

# Investigative Physics

## Module 2: Activity Units for Physics 132

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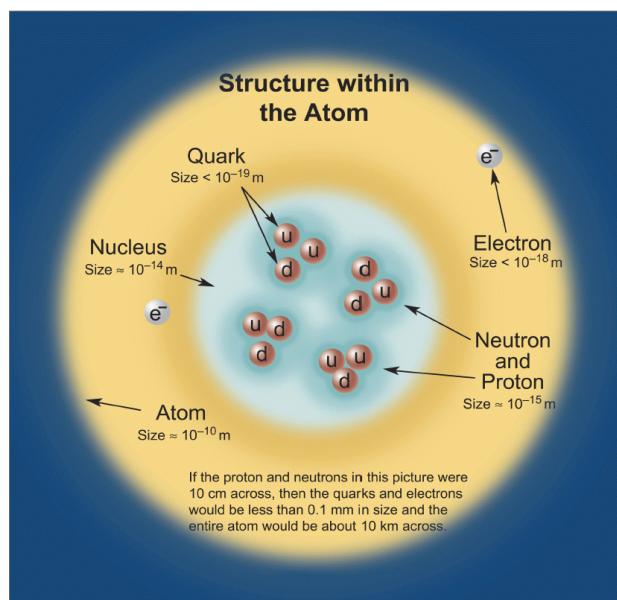
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### Abstract

The exercises in this manual have been developed to support an investigative physics course that emphasizes active learning. Some of these units have been taken from the Workshop Physics project at Dickinson College and the Tools for Scientific Thinking project at Tufts University and modified for use at the University of Richmond. Others have been developed locally.

The units are made up of activities designed to guide your investigations in the laboratory. The written work will consist primarily of documenting your class activities by filling in the entries in the spaces provided in the units. The entries consist of observations, derivations, calculations, and answers to questions. Although you may use the same data and graphs as your partner(s) and discuss concepts with your classmates, all entries should reflect your own understanding of the concepts and the meaning of the data and graphs you are presenting. Thus, each entry should be written in your own words. Indeed, it is very important to your success in this course that your entries reflect a sound understanding of the phenomena you are observing and analyzing.

We wish to acknowledge the support we have received for this project from the University of Richmond and the Instrumentation and Laboratory Improvement program of the National Science Foundation. Also, we would like to thank our laboratory directors for their invaluable technical assistance.



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Cover art: The atom consists of smaller electrons and a nucleus consisting of protons and neutrons. The protons and neutrons, in turn, are formed from objects called quarks and gluons. Courtesy, the American Institute of Physics.

## Contents

1 Heat, Temperature, and Internal Energy	5
2 Calorimetry	7
3 Heat of Vaporization of Nitrogen	11
4 Heat of Vaporization of Nitrogen	13
5 Boyle's Law	14
6 Charles' Law	17
7 The P-T Relationship of a Gas	20
8 Impulse, Momentum, and Interactions	22
9 Kinetic Theory of Ideal Gases	26
10 Applying the Kinetic Theory	31
11 Einstein Solid	36
12 Entropy and Temperature	45
13 Music to Our Ears: Standing Waves in Strings	52
14 Resonance in Tubes	58
15 Electrostatics	60
16 The Electrical and Gravitational Forces	64
17 The Electric Field Near a Charged Rod	69
18 The Electric Potential	71
19 Electric Fields and Equipotential Lines	78
20 The Electric Field and the Electric Potential I	81
21 The Electric Field and the Electric Potential II	85
22 The Electric Field and the Electric Potential III	88
23 The Electric Field and the Electric Potential IV	91

24 The Electric Field of the Atomic Nucleus	94
25 The Charge Distribution of the H <sub>2</sub> O Molecule	97
26 Capacitance Series Circuit	101
27 Ohm's Law	103
28 Kirchhoff's Rules	108
29 RC Circuits	111
30 Magnetism I	114
31 Magnetism II	117
32 Magnetism III	120
33 Magnetic Field of the Earth	123
34 Centripetal Force	126
35 Weighing an Electron	132
36 Nuclear Decay and Radiocarbon Dating	139
37 Electromagnetic Induction	148
38 Electromagnetic Induction II	154
39 The LR Circuit	159
40 Refraction of Light	164
41 Refraction at Spherical Surfaces: Thin Lenses	167
42 The Interference of Light	169
43 Diffraction of Light	173
44 Diffraction 2	179
45 The Diffraction Grating	181
46 Conservation of Angular Momentum	182
47 The Optical Spectrum of Hydrogen	185

<b>48 A Theory for the Hydrogen Atom</b>	<b>188</b>
<b>49 Where's the Electron?</b>	<b>195</b>
<b>50 Galilean Relativity</b>	<b>202</b>
<b>51 The Twins Paradox</b>	<b>206</b>
<b>A Treatment of Experimental Data</b>	<b>209</b>
<b>B Introduction to DataStudio</b>	<b>211</b>
<b>C Introduction to Excel</b>	<b>213</b>
C.1 Data and formulae . . . . .	213
C.2 Graphs . . . . .	213
C.3 Making Histograms . . . . .	214
C.4 LINEST . . . . .	216
<b>D Video Analysis Using ViseoPoint</b>	<b>217</b>
<b>E Video Analysis Using Tracker</b>	<b>219</b>
<b>F Instrumentation</b>	<b>222</b>
<b>G Nuclear Safety</b>	<b>226</b>

# 1 Heat, Temperature, and Internal Energy

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

**Objective**

To investigate the relationship between heat and temperature.

**Apparatus**

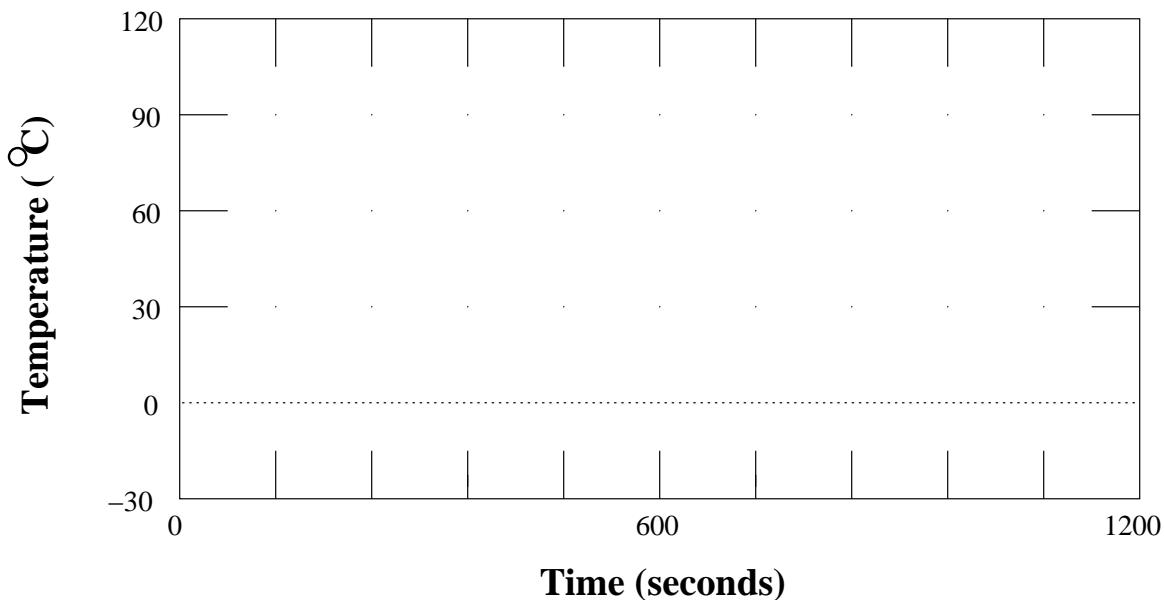
- Glass beaker
- Glass stirring rod
- Hot plate
- Ice
- Data Studio software and temperature probe
- Clamp and stand

**Temperature of a Substance as a Function of Heat Transfer**

As part of our quest to understand heat energy transfer, temperature, and internal energy of a substance, let's consider the temperature change as ice is changed to water and then to steam.

**Activity 1: Predicting T vs. t for Water**

Suppose you were to add heat at a constant rate to a container of ice water at  $0^{\circ}\text{C}$  until the water begins to boil. Sketch the predicted shape of the heating curve on the graph below using a dashed line. Mark the points at which the ice has melted and the water begins to boil.



**Activity 2: Measuring T vs. t for Water**

(a) To test your prediction:

1. Fill the glass beaker one third full of ice and set it on top of the hot plate.
2. Suspend the temperature probe so that the end is submerged in the ice but not touching the side or bottom of the beaker. You will need to use the clamp and stand to do this. BE SURE THE WIRE FROM THE TEMPERATURE PROBE IS NOT TOUCHING THE HOT PLATE OR GLASS BEAKER. Also, be sure the temperature probe is attached to port "A" of the Pasco interface. Let set for 60 seconds so temperature probe can come to equilibrium with the ice.
3. Open the *Heat, Temp, & Internal Energy ss* application in the 132 Workshop folder under *Physics applications*.
4. Turn on the hot plate to about 6 and click the **Start** button on the monitor to begin recording data. The temperature of the water will be recorded on the graph shown on the monitor. Stir often to keep ice against the temperature probe.
5. When ice begins to turn to water, stir at about 1 minute intervals. When all ice is melted, there is no need to stir any further.
6. After the water begins to boil, keep hot plate on for a minute or two as temperature flattens at 100 degrees.
7. Stop data collection using the **Stop** button and turn off hot plate.
8. Sketch the shape of the measured heating curve on the above graph using a solid line. Ignore small variations due to noise and uneven heating. Mark the points at which the ice has melted and the water begins to boil.
9. Print your graph and attach to this unit.

(b) Does your prediction agree with the measured heating curve? If not, what are the differences?

(c) What is the relationship between the temperature and the added heat while the ice is melting?

(d) What is the relationship between the temperature and the added heat after the ice has melted, but before the water begins to boil?

(e) What is the relationship between the temperature and the added heat while the water is boiling?

(f) If there are regions of the heating curve in which the temperature is not changing, what do you think is happening to the added heat in these regions?

## 2 Calorimetry

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

**Purpose**

- To learn to use a method for measuring heat called calorimetry.
- To use calorimetry to determine the specific heat of aluminum and the heat of fusion of ice.

**Apparatus**

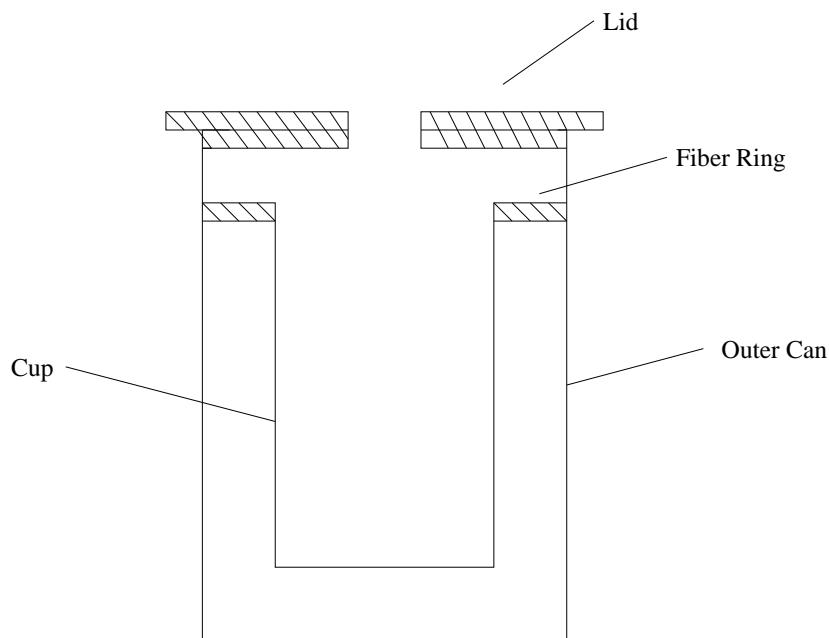
- Hypsometer
- Hot plate
- Aluminum pellets
- Plastic beaker
- Ice
- Compact scale (for measuring masses)
- Calorimeter
- Temperature probe
- Glass stirring rod
- Clamp and stand
- Safety goggles
- *DataStudio* software (Calorimetry)

**Introduction**

Calorimetry is a method for measuring heat. As applied in this experiment, the method involves the mixing together of substances initially at two different temperatures. The substances at the higher temperature lose heat and the substances at the lower temperature gain heat until thermal equilibrium is reached.

**Activity 1: Statement of Conservation of Energy**

If no heat is transferred to the surroundings, what is the relationship between the heat lost by the substances initially at high temperature and the heat gained by the substances initially at low temperature? Note: This is simply a statement of conservation of energy.



### Experimental Equipment

A calorimeter, shown in the above figure, is used in this experiment to minimize the exchange of heat between the system and the surroundings. The inner calorimeter cup is thermally insulated from the surroundings by suspending it on a ring of material with low heat conductivity and surrounding it with a layer of air. Also the cup is shiny to minimize radiation loss. Hence, if the mixture of substances is placed inside the calorimeter cup, the heat lost to or gained from the surroundings can be ignored, and the above relationship can be used. The only part of the calorimeter which is involved in the calculation is the inner calorimeter cup which contains water and in which an exchange of heat between the hot and cold bodies takes place. The cup will undergo the same temperature change as the contained water. Of course, an instrument will have to be introduced to measure the temperature of the system, but the heat gained or lost by the instrument is small and can be ignored.

#### Activity 2: Specific Heat of Aluminum

- Fill the hypsometer (boiler) at least half full of water and start heating the water.
- Determine and record the mass of the hypsometer cup,  $m_h$ . Then fill it about half full with dry aluminum pellets. Determine and record the mass of the cup and pellets,  $m_{hp}$ , and calculate the mass of the pellets,  $m_p$ . Record the measurements in the space below.
- Fill the plastic beaker with ice water. Open the *Calorimetry ss* application in the 132 Workshop folder in the **Start** menu and start collecting data. To make sure the temperature probe is working properly place it in the ice water and check that it is reading approximately  $0^\circ\text{C}$ . If not, then consult your instructor.
- Place the hypsometer cup in the top of the hypsometer and put the temperature probe into the middle of the pellets. To do this, remove the pellets from the cup, place the temperature probe in the proper position (using the clamp and stand), then return the pellets to the cup.
- Determine and record the mass of the calorimeter cup,  $m_c$ . Fill this cup about half full of cold tap water. Determine and record the mass of the cup and water,  $m_{cw}$ , and calculate the mass of the water,  $m_w$ . Then place the calorimeter cup in the outer can and put the lid on.

(f) When the temperature of the pellets becomes constant, at or near  $100^{\circ}\text{C}$ , record the temperature of the pellets as  $T_p$ . Remove the probe from the pellets and put it in the cold water in the calorimeter cup. When the temperature of the water levels off, record it as  $T_w$ .

(g) Now, quickly but carefully, pour the pellets into the water in the calorimeter cup. Stir the water occasionally with the glass stirring rod and monitor the temperature of the mixture. When the temperature levels off, record this value as  $T$ . Click the **Stop** button on the monitor, print your graph of temperature as a function of time and include it in this unit.

(h) Write the complete heat equation and solve for the unknown specific heat of the metal pellets. The specific heat of the calorimeter cup is  $900 \text{ J/kg}^{\circ}\text{C}$ .

(i) Look up the accepted value for the specific heat of aluminum and calculate the percent difference between this value and the one you determined above. Do the two values agree within experimental uncertainties? Comment on possible sources of error.

### Activity 3: Specific Heat of Another Metal

(a) Repeat steps 2(a)-2(i) with pellets of a different metal besides aluminum. Record the type of metal, the mass of the pellets, the temperature of the pellets just before you pour them in the cold water, and the temperature of the combined pellets, water, and cup.

(b) Use the equation you derived above for the unknown specific heat of the new metal. The specific heat of the calorimeter cup is 900 J/kg-°C.

(c) Look up the accepted value for the specific heat of your new metal and calculate the percent difference between this value and the one you determined above.

#### **Activity 4: Average and Standard Deviation**

Consult the other lab groups in class and record their values of the specific heat of aluminum and the second metal below. Calculate the average and standard deviation for each metal.

#### **Activity 5: Heat of Fusion of Ice**

(a) The heat of fusion of ice is found experimentally as follows: A known mass of warm water is placed in the calorimeter cup and its temperature recorded. A known mass of ice at 0°C (with no water) is added to the water and allowed to melt. The final temperature of the mixture after the ice has melted is recorded. Perform the experiment and record the data in the space below.

(b) Write the complete heat equation and solve for the unknown heat of fusion of ice.

### 3 Heat of Vaporization of Nitrogen

Name \_\_\_\_\_

Section \_\_\_\_\_

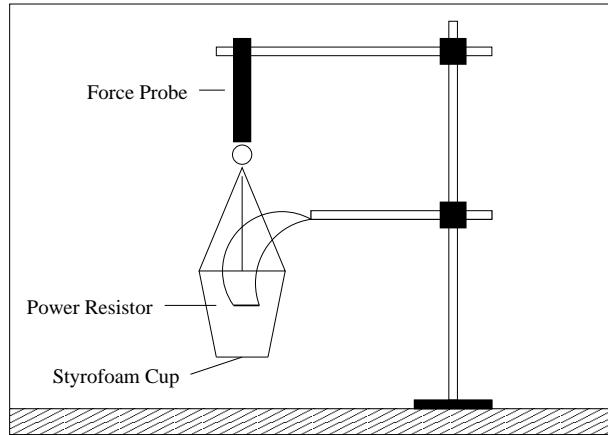
Date \_\_\_\_\_

**Objective**

To measure the heat of vaporization of nitrogen.

**Apparatus**

- Force probe
- Lab stand
- Styrofoam cup
- Liquid nitrogen
- Power resistor
- Power supply
- Wires
- *DataStudio* Software (NitrogenVap)

**Introduction**

The experimental setup is shown in the figure above. The styrofoam cup is filled with liquid nitrogen. The force transducer is used as an electronic balance to measure the mass of the liquid nitrogen as a function of time. The power resistor submerged in the liquid nitrogen converts electrical energy into heat. The power, or rate at which electrical energy is converted to heat, is determined from the voltage and current. The power supply will display both of these values. (Note: Power supply is not shown in the figure.)

**Activity 1: Measuring the Heat of Vaporization of Nitrogen**

(a) Perform the following steps:

1. Remove the load from the force probe and press the TARE button.
2. Adjust the power resistor so that it is as far down in the cup as possible without pushing down on the cup.
3. Fill the cup with liquid nitrogen.

4. Turn on the power supply.. Adjust the voltage and current controls on the power supply so that the current is 2.0 A. Record the values of the current, I, and the voltage, V, in the space below. Then turn off the power supply and refill the cup with liquid nitrogen if necessary.
  
  
  
  
  
  
  5. Open the *NitrogenVap* application in the 132 Workshop folder in the **Start** menu.
  6. The computer will measure the output of the force transducer every second. Start collecting data by clicking on the Start button and recording the force versus time for at least four minutes. When you are finished make a linear fit to the data using the Fit menu. The slope is related to the rate of evaporation of the liquid nitrogen. Calculate the rate of mass loss from this measured slope and enter the result in the space below. Print the graph of force versus time and attach a copy to the unit.
  
  
  
  
  
  
  7. Turn on the power supply and repeat step 6.
  8. Turn off the power supply and repeat step 6.
- (b) Subtract the average of the absolute values of the two slopes when power was off from the absolute value of the slope when power was on. This is the net rate of evaporation caused by the energy supplied to the heater.
- (c) Calculate the heat of vaporization,  $L_v$ , which is given by
- $$L_v = \frac{\text{power to heater}}{\text{net rate of evaporation}} = \frac{VI}{\text{net rate of evaporation}}$$
- (d) Look up the accepted value for the heat of vaporization of nitrogen and calculate the percent difference. Do the two values agree within experimental uncertainties? Comment on possible sources of error.
- (e) Was the temperature of the liquid nitrogen changing during this experiment? Explain.

## 4 Heat of Vaporization of Nitrogen

Name \_\_\_\_\_

Section \_\_\_\_\_

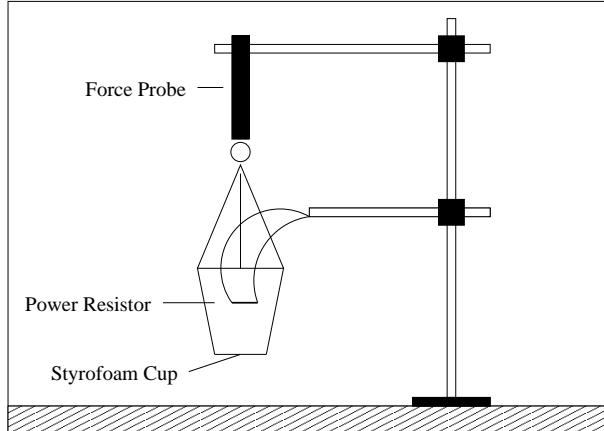
Date \_\_\_\_\_

### Objective

To measure the heat of vaporization of nitrogen.

### Apparatus

- Force probe (with calibration weights)
- Lab stand
- Styrofoam cup
- Liquid nitrogen
- Power resistor
- Power supply
- Digital multimeter
- Wires
- *DataStudio* Software (NitrogenVap)



Your task is to measure the latent heat of vaporization for nitrogen with the equipment given. Your basic strategy will be to use a resistor “heater” to put a known amount of thermal energy into a container of liquid nitrogen, and to simultaneously use a force probe to measure the amount of nitrogen changed from a liquid into a gas.

Some points to keep in mind:

1. You can measure three quantities that can help you find the power dissipated in the resistor: I, V, and R. Be aware that the “R” may change drastically with temperature.
2. Some liquid nitrogen will boil away even with your heater turned off, just due to lousy thermal insulation. You’ll have to account for this somehow.
3. Check your force measurements with known weights to make sure your force probe is properly calibrated and not giving completely bogus results.
4. You will probably have to assume that the nitrogen gas given off is at exactly the same temperature as the liquid, even though it could be slightly hotter. Can you think of some good ways to either minimize this problem or account for it somehow?
5. Yes, you will need to estimate your uncertainty for this experiment. Does the accepted value for the latent heat of nitrogen fall within the range of your uncertainty?

