStudio* software (Position & Velocity Graphs application)
- Wooden board

### Activity 1: Predicting Velocity Graphs from Position Graphs

(a) Carefully study the position graph shown below and predict the velocity vs. time graph that would result from the motion. Using a dashed line, sketch your prediction of the corresponding velocity vs. time graph on the velocity axes.



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<sup>5</sup>1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

(b) After each person in your group has sketched a prediction, test your prediction by matching the position vs. time graph shown. When you have made a good duplicate of the position graph, sketch your actual graph over the existing position vs. time graph. Use a solid line to draw the actual velocity graph on the same graph with your prediction. (Do not erase your prediction).

(c) How would the position graph be different if you moved faster? Slower?

(d) How would the velocity graph be different if you moved faster? Slower?

### Activity 2: Average Velocity Calculations

(a) Find your average velocity during the time you were moving from your velocity graph in the previous activity. To do this, use the Smart Tool on the graph menu bar (sixth box along the top of the graph window) to determine the velocity at ten points on the left part of the graph. Record these ten values, then average them to get an estimate of the average velocity (for the time you were moving).

Velocity values (m/s) \_\_\_\_\_

Average value of the velocity: \_\_\_\_\_ m/s

Standard deviation: \_\_\_\_\_ m/s

Average velocity with uncertainty: \_\_\_\_\_ m/s

(b) Average velocity during a particular time interval can also be calculated as the change in position divided by the change in time. (The change in position is often called the displacement.) By definition, this is also the slope of the position vs. time graph for that time period. Use the cursors on the position vs. time graph to read the position and time coordinates for two typical points while you were moving. (For a more accurate answer, use two points as far apart as possible but still typical of the motion, and within the time interval over which you took velocity readings in part (a).) Record the coordinates of the points below.

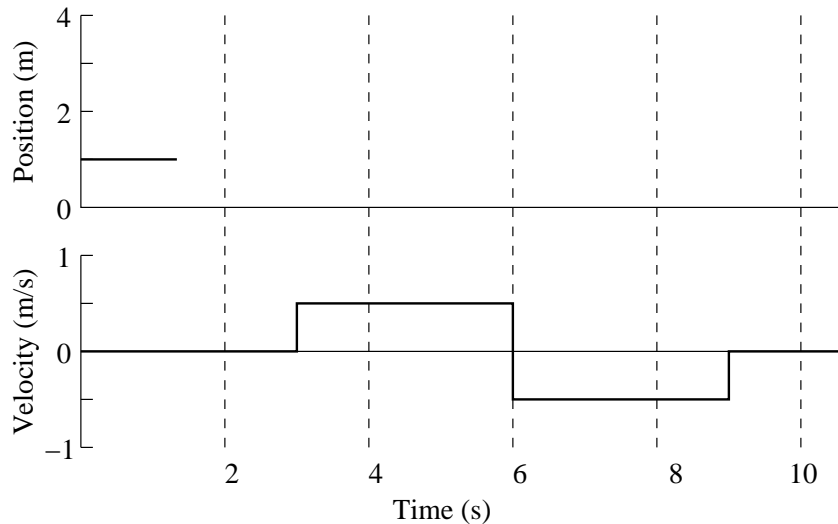
(c) Calculate the change in position (displacement) between the two points in part (b). Also calculate the corresponding change in time (time interval). Divide the change in position by the change in time to calculate the average velocity. Show your calculations below.

(d) Is the average velocity positive or negative? Is this what you expected?

(e) Does the average velocity you just calculated from the position graph agree with the average velocity you estimated from the velocity graph? Do you expect them to agree? How would you account for any differences?

### Activity 3: Finding Position from a Velocity Graph

(a) Carefully study the velocity graph that follows. Using a dashed line, sketch your prediction of the corresponding position graph on the top set of axes. (Assume that you started at the 1-meter mark.)



(b) After each person has sketched a prediction, do your group's best to duplicate the bottom (velocity vs. time) graph by walking. When you have made a good duplicate of the velocity vs. time graph, draw your actual result over the existing velocity vs. time graph. Use a solid line on the top graph to draw the actual position vs. time graph on the same axes with your prediction. Do not erase your prediction.

(c) How can you tell from a velocity vs. time graph that the moving object has changed direction?

(d) What is the velocity at the moment the direction changes?

(e) Is it possible to actually move your body (or an object) to make vertical lines on a position vs. time graph? Why or why not? What would the velocity be for a vertical section of a position vs. time graph?

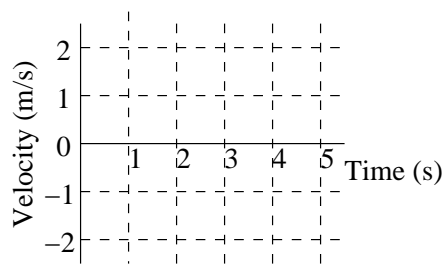
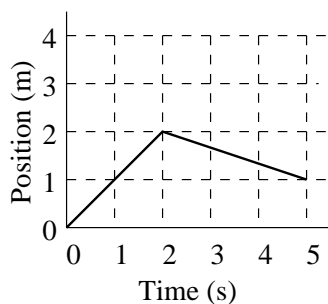
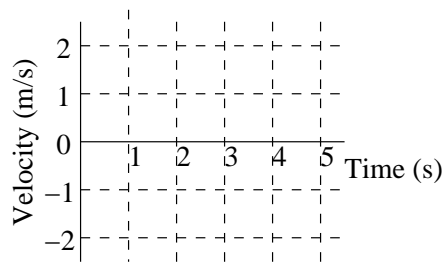
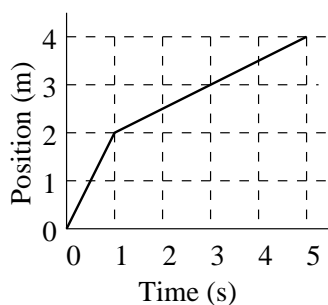
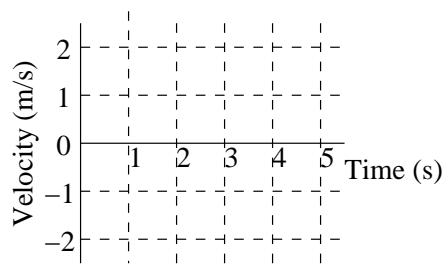
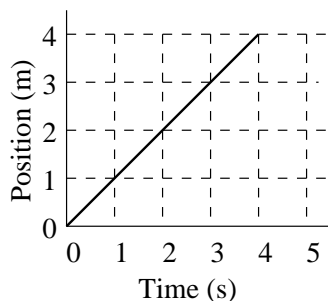
(f) How can you tell from a position vs. time graph that your motion is steady (motion at a constant velocity)?

(g) How can you tell from a velocity vs. time graph that your motion is steady (constant velocity)?

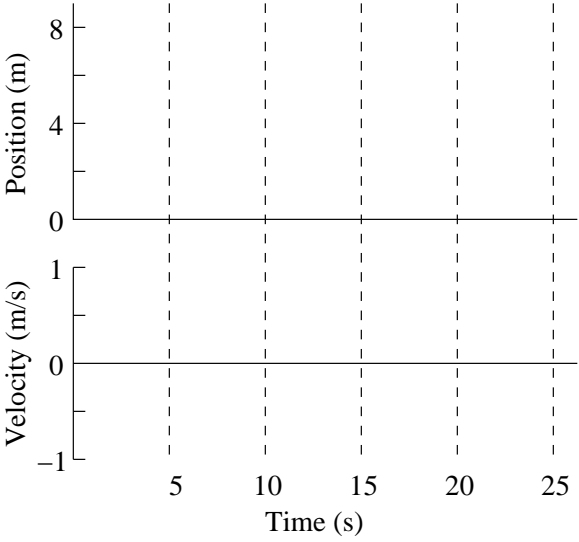


## Homework

1. Draw the velocity graphs for an object whose motion produced the position-time graphs shown below on the left. Position is in meters and velocity in meters per second. Note: Unlike most real objects, you can assume these objects can change velocity so quickly that it looks instantaneous with this time scale.



2. Draw careful graphs below of position and velocity for a cart that (a) moves away from the origin at a slow and steady (constant) velocity for the first 5 seconds; (b) moves away at a medium-fast, steady (constant) velocity for the next 5 seconds; (c) stands still for the next 5 seconds; (d) moves toward the origin at a slow and steady (constant) velocity for the next 5 seconds; (e) stands still for the last 5 seconds.



## 8 Changing Motion<sup>6</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

- To learn how to relate graphs of acceleration vs. time to the motions they represent.
- To understand the relationship between position vs. time, velocity vs. time, and acceleration vs. time graphs.

### Velocity and Acceleration Graphs

We are interested in having you learn to describe simple motions in which the velocity of an object is changing. In order to learn to describe motion in more detail for some simple situations, you will be asked to observe and describe the motion of a dynamics cart on a track. Although graphs and words are still important representations of these motions, you will also be asked to draw velocity vectors, arrows that indicate both the direction and speed of a moving object. Thus, you will also learn how to represent simple motions with velocity diagrams.

In the last session, you looked at position vs. time and velocity vs. time graphs of the motion of your body as you moved at a “constant” velocity. The data for the graphs were collected using a motion detector. Your goal in this session is to learn how to describe various kinds of motion in more detail. It is not enough when studying motion in physics to simply say that “the object is moving toward the right” or “it is standing still.” You have probably realized that a velocity vs. time graph is better than a position vs. time graph when you want to know how fast and in what direction you are moving at each instant in time as you walk. When the velocity of an object is changing, it is also important to know how it is changing. The rate of change of velocity is known as the acceleration.

In order to get a feeling for acceleration, it is helpful to create and learn to interpret velocity vs. time and acceleration vs. time graphs for some relatively simple motions of a cart on a track. You will be observing the cart with the motion detector as it moves at a constant velocity and as it changes its velocity at a constant rate. Use the **P, V & A Graphs** application by going to **Start** → **Programs** → **Physics Applications** → **131 Workshop** → **P, V & A Graphs** for all of the activities in this unit.

### Apparatus

- *Science Workshop 750 Interface*
- Ultrasonic motion detector
- *DataStudio* software (P, V & A Graphs application)
- Dynamics cart and track
- Lab stand to incline the track

### Graphing a Constant Velocity Cart Motion

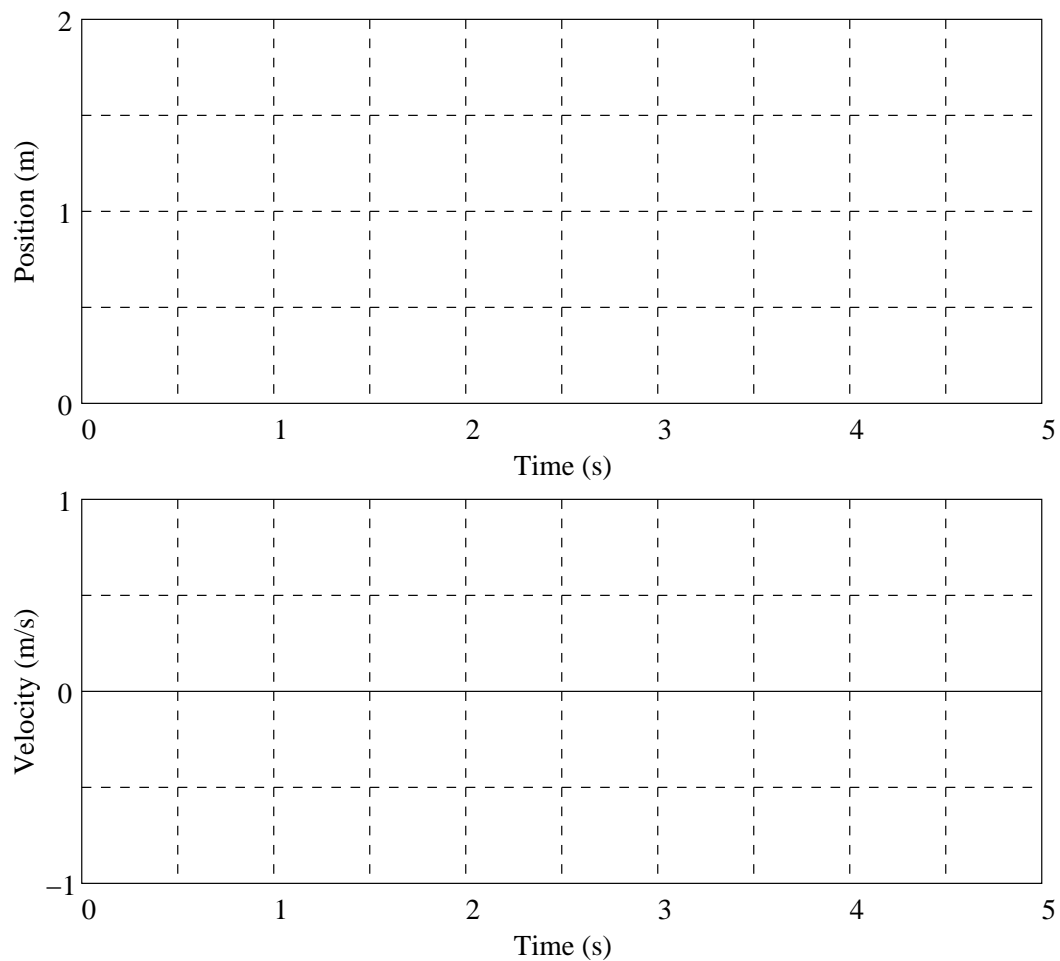
Let’s start by giving the cart a push along the level track and graphing its motion.

#### Activity 1: Position, Velocity and Acceleration Graphs of Constant Velocity

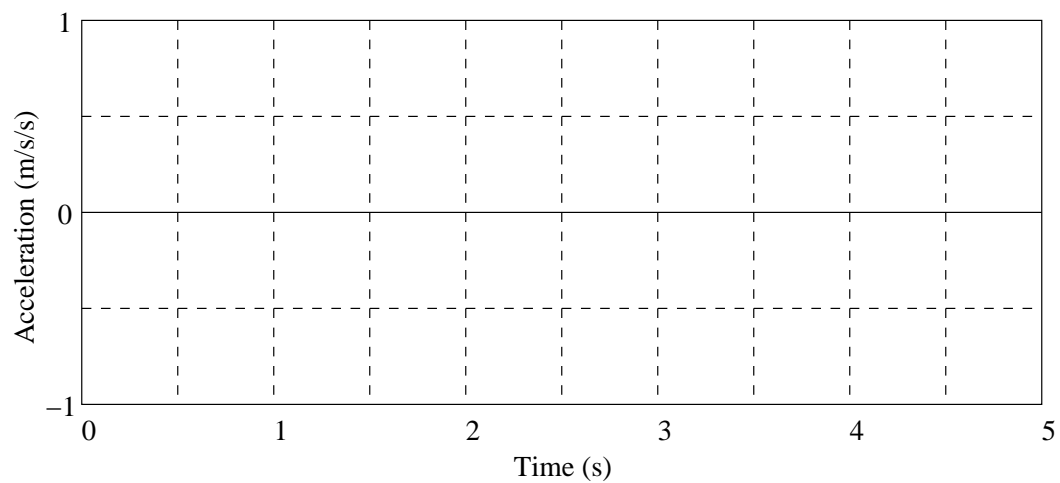
(a) Based on your observations of the motions of your body in the last session, how should the position and velocity graphs look if you move the cart at a constant velocity away from the motion detector starting at the 0.5 meter mark? Sketch your predictions with dashed lines on the axes that follow.

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<sup>6</sup>1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.



(b) Acceleration is defined as the time rate of change of velocity. Sketch your prediction of the cart acceleration on the axes that follow using a dashed line.



(c) Test your prediction. Be sure that the cart is never closer than 0.15 meter from the motion detector. Try

several times until you get a fairly constant velocity. Sketch your results with solid lines on the axes shown above. The acceleration vs. time graphs will exhibit small fluctuations due to irregularities in the motion of the cart. You should ignore these fluctuations and draw smooth patterns.

(d) Did your graphs agree with your predictions? What characterizes constant velocity motion on a position vs. time graph?

(e) What characterizes constant velocity motion on a velocity vs. time graph?

(f) What characterizes constant velocity motion on an acceleration vs. time graph?

### Finding Accelerations

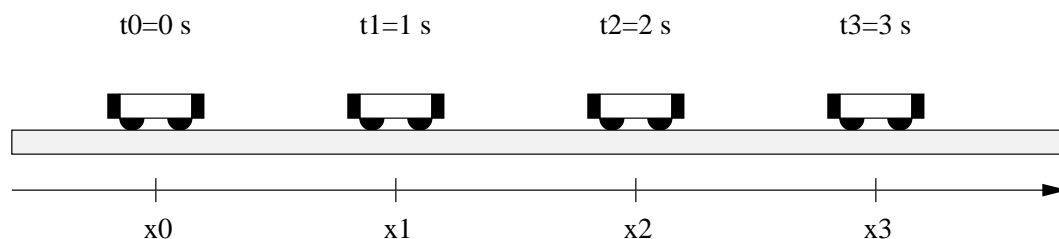
To find the average acceleration of the cart during some time interval (the average time rate of change of its velocity), you must measure its velocity at two different times, calculate the difference between the final value and the initial value and divide by the time interval.

To find the acceleration vector from two velocity vectors, you must first find the vector representing the change in velocity by subtracting the initial velocity vector from the final one. Then you divide this vector by the time interval.

### Activity 2: Representing Acceleration

(a) Calculate the average acceleration during some time interval from your velocity graph in Activity 1. Does the result agree with your acceleration graph in Activity 1?

(b) The diagram below shows the positions of the cart at equal time intervals. (This is like taking snapshots of the cart at equal time intervals.) At each indicated time, sketch a vector above the cart which might represent the velocity of the cart at that time while it is moving at a constant velocity away from the motion detector.



(c) Explain how you would find the vector representing the change in velocity between the times 1.0 s and 2.0 s in the diagram above. From this vector, what value would you calculate for the acceleration? Explain. Is this value in agreement with the acceleration graph you obtained in Activity 1?

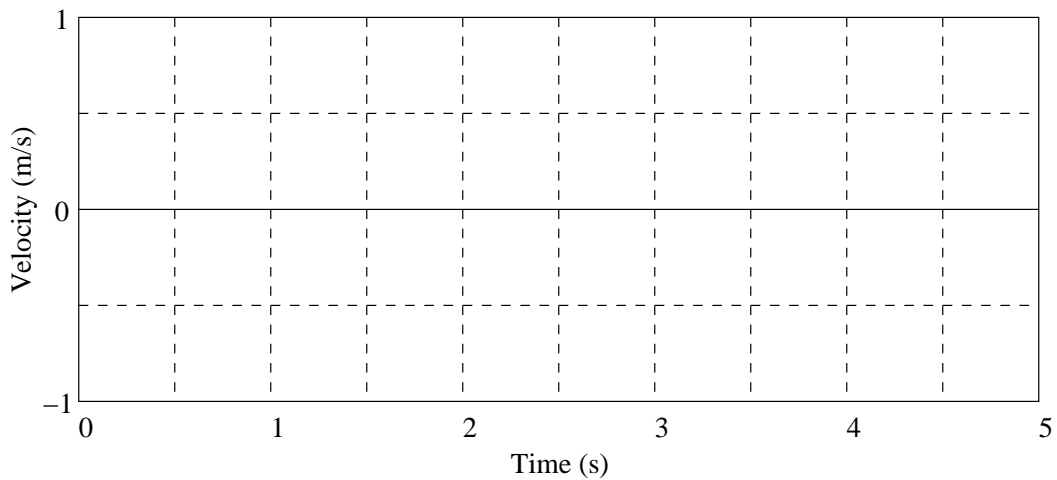
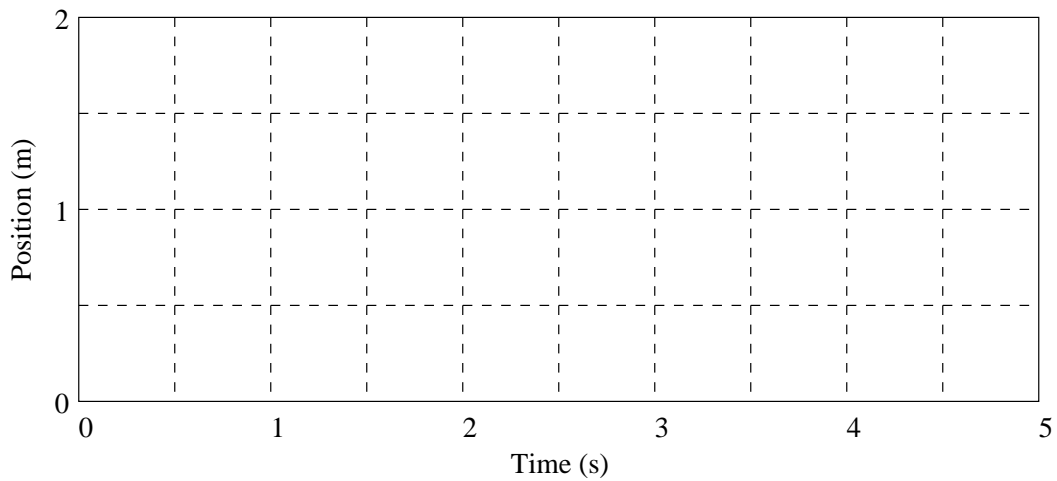
**Speeding Up at a Moderate Rate**

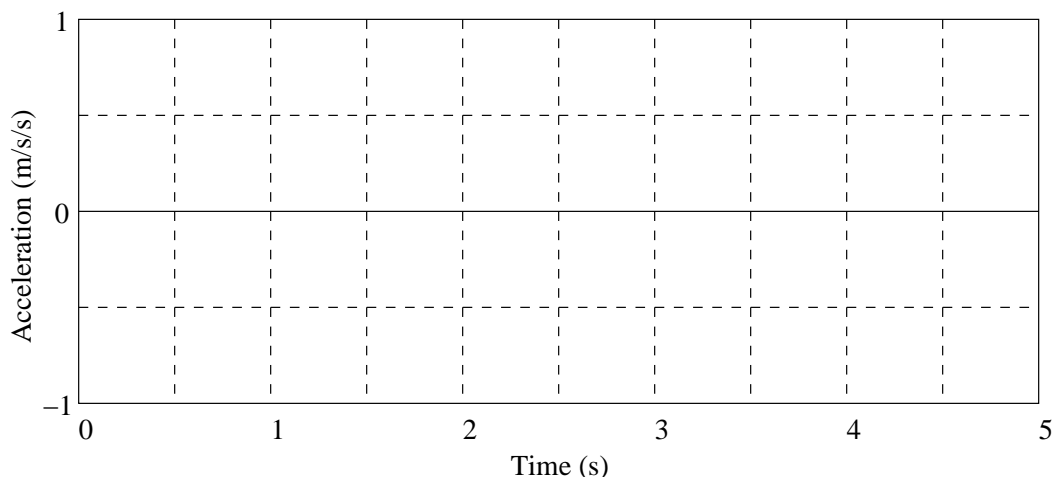
In the next activity you will look at velocity and acceleration graphs of the motion of a cart when its velocity is changing. You will be able to see how these two representations of the motion are related to each other when the cart is speeding up.

In order to get your cart speeding up smoothly use the lab stand to raise the track several centimeters at the end where the motion detector is mounted.

**Activity 3: Graphs Depicting Speeding Up**

(a) Predict the shape of the position, velocity, and acceleration vs. time graphs for the cart moving away from the sensor and speeding up. Sketch your predictions on the following axes using dashed lines.





(b) Create graphs of the motion of your cart as it moves away from the detector and speeds up. Sketch the graphs neatly on the above axes using solid lines.

(c) How does your position graph differ from the position graphs for steady (constant velocity) motion?

(d) What feature of your velocity graph signifies that the motion was away from the detector?

(e) What feature of your velocity graph signifies that the cart was speeding up? How would a graph of motion with a constant velocity differ?

(f) During the time that the cart is speeding up, is the acceleration positive or negative? How does speeding up while moving away from the detector result in this sign of acceleration? Hint: Remember that acceleration is the rate of change of velocity. Look at how the velocity is changing.

(g) How does the velocity vary in time as the cart speeds up? Does it increase at a steady rate or in some other way?

(h) How does the acceleration vary in time as the cart speeds up? Is this what you expect based on the velocity graph? Explain.

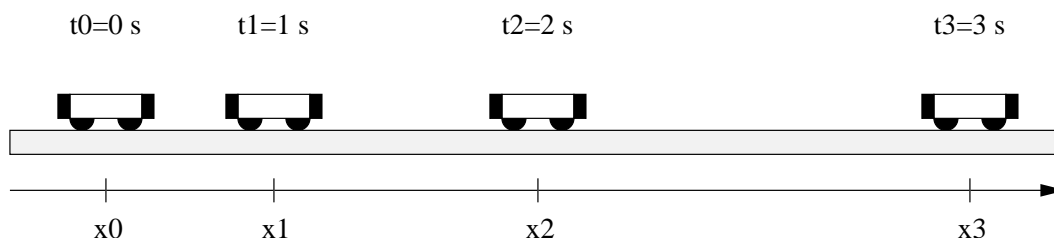
(i) Do not delete the graphs from the computer screen. They will be used in Activity 5.

**Using Vectors to Describe the Acceleration**

Let's return to the Vector Diagram representation and use it to describe the acceleration.

**Activity 4: Acceleration Vectors**

(a) The diagram that follows shows the positions of the cart at equal time intervals. At each indicated time, sketch a vector above the cart which might represent the velocity of the cart at that time while it is moving away from the motion detector and speeding up.



(b) Show below how you would find the approximate length and direction of the vector representing the change in velocity between the times 1.0 s and 2.0 s using the diagram above. No quantitative calculations are needed. Based on the direction of this vector and the direction of the positive x-axis, what is the sign of the acceleration? Does this agree with your answer to Activity 3 (f)?

**Measuring Acceleration**

In this investigation you will analyze the motion of your accelerated cart quantitatively. This analysis will be quantitative in the sense that your results will consist of numbers. You will determine the cart's acceleration from the slope of your velocity vs. time graph and compare it to the average acceleration read from the acceleration vs. time graph. You can display actual values for your acceleration and velocity data using the Smart Tool on the graph window menu.

**Activity 5: Calculating Accelerations**

(a) List 10 of the typical accelerations of the cart measured in Activity 3. Use the Smart Tool on the acceleration vs. time graph to get these values. (Only use values from the portion of the graph after the cart was released and before you stopped it.)

(b) Calculate the average value of the acceleration and record it below. Also calculate the standard deviation and write the acceleration with its uncertainty.



(c) The average acceleration during a particular time period is defined as the change in velocity divided by the change in time. This is the average rate of change of velocity. By definition, the rate of change of a quantity graphed with respect to time is also the slope of the curve. Thus the (average) slope of an object's velocity vs. time graph is the (average) acceleration of the object.

Use the Smart Tool to read the velocity and time coordinates for two typical points on the velocity vs. time graph. For a more accurate answer, use two points as far apart in time as possible but still during the time the cart was speeding up. Record the points in the space below.

(d) Calculate the change in velocity between points 1 and 2. Also calculate the corresponding change in time (time interval). Divide the change in velocity by the change in time. This is the average acceleration. Show and then summarize your calculations below.

(e) Is the acceleration positive or negative? Is this what you expected?

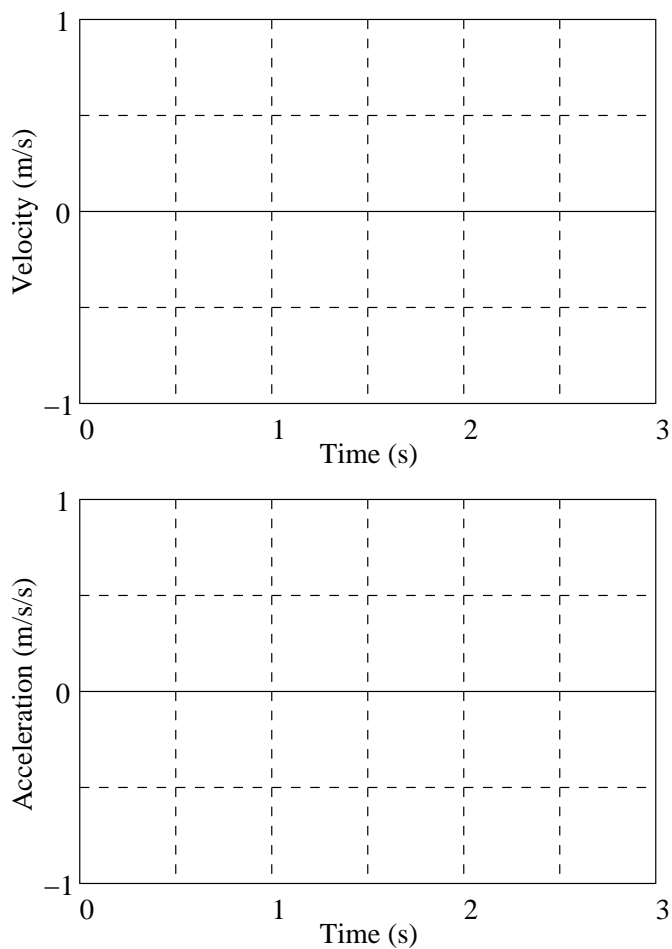
(f) Does the average acceleration you just calculated agree with the average acceleration you calculated from the acceleration vs. time graph? Do you expect them to agree? How would you account for any differences?

### **Speeding Up at a Faster Rate**

Suppose that you accelerate your cart at a faster rate by raising the end of the track several more centimeters. How would your velocity and acceleration graphs change?

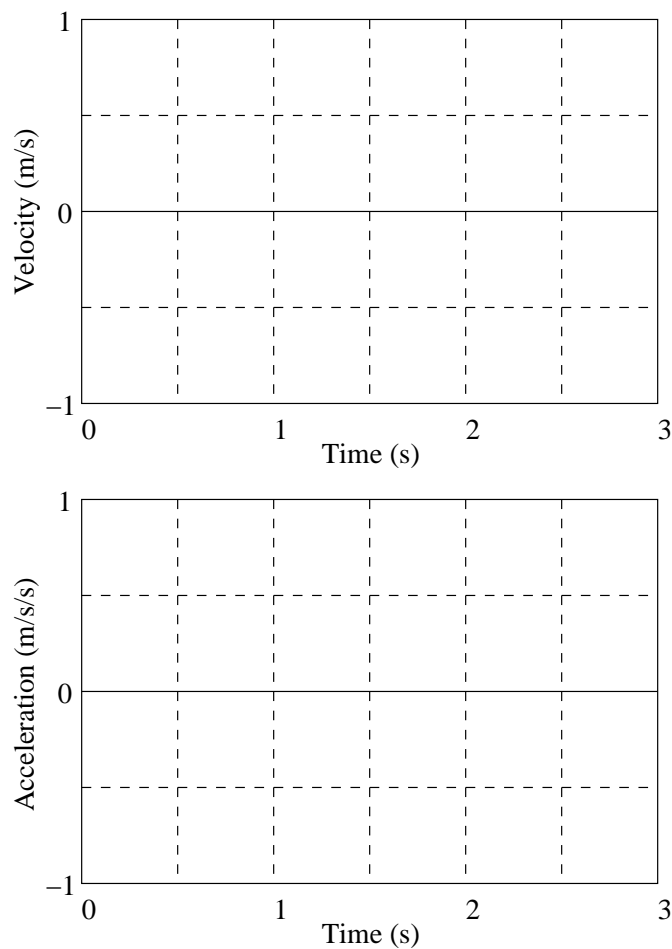
### **Activity 6: Graphs for a Greater Acceleration**

(a) Resketch the velocity and acceleration graphs you found in Activity 3 using the axes that follow.



(b) In the previous set of axes, use a dashed line or another color to sketch your predictions for the general graphs that depict a cart speeding up at a faster rate. Exact predictions are not expected. We just want to know how you think the general shapes of the graphs will change.

(c) Test your predictions by accelerating the cart with the end of the track raised several centimeters more than in Activity 3. Repeat if necessary to get nice graphs and then sketch the results using the axes that follow.

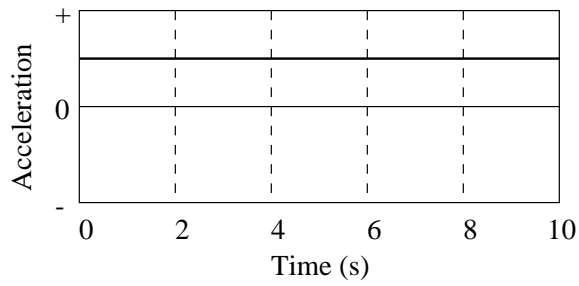


(d) Did the general shapes of your velocity and acceleration graphs agree with your predictions? How is the greater magnitude (size) of acceleration represented on a velocity vs. time graph?

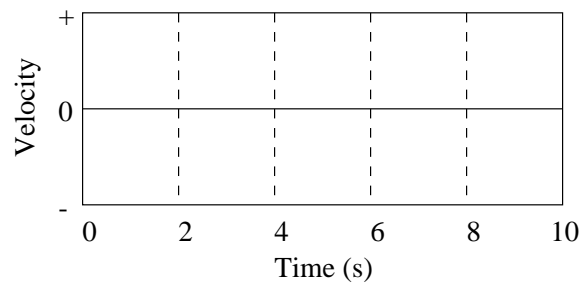
(e) How is the greater magnitude (size) of acceleration represented on an acceleration vs. time graph?

### Homework

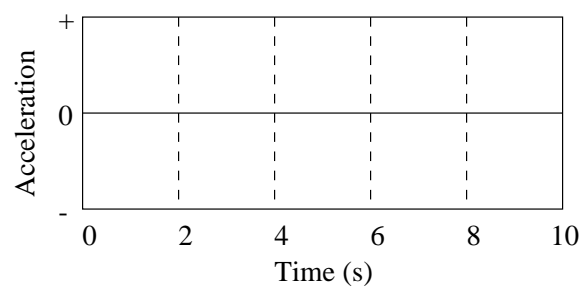
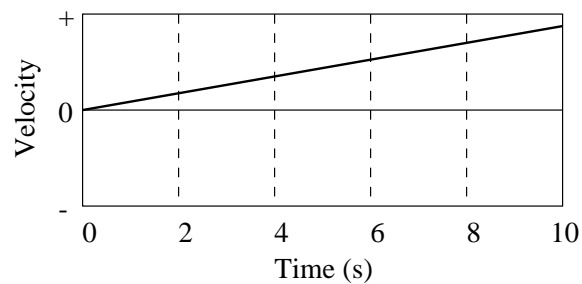
1. An object moving along a line (the + position axis) has the acceleration-time graph shown below. Describe how might the object move to create this graph if it is moving away from the origin?

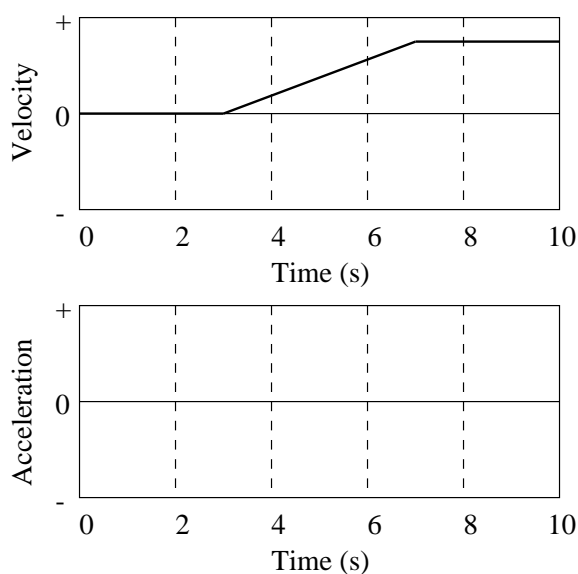


2. Sketch on the axes below a velocity-time graph that goes with the above acceleration-time graph.

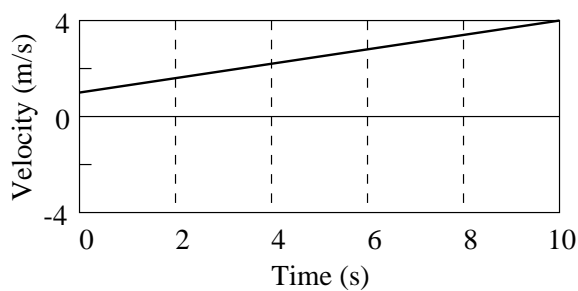


3. For each of the velocity-time graphs below, sketch the shape of the acceleration-time graph that goes with it.



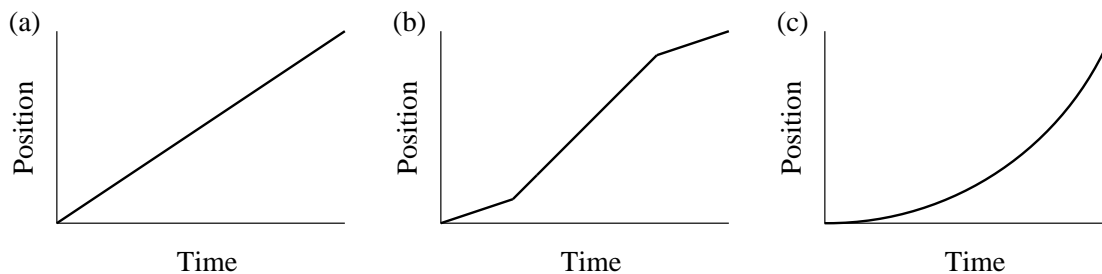


4. The following is a velocity-time graph for a car.



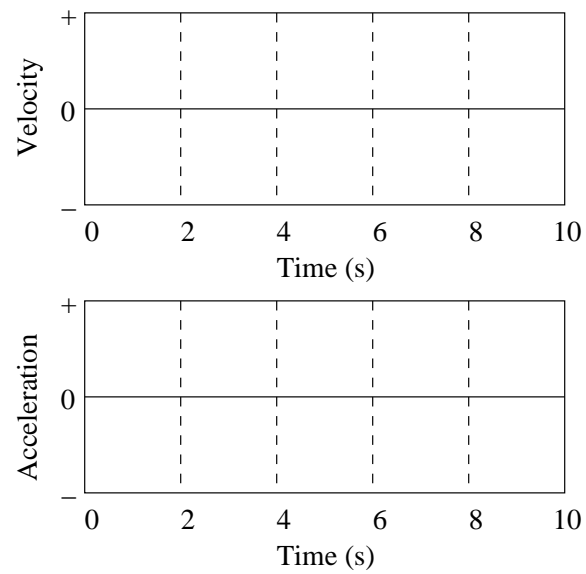
What is the average acceleration of the car? Show your work below.

5. Which position-time graph below could be that for a cart that is steadily accelerating away from the origin?

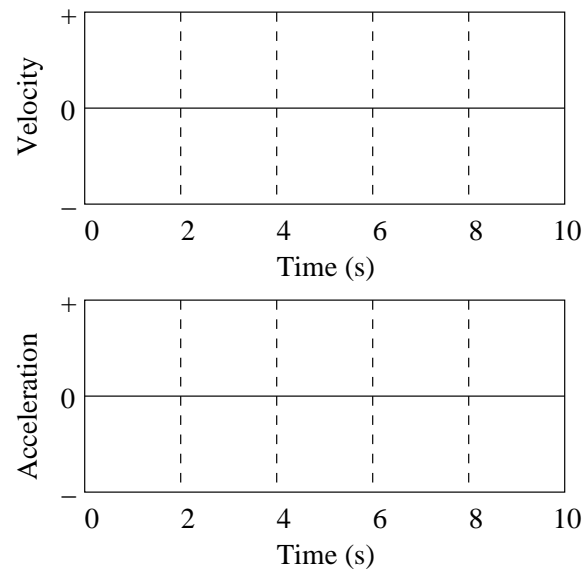


A car can move along a line (the + position axis). Sketch velocity-time and acceleration-time graphs which correspond to each of the following descriptions of the car's motion.

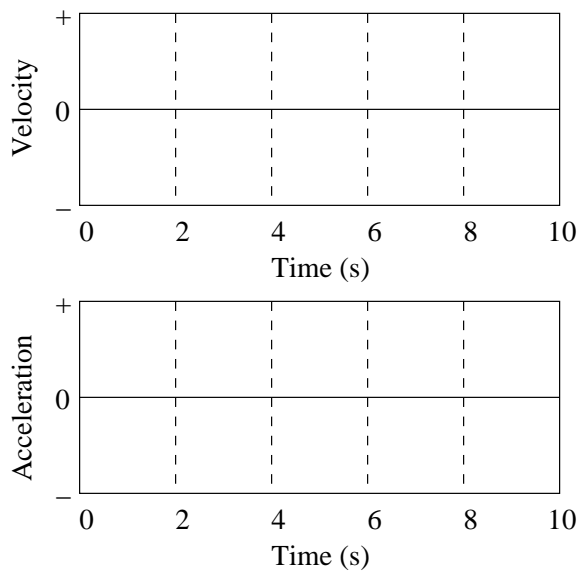
6. The car starts from rest and moves away from the origin increasing its speed at a steady rate.



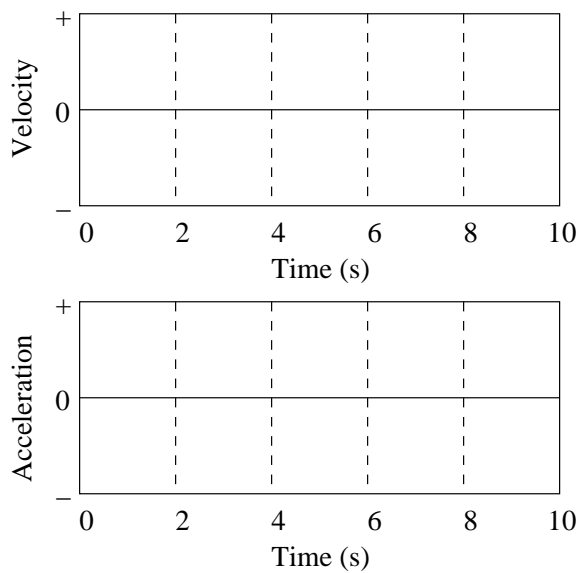
7. The car is moving away from the origin at a constant velocity.



8. The car starts from rest and moves away from the origin increasing its speed at a steady rate twice as large as in (6) above.



9. The car is moving away from the origin at a constant velocity twice as large as in (7) above.



## 9 Slowing Down, Speeding Up, and Turning<sup>7</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

- To learn how to relate graphs of acceleration vs. time to the motions they represent.
- To understand the relationship between velocity vs. time and acceleration vs. time graphs.

### About Slowing Down, Speeding Up and Turning

In the previous session, you explored the characteristics of position vs. time, velocity vs. time and acceleration vs. time graphs of the motion of a dynamics cart. In the cases examined, the cart was always moving away from a motion detector, either at a constant velocity or with a constant acceleration. Under these conditions, the velocity and acceleration are both positive. You also learned how to find the magnitude of the acceleration from velocity vs. time and acceleration vs. time graphs, and how to represent the velocity and acceleration using vectors.

In the motions you studied in the last session, the velocity and acceleration vectors representing the motion of the cart both pointed in the same direction. In order to get a better feeling for acceleration, it will be helpful to examine velocity vs. time and acceleration vs. time graphs for some slightly more complicated motions of a cart on an inclined track. Again you will use the motion detector to observe the cart as it changes its velocity at a constant rate. Only this time the motion may be toward the detector, and the cart may be speeding up or slowing down.

