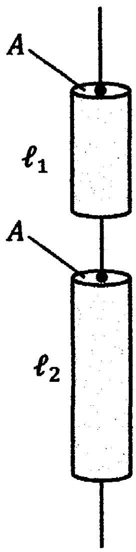
SOLUTIONS

Some random exercises with circuits, resistors, etc.

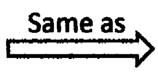
Part 1.

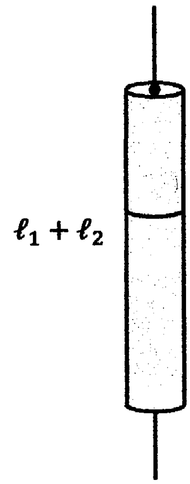
Suppose you have two "resistors" in series, which are both cylinders of material with the same cross sectional area A and resistivity ρ, and different lengths ℓ_1 and ℓ_2. Write their resistances below.



$R_1 = \frac{\rho \ell_1}{A}$

$R_2 = \frac{\rho \ell_2}{A}$





Write R_{eq} in terms of $\ell_1, \ell_2, \rho,$ and A:

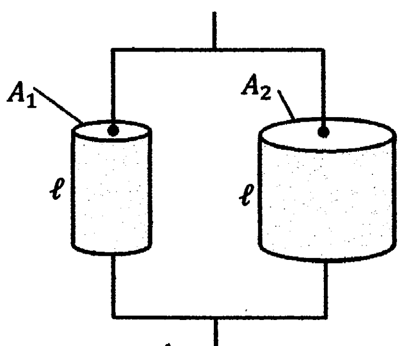
$R_{eq} = \frac{\rho(\ell_1+\ell_2)}{A}$

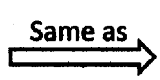
Use the result above to write R_{eq} in terms of R_1, and R_2.

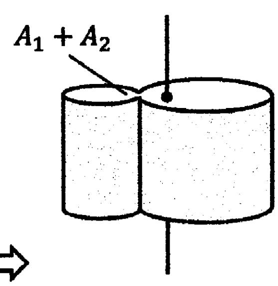
$R_{eq} = \frac{\rho \ell_1}{A} + \frac{\rho \ell_2}{A}$

$= R_1 + R_2$

Suppose you have two "resistors" in parallel, which are both cylinders of material with the same cross sectional length ℓ and resistivity ρ, and different Areas A_1 and A_2. Write their resistances below.







$R_1 = \frac{\rho \ell}{A_1}$

$R_2 = \frac{\rho \ell}{A_2}$

Write R_{eq} in terms of ℓ, ρ, A_1 and A_2:

$R_{eq} = \frac{\rho \ell}{(A_1+A_2)}$

Use the result above to write R_{eq} in terms of R_1, and R_2. (Hint: start by writing $1/R_{eq}$.)

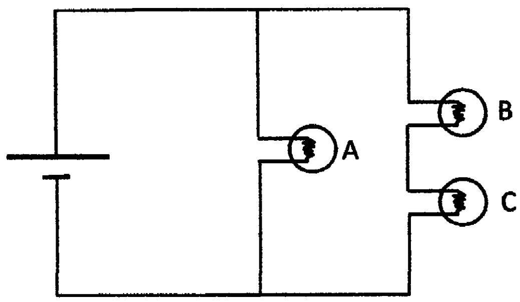
$$\frac{1}{R_{eq}} = \frac{A_1 + A_2}{\rho \ell} = \frac{A_1}{\rho \ell} + \frac{A_2}{\rho \ell}$$

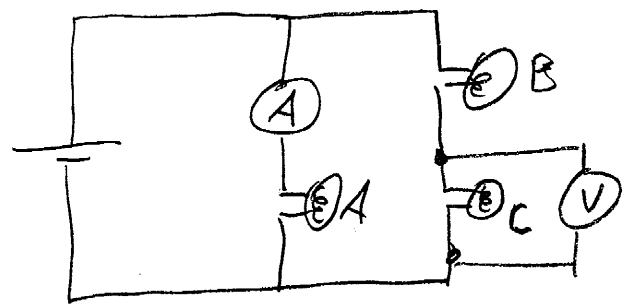
$$= \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

Part 2.