Good Luck! :-)

## 5 Boyle's Law

Name \_\_\_\_\_

Section \_\_\_\_\_

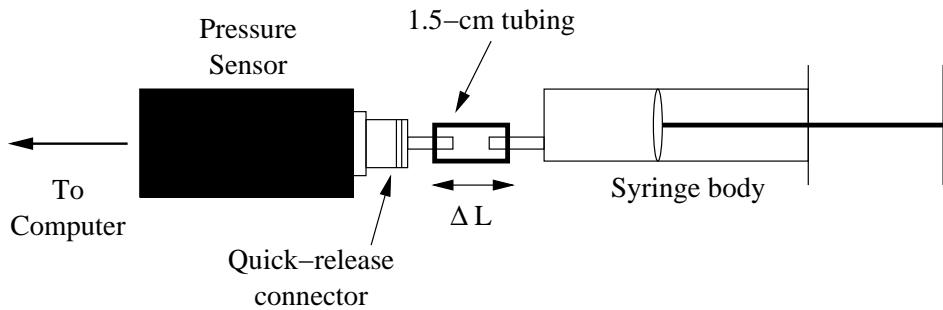
Date \_\_\_\_\_

### Objective

To investigate the relationship between the pressure and volume of a gas.

### Apparatus

- DataStudio 750 Interface
- Pasco Pressure Sensor
- Syringe
- Tubing



Apparatus for Boyle's Law.

### Introduction

The behavior of a gas can be described in terms of the macroscopic quantities: temperature (T), pressure (P), and volume (V). The relationship between these quantities is given by the equation of state of the gas. A real gas behaves approximately as an ideal gas if it is far from liquefaction. In that case, the equation of state of an ideal gas can be used to describe a real gas. For a given mass of a gas, if one of the quantities P, T, or V is changed, a change in the other two quantities probably will result. However, if one of the quantities is kept constant, the relationship between the other two can be studied. The relationship between pressure and volume of an ideal gas is called Boyle's law.

The experimental apparatus is shown in the figure above. The gas is air contained in a syringe that has markings on its side to measure the volume of the syringe. A short tube connects the syringe with a pressure sensor that measures the pressure in the tube and converts that measurement into a signal that can be read by the DataStudio interface.

### Activity 1: Relationship Between P and V of a Gas

(a) Check that there are no leaks in the apparatus by trying to compressing the syringe from the 20.0 ml position to the 10.0 ml position. It should become increasingly difficult to push the plunger as the volume decreases. If this is not the case, check the couplings for fit. If no problem is obvious, then consult your instructor.

(b) The initial volume of air in the syringe should be set at 20.0 ml. If your syringe is set to some other value, disconnect the quick release connector from the sensor by gently rotating it in the counter-clockwise direction as

you look from the syringe toward the pressure sensor. Next, move the piston to the 20.0 ml position, and then re-connect the quick release connector to the pressure sensor.

(c) **Data Recording.** Open the Boyle's Law activity located in the 132 Workshop Folder under the **Start** menu. Click on the window labeled *Volume and Pressure Table*. This is where your data will be displayed as you record it. This table display will show the values of the gas volume in the syringe which you will set by moving the piston to the appropriate marking on the syringe. You will record the pressure at each of these settings with the pressure sensor. To begin recording data, make sure the piston is at the 20-ml setting, and click the Start button. The Start button will change to a Keep button and the table display will show the value of the pressure next to the first volume value (20 ml) in the table. The reading in the pressure column should be colored red. Click the Keep button to record this pressure (notice the reading in the Pressure column beside the 20-ml entry changes from red to black). The next setting for the volume (18 ml) will appear in the Volume column of the data table display.

NOTE: For the first pressure reading at 20 ml, the air in the syringe will be in thermal equilibrium with the environment. This will not be the case immediately after compressing the syringe for the next reading. Therefore, you must allow a couple of seconds for the system to return to thermal equilibrium after you compress the syringe and before clicking on Keep to record pressure values.

(d) Compress the syringe to the next value of the volume as listed in the data display table (i.e., the window labeled *Volume and Pressure Table*) and wait a couple of seconds for the system to reach thermal equilibrium. Once thermal equilibrium is reached, click Keep to record the pressure. The data table display will automatically change to show the next value of the volume at which the pressure will be measured.

(e) Repeat step (d) for the remaining values of the volume listed in the table display. In other words, continue taking pressure measurements at the prescribed volume values in the data table display by moving the piston to the prescribed value and clicking on Keep after thermal equilibrium is reached. After you record the pressure for the last volume (8 ml), click the small, red box next to the Keep button (this is the stop button) to end data recording.

(f) **Analysis.** What happened to the pressure when the volume was reduced from 20 ml to 8 ml? From looking at the data, do the pressure and volume seem to be directly or inversely proportional? Explain.

(g) Copy your data into a spreadsheet and plot pressure versus volume (including a title and axis labels with units). Next, fit your data with a trendline using a power function. Record the result here. What is the power of V? Print the graph and include it with this unit.

(h) If pressure and volume are inversely proportional, then what can you say about the product of pressure and volume? Explain.

(i) The measurements you have made are V in milliliters (ml) and P in kilopascals (kPa). What are the corresponding units of PV? Show how you get this here (the result must be in energy units):

(j) Construct a table in the space below with the column headings: V (ml), P (kPa), and PV. Include the units from above in the last column. Enter the results for P and V in this new table and calculate PV for each set of readings. Determine the mean value and the standard deviation  $\sigma$  for PV. Record the results in the form  $PV = \text{Mean} \pm \sigma$ . Be sure to include the proper units. What does this result tell you about the product PV?

(k) Is there a trend in the values of PV as the volume decreases? If so, what do you suppose causes that trend?

## 6 Charles' Law

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_ +

### Objectives

To investigate the relationship between volume and temperature for a constant mass of gas at constant pressure and determine the value of absolute zero.

### Apparatus

- Charles law apparatus with stand
- Temperature sensor
- Air chamber and tubing
- Hot plate
- Glass beaker
- Clamp and stand

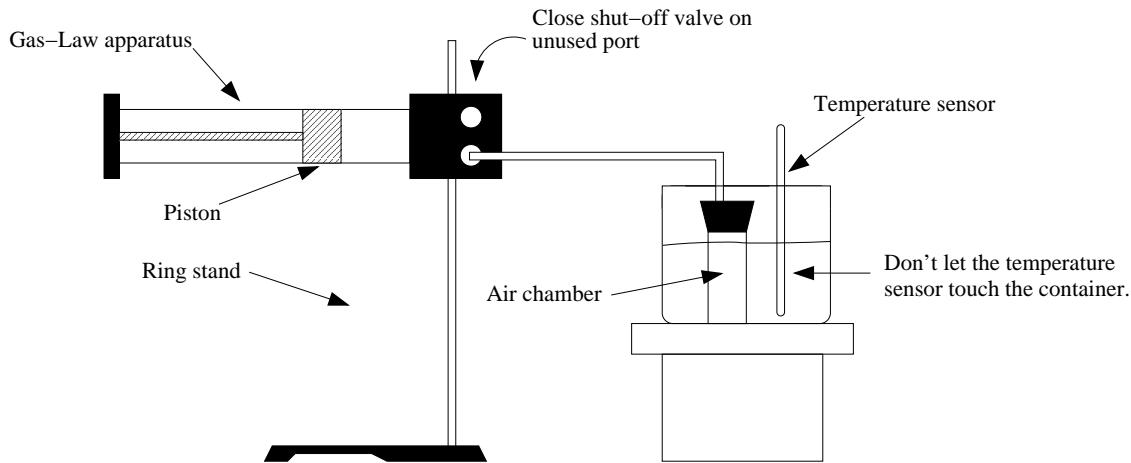


Figure 1: Charles' Law apparatus.

### Introduction

The behavior of a gas can be described in terms of the macroscopic quantities: temperature (T), pressure (P), and volume (V). The relationship between these quantities is given by the equation of state of the gas. A real gas behaves approximately as an ideal gas if it is far from liquefaction. In that case, the equation of state of an ideal gas can be used to describe a real gas. For a given mass of a gas, if one of the quantities P, T, or V is changed, a change in the other two quantities probably will result. However, if one of the quantities is kept constant, the relationship between the other two can be studied. The relationship between temperature and volume of an ideal gas is called Charles' law.

The experimental apparatus is shown in the figure above and consists of an air chamber containing dry air. The pressure on the air in the chamber is due to atmospheric pressure applied through the movable piston.

### Activity 1: V-T Relationship for a Gas

- (a) Check that there are no leaks in the apparatus by trying to compressing the piston from the 100 mm position to the 10 mm position. It should become increasingly difficult to push the plunger as the volume decreases. If this is not the case, check the couplings for fit. If no problem is obvious, then consult your instructor.

(b) Open the *Charles' Law ss* activity in the 132 Workshop Folder under the **Start** menu. Click on the window labeled *Charles' Law Table*. This is where your data will be displayed as you record it. This table display will show the values of the gas temperature in the air chamber and the entry number. The data-taking procedure you will follow is described here first. One member of your team will heat the air chamber in the flask on the hot plate and call out the position of the piston. Another member will record the position settings by hand in the table below and click the **Keep** button on the *DataStudio* interface to record the temperature for that entry. To begin recording data, make sure the piston is at the low end of the scale, and click the **Start** button on the *DataStudio* interface. The **Start** button will change to a **Keep** button and the table display will show the value of the temperature next to the first entry in the table. The reading in the temperature column should be colored red. Click the **Keep** button to record this temperature (notice the reading in the Temperature column beside the entry number changes from red to black). The next entry number will appear in the Entry column of the data table display.

- (h) Repeat steps c-h to obtain a second  $V-T$  curve. Record your data in the table below along with the fit to the  $V-T$  data.

## Activity 2: Absolute Zero and the Kelvin Scale

(a) The absolute zero of temperature can be defined as the temperature at which the volume of an ideal gas is zero. Determine absolute zero from the equation of your graph by setting  $V = 0$  and solving for  $T$ .

(b) Determine the percent difference between your value of absolute zero and the accepted value of  $-273^{\circ}\text{C}$ . Are you happy or sad?

(c) Record the results from the other groups in class. Obtain an average and standard deviation and record it here. Are your results consistent with the class average? Explain.

## 7 The P-T Relationship of a Gas

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_ +

### Objectives

To investigate the relationship between pressure and temperature for a constant mass of gas at constant volume and determine the value of absolute zero.

### Apparatus

- DataStudio 750 Interface
- Pressure sensor
- Temperature sensor
- Air chamber and tubing
- Hot plate
- Glass beaker
- Clamp and stand
- Plastic beaker

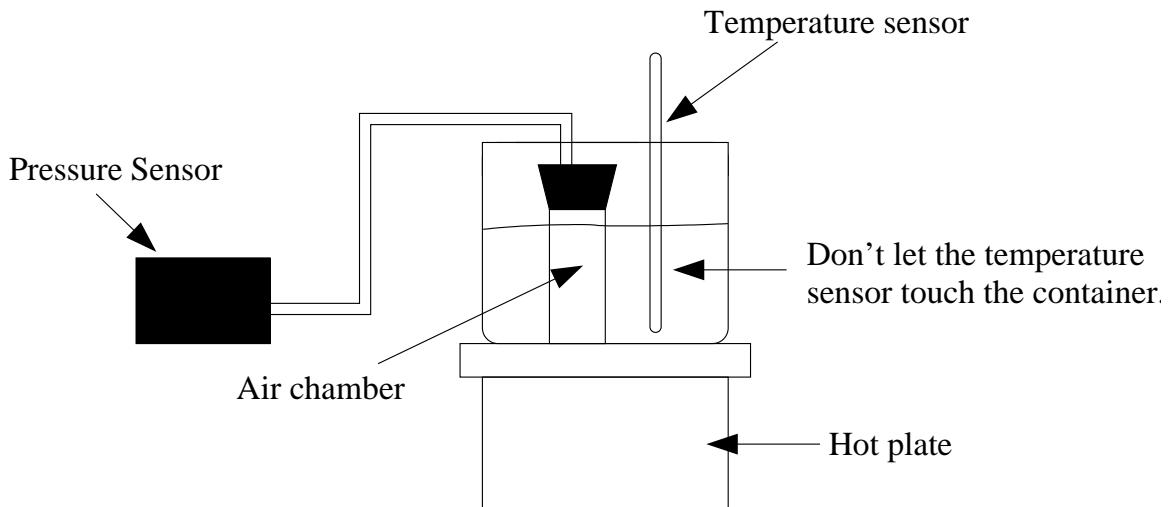


Figure 2: P-T apparatus.

### Introduction

The behavior of a gas can be described in terms of the macroscopic quantities: temperature (T), pressure (P), and volume (V). The relationship between these quantities is given by the equation of state of the gas. A real gas behaves approximately as an ideal gas if it is far from liquefaction. In that case, the equation of state of an ideal gas can be used to describe a real gas. For a given mass of a gas, if one of the quantities P, T, or V is changed, a change in the other two quantities probably will result. However, if one of the quantities is kept constant, the relationship between the other two can be studied. The relationship between temperature and pressure of an ideal gas is known as Gay-Lussac's law.

The experimental apparatus is shown in the figure above and consists of an air chamber containing dry air. The volume of the gas is fixed. The temperature and pressure sensors are connected to the 750 Interface. Be sure the temperature sensor is connected to port A and the pressure sensor to port B. Also be sure the cables from the sensors are not touching the hot plate.

#### Activity 1: P-T Relationship for a Gas

- (a) The equipment should be set when you arrive in lab. The air chamber is set inside the glass beaker on top of the hot plate. The retemperature sensor is set so that it is not touching the side or bottom of the beaker. Use the plastic beaker to fill the glass beaker about 3/4 full of cold tap water.
- (b) Open the *P-T ss* activity in the 132 Workshop Folder under the **Start** menu. Click on the window labeled *Temperature and Pressure Table*. This is where your data will be displayed as you record it. This table display will show the values of the gas pressure in the air chamber and the temperature of the heat bath. To begin recording data click the **Start** button on the *DataStudio* interface. The **Start** button will change to a **Keep** button and the table display will show the values of the temperature and pressure. Click the **Keep** button to record this temperature and pressure.
- (c) Turn the hot plate on high. As the temperature rises, click the **Keep** button when the temperature is 5-7° above its first value. Continue recording the temperature and pressure at 5-7° intervals (by clicking the **Keep** button) until the water is close to boiling. You can monitor the temperature on the temperature versus time plot to the right (on the monitor) or by watching the temperature in the *Temperature and Pressure Table*. After your last reading, click the small red box next to the **Keep** button (this is the stop button) to end data recording. Turn off the hot plate.
- (d) How are the pressure and temperature related? Print your data table, enter the data in *Excel* and plot pressure vs temperature on a linear graph, showing the equation of the graph. Print this graph and add it to this unit. Write the equation here as pressure (P) as a function of temperature (T), including UNITS on the constants.

#### Activity 2: Absolute Zero and the Kelvin Scale

- (a) The absolute zero of temperature can be defined as the temperature at which the pressure of an ideal gas is zero. Determine absolute zero from the equation of your graph by setting  $P = 0$  and solving for  $T$ .
- (b) Record the results from the other groups in the class. Obtain an average and standard deviation and record it here. Are your results consistent with the class average? Explain.
- (c) Does there appear to be a systematic error in the class results relative to the accepted value of -273°C? If so, what do you suppose causes the error? Think about this and try to explain.

## 8 Impulse, Momentum, and Interactions<sup>1</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

- To verify the relationship between impulse and momentum experimentally.
- To study the forces between objects that undergo collisions and other types of interactions in a short time period.

### Apparatus

Dynamics cart with flag and track	Force probe	Motion sensor
<i>Science Workshop 750 Interface</i>	compact scale	<i>DataStudio</i>

### The Impulse-Momentum Theorem

When an object collides with something the force on it 'pushes' for a while and eventually stops. If the force  $\vec{F}$  is constant, we can get some idea of the total 'push' by multiplying by the time  $\Delta t$  the force acted. This product is called the impulse  $\vec{I} = \vec{F}\Delta t$  and it measures the total 'kick' received by an object. Real collisions, like those between eggs and hands, a Nerfball and a wall, or a falling ball and a table top are tricky to study because  $\Delta t$  is so small and the collision forces are not really constant over the time the colliding objects are in contact. Thus, we cannot calculate the impulse as  $\vec{F}\Delta t$ . Before we study more realistic collision processes, let's redo the theory for a variable force. In a collision, according to Newton's second law, the force exerted on a falling ball by the table top at any infinitesimally small instant in time is given by

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \quad [Eq. 1]$$

To describe a general collision that takes place between an initial time  $t_i$  and a final time  $t_f$ , we must take the integral of both sides of the equation with respect to time. This gives

$$\int_{t_i}^{t_f} \mathbf{F} dt = \int_{t_i}^{t_f} \frac{d\mathbf{p}}{dt} dt = (\mathbf{p}_f - \mathbf{p}_i) = \Delta\mathbf{p} \quad [Eq. 2]$$

Impulse is a vector quantity defined by the equation

$$\text{Area} = \int_{t_i}^{t_f} \mathbf{F} dt \quad [Eq. 3]$$

By combining equations [2] and [3] we can formulate the impulse-momentum theorem in which

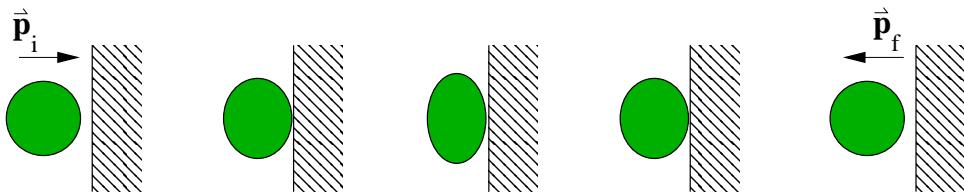
$$\mathbf{I} = \Delta\mathbf{p} \quad [Eq. 4]$$

If you have a fairly smooth graph of how the force  $F$  varies as a function of time, the impulse integral can be calculated as the area under the  $F$ - $t$  curve.

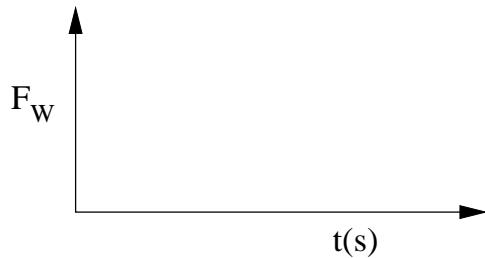
Let's see qualitatively what an impulse curve might look like in a real collision in which the forces change over time during the collision. In particular, let's consider the collision of a Nerfball with a wall as shown below.

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**Activity 1: Predicting Collision Forces That Change**

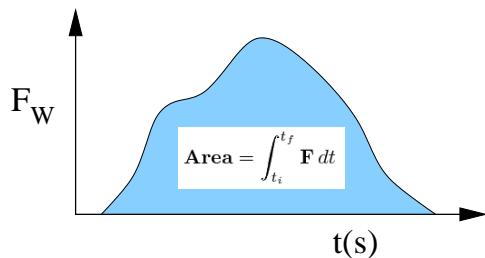
- (a) Suppose a tennis ball is barreling toward a wall and collides with it. If friction is neglected, what is the net force exerted on the object just before it starts to collide?
- (b) When will the magnitude of the force on the ball be a maximum?
- (c) Roughly how long does the collision process take? Half a second? Less? Several seconds?
- (d) Attempt a rough sketch of the shape of the force the wall exerts on a moving object during a collision.

**Verification of the Impulse-Momentum Theorem**

To verify the impulse-momentum theorem experimentally we must show that for an actual collision involving a single force on an object the equation

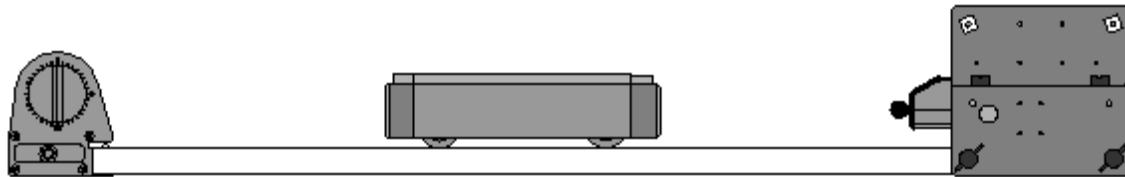
$$\int_{t_i}^{t_f} \mathbf{F} dt = \Delta \mathbf{p}$$

holds, where the impulse integral can be calculated by finding the area under the  $F$  vs.  $t$  curve.



In this experiment you will investigate this theorem by measuring the impulse and the change in momentum of a cart undergoing a one-dimensional collision. The experimental setup is shown in the figure below. The end of the track with the motion detector should be raised about 1.5 cm so that, when released, the cart will collide

with the force probe. The force probe will measure the force as a function of time during the collision. The motion detector is used to measure the velocity of the glider before and after the collision. You will use the Impulse-Momentum application to make these measurements.



### Activity 2: Verification of the Impulse-Momentum Theorem

- Measure and record the mass of the cart,  $m$ , using the compact scale.
- Calibrate the force probe (see *Calibrating Force Sensors* in **Appendix F: Instrumentation**).
- Construct a data table in the space below with the column headings Trial #, Area,  $v_i$ , and  $v_f$ . Make enough room to record five trials.

Trial #	Area	$v_i$	$v_f$

- Position the force probe part way up the track (to be closer to the motion sensor). Open the **Impulse-Momentum** application in the **131 Workshop** submenu.
- Set the cart on the track about mid-way between the motion sensor and the force probe. Start recording data and release the cart. Stop recording data after the cart collides with the force probe and bounces back. The computer will then display graphs of velocity and force versus time.
- Determine the area under the force vs. time graph and record the value in your data table. See **Appendix B: Introduction to DataStudio** for instructions on how to determine the area under a curve.
- Use the smart tool to find the velocity just before the collision and the velocity just after the collision from the velocity versus time graph. Record these values in your data table.
- Repeat parts (e) through (g). For these trials, the area function and the smart tool must be turned on and off for each trial. Print the graphs for one of your trials and include it with this report.

- (i) Construct another data table below with the column headings Trial #,  $I$ ,  $\Delta p$ , Diff., and Percent diff. For each trial, calculate and record the impulse,  $I$ , and the change in momentum,  $\Delta p$ , in kg m/s. Also, determine the difference between  $I$  and  $\Delta p$  for each trial, and the percent difference. Also, show a sample calculation of  $I$  and  $\Delta p$  for one of your trials.



(j) What do you expect for the values in the last column of your table (Percent diff)? Make a histogram of your results in that column and calculate the average and standard deviation. For information on making histograms, see **Appendix C**. For information on calculating the average and standard deviation, see **Appendix A**. Record the average and standard deviation here. Attach the histogram to this unit. Is your data consistent with your expectation? Be quantitative in your answer.

(k) Do your results verify the impulse-momentum theorem? Explain quantitatively.

(l) What does the histogram of your data tell you? Be quantitative in your answer.