### Apparatus

- *Science Workshop 750 Interface*
- Ultrasonic motion detector
- *DataStudio* software (P, V & A Graphs application)
- Dynamics cart and track
- Lab stand to incline the track

### Slowing Down and Speeding Up

In this activity you will look at a cart moving along an inclined track and slowing down. A car being brought to rest by the steady action of brakes is a good example of this type of motion. Later you will examine the motion of the cart toward the motion detector and speeding up. In both cases, we are interested in the shapes of the velocity vs. time and acceleration vs. time graphs, as well as the vectors representing velocity and acceleration.

Let's start with the creation of velocity and acceleration graphs of when it is moving away from the motion detector and slowing down. To do this activity, the track should be inclined with a lab stand at one end and the motion detector set up at the lower end of the track. Adjust the lab stand so that the track is raised a few centimeters at the opposite end from where the motion detector is located. Now when you give the cart a push away from the motion detector, it will slow down after it is released. In this activity you will examine the velocity and acceleration of this motion.

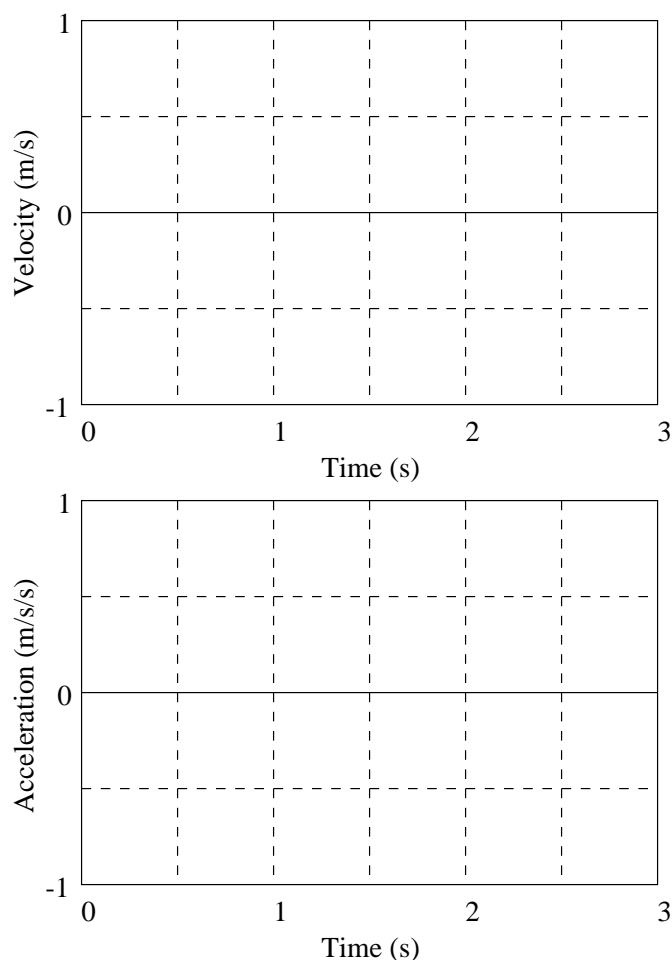
### Activity 1: Graphs Depicting Slowing Down

(a) If you give the cart a push away from the motion detector and release it, will the acceleration be positive, negative or zero (after it is released)? Sketch your predictions for the velocity vs. time and acceleration vs. time graphs on the axes below using dashed lines.

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<sup>7</sup>1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.





(b) To test your predictions, open the **P, V & A Graphs** application as you did in the previous experiment. Locate the cart 0.15 m from the motion detector and gently push the cart away from the motion detector once it starts clicking. Catch the cart before it turns around or hits the end of the track.

Draw the results on the axes above using solid lines for the part of the motion after the cart is released. You may have to try a few times to get a good run. The acceleration vs. time graphs will exhibit small fluctuations due to irregularities in the motion of the cart. You should ignore these fluctuations and draw smooth patterns.

(c) Did the shapes of your velocity and acceleration graphs agree with your predictions? How is the sign of the acceleration represented on a velocity vs. time graph?

(d) How is the sign of the acceleration represented on an acceleration vs. time graph?

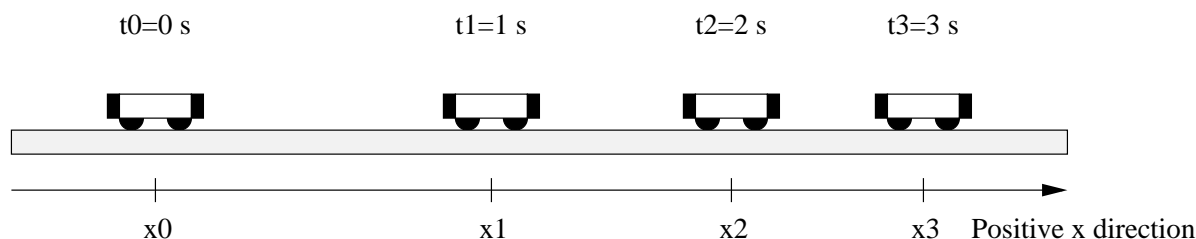
(e) Is the sign of the acceleration what you predicted? How does slowing down while moving away from the detector result in this sign of acceleration? Hint: Remember that acceleration is the rate of change of velocity. Look at how the velocity is changing.

### Constructing Acceleration Vectors for Slowing Down

Let's consider a diagrammatic representation of a cart which is slowing down and use vector techniques to figure out the direction of the acceleration.

#### Activity 2: Vector Diagrams for Slowing Down

(a) The diagram that follows shows the positions of the cart at equal time intervals. (This is like taking snapshots of the cart at equal time intervals.) At each indicated time, sketch a vector above the cart which might represent the velocity of the cart at that time while it is moving away from the motion detector and slowing down.



(b) Show below how you would find the vector representing the change in velocity between the times 1 s and 2 s in the diagram above. Based on the direction of this vector and the direction of the positive x-axis, what is the sign of the acceleration? Does this agree with your answer to Question (e) in Activity 1?

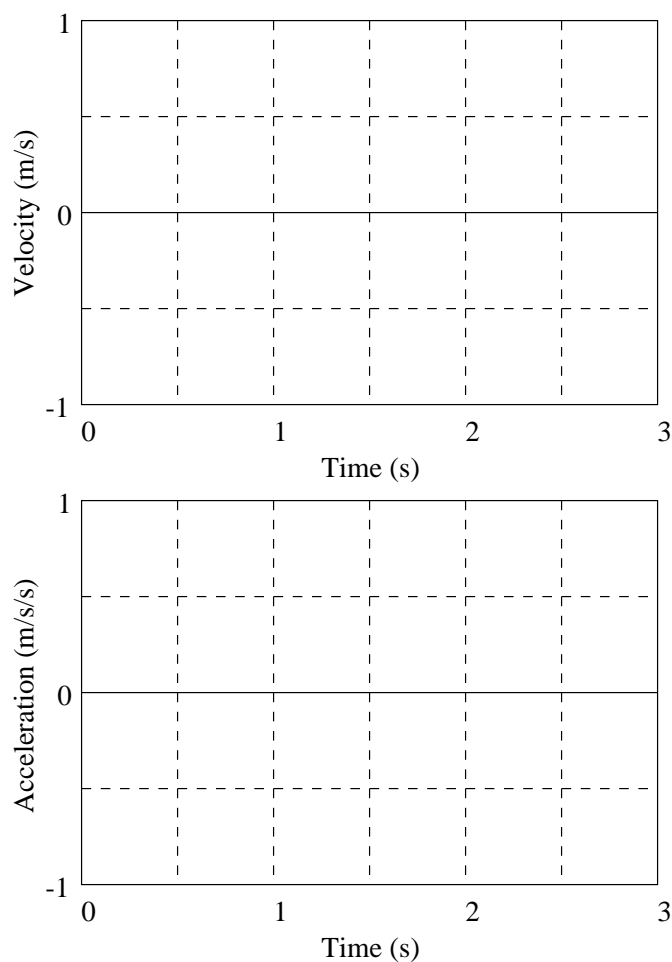
(c) Based on your observations in this activity and in the last session, state the general rules to predict the sign and direction of the acceleration if you know the sign of the velocity (i.e., the direction of motion) and whether the object is speeding up or slowing down.

### Speeding Up Toward the Motion Detector

Let's investigate another common situation. Suppose the cart is allowed to speed up when traveling toward the motion detector. What will be the direction of the acceleration? Positive or negative?

#### Activity 3: Graphs Depicting Speeding Up

(a) Use the general rules that you stated in Activity 2 to predict the shapes of the velocity and acceleration graphs. Sketch your predictions using dashed lines on the axes that follow.



(b) Test your predictions by releasing the cart from rest at the raised end of the track after the motion detector starts clicking. Catch the cart before it gets too close to the detector.

Draw the results using solid lines on the axes above. You may have to try a few times to get a good run.

(c) How does your velocity graph show that the cart was moving toward the detector?

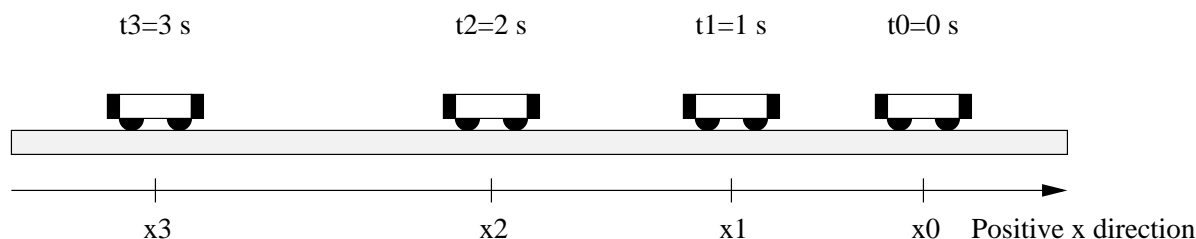
(d) During the time that the cart was speeding up, is the acceleration positive or negative? Does this agree with your prediction? Explain how speeding up while moving toward the detector results in this sign of acceleration. Hint: Think about how the velocity is changing.

### Constructing Acceleration Vectors for Speeding Up

Let's consider a diagrammatic representation of a cart which is speeding up and use vector techniques to figure out the direction of the acceleration.

**Activity 4: Vector Diagrams for Speeding Up**

(a) The diagram that follows shows the positions of the cart at equal time intervals. (This is like taking snapshots of the cart at equal time intervals.) At each indicated time, sketch a vector above the cart which might represent the velocity of the cart at that time while it is moving toward the motion detector and speeding up.



(b) Show below how you would find the vector representing the change in velocity between the times 1s and 2 s in the diagram above. Based on the direction of this vector and the direction of the positive x-axis, what is the sign of the acceleration? Does this agree with your answer to Question (d) in Activity 3?

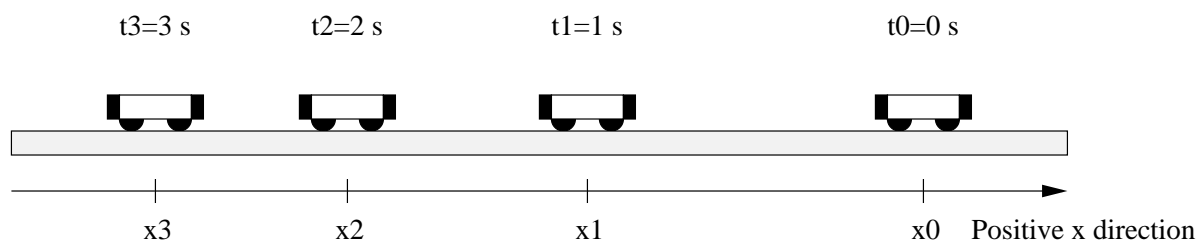
**Moving Toward the Detector and Slowing Down**

There is one more possible combination of velocity and acceleration for the cart, that of moving toward the detector while slowing down.

**Activity 5: Slowing Down Toward the Detector**

(a) Use your general rules to predict the direction and sign of the acceleration when the cart is slowing down as it moves toward the detector. Explain why the acceleration should have this direction and this sign in terms of the velocity and how the velocity is changing.

(b) The diagram below shows the positions of the cart at equal time intervals for slowing down while moving toward the detector. At each indicated time, sketch a vector above the cart which might represent the velocity of the cart at that time while it is moving toward the motion detector and slowing down.



(c) Show below how you would find the vector representing the change in velocity between the times 1s and 2 s in the diagram above. Based on the direction of this vector and the direction of the positive x-axis, what is the sign of the acceleration? Does this agree with the prediction you made in part (a)?

### Acceleration and Turning Around

In the last session and in the first activity in this session, you looked at velocity vs. time and acceleration vs. time graphs for a cart moving in one direction with a changing velocity. In this investigation you will look at what happens when the cart slows down, turns around and then speeds up.

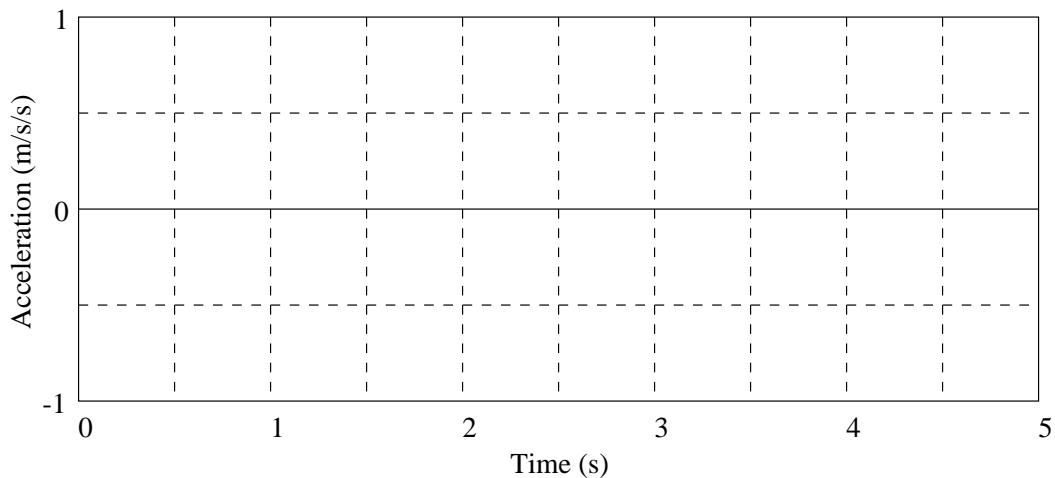
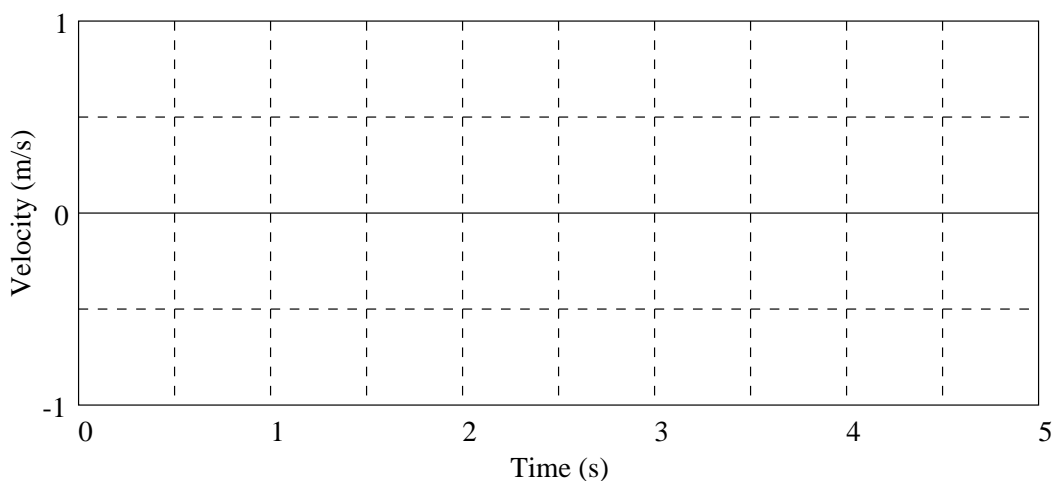
To practice this motion you should position the cart 0.15 m from the detector and give the cart a gentle push away from the motion detector. It should move up the track, slow down, reverse direction and then move back down toward the detector. Try it without activating the motion detector! Be sure that the cart does not hit the end of the track before it turns around.

### Activity 6: Reversing Direction

(a) For each part of the motion-away from the detector, at the turning point, and toward the detector, predict in the table that follows whether the velocity will be positive, zero or negative. Also indicate whether the acceleration is positive, zero or negative.

	Moving Away	Turning Around	Moving Toward
Velocity			
Acceleration			

(b) Sketch the predicted shapes of the velocity vs. time and acceleration vs. time graphs of this entire motion on the axes that follow using dashed lines.



(c) Test your predictions by making graphs of the motion. Use the procedures you used in the slowing down and speeding up activities. You may have to try a few times to get a good run. When you get a good run, sketch both graphs on the axes above using solid lines.

(d) Did the cart have a zero velocity at any point in the motion? (Hint: Look at the velocity graph. What was the velocity of the cart at the end of the ramp?) Does this agree with your prediction? How much time did it spend at zero velocity before it started back toward the detector?

(e) According to your acceleration graph, what is the acceleration at the instant the cart comes to rest? Is it positive, negative or zero? Does this agree with your prediction?

(f) Explain the observed sign of the acceleration at the end of the ramp. (Hint: Remember that acceleration is the rate of change of velocity.)

(g) Print a copy of the velocity and acceleration graphs for each person in your group and add the graphs to this unit.

(h) Notice that the slope of the velocity graph is not quite the same for positive velocities as it is for negative velocities. (This difference can also be seen on the acceleration graph.) What accounts for this difference?

### Tossing a Ball

Suppose you throw a ball up into the air. It moves upward, reaches its highest point and then moves back down toward your hand. We will now consider what can be said about the directions of its velocity and acceleration vectors at various points.

#### Activity 7: The Rise and Fall of a Ball

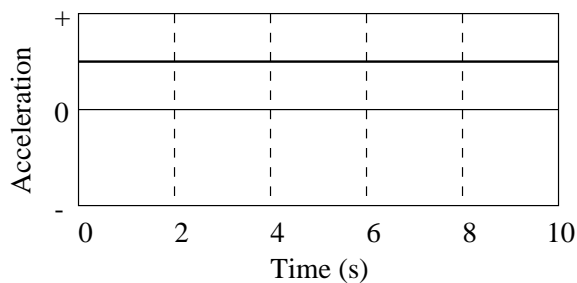
(a) Consider the ball toss carefully. Assume that upward is the positive direction. Indicate in the table that follows whether the velocity is positive, zero or negative during each of the three parts of the motion. Also indicate if the acceleration is positive, zero or negative. Hint: Remember, to find the acceleration you must look at the change in velocity.

	Moving Up (After Release)	At Highest Point	Moving Down
Velocity			
Acceleration			

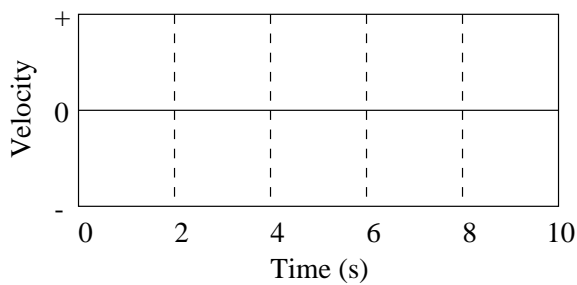
(b) In what ways is the motion of the ball similar to the motion of the cart which you just observed?

**Homework**

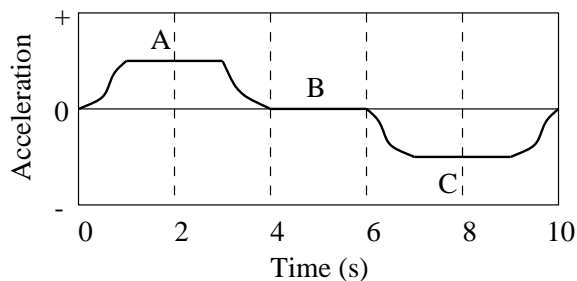
1. An object moving along a line (the + position axis) has the acceleration-time graph below. How might the object move to create this graph if it is moving toward the origin?



2. Sketch on the axes below the velocity-time graph that goes with the above acceleration-time graph.



3. How would an object move to create each of the three labeled parts of the acceleration-time graph shown below?

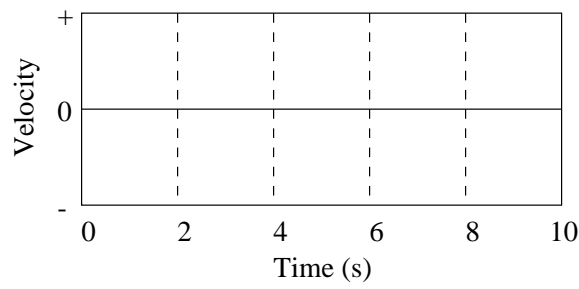


A:

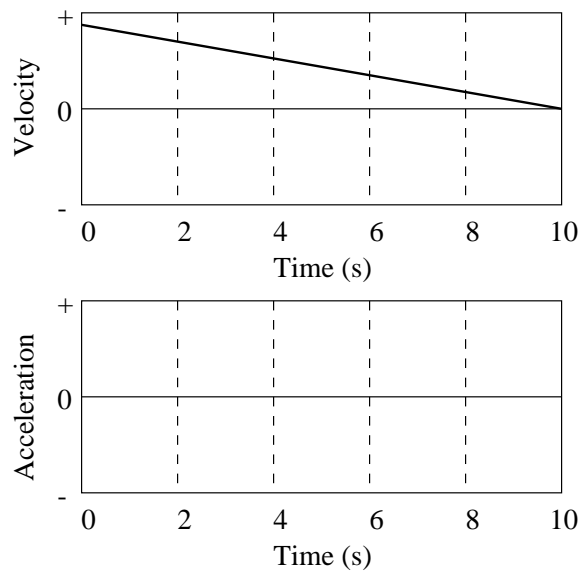
B:

C:

4. Sketch below a velocity-time graph which might go with the acceleration-time graph in question (3).



5. Sketch the shape of the acceleration-time graph that goes with the velocity-time graph shown below.

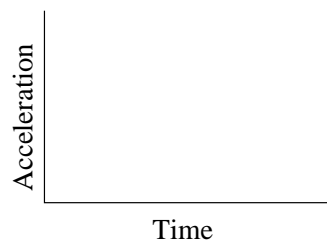
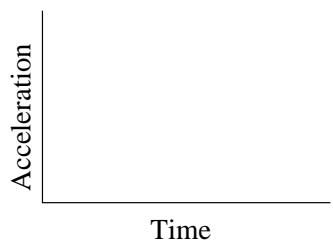
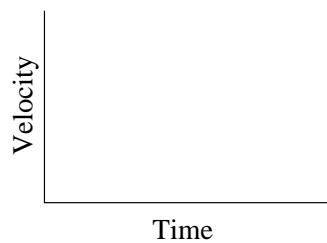
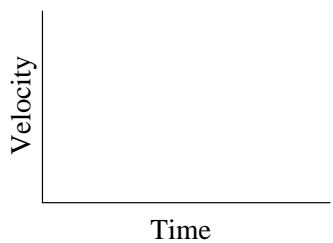
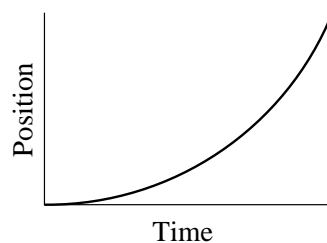
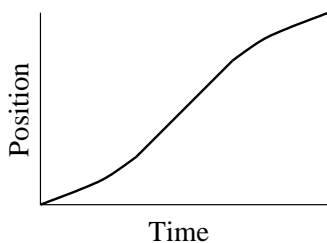
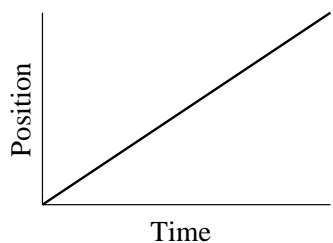


6. A car moves along a line [the + position axis]. Fill in the table below with the sign (+ or -) of the velocity and acceleration of the car for each of the motions described.

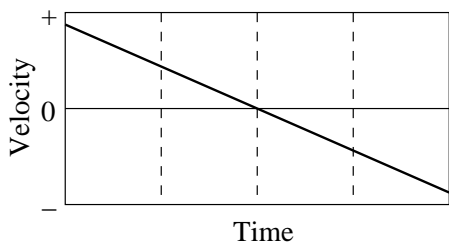
	Position	Velocity	Acceleration Speeding Up	Acceleration Slowing Down
Car Moves Away from the Origin	+			
Car Moves Toward the Origin	+			



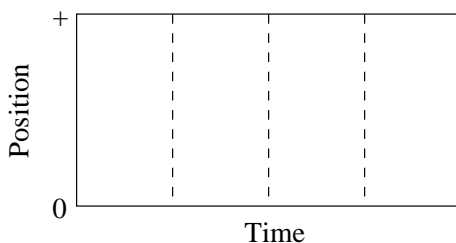
7. For each of the position-time graphs shown, sketch below it the corresponding velocity-time and acceleration-time graphs.



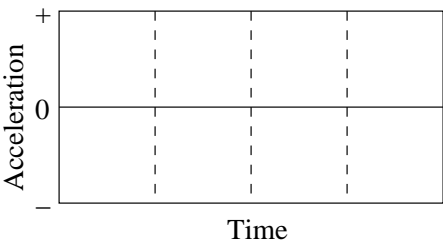
8. Describe how you would move to produce the velocity-time graph shown below.



9. Sketch a position-time graph corresponding to the velocity-time graph above.

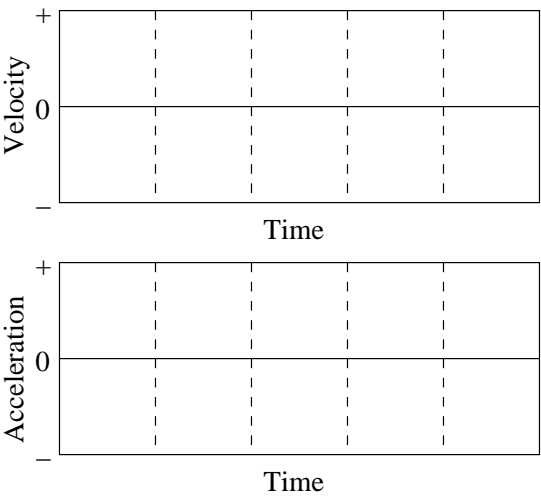


10. Sketch an acceleration-time graph corresponding to the velocity-time graph above.

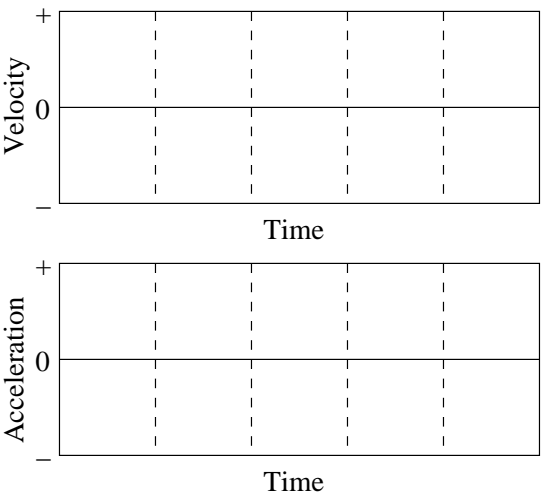


A car can move in either direction along a line (the + position axis). Sketch velocity-time and acceleration-time graphs that correspond to each of the following descriptions of the car’s motion.

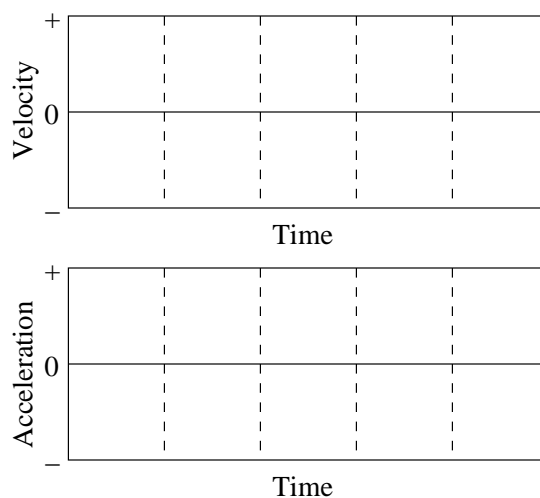
11. The car is moving toward the origin at a constant velocity.



12. The car starts from rest and moves toward the origin, speeding up at a steady rate.

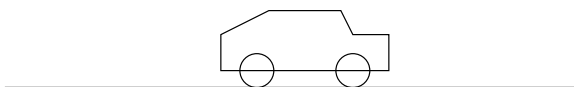


13. A ball is tossed in the air. It moves upward, reaches its highest point and falls back downward. Sketch a velocity-time and an acceleration-time graph for the ball from the moment it leaves the thrower's hand until the moment just before it reaches her hand again. Consider the positive direction to be upward.

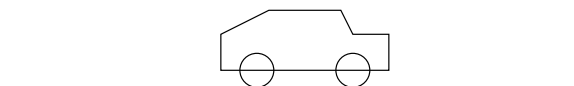


14. Each of the pictures below represents a car driving down a road. The motion of the car is described. In each case, draw velocity and acceleration vectors above the car which might represent the described motion. Also specify the sign of the velocity and the sign of the acceleration. (The positive direction is toward the right.)

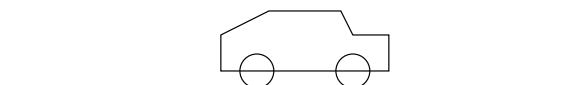
(a) The driver has stepped on the accelerator and the car is just starting to move forward.



(b) The car is moving forward. The brakes have been applied. The car is slowing down, but has not yet come to rest.

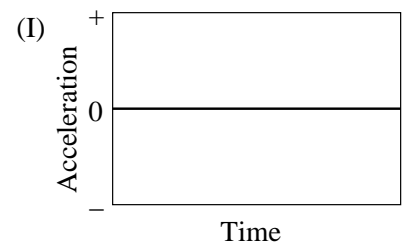
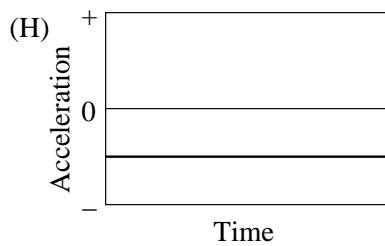
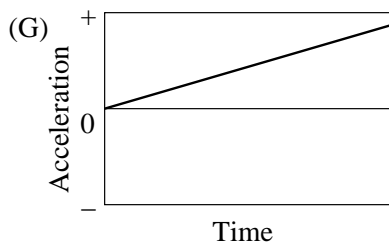
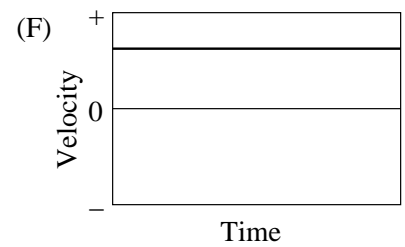
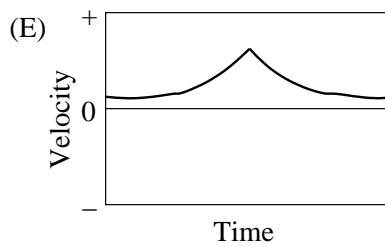
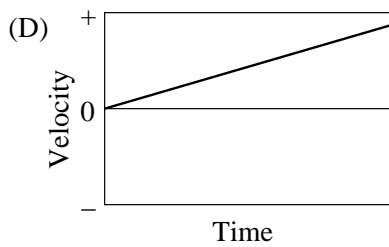
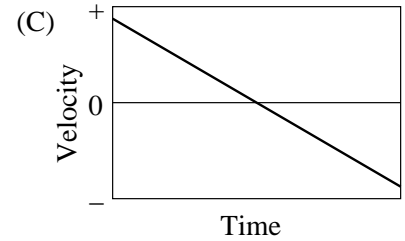
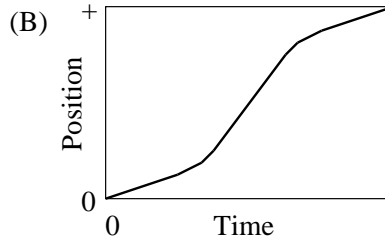
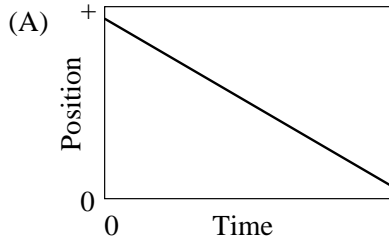


(c) The car is moving backward. The brakes have been applied. The car is slowing down, but has not yet come to rest.



The following graphs represent the motions of objects along the positive position axis. Notice that the motion of the objects is represented by position, velocity, or acceleration graphs.

Answer the following questions. You may use a graph more than once or not at all, and there may be more correct choices than blanks. If none of the graphs is correct, answer none.



15. Pick one graph that gives enough information to indicate that the velocity is always negative. \_\_\_\_\_

Pick three graphs that represent the motion of an object whose velocity is constant. \_\_\_\_\_

16. \_\_\_\_\_ 17. \_\_\_\_\_ 18. \_\_\_\_\_

19. Pick one graph that definitely indicates an object has reversed direction. \_\_\_\_\_

20. Pick one graph that might possibly be that of an object standing still. \_\_\_\_\_

Pick 3 graphs that represent the motion of objects whose acceleration is changing. \_\_\_\_\_

21. \_\_\_\_\_ 22. \_\_\_\_\_ 23. \_\_\_\_\_

Pick a velocity graph and an acceleration graph that could describe the motion of the same object during the time shown. \_\_\_\_\_

24. Velocity graph. \_\_\_\_\_ 25. Acceleration graph. \_\_\_\_\_

## 10 Gravity and Free Fall<sup>8</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

To explore the phenomenon of gravity and study the nature of motion along a vertical line near the earth's surface.

### Overview

When an object falls close to the surface of the earth, there is no obvious force being applied to it. Whatever is causing it to move is invisible. Most people casually refer to the cause of falling motions as the action of "gravity." What is gravity? Can we describe its effects mathematically? Can Newton's laws be interpreted in such a way that they can be used for the mathematical prediction of motions that are influenced by gravity? In this investigation we will study the phenomenon of gravity for vertical motion. You will need:

### Apparatus

- A tennis ball.
- A movie scaling ruler.
- A video analysis system (*VideoPoint*).
- Graphing software (*Excel*).

### Vertical Motion: Describing How Objects Rise and Fall

Let's begin the study of the phenomenon of gravity by predicting the nature of the motion of an object, such as a tennis ball, when it is tossed up and then allowed to fall vertically near the surface of the earth. This is not easy since motion happens pretty fast! To help you with this prediction you should toss a ball in the laboratory several times and see what you think is going on.

#### Activity 1: Predicting the Motion of a Tossed Ball

(a) Toss a ball straight up a couple of times and then describe how you think it might be moving when it is moving upward. Some possibilities include: (1) rising at a constant velocity; (2) rising with an increasing acceleration; (3) rising with a decreasing acceleration; or (4) rising at a constant acceleration. What do you think? (You may want to review Activity 7 of Experiment 9.)

(b) Explain the basis for your prediction.

(c) Now describe how you think the ball might be moving when it is moving downward. Some possibilities include: (1) falling at a constant velocity; (2) falling with an increasing acceleration; (3) falling with a decreasing acceleration; or (4) falling at a constant acceleration. What do you think?

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(d) Each group in the laboratory will measure the acceleration of a ball. Should these different measurements be the same? Why or why not?

(d) Explain the basis for your prediction.

(e) Do you expect the acceleration when the ball is rising to be different in some way from the acceleration when the ball is falling? Why or why not?

(f) What do you think that the acceleration will be at the moment when the ball is at its highest point? Why?

The motion of a tossed ball is too fast to observe carefully by eye without the aid of special instruments. In the next activity we will use a video analysis system to study the motion of a freely falling object. You will use a sequence of these video frames and mathematical modeling techniques to find an equation that describes the fall.

### Activity 2: Analyzing the Motion of a Tennis Ball

(a) Make a movie of a tennis ball in flight by following these steps.

1. Turn on the video camera and center the field of view on the region where you will toss the ball. This region should be about 2 meters from the camera to get a large enough area for the flight of the ball. Place a ruler or meter stick somewhere in the field of view where it won't interfere with the motion. This ruler will be used later to determine the scale.
2. Make a movie of the tennis ball being dropped from rest. Make sure most of the trajectory is visible to the camera. See **Appendix D: Video Analysis** for details on making the movie. **NOTE:** The procedure described in the first part of **Appendix D (Making a Movie)** must be followed step by step in order to get good results.

**IMPORTANT:** Do NOT save the movie to your netfiles space. Save it to the DESKTOP as indicated in **Appendix D** number 2. (Before logging out later you can save the movie file to your own space on Saturn.)

(b) Determine the vertical position,  $y$ , of the tennis ball at different times during the motion. To do this follow the instructions in the second section of **Appendix D: Video Analysis - Analyzing the Movie**.

(c) Use *Excel* to plot a graph of  $y$  vs.  $t$ . See **Appendix C: Introduction to Excel** for details.

(d) What is the initial value of  $y$  (usually denoted  $y_0$ ) (from your data table)?

(e) By examining your data table, calculate the approximate value of the initial velocity of the ball in the  $y$ -direction (from the first two lines of data). Include the sign of the velocity and its units. (Use the convention that on the  $y$ -axis up is positive and down is negative.)

(f) Examine the graph of your data. What does the nature of this motion look like? Constant velocity, constant acceleration, an increasing or decreasing acceleration? How does your observation compare with the prediction you made earlier in this unit for the ball on its way down?

(g) Using the convention that on the  $y$ -axis up is positive and down is negative, is the acceleration positive or negative (i.e., in what direction is the magnitude of the velocity increasing)?

(h) If you think the object is undergoing a constant acceleration, use the fitting capability of *Excel* (see **Appendix C: Introduction to Excel** for details) to find an equation that describes  $y$  as a function of  $t$  as the ball drops. Hints: (1) You might try to model the system with a second order equation like the kinematic equation for uniformly accelerated motion. (2) Write the equation of motion in the space below. Then use coefficients of the best-fit equation to find the values of  $a$ ,  $v_0$  and  $y_0$  with the appropriate units. **Note: The numbers from your graph should be rounded off to no more than 3 significant figures.** Also, since the acceleration is caused by gravity, our notation for it will be  $g$  rather than just  $a$ . When you have found a good fit to the data, print it and attach a copy to this unit.

1. The equation of motion with proper units is:  $y =$
2. The acceleration with proper sign and units is:  $g =$
3. The initial velocity with proper sign and units is:  $v_0 =$
4. The initial position with proper sign and units is:  $y_0 =$
5. The initial position and velocity of the ball depend on the details of your throwing motion. The acceleration does not. It depends only on the motion of ball after you have released it. Go around to the other groups in the lab and ask them for the value of the acceleration they obtained. Make a histogram of your results and calculate the average and standard deviation of the acceleration for the whole class. For information on making histograms, see **Appendix C**. For information on calculating the average and standard deviation, see **Appendix A**. Record the average and standard deviation here. Attach the histogram to this unit.

7. What does the histogram of the class data tell you? Be quantitative in your answer.



## 11 Projectile Motion<sup>9</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

To understand the experimental and theoretical basis for describing projectile motion as the superposition of two independent motions: (1) a body falling in the vertical direction, and (2) a body moving in the horizontal direction with no forces.

### Apparatus

- A tennis ball.
- A movie scaling ruler.
- A video analysis system (*VideoPoint*).
- Graphing and curve fitting software (*Excel*).

### Activity 1: Predicting the Two-Dimensional Motion of a Tossed Ball

(a) Toss a tennis ball up at an angle of about  $60^\circ$  with the horizon a couple of times. Sketch the motion and describe it in words below. What is the shape of the trajectory?

(b) Let's consider the horizontal and vertical components of the motion separately. What do you think is the horizontal motion of the ball? Is it motion with constant velocity? Constant acceleration? Or is it some other kind of motion? Hint: What is the force acting on the ball in the horizontal direction (after it is released)?

(c) What do you think is the vertical motion of the ball? Is it motion with constant velocity? Constant acceleration? Or is it some other kind of motion? Hint: What is the force acting on the ball in the vertical direction (after it is released)?

The two-dimensional motion of a tossed ball is too fast to observe carefully by eye without the aid of special instruments. In the next activity we will use a video analysis system to study the motion of a small ball launched at an angle of about  $60^\circ$  with respect to the horizontal. You are to use the video analysis software and mathematical modeling techniques to find the equations that describe: (a) the trajectory ( $y$  vs.  $x$ ), (b) the horizontal motion ( $x$  vs.  $t$ ), and (c) the vertical motion ( $y$  vs.  $t$ ) of the projectile.

### Activity 2: Analyzing Projectile Motion

(a) Make a movie of a tennis ball in flight by following these steps.

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1. Set up the video camera and center the field of view on the region where you will toss the ball. This region should be about 2 meters from the camera to get a large enough area for the flight of the ball. Place a ruler or meter stick somewhere in the field of view close to the plane of the motion of the ball where it won't interfere with the motion. This ruler will be used later to determine the scale.
2. Make a movie of the tennis ball flying through the air with a significant component of its initial velocity in the horizontal direction (i.e., don't toss it straight up). Make sure most of the complete trajectory is visible to the camera. See **Appendix D: Video Analysis** for details on making the movie.

(b) Determine the position of the projectile at different times during the motion. To do this task follow the instructions in the second section of **Appendix D: Video Analysis** for recording and calibrating the position data. When you are analyzing the movie place the origin at the position of the ball in your first frame. Do this by clicking on the arrow symbol on the menu bar to the left of the movie frame. The cursor will take the shape of an arrow when you place it over the frame. Move the point of the cursor's arrow to the origin. Click and drag the cursor and move the origin to the position of the ball in the first frame and release. This sets the origin at the location of the ball at the initial time. When you have placed the origin at the desired spot, click on the circular icon at the top of the menu bar to the left of the movie frame. This returns the cursor to a circle that marks the position of the ball. When you have finished marking the ball's position, export the data into an *Excel* file as described in **Appendix D**.

(c) Open your data in *Excel*. Launch *Excel*. See **Appendix C: Introduction to Excel** for details on using *Excel*. Make a plot of the vertical position ( $y$ ) versus the horizontal position ( $x$ ). Determine the equation that describes the trajectory of the projectile. When you have found a good fit to the data, print the graph and attach a copy to the unit. Write the equation for the trajectory of the projectile in the space below. What is the shape of the trajectory? Does the result agree with your earlier prediction?

(d) Determine the equation that describes the horizontal motion of the projectile by plotting the horizontal position ( $x$ ) versus time ( $t$ ) using *Excel*. Find a good fit to the data with *Excel*. When you have found a good fit to the data, print the graph and attach a copy to the unit. What kind of motion is it? Does the result agree with your earlier prediction? **Note:** As in the previous experiment, numbers from your graph should be rounded off to no more than 3 significant figures.

