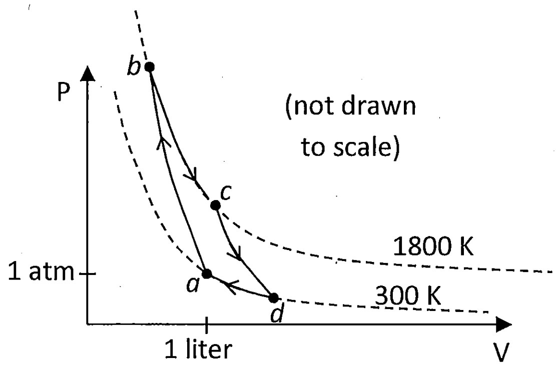
SOLUTIONS

Thermodynamics Worksheet #2: An ideal gas in a Carnot cycle



	P	V	nR	T
a	10^5 Pa	10^{-3} m^3	$\frac{1}{3}$ J/K	300 K
b	52.91×10^6	1.134×10^{-5}	$\frac{1}{3}$ J/K	1800 K
c	35.46×10^6	1.69×10^{-5}	$\frac{1}{3}$ J/K	1800 K
d	6.70×10^4	1.492×10^{-3}	$\frac{1}{3}$ J/K	300 K

As in the previous worksheet, we start with a sample of diatomic N_2 gas at pressure $P_a = 10^5$ N/m^2 (about 1 atm), volume $V_a = 1$ liter, and temperature $T_a = 300$ K. From before, this gives $nR = \left(\frac{1}{3}\right)$ J/K, or $n = 0.0401$moles. As before, we heat the gas to a maximum temperature of 1800 K, but here we do so in one step, a single adiabatic compression.

1. Recalling that PV^γ is constant for an adiabatic process, where $\gamma = C_P/C_V$, what is the final volume V_b? (Answer: $V_b = 1.134 \times 10^{-5}$ m^3.)

$$P_b V_b^\gamma = P_a V_a^\gamma$$

$$V_b^\gamma = \frac{P_a V_a^\gamma}{P_b}$$

substitute $P_b = \frac{nRT_b}{V_b}$

$$V_b^\gamma = \frac{P_a V_a^\gamma}{nRT_b/V_b}$$

$$V_b^{\gamma-1} = \frac{P_a V_a^\gamma}{nRT_b}$$

$$V_b = \left(\frac{P_a V_a^\gamma}{nRT_b}\right)^{\frac{1}{\gamma-1}} \quad \text{where } \gamma = \frac{C_P}{C_V} = \frac{7}{5}$$

$$V_b = 1.134 \times 10^{-5} \text{ m}^3$$

(see next page for alternate solution.)

2. What are ΔE_{ab}, W_{ab}, and Q_{ab}? Go ahead and start filling out the table on the next page if you like. Also, the table at the top of this page may help you keep your thoughts organized.

$Q_{ab} = 0$, because it's adiabatic.

$$\Delta E_{ab} = \frac{5}{2} nR\Delta T = 1250\text{ J}$$

$$W = \Delta E_{ab} = 1250\text{ J}$$

1. (Alternate Solution) (Kudos to Arsalan for this!)

$$P_a V_a^\gamma = P_b V_b^\gamma$$

$$\frac{P_a}{P_b} = \left(\frac{V_b}{V_a}\right)^\gamma$$

$$\frac{nRT_a/V_a}{nRT_b/V_b} = \left(\frac{V_b}{V_a}\right)^\gamma$$

$$\frac{T_a}{T_b} = \left(\frac{V_b}{V_a}\right)^{\gamma-1}$$

$$\left(\frac{T_a}{T_b}\right)^{\frac{1}{\gamma-1}} = \frac{V_b}{V_a}$$

$$V_b = V_a \left(\frac{T_a}{T_b}\right)^{\frac{1}{\gamma-1}} = (10^{-3}\,m^3)\left(\frac{300\,K}{1800\,K}\right)^{\frac{1}{\frac{7}{5}-1}}$$

$$V_b = 1.134 \times 10^{-5}\,m^3$$

3. In the process $b \to c$, the gas is expanded isothermally to a new volume, $V_c = 1.6918 \times 10^{-5}$ m^3. Calculate ΔE_{bc}, W_{bc}, and Q_{bc} for this process. (This particular value for V_c makes the numbers in the table turn out pretty. You'll see.)

$\Delta E_{bc} = 0$, because process is isothermal.

$$W_{bc} = -\int P\,dV = -\int \frac{nRT}{V}\,dV = -nRT\int_{V_b}^{V_c} \frac{1}{V}\,dV = -nRT\ln\frac{V_c}{V_b} = -240\text{J}$$

$$Q_{bc} = -W_{bc} = +240\text{J}$$

4. Now the gas is expanded adiabatically back to $T_d = 300$ K. Find V_d, and also find ΔE_{cd}, W_{cd}, and Q_{cd}.

As in problem 1: $$V_d = \left(\frac{P_c V_c}{nRT_d}\right)^{\frac{1}{\gamma-1}} = 1.492\times10^{-3}\,m^3$$

or $$V_d = V_c\left(\frac{T_c}{T_d}\right)^{\frac{1}{\gamma-1}} = 1.492\times10^{-3}\,m^3$$

$$\Delta E_{cd} = \frac{5}{2}nR\Delta T = -1250\text{J}$$

$Q_{cd} = 0$, because adiabatic. $W_{cd} = \Delta E_{cd} = -1250\text{J}$

5. Finally, the gas is compressed isothermally back to V_a. Find ΔE_{da}, W_{da}, and Q_{da}.

$$W_{da} = -\int P\,dV = -nRT\int_{V_d}^{V_a}\frac{dV}{V} = -nRT\ln\frac{V_a}{V_d} = +40\text{J}$$

$\Delta E_{cd} = 0$; $Q = -W = -40\text{J}$

6. If you haven't done so already, complete the following table:

	ΔE	W	Q
$a \to b$	+1250 J	+1250 J	0
$b \to c$	0	−240 J	+240 J
$c \to d$	−1250 J	−1250 J	0
$d \to a$	0	+40 J	−40 J
NET:	0	−200 J	+200 J

7. Compare the table on page 2 of this worksheet with the table on the previous thermodynamics worksheet, for a rectangular cycle.

a) What is the Net work done by the gas in each case?

$W_{\substack{NET \\ RECT}} = +200\,J$ $\qquad$ $W_{\substack{NET \\ CARNOT}} = +200\,J$

b) What is the total heat Q_{IN} put into the gas from the hot reservoir?

$\left|Q_{\substack{IN \\ RECT}}\right| = 1400\,J + 250\,J$ $\qquad$ $\left|Q_{\substack{IN \\ CARNOT}}\right| = 240\,J$

$\left|Q_{\substack{in \\ Rect}}\right| = 1650\,J$

c) What is the total heat Q_{OUT} dumped into the cold reservoir in each process?

$\left|Q_{\substack{OUT \\ RECT}}\right| = 1450\,J$ $\qquad$ $\left|Q_{\substack{OUT \\ CARNOT}}\right| = 40\,J$

d) Which heat engine is more efficient? That is, which heat engine does the most work per ton of coal burned?

For this Carnot cycle, $\dfrac{W_{NET}}{Q_{in}} = \dfrac{200\,J}{240\,J} = 0.83.$

For the rectangular cycle, $\dfrac{W_{NET}}{Q_{in}} = \dfrac{200\,J}{1650\,J} = 0.12.$

The Carnot cycle is much more efficient.