(m) Is there any indication of a systematic uncertainty? What are the possible sources of error?

## 9 Kinetic Theory of Ideal Gases<sup>2</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

To derive a relationship between the macroscopic properties of an ideal gas and the microscopic motion of the unseen atoms that make up the gas. To do this activity you will need:

- A computer with an atomic and molecular motion simulation

### Introduction

Do you believe in atoms? Our forefathers believed in the reality of witches. In fact, they thought that they had good evidence that witches existed, good enough evidence to accuse some people of being witches. We believe in atoms. Are we truly more scientific than they were?

#### Activity 1: Why Atoms!?

(a) List reasons why you do or do not believe that matter consists of atoms and molecules, even though you have never seen them with your own eyes.

(b) What happens when heat energy is being transferred into a substance? If you believe that substances are made of atoms and molecules, how would you use their existence to explain the change in volume of a heated gas?

### Models of Pressure Exerted by Molecules

So far in physics we have talked about matter as if it were continuous. We didn't need to invent aluminum atoms to understand how a ball rolled down the track. But ever since the time of the fifth century B.C. Greek philosophers Leucippus and Democritus, some thinkers have believed in "atomism", a picture of the universe in which everything is made up of tiny "eternal" and "incorruptible" particles, separated by "the void". Today, we think of these particles as atoms and molecules.

In terms of every day experience molecules and atoms are hypothetical entities. In just the past 40 years or so, scientists have been able to "see" molecules using electron microscopes and field ion microscopes. But long before atoms and molecules could be "seen" experimentally, nineteenth century scientists such as James Clerk Maxwell and Ludwig Boltzmann in Europe and Josiah Willard Gibbs in the United States used these imaginary microscopic entities to construct models that made the description and prediction of the macroscopic behavior of thermodynamic systems possible. Is it possible to describe the behavior of an ideal gas that obeys the first law of thermodynamics as a collection of moving molecules? To answer this question, let's observe the pressure exerted by a hypothetical molecule undergoing elastic collisions with the walls of a 3D box. By using the laws of mechanics we can derive a mathematical expression for the pressure exerted by the molecule as a function of the volume of the box. If we then define temperature as being related to the average kinetic energy of the molecules in an ideal gas, we can show that kinetic theory is compatible with the ideal gas law and the first law of thermodynamics. This compatibility doesn't prove that molecules exist, but allows us to say that the molecular model would enable us to explain the experimentally determined ideal gas law.

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<sup>2</sup>1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

### Atomic Motion and Pressure

Consider a spherical gas molecule that has velocity  $\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$  and makes perfectly elastic collisions with the walls of a three-dimensional, cubical box of length, width, and height  $l$ . Start the program called "Atoms in Motion" (in "Physics Applications"). You will see a screen like the one shown below. Experiment with it for a few moments. The *Run* and *Stop* buttons control the processing of the simulation of the gas atoms while the *Step* button allows you to watch the 'movie' one frame at a time.

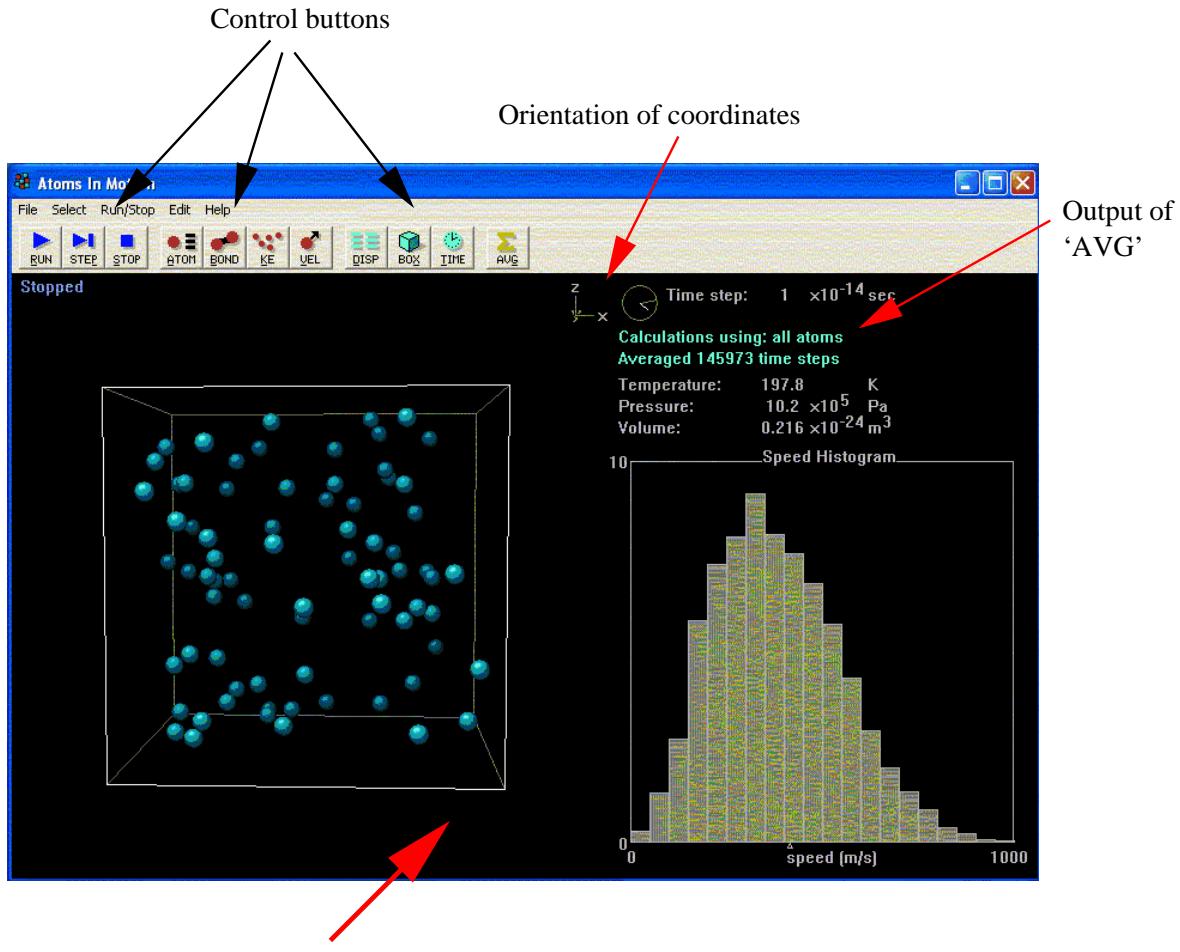


Figure 3: *Atoms in Motion* window.

Can we use the concept of molecules behaving like little billiard balls to explain why the ideal gas law relationship might hold? In the next activity you are to pretend you are looking under a giant microscope at a single spherical molecule as it bounces around in a three-dimensional box by means of elastic collisions and that you can time its motion and measure the distances it moves as a function of time.

If the molecule obeys Newton's Laws, you can calculate how the average pressure that the molecule exerts on the walls of its container is related to the volume of the box. The questions we have to consider are the following. What is the momentum change as the molecule bounces off a wall? How does this relate to the change in the velocity component perpendicular to the wall? How often will our molecule "hit the wall" as a function of its component of velocity perpendicular to the wall and the distance between opposite walls? What happens when the molecule is more energetic and moves even faster? Will the results of your calculations based on mechanics be compatible with the ideal gas law?

### Activity 2: The Theory of Atomic Motion

- (a) Stop the simulation if it's running and set the number of molecules to one. Do this by clicking on the *ATOM* button and getting a dialog box. Enter '1' for the number of Type A atoms and zero for all the others. Record the mass of the Type A atom. Click *OK* and the cube should now contain a single atom. If not, consult your instructor.
- (b) The orientation of coordinates can be seen just above the right-hand corner of the cube (consult Figure 1 also). Suppose the molecule moves a distance  $2l$  (across the cube and back) in the  $x$ -direction in a time  $\Delta t_x$ . What is the equation needed to calculate its  $x$ -component of velocity in terms of  $l$  and  $\Delta t_x$ ?
- (c) Suppose the molecule moves a distance  $2l$  in the  $y$ -direction in a time  $\Delta t_y$ . What is the equation needed to calculate its  $y$ -component of velocity in terms of  $l$  and  $\Delta t_y$ ?
- (d) Suppose the molecule moves a distance  $2l$  in the  $z$ -direction in a time  $\Delta t_z$ . What is the equation needed to calculate its  $z$ -component of velocity in terms of  $l$  and  $\Delta t_z$ ?
- (e) We will now measure the average time  $\Delta t_y$  for one complete round trip from the left side of the cube to the right side and back again. Click *AVG* and you will see some information printed in the color blue on the right-hand side of the *Atoms-in-Motion* window (see also Figure 1). The simulation takes small steps in time and calculates the positions of the atoms at the end of each time step. The number of these time steps taken is shown on the right-hand side and the size of each time step is printed at the top, right-hand-side of the window. Using the *Step* button let the atom in the cube move until it bounces off the left wall of the cube. Stop the motion and record the number of the time step in the space below.
- (f) Now run or step the simulation until the atom bounces across the cube, hits the right-hand wall, comes back and strikes the left hand wall again. Stop the motion and record the number of the time step. Calculate  $\Delta t_y$  and record it below.
- (g) Each side of the cube has a length  $l = 50 \times 10^{-10}$  m. Combine this with the previous result to determine  $v_y$  and record it.

- (h) Repeat the above procedure for the top and bottom walls of the cube to get  $v_z$ .
- (i) Rotate the cube by clicking and dragging one of the corners of the cube. Spin it until you can see the atom bounces between the walls in the direction of the  $x$  coordinate. Measure the  $x$  component of the speed of the atom using the same procedure as before.
- (j) Write the expression for  $v_{total}$  in terms of the  $x$ ,  $y$ , and  $z$  components of velocity. (Hint: This is an application of the 3-dimensional Pythagorean theorem.) Determine  $v_{total}$  for your atom. We will use these results in a little while to calculate the pressure exerted by our one-atom ‘gas’.
- (k) Record the value of the pressure and temperature for your ‘gas’ (as printed on the screen).
- (l) If the collisions with the wall perpendicular to the  $x$  direction are elastic, show that the force exerted on that wall for each collision is just  $F_x = 2m \frac{v_x}{\Delta t_x}$  where  $m$  is the mass of the particles and  $\Delta t_x$  the mean interval between collisions with the wall. (Hint: Think of the form of Newton’s second law in which force is defined in terms of the change in momentum per unit time so that  $F = \frac{\Delta p}{\Delta t}$ .) **Warning:** Physicists too often use the same symbol to stand for more than one quantity. In this case, note that  $\Delta p$  (where “p” is in lower case) indicates the change in *momentum*, not pressure.
- (m) Substitute the expression from part (b) for  $\Delta t_x$  to show that
- $$F_x = \frac{mv_x^2}{l}$$
- (n) We have assumed from the beginning that we have a cubical box of edge length  $l$ . Show that the pressure on the wall perpendicular to the  $x$  axis caused by the force  $F_x$  due to *one* molecule is described by the following expression.

$$P = \frac{mv_x^2}{l^3}$$

(o) Now if we write the volume of our box as  $V = l^3$ , the pressure becomes

$$P = \frac{mv_x^2}{V}$$

(p) Now suppose that there are not one but  $N$  molecules in the box. What is the pressure on the wall now?

(q) You have seen earlier that the three components of velocity for your single molecule were all different. But averaged over *many* molecules, the three components will have equal speeds. In other words,  $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$  where the bars indicate averages. If this is the case, then use the result of part (j) to show that

$$\overline{v_x^2} = \frac{\overline{v_{total}^2}}{3}$$

(r) Now show that

$$P = N \frac{m\overline{v_{total}^2}}{3V}$$

(s) Finally, since the average kinetic energy of a molecule is just

$$\langle E_{kin} \rangle = \frac{1}{2} m \overline{v_{total}^2}$$

show that the pressure in the box can be written in the following way.

$$P = \frac{2N \langle E_{kin} \rangle}{3V}$$

(t) Use the previous result to calculate the pressure using  $v_{total}$ , the mass of the atom,  $N$  and  $V$  for your one-atom gas. Compare your result with the pressure you recorded above from the output of the simulation (part (k) above). Do they agree? How would you account for any difference?

## 10 Applying the Kinetic Theory<sup>3</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

To derive the relationship between temperature and the kinematic properties of the monatomic molecules of an ideal gas. We will also calculate the specific heat per mole of an ideal, monatomic gas at constant volume using the kinetic theory and compare the prediction with data. To do this activity you will need:

- A computer with an atomic and molecular motion simulation

### Kinetic Energy, Internal Energy, and Temperature

We have hypothesized the existence of non-interacting molecules to provide the basis for a particle model of ideal gas behavior. We have shown that the pressure of such a gas can be related to the average kinetic energy of each molecule:

$$P = \frac{2N\langle E_{kin} \rangle}{3V} \text{ or } PV = \frac{2}{3}N\langle E_{kin} \rangle$$

Pressure increases with kinetic energy per molecule and decreases with volume. This result makes intuitive sense. The more energetic the motions of the molecules, the more pressure we would expect them to exert on the walls. Increasing the volume of the box decreases the frequency of collisions with the walls, since the molecules will have to travel longer before reaching them, so increasing volume should decrease pressure if  $\langle E_{kin} \rangle$  stays the same.

### The Molar Specific Heat

The kinetic theory of gases uses the atomic theory to relate the macroscopic properties of gases to the microscopic features of the atoms and molecules that make up the gas. In this laboratory we will extend the calculations that we have made so far to include the molar specific heat of an ideal, monatomic gas. The success of that extension of the theory depends on how well the calculations reproduce the measured heat capacities of a variety of real (not ideal) gases.

### Activity 1: Experimenting with the Gas Simulation Program

Open the *Atoms in Motion* program (in *Physics Applications*) on the *Start* menu. We are first going to explore the relationship between pressure and volume in our kinetic theory using the simulation.

(a) According to the ideal gas law  $PV = nRT = Nk_B T$ , where  $R$  is the universal gas constant and  $k_B$  is Boltzmann's constant. What should happen to the pressure of an ideal gas as its volume increases or decreases?

(b) We now want to run a more realistic simulation. Under the *ATOM* menu set the number of Type A atoms to 50 and set all the others to zero. Click on the *BOX* button and a new dialog box will appear. Check the box beside 'Floor conducts heat' and set the temperature to 200 K. Notice at the top that the box width is  $l = 50 \times 10^{-10} \text{ m}$ . We have now set up a situation where one side of the cube is held at a constant temperature (*e.g.* it's sitting on a stove) so the collisions of the atoms with the floor are no longer elastic. The remaining sides of the cube do not transfer any energy (they're insulated) so elastic collisions still occur at those walls.

Start the simulation and make sure you are averaging the pressure over many time steps. You should see the number of averaged time steps increasing on the right-hand side of the *Atoms-in-Motion* window. If you don't see this information, click on *AVG* and it should appear.

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(c) What happens to the pressure? What happens to the temperature of the gas? You will find that it can take several minutes of computer time for the temperature of the gas to reach equilibrium with the floor. Once the gas temperature is within  $8 - 10\text{ K}$  of the floor temperature, we can consider the gas and the floor to be in thermal equilibrium. Record the volume, pressure and temperature of the gas in the first line of the table below.

(d) Record the volume, pressure and temperature of the gas for five more volumes of the cube. Change the volume of the cube using the *BOX* menu and increasing the box width in increments of  $10 \times 10^{-10}\text{ m}$ . The volume is printed on the *Atoms-in-Motion* window. In each case, wait until the temperature is within  $8 - 10\text{ K}$  of the floor temperature (the closer the better). Plot your results including a trend line with a power function and attach the graph to this unit. Are your results consistent with the ideal gas law and your prediction in part (a)? Are they consistent with the results of Experiment 4 (Boyle's law)?

Volume of Box	Average Pressure	Temperature

(e) In the procedure above you should have found the pressure to be inversely proportional to the volume. How could you modify your plot to show the pressure is proportional to  $1/V$ ? Make such a plot and fit it with a linear function. How close is your data to following a straight line? Attach the plot to this unit.

(f) According to the ideal gas law  $PV = nRT = Nk_B T$ . What should happen to the pressure of an ideal gas as the number of particles increases or decreases? We will explore this idea with the simulation next.

(g) Start off with the gas parameters from the last 'run' of the simulation. Record the number of atoms, temperature, and pressure in the table below. Use the *ATOM* menu to change the number of atoms (or molecules) in the cube. Start the simulation. What happens to the pressure? Record the pressure and the number of molecules for four more values of the number of molecules and plot your results. Attach the plot to this unit. Are your results consistent with the ideal gas law and your prediction in part (f)?

Number of Molecules	Average Pressure	Temperature

### Kinetic Theory and the Definition of Temperature

The model of an ideal gas we have just derived requires that

$$PV = \frac{2}{3}N \langle E_{kin} \rangle$$

But we have determined experimentally the ideal gas law:

$$PV = Nk_B T$$

What can we say about the average kinetic energy per molecule for an ideal gas? You can derive a relationship between temperature and the energy of molecules that serves as a microscopic (i.e. molecular) definition of temperature.

### Activity 2: Microscopic Definition of Temperature

(a) From the two equations above, derive an expression relating  $\langle E_{kin} \rangle$  and  $T$ . Show the steps in your derivation.

(b) In general, molecules can store energy by rotating or vibrating, but for an ideal gas of *point* particles (monatomic gas), the only possible form of kinetic energy is the translational motion of the particles. If we can ignore potential energy due to gravity or electrical forces, then the internal energy  $E_{int}$  of a gas of  $N$  particles is  $E_{int} = N \langle E_{kin} \rangle$ . Use this and the formulas above to show that for an ideal gas of point particles,  $E_{int}$  depends only on  $N$  and  $T$ . Derive the equation that relates  $E_{int}$ ,  $N$  and  $T$ . Show the steps.

The microscopic and the macroscopic definitions of temperature are equivalent. The microscopic definition of temperature which you just derived is fundamental to the understanding of all thermodynamics!

### Activity 3: Calculating the Molar Specific Heat

In this section we will generate a series of equations that we will then bring together in order to predict the molar specific heat at constant volume.

(a) Consider an ideal gas in a rigid container that has a fixed volume. How is the molar specific heat defined in terms of the heat added  $Q$ ?

(b) If the gas is heated by an amount  $Q$ , then how much work is done against the fixed container? Recall the first law of thermodynamics and incorporate this result into your statement of the first law.

(c) Now use the equations of parts (a) and (b) to relate the change in internal energy  $\Delta E_{int}$  to the molar specific heat.

(d) Write down an expression for the change in internal energy of the ideal gas in terms of  $\langle E_{kin} \rangle$ . (Suggestion: see part (b) of Activity 2.) How is  $\langle E_{kin} \rangle$  related to the temperature? Incorporate this relationship into your expression for the change in the internal energy. You should find that

$$\Delta E_{int} = \frac{3}{2} N k_B \Delta T$$

where  $k_B$  is Boltzmann's constant and  $N$  is the number of molecules in the gas.

(e) Use the equations from parts (c) and (d) to relate the molar specific heat to the number of particles  $N$  and Boltzmann's constant  $k_B$ . You should find that

$$n C_V = \frac{3}{2} N k_B$$

where  $n$  is the number of moles.

(f) How is the number of molecules in the gas  $N$  related to the number of moles  $n$  and Avogadro's number  $N_A$ ? Use this expression and the result of part (e) to show

$$C_V = \frac{3}{2}N_A k_B \text{ or } \frac{C_V}{N_A k_B} = \frac{3}{2}$$

Since  $N_A k_B = R$ , this can be written as

$$C_V = \frac{3}{2}R \text{ or } \frac{C_V}{R} = \frac{3}{2}$$

#### Activity 4: Comparing Calculations and Data

We now want to compare our calculation of the molar specific heat of an ideal, monatomic gas with the measured molar specific heats of some real gases. The table below lists some of those measurements.

Molecule	$\frac{C_V}{R}$	Molecule	$\frac{C_V}{R}$
He	1.50	CO	2.52
Ar	1.50	$\text{Cl}_2$	3.08
Ne	1.51	$\text{H}_2\text{O}$	3.25
Kr	1.49	$\text{SO}_2$	3.77
$\text{H}_2$	2.48	$\text{CO}_2$	3.42
$\text{N}_2$	2.51	$\text{CH}_4$	3.25
$\text{O}_2$	2.53		

(a) Has our theoretical calculation been successful at all? Which gases appear to be consistent with our calculation? Which gases are not? How do these two groups of real gases differ?

(b) Can you suggest an explanation for the partial success of the theory? Which one of the original assumptions that went into our kinetic theory might be wrong? (See Activity 2(b).)

## 11 Einstein Solid

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

To develop a quantum-mechanical model of an elemental solid (*e.g.* aluminum) and introduce the ideas of statistical mechanics.

### Overview of the Model

One of the earliest successful applications of quantum mechanics was done by Albert Einstein in 1907 when he developed a model of an elemental solid (*i.e.* one that consists of a single element from the periodic table like aluminum, lead, *etc.*). We start by assuming that each atom in the solid is bound in a square lattice with each of six neighbors. Each bond is treated as a simple spring so the mechanical energy for a single atom is

$$E = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2) + \frac{1}{2}k(x^2 + y^2 + z^2) \quad (1)$$

where  $k$  is the spring constant of the bond, the coordinates  $x$ ,  $y$ , and  $z$  are relative to the equilibrium position of the atom and  $v_x$ ,  $v_y$ , and  $v_z$  are the components of the velocity. Einstein used an idea pioneered by Max Planck in 1901 and guessed the energy in the solid came in discrete pieces or quanta that were all the same size. Adding or removing these quanta heated or cooled the solid. Many years later the quantum mechanical energy  $E$  for a mass on a spring was found to be

$$E = (n_x + n_y + n_z + \frac{3}{2})\hbar\omega \quad (2)$$

where  $\hbar$  is Planck's constant,  $\omega$  is related to the spring constant  $k$  of the bond mentioned above, and  $n_x$ ,  $n_y$ , and  $n_z$  represent the number of quanta associated with each degree of freedom of the spring. The degrees of freedom here correspond to the three possible directions each atom can vibrate. The size of each energy quantum is  $\epsilon = \hbar\omega$ . The total number of energy quanta in the solid is labeled  $q_A$  so the internal energy is  $E_{int} = q_A\epsilon$ . We then assume that all microstates of the solid have an equal probability of being populated. A microstate is a specific arrangement of the quanta on the atoms in the solid.

### Activity 1: The Statistics of Matter

Before you embark on building the model of the Einstein solid consider some ideas from your previous study of gases. You will make some predictions here about the statistical nature of matter that you can refer back to later on in this unit.