1. The equation for the horizontal component of the motion with proper units is:  $x =$
2. The horizontal component of the acceleration with proper sign and units is:  $a_x =$
3. The horizontal component of the initial velocity with proper sign and units is:  $v_{0x} =$
4. The initial  $x$  position with proper units is:  $x_0 =$

(e) Determine the equation that describes the vertical motion of the projectile by plotting the vertical position ( $y$ ) versus time ( $t$ ). Find a good fit to the data. When you have found a good fit to the data, print the graph and attach a copy to the unit. What kind of motion is it? Does the result agree with your earlier prediction?

1. The equation for the vertical component of the motion with proper units is:  $y =$
  2. The vertical component of the acceleration with proper sign and units is:  $a_y =$
  3. The vertical component of the initial velocity with proper sign and units is:  $v_{0y} =$
  4. The initial  $y$  position with proper units is:  $y_0 =$
- (f) Does it appear that projectile motion is simply the superposition of two types of motion that we have already studied? Explain.
- (g) Go around to the other groups in the lab and ask them for their measured values of the horizontal and vertical accelerations. Make a histogram of your results for each component and calculate the average and standard deviation of each one. For information on making histograms, see **Appendix C**. For information on calculating the average and standard deviation, see **Appendix A**. Record the average and standard deviation here. Attach the histogram to the unit.
- (h) What is your expectation for the vertical acceleration of the ball? Is your data consistent with your expectation? Is the acceleration the same for the entire class? Use the average and standard deviation for the class to quantitatively answer these questions.
- (i) What is your expectation for the horizontal acceleration of the ball? Is your data consistent with your expectation? Is this acceleration the same for the entire class? Use the average and standard deviation for the class to quantitatively answer these questions.
- (j) What do the histograms of the class data for each component of the acceleration tell you? Be quantitative in your answer.

## 12 Force and Motion I<sup>10</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

- To learn how to use a force probe to measure force.
- To understand the relationship between forces applied to an object and its motions.
- To find a mathematical relationship between the force applied to an object and its acceleration.

### Overview

In the previous labs, you have used a motion detector to display position-time, velocity-time and acceleration-time graphs of the motion of different objects. You were not concerned about how you got the objects to move, i.e., what forces (pushes or pulls) acted on the objects. From your experiences, you know that force and motion are related in some way. To start your bicycle moving, you must apply a force to the pedal. To start up your car, you must step on the gas pedal to get the engine to apply a force to the road through the tires.

But, exactly how is force related to the quantities you used in the previous unit to describe motion — position, velocity and acceleration? In this unit you will pay attention to forces and how they affect motion. You will apply forces to a cart, and observe the nature of its resulting motion graphically with a motion detector.

### Apparatus

- Force probe
- Variety of hanging masses
- CS2000 compact scale (for measuring mass)
- Low friction pulley and string
- Motion detector
- Dynamics cart and track
- *Science Workshop 750 Interface*
- *DataStudio* software (V, A & F Graphs application)

### Measuring Forces

In this investigation you will use a force probe (also called a force sensor) to measure forces. The force probe puts out a voltage signal proportional to the force applied to the arm of the probe. Physicists have defined a standard unit of force called the newton, abbreviated N. For your work on forces and the motions they cause, it will be more convenient to have the force probe read directly in newtons rather than voltage. Before the force probe is used it must be calibrated. To calibrate the force probe, see *Calibrating Force Sensors* in **Appendix E: Instrumentation**.

### Motion and Force

Now you can use the force probe to apply measured amounts of force to an object. You can also use the motion detector, as in the previous units, to examine the motion of the object. In this way you will be able to establish the relationship between motion and force.

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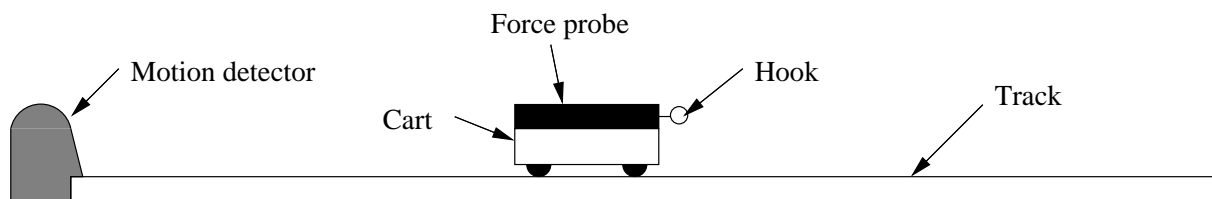


Figure 1: Equipment setup for qualitative measurements of force and motion.

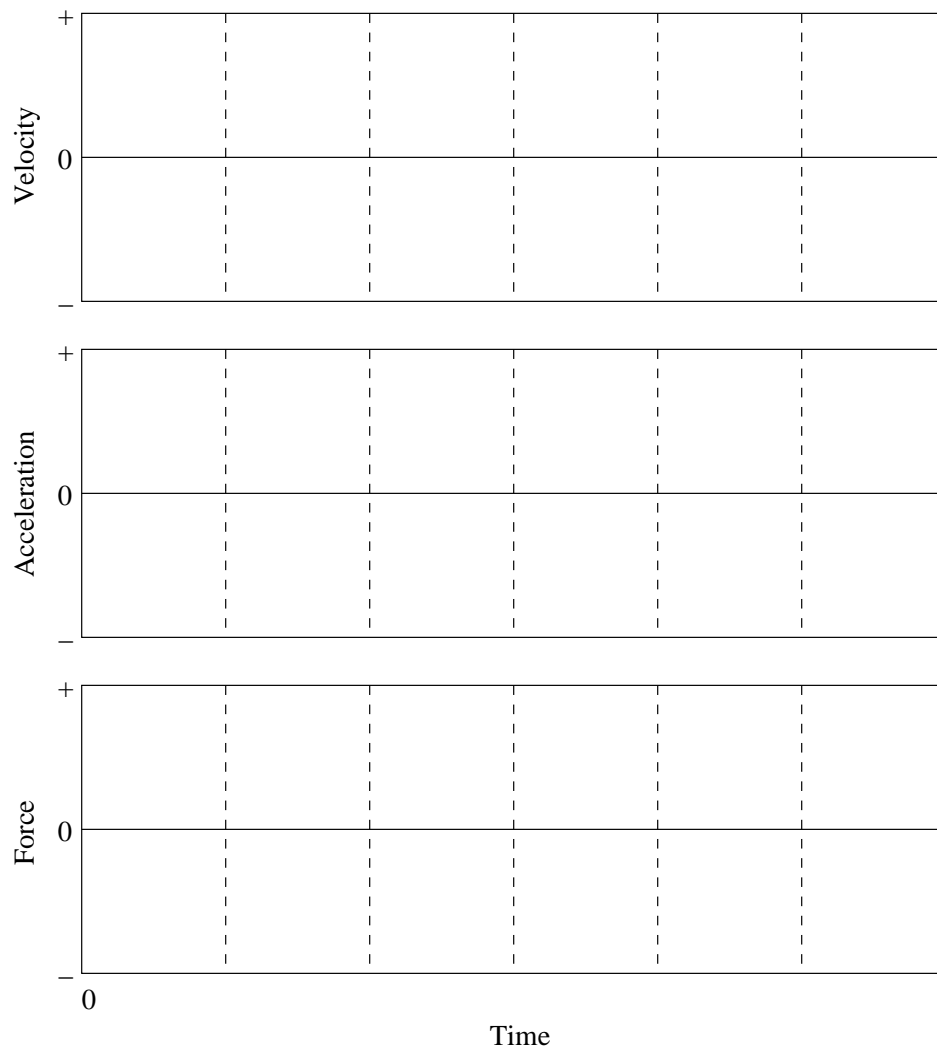
### Activity 1: Pushing and Pulling a Cart

In this activity you will move a cart by pushing and pulling it with your hand. You will measure the force, velocity and acceleration. Then you will be able to look for relationships between the applied force and the motion quantities, to see which is (are) related to force.

(a) Set up the cart, force probe and motion detector on the level track as shown in Figure 1. Measure and record the mass of the cart and force probe assembly (using the compact scale).

(b) Suppose you grasp the hook on the force probe and move the cart forwards and backwards in front of the motion detector. Do you think that either the velocity or the acceleration graph will look like the force graph? Is either of these motion quantities related to force? That is to say, if you apply a changing force to the cart, will the velocity or acceleration change in the same way as the force?

(c) To test your predictions, open the V, A & F Graphs application. Grasp the hook on the force probe and start acquiring data. When you hear the clicks, pull the cart away from the motion detector, and quickly stop it. Then push it back towards the motion detector, and again quickly stop it. Be sure that the cart never gets closer than 0.15 m away from the detector and be careful of the wires. Repeat until you get a good run, and adjust the sampling time and scale of the axes if necessary. Sketch your graphs on the axes that follow.



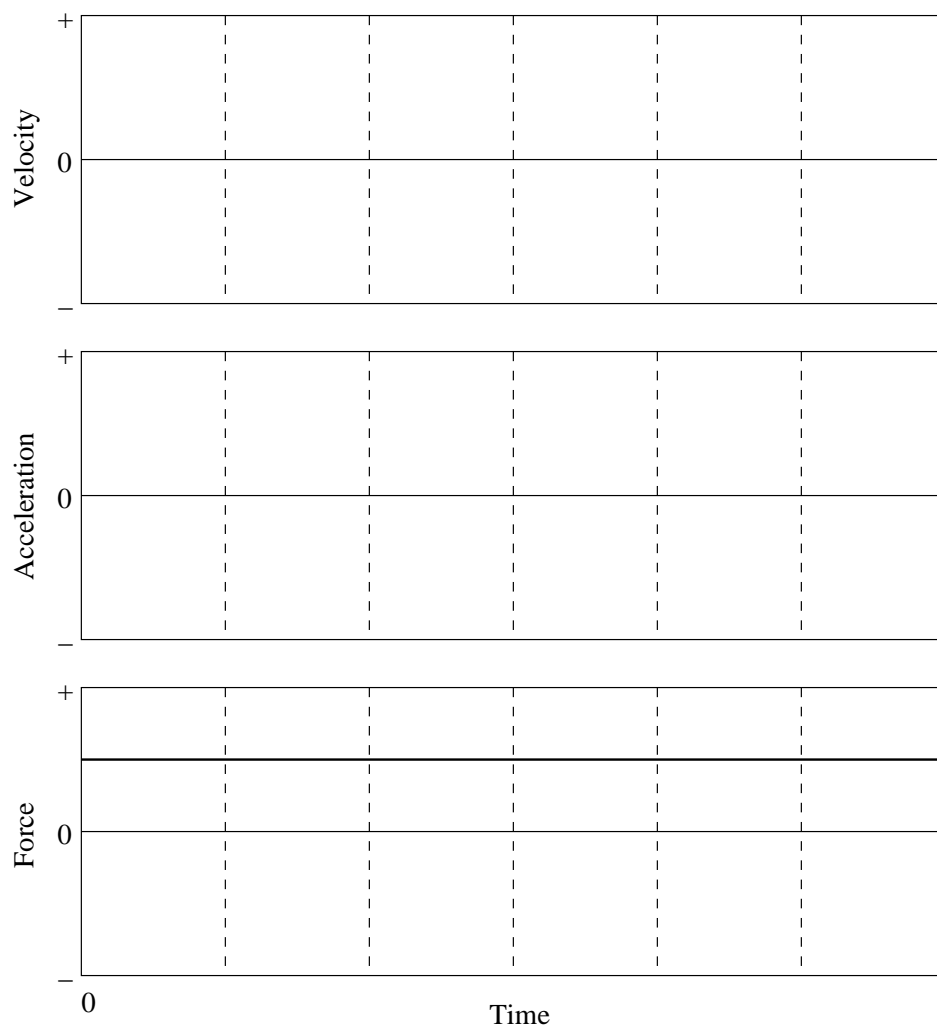
(d) Does either graph—velocity or acceleration—resemble the force graph? Which one? Explain.

(e) Based on your observations, does it appear that either the velocity or acceleration of the cart might be related to the applied force? Explain.

**Activity 2: Speeding Up**

You have seen in the previous activity that force and acceleration seem to be related. But just what is the relationship between force and acceleration?

(a) Suppose you have a cart with very little friction, and that you pull this cart with a constant force as shown below on the force-time graph. Sketch on the axes below the velocity-time and acceleration-time graphs of the cart's motion.



(b) Describe in words the predicted shape of the velocity vs. time and acceleration vs. time graphs for the cart.

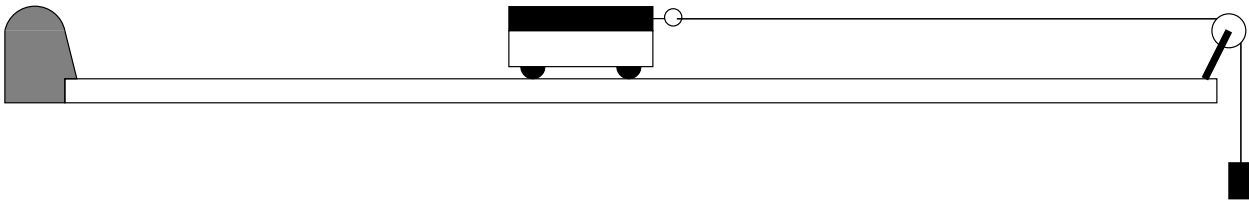
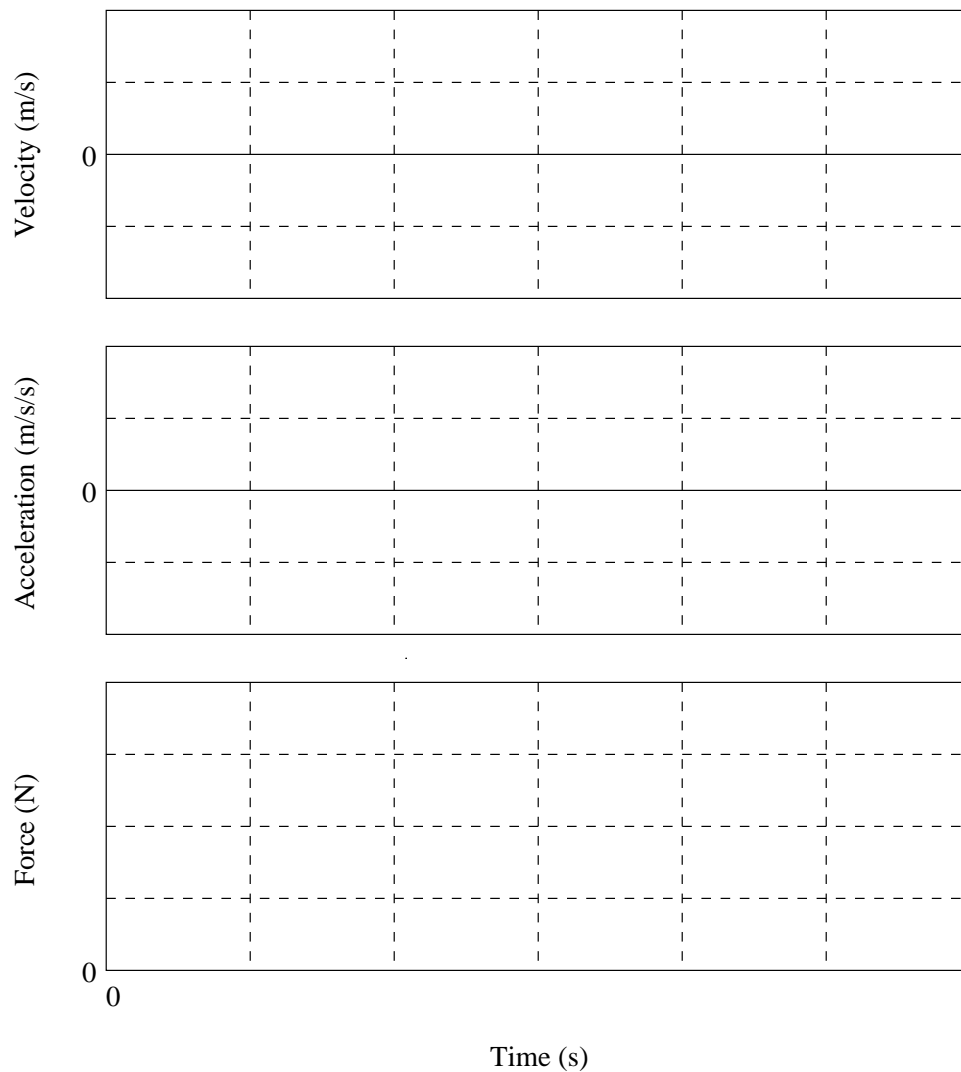


Figure 2: Equipment setup for quantitative measurements of force and motion.

(c) Test your predictions. Set up the pulley, cart, string, motion detector and force probe as shown in Figure 2. The cart should be the same mass as before. Zero the force probe. Hang 50 g from the end of the string. Start the data acquisition. Release the cart when you hear the clicks of the motion detector. Be sure that there is enough slack in the force probe cables to complete the motion and catch the cart before it crashes into the pulley. Repeat until you get good graphs in which the cart is seen by the motion detector over the entire motion. Sketch the actual velocity, acceleration and force graphs for the motion of interest on the axes below and indicate the scale on the axes. Draw smooth graphs; don't worry about small bumps.



(d) Is the force which is applied to the cart by the string constant, increasing or decreasing? Explain based on your graph.



(e) How does the acceleration graph vary in time? Does this agree with your prediction? What kind of acceleration corresponds to a constant applied force?

(f) How does the velocity graph vary in time? Does this agree with your prediction? What kind of velocity corresponds to a constant applied force?

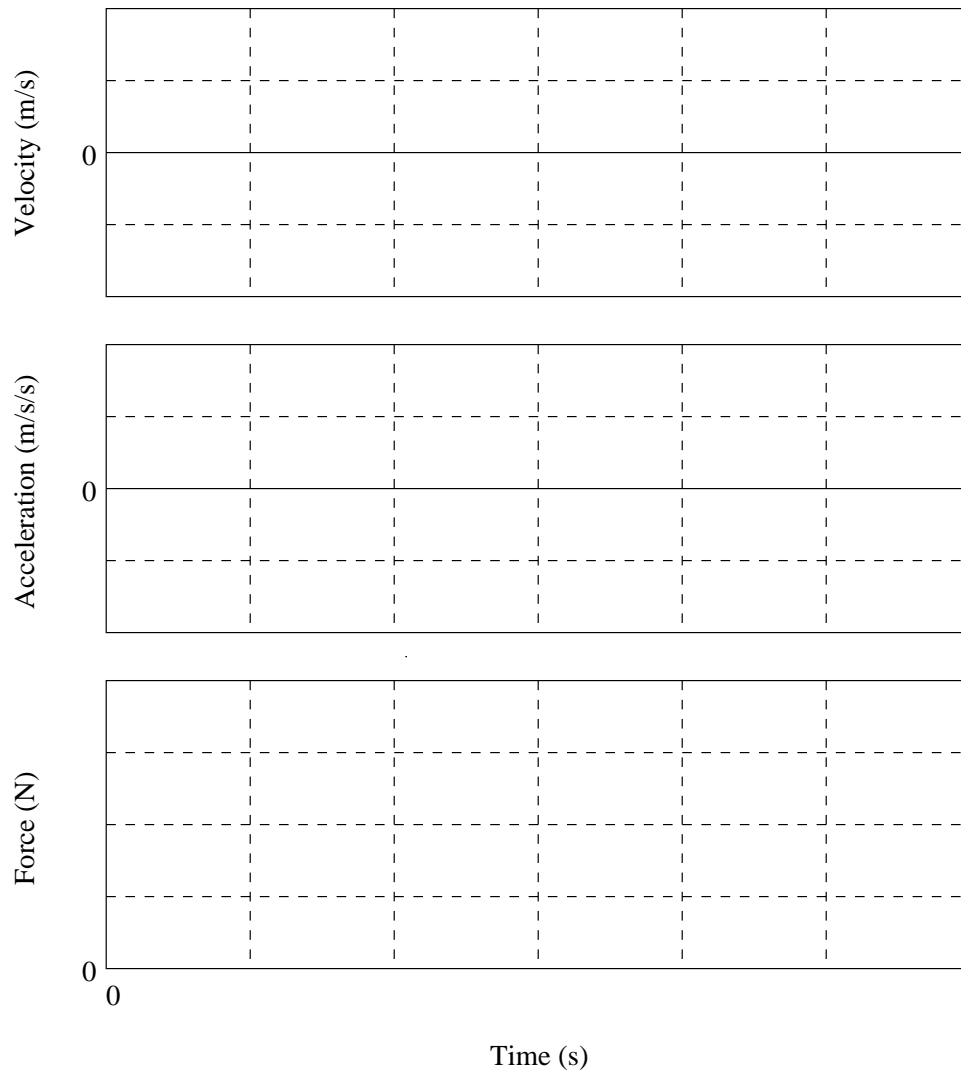
(g) Use the Smart Tool to determine the average force and the average acceleration and record them below. Find the mean values only during the time interval when the force and acceleration are nearly constant. See Appendix B for details on the use of the Smart Tool feature of DataStudio.

### **Activity 3: Acceleration from Different Forces**

In the previous activity you have examined the motion of a cart with a constant force applied to it. But, what is the relationship between acceleration and force? If you apply a larger force to the same cart (same mass as before) how will the acceleration change? In this activity you will try to answer these questions by applying different forces to the cart, and measuring the corresponding accelerations.

(a) Suppose you pulled the cart with a force about twice as large as before. What would happen to the acceleration of the cart? Explain.

(b) Test your prediction by replacing the 50-g mass with a 100-g mass and creating graphs of the motion as before. Repeat until you have a good run. Sketch the results on the axes that follow. Don't forget to put the scale on the axes.



(c) Use the Smart Tool to find the average force and the average acceleration and record them below. Find the mean values only during the time interval when the force and acceleration are nearly constant.

(d) How did the force applied to the cart compare to that with the smaller force in Activity 2?

(e) How did the acceleration of the cart compare to that caused by the smaller force in Activity 2? Did this agree with your prediction? Explain.

**Activity 4: The Relationship Between Acceleration and Force**

If you accelerate the same cart (same mass) with another force, you will then have three data points—enough data to plot a graph of acceleration vs. force. You can then find the mathematical relationship between acceleration and force.

(a) Accelerate the cart with a force roughly midway between the other two forces tried. Use a hanging mass about midway between those used in the last two activities. Record the mass below.

(b) Graph velocity, acceleration and force. Sketch the graphs on the axes in Activity 3 using dashed lines.

(c) Find the mean acceleration and force, as before, and record the values in the table below (in the Activity 4 line). Also, enter the values from the previous two activities in the table.

	Average Force (N)	Average Acceleration ( $\text{m/s}^2$ )
Activity 2		
Activity 4		
Activity 3		

(d) Plot the average force applied to the cart as a function of the average acceleration of the cart by fitting the data with a linear function. Label and print the graph showing the best fit, and add it to this unit.

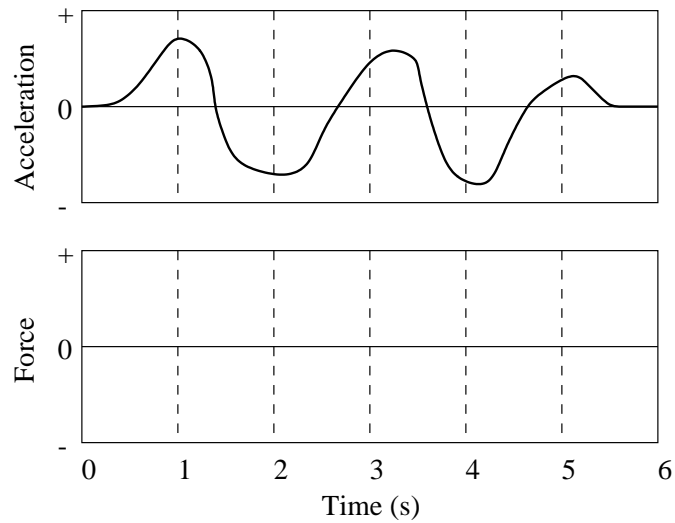
(e) Does there appear to be a simple mathematical relationship between the acceleration of a cart (with fixed mass) and the force applied to the cart (measured by the force probe mounted on the cart)? Write down the equation you found and describe the mathematical relationship in words.

(f) What is the slope of the graph? How does it compare with the mass of the cart and force probe assembly as measured in Activity 1?

Comment: The relationship which you have been examining between the acceleration of the cart and the applied force is known as Newton's Second Law,  $F = ma$ .

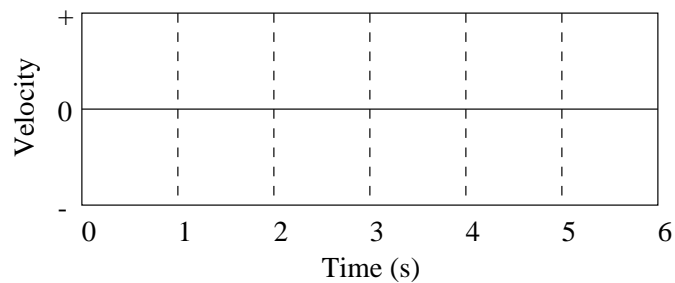
**Homework**

1. A force is applied which makes an object move with the acceleration shown below. Assuming that friction is negligible, sketch a force-time graph of the force on the object on the axes below.

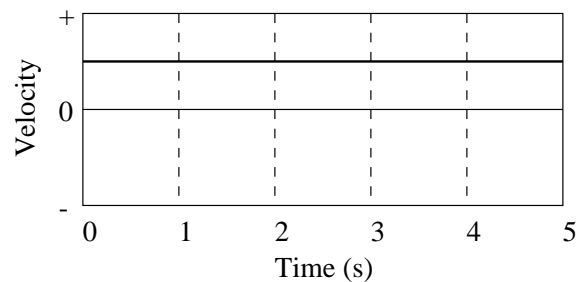


Explain your answer:

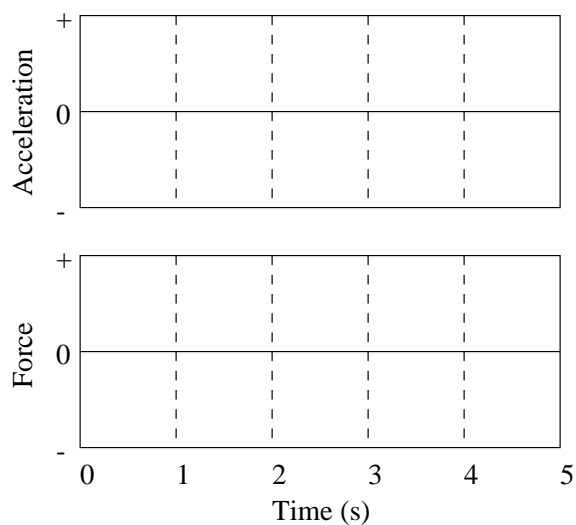
2. Roughly sketch the velocity-time graph for the object in question 1 on the axes below.



3. A cart can move along a horizontal line (the + position axis). It moves with the velocity shown below.



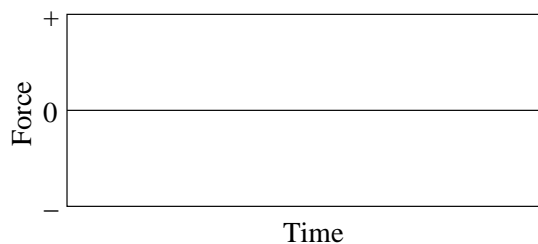
Assuming that friction is so small that it can be neglected, sketch on the axes that follow the acceleration-time and force-time graphs of the cart's motion.



Explain both of your graphs.

Questions 4-6 refer to an object which can move in either direction along a horizontal line (the + position axis). Assume that friction is so small that it can be neglected. Sketch the shape of the graph of the force applied to the object which would produce the motion described.

4. The object moves away from the origin with a constant acceleration.



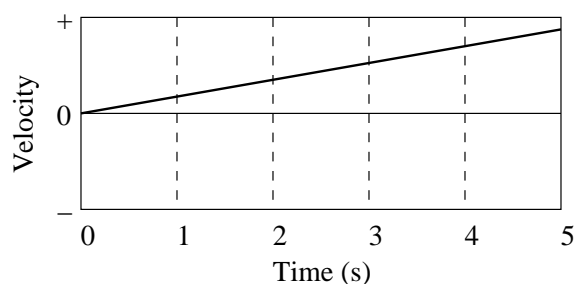
5. The object moves toward the origin with a constant acceleration.



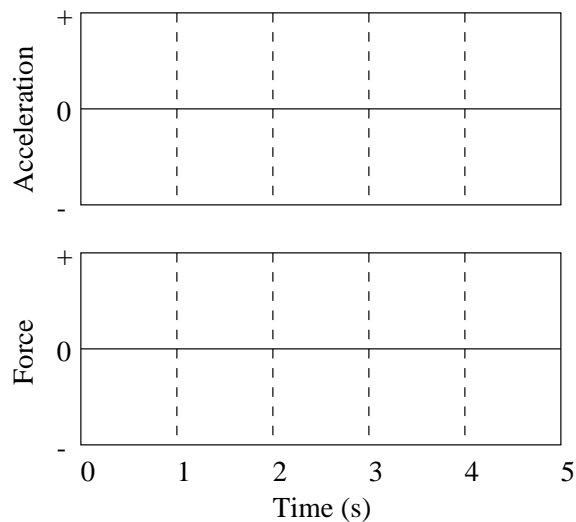
6. The object moves away from the origin with a constant velocity.



Questions 7 and 8 refer to an object which can move along a horizontal line (the + position axis). Assume that friction is so small that it can be ignored. The object's velocity-time graph is shown below.

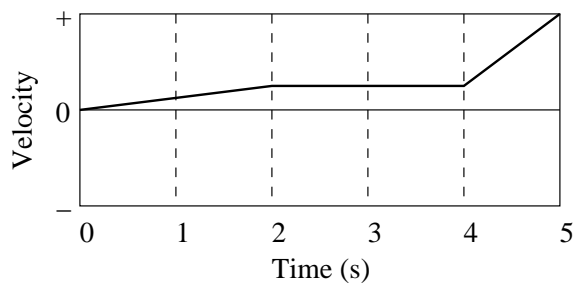


7. Sketch the shapes of the acceleration-time and force-time graphs on the axes below.

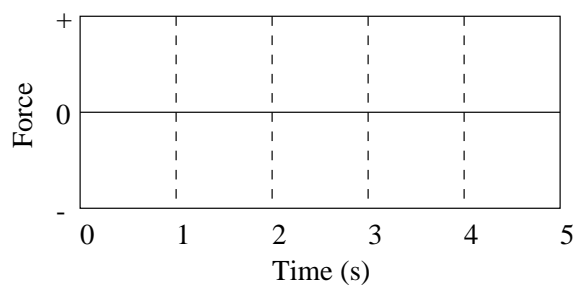
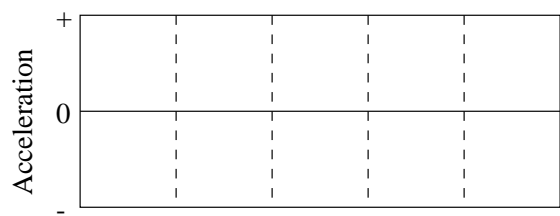


8. Suppose that the force applied to the object were twice as large. Sketch with dashed lines on the same axes above the force, acceleration, and velocity.

Questions 9 refer to an object which can move along a horizontal line (the + position axis). Assume that friction is so small that it can be ignored. The object's velocity-time graph is shown below.



9. Sketch the shapes of the acceleration and force graphs on the axes below.



## 13 Force and Motion II<sup>11</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

To understand the relationship between the direction of the force applied to an object and the direction of the acceleration of the object.

### Overview

In the previous lab you examined the one-dimensional motions of an object caused by a single force applied to the object. You have seen that when friction is so small that it can be ignored, a single constant applied force will cause an object to have a constant acceleration. (The object will speed up at a steady rate.)

Under these conditions, you have seen that the acceleration is proportional to the applied force, if the mass of the object is not changed. You saw that when a constant force is applied to a cart, the cart speeds up at a constant rate so that it has a constant acceleration. If the applied force is made larger, then the acceleration is proportionally larger. This allows you to define force more precisely not just in terms of the stretches of rubber bands and springs, but as the entity (the “thing”) that causes acceleration.

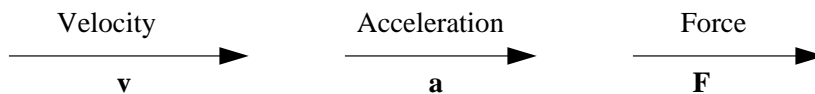
The goal of this lab is to continue to develop the relationship between force and acceleration, an important part of the first two of Newton’s famous laws of motion. You will explore motions in which the applied force (and hence the acceleration of the object) is in a different direction than the object’s velocity. In this case the object is slowing down in the sense that its speed is decreasing.

### Apparatus

- *Science Workshop 750 Interface*
- Force probe
- Variety of hanging masses
- Low friction pulley and string
- Motion detector
- Dynamics cart (with flag) and track
- *DataStudio* software (V, A & F Graphs application)

### Speeding Up and Slowing Down

So far you have looked at cases where the velocity, force and acceleration all have the same sign (all positive). That is, the vectors representing each of these three vector quantities all point in the same direction. For example, if the cart is moving toward the right and a force is exerted toward the right, then the cart will speed up. Thus the acceleration is also toward the right. The three vectors can be represented as:



If the positive x direction is toward the right, then you could also say that the velocity, acceleration and also force are all positive. In this investigation, you will examine the vectors representing velocity, force and acceleration for other motions of the cart. This will be an extension of your earlier observations of changing motion.

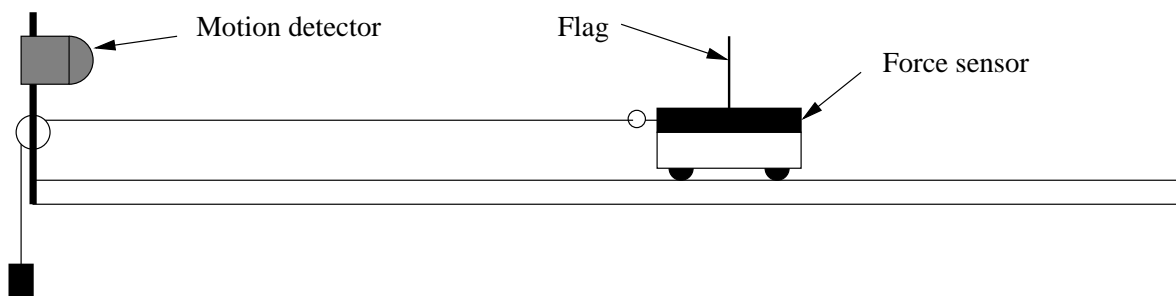
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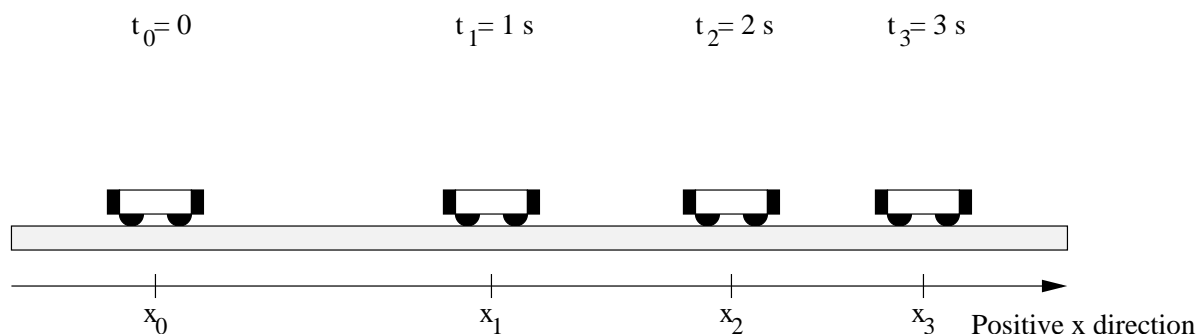


**Activity 1: Slowing Down**

Set up the cart, pulley, hanging mass and motion detector as shown below. Now when you give the cart a push away from the motion detector, it will slow down after it is released. In this activity you will examine the acceleration and the applied force.



(a) Suppose that you position the cart 0.15 m from the motion detector and give it a push away from the motion detector and release it. Draw below vectors which might represent the velocity, force and acceleration of the cart at each time after it is released and is moving toward the right. Be sure to mark your arrows with  $\mathbf{v}$ ,  $\mathbf{a}$ , or  $\mathbf{F}$  as appropriate.

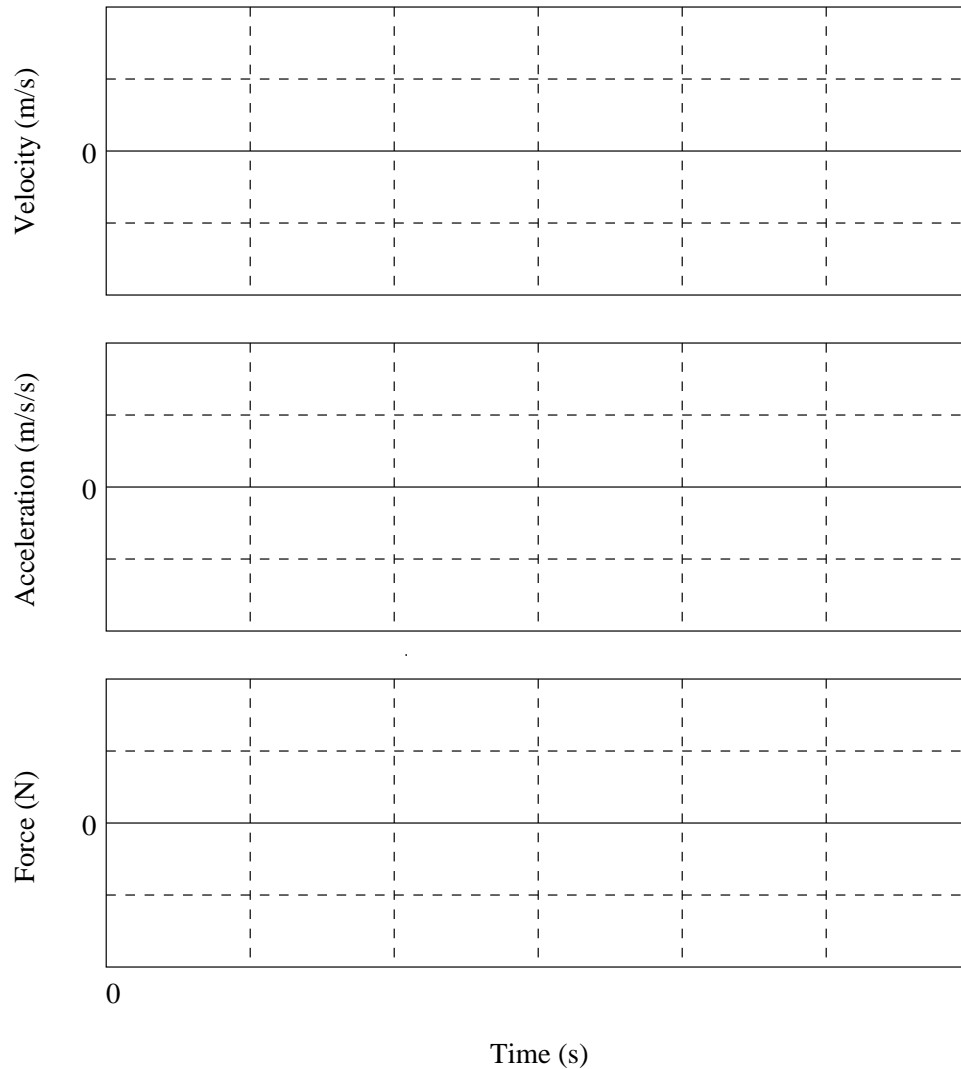


(b) If the positive  $x$  direction is toward the right, what are the signs of the velocity, force and acceleration after the cart is released and is moving toward the right?

(c) To test your predictions:

1. Calibrate the force probe (see *Calibrating Force Sensors* in **Appendix E: Instrumentation**).
2. Hang a 50-g mass from the end of the string.
3. Use the V, A & F Graphs application to graph the motion.
4. Test to be sure that the motion detector sees the cart during its complete motion, and that the string and force probe cable are not interfering with the motion detector. You may need to move the motion detector to the side slightly so that it does not see the string. Also make sure that the cables to the motion detector do not impede the motion of the cart. Remember that the back of the cart must always be at least 0.15 meter from the motion detector.
5. Start recording data. When the motion detector starts clicking, give the cart a short push away from the motion detector and then let it go. Stop the cart before it reverses its direction. Repeat until you get a good run.

6. Sketch your velocity, acceleration and force graphs on the axes below. Label the time scale on these axes. Indicate with an arrow the time when the push stopped.



(d) Did the signs of the velocity, force and acceleration agree with your predictions? If not, can you now explain the signs?

(e) Did the velocity and acceleration both have the same sign? Explain these signs based on the relationship between acceleration and velocity.

(f) Did the force and acceleration have the same sign? Were the force and acceleration in the same direction? Explain.

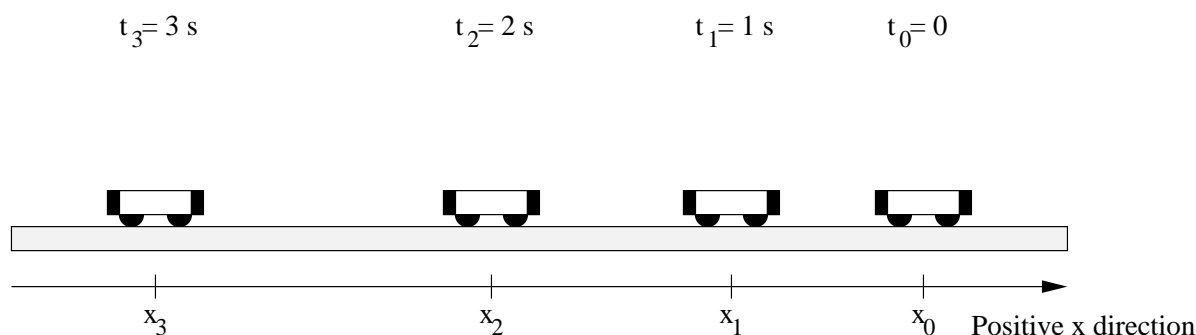
(g) Based on your observations, draw below vectors which might represent the velocity, force and acceleration for the cart at the same instant in time. Do these agree with your predictions? If not, can you now explain the directions of the vectors?

(h) After you released the cart, was the force applied by the falling mass constant, increasing or decreasing? Explain why this kind of force is necessary to cause the observed motion of the cart.

### Activity 2: Speeding Up Toward the Motion Detector

Using the same setup as in the last activity, you can start with the cart at the opposite end of the table from the motion detector and release it from rest. It will then be accelerated toward the motion detector as a result of the force applied by the falling mass.

(a) Suppose that you release the cart from rest and let it move toward the motion detector. Draw on the diagram below vectors which might represent the velocity, force and acceleration of the cart at each time after it is released and is moving toward the left. Be sure to mark your arrows with  $\mathbf{v}$ ,  $\mathbf{a}$ , or  $\mathbf{F}$  as appropriate.



(b) What are the signs of the velocity, force and acceleration after the cart is released and is moving toward the motion detector? (The positive  $x$  direction is toward the right.)

