

First, let's see how a change in \vec{B} will affect \vec{E} .

We'll make a rectangular loop Δx wide by ℓ high, and evaluate Faraday's law:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{\partial \Phi_B}{\partial t}$$

Go around counter-clockwise, starting on left leg:

$$-\underbrace{E(x)}_{\text{"E of x," not "E times x"}} \ell + 0\Delta x + \underbrace{E(x + \Delta x)}_{\text{"E of x + Δx," not "E times x + Δx"}} \ell + 0\Delta x = -\frac{\partial(B\Delta x\ell)}{\partial t}$$

"E of x," not
"E times x"

$$E(x + \Delta x) = E(x) + \frac{\partial E}{\partial x} \Delta x \quad (\text{from your calc class})$$

$$-E(x)\ell + \left(E(x) + \frac{\partial E}{\partial x} \Delta x\right)\ell = -\frac{\partial(B\Delta x\ell)}{\partial t}$$

$$-E(x)\ell + E(x)\ell + \frac{\partial E}{\partial x} \Delta x \ell = -\Delta x \ell \frac{\partial B}{\partial t}$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

Call this equation (1)