(a) Consider a gas in a container. Would it violate Newton's Laws or any other physical law if all the particles in the gas collided in such a way that all of the gas particles ended up in the bottom half of the container leaving the top half empty?

(b) Is such a scenario likely? Explain.

(c) If you started out with all the gas in the bottom half of the container how likely is it to stay there?

The questions you answered above are addressing the notion of irreversibility. Many processes in nature appear to proceed in one 'direction' only. When you add milk to coffee it disperses throughout the coffee. After it is dispersed, the milk never re-concentrates into a blob of milk in the middle of the coffee. These processes go from

a more orderly configuration (a concentrated drop of milk) to a disordered state (milk spread throughout the liquid). The reverse never happens. We will return to this notion again in this laboratory.

**Activity 2: Calculating the Multiplicity of Some ‘Solids’**

(a) You will first calculate the configurations of the quanta (the microstates) for a VERY simple solid consisting of a single atom! The number of atoms for solid  $A$  is  $N_A = 1$  so there are three degrees of freedom  $N_a = 3$  because there is one degree of freedom for each spatial direction. The atom’s vibration can be decomposed into three components, one for each direction. Let the ‘solid’ contain two quanta of energy so  $q_A = 2$ . Make a table with the headings  $n_1$ ,  $n_2$ , and  $n_3$  and in each row enter one arrangement of the two quanta. Each row in the table is a microstate. Make a table with all of the possible microstates. The multiplicity  $\Omega_A$  of the system is the number of all possible microstates. What is your multiplicity? Record it here.

(b) You can calculate the multiplicity  $\Omega_A$  using the expression

$$\Omega(N_A, q_A) = \frac{(q_A + 3N_A - 1)!}{q_A!(3N_A - 1)!} \quad (3)$$

Make the calculation for  $N_A = 1$  and  $q_A = 2$ . Does this agree with your result in part 2.a?

(c) Now do the same thing for a different ‘solid’. This time for solid  $B$ , let  $N_B = 2$  (two whole atoms!) and  $q_B = 1$ . How many degrees of freedom does solid  $B$  have? Make a table analogous to the one in part 2.a on the same sheet as before. What is the multiplicity of solid  $B$ ? Record it here. Use the expression in Activity 2.b to check your calculation.

**Activity 3: Putting the ‘Solids’ Together**

When two solids are brought together heat/energy can flow between the two objects. For the model of the Einstein solid you are building this corresponds to energy quanta ( $\hbar\omega$ ) moving from atom to atom and occupying different microstates of the combined system.

(a) Now bring the your solids  $A$  and  $B$  ‘together’ into a single system. What is  $N_{AB}$  the total number of atoms? What is the number of degrees of freedom of the combined system?

- (b) What is the total number of energy quanta  $q_{AB}$  for the combined system?
- (c) The system is in its initial macrostate. A macrostate is a configuration of the system defined here by the total number of atoms and quanta in each solid. In this case the macrostate is defined by  $N_A = 1$ ,  $q_A = 2$ ,  $N_B = 2$ , and  $q_B = 1$ . What is the total multiplicity  $\Omega_{AB}$  for the combined system with  $q_A = 2$  and  $q_B = 1$  in its initial macrostate?
- (d) Now take the energy quantum in solid  $B$  and put it in solid  $A$ , *i.e.*, let heat flow from solid  $B$  into solid  $A$ . This is now a macrostate where  $q_A = 3$  and  $q_B = 0$ . What is the new multiplicity  $\Omega_A$  for solid  $A$  and the multiplicity  $\Omega_B$  for solid  $B$ ?
- (e) What is the multiplicity  $\Omega_{AB}$  for the combined system (solids  $A$  and  $B$ )?
- (f) Remember that a macrostate is defined by the combination of  $N_A$ ,  $N_B$ ,  $q_A$ , and  $q_B$ . Which macrostate had the greatest multiplicity,  $(q_A = 2, q_B = 1)$  or  $(q_A = 3, q_B = 0)$  (remember that  $N_A$  and  $N_B$  are the same in each configuration so we don't list those parameters here)?
- (g) If the energy quanta can move from atom to atom which macrostate ( $q_A = 2, q_B = 1$ ) or ( $q_A = 3, q_B = 0$ ) is most probable? Why?
- (h) If you started out in the  $(q_A = 3, q_B = 0)$  macrostate is it more likely that you will remain in that macrostate or evolve to the  $(q_A = 2, q_B = 1)$  macrostate? Why?

What you have discovered is a version of the irreversibility mentioned earlier, One macrostate ( $q_A = 2, q_B = 1$ ) is preferred over the other because it has more microstates than the other. This result depends critically on your assumption that all states are equally populated.

### Activity 4: Using *StatMech* For More Complex Cases

You should have found in the previous activity that the ( $q_A = 2$ ,  $q_B = 1$ ) macrostate was more likely to occur and the process proposed in part 3.d is relatively unlikely. In other words, it is more likely for energy to be spread evenly throughout the system. This is good news because it means the statistical picture we are painting is consistent with reality. Remember what happens to the blob of milk in the coffee.

(a) You should realize that making the sorts of calculations you did in Activity 3 above would become rather painful for say  $N_A = 300$  atoms. In order to push the model further you will use a software package called *StatMech* to perform the same calculations. To run the program go to the **Physics Applications** menu and click on **StatMech**. You should see a window like the one below. The top of the window has several entry boxes

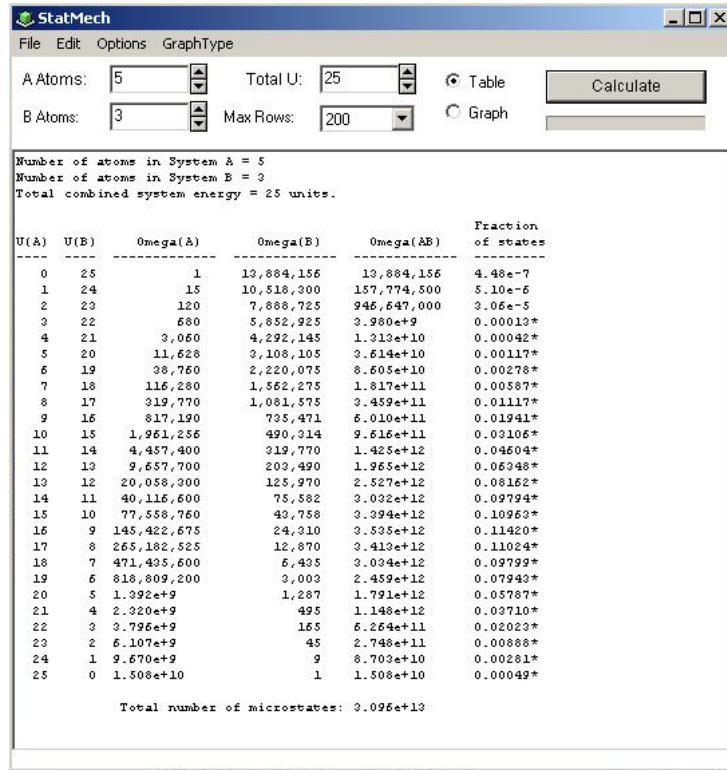


Figure 1: The **StatMech** window showing the table of multiplicities for each microstate. Each row corresponds to a different value of  $q_A$ .

where you can set the number of atoms ( $N_A$  and  $N_B$ ) and the total number of energy quanta in the system  $U$ . The parameter  $U$  is the total internal energy *int* in the system in units of  $\epsilon = \hbar\omega$ . It is equivalent to the sum  $q_{AB} = q_A + q_B$ . You can also set the number of rows of microstates to print out or choose to view a graph instead of the table. To test the operation of *StatMech* redo the calculations of the microstates that you did in Activity 3. Make sure your results in Activity 3 agree with the output of *StatMech*. You will also see there are other macrostates that were ignored in Activity 3 for simplicity.

(b) Now run *StatMech* for the case where  $N_A = 10$ ,  $N_B = 20$ ,  $U = 500$ . What is the value of  $q_A$  for the most probable macrostate? Record it here. Click on the button at the top of the *StatMech* window and choose graph. You will see a graph of the table and it should look something like Figure 2. The vertical axis is the probability of a particular macrostate divided by the maximum probability of any macrostate. The horizontal axis is  $U_A/U_{TOT}$  where  $U_A$  is the energy of solid A in units of  $\epsilon$  (equivalent to  $q_A$ ) and  $U_{TOT}$  is the total internal energy of the solid in units of  $\epsilon$  (equivalent to the total number of quanta  $q_{AB}$ ). What is the value of  $U_A/U_{TOT}$  for the most probable state? How is this value related to the value of  $q_A$  for the most probable macrostate? Also, explain in words what this plot is showing you.

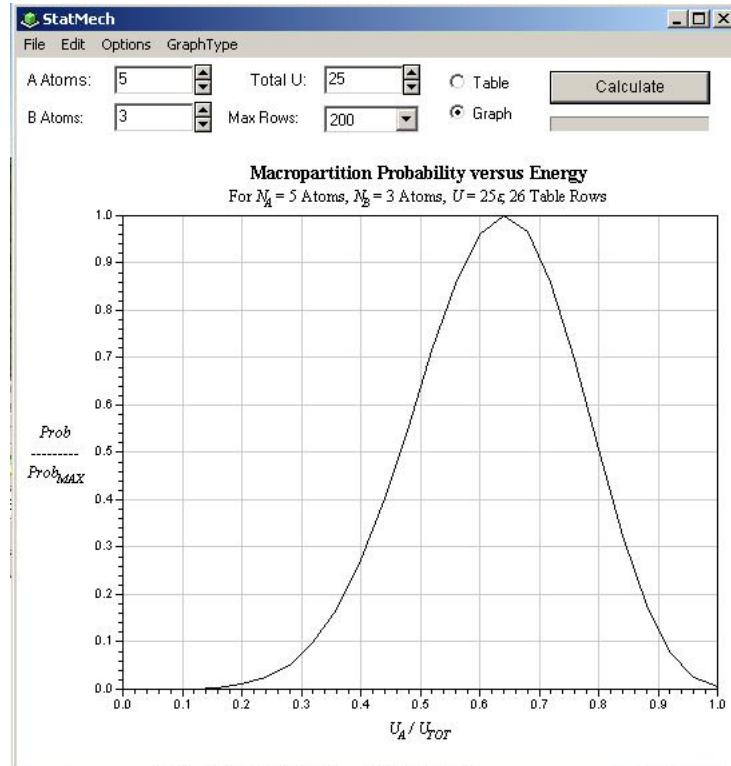


Figure 2: The StatMech window showing a graph of the multiplicities as a function of  $E_A/E_{int}$  where  $E_A = q_A\hbar\omega$  and  $E_{int} = q_{AB}\hbar\omega = (q_A + q_B)\hbar\omega$ .

(c) How wide is the distribution of microstates? Measure this number by estimating the full-width-half-maximum (FWHM) from your graph. Do this by finding the largest value on the vertical axis, divide that value by two, and find the two points on either side of the peak where the distribution is equal to that half-maximum. Take the difference between these two points along the horizontal and this is the FWHM. Record your result here.

(d) Now repeat steps 4.b-c with  $U = 100,000$  and  $N_A$  and  $N_B$  at their last values. What is the most probable value of  $U_A/U_{TOT}$  and the FWHM? How have things changed?

(e) Keeping  $U = 100,000$  now repeat steps 4.b-c, but this time double the values of  $N_A$  and  $N_B$ . Record the most probable macrostate and the FWHM. Repeat this doubling of the number of atoms in each solid while keeping  $U$  fixed at least 3-4 times. Record the most probable macrostate and the FWHM each time along with  $N_A$  and  $N_B$ .

(f) How does the value of  $q_A$  for the most probable macrostate change as the number of atoms increases?

(g) How does the FWHM change as the number of atoms increases?

### Activity 5: Irreversibility

You will now use the results from the previous Activity to delve into some of the implications of the statistical mechanics of the Einstein solid.

(a) As the number of atoms increases, what happens to the probability for finding the system in a macrostate different from the most probable one? Use the results of your calculations to explain your answer.

(b) When the system is in a macrostate far from the most probable one, what is the most likely thing to happen as energy or heat flows around the system?

(c) For the last calculation in Activity 4 what is the probability of the state with the minimum value of  $q_A$ ? In other words what is the probability that all of the quanta would end up all in solid  $B$ ? What is the probability of the most probable macrostate?

(d) Go back to the questions in Activity 1 and look at your answers. Do they still appear to be correct? A situation where all of the gas particles end up in one part of the container is a macrostate of the system analogous to the situation in Activity 5.c where all of the quanta end up in one of the solids and not the other. Answer those questions in Activity 1 again in terms of microstates, macrostates, and probability.

The behavior you are seeing here is for an Einstein solid, but is actually typical for most macroscopic systems. These systems have a large number of atoms or molecules with a variety of different energy states available.

They evolve to the most probable macrostate and there is essentially no chance to occupy a state far from the most probable one. When two materials are first put in thermal contact they may be far from the most probable macrostate, but they equilibrate at that most probable one (where the temperatures are equal) and never go back. This is irreversibility.

**Activity 6: Homework Problems** (E - exercise, P - problem)

1. (E) Consider the following ‘gas’. It consists of four atoms in a cubical box. At any instant, there is a 50% chance of each atom being in the left half of the box ( $L$ ) or the right half ( $R$ ). Make a table showing all the microstates of this system. (Hint: There are 16.) How many macrostates are there? How many microstates are in each macrostate?
2. (E) Show that for  $N$  gas atoms in a box, the number of possible microstates is  $2^N$  when microstates are defined by whether a given molecule is in the left half of the box or the right half of the box. The volumes of each half are equal.
3. (E) Imagine that we have an ideal gas consisting of 15 molecules. We can flip the signs of each of the three velocity components of a given molecule  $w$  without changing its overall energy (and thus without changing the gas’s macrostate). How many possible patterns of sign choices are there?
4. (E) Calculate the multiplicity of an Einstein solid with  $N = 1$  and  $E_{int} = 6\epsilon$  by directly listing and counting the microstates. Check your work by using equation 3.
5. (E) Calculate the multiplicity of an Einstein solid with  $N = 1$  and  $E_{int} = 5\epsilon$  by directly listing and counting the microstates. Check your work by using equation 3.
6. (E) Use equation 3 to calculate the multiplicity of an Einstein solid with  $N = 4$  and  $E_{int} = 10\epsilon$ .
7. (E) Use equation 3 to calculate the multiplicity of an Einstein solid with  $N = 3$  and  $E_{int} = 15\epsilon$ .
8. (E) How many times more likely is that the combined system of solids described in the table below will be found in macropartition 3:3 than in macropartition 0:6, if the fundamental assumption is true?
9. (E) How many times more likely is it that the combined system of solids describe in the table below will not be found in macropartition 3:3 than it is to be found in macropartition 0:6, if the fundamental assumption is true?
10. (E) Consider the system consisting of a pair of Einstein solids in thermal contact. A certain macropartition has a multiplicity of  $3.7 \times 10^{1024}$ , while the total number of microstates available to the system in all macropartitions is  $5.9 \times 10^{1042}$ . If we look at the system at a given instant of time, what is the probability that we will find it to be in our certain macropartition?
11. (E) Consider the system consisting of a pair of Einstein solids in thermal contact. A certain macropartition has a multiplicity of  $1.2 \times 10^{346}$ , while the total number of microstates available to the system in all macropartitions is  $5.9 \times 10^{362}$ . If we look at the system at a given instant of time, what is the probability that we will find it to be in our certain macropartition?
12. (E) Consider the system consisting of a pair of Einstein solids in thermal contact. Imagine that it is initially in a macropartition that has a multiplicity of  $8.8 \times 10^{123}$ . The adjacent macrostate closer to the equilibrium macrostate has a multiplicity of  $4.2 \times 10^{1234}$ . If we look at the system a short later, how many times more likely is it to have moved to the second macropartition than to have stayed with the first?

Macropartition	$E_A$	$E_B$	$\Omega_A$	$\Omega_B$	$\Omega_{AB}$
0:6	0	6	1	28	28
1:5	1	5	3	21	63
2:4	2	4	6	15	90
3:3	3	3	10	10	100
4:2	4	2	15	6	90
5:1	5	1	21	3	63
6:0	6	9	28	1	28
Total =					462

Table 1: Possible macropartitions for  $N_A = 1$ ,  $N_B = 1$ ,  $E_{int} = 6\epsilon$ .

13. (E) Consider the system consisting of a pair of Einstein solids in thermal contact. Imagine that it is initially in a macropartition that has a multiplicity of  $7.6 \times 10^{3235}$ . The adjacent macropartition closer to the equilibrium macropartition has a multiplicity of  $4.1 \times 10^{3278}$ . If we look at the system a short time later, how many times more likely is it to have moved to the second macropartition than to have stayed with the first?
14. (P) Suppose you put 100 pennies in a cup, shake it up, and toss them all into the air. (a) After landing, how many different head-tail arrangements (microstates) are possible for the hundred pennies? (b) What is the probability of finding exactly 50 heads? (c) 49 heads? (d) 1 head?
15. (P) You ask your roommate to clean up a mess he or she made in your room. Your roommate refuses, because cleaning up the mess would violate the second law of thermodynamics, and campus security's record of your roommate's legal violation is already excessive. Gently but firmly explain why complying will not put your roommate at risk of such an infraction.
16. (P) The classic statement of Murphy's law reads, 'If something can go wrong, it will.' Explain how this is really a consequence of the second law of thermodynamics. (Hint: What is the entropy of 'wrong' in a given context compared to the entropy of 'right'?)
17. (P) Run the *StatMech* program to answer the questions below.
- (a) For two Einstein solids in contact with  $N_A = N_B = 100$  and  $E_{int} = 200\epsilon$  answer the following questions. (1) How many times more likely is the system to be found in the center macropartition than in the extreme macropartition where  $E_A = 0$  and  $E_B = 200\epsilon$  (2) What is the range of values that  $E_A$  is likely to have more than 99.98% of the time? (3) if  $E_A$  were initially to have the extreme value 0, how many times more likely is it to move to the next macropartition nearer the center than to remain in the extreme one?
- (b) Answer the same question as in (a) for a run where you scale everything up by a factor of 10, so that  $N_A = N_B = 1000$  and  $E_{int} = 2000\epsilon$ .
- (c) Answer the same question as in (a) for a run where  $N_A = N_B = 1000$  and  $E_{int} = 200\epsilon$ . Comment on the effect that increasing just the size of the system by a factor of 10 has on these answers.
- (d) Answer the same question as in (a) for a run where  $N_A = N_B = 100$  and  $E_{int} = 2000\epsilon$ . Comment on the effect that increasing just the energy available to the system by a factor of 10 has on these answers.
18. (P) Consider two Einstein solids in thermal contact. The solids have different values of  $N$  but are identical in all other respects. It is plausible, since every atom in the combined system is identical, that in equilibrium

the energy will be distributed among the solids in such a way that the average energy per atom is the same. Use *StatMech* to test this hypothesis in the situation where  $E_{int} = 1000\epsilon$  and  $N_A$  and  $N_B$  have various different values such that  $N_A + N_B = 1000$ . (Set **Max Rows** to 1000 so that you can see every macropartition).

- (a) Is it true in most cases that in the most probable macropartition the solids have energies such that the average energy per atom in each is the same? Is it strictly true in every case? Answer these questions by discussing the values  $N_A$  and  $N_B$  you tested, and whether the actual most probable macropartition is the same as that predicted by the hypothesis.
  - (b) In any case where the hypothesis does not work, does increasing both  $N_A$  and  $N_B$  by a factor of 10 or 100 (but leaving  $U$  alone) yield a result more or less consistent with the hypothesis?
  - (c) Speculate as to the value of this hypothesis in the large- $N$  limit.
19. (P) For the following questions, you will find that using *StatMech* is by far the fastest way to calculate the multiplicity.
- (a) What is the entropy of an Einstein solid with 5 atoms and an energy of  $15\epsilon$ ? Express your answer as a multiple of  $k_b$ .
  - (b) What is the entropy of an Einstein solid with 50 atoms and an energy of  $100\epsilon$ ? Express your answer as a multiple of  $k_b$ .
20. (P) A certain macropartition of two Einstein solids has an entropy of  $305.2k_b$ . The next macropartition closer to the most probable one has an entropy of  $335.5k_b$ . If the system is initially in the first macropartition and we check it again later, how many times more likely is it to have moved to the other than to have stayed in the first?
21. (P) My calculator cannot display  $e^x$  for  $x > 230$ . One can calculate  $e^x$  for larger values of  $x$  as follows. Define  $y$  such that  $x = y \ln 10$ . This means that  $e^x = e^{y \ln 10} = (e^{\ln 10})^y = 10^y = 10^{x \ln 10}$ . Note that we can calculate 10 raised to a non-integer power (for example, 103.46) as follows:  $10^{3.46} = 10^{3+0.46} = 10^3(10^{0.46}) = 2.9 \times 10^3$ . Use these techniques to solve the following problem. The entropy of the most probably macropartition for a certain system of Einstein solids is  $6025.3k_b$ , while the entropy of an extreme macropartition is only  $5755.4k_b$ . What is the probability of finding the system at a given time in the extreme macropartition compared to that of finding it in the most probable macropartition?
22. (P) In principle, the entropy of an isolated system decreases a little bit whenever random processes cause its macropartition to fluctuate away from the most probable macropartition. We can certainly see this with small systems. But is this really a possibility for a typical macroscopic system? Imagine that we can measure the entropy of a system of two solids to within 2 parts in 1 billion. This means that we could just barely distinguish a system that has an entropy of  $4.99999999$  J/K (eight 9s!) from one that has  $5.00000000$  J/K. (This is a reasonable entropy for a macroscopic system).
- (a) Imagine that the entropy of the equilibrium macropartition is  $5.00000000$  J/K. Show that the approximate probability that at any given time later we will find the system in a macropartition with entropy  $4.99999999$  J/K (*i.e.*, with an entropy that is only barely measurably smaller) is about 10315,000,000,000,000 times smaller than the probability we will still find it to have entropy  $5.00000000$  J/K. (Hint: See problem 17.)
  - (b) Defend the statement that the entropy of an isolated system in thermal equilibrium never decreases.

## 12 Entropy and Temperature

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

**Objective**

To explore the connection between the fundamental definition of entropy and temperature.