(c) Test your predictions. Use a hanging mass of 100 g. Start recording data. When you hear the motion detector, release the cart from rest as far away from the motion detector as possible. Catch the cart before it hits the motion detector. Repeat until you get a good run. Sketch your graphs on the above axes with dashed lines.

(d) Which of the signs – velocity, force and/or acceleration – are the same as in the previous activity (where the cart was slowing down and moving away), and which are different? Explain any differences in terms of the differences in the motion of the cart.

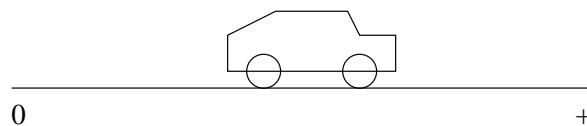
(e) Based on your observations, draw below vectors which might represent the velocity, force and acceleration for the cart at the same instant in time. Do these agree with your predictions? If not, can you now explain the directions of the vectors?

(f) Write down a simple rule in words which describes the relationship between the direction of the applied force and the direction of the acceleration for any motion of the cart.

(g) Is the direction of the velocity always the same as the direction of the force? Is the direction of the acceleration always the same as the direction of the force? In terms of its magnitude and direction, what is the effect of a force on the motion of an object?

### Homework

Questions 1-6 refer to a toy car which can move in either direction along a horizontal line (the  $+$  position axis).



Assume that friction is so small that it can be ignored. Sketch the shape of the graph of the applied force which would keep the car moving as described in each statement.

1. The toy car moves away from the origin with a constant velocity.



2. The toy car moves toward the origin with a constant velocity.



3. The toy car moves away from the origin with a steadily decreasing velocity (a constant acceleration).



4. The toy car moves away from the origin, speeds up and then slows down.



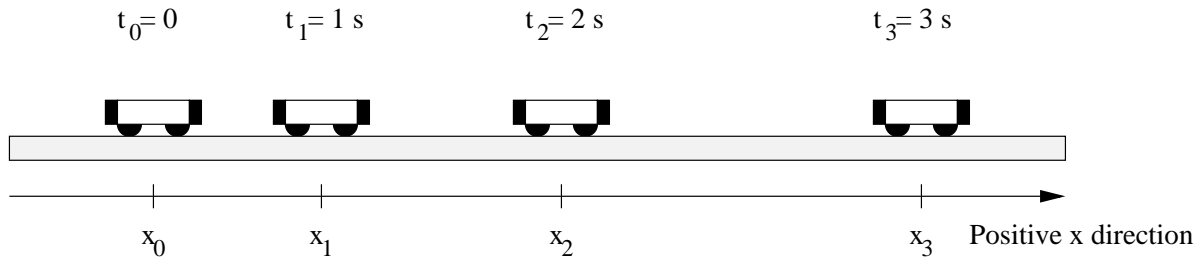
5. The toy car moves toward the origin with a steadily increasing speed ( a constant acceleration).



6. The toy car is given a push away from the origin and released. It continues to move with a constant velocity. sketch the force after the car is released.



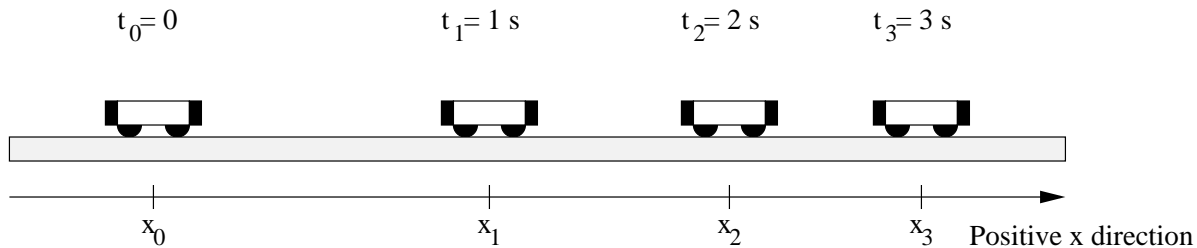
7. A cart is moving toward the right and speeding up, as shown in the diagram below. Draw arrows above the cart representing the magnitudes and directions of the net (combined) forces you think are needed on the cart at the times shown to maintain its motion with a steadily increasing velocity.



Explain the reasons for your answers.

8. If the positive direction is toward the right, what is the sign of the force at  $t = 2\text{ sec}$  in question 7? Explain.

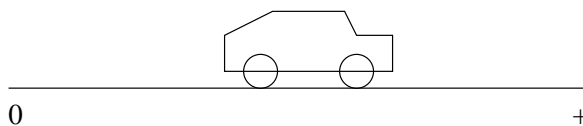
9. A cart is moving toward the right and slowing down, as shown in the diagrams below. Draw arrows above the cart representing the magnitudes and directions of the net (combined) forces you think are needed on the cart at  $t = 0\text{ s}$ ,  $t = 1\text{ s}$ , etc. to maintain its motion with a steadily decreasing velocity.



Explain the reasons for your answers.

10. If the positive direction is toward the right, what is the sign of the force at  $t = 2\text{ sec}$  in question 9? Explain.

11. A toy car can move in either direction along a horizontal line (the + position axis).



Assume that friction is so small that it can be ignored. A force toward the right of constant magnitude is applied to the car. Sketch on the axes below using a solid line the shape of the acceleration-time graph of the car.



Explain the shape of your graph in terms of the applied force.

## 14 Newton's 3rd Law, Tension, and Normal Forces<sup>12</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

- To investigate Newton's 3rd law of motion.
- To explore the characteristics of two different types of forces: tension (in strings, ropes, springs, and chains), and normal forces (which support objects that are in contact with solid surfaces).

### Apparatus

- Spring scales (2)
- Variety of masses
- Rubber band
- Various lengths of string
- Pulleys (2)

### An Introduction to Newton's Third Law

In order to apply Newton's laws to complex situations with strings, pulleys, inclined planes and so forth, we need to consider a third force law formulated by Newton having to do with the forces of interaction between two objects. In order to "discover" some simple aspects of the third law, you should make some straightforward observations using 2 spring scales and a set of masses.



### Activity 1: Newton's 3rd Law Forces of Interaction

Set up the situations shown in the diagram above and see if there are any circumstances in which the object that is pulling and the object that is being pulled exert different forces on each other. Describe your conclusions below. Note: You can drag the mass set block across the table for your dynamic observations.

In contemporary English, Newton's third law can be stated as follows:

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### Newton's Third Law

*If one object exerts a force on a second object, then the second object exerts a force back on the first object which is equal in magnitude and opposite in direction to that exerted on it by the first object.*

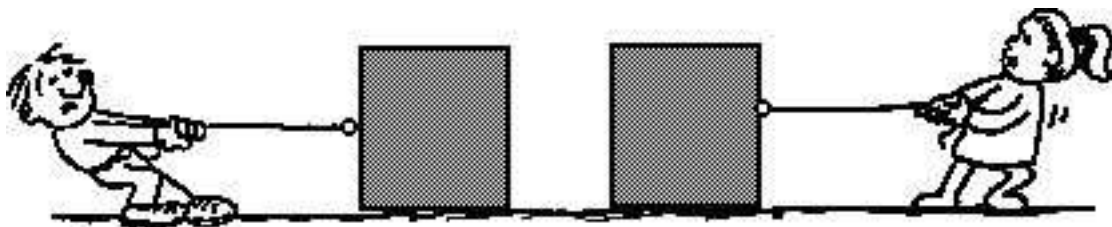
In mathematical terms, using vector notation, we would say that the forces of interaction of object 1 on object 2 are related to the forces of interaction of object 2 on object 1 as follows:

$$\vec{F}_{12} = -\vec{F}_{21}$$


Newton actually formulated the third law by studying the interactions between objects when they collide. It is difficult to understand the significance of this law fully without first studying collisions. Thus, we will consider this law again in the study of collision processes.

### Tension Forces

When you pull on one end of a rope attached to a crate, a force is transmitted down the rope to the crate. If you pull hard enough the crate may begin to slide. Tension is the name given to forces transmitted in this way along devices that can stretch such as strings, ropes, rubber bands, springs, and wires.



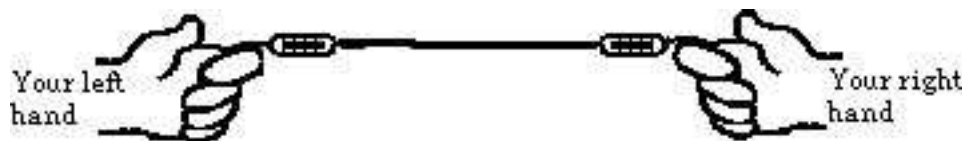
The end of the rope which is tied to the crate can apply a force to the crate only if you first pull on the other end of the rope.

In order to analyze situations in which objects are attached by strings, rubber bands, or ropes it is necessary to understand some attributes of tension forces. We need to answer the following related questions:

1. (a) What is the mechanism for creating tension in strings, ropes, and rubber bands? (b) If a string exerts a tension force on an object at one end, what is the magnitude and direction of the tension force it exerts on another object at its other end?
2. What happens to the magnitude and directions of the tension forces at each end of a string and in the middle of that string when the direction of the string is changed by a post or pulley?
3. Can a flexible force transmitter (like a string) support a lateral (or sideways) force?

### Mechanisms for Tension and the Direction of Forces

For these observations you should stretch a rubber band and then a string between your hands as shown in the diagram below. First, just feel the directions of the forces. Then add the spring scales and both feel and measure the forces.



**Activity 2: Tension Mechanisms & Force Directions**

(a) Pull on the two ends of a rubber band. (Forget about the spring scales for now). Does the rubber band stretch? What is the direction of the force applied by the rubber band on your right hand? On your left hand?

(b) Does the magnitude of the forces applied by the rubber band on each hand feel the same?

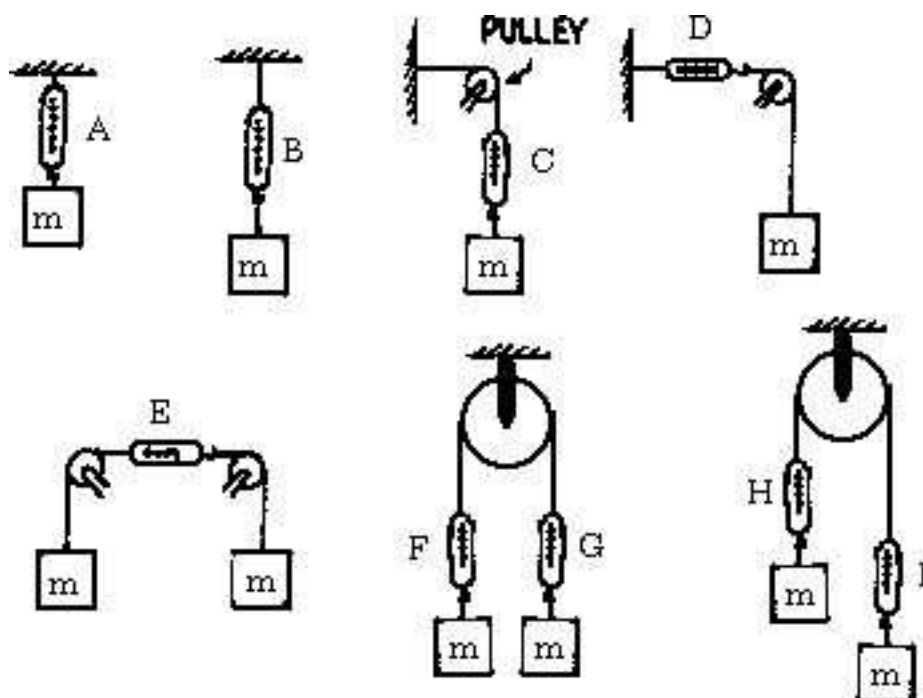
(c) Repeat this activity with a string instead of a rubber band. This time, use a spring scale at each end to measure the forces at the ends of the string. Does the string stretch? (Look carefully!)

(d) If you pull by the same amount on the string as you did on the rubber band, does substituting the string for the rubber band change anything about the directions and magnitudes of the tension forces exerted on each hand?

(e) If the forces caused by the string on your left and right hands respectively are given by  $F_{T1}$  and  $F_{T2}$ , what is the equation that relates these two forces?

**Tension Forces when a String Changes Direction**

Suppose you were to hang equal masses of  $m = 0.5$  kg in the various configurations shown below. Predict and measure the tension in the string for each of the following situations.



### Activity 3: Tension and Direction Changes

(a) For each configuration shown above, predict the reading in newtons on each of the spring scales; these readings indicate the forces that are transmitted by the tensions at various places along the string. Then measure all of the forces and record their values. Note: Remember that  $m = 0.5 \text{ kg}$ .

#### Predicted Force Magnitudes

$F_A = \underline{\hspace{2cm}} \text{ N}$

$F_B = \underline{\hspace{2cm}} \text{ N}$

$F_C = \underline{\hspace{2cm}} \text{ N}$

$F_D = \underline{\hspace{2cm}} \text{ N}$

$F_E = \underline{\hspace{2cm}} \text{ N}$

$F_F = \underline{\hspace{2cm}} \text{ N}$

$F_G = \underline{\hspace{2cm}} \text{ N}$

$F_H = \underline{\hspace{2cm}} \text{ N}$

$F_I = \underline{\hspace{2cm}} \text{ N}$

#### Measured Force Magnitudes

$F_A = \underline{\hspace{2cm}} \text{ N}$

$F_B = \underline{\hspace{2cm}} \text{ N}$

$F_C = \underline{\hspace{2cm}} \text{ N}$

$F_D = \underline{\hspace{2cm}} \text{ N}$

$F_E = \underline{\hspace{2cm}} \text{ N}$

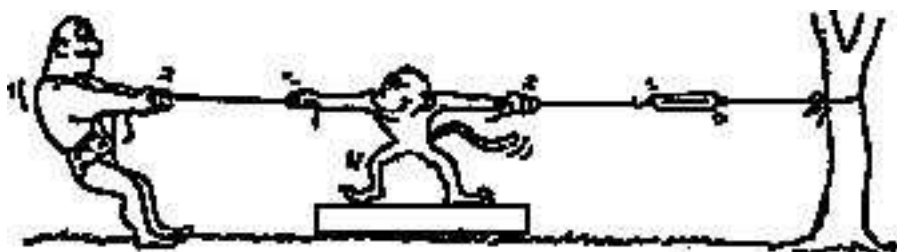
$F_F = \underline{\hspace{2cm}} \text{ N}$

$F_G = \underline{\hspace{2cm}} \text{ N}$

$F_H = \underline{\hspace{2cm}} \text{ N}$

$F_I = \underline{\hspace{2cm}} \text{ N}$

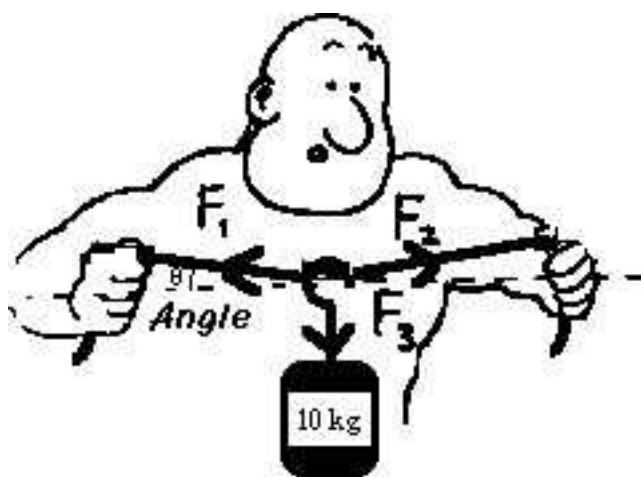
(b) Based on Newton's third law and the observations you just made, answer the following questions using vector notation. If the muscle man in the diagram below is pulling to the left on a rope with a force of  $\mathbf{F} = -(150 \text{ N})\mathbf{i}$ .



- (1) What is the magnitude and direction of the force that the rope is exerting on the man? \_\_\_\_\_
  - (2) What force is the left-hand rope exerting on the monkey's right arm? \_\_\_\_\_
  - (3) What force is the spring scale experiencing on its left end? \_\_\_\_\_
  - (4) What force is the spring scale experiencing on its right end? \_\_\_\_\_
  - (5) What is the reading on the spring scale? \_\_\_\_\_
  - (6) What force is the rope exerting on the tree? \_\_\_\_\_
  - (7) What force is the tree exerting on the rope? \_\_\_\_\_
- (c) Summarize what your observations reveal about the nature of tension forces everywhere along a string.

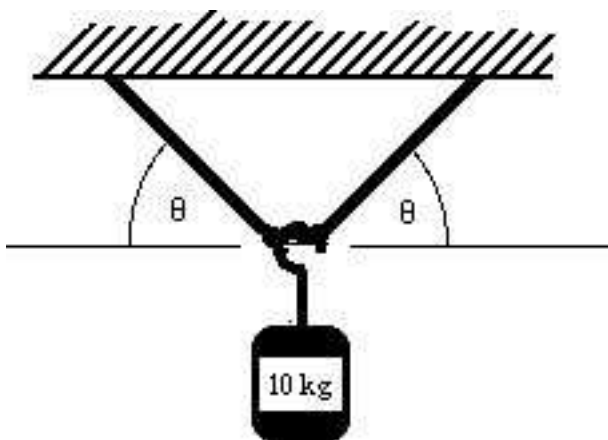
### Can a String Support Lateral Forces?

Take a look at the diagram below. Can the strongest member of your group stretch a string or rope so that it is perfectly horizontal when a 10 kg mass is hanging from it? In other words, can the string provide a force that just balances the force exerted by the mass?



### Activity 4: Can a String Support a Lateral Force?

- (a) Draw a vector diagram showing the directions of the forces exerted by the strings on the mass hook in the diagram below. What would happen to the direction of the forces as  $\theta$  goes to zero? Do you think it will be possible to support the mass when  $\theta = 0$ ? Why?



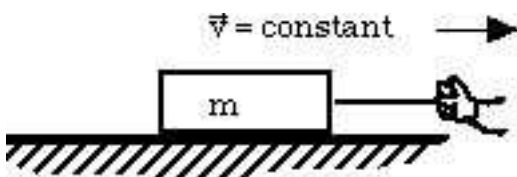
(b) Now, experiment with holding a mass horizontally with a string. What do you conclude about the ability of a string to support a mass having a force which is perpendicular to the direction of the string?

### Normal Forces

A book resting on a table does not move; neither does a person pushing against a wall. According to Newton's first law the net force on the book and on the person's hand must be zero. We have to invent another type of force to explain why books don't fall through tables and hands don't usually punch through walls. The force exerted by any surface always seems to act in a direction perpendicular to that surface; such a force is known as a normal force.

### Activity 5: Normal Forces

(a) The diagram below shows a block sliding along a table near the surface of the earth at a constant velocity. According to Newton's first law, what is the net force on the block? In other words, what is the vector sum of all the forces on the block?

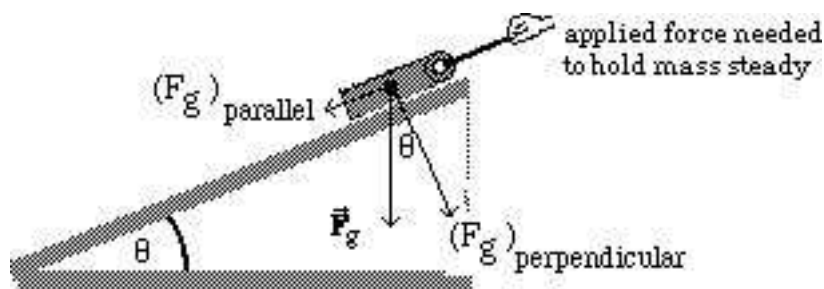


(b) The net force is made up of four forces. In what direction does each one act? Draw a diagram indicating the direction of each of the forces.

### Gravitational Force on a Mass on an Incline

Suppose that a block of mass  $m$  is perched on an incline of angle  $\theta$  as shown in the diagram below. Also suppose that you know the angle of the incline and the magnitude and direction of the gravitational force vector. What

do you predict the magnitude of the components of the force vector will be parallel to the plane? Perpendicular to the plane?



**Activity 6: Components of  $F_g$  on an Incline**

(a) The angle that the incline makes with the horizontal and the angle between  $F_{\text{perpendicular}}$  and  $F_g$  are the same. Explain why.

(b) Choose a coordinate system with the x-axis parallel to the plane with the positive direction up the plane. Using normal mathematical techniques for finding the components of a vector, find the values of  $F_{\text{parallel}}$  and  $F_{\text{perpendicular}}$  as a function of  $F_g$  and the angle of the incline  $\theta$ .

(c) What is the equation for the magnitude of the normal force exerted on the block by the surface of the incline? Hint: Use Newton's first law and the knowledge that the block is not moving in a direction perpendicular to the plane.

## 15 The Electrical and Gravitational Forces<sup>13</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

I began to think of gravity extending to the orb of the moon, and . . . I deduced that the forces which keep the planets in their orbs must be reciprocally as the squares of their distances from the centers about which they revolve: and thereby compared the force requisite to keep the moon in her orb with the force of gravity at the surface of the earth, and found them to answer pretty nearly. All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention, and minded mathematics and philosophy more than at any time since. — Isaac Newton

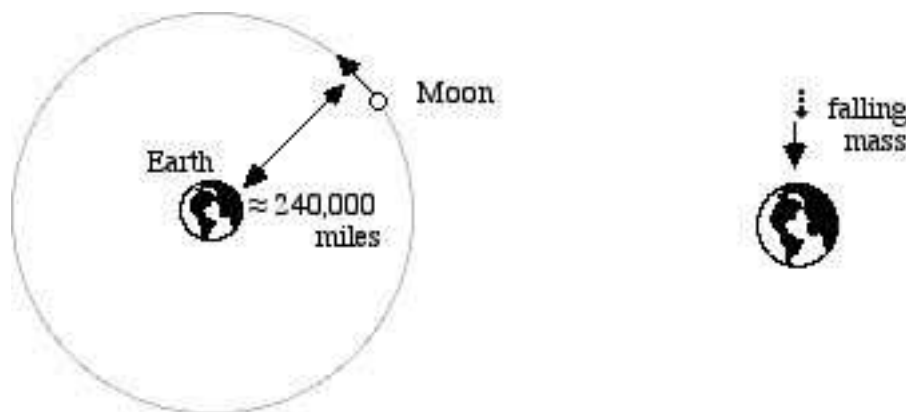
### Objective

To understand the similarities of the gravitational and electrical forces.

### Overview

The enterprise of physics is concerned ultimately with mathematically describing the fundamental forces of nature. Nature offers us several fundamental forces, which include a strong force that holds the nuclei of atoms together, a weak force that helps us describe certain kinds of radioactive decay in the nucleus, the force of gravity, and the electromagnetic force.

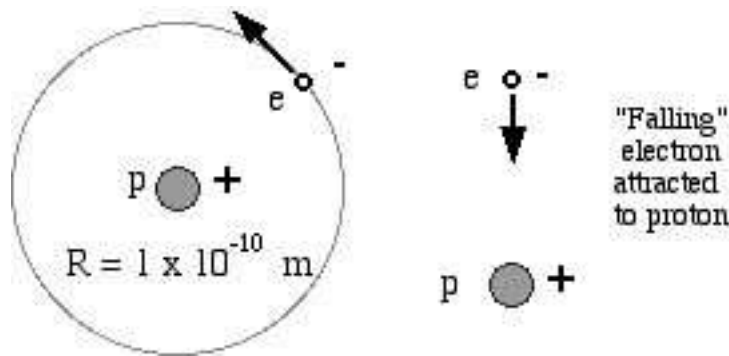
*Two kinds of force dominate our everyday reality: the gravitational force acting between masses and the Coulomb force acting between electrical charges.* The gravitational force allows us to describe mathematically how objects near the surface of the earth are attracted toward the earth and how the moon revolves around the earth and planets revolve around the sun. The genius of Newton was to realize that objects as diverse as falling apples and revolving planets are both moving under the action of the same gravitational force.



Similarly, the Coulomb force allows us to describe how one charge “falls” toward another or how an electron orbits a proton in a hydrogen atom.

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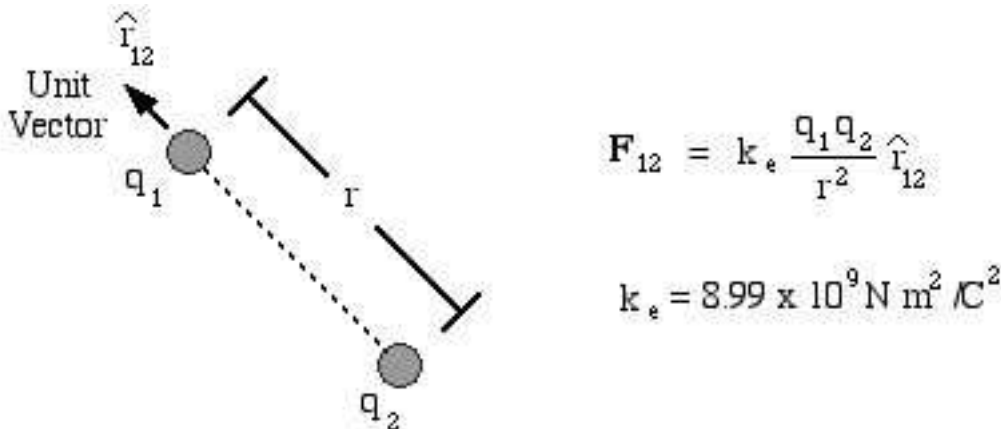


The fact that both the Coulomb and the gravitational forces lead to objects falling and to objects orbiting around each other suggests that these forces might have the same mathematical form.

In this unit we will explore the mathematical symmetry between electrical and gravitational forces for two reasons. First, it is beautiful to behold the unity that nature offers us as we use the same type of mathematics to predict the motion of planets and galaxies, the falling of objects, the flow of electrons in circuits, and the nature of the hydrogen atom and of other chemical elements. Second, what you have already learned about the influence of the gravitational force on a mass can be applied to aid your understanding of the forces on charged particles.

### Activity 1: Comparison of Electrical and Gravitational Forces

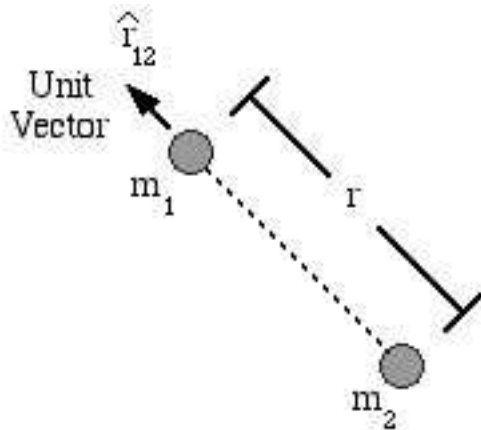
Let's start our discussion of this comparison with the familiar expression of the Coulomb force exerted on charge 1 by charge 2.



Charles Coulomb did his experimental investigations of this force in the 18th century by exploring the forces between two small charged spheres. Much later, in the 20th century, Coulomb's law enabled scientists to design cyclotrons and other types of accelerators for moving charged particles in circular orbits at high speeds.

Newton's discovery of the universal law of gravitation came the other way around. He thought about orbits first. This was back in the 17th century, long before Coulomb began his studies. A statement of Newton's universal law of gravitation describing the force experienced by mass 1 due to the presence of mass 2 is shown below in modern mathematical notation:





$$\mathbf{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{r}_{12}$$

$$G = 6.67 \times 10^{-11} \text{ N m}^2 / \text{kg}^2$$

About the time that Coulomb did his experiments with electrical charges in the 18th century, one of his contemporaries, Henry Cavendish, did a direct experiment to determine the nature of the gravitational force between two spherical masses in a laboratory. This confirmed Newton's gravitational force law and allowed him to determine the gravitational constant,  $G$ . A fact emerges that is quite amazing. Both types of forces, electrical and gravitational, are very similar. Essentially the same mathematics can be used to describe orbital and linear motions due to either electrical or gravitational interactions of the tiniest fundamental particles or the largest galaxies. This statement needs to be qualified a bit when electrons, protons and other fundamental particles are considered. A new field called quantum mechanics was developed in the early part of the last century to take into account the wave nature of matter, which we don't actually study in introductory physics. However, even in wave mechanical calculations electrical forces like those shown above are used.

### Activity 2: The Electrical vs. the Gravitational Force

Examine the mathematical expression for the two force laws.

(a) What is the same about the two force laws?

(b) What is different? For example, is the force between two like masses attractive or repulsive? How about two like charges? What part of each equation determines whether the like charges or masses are attractive or repulsive?

(c) Do you think negative mass could exist? If there is negative mass, would two negative masses attract or repel?

### Which Force is Stronger— Electrical or Gravitational?

Gravitational forces hold the planets in our solar system in orbit and account for the motions of matter in galaxies. Electrical forces serve to hold atoms and molecules together. If we consider two of the most common

fundamental particles, the electron and the proton, how do their electrical and gravitational forces compare with each other?

Let's peek into the hydrogen atom and compare the gravitational force on the electron due to interaction of its mass with that of the proton to the electrical force between the two particles as a result of their charge. In order to do the calculation you'll need to use some well known constants.

Electron:  $m_e = 9.1 \times 10^{-31}$  kg,  $q_e = -1.6 \times 10^{-19}$  C

Proton:  $m_p = 1.7 \times 10^{-27}$  kg,  $q_p = +1.6 \times 10^{-19}$  C

Distance between the electron and proton:  $r = 1.0 \times 10^{-10}$  m

### Activity 3: The Electrical vs. the Gravitational Force in the Hydrogen Atom

- (a) Calculate the magnitude of the electrical force on the electron. Is it attractive or repulsive?
  
  
  
  
  
  
  
  
  
  
- (b) Calculate the magnitude of the gravitational force on the electron. Is it attractive or repulsive?
  
  
  
  
  
  
  
  
  
  
- (c) Which is larger? By what factor (i.e., what is the ratio)?
  
  
  
  
  
  
  
  
  
  
- (d) Which force are you more aware of on a daily basis? If your answer does not agree with that in part (c), explain why.

### Activity 4: The Gravitational Force of the Earth

- (a) Use Newton's law of universal gravitation to show that the magnitude of the acceleration due to gravity on an object of mass  $m$  at a height  $h$  above the surface of the earth is given by the following expression.

$$\frac{GM_e}{(R_e + h)^2}$$

Hint: Because of the spherical symmetry of the Earth you can treat the mass of the Earth as if it were all concentrated at a point at the Earth's center.

(b) Calculate the acceleration due to gravity of a mass  $m$  at the surface of the earth ( $h=0$ ). The radius of the earth is  $R_e \approx 6.38 \times 10^3$  km and its mass  $M_e \approx 5.98 \times 10^{24}$  kg. Does the result look familiar? How is this acceleration related to the gravitational acceleration  $g$ ?

(c) Use the equation you derived in part (a) to calculate the acceleration due to gravity at the ceiling of the room you are now in. How does it differ from the value at the floor? Can you measure the difference in the lab using the devices available?

(d) Suppose you travel halfway to the moon. What is the new value of the acceleration due to gravity (neglecting the effect of the moon's pull)? (Recall that the earth-moon distance is about 384,000 km.)

(e) Is the gravitational acceleration “constant,”  $g$ , really a constant? Explain.

(f) In part (d) you showed that there is a significant gravitational attraction halfway between the earth and the moon. Why, then, do astronauts experience “weightlessness” when they are orbiting a mere 120 km above the earth?

## 16 Centripetal Force<sup>14</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

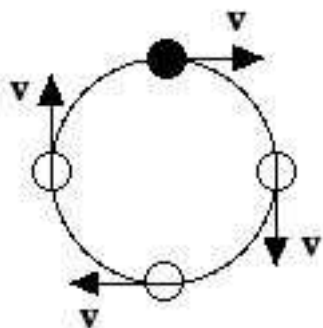
### Objective

To explore the phenomenon of uniform circular motion and the accelerations and forces needed to maintain it.

### Overview

In a previous unit you began the study of the application of Newton's laws to projectile motion. In this unit we are going to consider the application of Newton's laws to another phenomenon in two dimensions. Since Newton's laws can be used to predict types of motion or the conditions for no motion, their applications are useful in many endeavors including human body motion, astrophysics, and engineering.

You will explore uniform circular motion, in which an object moves at a constant speed in a circle. In particular, you will develop a mathematical description of centripetal acceleration and the force needed to keep an object moving in a circle.



### Apparatus

- An airplane.
- A video analysis system (VideoPoint).
- A spring scale.
- Graphing software (Excel).

### Moving in a Circle at a Constant Speed

When a race car speeds around a circular track, or when David twirled a stone at the end of a rope to clobber Goliath, or when a planet like Venus orbits the sun, they undergo uniform circular motion. Understanding the forces which govern orbital motion has been vital to astronomers in their quest to understand the laws of gravitation.

But we are getting ahead of ourselves, for as we have done in the case of linear and projectile motion we will begin our study by considering situations involving external applied forces that lead to circular motion in the absence of friction. We will then use our belief in Newton's laws to see how the circular motions of the planets can be used to help astronomers discover the laws of gravitation.



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<sup>14</sup>1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material have been modified locally and may not have been classroom tested at Dickinson College.

Let's begin our study with some very simple considerations. Suppose an astronaut goes into outer space, ties a ball to the end of a rope, and spins the ball so that it moves at a constant speed.

**Activity 1: Uniform Circular Motion**

(a) Consider the figure above. What is the speed of a ball that moves in a circle of radius  $r = 2.5$  m if it takes 0.50 s to complete one revolution?

(b) The speed of the ball is constant! Would you say that this is accelerated motion?

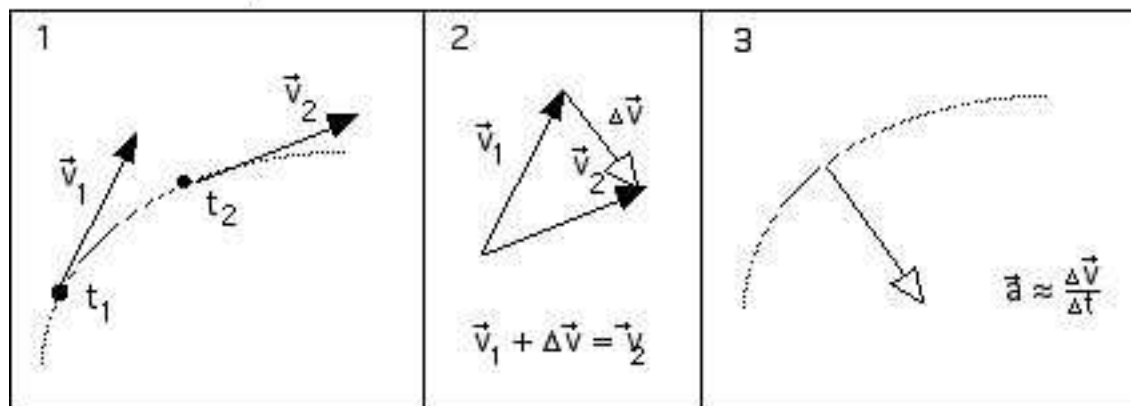
(c) What is the definition of acceleration? (Remember that acceleration is a vector!)

(d) Are velocity and speed the same thing? Is the velocity of the ball constant? (Hint: Velocity is a vector quantity!)

(e) In light of your answers to (c) and (d), would you like to change your answer to part (b)? Explain.

**Using Vectors to Diagram How Velocity Changes**

By now you should have concluded that since the direction of the motion of an object undergoing uniform circular motion is constantly changing, its velocity is also changing and thus it is accelerating. We would like you to figure out how to calculate the direction of the acceleration and its magnitude as a function of the speed  $v$  of an object such as a ball as it revolves and as a function of the radius of the circle in which it revolves. In order to use vectors to find the direction of velocity change in circular motion, let's review some rules for adding velocity vectors.

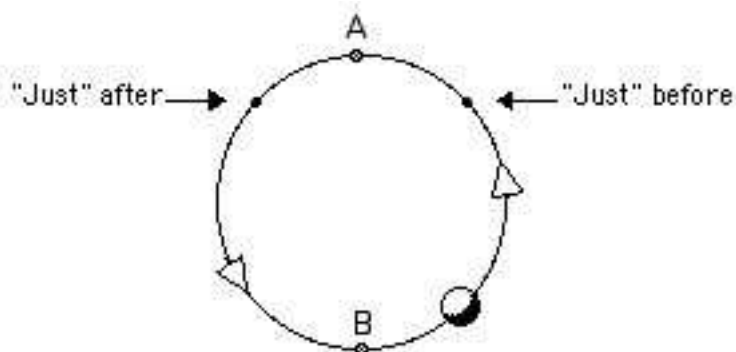


1. To Draw Velocities: Draw an arrow representing the velocity,  $\mathbf{v}_1$ , of the object at time  $t_1$ . Draw another arrow representing the velocity,  $\mathbf{v}_2$ , of the object at time  $t_2$ .
2. To Draw Velocity Change: Find the change in the velocity  $\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$  during the time interval described by  $\Delta t = t_2 - t_1$ . Start by using the rules of vector sums to rearrange the terms so that  $\mathbf{v}_1 + \Delta\mathbf{v} = \mathbf{v}_2$ . Next place the tails of the two velocity vectors together halfway between the original and final location of the object. The change in velocity is the vector that points from the head of the first velocity vector to the head of the second velocity vector.
3. To Draw Acceleration: The acceleration equals the velocity change  $\Delta\mathbf{v}$  divided by the time interval  $t$  needed for the change. Thus,  $\mathbf{a}$  is in the same direction as  $\Delta\mathbf{v}$  but is a different length (unless  $\Delta t = 1$ ). Thus, even if you do not know the time interval, you can still determine the direction of the acceleration because it points in the same direction as  $\Delta\mathbf{v}$ .

The acceleration associated with uniform circular motion is known as centripetal acceleration. You will use the vector diagram technique described above to find its direction.

### Activity 2: The Direction of Centripetal Acceleration

(a) Determine the direction of motion of the ball shown below if it is moving counter-clockwise at a constant speed. Note that the direction of the ball's velocity is always tangential to the circle as it moves around. Draw an arrow representing the direction and magnitude of the ball's velocity as it passes the dot just before it reaches point A. Label this vector  $\mathbf{v}_1$ .



(b) Next, use the same diagram to draw the arrow representing the velocity of the ball when it is at the dot just after it passes point A. Label this vector  $\mathbf{v}_2$ .

(c) Find the direction and magnitude of the change in velocity as follows. In the space below, make an exact copy of both vectors, placing the tails of the two vectors together. Next, draw the vector that must be added to

vector  $\mathbf{v}_1$  to add up to vector  $\mathbf{v}_2$  ; label this vector  $\Delta\mathbf{v}$ . Be sure that vectors  $\mathbf{v}_1$  and  $\mathbf{v}_2$  have the same magnitude and direction in this drawing that they had in your drawing in part (a)!

(d) Now, draw an exact copy of  $\Delta\mathbf{v}$  on your sketch in part (a). Place the tail of this copy at point A. Again, make sure that your copy has the exact magnitude and direction as the original  $\Delta\mathbf{v}$  in part (c).

(e) Now that you know the direction of the change in velocity, what is the direction of the centripetal acceleration,  $\mathbf{a}_c$ ?

(f) If you re did the analysis for point B at the opposite end of the circle, what do you think the direction of the centripetal acceleration,  $\mathbf{a}_c$ , would be now?

(g) As the ball moves on around the circle, what is the direction of its acceleration?

(h) Use Newton's second law in vector form ( $\sum \mathbf{F} = m\mathbf{a}$ ) to describe the direction of the net force on the ball as it moves around the circle.

(i) If the ball is being twirled around on a string, what is the source of the net force needed to keep it moving in a circle?

**Using Mathematics to Derive How Centripetal Acceleration Depends on Radius and Speed**

You haven't done any experiments yet to see how centripetal acceleration depends on the radius of the circle and the speed of the object. You can use the rules of mathematics and the definition of acceleration to derive the relationship between speed, radius, and magnitude of centripetal acceleration.

### Activity 3: How Does $a_c$ Depend on $v$ and $r$ ?

(a) Do you expect you would need more centripetal acceleration or less centripetal acceleration to cause an object moving at a certain speed to rotate in a smaller circle? In other words, would the magnitude,  $a_c$ , have to increase or decrease as  $r$  decreases if circular motion is to be maintained? Explain.

(b) Do you expect you would need more centripetal acceleration or less centripetal acceleration to cause an object to rotate at a given radius  $r$  if the speed  $v$  is increased? In other words, would the magnitude,  $a_c$ , have to increase or decrease as  $v$  increases if circular motion is to be maintained? Explain.

You should have guessed that it requires more acceleration to move an object of a certain speed in a circle of smaller radius and that it also takes more acceleration to move an object that has a higher speed in a circle of a given radius. Let's use the definition of acceleration in two dimensions and some accepted mathematical relationships to show that the magnitude of centripetal acceleration should actually be given by the equation

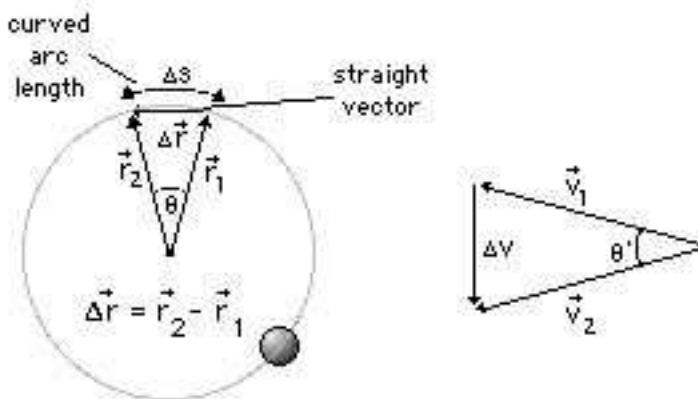
$$a_c = \frac{v^2}{r} \quad [Eq. 1]$$

In order to do this derivation you will want to use the following definition for acceleration

$$\langle \mathbf{a} \rangle = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t_2 - t_1} = \frac{\Delta \mathbf{v}}{\Delta t} \quad [Eq. 2]$$

### Activity 4: Finding the Equation for $a_c$

(a) Refer to the diagram below. Explain why, at the two points shown on the circle, the angle between the position vectors at times  $t_1$  and  $t_2$  is the same as the angle between the velocity vectors at times  $t_1$  and  $t_2$ . Hint: In circular motion, velocity vectors are always perpendicular to their position vectors.



(b) Since the angles are the same and since the magnitudes of the displacements never change (i.e.,  $r = r_1 = r_2$ ) and the magnitudes of the velocities never change (i.e.,  $v = v_1 = v_2$ ), use the properties of similar triangles to explain why  $\frac{\Delta v}{v} = \frac{\Delta r}{r}$ .



(c) Now use the equation in part (b) and the definition of  $\langle a \rangle$  to show that  $\langle a_c \rangle = \frac{\Delta v}{\Delta t} = \frac{(\Delta r)}{(\Delta t)} \frac{v}{r}$ .

(d) The speed of the object as it rotates around the circle is given by  $v = \frac{\Delta s}{\Delta t}$ . Is the change in arc length,  $\Delta s$ , larger or smaller than the magnitude of the change in the position vector,  $\Delta r$ ? Explain why the arc length change and the change in the position vector are approximately the same when  $t$  is very small (so that the angle  $\theta$  becomes very small) i.e., why is  $\Delta s \simeq \Delta r$ ?