The circuit diagram below shows a battery connected to three light bulbs:





(a) In the space above, draw a diagram showing how to add a current meter and a voltmeter to measure the current through bulb A, and the voltage across bulb C.

(b) In general, you hope that when you connect current and voltage meters, they don't actually change any currents or voltages in your circuit. Ideally, would you want the resistance of your current meter to be very small, very large, or somewhere in the middle?

Ideally, want $R_{Ammeter}$ small.

(c) Ideally, would you want the resistance of your voltmeter to be very small, very large, or somewhere in the middle?

Ideally, want $R_{Voltmeter}$ large

(d) You have two DMMs at your station. Use one of them as a resistance meter (on "Ω") to measure the resistance of the other meter when it is used as a current meter (on all four ranges) and as a voltmeter. Write your results below. (Results here are for the black BK meters)

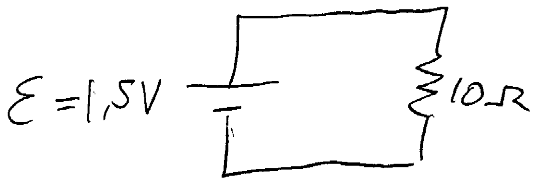
$R_{Voltmeter} > 40\ M\Omega$

$$R_{Ammeter} = \begin{cases} \sim 0.1\ \Omega & \text{on 20A scale} \\ \sim 2.0\ \Omega & \text{on 400mA scale} \\ \sim 20\ \Omega & \text{on 40mA scale} \end{cases}$$

Did your measurements agree with your predictions in parts (b) and (c)? Why, yes! ☺

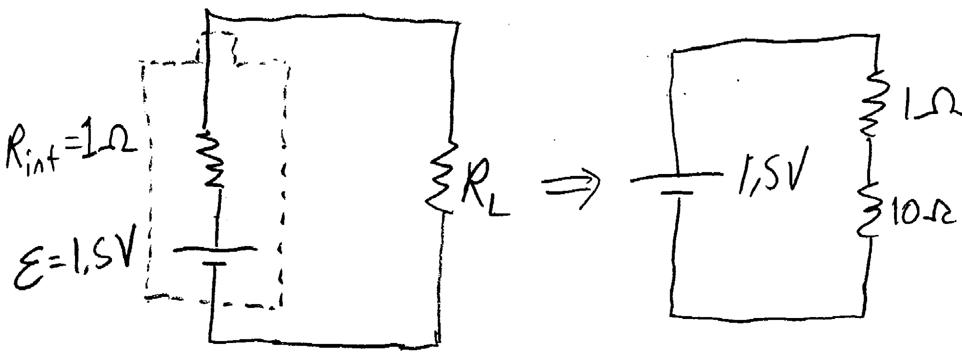
Part 3.

An ideal battery would always produce a constant EMF $\mathcal{E}$ across the "load" resistance. (The "load" might be a motor or a light bulb, for instance.) What would be the current through a load resistance of 10 Ω, if $\mathcal{E}$ = 1.5 Volts?



$$I = \frac{\mathcal{E}}{R_L} = \frac{1.5\,V}{10\,\Omega} = 0.15 \text{ Amps}$$

A typical 1.5 volt alkaline battery might have an additional "internal" resistance R_{int} = 1Ω What would be the current through a 10 Ω load resistance in that case?



$R_{eq} = 11\,\Omega$

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{1.5\,V}{11\,\Omega} = 0.136A$$

For the non-ideal battery above, with $\mathcal{E}$ = 1.5 Volts and R_{int} = 1 Ω, what would be the voltage across a 10 Ω load resistance?

$$\Delta V_{R_L} = I_{R_L} R = (0.136A)(10\,\Omega) = 1.36 \text{ Volts}$$

What current would the non-ideal battery above produce if you short circuited the two terminals together with a wire? What would be the short-circuit current of an "ideal" battery?

Assuming the wire has zero resistance, the battery above would produce a current of

$$I = \frac{1.5\,V}{1\,\Omega} = 1 \text{ Amp.}$$

Since a mythical "ideal" battery has $R_{int} = 0$, it would apparently produce an infinite current, which is of course nonsense.