**Overview**

Consider the microscopic definition of the entropy of a system

$$S = k_B \ln \Omega \quad (1)$$

where  $k_B$  is Boltzmann's constant and  $\Omega$  is the multiplicity or number of microstates. A microstate is defined by a particular arrangement of energy quanta among the atoms. A macrostate is defined by the total number of energy quanta  $q$  and the number of atoms  $N$ . We are building a model of an elemental solid (*e.g.*, like aluminum) where the total internal energy in the solid  $E_{int}$  is described by

$$E_{int} = q\hbar\omega \quad (2)$$

where  $\hbar$  is Planck's constant divided by  $2\pi$  and  $\omega$  is a constant that characterizes the strength of the bonds between the atoms. The parameter  $q$  is the total number of quanta in the system and is a constant. These quanta are statistically distributed over the  $N$  atoms of the solid so all possible states of the system are equally likely and the multiplicity  $\Omega$  is

$$\Omega = \frac{(q + 3N - 1)!}{q!(3N - 1)!} \quad . \quad (3)$$

This model of an elemental solid is called an Einstein solid.

We want to find a connection between the entropy defined in Equation 1 and the temperature. Recall how temperature is usually defined relative to some properties of matter like the freezing and boiling points of water. You are developing the microscopic picture of entropy, but it won't be successful until you can connect it to the observed behavior of bulk matter and our familiar notions of temperature.

**Activity 1: The Entropy of Einstein Solids in Thermal Equilibrium**

(a) To start connecting the entropy to the temperature you have to study the behavior of the entropy as the energy changes. To do this we will study two Einstein solids ( $A$  and  $B$ ) in thermal equilibrium with each other. Their total internal energy will be

$$E_{int} = q_{AB}\hbar\omega = (q_A + q_B)\hbar\omega \quad (4)$$

where  $q_A$  and  $q_B$  are the numbers of energy quanta in each solid and  $q_{AB}$  is their sum.

Use the program *StatMech* (see the Physics Applications menu) for the configuration where you choose  $N_A > 100$ ,  $N_B > 80$  and  $U > 400$ . The label  $U$  in the *StatMech* window refers to the total number of energy quanta in the system in units of  $\hbar\omega$  and is equivalent to  $q_{AB}$  here. An example of the output of *StatMech* is shown in Figure 1. The first two columns in the lower panel of Figure 1 represent  $U(A)$  and  $U(B)$ , the energies in each individual solid (again in units of  $\hbar\omega$ ) and are equivalent to  $q_A$  and  $q_B$ . After you perform the calculation with *StatMech* scan quickly down the column labeled 'Omega(AB)'. If any of the exponents you see exceed the value 307, then run the calculation again with smaller inputs until no exponent exceeds 307. This limitation is a restriction on MicroSoft *Excel* that you will use later to make plots. Record your values of  $N_A$ ,  $N_B$ , and  $U$ .

(b) Now generate plots of  $S_{AB} = S_A + S_B$ ,  $S_A$ , and  $S_B$  from the *StatMech* table. You can do this with *Excel*, but there are some intermediate steps necessary. Start Microsoft *Word* first. Next, go to the *StatMech* window, highlight the table, copy it (see the *Edit* menu on the *StatMech* window), and paste it into the *Word* document. In *Word* edit out all the commas (',') and asterisks (\*) in the file (use the *Replace* option under the *Edit*

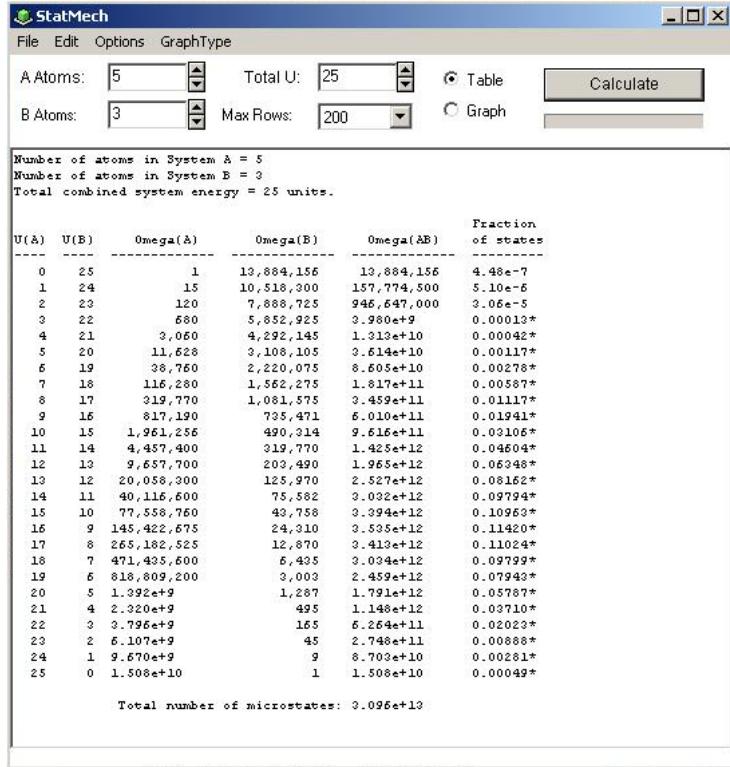


Figure 1: The StatMech window showing the table of multiplicities for each microstate. Each row corresponds to a different value of  $q_A$ .

menu). Save the Word file, but save it as a plain text ('.txt') file. You can now open the file in *Excel*. When you open the file, *Excel* pops up a *Text Import Wizard* that will guide you through the format of the input file. The defaults usually seem to work. Use *Excel* to calculate and plot on one graph  $S_{AB}$ ,  $S_A$ , and  $S_B$  as a function of  $q_A$ . Print out your plot and attach it to this unit.

(c) What is  $q_A$  for the most probable macrostate? What mathematical condition can you impose on the total entropy  $S_{AB}$  to determine the most probable macrostate? How do you think the temperatures of solids  $A$  and  $B$  are related at the most probable microstate?

(d) How are the slopes of  $S_A$  and  $S_B$  related to one another at the most probable macrostate?

(e) How is  $E_A$ , the energy in solid A related to  $q_A$ ? How is  $E_B$ , the energy in solid B related to  $q_A$ ? Remember that  $q_{AB}$  is a constant and  $q_{AB} = q_A + q_B$ . Calculate the differentials  $dE_A$  and  $dE_B$  and rewrite the answer in part 1.d in terms of  $dS_A/dE_A$  and  $dS_B/dE_B$ .

### Activity 2: Relating Entropy and Temperature

(a) Using the spreadsheet you generated in Activity 1, calculate  $dS_A/dq_A$  as a function of  $q_A$  and plot it. You can do this to an adequate approximation by doing taking the difference between  $S_A$  at adjacent values/rows of  $q_A$ . To do this suppose your spreadsheet has the values of  $S_A$  in column H. The Excel syntax for estimating the derivative for the first value of  $q_A$  (the first row) is ‘=(H2-H1)/1.0’ where H2 is the value in the second row and H1 is the value in the first row. The numerator of one is redundant, but it shows you are approximating the derivative using the data from points that differ by 1.0 in  $q_A$ , the number of quanta in solid A. The syntax for  $dS_A/dq_A$  for the second value of  $q_A$  is ‘=(H3-H2)/1.0’ and so on. Do the same for  $dS_B/dq_A$  and  $dS_{AB}/dq_A$ . Does the slope of  $S_{AB}$  pass through zero at the correct spot (recall part 1.c)? How are  $dS_A/dq_A$  and  $dS_B/dq_A$  related at the most probable macrostate. Does your plot agree with that result?

(b) If the energy  $E_A$  and  $q_A$  of solid A increases what should the temperature of solid A do? If the energy  $E_A$  of solid A increases what happens to  $dS_A/dq_A$  in your plot? Do the temperature and  $dS_A/dq_A$  change in the same way or in a different way as  $E_A$  increases?

(c) We want to come up with a relationship between temperature and the entropy. From the results above (parts 1.a-e) you should have found

$$\frac{dS_A}{dE_A} = \frac{dS_B}{dE_B} \quad (5)$$

and

$$T_A = T_B \quad (6)$$

for the most probable macrostate. This means there is some function of the temperature  $T$  such that

$$f(T) = \frac{dS}{dE} \quad (7)$$

for each solid that will be equal at equilibrium. We want  $f(T)$  to behave like the temperatures we are accustomed to using. In other words, as the energy in the solid increases  $T$  should increase. Recall part 2.b and the behavior of  $dS/dE$  as  $T$  increases. Try to guess a mathematical form of  $f(T)$  that acts like ‘normal’ temperatures and one that doesn’t. Explain your reasoning.

Solid	$dE/dT$ per mole	Solid	$dE/dT$ per mole

Table 2: Molar specific heats ( $(1/n)dE/dT$ ) for several elemental solids.

- (d) How would you choose which of the forms you guessed in part 2.c is the correct one?

### Activity 3: Determining $f(T)$ and the Heat Capacity

In the previous Activity you should have found that the mathematical form of  $f(T)$  has to be something like  $1/T^n$  where  $n$  is some positive number. This is necessary because your graphs should show that as the energy  $E_A$  (and the number of quanta  $q_A$ ) of the solid increases  $f(T) = dS/dE$  goes down. To make sure the temperature  $T$  behaves reasonable (and goes up with  $E_A$  and  $q_A$ )  $f(T)$  must be some inverse of function of  $T$ . To decide exactly which function is right requires comparing Equation 7 or some result from it to some data.

Recall the table of heat capacities we generated in the laboratory entitled *Calorimetry*. Use those results to fill in Table 1 making sure that you are using molar heat capacities. The heat capacities are constant with respect to temperature and are similar in value to one another. These are the data that will help us determine  $f(T)$ . To calculate  $dE/dT$  we must find a relationship between  $E$  and  $T$  for the Einstein solid.

- (a) Start with Equations 1 and 7 and the chain rule and show the following.

$$\frac{dS}{dE} = k_B \frac{1}{\Omega} \frac{d\Omega}{dE} \quad (8)$$

- (b) Use Equation 2 to show

$$dE = \hbar\omega dq \quad \text{and} \quad \frac{d\Omega}{dE} = \frac{1}{\hbar\omega} \frac{d\Omega}{dq} \quad . \quad (9)$$

(c) Starting with Equations 1 and 3 one can show

$$\frac{d\Omega}{dq} = \frac{3N\hbar\omega}{E}\Omega \quad . \quad (10)$$

Combine this equation (number 10) and the results from 3.a-b to get a relationship for  $dS/dE$  for the Einstein solid in terms of  $N$  and  $E$ . Set that expression equal to  $f(T)$ , and solve for  $E$  the internal energy. It is the derivative of this last equation  $((1/n)dE/dT)$  that will give you the molar specific heat. What function of  $f(T)$  will give a result that is independent of temperature when you take the derivative with respect to  $T$  of your expression for the internal energy  $E$ ?

(d) What is the final form of Equation 7 and  $f(T)$ ?

(e) Determine the mean and standard deviation of the heat capacity of the elemental solids in Table 1. Calculate the molar specific heat  $((1/n)dE/dT)$  for the Einstein solid using your results from parts 3.c-d. Is the molar specific heat for the model of the Einstein solid consistent with the measured ones?

#### Activity 4: The Second Law of Thermodynamics

(a) Go back to your plots of the entropy as a function of  $q_A$  from part 1.b. Consider two Einstein solids that are brought together at a value of  $q_A$  that is higher than the equilibrium one at the most probable macrostate. Choose a value of  $q_A$  that is halfway between the most probable value and the maximum. Once the two Einstein solids are in contact, how will the system evolve? What happens to  $S_A$  and  $S_B$ ? Do they go up, down, or stay the same? What happens to the total entropy  $S_{AB}$ ? In fact, based on your plot from part 2.b, is there any circumstance where  $S_{AB}$  will not increase?

(c) To vividly see what happens when Einstein solids come in thermal contact, run the program *equilib.exe*

available in the **Physics Applications** menu. This program starts with two Einstein solids with all the energy quanta in solid  $B$ . It simulates the evolution of the two solids as they march toward thermal equilibrium. Click **Evolve** to see the simulation run.

What you should have discovered in the previous part is that the entropy of the combined systems always increases regardless of what configuration the Einstein solids are in when they come in contact. The system always evolves to the most probable, most disordered macrostate where the temperatures will be equal and the entropy is a maximum. The energy quanta are most spread out. This result is stated in several different ways, but the most succinct is simply  $\Delta S > 0$  for an isolated system. The entropy of an isolated system always increases. This is called the Second Law of Thermodynamics.

#### Activity 4: Homework Problems

1. (E) An object's entropy is measured to increase by  $0.1 \text{ J/K}$  when we add  $35 \text{ J}$  of energy. What is its approximate temperature? (Assume that the object's temperature does not change much when we add the  $35 \text{ J}$ .)
2. (E) A certain Einstein solid's entropy changes from  $305.2k_b$  to  $338.1k_b$  when we add 1 unit  $\epsilon$  of energy. What is the value (and units) of  $k_b T/\epsilon$  for this solid? If  $\epsilon = 1.0 \text{ eV}$ , what is its temperature  $T$ ?
3. (E) Does it make sense to talk about the temperature of a vacuum? If so, how could you measure or calculate it? If not, why not?
4. (E) An Einstein solid in a certain macrostate has a multiplicity of  $3.8 \times 10^{280}$ . What is its entropy (expressed as a multiple of  $k_B$ )?
5. (E) A pair of Einstein solids in a certain macropartition has multiplicities of  $4.2 \times 10^{320}$  and  $8.6 \times 10^{132}$ . What are the entropies of each solid? What is the total entropy of the system in this macropartition? (Express entropies as multiples of  $k_b$ .)
6. (E) Is it really true that the entropy of an isolated system consisting of two Einstein solids never decreases? (Consider a pair of very small solids.) Why is this statement more accurate for large systems than for small systems? Explain in your own words.
7. (P) In lab we argued on fairly fundamental grounds that  $dS/dU = f(T)$ . In principle, we could define  $f(T)$  to be anything that we like: this would amount to defining temperature and its scale. Still, some definitions would violate deeply embedded preconceptions about the nature of temperature. For example, the simplest definition of temperature would be  $dS/dU = T_{new}$ . Show that this definition
  - (a) Would imply that  $T_{new}$  has units of  $K^{-1}$  and
  - (b) Would imply that heat would flow spontaneously from objects with low  $T_{new}$  to objects with high  $T_{new}$ . This would imply that objects with low values of  $T_{new}$  are hot, while objects with high values  $T_{new}$  are cold (we might want to call  $T_{new}$  so defined *coolness* instead of *temperature*). While we could define temperature in this way, it would really fly in the face of convention (if not intuition).
  - (c) If we did define coolness  $T_{new}$  in this way, what ordinary temperature  $T$  would an object with absolutely zero coolness ( $T_{new} = 0$ ) have? What about something that is infinitely cool ( $T_{new} = \infty$ )?
8. (P) Imagine that the entropy of a certain substance as a function of  $N$  and  $U$  is given by the formula  $S = Nk_b \ln U$ . Using the definition of temperature, show that the thermal energy of this substance is related to its temperature by the expression  $U = Nk_b T$ .
9. (P) Imagine that the multiplicity of a certain substance is given by  $\Omega(U, N) = Ne^{\sqrt{NU/\epsilon}}$ , where  $\epsilon$  is some unit of energy. How would the energy of an object made out of this substance depend on its temperature? Would this be a ‘normal’ substance in our usual sense of temperature?
10. (P) Consider an Einstein solid having  $N = 20$  atoms.

- (a) What is the solids temperature when it has an energy of  $10\epsilon$ , assuming that  $\epsilon = h\omega = 0.02\text{eV}$ ? Calculate this directly from the definition of temperature by finding  $S$  at  $10\epsilon$  and  $11\epsilon$ , computing  $dS/dU \approx [S(11\epsilon) - S(10\epsilon)]/\epsilon$ , and then applying the definition of temperature. (You will find that your work will go faster if you use *StatMech* to tabulate the multiplicities.)
- (b) How does this compare with the result from the formula  $U = 3Nk_bT$  (which is only accurate if  $N$  is large and  $U/3N\epsilon > 1$ )?
- (c) If you have access to *StatMech*, repeat for  $N = 200$  and  $U = 100\epsilon$ . (Hint: If your calculator cannot handle numbers in excess of  $10^{100}$ , use the fact that  $\ln(a \times 10^b) = \ln a + b \ln 10$ ).

11. (P) A newly-created material has a multiplicity

$$\Omega = \alpha NE$$

where  $N$  is the number of atoms in the solid,  $E$  is the total, internal energy in the solid, and  $\alpha$  is a constant of proportionality.

- (a) How does the temperature of the new material depend on the internal energy?
- (b) What is the molar heat capacity for this solid?
- (c) Could this material really exist? Why or why not?

12. (P) A newly-created material has a multiplicity

$$\Omega = \beta ME^2$$

where  $N$  is the number of atoms in the solid,  $E$  is the total, internal energy in the solid, and  $\alpha$  is a constant of proportionality.

- (a) How does the temperature of the new material depend on the internal energy?
- (b) What is the molar heat capacity for this solid?
- (c) Could this material really exist? Why or why not?

## 13 Music to Our Ears: Standing Waves in Strings

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

**Aim:**

- to study the natural modes of vibration of a stretched string

**Equipment:**

- string vibrator and power supply
- inelastic braided string
- 2 clamps
- superpulley
- mounting rod for the superpulley
- mass and hanger set
- balance
- tape measure

How do we make musical sounds? To make a sound , we need something that vibrates. If we want to make musical notes you usually need the vibration to have an almost constant frequency: that means stable pitch. We also want a frequency that can be easily controlled by the player. In electronic instruments this is done with electric circuits or with clocks and memories. In non-electronic instruments, the stable, controlled vibration is produced by a standing wave. Here we discuss the way strings work. This is also a good introduction for studying wind instruments, because vibrating strings are easier to visualise than the vibration of the air in wind instruments, but the math is very similar.

**Introduction**

Waves are oscillations in an elastic medium:

- your own vocal cords (the medium) vibrating as air is forced over them by your lungs;
- a stretched string (the medium) on a musical instrument vibrating as it is bowed, hammered or plucked;
- pressure oscillations in a column of air (the medium) in a wind instrument, organ pipe or your own oral and nasal cavities.

In each case the medium has an equilibrium state, and when displaced or otherwise perturbed from that state, experiences a force which tends to restore it to equilibrium. For small perturbations, the restoring force is proportional to the displacement and the medium becomes a simple harmonic oscillator.

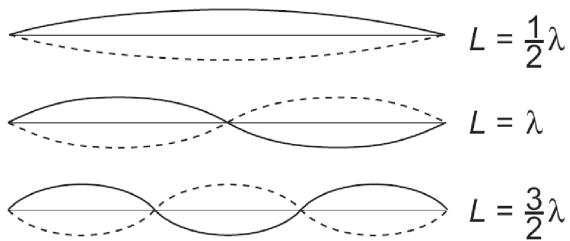
There are also waves that do not require a medium, namely electromagnetic waves, which will be discussed later in the course.

**Background**

Standing waves (stationary waves) are produced by the interference of two traveling waves, both of which have the same wavelength, speed and amplitude, but travel in opposite directions through the same medium. The necessary conditions for the production of standing waves can be met in the case of a stretched string by having waves set up by some vibrating body, reflected at the end of the string and then interfering with the oncoming waves.

One characteristic of every standing wave pattern is that there are points along the medium which appear to be standing still. These points, sometimes described as points of no displacement, are referred to as nodes. There are other points along the medium which undergo vibrations between a large positive and large negative displacement. These are the points which undergo the maximum displacement during each vibrational cycle of the standing wave. In a sense, these points are the opposite of nodes, and so they are called antinodes. A standing wave pattern always consists of an alternating pattern of nodes and antinodes. When a standing wave pattern is established in a medium, the nodes and the antinodes are always located at the same position along the medium; they are "standing still." It is this characteristic which has earned the name "standing wave."

A stretched string has many natural modes of vibration (three examples are shown below). If the string is fixed at both ends then there must be a node at each end. It may vibrate as a single segment, in which case the length ( $L$ ) of the string is equal to  $1/2$  the wavelength ( $\lambda$ ) of the wave. It may also vibrate in two segments with a node at each end and one node in the middle; then the wavelength is equal to the length of the string. It may also vibrate with a larger integer number of segments. In every case, the length of the string equals some integer number of half wavelengths.



If you drive a stretched string at an arbitrary frequency, you will probably not see any particular mode; many modes will be mixed together. But, if the tension and the string's length are correctly adjusted to the frequency of the driving vibrator, one vibrational mode will occur at a much greater amplitude than the other modes. For any wave with wavelength ( $\lambda$ ) and frequency  $f$ , the speed,  $v$ , is:

$$v = \lambda f \quad (11)$$

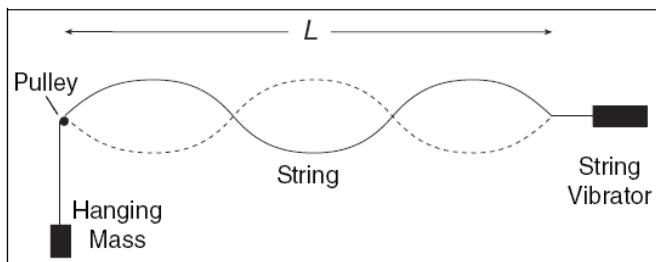
The speed of a wave on a string is also given by:

$$v = \sqrt{\frac{T}{\mu}} \quad (12)$$

where  $T$  is the tension in the string and  $\mu$  is the linear density (mass/length) of the string.

In this experiment, standing waves are set up in a stretched string by the vibrations of an electrically-driven String Vibrator. The tension in the string equals the weight of the masses suspended over the pulley. You can alter the tension by changing the masses.  $L$  is the length of the string and  $n$  is the number of segments. (Note that  $n$  is not the number of nodes). Since a segment is  $1/2$  wavelength then

$$\lambda = \frac{2L}{n} \quad n = 1, 2, 3, \dots \quad (13)$$

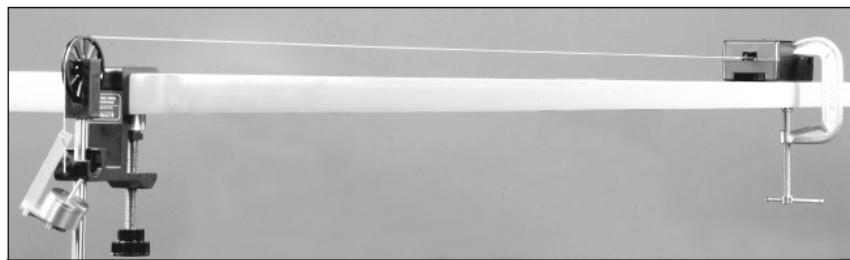


**Procedure**

1. Measure the exact length of a piece of string several meters long. Measure the mass of the string and calculate the linear density (mass/length). (If your balance is not precise enough to measure that length of string, use a longer piece of string to calculate the linear density.)