(e) If  $\Delta s \simeq \Delta r$ , then what is the equation for the speed in terms of  $\Delta r$  and  $\Delta t$ ?

(f) Using the equation in part (c), show that as  $\Delta t \rightarrow 0$ , the instantaneous value of the centripetal acceleration is given by Eq. 1.

(g) If the object has a mass  $m$ , what is the equation for the magnitude of the centripetal force needed to keep the object rotating in a circle (in terms of  $v$ ,  $r$ , and  $m$ )? In what direction does this force point as the object rotates in its circular orbit?

### Experimental Test of the Centripetal Force Equation

The theoretical considerations in the last activity should have led you to the conclusion that, whenever you see an object of mass  $m$  moving in a circle of radius  $r$  at a constant speed  $v$ , it must at all times be experiencing a net centripetal force directed toward the center of the circle that has a magnitude of

$$F_c = ma_c = m \frac{v^2}{r} \quad [Eq. 3]$$

Let's check this out. Does this rather odd equation really work for an external force?

To test the validity of the derivation we must compare it to experience. We will use a "toy" airplane suspended from a string and flying in a circular path. We will use the video analysis system to measure the properties of the motion and determine the horizontal and vertical components of the force exerted on the airplane using Equation 3. We will compare that result with a direct measurement of the tension in the string.

#### Activity 5: Verifying the $F_c$ Equation

(a) Measure the mass of the airplane and the length of the portion of the string that hangs below the horizontal post with the hole in it. Record the values below.

(b) The airplane is suspended from a spring scale. The string should pass straight down from the scale through the small hole in the horizontal post. The camera should be placed about 1 meter above the airplane. Turn on the camera and center the spring scale in the frame by using the pendulum. Is the reading of the spring scale consistent with the mass of the airplane? Mount a ruler somewhere in the camera's field of view to serve as a scaling object. Launch the plane into uniform circular motion. When the motion is steady record a movie of at least one complete revolution and record the reading of the spring scale. See **Appendix D: Video Analysis** for details.

$$F_{scale} =$$

(c) Determine the position of the airplane during one complete revolution. To do this task follow the instructions in the second section of **Appendix D: Video Analysis** for recording and calibrating a data file. As you analyze the movie frame by frame, estimate to the nearest fraction of a frame the number of frames for one complete revolution. Record your result below. When you calibrate the time and position data, note the number of frames per second of the movie and convert that number to the time interval between frames,  $\Delta t$ . Record this result below. Now you can determine the time interval for one complete revolution and record it below. The resulting data file should contain three columns with the values of time, x-position, and y-position.

$$N_{frame} =$$

$$\Delta t =$$

$$t_{rev} =$$

(d) Make a graph of the trajectory of the airplane during one full revolution. See **Appendix C: Introduction to Excel** for details on using the graphing software. When you make your plot make sure the x and y axes cover the same size range; otherwise, you will distort the path of the airplane. Print your plot and attach it to your write-up.

1. Is the motion circular? What is your evidence?

2. What is the radius of the motion?  $r =$

(e) To test the validity of the expression for  $F_c$  we must know the speed. Use the measurements of the radius of the airplane's trajectory and the time for one complete revolution to calculate the average speed.

$$v_{ave} =$$

(f) Use your results for the mass, the average velocity of the airplane, and the radius of the circular motion to predict the centripetal force exerted by the string.

$$F_c =$$

(g) We need one more piece of the puzzle to predict the tension in the string, namely, the vertical component of the force exerted on the airplane by the string. Recall that we know  $r$ , the radius of the airplane's circular path, and  $R$ , the total length of the string that is actually rotating.

1. Using these two distances ( $r$  and  $R$ ), calculate the angle the string makes with the horizontal.

$$\theta =$$

2. We determined the horizontal component of the force on the airplane in part (f). Now with the angle  $\theta$  generate an expression for the vertical component exerted by the string and calculate it. Make a vector diagram of the different components. Generate an expression for the total force acting on the airplane due to the string and calculate the result.

$$F_y =$$

$$F_{plane} =$$

- (h) Compare your result for  $F_{plane}$  with the measurement of the spring scale  $F_{scale}$ . Within experimental uncertainty, how well does your data support the hypothesis that  $F_c = mv^2/r$ ?

- (i) Discuss the major sources of uncertainty in this experiment.

## 17 Work and Power<sup>15</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

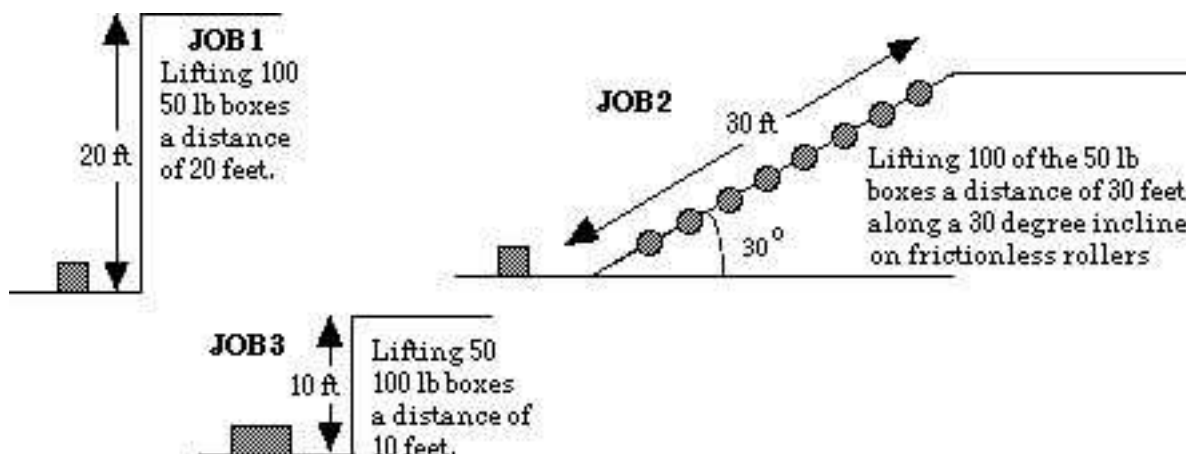
- To extend the intuitive notion of work as physical effort to a formal mathematical definition of work as a function of force and distance.
- To understand the concept of power and its relationship to work.

### Apparatus

- Spring scale
- Wooden block with hook
- Variety of masses

### The Concept of Physical Work

Suppose you are president of the Richmond Load 'n' Go Co. A local college has three jobs available and will allow you to choose which one you want before offering the other two jobs to rival companies. All three jobs pay the same amount of money. Which one would you choose for your crew?



### Activity 1: Choosing Your Job

Examine the descriptions of the jobs shown in figure above. Which one would you be most likely to choose? Least likely to choose? Explain the reasons for your answer.

You obviously want to do the least amount of work for the most money. Before you reconsider your answers later in this unit, you should do a series of activities to get a better feel for what physicists mean by work and how the president of Load 'n' Go can make top dollar.

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In everyday language we refer to doing work whenever we expend effort. In order to get an intuitive feel for how we might define work mathematically, you should experiment with moving your textbook back and forth along a table top and a rougher surface such as a carpeted floor.

**Activity 2: This is Work!**

(a) Pick a distance of a meter or so. Sense how much effort it takes to push a heavy book that distance. How much more effort does it take to push it twice as far?

(b) Pile another similar book on top of the original one and sense how much effort it takes to push the two books through the distance you picked. Comment below.

(c) From your study of sliding friction, what is the relationship between the mass of a sliding object and the friction force it experiences? On the basis of your experience with sliding friction, estimate how much more force you have to apply to push two books compared to one book.

(d) If the “effort” it takes to move an object is associated with physical work, guess an equation that can be used to define work mathematically when the force on an object and its displacement (i.e., the distance it moves) lie along the same line.

In physics, work is not simply effort. In fact, the physicist’s definition of work is precise and mathematical. In order to have a full understanding of how work is defined in physics, we need to consider its definition in a very simple situation and then enrich it later to include more realistic situations.

**A Simple Definition of Physical Work:** If an object that is moving in a straight line experiences a constant force in the direction of its motion during the time it is undergoing a displacement, the work done by the external force,  $F_{ext}$ , is defined as the product of the force and the displacement of the object,

$$W = F_{ext}\Delta x$$

where  $W$  represents the work done by the external force,  $F_{ext}$  is the magnitude of the force, and  $\Delta x$  is the displacement of the object.

What if the force of interest and the displacement are in opposite directions? For instance, what about the work done by the force of sliding friction,  $F_f$ , when a block slides on a rough surface? The work done by the friction force is

$$W_f = -F_f \Delta x$$

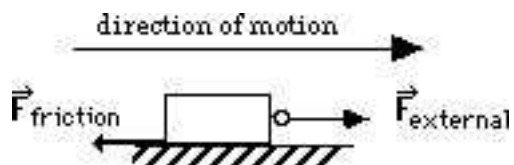
### Activity 3: Applying the Physics Definition of Work

(a) Does effort necessarily result in physical work? Suppose two guys are in an evenly matched tug of war. They are obviously expending effort to pull on the rope, but according to the definition of physical work, are they doing any physical work? Explain.



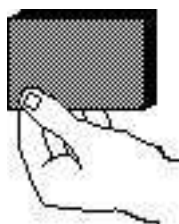
(b) A wooden block with a mass of .30 kg is pushed along a sheet of ice that has no friction with a constant external force of 10 N which acts in a horizontal direction. After it moves a distance of 0.40 m how much work has been done on the block by the external force?

(c) The same wooden block with a mass of .30 kg is pushed along a table with a constant external force of 10 N which acts in a horizontal direction. It moves a distance of 0.40 m. However, there is a friction force opposing its motion. Assume that the coefficient of sliding friction,  $\mu_k$ , is 0.20.



1. According to the definition of work done by a force, what is the work associated with the external force? Is the work positive or negative? Show your calculation.
2. According to our discussion above of the work done by a friction force, what is the work associated with the friction force? Is the work positive or negative? Show your calculation.

(d) Suppose you lift a 0.3 kg object through a distance of 1.0 m at a constant velocity.



1. What is the work associated with the force that the earth exerts on the object? Is the work positive or negative? Show your calculation.
2. What is the work associated with the external force you apply to the object? Is the work positive or negative? Show your calculation.

### **Pulling at an Angle What Happens When the Force and the Displacement Are Not Along the Same Line?**

Let's be more quantitative about measuring force and distance and calculating the work. How should work be calculated when the external force and the displacement of an object are not in the same direction?



To investigate this, you will use a spring scale to measure the force necessary to slide a block along the table at a constant speed. Before you make your simple force measurements, you should put some weights on your block so that it slides along a smooth surface at a constant velocity even when it is being pulled with a force that is 30 or 60 degrees from the horizontal.

#### **Activity 4: Calculating Work**

(a) Hold a spring scale horizontal to the table and use it to pull the block a distance of 0.5 meters along the horizontal surface in such a way that the block moves at a constant speed. Record the force in newtons and the distance in meters in the space below and calculate the work done on the block in joules. (Note that there is a special unit for work, the joule, or J for short. One joule is equal to one newton times one meter, i.e.,  $J = N \cdot m$ .)

(b) Repeat the measurement, only this time pull on the block at a  $30^\circ$  angle with respect to the horizontal. Pull the block at about the same speed. Is the force needed larger or smaller than you measured in part (a)?

(c) Repeat the measurement once more, this time pulling the block at a  $60^\circ$  angle with respect to the horizontal. Pull the block at about the same speed as before.

(d) Assuming that the actual physical work done in part (b) is the same as the physical work done in part (a) above, how could you enhance the mathematical definition of work so that the forces measured in part (b) could be used to calculate work? In other words, use your data to postulate a mathematical equation that relates the physical work,  $W$ , to the magnitude of the applied force,  $F$ , the magnitude of the displacement,  $\Delta s$ , and the angle,  $\theta$ , between  $F$  and  $\Delta s$ . Explain your reasoning. Hint:  $\sin 30^\circ = .500$ ,  $\sin 60^\circ = .866$ ,  $\cos 30^\circ = .866$ ,  $\cos 60^\circ = .500$ .

### Work as a Dot Product

Review the definition of dot (or scalar) product as a special product of two vectors in your textbook, and convince yourself that the dot product can be used to define physical work in general cases when the force is constant but not necessarily in the direction of the displacement resulting from it.

$$W = \mathbf{F} \cdot \Delta \mathbf{s}$$

### Activity 5: How Much Work Goes with Each Job?

(a) Re-examine the descriptions of the jobs shown in the figure on the first page of this section. What is the minimum physical work done in job 1? Note that the data are given in British units, so the work will be expressed in foot pounds (ft lbs), not newton meters (joules).

(b) What is the minimum physical work done in job 2?

(c) What is the minimum physical work is done in job 3?



(d) Was your original intuition about which job to take correct? Which job should Richmond Load 'n' Go try to land?

### The Concept of Power

People are interested in more than physical work. They are also interested in the rate at which physical work can be done. Average power,  $\langle P \rangle$ , is defined as the ratio of the amount of work done,  $\Delta W$ , to the time interval,  $\Delta t$ , it takes to do the work, so that

$$\langle P \rangle = \frac{\Delta W}{\Delta t}.$$

Instantaneous power is given by the derivative of work with respect to time, or

$$P = \frac{dW}{dt}.$$

If work is measured in joules and time in seconds then the fundamental unit of power is in joules/second where 1 joule/second equals one watt. However, a more traditional unit of power is the horsepower, which represents the rate at which a typical work horse can do physical work. It turns out that *1 horsepower (or hp) = 746 watts = 746 joules/second*.

Those of you who are car buffs know that horsepower is a big deal in rating high performance cars. The hp in a souped-up car is in the hundreds. How does your stair climbing ability stack up? Let's see how long it takes you to climb the two stories of stairs in the science center.

### Activity 6: Rate the Horsepower in Your Legs

(a) Determine the time it takes you to climb the two flights of stairs in the science center. Then measure the height of the climb and compute the work done against the force of gravity.

(b) Compute the average power,  $\langle P \rangle$ , you expended in hp. How does this compare to the horsepower of your favorite automobile? If you're not into cars, how do you stack up against a horse?

## 18 Conservation of Mechanical Energy<sup>16</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

- To understand the concept of potential energy.
- To investigate the conditions under which mechanical energy is conserved.

### Overview

The last unit on work and energy culminated with a mathematical proof of the work-energy theorem for a mass falling under the influence of the force of gravity. We found that when a mass starts from rest and falls a distance  $y$ , its final velocity can be related to  $y$  by the familiar kinematic equation

$$v_f^2 = v_i^2 + 2gy \quad \text{or} \quad gy = \frac{1}{2}(v_f^2 - v_i^2) \quad [Eq. 1]$$

where  $v_f$  is the final velocity and  $v_i$  is the initial velocity of the mass.

We believe this equation is valid because: (1) you have derived the kinematic equations mathematically using the definitions of velocity and constant acceleration, and (2) you have verified experimentally that masses fall at a constant acceleration. We then asked whether the transformation of the mass from a speed  $v_i$  to a speed  $v_f$  is related to the work done on the mass by the force of gravity as it falls.

The answer is mathematically simple. Since  $F_g = mg$ , the work done on the falling object by the force of gravity is given by

$$W_g = F_g y = mgy \quad [Eq. 2]$$

But according to Equation 1,  $gy = \frac{1}{2}v_f^2 - \frac{1}{2}v_i^2$ , so we can re-write Equation 2 as

$$W_g = mgy = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \quad [Eq. 3]$$

The  $\frac{1}{2}mv_f^2$  is a measure of the motion resulting from the fall. If we define it as the energy of motion, or, more succinctly, the kinetic energy, we can define a work-energy theorem for falling objects:

$$W = \Delta K \quad [Eq. 4]$$

or, the work done on a falling object by the earth is equal to the change in its kinetic energy as calculated by the difference between the final and initial kinetic energies.

If external work is done on the mass to raise it through a height  $y$  (a fancy phrase meaning “if some one picks up the mass”), it now has the potential to fall back through the distance  $y$ , gaining kinetic energy as it falls. Aha! Suppose we define *potential energy* to be *the amount of external work,  $W_{ext}$ , needed to move a mass at constant velocity through a distance  $y$  against the force of gravity*. Since this amount of work is positive while the work done by the gravitational force has the same magnitude but is negative, this definition can be expressed mathematically as

$$U = W_{ext} = mgy \quad [Eq. 5]$$

Note that when the potential energy is a maximum, the falling mass has no kinetic energy but it has a maximum potential energy. As it falls, the potential energy becomes smaller and smaller as the kinetic energy increases. The kinetic and potential energy are considered to be two different forms of mechanical energy. What about the total mechanical energy, consisting of the sum of these two energies? Is the total mechanical energy constant

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during the time the object falls? If it is, we might be able to hypothesize a law of conservation of mechanical energy as follows: *In some systems, the sum,  $E$ , of the kinetic and potential energy is a constant.* This hypothesis can be summarized mathematically by the following statement.

$$E = K + U = \text{constant} \quad [\text{Eq. 6}]$$

The idea of mechanical energy conservation raises a number of questions. Does it hold quantitatively for falling masses? How about for masses experiencing other forces, like those exerted by a spring? Can we develop an equivalent definition of potential energy for the mass-spring system and other systems and re-introduce the hypothesis of conservation of mechanical energy for those systems? Is mechanical energy conserved for masses experiencing frictional forces, like those encountered in sliding?

In this unit, you will explore whether or not the mechanical energy conservation hypothesis is valid for a falling mass.

### Activity 1: Mechanical Energy for a Falling Mass

Suppose a ball of mass  $m$  is dropped from a height  $h$  above the ground.

(a) Where is  $U$  a maximum? A minimum?

(b) Where is  $K$  a maximum? A minimum?

(c) If mechanical energy is conserved what should the sum of  $K + U$  be for any point along the path of a falling mass?

### Mechanical Energy Conservation

How do people in different reference frames near the surface of the earth view the same event with regard to mechanical energy associated with a mass and its conservation? Suppose the president of your college drops a 2.0-kg water balloon from the second floor of the administration building (10.0 meters above the ground). The president takes the origin of his or her vertical axis to be even with the level of the second floor. A student standing on the ground below considers the origin of his coordinate system to be at ground level. Have a discussion with your classmates and try your hand at answering the questions below.

### Activity 2: Mechanical Energy and Coordinate Systems

(a) What is the value of the potential energy of the balloon before and after it is dropped according to the president? According to the student? Show your calculations and don't forget to include units!

The president's perspective is that  $y = 0.0$  m at  $t = 0$  s and that  $y = -10.0$  m when the balloon hits the student):

$$U_i =$$

$$U_f =$$

The student's perspective is that  $y = 10.0$  m at  $t = 0$  s and that  $y = 0.0$  m when the balloon hits:

$$U_i =$$

$$U_f =$$

Note: If you get the same potential energy value for the student and the president, you are on the wrong track!

(b) What is the value of the kinetic energy of the balloon before and after it is dropped according to the president? According to the student? Show your calculations. Hint: Use a kinematic equation to find the velocity of the balloon at ground level.

President's perspective:

$$K_i =$$

$$K_f =$$

Student's perspective:

$$K_i =$$

$$K_f =$$

Note: If you get the same values for both the student and the president for values of the kinetic energies you are on the right track!

(c) What is the value of the total mechanical energy of the balloon before and after it is dropped according to the president? According to the student? Show your calculations. Note: If you get the same values for both the student and the president for the total energies you are on the wrong track!!!!

President's perspective:

$$E_i =$$

$$E_f =$$

Student's perspective:

$$E_i =$$

$$E_f =$$

(d) Why don't the two observers calculate the same values for the mechanical energy of the water balloon?

(e) Why do the two observers agree that mechanical energy is conserved?

## 19 Momentum and Momentum Change<sup>17</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

- To understand the definition of momentum and its vector nature as it applies to one-dimensional collisions.
- To reformulate Newton's second law in terms of change in momentum, using the fact that Newton's "motion" is what we refer to as momentum.
- To develop the concept of impulse to explain how forces act over time when an object undergoes a collision.
- To use Newton's second law to develop a mathematical equation relating impulse and momentum change for any object experiencing a force.

### Overview

In the next few units we will explore the forces of interaction between two or more objects and study the changes in motion that result from these interactions. We are especially interested in studying collisions and explosions in which interactions take place in fractions of a second or less. Early investigators spent a considerable amount of time trying to observe collisions and explosions, but they encountered difficulties. This is not surprising, since the observation of the details of such phenomena requires the use of instrumentation that was not yet invented (such as the high speed camera). However, the principles of the outcomes of collisions were well understood by the late seventeenth century, when several leading European scientists (including Sir Isaac Newton) developed the concept of "quantity of motion" to describe both elastic collisions (in which objects bounce off each other) and inelastic collisions (in which objects stick together). These days we use the word momentum rather than motion in describing collisions and explosions.

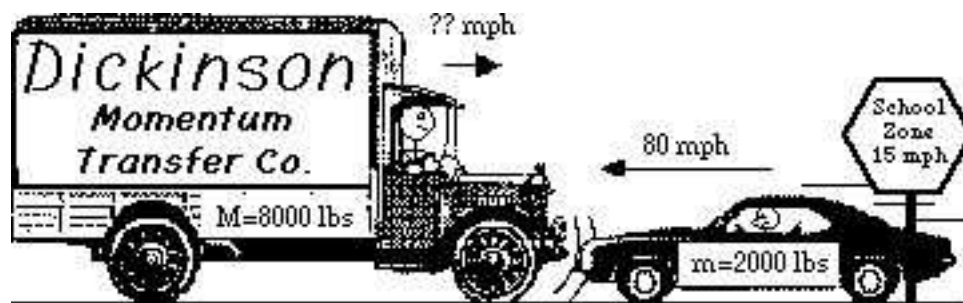
We will begin our study of collisions by exploring the relationship between the forces experienced by an object and its momentum change. It can be shown mathematically from Newton's laws and experimentally from our own observations that the integral of force experienced by an object over time is equal to its change in momentum. This time-integral of force is defined as a special quantity called impulse, and the statement of equality between impulse and momentum change is known as the impulse-momentum theorem.

### Apparatus

- Dynamics carts (2) and track

### Defining Momentum

In this session we are going to develop the concept of momentum to predict the outcome of collisions. But you don't officially know what momentum is because we haven't defined it yet. Let's start by predicting what will happen as a result of a simple one-dimensional collision. This should help you figure out how to define momentum to enable you to describe collisions in mathematical terms.



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It's early fall and you are driving along a two lane highway in a rented moving van. It is full of all of your possessions so you and the loaded truck were weighed in at 8000 lbs. You have just slowed down to 15 MPH because you're in a school zone. It's a good thing you thought to do that because a group of first graders is just starting to cross the road. Just as you pass the children you see a 2000 lb sports car in the oncoming lane heading straight for the children at about 80 MPH. What a fool the driver is! A desperate thought crosses your mind. You figure that you just have time to swing into the oncoming lane and speed up a bit before making a head-on collision with the sports car. You want your truck and the sports car to crumple into a heap that sticks together and doesn't move. Can you save the children or is this just a suicidal act? For simulated observations of this situation you can use two carts of different masses set up to stick together in trial collisions.

**Activity 1: Can You Stop the Car?**

(a) Predict how fast you would have to be going to completely stop the sports car. Explain the reasons for your prediction.

(b) Try some head on collisions with the carts of different masses to simulate the event. Describe some of your observations. What happens when the less massive cart is moving much faster than the more massive cart? Much slower? At about the same speed?

(c) Based on your intuitive answers in parts (a) and (b) and your observations, what mathematical definition might you use to describe the momentum (or motion) you would need to stop an oncoming vehicle traveling with a known mass and velocity?

Just to double check your reasoning, you should have come to the conclusion that momentum is defined by the vector equation

$$\mathbf{p} = m\mathbf{v}.$$

**Expressing Newton's Second Law Using Momentum**

Originally Newton did not use the concept of acceleration or velocity in his laws. Instead he used the term "motion," which he defined as the product of mass and velocity (a quantity we now call momentum). Let's examine a translation from Latin of Newton's first two laws (with some parenthetical changes for clarity).

*Newton's First Two Laws of Motion*

1. *Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed on it.*
2. *The (rate of) change of motion is proportional to the motive force impressed: and is made in the direction in which that force is impressed.*

The more familiar contemporary statement of the second law is that the net force on an object is the product of its mass and its acceleration where the direction of the force and of the resulting acceleration are the same. Newton's statement of the law and the more modern statement are mathematically equivalent, as you will show.

**Activity 2: Re-expressing Newton's Second Law**

(a) Write down the contemporary mathematical expression for Newton's second law relating net force to mass and acceleration. Please use vector signs and a summation sign where appropriate.

(b) Write down the definition of instantaneous acceleration in terms of the rate of change of velocity. Again, use vector signs.

(c) It can be shown that if an object has a changing velocity and a constant mass then  $m \frac{d\mathbf{v}}{dt} = \frac{d(m\mathbf{v})}{dt}$ . Explain why.

(d) Show that  $\sum \mathbf{F} = m\mathbf{a} = \frac{d\mathbf{p}}{dt}$ .

(e) Explain in detail why Newton's statement of the second law and the mathematical expression  $\sum \mathbf{F} = \frac{d\mathbf{p}}{dt}$  are two representations of the same statement, i.e., are logically equivalent.

**Momentum Change and Collision Forces***What's Your Intuition?*

You are sleeping in your sister's room while she is away at college. Your house is on fire and smoke is pouring into the partially open bedroom door. The room is so messy that you cannot get to the door. The only way to close the door is to throw either a blob of clay or a super ball at the door — there's not enough time to throw both.

**Activity 3: What Packs the Biggest Wallop-A Clay Blob or a Super ball?**

Assuming that the clay blob and the super ball have the same mass, which would you throw to close the door: the clay blob (which will stick to the door) or the super ball (which will bounce back with almost the same velocity it had before it collided with the door)? Give reasons for your choice, using any notions you already have or any new concepts developed in physics such as force, momentum, Newton's laws, etc. Remember, your life depends on it!

**Momentum Changes**

It would be nice to be able to use Newton's formulation of the second law of motion to find collision forces, but it is difficult to measure the rate of change of momentum during a rapid collision without special instruments.

However, measuring the momenta of objects just before and just after a collision is usually not too difficult. This led scientists in the seventeenth and eighteenth centuries to concentrate on the overall changes in momentum that resulted from collisions. They then tried to relate changes in momentum to the forces experienced by an object during a collision. In the next activity you are going to explore the mathematics of calculating momentum changes.

**Activity 4: Predicting Momentum Changes**

Which object undergoes the most momentum change during the collision with a door: the clay blob or the super ball? Explain your reasoning carefully.

Let's check your reasoning with some formal calculations of the momentum changes for both inelastic and elastic collisions. This is a good review of the properties of one-dimensional vectors. Recall that momentum is defined as a vector quantity that has both magnitude and direction. Mathematically, momentum change is given by the equation

$$\Delta \mathbf{p} = \mathbf{p}_f - \mathbf{p}_i$$

where  $\mathbf{p}_i$  is the initial momentum of the object just before and  $\mathbf{p}_f$  is its final momentum just after a collision.

**Activity 5: Calculating 1D Momentum Changes**

(a) Suppose a dead ball (or clay blob) is dropped on a table and “sticks” in such a way that it has an initial momentum just before it hits of  $\mathbf{p}_i = -p_{iy}\hat{\mathbf{j}}$  where  $\hat{\mathbf{j}}$  is a unit vector pointing along the positive y axis. Express the final momentum of the dead ball in the same vector notation.

(b) What is the change in momentum of the clay blob as a result of its collision with the table? Use the same type of unit vector notation to express your answer.

(c) Suppose that a live ball (or a super ball) is dropped on a table and “bounces” on the table in an elastic collision so that its speed just before and just after the bounce are the same. Also suppose that just before it bounces it has an initial momentum  $\mathbf{p}_i = -p_{iy}\hat{\mathbf{j}}$ , where  $\hat{\mathbf{j}}$  is a unit vector pointing along the positive y-axis. What is the final momentum of the ball in the same vector notation? Hint: Does the final  $\mathbf{p}$  vector point along the +y or -y axis?

(d) What is the change in momentum of the ball as a result of the collision? Use the same type of unit vector notation to express your result.

(e) The answer is not zero. Why? How does this result compare with your prediction? Discuss this situation.

(f) Suppose the mass of each ball is 0.2 kg and that they are dropped from 1 m above the table. Using this value for mass of the balls and a calculated value for the velocity of each of the balls just before they hit the table,



you can calculate the momentum just before the collision  $\mathbf{p}_i$  for each of the balls. Also calculate the momentum of the balls just after the collision  $\mathbf{p}_f$  and the change in momentum  $\Delta\mathbf{p}$  for each ball. Show your calculations in the space below.

### **Applying Newton's Second Law to the Collision Process (The Egg Toss)**

Suppose somebody tosses you a raw egg and you catch it. In physics jargon, one would say (in a very official tone of voice) that “the egg and the hand have undergone an inelastic collision.” What is the relationship between the force you have to exert on the egg to stop it, the time it takes you to stop it, and the momentum change that the egg experiences? You ought to have some intuition about this matter. In more ordinary language, would you catch an egg slowly or fast?

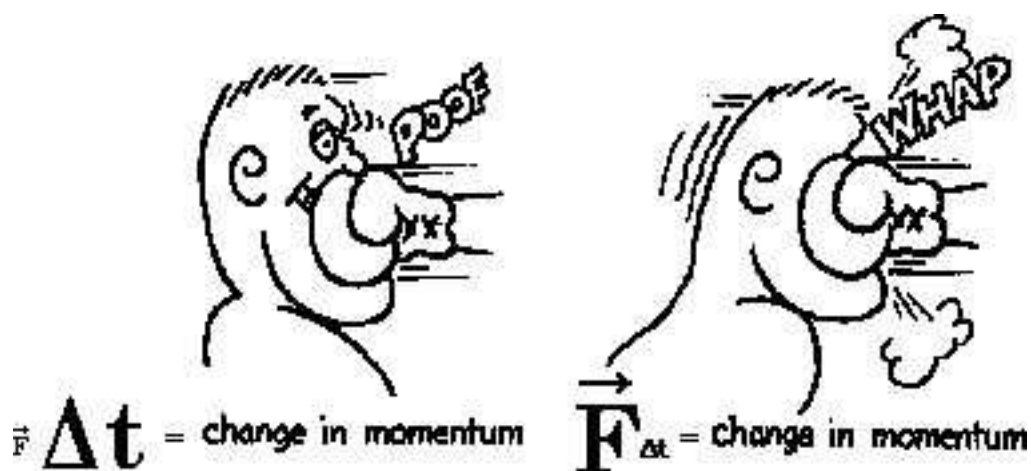
### **Activity 6: Momentum Changes and Average Forces on an Egg: What's Your Intuition?**

(a) If you catch an egg of mass  $m$  that is heading toward your hand at speed  $v$ , what is the magnitude of the momentum change that it undergoes?

(b) Does the total momentum change differ if you catch the egg more slowly or is it the same?

(c) Suppose the time you take to bring the egg to a stop is  $\Delta t$ . Would you rather catch the egg in such a way that  $\Delta t$  is small or large? Why?

(d) What do you suspect might happen to the average force you exert on the egg while catching it when  $\Delta t$  is small?



You can use Newton's second law to derive a mathematical relationship between momentum change, force, and collision times for objects. This derivation leads to the impulse-momentum theorem. Let's apply Newton's second law to the egg catching scenario.

#### Activity 7: Force and Momentum Change

(a) Sketch an arrow representing the magnitude and direction of the force exerted by your hand on the egg as you catch it.



(b) Write the mathematical expression for Newton's second law in terms of the net force and the time rate of change of momentum. (See Activity 2(e) for details.)

(c) Explain why, if  $\mathbf{F}$  is a constant during the collision lasting a time  $\Delta t$ , then  $\frac{d\mathbf{p}}{dt} = \frac{\Delta\mathbf{p}}{\Delta t}$ .

(d) Show that for a constant force  $\mathbf{F}$  the change in momentum is given by  $\Delta\mathbf{p} = \mathbf{F} \Delta t$ . Note that for a constant force, the term  $\mathbf{F} \Delta t$  is known as the impulse given to one body by another.

## 20 Introduction to Rotation<sup>18</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

To understand the definitions of angular velocity and angular acceleration and the kinematic equations for rotational motion on the basis of observations. We will discover the relationship between linear velocity and angular velocity and between linear acceleration and angular acceleration.

### Apparatus

- A rotator consisting of an axle, a metal disk, and a fixture to hold the disk.
- A stopwatch.
- A meter stick, drawing compass, flexible ruler, protractor, and some string.

### Overview

Earlier in the course, we studied centripetal force and acceleration, which characterize circular motion. In general, however, we have focused on studying motion along a straight line as well as the motion of projectiles. We have defined several measurable quantities to help us describe linear and parabolic motion, including position, velocity, acceleration, force, and mass. In the real world, many objects undergo circular motion and/or rotate while they move. The electron orbiting a proton in a hydrogen atom, an ice skater spinning, and a hammer that tumbles about while its center of mass moves along a parabolic path are just three of many rotating objects.

Since many objects undergo rotational motion it is useful to be able to describe their motions mathematically. The study of rotational motion is also very useful in obtaining a deeper understanding of the nature of linear and parabolic motion.

We are going to try to define several new quantities and relationships to help us describe the rotational motion of rigid objects, i.e., objects that do not change shape. These quantities will include angular velocity, angular acceleration, rotational inertia and torque. We will then use these new concepts to develop an extension of Newton's second law to describe rotational motion for masses more or less concentrated at a single point in space (e.g., the electron in the hydrogen atom) and for extended objects (like the tumbling hammer).

### Rigid vs. Non-rigid Objects

We will begin our study of rotational motion with a consideration of some characteristics of the rotation of rigid objects about a fixed axis of rotation. The motions of objects, such as clouds, that change size and shape as time passes are hard to analyze mathematically. In this unit we will focus primarily on the study of the rotation of particles and rigid objects around an axis that is not moving. A rigid object is defined as an object that can move along a line or can rotate without the relative distances between its parts changing.

Shown in the figure below are examples of a non-rigid object in the form of a cloud that can change shape and of a rigid object in the form of an empty coffee cup that does not change shape.



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A hammer tossed end over end and an empty coffee cup are examples of rigid objects. A ball of clay that deforms permanently in a collision and a cloud that grows are examples of non-rigid objects.

### A Puzzler

Use your imagination to solve the rotational puzzler outlined below. It's one that might stump someone who hasn't taken physics.

#### Activity 1: Horses of a Different Speed

You are on a white horse, riding off at sunset with your beau on a chestnut mare riding at your side. Your horse has a speed of 4.0 m/s and your beau's horse has a speed of 3.5 m/s, yet he/she constantly remains at your side. Where are your horses? Make a sketch to explain your answer.



### Review of the Geometry of Circles

Remember way back before you came to college when you studied equations for the circumference and the area of a circle? Let's review those equations now, since you'll need them a lot from here on in.

#### Activity 2: Circular Geometry

(a) What is the equation for the circumference,  $C$ , of a circle of radius  $r$ ? What are the units of  $C$ ?

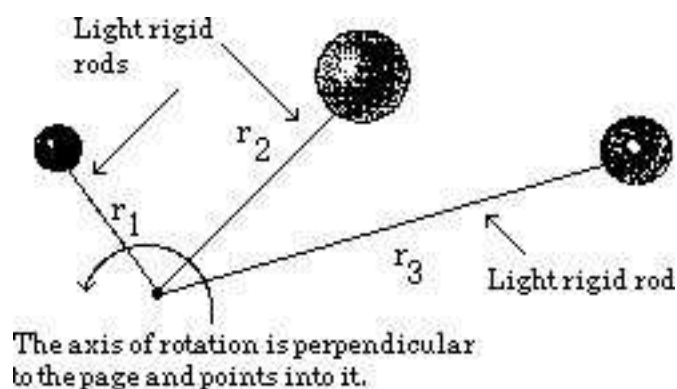


(b) What is the equation for the area,  $A$ , of a circle of radius  $r$ ? What are the units of  $A$ ?

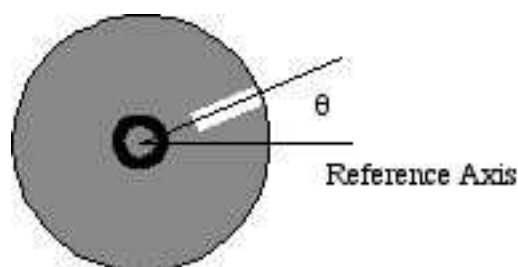
(c) If someone told you that the area of a circle was  $A = r$ , how could you refute them immediately? What's wrong with the idea of area being proportional to  $r$ ?

### Distance from an Axis of Rotation and Speed

Let's begin our study by examining the rotation of objects about a common axis that is fixed. What happens to the speeds of different parts of a rigid object that rotates about a common axis? How does the speed of the object depend on its distance from an axis? You should be able to answer this question by observing the rotational speed of the rotator at each experimental station.



Place the disk in the fixture and slowly rotate it a constant speed. The figure below shows the rotator and the definition of angular displacement.



### Activity 3: Spinning the Rotator: Speed vs. Radius

(a) Measure how long it takes the white marker to sweep through a known angle. Record the time and the angle in the space below.

(b) Calculate the distance of the paths traced out by the outer edge of the white marker and the inner edge as it rotated through the angle you just recorded. (Note: What do you need to measure to perform this calculation?) Record your data below.

(c) Calculate the average speed of the outer edge of the white marker and the average speed of the inner edge of the marker. How do they compare?

(d) Do the speeds seem to be related in any way to the distances of inner and outer edges of the white marker from the axis of rotation? If so, describe the relationship mathematically.

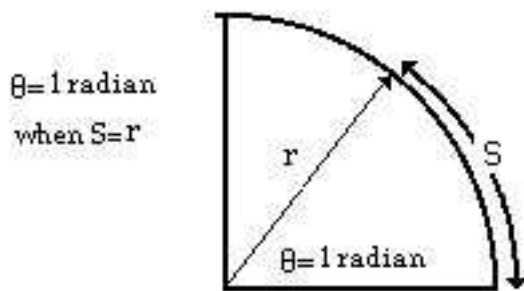
(e) As the disk rotates, does the distance from the axis of rotation to the outer edge of the white marker change?

- (f) As the disk rotates, does the distance from the axis of rotation to the inner edge of the white marker change?
- (g) At any given time during the rotation, is the angle between the reference axis and the inner edge of the white marker the same as the angle between the axis and the outer edge of the white marker, or do the angles differ?
- (h) At any given time during the rotation, is the rate of change of the angle between the reference axis and the inner edge of the white marker the same as the rate of change of the angle between the axis and the outer edge, or do the rates differ?
- (i) What happens to the linear velocity,  $\mathbf{v}$ , of the outer edge of the marker as it rotates at a constant rate? Hint: What happens to the magnitude of the velocity, i.e., its speed? What happens to its direction?
- (j) Is the outer edge of the white marker accelerating? Why or why not?

### Radians, Radii, and Arc Lengths

An understanding of the relationship between angles in radians, angles in degrees, and arc lengths is critical in the study of rotational motion. There are two common units used to measure angles: degrees and radians.

1. A degree is defined as  $1/360$ th of a rotation in a complete circle.
2. A radian is defined as the angle for which the arc along the circle is equal to its radius as shown in the figure below.



In the next series of activities you will be relating angles, arc lengths, and radii for a circle.

**Activity 4: Relating Arcs, Radii, and Angles**

(a) Let's warm up with a review of some very basic mathematics. What should the constant of proportionality be between the circumference of a circle and its radius? Write the appropriate equation.

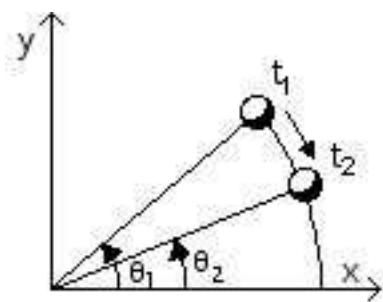
(b) Approximately how many degrees are in one radian? Let's do this experimentally. Using the compass draw a circle and measure its radius. Then, use the flexible ruler to trace out a length of arc,  $s$ , that has the same length as the radius. Next measure the angle in degrees that is subtended by the arc.

(c) Theoretically, how many degrees are in one radian? Please calculate your result to three significant figures. Using the equation for the circumference of a circle as a function of its radius and the constant  $\pi = 3.1415927\dots$ , figure out a general equation to find degrees from radians. **Hint:** How many times does a radius fit onto the circumference of a circle? How many degrees fit in the circumference of a circle?

(d) If an object moves 30 degrees on the circumference of a circle of radius 1.5 m, what is the length of its path?

(e) If an object moves 0.42 radians on the circumference of a circle of radius 1.5 m, what is the length of its path?

(f) Remembering the relationship between the speed of the outer edge of the rotator and the distance,  $r$ , from the rotator's axis the outer edge, what equations would you use to define the magnitude of the average "angular" velocity,  $\langle\omega\rangle$ ? **Hint:** In words,  $\langle\omega\rangle$  is defined as the amount of angle swept out by the object per unit time. Note that the answer is not simply  $\theta/t$ !



- (g) How many radians are there in a full circle consisting of 360 degrees?
- (h) When an object moves in a complete circle in a fixed amount of time, what quantity (other than time) remains unchanged for circles of several different radii?

### Relating Linear and Angular Quantities

It's very useful to know the relationship between the variables  $s$ ,  $v$ , and  $a$ , which describe linear motion and the corresponding variables  $\theta$ ,  $\omega$ , and  $\alpha$ , which describe rotational motion. You now know enough to define these relationships.

#### Activity 5: Linear and Angular Variables

- (a) Using the definition of the radian, what is the general relationship between a length of arc,  $s$ , on a circle and the variables  $r$  and  $\theta$  in radians.
- (b) Assume that an object is moving in a circle of constant radius,  $r$ . Using the relationship you found in part (a) above, take the derivative of  $s$  with respect to time to find the velocity of the object. Show that the magnitude of the linear velocity,  $v$ , is related to the magnitude of the angular velocity,  $\omega$ , by the equation  $v = \omega r$ .
- (c) Assume that an object is accelerating in a circle of constant radius,  $r$ . Using the relationship you found in part (b) above, take the derivative of  $v$  with respect to time to find the tangential acceleration of the object. Show that the linear acceleration,  $a_t$ , tangent to the circle is related to the angular acceleration,  $\alpha$ , by the equation  $a_t = \alpha r$ .

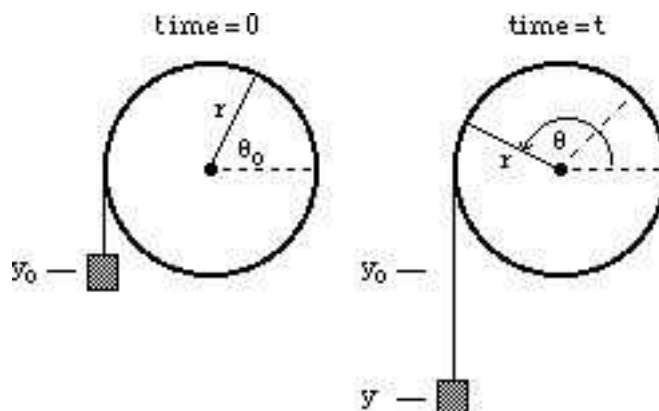
### The Rotational Kinematic Equations for Constant $\alpha$

The set of definitions of angular variables are the basis of the physicist's description of rotational motion. We can use them to derive a set of kinematic equations for rotational motion with constant angular acceleration that are similar to the equations for linear motion.

#### Activity 6: The Rotational Kinematic Equations



The figure below shows a massless string wound around a spool of radius  $r$ . The mass falls with a constant acceleration,  $a$ . Refer to this figure and the results of Activity 5 to answer the following questions.



(a) What is the equation for  $\theta$  in terms of  $y$  and  $r$ ?

(b) What is the equation for  $\omega$  in terms of  $v$  and  $r$ ?

(c) What is the equation for  $\alpha$  in terms of  $a$  and  $r$ ?

(d) Consider the falling mass in the figure above. Suppose you are standing on your head so that the positive  $y$ -axis is pointing down. Using the relationships between the linear and angular variables in parts (a), (b), and (c), derive the rotational kinematic equations for constant accelerations for each to the linear kinematic equations listed below. **Warning:** Don't just write the analogous equations! Show the substitutions needed to derive the equations on the right from those on the left.

1.  $v = v_0 + at$   $\omega =$

2.  $y = y_0 + v_0 t + \frac{1}{2}at^2$   $\theta =$

3.  $v^2 = v_0^2 + 2ay$   $\omega^2 =$

## 21 Conservation of Angular Momentum

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

To test the Law of Conservation of Angular Momentum and to explore the applicability of angular momentum conservation among objects that experience no external torques.

### Apparatus

- A Rotating Disk System
- A mass of 1 kg
- A meter stick and a ruler
- A small water bubble level
- A video analysis system (*VideoPoint*)

### Overview

As a consequence of Newton's laws, angular momentum like linear momentum is believed to be conserved in isolated systems. This means that, no matter how many internal interactions occur, the total angular momentum of any system should remain constant if there are no external torques. When one of the objects gains some angular momentum another part of the system must lose the same amount. If angular momentum isn't conserved, then we believe that there is some outside torque acting on the system. By expanding the boundary of the system to include the source of that torque we can always preserve the Law of Angular Momentum Conservation.

In this unit you will test the notion of the conservation of angular momentum. As in the test of the conservation of linear momentum, we will investigate what happens when two bodies undergo a "rotational" collision. You will drop a large weight onto a rotating disk and determine the moment of inertia, the angular speed, and finally, the angular momentum of the rotator-disk-weight system before and after this perfectly inelastic collision.

### Activity 1: The Moment of Inertia Before and After the Collision

(a) Calculate the theoretical value of the moment of inertia of the metal disk using basic measurements of its radius and mass. Be sure to state units and show the expression you used!

 $r_d =$  $M_d =$  $I_d =$ 

(b) The rotating fixture that holds the disk has a complex shape. We have determined its moment of inertia without the disk and recorded the result. Record that value here. Be sure to state units.

 $I_f =$ 

(c) After dropping the weight on the rotating disk, the system will have a new moment of inertia. Derive a formula for the moment of inertia of a cylindrical-shaped weight of mass  $m_w$  and radius  $r_w$  revolving about the origin at a distance,  $r_r$ . (You will have to use the parallel axis theorem to do this.)

 $I_w =$

(d) Measure the mass of the weight and use a vernier caliper to measure its diameter.

$$m_w =$$

$$r_w =$$

(e) Come up with a formula for the moment of inertia,  $I$ , of the whole system before and after the collision and calculate the moment of inertia before the collision only. (The moment of inertia after the collision will be determined AFTER you do the experiment.) Don't forget to include the units.

$$I_{\text{before}} =$$

$$I_{\text{after}} =$$

### Activity 2: Making a Movie of the Collision

(a) Place the video camera about 1 m above the rotator, align the camera with the center of the rotator using the pendulum, and center the rotator in the field of view of the camera by viewing it with the *VideoPoint Capture* software. Place a ruler of known length in the field of view of the camera and parallel to one side of the frame. Check that the rotator is flat with the small water-bubble level.

(b) Give the rotator a push and begin recording its motion with the video camera. See **Appendix D: Video Analysis** for details. While the rotator is moving hold the 1 kg weight near the rim of the metal disk and close to, but not quite touching, the surface of the moving metal disk. After at least one revolution of the metal disk drop the 1-kg mass onto the disk and record the motion of the disk for at least one revolution afterward.

(c) Determine the distance of the center of the weight you dropped from the center of the rotator  $r_r$ . To do this, measure the distance from the center of the rotator to the edge of the weight  $r_{\text{edge}}$  and use the result from Activity 1 part (d) for the diameter of the weight. Calculate the distance from the origin to the center of the weight  $r_r$ . Use these results and those from Activity 1 part (e) to calculate the final moment of inertia.

$$r_{\text{edge}} =$$

$$r_w =$$

$$r_r =$$

$$I_{\text{after}} =$$

(d) Determine the angular speed before and after the collision. To do this task see the instructions in **Appendix D: Video Analysis** for creating and analyzing a movie file.

1. Find the last frame before you dropped the weight on the rotator and click on the position of the white marker on the metal disk. Under the **Edit** menu highlight **Leave/Hide Trails**. Now go backward through the film until the rotator has gone through one full rotation. Estimate to the nearest fraction of a frame how many frames there are in one revolution. You also need to know the time between frames  $\Delta t_{\text{frame}}$ , which you can get from the data table in *VideoPoint*.

$$N_{\text{before}} =$$

$$\Delta t_{\text{frame}} =$$

Calculate the time for one revolution before the collision and the angular speed.

$$t_{\text{before}} =$$

$$\omega_{\text{before}} =$$

2. We now follow a similar procedure to determine the angular speed after the collision. Under the **Edit** Menu highlight **Clear All...** to get rid of your previous results. Find the first frame after you dropped

the weight on the rotator and click on the position of the white marker. Now click forward and estimate to the nearest fraction of a frame the number of frames in one full revolution.

$$N_{\text{after}} = \qquad \Delta t_{\text{frame}} =$$

Calculate the time for one revolution and the angular speed after the collision.

$$t_{\text{after}} = \qquad \omega_{\text{after}} =$$

(e) Calculate the angular momentum before and after the collision. Calculate the percent difference between the two results. Is angular momentum conserved?

$$L_{\text{before}} =$$

$$L_{\text{after}} =$$

(f) Would the procedure you followed above change if the weight was moving horizontally at a constant velocity when you dropped it? If it changed, what would be different?

## 22 Hooke's Law

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives:

- To explore the nature of elastic deformation and restoring forces

### Introduction

There is no such thing as a perfectly rigid body. The stiffest of metal bars can be twisted, bent, stretched, and compressed. Delicate measurements show that even small forces cause these distortions. Under certain circumstances (typically, when the forces are not too large), a body deformed by forces acting upon it will return to its original size and shape when the forces are removed, a capacity known as elasticity. Permanent distortion from large forces is referred to as plastic deformation. In this lab, you will stay within the elastic limit.

### Apparatus:

- two springs and supports
- collection of masses
- 2-meter stick

### Activity:

1. Suspend one of the springs from the support. Using the meter stick, observe the position of the lower end of the spring and record the value in the table below.
2. Hang 100 grams from the lower end of the spring and again record the position of this end.
3. Repeat 3 with loads of 200, 300, 400, and 500 grams hung from the spring.
4. Repeat 2, 3 and 4 with the second spring.

Mass suspended from spring 1 (g)	Position reading (m)	Elongation (m)	Mass suspended from spring 2 (g)	Position reading (m)	Elongation (m)
0		0	0		0
100			100		
200			200		
300			300		
400			400		
500			500		

5. Determine the elongation produced by each load.
6. Plot a graph using the values of the elongation as the abscissas (x values) and the forces due to the corresponding loads as ordinates (y values) for each spring. Make sure you use compatible units. Write the equation for each curve in the space below.

**Questions:**

1. What do your graphs show about the dependence of each spring's elongation upon the applied force?
2. List the proportionality constant (including proper units) for each spring in the space below.
3. The slope of each line (the proportionality constant) is known as the force constant,  $k$ . Which spring has the larger force constant? Express in your own words the physical implications of different size force constants.

## 23 Periodic Motion

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

- To learn directly about some of the characteristics of periodic motion period, frequency, and amplitude.
- To investigate the relationships between position, velocity, acceleration, and force in simple harmonic motion.
- To investigate energy in simple harmonic motion.

### Introduction

Periodic motion is motion that repeats itself. You can see the repetition in the position-, velocity-, or acceleration-time graphs. The length of time to go through one cycle and begin to repeat the motion is called the period. The number of cycles in each second is called the frequency. The unit of frequency, cycles per second, is given a special name — Hertz.

### Motion of a Mass Hanging from a Spring

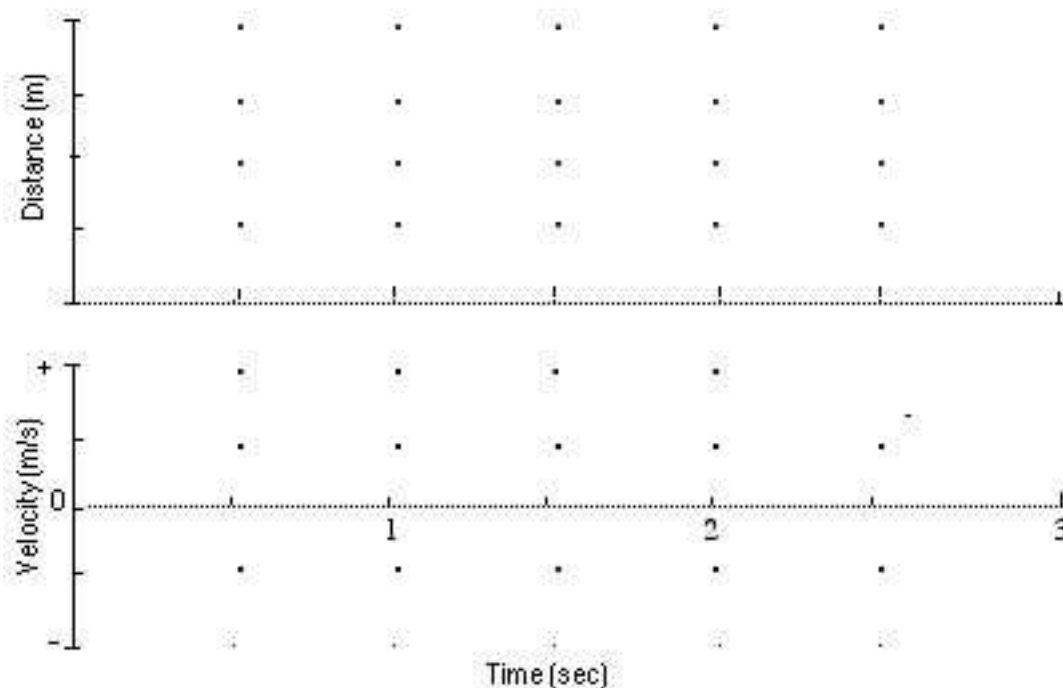
In this unit you will investigate the motion of a mass hanging from a spring.

### Apparatus

- Large spring
- Force probe
- Motion detector
- Variety of masses
- *Science Workshop 750 Interface*
- *DataStudio* software (Position & Velocity Graphs application and SHM application)

### Activity 1: Periodic Motion of a Mass-Spring System

(a) Open the **Position & Velocity Graphs** application in the **131 Workshop** menu. Hang the large spring from the force probe hook with the large diameter coils down and hang a 200-g mass from the spring. Place the motion detector facing up directly below the spring. Pull the mass downward about 15 cm, and let go. Adjust the height of the support so that the mass comes no closer than 0.15 m to the detector. Record data for a few seconds to display position-time and velocity-time graphs of the motion. Sketch the graphs on the axes below.



**Comment:** Note that when an object returns to the same position, it does not necessarily mean that a cycle is ending. It must return to the same position, and the velocity and acceleration must also return to the same values in both magnitude and direction for this to be the start of a new cycle.

(b) Label the graphs above with: “B” at the Beginning of a cycle and “E” at the End of the same complete cycle. “A” on each spot where the mass is moving Away from the detector fastest. “T” on each spot where the mass is moving Toward the detector fastest. “S” on each spot where the mass is standing Still. “F” where the mass is Farthest from the motion detector. “C” where the mass is Closest to the motion detector.

(c) Do the position and velocity graphs appear to have the same period? Do their peaks occur at the same times? If not, how are the peaks related in time?

(d) Use the Smart Tool to measure the period and frequency of the motion. (For better accuracy, measure the total time over as many cycles as possible and divide by the number of cycles.)

(e) Using the Smart Tool, determine and record the maximum displacement. Then record data with the mass at rest to find the equilibrium position. Draw a straight line on your position graph indicating the equilibrium position in terms of the distance from the motion detector. Calculate and record the amplitude of the motion (the difference between the maximum displacement and the equilibrium position).



**Simple Harmonic Motion**

The motion of a mass hanging from a spring that you looked at in Activity 1 is a close approximation to a kind of periodic motion called simple harmonic motion (sometimes abbreviated SHM).

**Activity 2: What Factors Determine the Period of the Mass-Spring System?**

What can you do to change the period of the SHM of the mass-spring system? What will happen to the period if you increase the amplitude? Increase the mass? Increase the spring constant (use a stiffer spring)?

**Activity 3: The Period of SHM and the Amplitude**

(a) Repeat the procedure of Activity 1, but with a different starting position (other than 15 cm). (Warning: Do not make the amplitude so large that the mass comes closer than 0.15 m from the motion detector.) When you have good graphs, find and record the period and the amplitude using the methods described in Activity 1.

(b) Take ratios of the period and the amplitude of Activity 1 to those determined here.

(c) Is there evidence that the period depends on amplitude? (Did the change in amplitude result in a comparable change in period?) Explain. How does this compare with your prediction?

**Activity 4: The Dependence of the Period of a SHM on the Mass**

(a) Carefully measure the period for two other masses. Record the masses and the measured periods in a table in the space below along with the mass and period from Activity 1.

(b) Does the period depend on the mass? Does it increase or decrease as mass is increased?

(c) Determine the mathematical relationship between the period  $T$  and the mass  $m$  by finding a function that fits the data. Write the equation that provides the best fit to the data in the space below.

**Comment:** You should have found that  $T$  is independent of amplitude and proportional to  $\sqrt{m}$ . The actual expression for the period is

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

### Velocity, Acceleration, Force and Energy

In this investigation you will look more carefully at the distance, velocity and acceleration graphs for simple harmonic motion. You will also look at the force graph, and will examine the energy associated with simple harmonic motion.

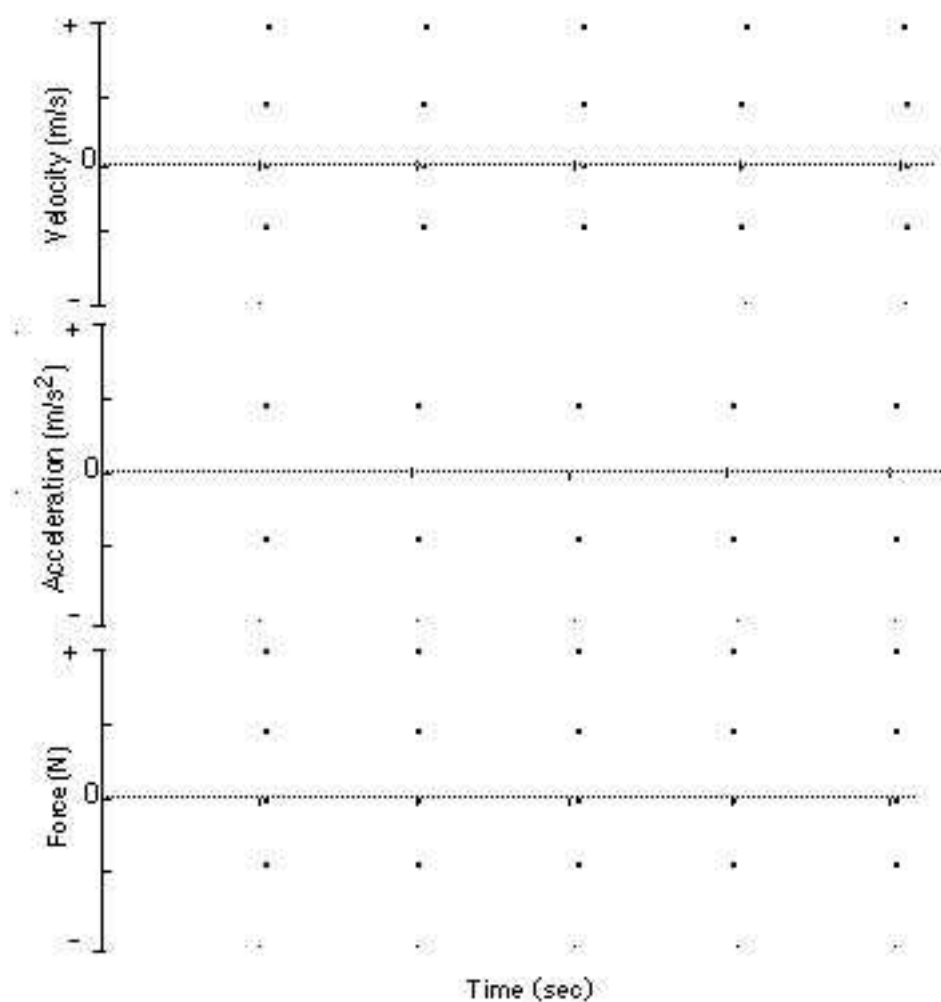
### Activity 5: Determination of the Spring Constant

Measure the distance the spring stretches for four different masses and use these data to determine the spring constant,  $k$ . Record your data and the result for  $k$  in the space below.

### Activity 6: Velocity, Acceleration, and Force for SHM

(a) Consider the motion you looked at in Activity 1 when the mass was 200 g and the initial position was 15 cm. Sketch the position and velocity graphs that you observed on the axes below using dashed lines.





(b) Based on what you know about the relationships between velocity, acceleration, and force, use dashed lines to sketch your predictions for the acceleration and force graphs.

(c) Suspend the 200-g mass from the spring and open the **SHM** application in the **131 Workshop** menu. Start the mass oscillating with an amplitude of 15 cm and record data for a few seconds. When you have obtained good graphs, sketch the results on the above axes using solid lines.

(d) When the mass is at its maximum distance from the detector, is the velocity maximum, minimum or some other value according to your graphs? Does this agree with your predictions? Does this agree with your observations of the oscillating mass? Explain.

(e) When the mass has its maximum positive velocity, is its distance from the detector maximum, minimum, the equilibrium value or some other value according to your graphs? What about when it reaches maximum negative velocity? Does this agree with your predictions? Does this agree with your observations of the oscillating mass? Explain.

(f) According to your graphs, for what distances from the detector is the acceleration maximum? For what distances is the acceleration zero? What is the velocity in each of these cases?

(g) Compare the force and acceleration-time graphs. Describe any similarities. Does the force graph agree with your prediction?

(h) From your graphs, what would you say is the relationship between force and acceleration?

(i) Compare the force and distance(position)-time graphs. What would you say is the relationship between force and position?

### Activity 7: Energy of a Mass Undergoing SHM

Now use the graphs from the last activity to examine the energy relationships in simple harmonic motion.

(a) At what points is the kinetic energy of the mass zero? Label these points on your distance and velocity graphs above with a K.

(b) Calculate the elastic potential energy due to the spring at one of these points. Label the point you use on your velocity and distance graphs with a 1. Use  $U = \frac{1}{2}kx^2$ , where  $x$  is the distance from the equilibrium position and  $k$  is the force constant of the spring, which you have already measured. Use the Smart Tool to measure  $x$ . Show your data and calculations in the space below.

(c) At what points is the potential energy zero? Label these points with a P on your distance and velocity graphs.

(d) If you measured the kinetic energy at one of these points, what would you expect its value to be? Explain.

(e) Check your prediction. Calculate the kinetic energy at one of these points. Label the point you use on your velocity graph with a 2. Use  $K = \frac{1}{2}mv^2$ . Use the Smart Tool to determine  $v$ . Show your data and calculations in the space below.

- (f) Did your calculated kinetic energy agree with your prediction?
- (g) If you calculated the potential and kinetic energies at a point where neither of these was zero, what would you expect the total energy to be? Explain.
- (h) Check your prediction. Pick a point where the mass has both kinetic and potential energy and calculate them both. Label this point on your distance and velocity graphs with a 3. Show your calculations.
- (i) Does your result agree with your prediction? Does it appear that energy is conserved? Explain.

## 24 The Period of a Pendulum

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

**Objectives:**

- Study the influences on the motion of a simple pendulum
- Calculate the acceleration due to gravity from measurements of the period and length of a simple pendulum

**Introduction:**

The period of a simple pendulum is related to its length and the acceleration due to gravity according to the relationship:

$$T = 2\pi\sqrt{\frac{L}{g}}. \quad [Eq. 1]$$

$T$  is the period,  $L$  is the length, and  $g$  is the gravitational acceleration. This assumes the oscillations are small. Let's check this prediction experimentally.

**Apparatus:**

- string attached to stand
- collection of masses
- stop watch
- meter stick

**Activity:**

1. With  $L = 1.0$  m, place a 1-kg mass at the end of your pendulum. Time twenty-five (25) oscillations of amplitude not greater than 10 degrees. The period is the total time divided by the number of oscillations. Calculate the period and enter the relevant data into the table below.
2. Repeat 1 for masses of 500 g, 200 g, 100 g, 50 g, and 20 g.

Trial No.	Mass (kg)	Length[L] (m)	No. of Oscillations	Total Time (s)	Period[T] (s)
1					
2					
3					
4					
5					
6					

3. With the 200-g mass, fix the length  $L$  to be 1.5 m. Time twenty-five (25) oscillations of amplitude not more than 10 degrees. Calculate the period and period squared, and enter the relevant data into the table below.
4. Repeat 3 for pendulum lengths of 1.0, 0.7, 0.4, 0.25, and 0.15 meters.

Trial No.	Mass (kg)	Length[L] (m)	No. of Oscillations	Total Time (s)	Period[T] (s)	$T^2$ (s <sup>2</sup> )
7						
8						
9						
10						
11						
12						

5. Plot  $T$  as a function of mass from the first set of data and  $T^2$  as a function of  $L$  from the second set of data on SEPARATE graphs. NOTE: Be sure the  $T$  versus mass graph contains the origin. (If you don't know how to do this, consult your instructor.) Fit the data and determine the slopes of the lines of each graph. Be sure to include UNITS with each slope.

slope: period versus mass \_\_\_\_\_

slope: period<sup>2</sup> versus length \_\_\_\_\_

**Questions:**

1. Interpret the slope of the period versus mass line: What is the relationship between mass and period? How does the period depend on the mass?
2. Interpret the slope of the period<sup>2</sup> versus length line: What is the relationship between length and period? How does the period depend on pendulum length?
3. If the length of the pendulum were  $\frac{1}{16}$  its original length, by how much would its period change?
4. Using the relationship between length and period (equation 1) and the slope you measured for the  $T^2$  vs  $L$  graph, determine the acceleration due to gravity  $g$ . Calculate the percent difference between your value and the accepted value of  $[9.8 \frac{\text{m}}{\text{s}^2}]$ .

## 25 Resonance in Tubes

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives:

- Determine the resonant frequency for a tube open at one end.
- Determine tube lengths at resonance for a tube of variable length.
- Determine the velocity of sound in air in the laboratory (two ways).

### Introduction

The Economy Resonance Tube is designed for the study of resonance in columns of air. The tube set includes a movable inner tube with a closed end and an outer tube which is open at both ends. The inner tube also includes a measuring tape to easily find the length of the air column. To adjust the length of the closed tube, simply slide the inner tube until the desired length appears on the measuring tape. Open tube experiments can also be performed by removing the inner tube.

In order that the tube resonate, the frequency of the vibrating air must coincide with the natural frequency of the tube (which may be its fundamental or one of its overtones). For the Economy Resonance Tube, which is closed at one end, this requirement is met if the tube length is an odd number of quarter wavelengths of the sound waves produced by the source ( $L = \lambda/4, 3\lambda/4, 5\lambda/4$ , etc., where  $L$  is the length of the tube and  $\lambda$  is the wave length of the sound). Note that if the length of the tube is gradually increased while the source is vibrating, the distance between successive resonance positions is  $\lambda/2$ .

**Note:** Due to edge effects at the open end of a tube, the effective length of the tube depends on the radius of the opening. Thus,  $L_{eff} = L + 0.6r$ , where  $L_{eff}$  is the *effective* length,  $L$  is the length measured, and  $r$  is the tube radius.

### Apparatus:

- Economy Resonance Tube
- Open speaker
- Sine wave generator
- 2 banana plug leads
- Sound sensor
- Meter stick
- Data Studio 750 Interface
- Thermometer

Room Temperature ( $^{\circ}\text{C}$ ) \_\_\_\_\_ Tube radius (m) \_\_\_\_\_

### Activity 1: Fixed tube length

1. Connect the open speaker to the sine wave generator using standard banana plug leads.
2. Adjust the length of the tube to 50 cm (check with meter stick).
3. Place the tube in front of the speaker in such a way that the tube is open at one end (the speaker can be set at an angle relative to the tube length).
4. Set the sound sensor inside the tube and connect it to the Data Studio interface.
5. To activate the sound sensor, perform the following sequence: Start up *DataStudio* by going to *Start*  $\rightarrow$  *Programs*  $\rightarrow$  *Physics Applications*  $\rightarrow$  *DataStudio*. Click on *Create Experiment*, then *Setup*, then *Add Sensor or Instrument*. Scroll down to *Sound level sensor* and select, then click *OK*. Double click *Graph* at left. Click *Start* to begin taking data.



6. Start at a frequency of 50 Hz and increase until you find the frequency of the largest resonance (indicated by a peak on the sound level graph). This is the fundamental frequency. Record the result here:
7. The resonant frequencies for a tube open at one end are given by  $f = nv/4L$  where  $n$  is an odd integer,  $v$  is the velocity of sound and  $L$  is the effective tube length. From the fundamental frequency you just found, calculate the velocity of sound in air (using  $n = 1$ ) and record it here:

**Activity 2: Fixed frequency**

1. Adjust the tube length to 20 cm.
2. Set the speaker inside the open end of the tube so that it is closed at both ends.
3. Set the sine wave generator frequency to 600 Hz (with low amplitude).
4. Slowly move the inner tube to increase the effective length of the tube. Record the length of the tube when resonance is achieved:
5. Increase the length of the tube until three more resonance lengths are found for the constant frequency and record them here:
6. The maxima you have determined are spaced a distance  $\lambda/2$  apart, where  $\lambda$  is the wavelength. Find the differences between adjacent resonance lengths and calculate the average of the three values:
7. Find  $\lambda$  from your average value of  $\lambda/2$  and calculate the velocity of sound in air from  $v = f\lambda$ .
8. The velocity of sound in air at 0°C is 331.4 m/s. The temperature dependence of sound velocity in air is given by  $v(T) = 331.4 + 0.6T$ , where  $T$  is in °C and  $v$  is in m/s. Calculate an “accepted” value of the velocity of sound in air from this formula.
9. What is the percent difference between your experimental result and the “accepted” value?

## 26 Heat, Temperature, and Internal Energy

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

To investigate the relationship between heat and temperature.

### Apparatus

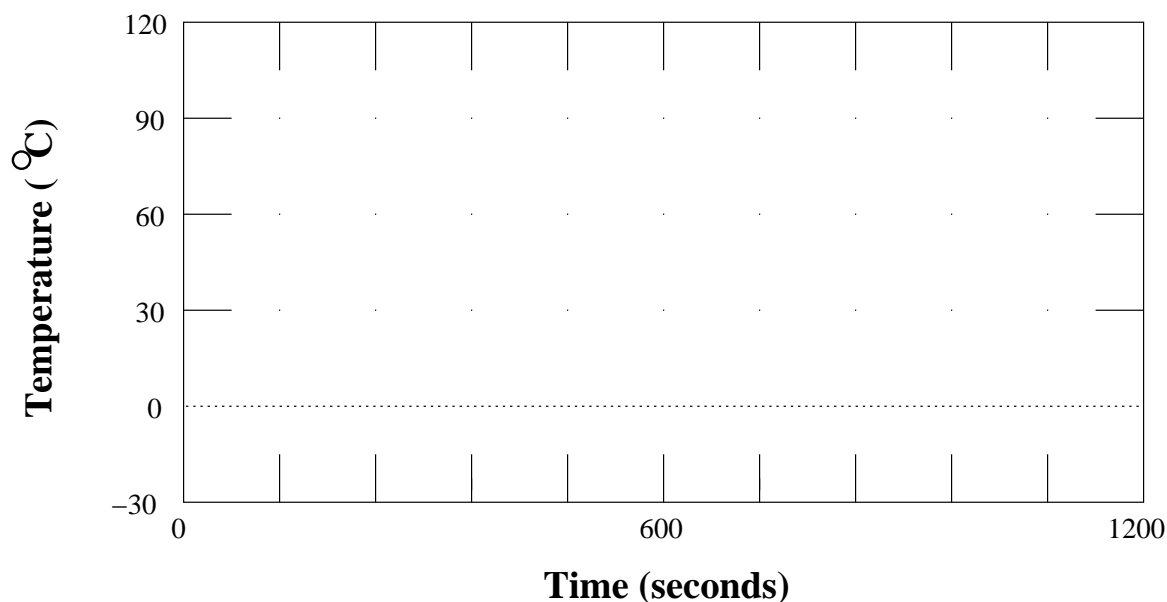
- Glass beaker
- Hot plate
- Ice
- Data Studio software and temperature probe
- Clamp and stand

### Temperature of a Substance as a Function of Heat Transfer

As part of our quest to understand heat energy transfer, temperature, and internal energy of a substance, let's consider the temperature change as ice is changed to water and then to steam.

#### Activity 1: Predicting T vs. t for Water

Suppose you were to add heat at a constant rate to a container of ice water at  $0^{\circ}\text{C}$  until the water begins to boil. Sketch the predicted shape of the heating curve on the graph below using a dashed line. Mark the points at which the ice has melted and the water begins to boil.



#### Activity 2: Measuring T vs. t for Water

(a) To test your prediction:

1. Fill the glass beaker at least half full of ice water and set it on top of the hot plate.

2. Suspend the temperature probe so that the end is submerged in the ice water but not touching the side or bottom of the beaker. You will need to use the clamp and stand to do this.
  3. Open the *Heat, Temp, & Internal Energy* application in the 132 Workshop folder on the **Start** menu.
  4. Turn on the hot plate and click the **Start** button on the monitor to begin recording data. The temperature of the water will be recorded on the graph shown on the monitor. While there is still ice, stir gently.
  5. After the water begins to boil, turn off the hot plate and stop collecting data using the **Stop** button on the monitor.
  6. Sketch the shape of the measured heating curve on the above graph using a solid line. Ignore small variations due to noise and uneven heating. Mark the points at which the ice has melted and the water begins to boil.
- (b) Does your prediction agree with the measured heating curve? If not, what are the differences?
- (c) What is the relationship between the temperature and the added heat while the ice is melting?
- (d) What is the relationship between the temperature and the added heat after the ice has melted, but before the water begins to boil?
- (e) What is the relationship between the temperature and the added heat while the water is boiling?
- (f) If there are regions of the heating curve in which the temperature is not changing, what do you think is happening to the added heat in these regions?

## 27 Calorimetry

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Purpose

- To learn to use a method for measuring heat called calorimetry.
- To use calorimetry to determine the specific heat of aluminum and the heat of fusion of ice.

### Apparatus

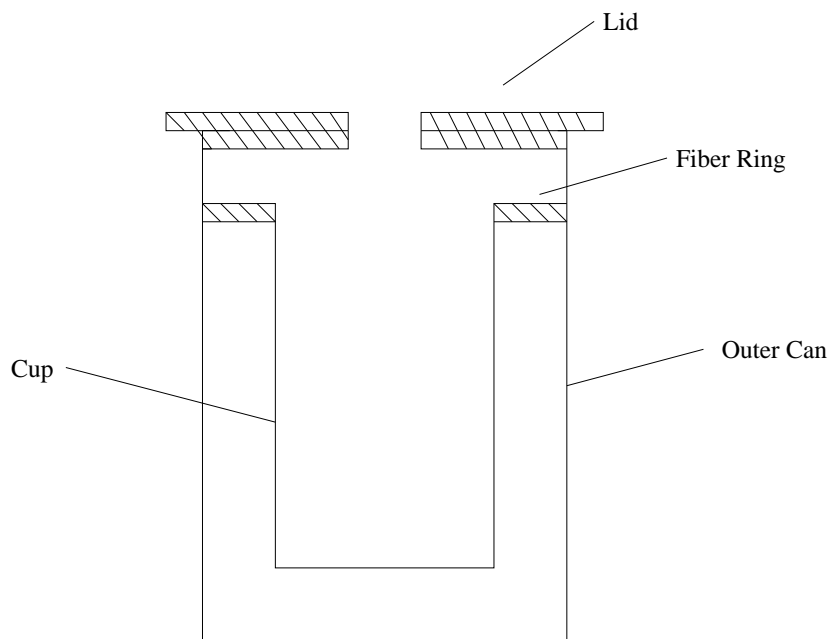
- Hypsometer and stand
- Hot plate
- Aluminum pellets
- Ice
- Compact scale (for measuring masses)
- Calorimeter
- Temperature probe
- Clamp and stand
- Safety goggles
- *DataStudio* software (Calorimetry)

### Introduction

Calorimetry is a method for measuring heat. As applied in this experiment, the method involves the mixing together of substances initially at two different temperatures. The substances at the higher temperature lose heat and the substances at the lower temperature gain heat until thermal equilibrium is reached.

### Activity 1: Statement of Conservation of Energy

If no heat is transferred to the surroundings, what is the relationship between the heat lost by the substances initially at high temperature and the heat gained by the substances initially at low temperature? Note: This is simply a statement of conservation of energy.



### Experimental Equipment

A calorimeter, shown in the above figure, is used in this experiment to minimize the exchange of heat between the system and the surroundings. The inner calorimeter cup is thermally insulated from the surroundings by suspending it on a ring of material with low heat conductivity and surrounding it with a layer of air. Also the cup is shiny to minimize radiation loss. Hence, if the mixture of substances is placed inside the calorimeter cup, the heat lost to or gained from the surroundings can be ignored, and the above relationship can be used. The only part of the calorimeter which is involved in the calculation is the inner calorimeter cup which contains water and in which an exchange of heat between the hot and cold bodies takes place. The cup will undergo the same temperature change as the contained water. Of course, an instrument will have to be introduced to measure the temperature of the system, but the heat gained or lost by the instrument is small and can be ignored.

### Activity 2: Specific Heat of Aluminum

- (a) Fill the hypsometer (boiler) at least half full of water and start heating the water.
- (b) Determine and record the mass of the hypsometer cup,  $m_h$ . Then fill it about half full with dry aluminum pellets. Determine and record the mass of the cup and pellets,  $m_{hp}$ , and calculate the mass of the pellets,  $m_p$ . Record the measurements in the space below.
- (c) Fill the plastic beaker with ice water. Open the *Calorimetry* application in the 132 Workshop folder in the **Start** menu and start collecting data. To make sure the temperature probe is working properly place it in the ice water and check that it is reading approximately  $0^\circ\text{C}$ . If not, then consult your instructor.
- (d) Place the hypsometer cup in the top of the hypsometer and put the temperature probe into the middle of the pellets. To do this, remove the pellets from the cup, place the temperature probe in the proper position (using the clamp and stand), then return the pellets to the cup.
- (e) Determine and record the mass of the calorimeter cup,  $m_c$ . Fill this cup about half full of cold tap water. Determine and record the mass of the cup and water,  $m_{cw}$ , and calculate the mass of the water,  $m_w$ . Then place the calorimeter cup in the outer can and put the lid on.

(f) When the temperature of the pellets becomes constant, at or near  $100^{\circ}\text{C}$ , record the temperature of the pellets as  $T_p$ . Remove the probe from the pellets and put it in the cold water in the calorimeter cup. When the temperature of the water levels off, record it as  $T_w$ .

(g) Now, quickly but carefully, pour the pellets into the water in the calorimeter cup. Stir the water occasionally with the temperature probe and monitor the temperature of the mixture. When the temperature levels off, record this value as  $T$ . Click the **Stop** button on the monitor, print your graph of temperature as a function of time and include it in this unit.

(h) Write the complete heat equation and solve for the unknown specific heat of the metal. The specific heat of the calorimeter cup is  $900 \text{ J/kg}\cdot^{\circ}\text{C}$ .

(i) Look up the accepted value for the specific heat of aluminum and calculate the percent difference between this value and the one you determined above. Do the two values agree within experimental uncertainties? Comment on possible sources of error.

### Activity 3: Specific Heat of Metals

(a) Repeat steps 2(a)-2(i) with pellets of a different metal besides aluminum. Record the the type of metal, the mass of the pellets, the temperature of the pellets just before you pour them in the cold water, and the temperature of the combined pellets, water, and cup.

(b) Use the equation you derived above for the unknown specific heat of the new metal. The specific heat of the calorimeter cup is  $900 \text{ J/kg}\cdot^{\circ}\text{C}$ .

(c) Look up the accepted value for the specific heat of your new metal and calculate the percent difference between this value and the one you determined above.

(d) Consult the other lab groups in class and record their values of the specific heat of aluminum and the second metal below. Calculate the average and standard deviation for each metal. Can you spot any trends in your data?

(e) The specific heats you measured above were in units of  $\text{J/kg}\cdot^{\circ}\text{C}$ . It is more illuminating to express the specific heat in units of  $\text{J/mole}\cdot^{\circ}\text{C}$ , proportional to the specific heat per atom. Do this for each of the averages and standard deviations you obtained in part 3(d) by multiplying the result for each metal by its molar mass. Record the results below. Can you spot any trends in your data now? What effect do the standard deviations have on your conclusion?

**Activity 4: Heat of Fusion of Ice**

(a) The heat of fusion of ice is found experimentally as follows: A known mass of warm water is placed in the calorimeter cup and its temperature recorded. A known mass of ice at  $0^{\circ}\text{C}$  (with no water) is added to the water and allowed to melt. The final temperature of the mixture after the ice has melted is recorded. Perform the experiment and record the data in the space below.

(b) Write the complete heat equation and solve for the unknown heat of fusion of ice.



## 28 Boyle's Law

Name \_\_\_\_\_

Section \_\_\_\_\_

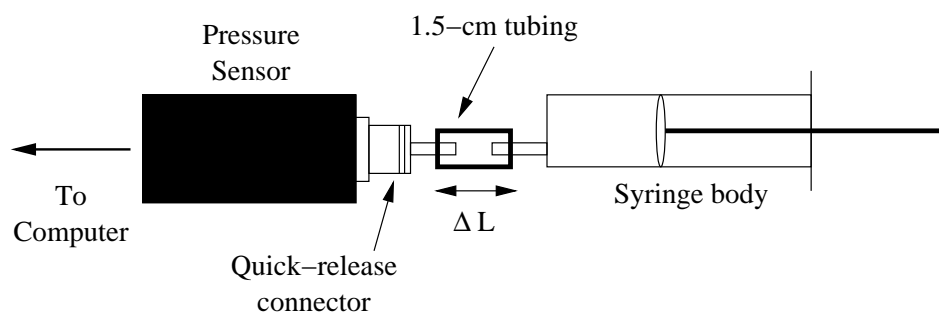
Date \_\_\_\_\_

### Objective

To investigate the relationship between the pressure and volume of a gas.

### Apparatus

- DataStudio 750 Interface
- Pasco Pressure Sensor
- Syringe
- Tubing



Apparatus for Boyle's Law.

### Introduction

The behavior of a gas can be described in terms of the macroscopic quantities: temperature ( $T$ ), pressure ( $P$ ), and volume ( $V$ ). The relationship between these quantities is given by the equation of state of the gas. A real gas behaves approximately as an ideal gas if it is far from liquefaction. In that case, the equation of state of an ideal gas can be used to describe a real gas. For a given mass of a gas, if one of the quantities  $P$ ,  $T$ , or  $V$  is changed, a change in the other two quantities probably will result. However, if one of the quantities is kept constant, the relationship between the other two can be studied. The relationship between pressure and volume of an ideal gas is called Boyle's law.

The experimental apparatus is shown in the figure above. The gas is air contained in a syringe that has marking on its side to measure the volume of the syringe. A short tube connects the syringe with a pressure sensor that measures the pressure in the tube and converts that measurement into a signal that can be read by the DataStudio interface.

### Activity 1: Relationship Between $P$ and $V$ of a Gas

(a) Check that there are no leaks in the apparatus by trying to compressing the syringe from the 20.0 ml position to the 10.0 ml position. It should become increasingly difficult to push the plunger as the volume decreases. If this is not the case, check the couplings for fit. If no problem is obvious, then consult your instructor.

(b) The initial volume of air in the syringe should be set at 20.0 ml. If your syringe is set to some other value, disconnect the quick release connector from the sensor by gently rotating it in the counter-clockwise direction as

you look from the syringe toward the pressure sensor. Next, move the piston to the 20.0 ml position, and then re-connect the quick release connector to the pressure sensor.

(c) **Data Recording.** Open the Boyle's Law activity located in the 132 Workshop Folder under the **Start** menu. Click on the window labeled *Volume and Pressure Table*. This is where your data will be displayed as you record it. This table display will show the values of the gas volume in the syringe which you will set by moving the piston to the appropriate marking on the syringe. You will record the pressure at each of these settings with the pressure sensor. To begin recording data, make sure the piston is at the 20-ml setting, and click the Start button. The Start button will change to a Keep button and the table display will show the value of the pressure next to the first volume value (20 ml) in the table. The reading in the pressure column should be colored red. Click the Keep button to record this pressure (notice the reading in the Pressure column beside the 20-ml entry changes from red to black). The next setting for the volume (18 ml) will appear in the Volume column of the data table display.

NOTE: For the first pressure reading at 20 ml, the air in the syringe will be in thermal equilibrium with the environment. This will not be the case immediately after compressing the syringe for the next reading. Therefore, you must allow one second for the system to return to thermal equilibrium after you compress the syringe and before clicking on Keep to record pressure values.

(d) Compress the syringe to the next value of the volume as listed in the data display table (i.e., the window labeled *Volume and Pressure Table*) and wait one second for the system to reach thermal equilibrium. Once thermal equilibrium is reached, click Keep to record the pressure. The data table display will automatically change to show the next value of the volume at which the pressure will be measured.

(e) Repeat step (d) for the remaining values of the volume listed in the table display. In other words, continue taking pressure measurements at the prescribed volume values in the data table display by moving the piston to the prescribed value and clicking on Keep after thermal equilibrium is reached. After you record the pressure for the last volume (8 ml), click the small, red box next to the Keep button (this is the stop button) to end data recording.

(f) **Analysis.** Click on the GraphDisplay to examine the plots of Syringe Volume Reading vs. Pressure, and the Volume to Pressure ratio (as a function of measuring time). Print the GraphDisplay and attach it to the unit. What happened to the pressure when the volume was reduced from 20 ml to 8 ml?

(g) From looking at the data, do the pressure and volume seem to be directly or inversely proportional? Explain.