2. Clamp the String Vibrator and pulley about 100 cm apart. Attach the string to the vibrating blade, run it over the pulley, and hang about 200 g of mass from it. Cut off the excess string.

3. Measure from the knot where the string attaches to the String Vibrator to the top of the pulley. This is the distance  $L$ . ( $L$  is NOT the total length of the string that you measured in 1.) Record the value here:  $L =$



4. Connect the AC power supply to the String Vibrator.

5. Adjust the tension by adding to or subtracting from the hanging mass so that the string vibrates in 2 segments. Adjust the tension further to achieve a “clean” node at the center. Also check the end of the vibrating blade; the point where the string attaches should be a node. It is more important to have a good node at the blade than it is to have the largest amplitude possible. However, it is desirable to have the largest amplitude possible while keeping a good node.

6. Record the hanging mass,  $m$ . How much uncertainty is there in your value? By how much can you change the hanging mass before you see an effect? Record the uncertainty.

**Part A: Speed of the Wave**

a. Calculate the tension (including the uncertainty) in the string.

- b.** Calculate the speed  $v_A$  (including uncertainty) of the wave from your observed values of tension ( $T$ ) and linear density ( $\mu$ ). Record your calculated value with the uncertainty and the correct number of significant figures.
- c.** Calculate the speed  $v_B$  from the wavelength ( $\lambda$ ) and frequency ( $f$ ). (In the U.S.  $f = 60.0$  Hz.)
- d.** Compare the two values ( $v_A$  and  $v_B$ ) of speed. What is the difference? How does the difference compare to the uncertainty that you determined in **b**?
- e.** Calculate the percentage by which  $v_A$  deviates from  $v_B$ .

$$\%Deviation = \frac{v_A - v_B}{v_B} \times 100\% \quad (14)$$

**Part B: Linear Density**

a. Produce standing waves of 3, 4, 5, etc. segments in the string by reducing the hanging mass. Get as many as you can. Create a table here with the headings  $n$  (number of segments),  $m$  (mass),  $T$  (tension in the string which is the same as  $mg$ ), and  $\frac{1}{n^2}$ . Be sure to include the result from Part A for  $n = 2$ .

b. For every value of mass, calculate the tension in the string, and include in the above table.

c. In *Excel*, plot a graph of  $T$  versus  $n$  including a trend line using a power function. Print the graph and include it with this unit. What is the power of  $n$ ?

d. For every value of  $n$ , calculate  $\frac{1}{n^2}$  and include in above table. In *Excel*, plot a graph of  $T$  versus  $\frac{1}{n^2}$ . Does the graph look linear? If so, include a linear trend line. Print the graph and include it with this unit.

e. Using the LINEST function in *Excel* (see **Appendix C**), find the slope of the graph and its uncertainty. Record the values here (including units):

f. Combine the equations in the **Background** section, and show that the tension can be written as:

$$T = 4\mu f^2 L^2 \frac{1}{n^2} \quad (15)$$

Thus the slope of a  $T$  versus  $\frac{1}{n^2}$  graph is  $4\mu f^2 L^2$ .

**g.** Use the slope from your graph to calculate the density,  $\mu$ , of the string. Also calculate the uncertainty in  $\mu$ , assuming the fractional uncertainty in  $\mu$  is the same as the fractional uncertainty in the slope.

**h.** Compare this measured value of string density to the accepted value. (You calculated the accepted value of  $\mu$  from the mass and length of the string at the beginning of the experiment). Does the accepted value fall within the uncertainty limits of your calculated value?

#### **Further Investigation**

If a strobe is available, observe the standing wave on a string with the strobe light. Draw a diagram explaining the motion of the string.

## 14 Resonance in Tubes

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives:

- Determine the resonant frequency for a tube open at one end.
- Determine tube lengths at resonance for a tube of variable length.
- Determine the velocity of sound in air in the laboratory (two ways).

### Introduction

The Economy Resonance Tube is designed for the study of resonance in columns of air. The tube set includes a movable inner tube with a closed end and an outer tube which is open at both ends. The inner tube also includes a measuring tape to easily find the length of the air column in the outer tube. To adjust the length of the outer tube, simply slide the inner tube until the desired length appears on the measuring tape. Open tube experiments can also be performed with the outer tube by removing the inner tube.

In order that the tube resonate, the frequency of the vibrating air must coincide with the natural frequency of the tube (which may be its fundamental or one of its overtones). For the Economy Resonance Tube, which is closed at one end, this requirement is met if the tube length is an odd number of quarter wavelengths of the sound waves produced by the source ( $L = \lambda/4, 3\lambda/4, 5\lambda/4$ , etc., where  $L$  is the length of the tube and  $\lambda$  is the wave length of the sound). Note that if the length of the tube is gradually increased while the source is vibrating, the distance between successive resonance positions is  $\lambda/2$ .

**Note:** Due to edge effects at the open end of a tube, the effective length of the tube depends on the radius of the opening. Thus,  $L_{eff} = L + 0.6r$ , where  $L_{eff}$  is the *effective* length,  $L$  is the length measured, and  $r$  is the tube radius.

### Apparatus:

- Economy Resonance Tube
- Open speaker
- Sine wave generator
- 2 banana plug leads
- Sound sensor
- Meter stick
- Data Studio 750 Interface
- Thermometer

Room Temperature (°C) \_\_\_\_\_ Tube radius (m) \_\_\_\_\_

### Activity 1: Fixed tube length

1. Connect the open speaker to the sine wave generator using standard banana plug leads.
2. Adjust the length of the OUTER (blue) tube to 80 cm (check with meter stick).
3. Place the tube in front of the speaker in such a way that the tube is open at one end (the speaker can be set at an angle relative to the tube length).
4. Set the sound sensor inside the tube at the open end and connect it to the Data Studio interface.
5. To activate the sound sensor, perform the following sequence: Start up *DataStudio* by going to *Start* → *Programs* → *Physics Applications* → *DataStudio*. Click on *Create Experiment*, then *Setup*, then *Add Sensor or Instrument*. Scroll down to *Sound level sensor* and select, then click *OK*. Double click *Graph* at left. Click *Start* to begin taking data.

6. The resonant frequencies for a tube open at one end are given by  $f = nv/4L$  where  $n$  is an odd integer,  $v$  is the velocity of sound and  $L$  is the effective tube length. Start at a frequency of 50 Hz and increase until you find three resonances (for  $n = 1, 3, 5$ ). These will be indicated by peaks on the sound level graph. You need to carefully determine the frequency associated with each peak. Record the results here:
  
7. From your data, plot a graph of  $f$  vs  $n$  (using *Excel*). Fit with a linear trendline. Print the graph (with title and axis labels) and include with this unit. Use the LINEST function in *Excel* to determine the slope of the graph and its uncertainty. From these values, calculate the velocity of sound in air, and its uncertainty, and record them here. Don't forget to use  $L_{eff}$  - see Introduction.

**Activity 2: Fixed frequency**

1. Adjust the outer tube length to 20 cm.
2. Set the speaker inside the open end of the tube so that it is closed at both ends. (Do not include sound sensor for this activity. You will just be listening for loudness.)
3. Set the sine wave generator frequency to 600 Hz (with low amplitude).
4. Slowly move the inner tube to increase the effective length of the tube. Listen for maximum loudness and record the length of the tube when resonance is achieved:
  
5. Increase the length of the tube until two more resonance lengths are found for the constant frequency and record them here:
  
6. The resonant frequencies for a tube closed at both ends are given by  $f = nv/2L$  where  $n = 1, 2, 3$ , etc. Solve this equation for  $L$  and plot  $L$  vs  $n$  (using *Excel*) where  $n = 1, 2$  and  $3$  for the three resonance lengths  $L$  you found in 4 and 5 above. Fit with a linear trendline. Use the LINEST function in *Excel* to determine the slope of the graph and its uncertainty. Use these values to determine the velocity of sound in air and its uncertainty and record them here:
  
7. The velocity of sound in air at 0°C is 331.4 m/s. The temperature dependence of sound velocity in air is given by  $v(T) = 331.4 + 0.6T$ , where  $T$  is in °C and  $v$  is in m/s. Calculate an “accepted” value of the velocity of sound in air from this formula.
  
8. Does the “accepted” value fall within the range of uncertainty of your experimental value?

## 15 Electrostatics

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To understand the basic phenomena of electric charges at rest.

### Introduction

Atoms consist of a central nucleus made up of protons and neutrons surrounded by one or more electrons. While the nuclei of solids are essentially localized, some of the electrons may be free to move about. A substance which has as many electrons as it has protons is said to be electrically neutral. Dissimilar objects have different affinities for electrons. When two such objects, initially neutral, are rubbed together, the friction may cause electrons to pass from one to the other. After separation, neither object is neutral. Each is said to have been "charged by friction". An isolated, electrified object becomes neutral again if its electron-proton balance is restored. A convenient means for accomplishing this is to connect the object to earth by means of a conductor, through which electrons readily travel. This process is called "grounding the body". Since an electrified object is referred to as "charged", grounding is also referred to as "discharging".

Substances through which electrons do not move easily are called "non-conductors", or "insulators". Experiment has shown that when rubber and wool are rubbed together, electrons pass from the wool to the rubber. The electrons remain on the surface of the rubber—a non-conductor—where they were transferred.

Rubbing a metal rod with a wool cloth can also transfer electrons. This rod, however, is a conductor and electrons pass through it to the experimenter and then to the earth. People, made mostly of salt water, are good conductors, as well. Metal that is isolated, however, can be electrified. This can be demonstrated with an electroscope, which has a metal knob connected to a stem from which a thin metal leaf hangs. An insulator prevents contact of these metal parts with the case, and consequently the earth.

### Apparatus

- electroscope
- rubber and glass rods
- wool and silk cloth
- plastic rod with metal disc mounted on one end

### Activity 1: Charging by Friction

1. Be sure the electroscope is discharged by touching the knob with your finger. Explain what happened and why you are convinced the electroscope is discharged.
2. **Prediction:** If you rub the knob of an electroscope with a wool cloth, what will be the state of the electroscope when you remove the cloth? Explain.

3. Gently and repeatedly rub the knob of the electroscope with the wool cloth for a couple of minutes. Remove the cloth. Note any differences in the electroscope from its appearance before you rubbed.
  
  
  
  
4. Explain what, if anything, happened.

**Activity 2: Charging by Contact**

1. Discharge the electroscope as before.
2. Charge the disc at the end of the plastic rod by friction with the wool cloth.
3. Does anything occur in the electroscope when you bring the disc close to the knob without touching it?
  
  
  
  
4. **Prediction:** What will happen to the electroscope if you repeatedly touch its knob with a freshly charged object?
  
  
  
  
5. Touch the disc to the knob; rub the disc again and again touch it to the knob; repeat this procedure two or three more times. Describe any changes to the electroscope.
  
  
  
  
6. Repeat the procedure above until the electroscope's leaf is at approximately a twenty degree angle with the stem.

**Activity 3: Kinds of Electrification**

1. Electrify one end of the rubber rod by wrapping the wool cloth around the rod, squeezing the wool against the rod, twisting the rod vigorously to ensure good contact, and separating the wool from the rod.
2. **Prediction:** What will happen when you bring the electrified end of the rubber rod toward, but not touching, the electroscope's knob? What will happen if you do the same with the wool cloth?
  
  
  
  
3. Bring the charged end of the rubber rod toward the knob, but do not touch it. Record what happens.

4. Repeat number 3 with the wool cloth.
5. What differences were there between the trial with the rod and the trial with the cloth?
6. How would you account for these differences?
7. **Note:** By definition, the electrical state of the rubber after being rubbed by the wool is negative. That is, an object that has an excess of electrons is said to be negatively charged. Realize that this is only a convention.
8. If the rubber is said to be negatively charged, in what electrical state is the wool cloth?
9. How can an electroscope be used to determine the nature of any charge?
10. Rub the end of the glass rod with the silk cloth and determine the charge of each (positive or negative) after they are separated.

#### Activity 4: Action of the Electroscope

1. **Discussion:** Two facts explain the rise or fall of the leaves of an electroscope: (a) Like charges repel (unlike charges attract); and (b) Free electrons move about in a conductor when an electric force acts upon them.
2. In Activity 3, the electroscope was negatively charged before either the rubber rod or the wool cloth was brought toward the knob. When the rubber rod approaches the knob (in Activity 3 number 3), in which direction do the free electrons in the electroscope move (up toward the knob or down toward the leaf)? Does the electron displacement increase or decrease the electrostatic force separating the leaf from the stem?
3. When the wool cloth approaches the knob (in Activity 3 number 4), which way do the free electrons move?

**Activity 5: Charging by Induction**

1. Discharge the electroscope.
2. **Prediction:** What will be the effect on the electroscope if you perform the following experiment: while grounding the electroscope with your finger, bring an electrified rubber rod near the knob, then take away your finger and then the rod (in that order)?
3. Carry out the experiment and describe the result.
4. Explain the result and why your prediction agreed or disagreed with it.
5. **Prediction:** Note that no electrons moved between the rod and the electroscope. What charge has been induced on the electroscope?
6. Test your prediction with the negatively charged rubber rod and the positively charged wool.
7. Does the test verify or contradict your prediction?

## 16 The Electrical and Gravitational Forces<sup>4</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

I began to think of gravity extending to the orb of the moon, and . . . I deduced that the forces which keep the planets in their orbs must be reciprocally as the squares of their distances from the centers about which they revolve: and thereby compared the force requisite to keep the moon in her orb with the force of gravity at the surface of the earth, and found them to answer pretty nearly. All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention, and minded mathematics and philosophy more than at any time since. - Isaac Newton

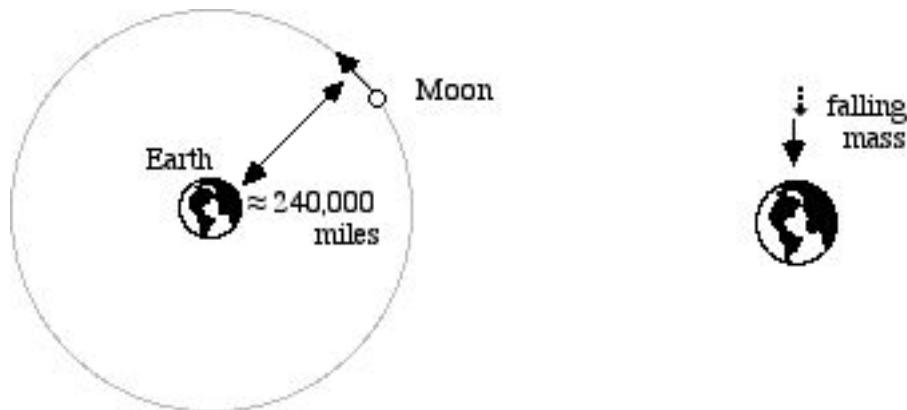
### Objective

To understand the similarities of the gravitational and electrical forces.

### Overview

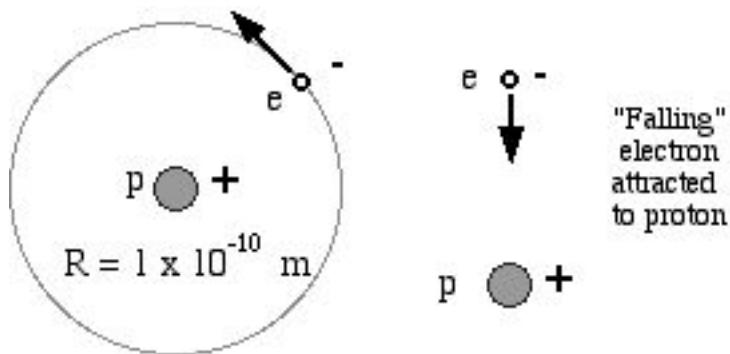
The enterprise of physics is concerned ultimately with mathematically describing the fundamental forces of nature. Nature offers us several fundamental forces, which include a strong force that holds the nuclei of atoms together, a weak force that helps us describe certain kinds of radioactive decay in the nucleus, the force of gravity, and the electromagnetic force.

*Two kinds of force dominate our everyday reality-the gravitational force acting between masses and the Coulomb force acting between electrical charges.* The gravitational force allows us to describe mathematically how objects near the surface of the earth are attracted toward the earth and how the moon revolves around the earth and planets revolve around the sun. The genius of Newton was to realize that objects as diverse as falling apples and revolving planets are both moving under the action of the same gravitational force.



Similarly, the Coulomb force allows us to describe how one charge "falls" toward another or how an electron orbits a proton in a hydrogen atom.

<sup>4</sup>1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material have been modified locally and may not have been classroom tested at Dickinson College.



The fact that both the Coulomb and the gravitational forces lead to objects falling and to objects orbiting around each other suggests that these forces might have the same mathematical form.

In this unit we will explore the mathematical symmetry between electrical and gravitational forces for two reasons. First, it is beautiful to behold the unity that nature offers us as we use the same type of mathematics to predict the motion of planets and galaxies, the falling of objects, the flow of electrons in circuits, and the nature of the hydrogen atom and of other chemical elements. Second, what you have already learned about the influence of the gravitational force on a mass can be applied to aid your understanding of the forces on charged particles.

#### Activity 1: Comparison of Electrical and Gravitational Forces

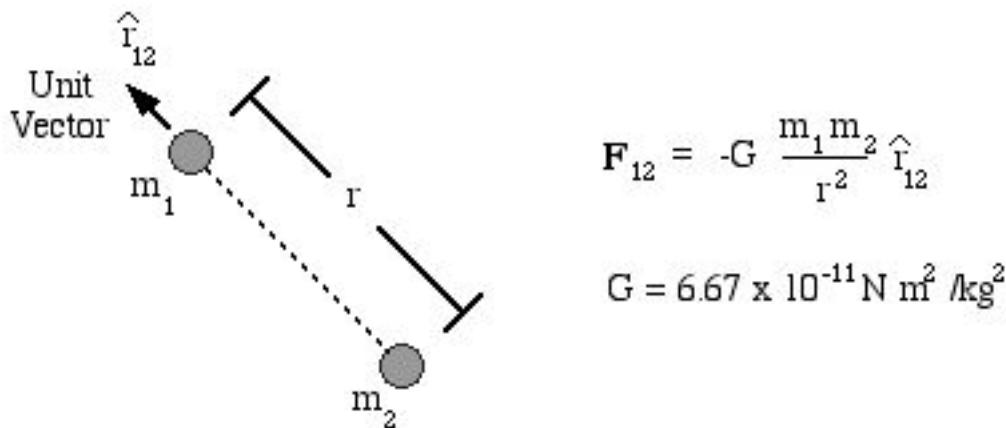
Let's start our discussion of this comparison with the familiar expression of the Coulomb force exerted on charge 1 by charge 2.

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

$$k_e = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$$

Charles Coulomb did his experimental investigations of this force in the 18th century by exploring the forces between two small charged spheres. Much later, in the 20th century, Coulomb's law enabled scientists to design cyclotrons and other types of accelerators for moving charged particles in circular orbits at high speeds.

Newton's discovery of the universal law of gravitation came the other way around. He thought about orbits first. This was back in the 17th century, long before Coulomb began his studies. A statement of Newton's universal law of gravitation describing the force experienced by mass 1 due to the presence of mass 2 is shown below in modern mathematical notation:



About the time that Coulomb did his experiments with electrical charges in the 18th century, one of his contemporaries, Henry Cavendish, did a direct experiment to determine the nature of the gravitational force between two spherical masses in a laboratory. This confirmed Newton's gravitational force law and allowed him to determine the gravitational constant, G. A fact emerges that is quite amazing. Both types of forces, electrical and gravitational, are very similar. Essentially the same mathematics can be used to describe orbital and linear motions due to either electrical or gravitational interactions of the tiniest fundamental particles or the largest galaxies. This statement needs to be qualified a bit when electrons, protons and other fundamental particles are considered. A new field called quantum mechanics was developed in the early part of this century to take into account the wave nature of matter, which we don't actually study in introductory physics. However, even in wave mechanical calculations electrical forces like those shown above are used.

#### Activity 2: The Electrical vs. the Gravitational Force

Examine the mathematical expression for the two force laws.

(a) What is the same about the two force laws?

(b) What is different? For example, is the force between two like masses attractive or repulsive? How about two like charges? What part of each equation determines whether the like charges or masses are attractive or repulsive?

(c) Do you think negative mass could exist? If there is negative mass, would two negative masses attract or repel?

#### Which Force is Stronger– Electrical or Gravitational?

Gravitational forces hold the planets in our solar system in orbit and account for the motions of matter in galaxies. Electrical forces serve to hold atoms and molecules together. If we consider two of the most common

fundamental particles, the electron and the proton, how do their electrical and gravitational forces compare with each other?

Let's peek into the hydrogen atom and compare the gravitational force on the electron due to interaction of its mass with that of the proton to the electrical force between the two particles as a result of their charge. In order to do the calculation you'll need to use some well known constants.

Electron:  $m_e = 9.1 \times 10^{-31} \text{ kg}$ ,  $q_e = -1.6 \times 10^{-19} \text{ C}$

Proton:  $m_p = 1.7 \times 10^{-27} \text{ kg}$ ,  $q_p = +1.6 \times 10^{-19} \text{ C}$

Distance between the electron and proton:  $r = 0.53 \times 10^{-10} \text{ m}$

### **Activity 3: The Electrical vs. the Gravitational Force in the Hydrogen Atom**

(a) Calculate the magnitude of the electrical force on the electron. Is it attractive or repulsive?

(b) Calculate the magnitude of the gravitational force on the electron. Is it attractive or repulsive?

(c) Which is larger? By what factor (i.e. what is the ratio)?

(d) Which force are you more aware of on a daily basis? If your answer does not agree with the result of part (c), explain why, i.e. why do we usually not experience electrical forces in our everyday lives?

### **Activity 4: The Gravitational Force of the Earth**

(a) Use Newton's law to show that the magnitude of the acceleration due to gravity on an object of mass  $m$  at a height  $h$  above the surface of the earth is given by the following expression:

$$\frac{GM_e}{(R_e + h)^2}$$

Hint: Because of the spherical symmetry of the Earth you can treat the mass of the Earth as if it were all concentrated at a point at the Earth's center.

(b) Calculate the acceleration due to gravity of a mass  $m$  at the surface of the earth ( $h=0$ ). The radius of the earth is  $R_e \approx 6.38 \times 10^3$  km and its mass  $M_e \approx 5.98 \times 10^{24}$  kg. Does the result look familiar? How is this acceleration related to the gravitational acceleration  $g$ ?

(c) Use the equation you derived in part (a) to calculate the acceleration due to gravity at the ceiling of the room you are now in. How does it differ from the value at the floor? Can you measure the difference in the lab using the devices available?