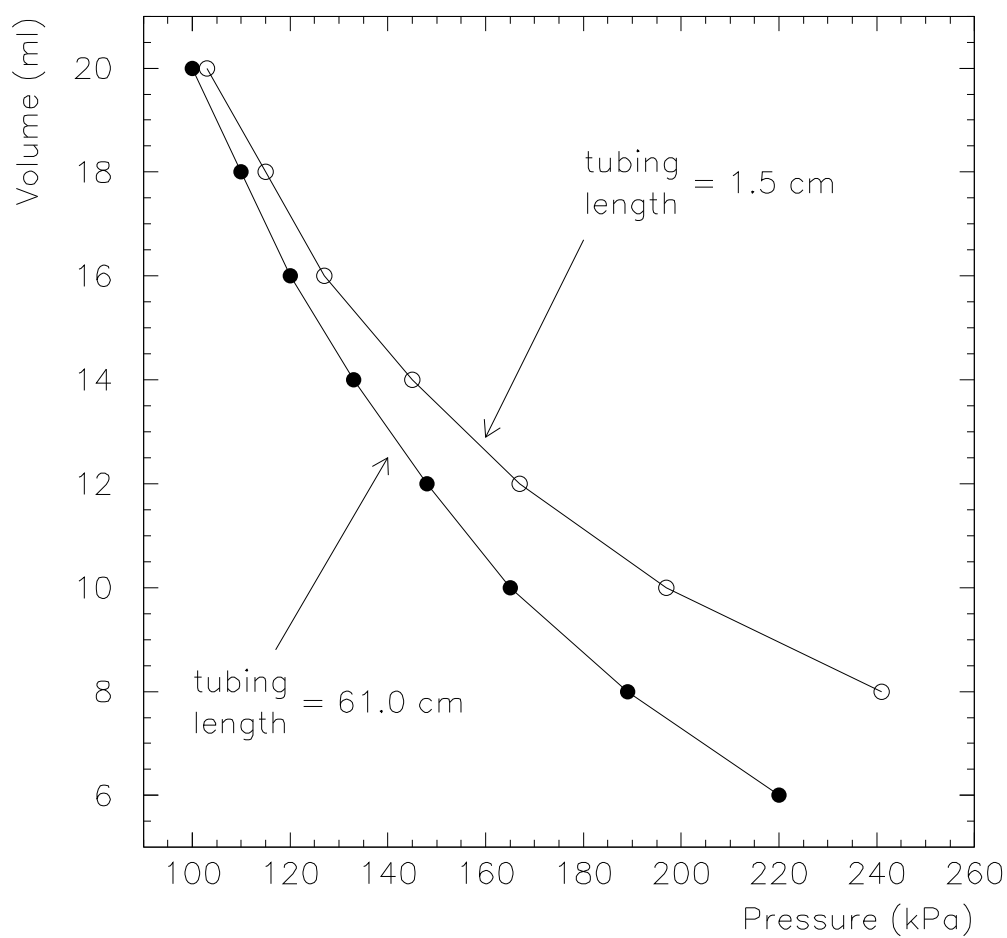
(h) Copy your data into a spreadsheet and plot pressure versus volume. Next, fit your data with some power of the volume. Record the result here. What should you get for the power? Why?

(i) If pressure and volume are inversely proportional, then what can you say about the product of pressure and volume? Explain.

(j) Construct a table in the space below with the column headings: V (ml), P (kPa), and PV. What are the units of the last column? Add your answer to the heading for that column. Enter the results for P and V in this new table and calculate PV for each set of readings. Determine the mean value and the standard deviation  $\sigma$  for PV. Record the results in the form  $PV = \text{Mean} \pm \sigma$ . What does this result tell you about the product PV? What does it tell you about the relationship between P and V? Explain.

(k) You may have noticed that the syringe plunger does not always return to the 20 ml mark at the end of a data run. Give three reasons why this may happen, and explain which is most likely and why.

(l) Examine the plot below with results from two different data runs. How do you explain the difference between the curves for the different tubing lengths ( $\Delta L$  in the diagram on page 11)?



Results of measurement with Boyle's Law apparatus  
different values of  $\Delta L$ , the tubing length.

## 29 Charles' Law

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_+

### Objectives

To investigate the relationship between volume and temperature for a constant mass of gas at constant pressure and determine the value of absolute zero.

### Apparatus

- Charles law apparatus with stand
- Temperature sensor
- Air chamber and tubing
- Hot plate
- Glass beaker
- Clamp and stand

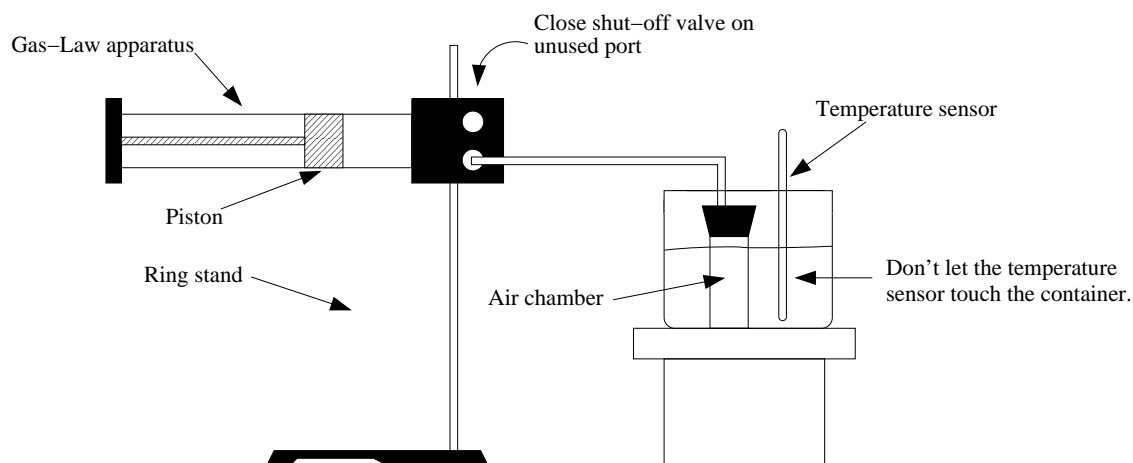


Figure 1: Charles' Law apparatus.

### Introduction

The behavior of a gas can be described in terms of the macroscopic quantities: temperature ( $T$ ), pressure ( $P$ ), and volume ( $V$ ). The relationship between these quantities is given by the equation of state of the gas. A real gas behaves approximately as an ideal gas if it is far from liquefaction. In that case, the equation of state of an ideal gas can be used to describe a real gas. For a given mass of a gas, if one of the quantities  $P$ ,  $T$ , or  $V$  is changed, a change in the other two quantities probably will result. However, if one of the quantities is kept constant, the relationship between the other two can be studied. The relationship between temperature and volume of an ideal gas is called Charles' law.

The experimental apparatus is shown in the figure above and consists of an air chamber containing dry air. The pressure on the air in the chamber is due to atmospheric pressure applied through the movable piston.

### Activity 1: V-T Relationship for a Gas

(a) Check that there are no leaks in the apparatus by trying to compressing the piston from the 100 mm position to the 10 mm position. It should become increasingly difficult to push the plunger as the volume decreases. If this is not the case, check the couplings for fit. If no problem is obvious, then consult your instructor.

(b) Open the *Charles' Law* activity in the 132 Workshop Folder under the **Start** menu. Click on the window labeled *Charles' Law Table*. This is where your data will be displayed as you record it. This table display will show the values of the gas temperature in the air chamber and the entry number. The data-taking procedure you will follow is described here first. One member of your team will heat the air chamber in the flask on the hot plate and call out the position of the piston. Another member will record the position settings by hand in the table below and click the **Keep** button on the *DataStudio* interface to record the temperature for that entry. To begin recording data, make sure the piston is at the low end of the scale, and click the **Start** button on the *DataStudio* interface. The **Start** button will change to a **Keep** button and the table display will show the value of the temperature next to the first entry in the table. The reading in the temperature column should be colored red. Click the **Keep** button to record this temperature (notice the reading in the Temperature column beside the entry number changes from red to black). The next entry number will appear in the Entry column of the data table display.

(c) Now, immerse the air chamber in a beaker of cold, tap water water and click **Start** on the *DataStudio* interface. You can monitor the temperature on the temperature versus time plot to the right. Make sure the set screw on the side of the piston is released.

(d) When the temperature is stable click **Keep** and that point will be recorded in the table. One team member should read off the piston position while the other writes it in the table at the same time.

(e) Now turn up the heat. The piston will move as the gas expands. Read out the position of the piston every one or two millimeters. The other team member will click **Keep** (recording the temperature) and record the piston position in the table.

(f) Repeat step (e) until the piston no longer moves or the water starts to boil.

(g) Calculate the volume of the apparatus for each piston position and plot this volume versus temperature. The diameter of the Gas-Law apparatus is written on its base.

Entry Number	Piston Position (mm)	Gas-Law Apparatus Volume (ml)	Temperature (°C)

(h) How are the volume and temperature related? Fit your data with the appropriate function and record the results here. Print your plot and attach it to this unit.

(h) Repeat steps c-h to obtain a second  $V$ - $T$  curve. Record your data in the table below along with the fit to the  $V$ - $T$  data.

Entry Number	Piston Position (mm)	Gas-Law Apparatus Volume (ml)	Temperature ( $^{\circ}\text{C}$ )

### Activity 2: Absolute Zero and the Kelvin Scale

(a) The absolute zero of temperature can be defined as the temperature at which the volume of an ideal gas is zero. Determine absolute zero from the equation of your graph by setting  $V = 0$  and solving for  $T$ .

(b) Determine the percent difference between your value of absolute zero and the accepted value of  $-273^{\circ}\text{C}$ . Are you happy or sad?

(c) Record the results from the other groups in class. Obtain an average and standard deviation and record it here. Are your results consistent with the class average? Explain.

## 30 The P-T Relationship of a Gas

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_+

### Objectives

To investigate the relationship between pressure and temperature for a constant mass of gas at constant volume and determine the value of absolute zero.

### Apparatus

- Pressure sensor
- Temperature sensor
- Air chamber and tubing
- Hot plate
- Glass beaker
- Clamp and stand

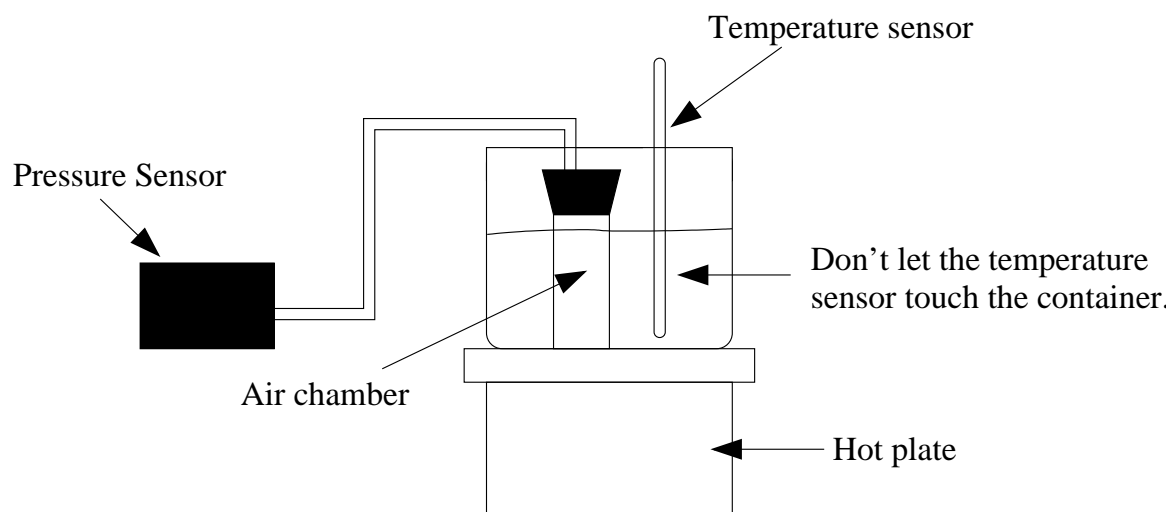


Figure 2: P-T apparatus.

### Introduction

The behavior of a gas can be described in terms of the macroscopic quantities: temperature (T), pressure (P), and volume (V). The relationship between these quantities is given by the equation of state of the gas. A real gas behaves approximately as an ideal gas if it is far from liquefaction. In that case, the equation of state of an ideal gas can be used to describe a real gas. For a given mass of a gas, if one of the quantities P, T, or V is changed, a change in the other two quantities probably will result. However, if one of the quantities is kept constant, the relationship between the other two can be studied. The relationship between temperature and pressure of an ideal gas has no particular name associated with it.

The experimental apparatus is shown in the figure above and consists of an air chamber containing dry air. The volume of the gas is fixed.

### Activity 1: P-T Relationship for a Gas



- (a) Fill the beaker  $\frac{3}{4}$  full with cold tap water and place it on the hot plate. Immerse the air chamber in the water so that most of the volume of the air chamber is submerged. The air chamber will have to be held in place with a clamp and stand or it will float to the top. Set the temperature sensor in the water in such a way that it is not touching the side or bottom of the beaker.
- (b) Open the *P-T* activity in the 132 Workshop Folder under the **Start** menu. Click on the window labeled *Temperature and Pressure Table*. This is where your data will be displayed as you record it. This table display will show the values of the gas pressure in the air chamber and the temperature of the heat bath. To begin recording data click the **Start** button on the *DataStudio* interface. The **Start** button will change to a **Keep** button and the table display will show the values of the temperature and pressure. Click the **Keep** button to record this temperature and pressure.
- (c) Turn the hot plate on high. As the temperature rises, click the **Keep** button when the temperature is  $5-7^{\circ}$  above its first value. Continue recording the temperature and pressure at  $5-7^{\circ}$  intervals (by clicking the **Keep** button) until the water is close to boiling. You can monitor the temperature on the temperature versus time plot to the right (on the monitor) or by watching the temperature in the *Temperature and Pressure Table*. After your last reading, click the small red box next to the **Keep** button (this is the stop button) to end data recording.
- (d) How are the pressure and temperature related? Print your data table, enter the data in *Excel* and plot pressure vs temperature on a linear graph, showing the equation of the graph. Print this graph and add it to this unit.

### Activity 2: Absolute Zero and the Kelvin Scale

- (a) The absolute zero of temperature can be defined as the temperature at which the pressure of an ideal gas is zero. Determine absolute zero from the equation of your graph by setting  $P = 0$  and solving for  $T$ .
- (b) Determine the percent difference between your value of absolute zero and the accepted value of  $-273^{\circ}\text{C}$ . Are you happy or sad?
- (c) Record the results from the other groups in class. Obtain an average and standard deviation and record it here. Are your results consistent with the class average? Explain.

## 31 Electrostatics

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To understand the basic phenomena of electric charges at rest.

### Introduction

Atoms consist of a central nucleus made up of protons and neutrons surrounded by one or more electrons. While the nuclei of solids are essentially localized, some of the electrons may be free to move about. A substance which has as many electrons as it has protons is said to be electrically neutral. Dissimilar objects have different affinities for electrons. When two such objects, initially neutral, are rubbed together, the friction may cause electrons to pass from one to the other. After separation, neither object is neutral. Each is said to have been “charged by friction”. An isolated, electrified object becomes neutral again if its electron-proton balance is restored. A convenient means for accomplishing this is to connect the object to earth by means of a conductor, through which electrons readily travel. This process is called “grounding the body”. Since an electrified object is referred to as “charged”, grounding is also referred to as “discharging”.

Substances through which electrons do not move easily are called “non-conductors”, or “insulators”. Experiment has shown that when rubber and wool are rubbed together, electrons pass from the wool to the rubber. The electrons remain on the surface of the rubber—a non-conductor—where they were transferred.

Rubbing a metal rod with a wool cloth can also transfer electrons. This rod, however, is a conductor and electrons pass through it to the experimenter and then to the earth. People, made mostly of salt water, are good conductors, as well. Metal that is isolated, however, can be electrified. This can be demonstrated with an electroscope, which has a metal knob connected to a stem from which a thin metal leaf hangs. An insulator prevents contact of these metal parts with the case, and consequently the earth.

### Apparatus

- electroscope
- rubber and glass rods
- wool and silk cloth
- plastic rod with metal disk mounted on one end

### Activity 1: Charging by Friction

1. Be sure the electroscope is discharged by touching the knob with your finger. Explain what happened and why you are convinced the electroscope is discharged.
2. **Prediction:** If you rub the knob of an electroscope with a wool cloth, what will be the state of the electroscope when you remove the cloth? Explain.

3. Gently and repeatedly rub the knob of the electroscope with the wool cloth for a couple of minutes. Remove the cloth. Note any differences in the electroscope from its appearance before you rubbed.
4. Explain what, if anything, happened.

### Activity 2: Charging by Contact

1. Discharge the electroscope as before.
2. Charge the plastic rod by friction with the wool cloth.
3. Does anything occur in the electroscope when you bring the disc close to the knob without touching it?
4. **Prediction:** What will happen to the electroscope if you repeatedly touch its knob with a freshly charged object?
5. Touch the disc to the knob; rub the disc again and again touch it to the knob; repeat this procedure two or three more times. Describe any changes to the electroscope.
6. Repeat the procedure above until the electroscope's leaf is at approximately a twenty degree angle with the stem.

### Activity 3: Kinds of Electrification

1. Electrify one end of the rubber rod by wrapping the wool cloth around the rod, squeezing the wool against the rod, twisting the rod vigorously to ensure good contact, and separating the wool from the rod.
2. **Prediction:** What will happen when you bring the electrified end of the rubber rod toward, but not touching, the electroscope's knob? What will happen if you do the same with the wool cloth?
3. Bring the charged end of the rubber rod toward the knob, but do not touch it. Record what happens.

4. Repeat number 3 with the wool cloth.
5. What differences were there between the trial with the rod and the trial with the cloth?
6. How would you account for these differences?
7. **Note:** By definition, the electrical state of the rubber after being rubbed by the wool is negative. That is, an object that has an excess of electrons is said to be negatively charged. Realize that this is only a convention.
8. If the rubber is said to be negatively charged, in what electrical state is the wool cloth?
9. How can an electroscope be used to determine the nature of any charge?
10. Rub the end of the glass rod with the silk cloth and determine the charge of each (positive or negative) after they are separated.

#### **Activity 4: Action of the Electroscope**

1. **Discussion:** Two facts explain the rise or fall of the leaves of an electroscope: (a) Like charges repel (unlike charges attract); and (b) Free electrons move about in a conductor when an electric force acts upon them.
2. When the wool cloth approaches the knob (in Activity 3 number 4), which way do the free electrons in the metal of the electroscope move (up toward the knob or down toward the leaf)?
3. In Activity 3, the electroscope was negatively charged before either the rod or the wool was brought toward the knob. For the case of the rod, in which direction do the free electrons in the electroscope move? Does the electron displacement increase or decrease the electrostatic force separating the leaf from the stem?

**Activity 5: Charging by Induction**

1. Discharge the electroscope.
2. **Prediction:** What will be the effect on the electroscope if you perform the following experiment: while grounding the electroscope with your finger, bring an electrified rubber rod near the knob, then take away your finger and then the rod (in that order)?
3. Carry out the experiment and describe the result.
4. Explain the result and why your prediction agreed or disagreed with it.
5. **Prediction:** Note that no electrons moved between the rod and the electroscope. What charge has been induced on the electroscope?
6. Test your prediction with the negatively charged rubber rod and the positively charged wool.
7. Does the test verify or contradict your prediction?

## 32 Electric Fields and Equipotential Lines

Name \_\_\_\_\_

Section \_\_\_\_\_

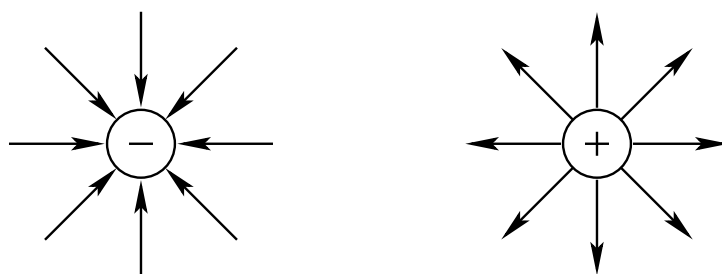
Date \_\_\_\_\_

### Objective

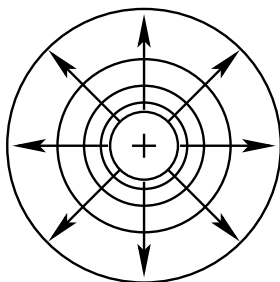
- To learn the shape of electric fields.

### Introduction

Charged objects exert an electrical force on other charged objects in proportion to the amount of charge each has, just as massive objects exert a gravitational force on one another in proportion to their masses. The magnitudes of both forces depend, too, on the distance between objects. However, whereas the gravitational force is always attractive, electrical forces may be either attractive or repulsive depending on the sign of the charges. It is convenient in understanding the nature of electrical forces to draw pictures of them. We represent the fields, which provide the magnitude and direction of the forces, as lines. We agree on a convention: the direction of the field is that of the force on an infinitesimal positive test charge. Thus, the lines of force originate on and come out of positive charges and are directed toward and terminate on negative charges (see figure below). The magnitude of the field, and therefore the force, is proportional to the density of the field lines.

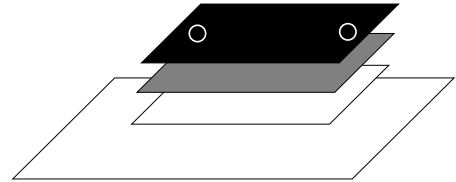
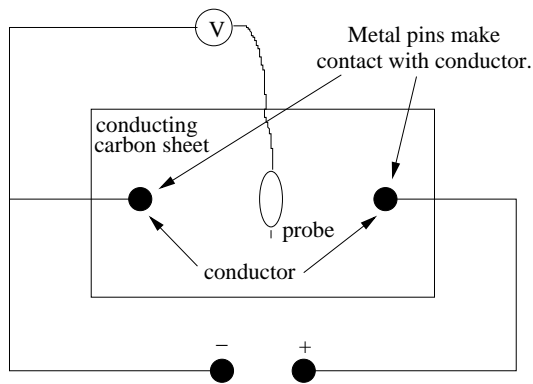


Please note that when the situation is electrostatic, 1) the electric field within a metal is zero, and 2) the electric field just outside the surface of a metal is perpendicular to the surface. If either of these conditions were altered, then there would be an electric current in the metal, which is not an electrostatic situation. Because an electric field exerts a force on a charge, work must be done to move a charged object along any of the field lines. On the other hand, movement perpendicular to the field lines requires no work. Such movement is said to be along an equipotential line.



In the figure above, the electric field for a positive point charge is shown as lines with arrows. The regions of equipotential (equipotential lines) are shown with circles. Notice that the equipotential lines are perpendicular to the electric field lines and that the density of equipotential lines is proportional to the electric field strength.

Electric field lines are difficult to measure directly, but potentials can be measured with a voltmeter. An electric field will arise in the space surrounding two separated charged conductors. With one lead of a voltmeter connected to one of the conductors and the other used as a probe, the potentials can be determined (see figure below).



### Apparatus

- Power supply
- Voltmeter
- Conducting sheets
- Carbon and white paper
- Wooden board and pins

### Activity 1: Field Lines for Two Point Charges

**Prediction:** Using the rules given in the introduction and given in the first set of figures, draw the electric lines for two oppositely charged point objects. Sketch the equipotential lines.

1. Find the conducting paper with the two silver circles on the front and lay it over a copy carbon and a sheet of paper on top of the wooden board.
2. Connect the positive output of the power supply to one of the circles and the negative to the other.
3. Connect the negative lead of the voltmeter to the negative conductor and use the positive lead as the probe.
4. With the power supply voltage turned on and set to 10 volts, probe lightly with the voltmeter to find a number of points on the carbon paper registering 8 volts. Push down each time you find a point so that marks will be made on the bottom paper.
5. Repeat for 6 volts, 4 volts, and 2 volts.
6. You should end up with a series of dots on your sheet of paper. Connect those associated with the same potential with smooth lines.

7. Recalling the relationship between electric field lines and equipotential lines, sketch in the electric field lines. (*Other group members can sketch copies of the same results.*)
8. Does your experimental result agree with your prediction? Explain.

**Activity 2: Field Lines for Parallel Plates**

**Prediction:** Draw what you think the field lines and equipotential lines between parallel plates will look like.

1. Carry out the instructions from Activity 1 to check your prediction.
2. Does your result agree with your prediction? Explain.

**Activity 3: Field Lines Between a Point Charge and a Plate**

**Prediction:** Draw what you think the field lines and equipotential lines between a point charge and a parallel plate will look like.

1. Map the field lines as before.
2. Does your result agree with your prediction? Explain.
3. If the potential is zero, must the electric field be zero as well?
4. What can you say about the electric field along an equipotential line?



## 33 Ohm's Law

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

- To investigate the most important principle in electronics.
- To determine how resistors in series and parallel add.

### Introduction

The rate at which electric charge flows through a conductor is called the electric current. In order to have a current, a potential difference, or voltage is necessary. We first want to determine the relationship between the potential difference at two ends of a conductor and the current flowing through it.

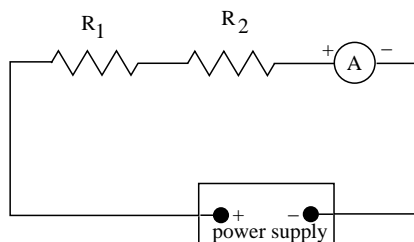
**Note:** Do not turn on a power supply until you are sure your circuit is correct. If you are at all unsure, please ask your instructor to approve your setup. Ammeters can be instantly and permanently ruined by an improper connection. Be sure to turn off the power supply before making any changes to the circuit.

### Apparatus

- power supply
- 2 rheostats
- ammeter
- voltmeter

### Activity 1: Ohm's Law

- Connect two rheostats (or variable resistors) in series as shown in the figure below. Set  $R_1$  at about the halfway point and  $R_2$  at the maximum. Connect an ammeter as shown. Also, connect a voltmeter across (that is, connect a probe to each side of)  $R_1$ .



- When sure of your circuit, turn on the power supply, and turn the voltage up all the way..
- Record the current through the circuit and the voltage across  $R_1$ .

- Reduce the resistance of  $R_2$  and record the current and voltage three more times by turning down the rheostat in approximately equal steps so that for the last time  $R_2$  is turned completely down.
- Turn off the power supply.
- Plot your four pairs of readings with the voltage on the vertical axis and the current on the horizontal axis.
- Fit a straight line to the points, using the origin as a fifth point.
- Is a straight line a good fit to the data? What does that say about the relationship between voltage and current?
- What are the value and meaning of the slope of this line? Write the equation of the line.
- Remove  $R_1$  from the rest of the circuit and use the ohmmeter option on the multimeter to measure the resistance of  $R_1$ . Does it agree with the slope you found? What is the percent difference? Replace  $R_1$ .
- What is the general relationship between voltage, current, and resistance? This is Ohm's Law.
- Why is the origin a legitimate point on the curve?

### Activity 2: Resistors in Series

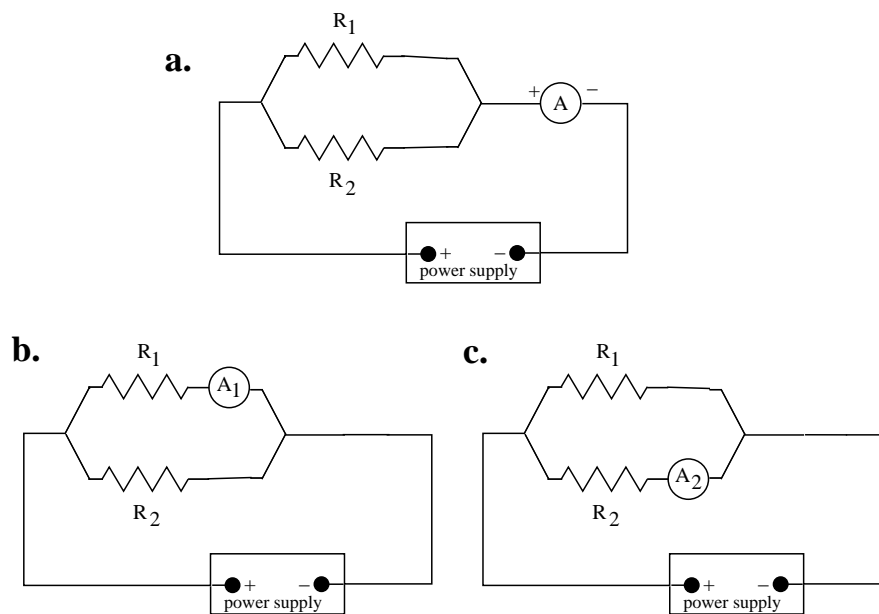
- Turn rheostat  $R_2$  to its maximum setting. Connect the multimeter across this resistor, being sure to set it for reading voltages.
- When you are sure the circuit is set, turn on the power supply and record the current and voltage. Turn off the power supply.

- **Prediction:** Based on your measurements, predict the resistance of  $R_2$ .
- Remove and measure the resistance of  $R_2$ . Record the percent difference between your prediction and measurement. Replace  $R_2$ .
- Was the current this time different from the first reading in Activity 1?
- What can you conclude about the current through two resistors in series?
- Connect the multimeter across both resistors, being sure to switch to voltage readout.
- When you are sure the circuit is correct, turn on the power supply and record the current and voltage. Turn off the power supply.
- Has the current changed?
- Has your previous conclusion been substantiated or refuted?
- How is the voltage just measured related to the first voltage measurements in Activities 1 and 2?
- What can you conclude about the voltage across resistors in series?

- Using your conclusions concerning voltages across and current through resistors in series and your formulation of Ohm's law, what can you conclude about the total resistance in a circuit having two resistors in series?

### Activity 3: Resistors in Parallel

- Connect the two rheostats in parallel as shown in figure **a** below, with the ammeter at the point marked A and the voltmeter across the two rheostats. Set the rheostats at about their halfway settings.



- When you are sure the circuit is set up correctly, turn on the power supply and record the total current through the circuit and the voltage drop across the parallel resistance combination. Turn off the power supply.
- Connect the ammeter to the point marked  $A_1$  in figure **b** above, without disturbing the rest of the circuit; apply power and record the current through  $R_1$  and the voltage reading. Turn off the power supply.
- Repeat the above measurements for  $R_2$ , connecting the ammeter at  $A_2$  as in figure **c** above.

- Using Ohm's Law, calculate the two resistances of the parallel connection and also the total resistance of the circuit. Check with the ohmmeter and determine the percent differences.
- What is the relationship between the total current and the current in each of the branches of the parallel circuit?
- What is the relationship between the total resistance of the parallel circuit and the resistance of each of the branches (you may want to look up in a reference what the correct relationship should be and see if your result agrees with it)?
- Determine, using Ohm's law, what the voltage was in each branch of the parallel circuit. Did it make any difference that you didn't reposition the voltmeter during this activity? On the basis of Ohm's law, does the result make sense?
- Can the total resistance of a series combination ever be less than the resistance of the largest resistor? Explain.
- Can the total resistance of a parallel combination ever be greater than the resistance of the smallest resistor? Explain.

## 34 Magnetism I

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

- To investigate the characteristics of magnets.
- To understand how a compass works.

### Introduction

The electric interaction, you probably know, is not the only one in which opposites attract and likes repel. Magnetic interactions have similar characteristics. All simple magnets, regardless of size, are bipolar: there are two magnetic poles. Consider this question, then: Can we talk about like and unlike as we do for electricity?

### Apparatus

- 2 bar magnets
- 2 cylindrical magnets
- rods and clamps
- wool cloth
- rubber rod
- string

### Activity 1: The Characteristics of Magnets

1. Feel the attraction between two magnets when pulled apart after having come together without effort on your part. Describe qualitatively in terms of strength and separation.
2. Feel the repulsion when one of them is turned around and pushed toward the other. Describe as in step 1.
3. Note and describe the difference in (strength and direction of) interactions between the ends and the middle.

### Activity 2: How a Compass Works

1. Identify geographic north and south.
2. Hang one of the cylindrical magnets horizontally from a horizontal rod.

3. When it comes to rest, along which geographical line does the magnet lie?
4. Which end (colored or uncolored) is the "north-seeking" end?
5. Remove the cylindrical magnet and repeat step 2 with the second cylindrical magnet. Answer, again, the questions above.
6. What happens when you bring the "north-seeking" end of the first magnet near the hanging one's north-seeking end?
7. What happens when you bring the first magnet's opposite end near the second's north-seeking end?
8. What about the first magnet's north-seeking end near the opposite end of the hanging one?
9. What happens when you bring the opposite ends near one another?
10. Define in your own words like and unlike poles?
11. What always happens between like poles?
12. What always happens between unlike poles?

13. Determine with a labelled bar magnet which end of your hanging magnet should be identified as the north pole and which the south.
14. Why do we identify one end of a magnet as the north pole and the other as the south?
15. In your own words, explain a compass.
16. In terms of magnetism, what is the earth?
17. Charge a rubber rod with the wool cloth and bring it near the ends of the suspended magnet; describe its effect on the magnet.
18. Does a south magnetic pole repel a negative electric charge?
19. Does a north magnetic pole attract a negative electric charge?



## 35 Magnetism II

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To investigate the magnetic field around a permanent magnet.

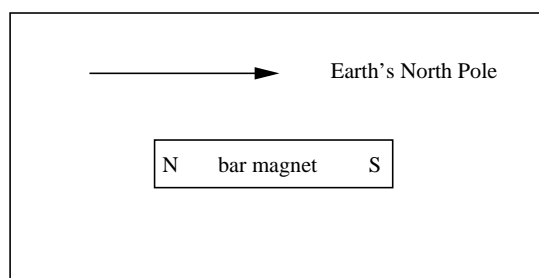
### Introduction

The magnetic field characterizes magnetic forces in much the same way that the electric field characterizes electric forces. At a given point in the region around a magnet, the strength of the field, similar to that of an electric field, is the force per unit north pole (one positive unit of magnetism), and the direction is indicated by the orientation of the north pole of a compass located at the point. On earth, the field mapped out around the magnet is actually the resultant of the field due to the magnet and the field due to the earth.

### Apparatus

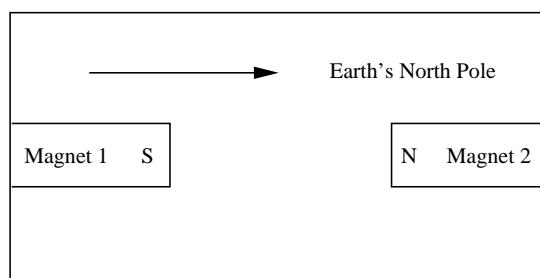
- 2 bar magnets
- small compass
- white paper and tape

### Activity 1: A Single Bar Magnet

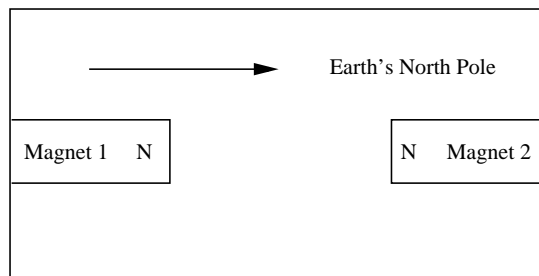


1. Center and tape a bar magnet on a piece of paper and orient it so that the magnet's south pole points to the earth's geographic north pole. Indicate the magnet's polarity and the direction of the earth's field.
2. Place the small compass near the north pole of the magnet and make a dot at each end of the needle using a pencil not encased in metal.
3. Move the compass forward until its south pole points at the previous north pole dot, and make a new dot at the north pole.
4. Repeat 3 until the series of dots reaches the south pole of the magnet or the edge of the paper.
5. In a similar manner, trace enough lines to map the magnetic field over the entire paper. Take points about 0.5 cm apart near the poles and about 2 cm apart near the middle of the magnet.
6. There are two points, called neutral points, near each end of the magnet where the magnet's field and the earth's field are equal and opposite and so cancel. At these points, the compass will align in no particular direction. Try to locate these points by tracing very carefully the lines of force in the neighborhood of the poles.

7. Do lines of force ever cross?
8. Where are the magnetic forces strongest? Weakest? How do the force lines indicate this? Does a line of force represent a constant force along its entire length?
9. Do the lines intersect the magnet at a particular angle (like the electric field lines near a conductor)? What does this imply about the source of a magnetic field as opposed to the surface charge of a conductor as the source of an electric field?

**Activity 2: Two Bar Magnets—Unlike Poles Facing One Another**

1. Set up two bar magnets on a sheet of paper as shown in the figure above. The magnets should be 8-10 cm apart.
2. Repeat steps 2 through 5 from the previous activity.
3. What sort of charge configuration produces an electric field that looks similar to the magnetic field you just identified?
4. What differences can you recognize?

**Activity 3: Two Bar Magnets—Like Poles Facing One Another**

1. Set up two bar magnets on a sheet of paper as shown in the figure above. The magnets should be 8-10 cm apart.
2. Repeat steps 2 through 5 from Activity 1 of this investigation.
3. Try to identify on your map a point at which the magnetic field is zero. Explain what causes this effect.
4. What sort of electric charge configuration would produce a similar field map?

## 36 Refraction of Light

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

To investigate the path traveled by light through a plate of plexiglass (a transparent solid material).

### Introduction

The speed of light depends on the medium in which it travels. In passing from one medium, at least some light energy is reflected. If the second medium is transparent, most of the light will pass into and through it. If the beam is not perpendicular to the boundary between the two media, it will bend as it enters, an effect known as refraction. The direction a single ray of light travels when refracted is given by Snell's law:

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

where

$i$  = incident angle

$r$  = refracted angle

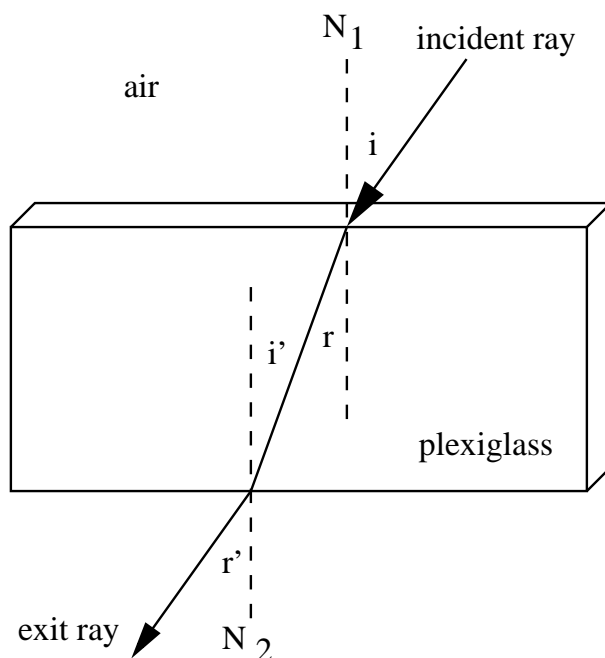
$v_1$  = light speed in medium 1

$v_2$  = light speed in medium 2

$n_1$  = index of refraction of medium 1

$n_2$  = index of refraction of medium 2

**Note:** All angles are measured from the normal to the boundary at the point the ray enters the medium. The index of refraction is the ratio of light's speed in a vacuum,  $c$ , to its speed in the medium,  $v$ :  $n = c/v$ . It is worth remembering that  $n_{air} \approx 1.00$ .



**Apparatus**

- light fence
- plexiglass block
- white paper, pins, and wood board
- protractor

**Activity**

1. Put a plexiglass plate at the center of a piece of paper. Outline its position. Identify a normal,  $N_1$ , perpendicular to an edge of the plate.
2. Arrange the light source apparatus so that the parallel rays of light cross the paper and are incident at a  $30^\circ$ - $35^\circ$  angle to the normal. Trace one of these rays.
3. Sight the corresponding ray as it emerges from the other side of the plexiglass. Trace this ray.
4. Construct the normal,  $N_2$ .
5. Measure and record  $i$ ,  $r$ ,  $i'$ , and  $r'$ . Compute and record  $n_{\text{plexiglass}}$ .
6. Repeat the above procedure for different incident angles between  $25^\circ$  and  $40^\circ$ .
7. Calculate and record an average  $n_{\text{plexiglass}}$ .
8. Does  $i = i'$ ? Explain.
9. Does  $r = r'$ ? Explain.
10. Are the incident and exit rays parallel? Explain.
11. What is the speed of light in the plexiglass?

12. Under what conditions would a refraction angle be greater than an incident angle?

## 37 Refraction at Spherical Surfaces: Thin Lenses

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To investigate thin lenses.

### Introduction

A lens converges or diverges light rays. It is a transparent material bounded, in the case of thin lenses, by spherical edges. The line between the centers of curvature of these edges is referred to as the principal axis. The principal focus is the point on the principal axis where parallel incident rays converge. The distance from the lens to this point is known as the focal length. The relation between the focal distance,  $f$ , the object distance,  $p$ , and the image distance,  $q$ , is:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}.$$

### Apparatus

- light fence
- converging (convex) lens in holder
- converging and diverging lenses (1 each) without holders
- optical bench
- light source
- white paper

### Investigation 1: The Converging Lens

#### Activity 1

- Arrange the light source apparatus so that the parallel rays of light cross a piece of paper.
  - Place a convex lens (without holder) on the paper perpendicular to the central ray. Outline its position and the path of the rays. Pay particular attention to the condition near the principal focus.
  - What is the focal length of this lens?
- 
- Include a sketch of the light rays in your lab book, showing the focal point and focal length of the lens.

#### Activity 2

- Place the light source in its bracket at one end of the optical bench. The arrow on the light source will be the object in this investigation. Measure and record its height,  $h_0$ .

- Place a converging lens in its holder on the optical bench 70 cm from the object (this is the object distance  $p$ ). Turn on the light source and position a piece of white paper so that a sharp image of the arrow appears on the paper. Measure the distance of the paper from the lens. This is the image distance  $q$ . Also measure the height of the image  $h_i$ . Record  $p$ ,  $q$ , and  $h_i$  in the first line of the following table.

$p$	$q$	$h_i$	$\frac{h_i}{h_0}$	$\frac{q}{p}$	$f$

- Move the lens to create four more object distances of 60, 50, 40, and 30 cm. In each case, measure the image distance and the height of the image and record in the above table. Calculate and record the ratios of the image and object heights,  $h_i/h_0$ , and the image and object distances,  $q/p$ . Record these values in the above table. You should now have the first five columns of the above table filled in.

- Calculate and record the focal length,  $f$ , for each observation. Show one of the calculations here:

- Determine an average focal length,  $f_{ave}$ .

- What is the relationship between the ratio of the image to object heights and the ratio of image and object distances? The first ratio is called the magnification.

- Replace the converging lens with a diverging one. Try to obtain a real image on a piece of white paper.
- Why can you not form a real image with a diverging lens?

## Investigation 2: Lenses in Combination (Optional)

### Activity

- Place a converging lens and a diverging lens together into the lens holder. Check to see that you can get a real image with this combination.

$p$	$q$	$h_i$	$\frac{h_i}{h_0}$	$\frac{q}{p}$	$f$



- Repeat the five sets of observations of Investigation 1, Activity 2, to get an equivalent focal length  $f_{ave}^{eq}$ .