(d) Suppose you travel halfway to the moon. What is the new value of the acceleration due to gravity (neglecting the effect of the moon's pull)? (Recall that the earth-moon distance is about 384,000 km.)

(e) Is the gravitational acceleration "constant",  $g$ , really a constant? Explain.

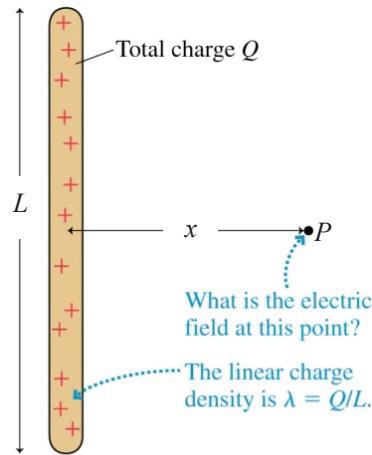
(f) In part (d) you showed that there is a significant gravitational attraction halfway between the earth and the moon. Why, then, do astronauts experience "weightlessness" when they are orbiting a mere 120 km above the earth?

## 17 The Electric Field Near a Charged Rod

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_


**Objective and statement of the problem:**

To calculate the electric field  $\vec{E}$  at a point  $P$  which is a distance  $x$  away from the midpoint of a thin rod of length  $L$  and total charge  $Q$ .

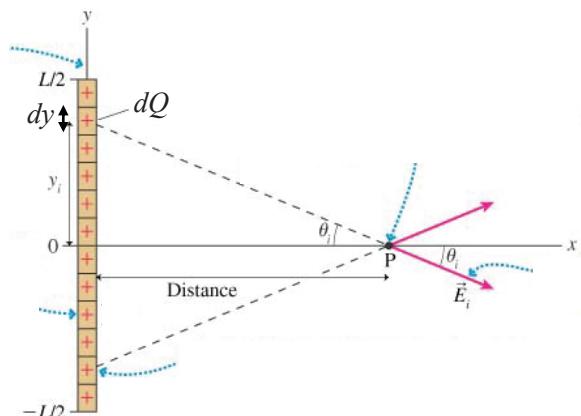
**Introduction and outline of solution:**

Think of the rod as a bunch of teeny-tiny little point charges  $dQ$  arranged in a row. You will solve this problem by calculating the sum (actually an integral) of the electric fields due to each point charge. In the second figure, below, we have added a set of coordinate axes, and defined a tiny bit of charge  $dQ$  that is on a tiny bit of length  $dy$ .

**Step 1:**

In words, what is the direction of the electric field at point  $P$  due to just one little bit of charge  $dQ$  shown?

What is the direction of the electric field at point  $P$  due to the whole rod?


**Step 2:**

What is the magnitude of the electric field  $|d\vec{E}|$  due to just the bit of charge  $dQ$ ? (Write the relevant distance in terms of  $x$  and  $y$ .)

$$|d\vec{E}| =$$

**Step 3:**

What is the magnitude of the  $x$ -component of  $d\vec{E}$ ? (It's similar to your answer above but with an extra  $\sin \theta$  or  $\cos \theta$ .)

$$d\vec{E}_x =$$

**Step 4:**

Rewrite the last step, giving  $\sin \theta$  or  $\cos \theta$  in terms of a ratio involving  $x$ ,  $y$ , and/or  $\sqrt{x^2 + y^2}$ .

$$d\vec{E}_x =$$

**Step 5:**

The linear charge density  $\lambda$  of the rod is given by  $\lambda = Q/L$ . Write  $dQ$  in terms of  $\lambda$  and  $dy$ , then in terms of  $Q$ ,  $L$  and  $dy$ .

$$dQ = \quad =$$

**Step 6:**

Combine your answers to steps 4 and 5 to write  $\vec{E}$  as a single integral over  $dy$  which you can solve. What are the limits of the integral?

$$\vec{E} = \int d\vec{E}_x =$$

**Step 7:**

Evaluate the integral.

$$\vec{E} =$$

*Note: one of the following might be helpful:*

$$\int \frac{1}{(y^2 + a^2)^{\frac{3}{2}}} dy = \frac{y}{a^2 \sqrt{y^2 + a^2}}$$

$$\int \frac{y}{(y^2 + a^2)^{\frac{3}{2}}} dy = \frac{-1}{\sqrt{y^2 + a^2}}$$

$$\int \frac{1}{y^2 + a^2} dy = \frac{1}{a} \tan^{-1} \frac{y}{a}$$

## 18 The Electric Potential<sup>5</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

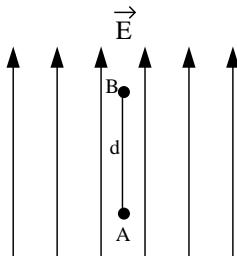
### Overview

It takes work to lift an object in the earth's gravitational field. Lowering the object releases the energy that was stored as potential energy when it was lifted. Last semester, we applied the term *conservative* to the gravitational force because it "releases" all of the stored energy. We found experimentally that the work required to move a mass in the gravitational field was path independent. This is an important property of any conservative force. Given the mathematical similarity between the Coulomb force and the gravitational force, it should come as no surprise that experiments confirm that an electric field is also conservative. This means that the work needed to move a charge from point A to point B is independent of the path taken between points. A charge could be moved directly between the two points or looped around and the work expended to take either path would be the same. Work done by an electric field on a test charge  $q$  traveling between points A and B is given by

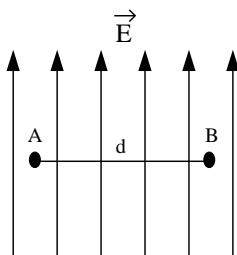
$$W = \int_A^B \vec{F} \cdot d\vec{s} = \int_A^B q \vec{E} \cdot d\vec{s}$$

### Activity 1: Work Done on a Charge Traveling in a Uniform Electric Field

- (a) A charge  $q$  travels a distance  $d$  from point A to point B; the path is parallel to a uniform electric field of magnitude  $E$ . What is the work done by the field on the charge? How does the form of this equation compare to the work done on a mass  $m$  traveling a distance  $d$  in the almost uniform gravitational field near the surface of the earth?



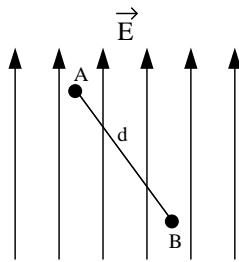
- (b) The charge  $q$  travels a distance  $d$  from point A to point B in a uniform electric field of magnitude  $E$ , but this time the path is perpendicular to the field lines. What is the work done by the field on the charge?



- (c) The charge  $q$  travels a distance  $d$  from point A to point B in a uniform electric field of magnitude  $E$ . The path lies at a  $45^\circ$  angle to the field lines. What is the work done by the field on the charge?

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### Potential Energy and Potential Difference

Recall that by definition the work done by a conservative force equals the negative of the change in potential energy, so that the change in potential energy of a charge moving from point *A* to point *B* under the influence of an electrical force is given by:

$$\Delta U = U_B - U_A = - \int_A^B q \vec{E} \cdot d\vec{s}$$

By analogy to the definition of the electric field, we are interested in defining the *electric potential difference*  $\Delta V = V_B - V_A$  as the change in electric potential energy  $\Delta U$  per unit charge. Formally, *the potential difference is defined as the work per unit charge that an external agent must perform to move a test charge from A to B without changing its kinetic energy*. The potential difference has units of joules per coulomb. Since 1 J/C is defined as one *volt*, the potential difference is often referred to as *voltage*.

### Activity 2: The Equation for Potential Difference

Write the equation for potential difference as a function of  $\vec{E}$ ,  $d\vec{s}$ , *A*, and *B*.

### The Potential Difference for a Point Charge

The simplest charge configuration that can be used to consider how voltage changes between two points in space is a single point charge. We will start by considering a single point charge and then move on to more complicated configurations of charge.

A point charge  $q$  produces an electric field that points radially outward in all directions for a positive charge, radially inward for a negative charge. The line integral equation for the potential difference can be evaluated to find the potential difference between any two points in space *A* and *B* (a line integral is one that follows a path through space).

It is common to choose the reference point for the determination of voltage to be set at infinity so that we are determining the work per unit charge that is required to bring a test charge from infinity to a certain point in space. Let's choose a coordinate system so that the point charge is conveniently located at the origin. In this case we will be interested in the potential difference between infinity and some point which is a distance  $r$  from the point charge. Thus, we can write the equation for the potential difference, or voltage, as

$$\Delta V = V_B - V_A = V_r - V_\infty = - \int_\infty^r \vec{E} \cdot d\vec{s}$$

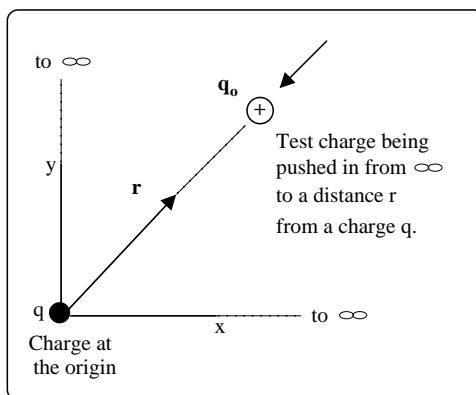
Often, when the reference point for the potential difference is at infinity, this difference is simply referred to as "the potential", and the symbol  $\Delta V$  is just replaced with the symbol  $V$ .

### Activity 3: Potential at a Distance $r$ from a Charge

Starting from the expression for the electric field of a point charge, show that, if *A* is at infinity and *B* is a distance  $r$  from a point charge  $q$ , then the potential  $V$  is given by the expression

$$V = \frac{kq}{r}$$

where  $k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$ .

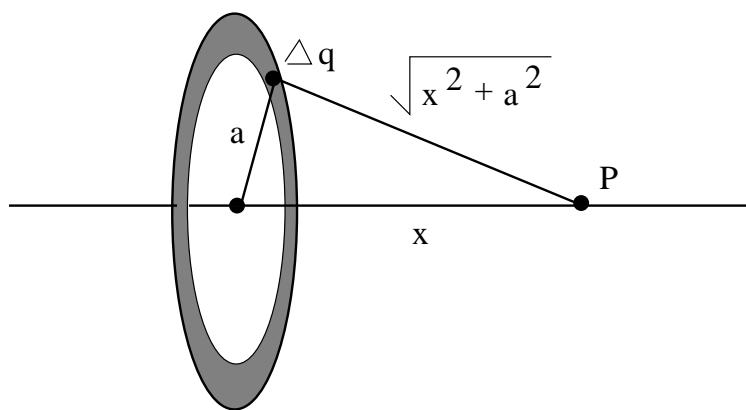


**Hint:** What is the mathematical expression for an  $E$ -field from a point charge?

#### Activities 4 and 5: Potential Due to Continuous Charge Distributions

The potential from a continuous charge distribution can be calculated several ways. Each method should yield approximately the same result. First, we can use an integral method in which the potential  $dV$  from each element of charge  $dq$  is integrated mathematically to give a total potential at the location of interest. Second, we can approximate the value of the potential  $V$  by summing up several finite elements of charge  $\Delta q$  by using a computer spreadsheet or hand calculations.

Again, let's consider a relatively simple charge distribution. In this case we will look at a ring with charge uniformly distributed on it. We will calculate the potential on the axis passing through the center of the ring as shown in the diagram below. (Later on you could find the potential difference from a disk or a sheet of charge by considering a collection of nested rings).



A ring of charge has a total charge of  $Q = 20\mu\text{C}$  (i.e.  $20 \times 10^{-6} \text{ C}$ ). The radius of the ring,  $a$ , is 30 cm. We want to find the electric field and the electric potential at a distance  $x$  from the ring along an axis that is perpendicular to the ring and passes through its center. Let's begin by calculating the potential in the next activity.

**Hints:** Since the potential is a scalar and not a vector we can calculate the potential at point  $P$  (relative to  $\infty$ ) for each of the charge elements  $\Delta q$  and add them to each other. This looks like a big deal but it is actually a trivial problem because all the charge elements are the same distance from point  $P$ .

**Activity 4: Numerical Estimate of the Potential due to a Charged Ring**

(a) Divide the ring into 20 elements, each of charge  $\Delta q = 1.0 \times 10^{-6} \text{ C}$  and calculate the total  $V$  at a distance of  $x = 20 \text{ cm}$  from the center of the ring using a spreadsheet program or by hand calculation. Summarize the result below. Be sure to attach a printout of your spreadsheet results.

**Activity 5: Calculation of the Potential due to a Charged Ring**

By following the steps below, you can use an integral to find a more exact value of the potential.

(a) Show that

$$V = k \int \frac{dq}{r} = k \int \frac{dq}{\sqrt{x^2+a^2}}$$

(b) Explain why

$$k \int \frac{dq}{\sqrt{x^2+a^2}} = \frac{k}{\sqrt{x^2+a^2}} \int dq$$

(i.e. explain why  $\sqrt{x^2 + a^2}$  can be pulled out of the integral).

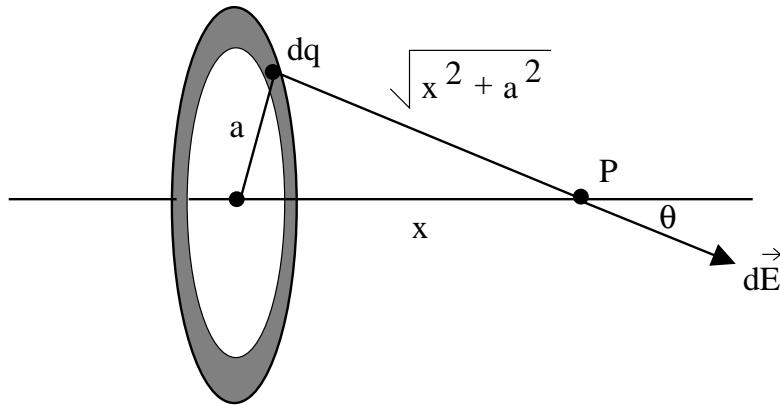
(c) Perform the integration in part (b) above. Then substitute values for  $a$ ,  $x$ , and  $Q$  into the resulting expression in order to obtain a more “exact” value for the potential.

(d) How does the “numerical” value that you obtained in Activity 4 compare with the “exact” value you obtained in (c)?

Now let's take a completely different approach to the same problem. If we can find the vector equation for the electric field at point P due to the ring of charge, then we can use the expression

$$\Delta V = V_r - V_\infty = - \int_{\infty}^r \vec{E} \cdot d\vec{s}$$

as an alternative way to find a general equation for the potential at point P.



#### Activity 6: $\Delta V$ from a Ring Using the E-field Method

- (a) Starting from the electric field of a point charge, show that the electric field at point P from the charged ring is given by

$$\vec{E} = \frac{kqx}{(x^2+a^2)^{3/2}} \hat{x}$$

**Hints:** (1) Why is there no y component of the E-field? (2)  $\cos \theta = \frac{x}{\sqrt{x^2+a^2}}$

- (b) To find  $\Delta V$  use the following equation.

$$\Delta V = V_r - V_\infty = - \int_{\infty}^r \vec{E} \cdot d\vec{s}$$

**Hint:** You will probably need to consult the integral tables in the appendix of your text. **Note:** The above expression for  $\vec{E}$  is only valid along the  $x$ -axis.

- (c) How does the result compare to that obtained in Activity 5 (c)?

### Equipotential Surfaces

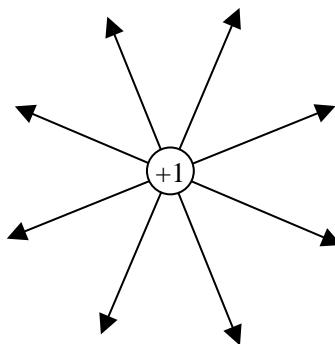
Sometimes it is possible to move along a surface without doing any work. Thus, it is possible to remain at the same potential energy anywhere along such a surface. If an electric charge can travel along a surface without doing any work, the surface is called an *equipotential surface*.

Consider the three different charge configurations shown below. Where are the equipotential surfaces? What shapes do they have?

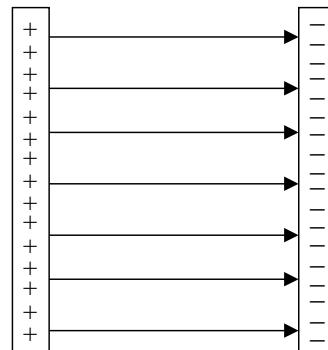
**Hint:** If you have any computer simulations available to you for drawing equipotential lines associated with electrical charges, you may want to check your guesses against the patterns drawn in one or more of the simulations.

#### Activity 7: Sketches of Electric Field Lines and Equipotentials

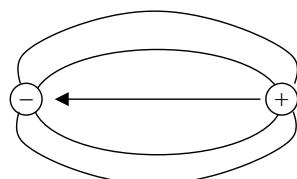
- (a) Suppose that you are a test charge and you start moving at some distance from the charge below (such as 4 cm). What path could you move along without doing any work, i.e.  $\vec{E} \cdot d\vec{s}$  is always zero? What is the shape of the equipotential surface? Remember that in general you can move in *three* dimensions.



- (b) Find some equipotential surfaces for the charge configuration shown below, which consists of two charged metal plates placed parallel to each other. What is the shape of the equipotential surfaces?



- (c) Find some equipotential surfaces for the electric dipole charge configuration shown below.



- (d) In general, what is the relationship between the direction of the equipotential lines you have drawn (representing that part of the equipotential surface that lies in the plane of the paper) and the direction of the electric field lines?

## 19 Electric Fields and Equipotential Lines

Name \_\_\_\_\_

Section \_\_\_\_\_

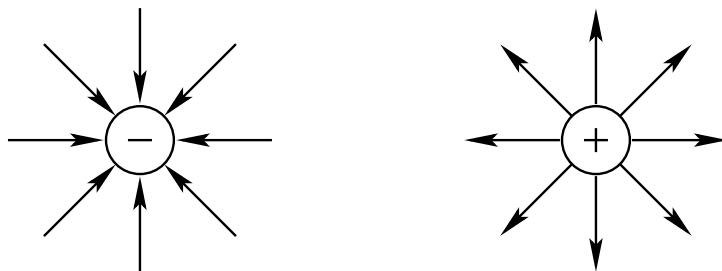
Date \_\_\_\_\_

### Objective

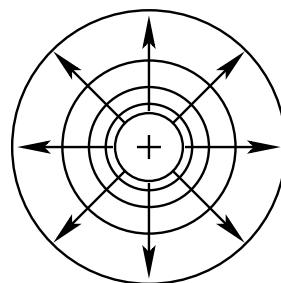
- To learn the shape of electric fields.

### Introduction

Charged objects exert an electrical force on other charged objects in proportion to the amount of charge each has, just as massive objects exert a gravitational force on one another in proportion to their masses. The magnitudes of both forces depend, too, on the distance between objects. However, whereas the gravitational force is always attractive, electrical forces may be either attractive or repulsive depending on the sign of the charges. It is convenient in understanding the nature of electrical forces to draw pictures of them. We represent the fields, which provide the magnitude and direction of the forces, as lines. We agree on a convention: the direction of the field is that of the force on an infinitesimal positive test charge. Thus, the lines of force originate on and come out of positive charges and are directed toward and terminate on negative charges (see figure below). The magnitude of the field, and therefore the force, is proportional to the density of the field lines.

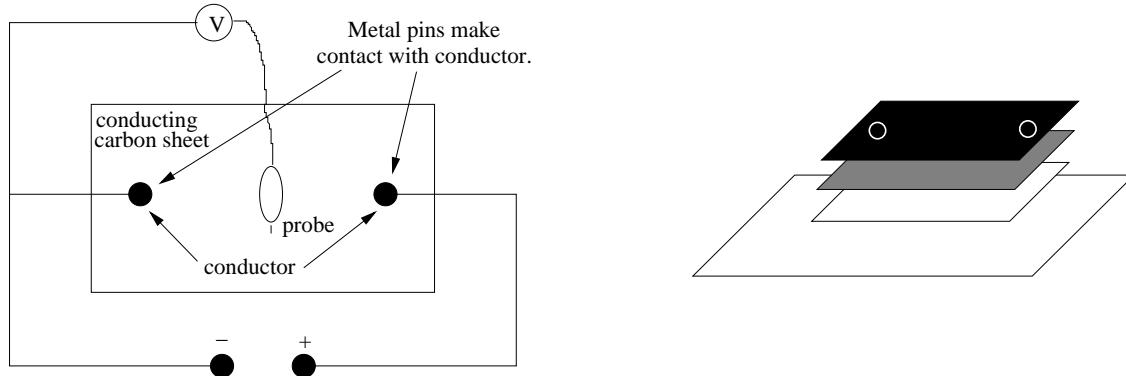


Please note that when the situation is electrostatic, 1) the electric field within a metal is zero, and 2) the electric field just outside the surface of a metal is perpendicular to the surface. If either of these conditions were altered, then there would be an electric current in the metal, which is not an electrostatic situation. Because an electric field exerts a force on a charge, work must be done to move a charged object along any of the field lines. On the other hand, movement perpendicular to the field lines requires no work. Such movement is said to be along an equipotential line.



In the figure above, the electric field for a positive point charge is shown as lines with arrows. The regions of equipotential (equipotential lines) are shown with circles. Notice that the equipotential lines are perpendicular to the electric field lines and that the density of equipotential lines is proportional to the electric field strength.

Electric field lines are difficult to measure directly, but potentials can be measured with a voltmeter. An electric field will arise in the space surrounding two separated charged conductors. With one lead of a voltmeter connected to one of the conductors and the other used as a probe, the potentials can be determined (see figure below).



### Apparatus

- Power supply
- Voltmeter
- Conducting sheets
- Carbon and white paper
- Cork board and pins

### Activity 1: Field Lines for Two Point Charges

**Prediction:** Using the rules given in the introduction and given in the first set of figures, draw the electric field lines for two oppositely charged point objects. Sketch the equipotential lines.

1. Find the conducting paper with the two silver circles on the front and lay it over a copy carbon and a sheet of paper on top of the cork board.
2. Connect the positive output of the power supply to one of the circles and the negative to the other. NOTE: Push pins must be pushed down HARD on the conducting ink of the circles.
3. Connect the negative lead of the voltmeter to the negative conductor and use the positive lead as the probe.
4. With the power supply voltage turned on and set to 20 volts, probe lightly with the voltmeter to find a number of points on the carbon paper registering 16 volts. Hold the probe at an angle so that more area of the point will be in contact with the conducting paper. This will minimize uncertainty in the reading. Push down each time you find a point so that marks will be made on the bottom paper.
5. Repeat for 12 volts, 8 volts, and 4 volts.