### Investigation 3: The Diverging Lens

#### Activity 1 (Optional)

- Using the relation:

$$\frac{1}{f^{eq}} = \frac{1}{f_1} + \frac{1}{f_2},$$

determine the focal length of the diverging lens,  $f_2$ . Use  $f_{ave}^{eq}$  for  $f^{eq}$  and  $f_{ave}$  for  $f_1$ .

#### Activity 2

- Repeat the procedure of Investigation 1, Activity 1, with a concave lens. Locate the principal focus by extending the refracted rays backwards.
- What is the focal length of this lens?
- Include a sketch of the light rays in your lab book, showing the focal point and focal length of the lens.

## 38 The Diffraction Grating

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To determine the wavelength of laser light using a diffraction grating.

### Introduction

Light bends (a bit) around corners. This phenomenon is called diffraction. Interference, or the overlap of waves, is the basis for diffraction. In a transmission grating, lines, about 4,000 to 8,000 per centimeter, are ruled onto glass. The unruled portions of the glass act as slits. The interference, and thus diffraction, which results from shining a beam of light through the grating permit the measurement of the wavelength of the light. The relevant relationship, known as the grating equation is:

$$n\lambda = d \sin \theta$$

where  $n$  is the order of the spectrum (the number of bright spots from the center),  $\lambda$  is the wavelength in nanometers ( $10^{-9}$  meters),  $d$  is the separation in nanometers between grating lines, and  $\theta$  is the angle of deviation from the light beam's original direction through the grating (the angle of diffraction).

### Activity

1. Record the separation between grating lines:  $d =$
2. Turn on the laser, being careful to avoid looking directly into the beam or shining it at anyone. Aim the light beam through the diffraction grating so that a horizontal series of dots appears on the wall. Adjust the positions of the laser and grating until you easily see at least two dots on either side of the brightest (central) dot.
3. Are the dots of the interference/diffraction pattern the same intensity? Describe the pattern you observe.
4. Measure and record the distance from the grating to the wall,  $L$ , as well as the distances from the central dot to the first dot to the right,  $x$ , and the first dot to the left,  $x'$ . Compute the average of these  $x$  values and record it as  $x_{ave}$ .
5. Compute and record a value for  $\theta$  by using the appropriate trigonometric relation between  $L$  and  $x_{ave}$ . Then, compute  $\sin \theta$ . Finally, compute the wavelength using equation above.
6. Repeat the procedure for three additional values of  $L$ .
7. Compute the average of your four determinations of the laser light's wavelength and compare it to the expected value of 632.8 nm. What is the percentage difference?

$L$ (cm)	$x$ (cm)	$x'$ (cm)	$x_{ave}$ (cm)	$\theta$ (rad)	$\sin \theta$	$\lambda$ (nm)

## 39 The Optical Spectrum of Hydrogen

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To determine the wavelengths of the visible lines in the hydrogen spectrum using a spectrometer and a diffraction grating.
- To determine the value of Rydberg's constant.
- To compare the predicted energy levels with the measured ones.

### Introduction

The spectral lines of the hydrogen spectrum that fall in the visible region are designated as the  $H_\alpha$ ,  $H_\beta$ ,  $H_\gamma$ , and  $H_\delta$  lines. All (there happen to be four of them) belong to the Balmer series. In general, the spectrum of hydrogen can be represented by Rydberg's formula:

$$\frac{1}{\lambda} = R_H \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad (1)$$

where  $n_f$  can be any positive integer and  $n_i$  takes on the values of  $n_f + 1$ ,  $n_f + 2$ ,  $n_f + 3$ , and so on and  $R_H$  is the Rydberg constant for hydrogen and equals  $1.097 \times 10^7 m^{-1}$ .

If one writes equation 1 twice—once, say for the  $H_\alpha$  wavelength  $\lambda_\alpha$ , and once for the  $H_\beta$  wavelength,  $\lambda_\beta$ , then one can eliminate  $R_H$ :

$$\frac{1}{n_\beta^2} = \frac{1}{n_f^2} - \left( \frac{\lambda_\alpha}{\lambda_\beta} \right) \left( \frac{1}{n_f^2} - \frac{1}{n_\alpha^2} \right) \quad (2)$$

Thus, once one finds  $\lambda_\alpha$  and  $\lambda_\beta$  so through trial and error one can determine the value of the three  $n$ 's in equation 2 (recall they all must be integers and  $(n_f < n_\alpha < n_\beta)$ ).

### Activity 1: Measuring Spectral Lines

Use the spectrometer to measure the angle (once on each side) for each line and get an average angle for each line. Calculate the wavelength of each line using the relation:

$$\lambda = d \sin \theta \quad (3)$$

where  $d$  is the diffraction grating spacing.

Diffraction grating spacing  $d =$  \_\_\_\_\_ Å

Line	$\theta_{left}$ (degrees, minutes)	$\theta_{right}$ (degrees, minutes)	$\theta_{average}$ (decimal degrees)	Wavelength (Å)	$n$	Color
$H_\alpha$						red
$H_\beta$						blue-green
$H_\gamma$						blue

**Activity 2: Calculating the Rydberg Constant**

Using pairs of measured wavelengths and guesses for  $n_f$  and one of the  $n_i$ 's, calculate the other  $n_i$  in equation 2. When this calculated number is close to an integer you may have the correct value for the other  $n$ 's. Once you have determined the proper  $n$ 's, calculate a value of  $R_H$  for each line and compare the average of these with the accepted value. Use your results to predict the wavelength of the next line in the series  $\lambda_\delta$ . Its measured value is 4101.2Å. How does your prediction compare?

$$R_\alpha =$$

$$R_\beta =$$

$$R_\gamma =$$

$$R_{average} = \quad \quad \quad \% \text{ difference} =$$

$$\lambda_\delta = \quad \quad \quad \% \text{ difference} =$$

Collect values of  $R_{average}$  from the other groups in the class and calculate an average and standard deviation. Record it below. How does this result compare with the accepted value? How does it compare with your individual measurement? Be quantitative in your answer.

**Activity 3: The Hydrogen Level Diagram**

The energy levels of hydrogen can be described with the equation

$$E_n = -\frac{13.6 \text{ eV}}{n^2} \quad (4)$$

where  $n$  is called the principle quantum number. The relationship between the wavelength of the emitted light and its energy is  $E = hc/\lambda$  where  $c$  is the speed of light and  $h$  is Planck's constant. Make an energy level diagram showing the transitions you believe you have measured. Calculate the transition energies (from  $E = hc/\lambda$ ) based on your measured wavelengths; also calculate the values of these transition energies predicted by equation 4. How do your measured transition energies compare with the predicted ones?

## 40 The Twins Paradox

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

To investigate some of the unusual implications of Einstein's special theory of relativity.

### Overview

Einstein's theory of special relativity leads to a variety of apparent paradoxes that depart radically from our everyday expectations. One of the most celebrated is the twins paradox, in which an identical twin makes a long interstellar journey while the other twin remains on the (roughly) stationary Earth. When the space-faring twin returns she finds her partner has aged considerably more than she has. In this unit you will explore the quantitative aspects of the paradox and some of the surprising consequences.

### Activity 1: Setting Things Up

Problems in special relativity are often very counterintuitive, so it is instructive to consider the situation non-relativistically first. Investigate this problem without applying any of the new ideas you have learned about the theory of special relativity.

One member of a pair of identical twins has decided to embark on a long space voyage. The two twins have lived their lives in close proximity to one another and are very similar in appearance. The adventurous twin boards a fast spacecraft and leaves the Earth behind at a speed of  $0.99c$  or 99% of the speed of light. The space-faring twin's itinerary is rather monotonous, and she simply travels at this constant speed for a time, turns around, and returns to the Earth at the same speed. She measures the time of her trip to be  $\Delta t_0$ . In the meantime the Earth-bound twin has seen twenty years pass by. We will refer to this time as  $\Delta t$ .

(a) In mathematical terms, what is the relationship between the times  $\Delta t_0$  and  $\Delta t$ ?

(b) Which time is associated with which twin?

(c) When the twins are reunited will their appearances differ?

### Activity 2: Applying Special Relativity

(a) Now we want to apply the lessons of special relativity. Time dilation implies that moving clocks run more slowly when observed by someone in a different inertial frame. For the twins paradox what does this imply about the time interval the space-faring twin measures during her trip? Will it be less than, equal to, or greater than the interval measured by the Earth-bound twin? Will the space-faring twin age more, less, or the same amount as the Earth-bound twin?

(b) What is the mathematical relationship between  $\Delta t_0$  and  $\Delta t$  according to the special theory of relativity?

- (c) How much time has passed on the Earth-bound twin's clock?
- (d) How much time has passed on the space-faring twin's clock?
- (e) If this result is inconsistent with your prediction above how should you resolve the contradiction?
- (f) How will the two twins' appearances differ, if at all? Is the difference only in the measurement of the time intervals or are there real physiological differences between the twins after the trip?
- (g) If the average speed of the space-faring twin was more like the typical orbital speed of the space shuttle (about 7.4 km/s) what would the time difference between the twins' clocks be?

### Activity 3: Graphical Analysis

(a) Find a mathematical relationship for the ratio of the time interval measured by the space-faring twin to the time interval measured by the Earth-bound twin. Show your work and record your result here.

(b) You will now use *Excel* to make a plot showing how this ratio behaves as a function of  $\beta$  (the speed expressed as a fraction of the speed of light). To do this, start up *Excel*, and create a column headed "beta." This column should contain the series of numbers 0,0.05,0.10,0.15,...,1. To create such a column of numbers, enter the first two rows and highlight them. Then grab the lower-right corner of the second cell with the mouse and drag down (in the same way as if you were dragging down a formula).

After you have created the  $\beta$  column, create a second column containing the ratio of time intervals (i.e., the relationship you found in part (a)). Use an *Excel* formula.

(c) Make a plot of the ratio of the time interval measured by the space-faring twin to the time interval measured by the Earth-bound twin. At what speed does the effect of time dilation become significant? Is there a limit to the ratio? Is there any reason to restrict the range of  $\beta$  to 0-1? Clearly state your reasoning. Print your plot and attach a copy to this unit.

(d) Consider the following scenario. As the space-faring twin's craft recedes from the Earth it is moving at a constant speed. Since no inertial frame can be considered "better" than any other there is nothing physically inconsistent with the view that the space-faring twin is observing the Earth recede from her at a constant velocity. Hence, the space-faring twin will observe clocks on the Earth to move slowly and the Earth-bound twin will age at a slower rate than the space-faring one. Is this reasoning flawed? How?

(e) If the scenario is not flawed how can it be that the space-faring twin was found to have aged less in the original problem?

## A Treatment of Experimental Data

### Recording Data

When performing an experiment, record all required original observations as soon as they are made. By “original observations” is meant what you actually see, not quantities found by calculation. For example, suppose you want to know the stretch of a coiled spring as caused by an added weight. You must read a scale both before and after the weight is added and then subtract one reading from the other to get the desired result. The proper scientific procedure is to record both readings as seen. Errors in calculations can be checked only if the original readings are on record.

All data should be recorded with units. If several measurements are made of the same physical quantity, the data should be recorded in a table with the units reported in the column heading.

### Significant Figures

A laboratory worker must learn to determine how many figures in any measurement or calculation are reliable, or “significant” (that is, have physical meaning), and should avoid making long calculations using figures which he/she could not possibly claim to know. *All sure figures plus one estimated figure are considered significant.*

The measured diameter of a circle, for example, might be recorded to four significant figures, the fourth figure being in doubt, since it is an estimated fraction of the smallest division on the measuring apparatus. How this doubtful fourth figure affects the accuracy of the computed area can be seen from the following example.

Assume for example that the diameter of the circle has been measured as .5264 cm, with the last digit being in doubt as indicated by the line under it. When this number is squared the result will contain eight digits, of which the last five are doubtful. Only one of the five doubtful digits should be retained, yielding a four-digit number as the final result.

In the sample calculation shown below, each doubtful figure has a short line under it. Of course, each figure obtained from the use of a doubtful figure will itself be doubtful. The result of this calculation should be recorded as 0.2771 cm<sup>2</sup>, including the doubtful fourth figure. (The zero to the left of the decimal point is often used to emphasize that no significant figures precede the decimal point. This zero is not itself a significant figure.)

$$(.5264 \text{ cm})^2 = .27709696 \text{ cm}^2 = 0.2771 \text{ cm}^2$$

*In multiplication and division, the rule is that a calculated result should contain the same number of significant figures as the least that were used in the calculation.*

*In addition and subtraction, do not carry a result beyond the first column that contains a doubtful figure.*

### Statistical Analysis

Any measurement is an intelligent estimation of the true value of the quantity being measured. To arrive at a “best value” we usually make several measurements of the same quantity and then analyze these measurements statistically. The results of such an analysis can be represented in several ways. Those in which we are most interested in this course are the following:

**Mean** - The mean is the sum of a number of measurements of a quantity divided by the number of such measurements. (In other words, the mean is the same thing as what people generally call the “average.”) It generally represents the best estimate of true value of the measured quantity.

**Standard Deviation** - The standard deviation ( $\sigma$ ) is a measure of the range on either side of the mean within which approximately two-thirds of the measured values fall. For example, if the mean is 9.75 m/s<sup>2</sup> and the standard deviation is 0.10 m/s<sup>2</sup>, then approximately two-thirds of the measured values lie within the range 9.65 m/s<sup>2</sup> to 9.85 m/s<sup>2</sup>. A customary way of expressing an experimentally determined value is: Mean  $\pm$   $\sigma$ , or (9.75  $\pm$  0.10) m/s<sup>2</sup>. Thus, the standard deviation is an indicator of the spread in the individual measurements, and a small  $\sigma$  implies high precision. Also, it means that the probability of any future measurement falling in this range is approximately two to one. The equation for calculating the standard deviation is

$$\sigma = \sqrt{\frac{\sum (x_i - \langle x \rangle)^2}{N - 1}}$$



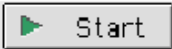

where  $x_i$  are the individual measurements,  $\langle x \rangle$  is the mean, and  $N$  is the total number of measurements.








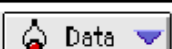


% Difference - Often one wishes to compare the value of a quantity determined in the laboratory with the best known or “accepted value” of the quantity obtained through repeated determinations by a number of investigators. *The % difference is calculated by subtracting the accepted value from your value, dividing by the accepted value, and multiplying by 100.* If your value is greater than the accepted value, the % difference will be positive. If your value is less than the accepted value, the % difference will be negative. The % difference between two values in a case where neither is an accepted value can be calculated by choosing either one as the accepted value.

## B Introduction to DataStudio

### Quick Reference Guide

Shown below is the quick reference guide for DataStudio.

What You Want To Do	How You Do It	Button
Start recording data	Click the 'Start' button or select 'Start Data' on the Experiment menu (or on the keyboard press CTRL - R (Windows) or Command - R (Mac))	
Stop recording (or monitoring) data	Click the 'Stop' button or select 'Stop Data' on the Experiment menu (or on the keyboard press CTRL - . (period) (Win) or Command - . (Mac))	
Start monitoring data	Select 'Monitor Data' on the Experiment menu (or on the keyboard press CTRL - M (Win) or Command - M (Mac))	<b>none</b>

On the Graph Display	In the Graph Toolbar	Button
Re-scale the data so it fills the Graph display window	Click the 'Scale to Fit' button.	
Pinpoint the x- and y-coordinate values on the Graph display	Click the 'Smart Tool' button. The coordinates appear next to the 'Smart Tool'.	
'Zoom In' or 'Zoom Out'	Click the 'Zoom In' or 'Zoom Out' buttons.	
Magnify a selected portion of the plotted data	Click the 'Zoom Select' button and drag across the data section to be magnified.	
Create a Calculation	Click the 'Calculate' button	
Add a text note to the Graph	Click the 'Note' button.	
Select from the Statistics menu	Click the Statistics menu button	
Add or remove a data run	Click the 'Add/Remove Data' menu button	
Delete something	Click the 'Delete' button	
Select Graph settings	Click the 'Settings' menu button	

### Selecting a Section of Data

1. To select a data section, hold the mouse button down and move the cursor to draw a rectangle around the data of interest. The data in the region of interest will be highlighted.
2. To unselect the data, click anywhere in the graph window.

### Fitting a Section of Data

1. Select the section of data to be fitted.

2. Click on the **Fit** button on the Graph Toolbar and select a mathematical model. The results of the fit will be displayed on the graph.
3. To remove the fit, click the **Fit** button and select the checked function type.

### **Finding the Area Under a Curve**

1. Use the **Zoom Select** button on the Graph Toolbar to zoom in around the region of interest in the graph. See the quick reference guide above for instructions.
2. Select the section of data that you want to integrate under.
3. Click the **Statistics** button on the Graph Toolbar and select **Area**. The results of the integration will be displayed on the graph.
4. To undo the integration, click on the **Statistics** button and select **Area**.

## C Introduction to Excel

Microsoft Excel is the spreadsheet program we will use for much of our data analysis and graphing. It is a powerful and easy-to-use application for graphing, fitting, and manipulating data. In this appendix, we will briefly describe how to use Excel to do some useful tasks.

### C.1 Data and formulae

Figure 1 below shows a sample Excel spreadsheet containing data from a made-up experiment. The experimenter was trying to measure the density of a certain material by taking a set of cubes made of the material and measuring their masses and the lengths of the sides of the cubes. The first two columns contain her measured results. **Note that the top of each column contains both a description of the quantity contained in that column and its units.** You should make sure that all of the columns of your data tables do as well. You should also make sure that the whole spreadsheet has a descriptive title and your names at the top.

In the third column, the experimenter has figured out the volume of each of the cubes, by taking the cube of the length of a side. To avoid repetitious calculations, she had Excel do this automatically. She entered the formula `=B5^3` into cell C5. Note the equals sign, which indicates to Excel that a formula is coming. The  $\wedge$  sign stands for raising to a power. After entering a formula into a cell, you can grab the square in the lower right corner of the cell with the mouse and drag it down the column, or you can just double-click on that square. (Either way, note that thing you're clicking on is the tiny square in the corner; clicking somewhere else in the cell won't work.) This will copy the cell, making the appropriate changes, into the rest of the column. For instance, in this case, cell C6 contains the formula `=B6^3`, and so forth.

Column D was similarly produced with a formula that divides the mass in column A by the volume in column C.

At the bottom of the spreadsheet we find the mean and standard deviation of the calculated densities (that is, of the numbers in cells D5 through D8). Those are computed using the formulae `=average(D5:D8)` and `=stdev(D5:D8)`.

### C.2 Graphs

Here's how to make graphs in Excel. For those who've used earlier version of Excel but not Excel 2007, note that the locations of some of the menu items have changed, although the basic procedure is similar.

First, use the mouse to select the columns of numbers you want to graph. (If the two columns aren't next to each other, select the first one, then hold down the control key while selecting the second one.) Then click on

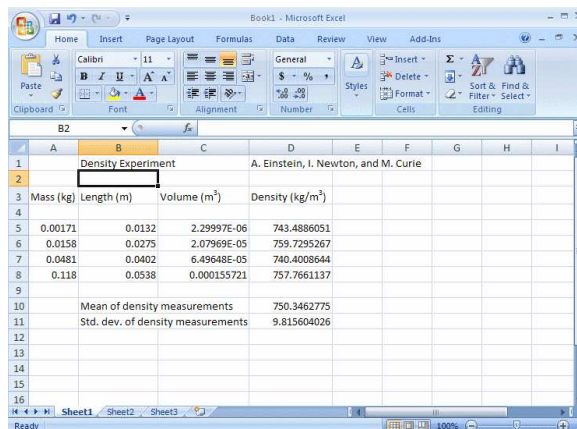



Figure 1: Sample Excel spreadsheet

the **Insert** tab at the top of the window. In the menu that shows up, there is a section called **Charts**. Almost all of the graphs we make will be scatter plots (meaning plots with one point for each row of data), so click on the **Scatter** icon. You'll see several choices for the basic layout of the graph. You usually want the first one, with an icon that looks like this . Click on this icon, and your graph should appear.

Next, you'll need to customize the graph in various ways, such as labeling the axes correctly. Everything you need to do this is in the **Chart Tools** menu, which should be visible in the upper right portion of the window. (If you don't see the words **Chart Tools**, try clicking on your newly-created graph, and it should appear.) The most useful items are under the **Layout** tab, so click on the word **Layout** under the **Chart Tools** menu. Here are some things to do under this menu:

- The most important item here is the **Axis Titles** menu. You can use this to add labels to the  $x$  and  $y$  axes of your graph, if it doesn't already have them. Edit the text inside of the two axis titles so that it indicates what's on the two axes of your graph *and the appropriate units*.
- It's probably a good idea to give your graph an overall title as well. The options for doing this are under **Chart Title** (not surprisingly).
- If the graph contains only one set of data points, you may wish to remove the legend that appears at the right side of the graph. After all, the information in the legend is probably already contained in the title and axis labels, so the legend just takes up space. Go to the **Legend** menu and click **None** to do this.
- Sometimes, you want your graph to contain a best-fit line passing through your data points. To do this, go to the **Trendline** menu. The easiest thing to do is to click on **More trendline options** at the bottom, which will bring up a dialog box with a bunch of choices. Excel can fit various kinds of curves through data points, but we almost always want a straight line, so you'll probably choose the **Linear** option. If you want to see the equation that describes this line, check the **Display Equation on chart** option near the bottom. Remember that Excel won't include the correct units on the numbers in this equation, but you should. Also, Excel will always call the two variables  $x$  and  $y$ , even though they might be something else entirely. Bear these points in mind when transcribing the equation into your lab notebook.

Sometimes, you may want to make a graph in Excel where the  $x$  column is to the right of the  $y$  column in your worksheet. In these cases, Excel will make the graph with the  $x$  and  $y$  axes reversed. There are at least two ways to fix this problem. Here's the simpler way: before you make your graph, make a copy of the  $y$  column in the worksheet and paste it so that it's to the right of the  $x$  column. Then follow the above procedure and everything will be fine. If you don't want to do that, here's another way. Click **Select data** (near the left-hand side under the **Chart Tools** menu). In the box that pops up, highlight **Series1** and click **Edit**. You should see a box that contains entries for **Series X values** and **Series Y values**. You want to swap the entries in those two windows. (But really, it's much easier to do it the first way.)

### C.3 Making Histograms

A histogram is a useful graphing tool when you want to analyze groups of data, based on the frequency at given intervals. In other words, you graph groups of numbers according to how often they appear. You start by choosing a set of 'bins', *i.e.*, creating a table of numbers that mark the edges of the intervals. You then go through your data, sorting the numbers into each bin or interval, and tabulating the number of data points that fall into each bin (this is the frequency). At the end, you have a visualization of the distribution of your data.

Start by entering your raw data in a column like the one shown in the left-hand panel of Figure 2. Look over your numbers to see what is the range of the data. If you have lots of values to sift through you might consider sorting your data in ascending or descending order. To do this task, choose the column containing your data by clicking on the letter at the top of the column, go to **Data** in the menubar, select **Sort**, and pick ascending or descending. The data will be rearranged in the order you've chosen and it will be easier to see the range of the data. For an example, see the middle column of data in the left-hand panel of Figure 2. Now to create your bins

pick a new column on your spreadsheet and enter the values of the bin edges. Make sure the bins you choose cover the range of the data. See the left-hand panel of Figure 2 again for an example.

You now have the ingredients for making the histogram. Go to **Data** in the menubar, select **Data Analysis**, and choose **Histogram**. You should see a dialog box like the one in the right-hand panel of Figure 2. Click in the box labeled **Input Range** and then highlight the column on the spreadsheet containing your data. Next, click in the box labeled **Bin Range** and highlight the column on the spreadsheet containing the bins. Under **Output Options**, select **New Worksheet Ply** and give the worksheet a name. Click **OK** in the **Histogram** dialog box. You should now see a new worksheet with columns labeled **Bin** and **Frequency** and a new tab at the bottom with the name you put in the **New Worksheet Ply** entry. See the left-hand panel in Figure 3. Your original data are still available on another worksheet (probably labeled **Sheet1**). Now highlight the **Bins** and **Frequency** columns by clicking and dragging across the column headings (the **A** and **B** at the top of the columns in the left-hand panel of Fig. 3). You can then make a graph by following the procedure in Appendix C.2 above. The only difference is that this time you will choose to make a **Column** graph instead of a **Scatter** graph. Make sure you properly label the axes including the units for each quantity. Results should look like the right-hand panel of Figure 3.

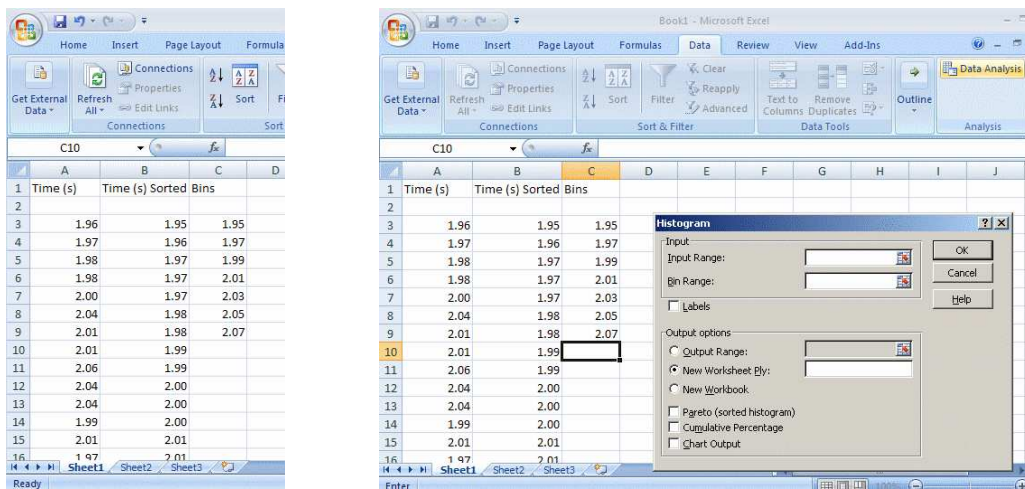


Figure 2: Column data and bins (left-hand panel) and dialog box (right-hand panel) for making a histogram in Excel.

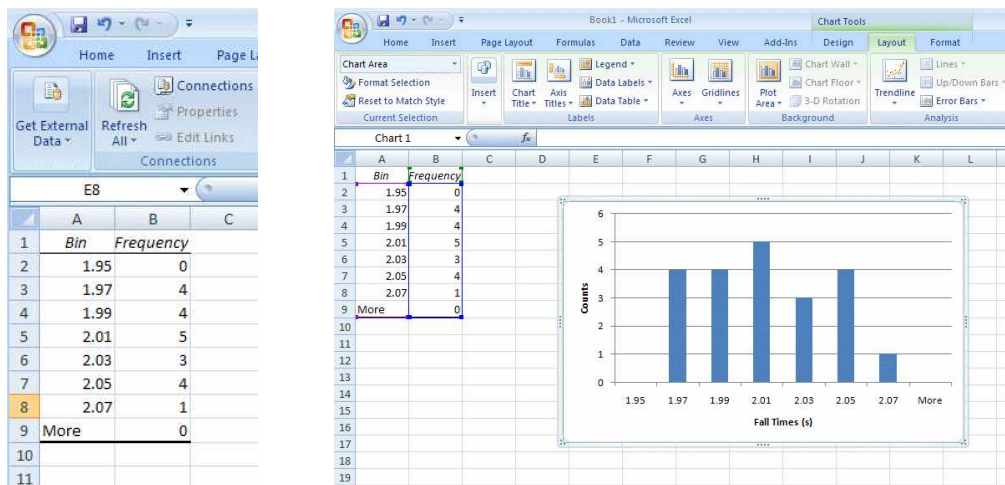


Figure 3: Newly-created worksheet (left-hand panel) and final plot (right-hand panel) for histogram worksheet in Excel.

## D Video Analysis

### Making a Movie

To make a movie, perform the following steps:

1. Start up *Videopoint Capture* by going to **Start** → **Programs** → **Physics Applications** → **VideoPoint** → **VP Capture**.
2. The program will first ask you to choose a file name and location for the video you are going to make. You should choose to put the file on the **Desktop** (so the data transfer will be fast enough).
3. Click on the **Capture rate** box and set the capture rate to 30 frames per second.
4. Go to the **Size & Colors** under the **Capture Options** menu, and choose the largest available size for the video.
5. Be sure the camera is at least 2 meters from the object you will be viewing. This constraint is required to reduce the effect of perspective for objects viewed near the edge of the field of view. Set up the camera so that its field of view is centered on the expected region where you will perform the experiment. When you perform the experiment only use the data when the object is in this central region.
6. Focus the camera by rotating the barrel on the outside of the lens until you have a clear picture.
7. Place a meter stick or some object of known size in the field of view where it won't interfere with the experiment. The meter stick should be the same distance away from the camera as the motion you are analyzing so the horizontal and vertical scales will be accurately determined. It should also be parallel to one of the sides of the movie frame.
8. One member of your group should perform the computer tasks while the other does the experiment.
9. To start recording your video, click **Record**. When you're done, click **Stop**.
10. The next step is to decide how much of the movie to save. Use the slider to step through the movie frame by frame. When you find the first frame you want to save, click **First**. When you find the last frame you want to save, click **Last**. You may want to save the entire movie, in which case **First** and **Last** really will be the first and last frames. Often, though, there will be "dead" time either at the beginning or the end of the movie, which you might as well cut out before saving.
11. After you've selected the range of frames you want to save, the button at the lower right should say **Keep**. Make sure that the box next to this button says **All**. (If it says **Double**, change it to **All**.) Then click **Keep** and **Save**. You will be asked to provide a file name. Pick something unique that you can easily identify. You will then see a quick replay of the movie as Videopoint converts and saves it.

### Analyzing the Movie

To determine the position of an object at different times during the motion, perform the following steps:

1. Start up *Videopoint* by going to **Start** → **Programs** → **Physics Applications** → **VideoPoint** → **VideoPoint 2.5**. Click **Open Movie**. You will see a dialog box. Set the **Files of type:** box to **All Files (\*.\*)** and select the file you created before. The file should have an extension like '.mov' or '.movvv'. Click **Open**.
2. **VideoPoint** will request the number of objects you want to track in the movie. Carefully read the instructions for the unit you are working on to find this number. Enter it in the space provided. You will now see several windows. (Note: You may have to move the movie window out of the way to see the other windows.) One contains the movie and has control buttons and a slider along the bottom of the frame to control the motion of the film. Experiment with these controls to learn their function. Another window below the movie frame (labeled **Table**) contains position and time data and a third window to the right of the frame (labeled **Coordinate Systems**) describes the coordinate system in use.

3. This is a good time to calibrate the scale. Go to a frame where an object of known size is clearly visible (see item 7 in the previous section). Under the **Movie** menu highlight **Scale Movie**. A dialog box will appear. Enter the length of the object and set **Scale Type** to **Fixed**. Click **Continue**. Move the cursor over the frame and click on the ends of scaling object.
4. You are now ready to record the position and time data. Go to the first frame of interest. Move the cursor over the frame and it will change into a small circle with an attached label. Place the circle over the object of interest in the frame and click. The  $x$  and  $y$  positions will be stored and the film advanced one frame. Move the circle over the position of the object in the frame and repeat. Continue this process until you have mapped out the motion of the object. If you entered more than one object to keep track of when you opened the movie, then you will click on all those objects in each frame before the film advances. Remember to restrict your analysis to the central region of the field of view to reduce any distortion created by the camera lens. For example, to study free-fall the ruler used for setting the scale should be in the central region of the camera's field of view. You should only take data while the object is falling from one end of the ruler to the other.
5. When you have entered all the points you want, go to the **File** menu and select **Export data**. This will allow you to save your data table as an Excel file. Save this file on the desktop (by clicking on the "Open" button, which actually doesn't open anything), and double-click on the saved file to start up Excel. You will now be able to continue your data analysis in Excel.
6. Once you have looked at your data in Excel and made sure everything looks OK, you can quit Videopoint Analysis. If you are sure you have exported your data correctly to Excel, there is no need to save in Videopoint.

### Changing the Origin

To change the position of the origin take the following steps.

1. Click on the arrow icon near the top of the menu bar to the left. The cursor will be shaped like an arrow when you place it on the movie frame.
2. Click at the origin (where the axes cross) and drag the origin to the desired location.
3. Click on the circle at the top of the menu bar to the left to return to the standard cursor for marking points on the film.

### Using a Moving Coordinate System

To record the position of an object and to change the coordinate system from frame to frame take the following steps.

1. Open the movie as usual and enter one object to record. First we have to select the existing origin and change it from a fixed one to a moving one. Click on the arrow near the top of the menu bar to the left. The cursor will have the shape of an arrow when you place it on the movie frame. Click on the existing origin (where the axes cross) and it will be highlighted.
2. Under the **Edit** menu drag down and highlight **Edit Selected Series**. A dialog box will appear. Click on the box labeled **Data Type** and highlight the selection **Frame-by-Frame**. Click OK.
3. Click on the circle at the top of the menu bar to the left to change the cursor back to the usual one for marking points. Go to the first frame of interest. When the cursor is placed in the movie frame it will be labeled with "Point S1." Click on the object of interest. The film will NOT advance and the label on the cursor will change to "Origin 1." Click on the desired location of the origin in that frame. The film will advance as usual. Repeat the procedure to accumulate the  $x$ - and  $y$ -positions relative to the origin you've defined in each frame.



## E Instrumentation

### Introduction

Being both quantitative and experimental, physics is basically a science of measurement. A great deal of effort has been expended over the centuries improving the accuracy with which the fundamental quantities of length, mass, time, and charge can be measured.

It is important that the appropriate instrument be used when measuring. Ordinarily, a rough comparison with a numerical scale, taken at a glance and given in round numbers, is adequate. Increasing precision, though, requires a more accurate scale read to a fraction of its smallest division. The “least count” of an instrument is the smallest division that is marked on the scale. This is the smallest quantity that can be read directly without estimating fractions of a division.

Even at the limit of an instrument’s precision, however, accidental errors — which cannot be eliminated — still occur. These errors result in a distribution of results when a series of seemingly identical measurements are made. The best estimate of the true value of the measured quantity is generally the arithmetic mean or average of the measurements.

Other errors, characteristic of all instruments, are known as systematic errors. These can be minimized by improving the equipment and by taking precautions when using it.

### Length Measurement

Three instruments will be available in this class for length measurements: a ruler (one- or two-meter sticks, for example), the vernier caliper, and the micrometer caliper.

#### *The Meter Stick*

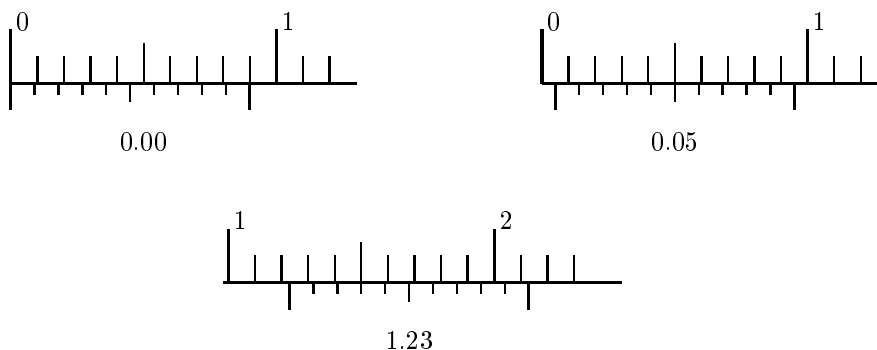
A meter stick, by definition, is 1 meter (m) long. Its scale is divided, and numbered, into 100 centimeters (cm). Each centimeter, in turn, is divided into 10 millimeters. Thus  $1\text{ cm} = 10^{-2}\text{ m}$ , and  $1\text{ mm} = 10^{-1}\text{ cm} = 10^{-3}\text{ m}$ .

When measuring a length with a meter stick, different regions along the scale should be used for the series of measurements resulting in an average value. This way, non-uniformities resulting from the meter stick manufacturing process will tend to cancel out and so reduce systematic errors. The ends of the stick, too, should be avoided, because these may be worn down and not give a true reading. Another error which arises in the reading of the scale is introduced by the positioning of the eyes, an effect known as parallax. Uncertainty due to this effect can be reduced by arranging the scale on the stick as close to the object being measured as possible.

#### *The Vernier Caliper*

A vernier is a small auxiliary scale that slides along the main scale. It allows more accurate estimates of fractional parts of the smallest division on the main scale.

On a vernier caliper, the main scale, divided into centimeters and millimeters, is engraved on the fixed part of the instrument. The vernier scale, engraved on the movable jaw, has ten divisions that cover the same spatial interval as nine divisions on the main scale: each vernier division is  $\frac{9}{10}$  the length of a main scale division. In the case of a vernier caliper, the vernier division length is 0.9 mm. [See figures below.]



Examples of vernier caliper readings

To measure length with a vernier caliper, close the jaws on the object and read the main scale at the position indicated by the zero-line of the vernier. The fractional part of a main-scale division is obtained from the first vernier division to coincide with a main scale line. [See examples above.]

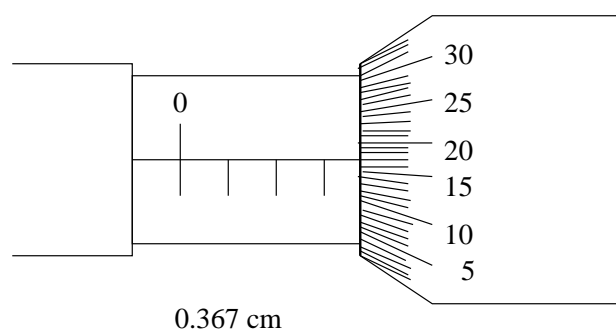
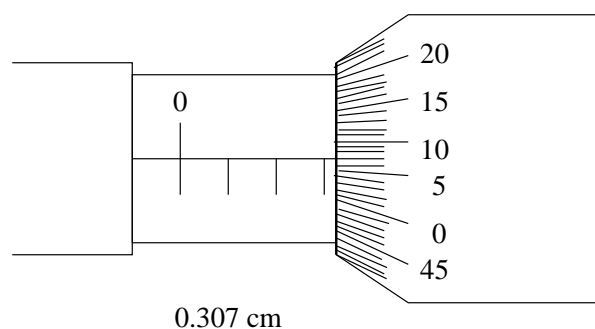
If the zero-lines of the main and vernier scales do not coincide when the jaws are closed, all measurements will be systematically shifted. The magnitude of this shift, called the zero reading or zero correction, should be noted and recorded, so that length measurements made with the vernier caliper can be corrected, thereby removing the systematic error.

### *The Micrometer Caliper*

A micrometer caliper is an instrument that allows direct readings to one hundredth of a millimeter and estimations to one thousandth of a millimeter or one millionth of a meter (and, hence, its name). It is essentially a carefully machined screw housed in a strong frame. To measure objects, place them between the end of the screw and the projecting end of the frame (the anvil). The screw is advanced or retracting by rotating a thimble on which is engraved a circular scale. The thimble thus moves along the barrel of the frame which contains the screw and on which is engraved a longitudinal scale divided in millimeters. The pitch of the screw is 0.5 mm, so that a complete revolution of the thimble moves the screw 0.5 mm. The scale on the thimble has 50 divisions, so that a turn of one division is  $\frac{1}{50}$  of 0.5 mm, or 0.01 mm.

Advance the screw until the object is gripped gently. Do not force the screw. A micrometer caliper is a delicate instrument.

To read a micrometer caliper, note the position of the edge of the thimble along the longitudinal scale and the position of the axial line on the circular scale. The first scale gives the measurement to the nearest whole division; the second scale gives the fractional part. It takes two revolutions to advance one full millimeter, so note carefully whether you are on the first or second half of a millimeter. The result is the sum of the two scales. (See examples below).



As with the vernier caliper, the zero reading may not be exactly zero. A zero error should be checked for and recorded, and measurements should be appropriately corrected.

### **Mass Measurement**

Three kinds of instruments will be available to determine mass: a digital scale and two types of balances. The operation of the first instrument is trivial, and so will not be explained here.

Please understand that with each of these instruments we are really comparing weights, not masses, but the proportionality of weight and mass allows the instruments to be calibrated for mass.

#### *The Equal-Arm Balance*

The equal-arm balance has two trays on opposite sides of a pivot. The total mass placed on one tray required to balance the object on the other gives the mass of the object. Most equal-arm balances have a slider, as well, that can move along a scale and allow for greater precision than the smallest calibrated mass available. Typically, this scale has 0.5 g divisions.

#### *The Triple-Beam Balance*

The triple-beam balance, so-called because of its three slider scales, can be read to 0.1 g and estimated to half that. With an object on the tray, the masses of the different scales are slid to notches until balanced. Get close with the larger masses first and then fine-adjust with the smallest slider.

### **Time Measurement**

Time measurements in this course will be made either with a computer or with a stop watch. This first is out of your control.

#### *The Stop Watch*

The stop watches you will use in class have a time range of from hours to hundredths of a second. There are two buttons at the top: a stop/start button and a reset button. The operation of these should be evident, although once the watch is reset, the reset button also starts the watch (but doesn't stop it). Please be aware of this feature.

### **Charge Measurements**

The magnitude of charge is among the most difficult measurements to make. Instead a number of indirect measurements are undertaken to understand electric phenomena. These measurements are most often carried out with a digital multimeter

#### *The Digital Multimeter*

The digital multimeters available for laboratory exercises have pushbutton control to select five ac and dc voltage ranges, five ac and dc current ranges, and six resistance ranges. The ranges of accuracy are 100 microvolts to 1200 volts ac and dc, 100 nanoamperes to 1.999 amperes ac and dc, and 100 milliohms to 19.99 megaohms.

To perform a DC voltage measurement, select the DCV function and choose a range maximum from one of 200 millivolts or 2, 20, 200, or 1200 volts. Be sure the input connections used are V- $\Omega$  and COMMON. The same is true for AC voltage, regarding range and inputs, but the ACV function button should be selected.

For DC current choose DC MA (for DC milliamperes), while for AC current choose AC MA. Your choices for largest current are 200 microamperes or 2, 20, 200, or 2000 milliamperes. Check that the input are connected to MA and COMMON.

There are two choices for resistance measurement: Kilohms ( $k\Omega$ ) and Megohms ( $20M\Omega$ ). The input connectors are the same as when measuring voltage, namely V- $\Omega$  and COMMON. The range switches do not function with the Megohm function, but one of the range buttons must be set. The maximum settings for Kilohm readings are  $200\Omega$  or 2, 20, 200, or  $2000k\Omega$ .

### Calibrating Force Sensors

1. Connect force sensor to Pasco interface (in port “A”).
2. Open V, A & F Graphs Application.
3. With NO force applied to force sensor, press TARE button on side of force sensor. This sets the sensor to zero. This is the ONLY time you will press the TARE button.
4. Click “Setup”.
5. Click “Calibrate Sensors”.
6. Set 1st calibration point to 0 newtons, press upper “Read from sensor” button.
7. Hang 200g from sensor, hold sensor still, set 2nd calibration point to 1.96 newtons, press lower “Read from sensor” button.
8. Click “OK”. Force sensor is now calibrated for the rest of your experiment. Close “Calibrate Sensors” window, close “Setup” window.
9. While still holding the force sensor still, press “Start”. Graph should now show a reading of 1.96 N. Press “Stop”.
10. Try hanging a different mass from the force sensor; press “Start” and check that it is reading correctly.