6. You should end up with a series of dots on your sheet of paper. Connect those associated with the same potential with smooth lines. Label each line with its associated voltage.
7. Recalling the relationship between electric field lines and equipotential lines, sketch in the electric field lines. (*Other group members can sketch copies of the same results.*)
8. Does your experimental result agree with your prediction? Explain.

**Activity 2: Field Lines for Parallel Plates**

**Prediction:** Draw what you think the field lines and equipotential lines between parallel plates will look like.

1. Carry out the instructions from Activity 1 to check your prediction.
2. Does your result agree with your prediction? Explain.

**Activity 3: Field Lines Between a Point Charge and a Plate**

**Prediction:** Draw what you think the field lines and equipotential lines between a point charge and a parallel plate will look like.

1. Map the field lines as before.
2. Does your result agree with your prediction? Explain.
3. If the potential is zero, must the electric field be zero as well?

## 20 The Electric Field and the Electric Potential I

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To investigate the electric field and potential of a point charge.

### Apparatus

- Electric field and potential simulation entitled *EMField*.

### Introduction

The direction of an electric field is the direction of the force on a tiny positive test charge placed in the region of space where the field is to be measured. If the magnitude of this test charge is infinitesimally small, so small that it will not displace or disturb the charges that are the source of the field, we can use the test charge to determine quantitatively the strength of the electric field. The strength of the electric field is taken to be the electric force,  $F$ , on the test charge divided by the magnitude of the test charge,  $q_t$ :  $E = \frac{F}{q_t}$ . The force (Coulomb's Law) between two charges,  $q_1$  and  $q_2$ , is  $F = k \frac{q_1 q_2}{r^2}$ , where  $k = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$ . The units of  $E$  are newtons per coulomb, so another way of describing the field strength is to say it is the force experienced by a unit positive test charge.

Recall from a previous laboratory exercise that the potential difference between two points A and B,  $V_B - V_A$ , is the work done carrying a unit positive charge from point A to point B. Also, the lines of force (the electric field lines) are always perpendicular to the equipotential lines, lines on which all points are at the same potential. In a static electric field, the electric potential difference between two points is a constant and does not depend on the path used for its computation. The absolute potential, as opposed to the potential difference, is the amount of work done in carrying a unit charge from infinity to point B. The magnitude of the absolute potential, then, is computed as the integral from infinity to the point B of the electric field.

### Investigation 1: A Single Charge

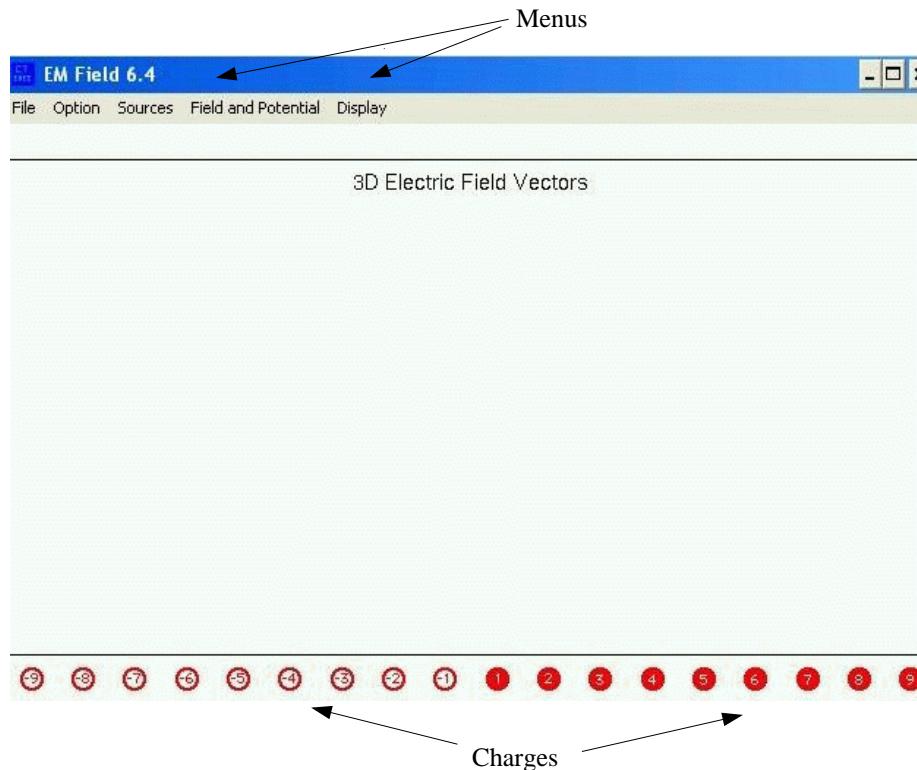
#### Activity 1: The Electric Field

(a) Start the program *EMField* from the *Physics Applications* menu. Click on the screen and you will see a screen with a set of instructions. Go to the **Sources** menu and select **3D Point Charges**. A blank ‘table top’ with a set of menu buttons at the top and bottom will appear. See Figure 1 below.

(b) Go to the **Display** menu and set *EMField* to **Show Grid** and **Constrain to Grid**. These choices will make the following investigation a bit easier to perform.

(c) Select the charge labeled “+4” from the available set by clicking on it and dragging it to the center of the table top.

(d) **Prediction:** You will take measurements of the field at different distances from the charge. You know the relative size of the charge (+4), but you don’t know the size of the charge in coulombs. Generate an expression for the magnitude of the field from an unknown charge with appropriate numerical constants and units. The only unknown in your result should be the charge in coulombs. How does the electric field depend on  $r$ , the distance from the point charge?

Figure 2: Table top for *EMField*.

- (e) Click anywhere in the table top and you will see an arrow drawn. The size and direction of the arrow represent the magnitude and direction of the electric field at that point due to the '+4' charge. In what direction does the arrow point? Click on the opposite side of the table top. In what direction does this arrow point? How is it related to the first arrow?
- (f) Click on many points so that you get a wide range of magnitudes from large (barely fits on the table top) to small (barely bigger than a dot).
- (g) Print the table top and use a ruler to measure the distance of each field point from the charge and the lengths of each of the arrows on your plot. Enter these data in the table below. Use the scale at the bottom of the table top to convert the length of each arrow into an electric field magnitude. The units of the scale electric field vector are  $1.0 \text{ N/C}$ .

Distance from Charge (cm)	Arrow Length (cm)	Measured E (N/C)

(h) **Prediction:** From Coulomb's Law, we expect the spatial variation of the field strength to obey a power law:  $|E| = Ar^n$ , where  $A$  and  $n$  are constants. What do you predict the value of  $n$  to be?

(i) Graph field strength as a function of  $r$ . Using the power fitting function, determine the power of the function,  $n$ , and record it here. Attach the plot to this unit.

(j) Does your result agree with your prediction? Explain any discrepancy.

### Activity 2: The Electric Potential

(a) Under the **Display** menu click on **Clean up Screen** to erase the electric field vectors.

(b) **Prediction:** You will now take measurements of the potential. How do you expect the electric potential to change with distance from the point charge?

(c) Click on the **Potential** option under the **Field and Potential** menu. Click on the table top and a marker will be placed at that point and labeled with the value of the potential there. Click on many spots on the table top from very close to the point charge to far away. When you are finished print the table top.

(d) Measure and record in the following table the values of the distance from the point charge and the potential.

Distance (cm)	Measured V (volts)

(e) **Prediction:** From Coulomb's Law and the definition of the electric potential, we expect the spatial variation of the potential to obey a power law:  $\Delta V = Br^m$ , where  $B$  and  $m$  are constants. What do you predict the value of  $m$  to be?

(f) Graph the voltage as a function of  $r$ . Using the power fitting function, determine the power of the function,  $m$ , and record it here.

(g) Does your result agree with your prediction? Explain any discrepancy.

### Activity 3: Field Lines and Equipotentials

(a) Under the **Display** menu click on **Clean up Screen** to erase the potential values.

(b) Click on **Field Lines** under the **Field and Potential** menu. Click on many spots on the table top all around the charge to create field lines. Why are they straight lines?

(c) Click on **Directional Arrows** under the **Field and Potential** menu. Click on many spots to create directional arrows for the field lines. (Remember, electric field is a vector quantity.) Why are they directed away from the charge?

(d) Click on **Equipotentials** under the **Field and Potential** menu. Click on many spots to create equipotential lines. Why are they all circular?

(e) What is the relationship between the field lines and the equipotentials at the points where they cross?

## 21 The Electric Field and the Electric Potential II

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To investigate the electric field and potential of a charge distribution.

### Apparatus

- Electric field and potential simulation entitled *EMField*.

### Introduction

In the previous unit (which we will refer to as Investigation 1) we studied the dependence of the electric field and the electric potential on  $r$ , the distance from a single charge. Now we will study the same ideas for a different charge distribution.

### Investigation 2: Four Symmetrically Arranged Charges

#### Activity 1: The Electric Field

(a) Start the program *EMField* from the *Physics Applications* menu or (if it's already running) use the options under the **Display** menu to clear the table top and delete any charges. Go to the **Sources** menu and select **3D Point Charges**. A blank ‘table top’ with a set of menu buttons at the top and bottom will appear (see Figure 1 in Investigation 1, the previous unit).

(b) Go to the **Display** menu and set *EMField* to **Show Grid** and **Constrain to Grid** if they are not already set. These choices will make the following investigation a bit easier to perform.

(c) Under **Sources**, click on **3D Point Charges**. Select the charge labeled “+4” from the available set by clicking on it. Add four individual charges, arranging them symmetrically within about 1 cm of the central point where the “+4” charge was located in Investigation 1 (the previous unit).

(d) **Prediction:** How will the electric field be oriented within the region of the four charges? How will the field be oriented outside the region of the four charges? How will the field depend on  $r$ , the distance from the center of the four charges, at large  $r$ ?

(e) Click anywhere in the table top and you will see an arrow drawn. The size and direction of the arrow represent the magnitude and direction of the electric field at that point due to the four charges. In what direction does the arrow point? Click on the opposite side of the table top. In what direction does this arrow point? How is it related to the first arrow?

(f) Click on many points so that you get a wide range of magnitudes from large (barely fits on the table top) to small (barely bigger than a dot).

(g) Print the table top and use a ruler to measure the lengths of each of the arrows on your plot, for points **outside** the region of the four charges. Enter this data in the following table. Use the scale at the bottom of the table top to convert the length of each arrow into an electric field magnitude. The units of the scale electric field vector are 1.0 N/C.

Distance from Charge Center (cm)	Arrow Length (cm)	Measured E (N/C)

(h) **Prediction:** From Coulomb's Law, we expect the spatial variation of the field strength to obey a power law:  $|E| = Ar^n$ , where  $A$  and  $n$  are constants. What do you predict the value of  $n$  to be?

(i) Graph your results. Using the power fitting function, determine the power of the function,  $n$ , and record it here. Attach the plot to this unit.

(j) Does your result agree with your prediction? Explain any discrepancy.

### Activity 2: The Electric Potential

(a) Under the **Display** menu click on **Clean up Screen** to erase the electric field vectors.

(b) **Prediction:** You will now take measurements of the potential. How do you expect the electric potential to change with distance from the center of the four charges?

(c) Click on the **Potential** option under the **Field and Potential** menu. Click on the table top and a marker will be placed at that point and labeled with the value of the potential there. Click on many spots on the table top from very close to the charges to far away. When you are finished print the table top.

(d) Measure and record in the following table the values of the distance from the center of the point charge region (for points outside the region of the four charges) and the potential.

Distance from Charge Center (cm)	Measured V (volts)

(e) **Prediction:** From Coulomb's Law and the definition of the electric potential, we expect the spatial variation of the potential to obey a power law:  $\Delta V = Br^m$ , where  $B$  and  $m$  are constants. What do you predict the value of  $m$  to be?

(f) Graph your results. Using the power fitting function, determine the power of the function,  $m$ , and record it here.

(g) Does your result agree with your prediction? Explain any discrepancy.

(h) How do your results for the power constants,  $n$  and  $m$ , of the four symmetrically-arranged charges compare with the power constants you determined in Investigation 1 (the previous unit) for the single point charge?

(i) What can you conclude about the field and potential effects due to a distribution of charge outside the region of the distribution (in relation to a single point charge)?

## 22 The Electric Field and the Electric Potential III

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To investigate the electric field and potential of an electric dipole.

### Apparatus

- Electric field and potential simulation entitled *EMField*.

### Introduction

In the previous units (which we will refer to as Investigation 1 and 2) we studied the dependence of the electric field and the electric potential on  $r$ , the distance from a charge distribution. Now we will study the same ideas for a charge distribution commonly found in physics and chemistry using the same methods we used before.

### Investigation 3: An Electric Dipole

#### Activity 1: The Electric Field

(a) Start the program *EMField* from the *Physics Applications* menu or use the options under the **Display** menu to clear the table top and delete any charges. Go to the **Sources** menu and select **3D Point Charges**. A blank ‘table top’ with a set of menu buttons at the top and bottom will appear.

(b) Go to the **Display** menu and set *EMField* to **Show Grid** and **Constrain to Grid** if they are not already set. These choices will make the following investigation a bit easier to perform.

(c) Clear the table top and build an electric dipole by placing two magnitude “8” charges of opposite sign a distance 4 cm apart near the lower left of the table top, oriented along a horizontal line. Put the positive charge on the left.

(d) **Prediction** How will the electric field be oriented between the two charges? How will the field be oriented outside the region of the two charges? How will the field depend on  $r$ , the distance from the midpoint of a line joining the two charges, at large  $r$ ?

(e) Click along a line perpendicular to the midpoint of a line joining the two charges. The size and direction of the arrow represent the magnitude and direction of the electric field at that point due to the dipole. In what direction does the arrow point?

(f) Click on many points along the same line so that you get a wide range of magnitudes from large (barely fits on the table top) to small (barely bigger than a dot).

(g) Print the table top and use a ruler to measure the lengths of each of the arrows on your plot and the distance from the midpoint of a line joining the two charges. Enter these data in the following table. Use the scale at the bottom of the table top to convert the length of each arrow into an electric field magnitude. The units of the scale electric field vector are  $1.0 \text{ N/C}$ .

Distance from Charge Center (cm)	Arrow Length (cm)	Measured E (N/C)

(h) **Prediction:** From Coulomb's Law, we expect the spatial variation of the field strength to obey a power law:  $|E| = Ar^n$ , where  $A$  and  $n$  are constants. What do you predict the value of  $n$  to be?

(i) Graph your results. Using the power fitting function, determine the power of the function,  $n$ , and record it here. Attach the plot to this unit.

(j) Does your result agree with your prediction? Explain any discrepancy.

(k) How do your results compare with the power law constants you found in Investigations 1 and 2? Explain.

### Activity 2: The Electric Potential

(a) Under the **Display** menu click on **Clean up Screen** to erase the electric field vectors.

(b) Reverse the two charges, i.e. put the positive charge on the right, keeping the same distance between the charges.

(c) **Prediction:** You will now take measurements of the potential. How do you expect the electric potential to change with distance from the electric dipole along the axis of the dipole (the line joining the two charges defines the axis)?

(d) Click on the **Potential** option under the **Field and Potential** menu. Click on the table top and a marker will be placed at that point and labeled with the value of the potential there. Click on many spots on the table top along the axis of the dipole (outside the dipole itself). When you are finished print the table top.

(e) Measure and record in the following table the values of the distance from the midpoint of a line joining the two charges and the potential.

Distance from Charge Center (cm)	Measured $\Delta V$ (volts)

(f) **Prediction:** From Coulomb's Law and the definition of the electric potential, we expect the spatial variation of the potential to obey a power law:  $\Delta V = Br^m$ , where  $B$  and  $m$  are constants. What do you predict the value of  $m$  to be?

(g) Graph your results. Using the power fitting function, determine the power of the function,  $m$ , and record it here.

(h) Does your result agree with your prediction? Explain any discrepancy.

(i) How do your results compare with the power law constants you found in Investigations 1 and 2? Explain.

### Activity 3: Equipotential Lines and Field Lines

(a) Under the **Display** menu click on **Clean up Screen** to erase the potential values.

(b) Under the **Field and Potential** menu, drag down to **Equipotentials**. Click the mouse on the table top and a line will be drawn representing the equipotential line with a label representing the value of the electric potential. [If the curve does not close (i.e., the last point drawn doesn't match up with the starting point), consult the instructor.] Map out the equipotential lines by moving the cursor across the table top away from each charge and clicking the mouse at regular intervals.

(c) What do these curves represent?

(d) Under the **Field and Potential** menu, click on **Field Lines**. The field lines of the charge distribution will be drawn. [If *EMField* takes a long time to draw one of the field lines, consult your instructor.] Print the result and attach it to this unit.

(e) How are the field lines and the equipotential lines related to one another at the points where they cross?

## 23 The Electric Field and the Electric Potential IV

Name \_\_\_\_\_

## Section \_\_\_\_\_

Date \_\_\_\_\_

## Objective

- To investigate the electric fields of atoms.

## Apparatus

- Electric field and potential simulation entitled *EMField*.

## Investigation 4: An “Atom”

## Activity 1: The Electric Field

(a) Start the program *EMField* from the *Physics Applications* menu or use the options under the **Display** menu to clear the table top and delete any charges. Go to the **Sources** menu and select **3D Point Charges**. A blank ‘table top’ with a set of menu buttons at the top and bottom will appear.

(b) Go to the **Display** menu and set *EMField* to **Show Grid** and **Constrain to Grid** if they are not already set. These choices will make the following investigation a bit easier to perform.

(c) Build an “atom” by symmetrically surrounding a “+36” charge with four (4) “-9” charges. Make the positive “nucleus” by clicking and dragging a “+9” charge to the same spot four times.

(d) **Prediction** How will the electric field be oriented inside the atom? How will the field be oriented outside the atom? How will the field depend on  $r$ , the distance from the nucleus of the atom at large  $r$ ?

(e) Click on the **Field** option under the **Field and Potential** menu. Next, click along a line to the right of the positive charge. What direction does the arrow point?

(f) Click on many points on the table top so that you get a wide range of magnitudes from large (barely fits on the table top) to small (barely bigger than a dot).

(g) Print the table top and use a ruler to measure the lengths of each of the arrows on your plot and the distance from the positive nucleus. Enter this data in the table below. Use the scale at the bottom of the table top to convert the length of each arrow into an electric field magnitude. The units of the scale electric field vector are  $1.0 \text{ N/C}$ .

(h) **Prediction:** From Coulomb's Law, we expect the spatial variation of the field strength to obey a power law:  $|E| = Ar^n$ , where  $A$  and  $n$  are constants. What do you predict the value of  $n$  to be?

(i) Graph your results. Using the power fitting function, determine the power of the function,  $n$ , and record it here. Attach the plot to this unit.

(j) Does your result agree with your prediction? Explain any discrepancy.

(k) How do your results compare with the power law constants you found in Investigation 3 for the electric dipole? Explain.

(l) How do your results compare with the power law constants you found in Investigations 1-2 for the positive charge distributions? Explain.

### Activity 2: The Electric Potential

(a) Under the **Display** menu click on **Clean up Screen** to erase the electric field vectors.

(b) **Prediction:** You will now take measurements of the potential. How do you expect the electric potential to change with distance from the positive nucleus?

(c) Click on the **Potential** option under the **Field and Potential** menu. Click on the table top and a marker will be placed at that point and labeled with the value of the potential there. Click on many spots on the table top along the line to the right of the positive charge. When you are finished print the table top.

(d) Measure and record in the table the values of the distance from the positive charge and the potential.

Distance (cm)	Measured $\Delta V$ (volts)

(e) **Prediction:** From Coulomb's Law and the definition of the electric potential, we expect the spatial variation of the potential to obey a power law:  $\Delta V = Br^m$ , where  $B$  and  $m$  are constants. What do you predict the value of  $m$  to be?

(f) Graph your results. Using the power fitting function, determine the power of the function,  $m$ , and record it here.

(g) Does your result agree with your prediction? Explain any discrepancy.

(h) How do your results compare with the power law constants you found in Investigation 3 for the electric dipole? Explain.

(i) How do your results compare with the power law constants you found in Investigations 1-2 for the positive charge distributions? Explain.

(j) Atoms, and therefore molecules, are composed of equal numbers of positive and negative charges (protons and electrons), each with its own electric field. In many circumstances, we can treat the whole atom as a neutral particle with no electric field (recall, for example, our treatment of ideal gas particles). How do the electric fields of different charges "cancel" out to reduce the magnitude of the electric field?

## 24 The Electric Field of the Atomic Nucleus

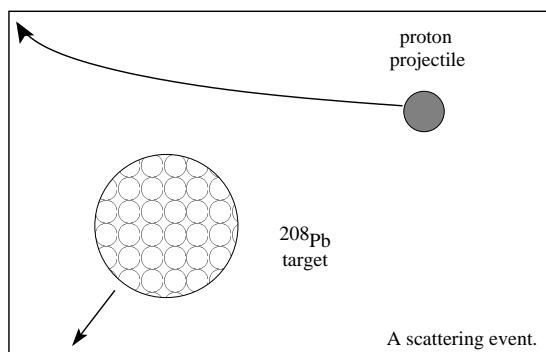
Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Introduction

In this exercise you will make a theoretical investigation of the electric field associated with the atomic nucleus. Our current understanding of the nucleus is the product of scattering experiments where a projectile (e.g. an electron, photon, or even another nucleus) is given an initial kinetic energy and collides with a target nucleus (see figure below).



The final energies and velocities of the products of the reaction are then measured and the nature of the interaction is inferred. There are several important features of the nucleus that can be learned from these experiments:

1. The nucleus is small, about 100,000 times smaller than the atom itself.
2. Most of the matter and all of the positive charge is located in this tiny object.
3. There exists a force, called the strong force, that is attractive, acts only within the volume of the nucleus, and is powerful enough to overcome the repulsion between the positive charges concentrated in the nucleus.

Below you will calculate the electric field of the  $^{208}\text{Pb}$  nucleus and the force between an incident proton and  $^{208}\text{Pb}$ . You will also calculate the work necessary to penetrate the nucleus.

### Activity 1: The Electric Field of $^{208}\text{Pb}$

(a) From scattering experiments like those described above one finds that the positive charge of  $^{208}\text{Pb}$  is uniformly distributed throughout the volume of the nucleus, which can be treated as a sphere of radius  $7.11 \times 10^{-15} \text{ m}$  or in units common to nuclear physics, 7.11 fm, where "fm" is known as a fermi. Calculate the total charge, Q, of the  $^{208}\text{Pb}$  nucleus.

(b) Calculate the volume charge density,  $\rho$ , of the  $^{208}\text{Pb}$  nucleus.

(c) Starting with Gauss' Law, generate an expression for the electric field,  $\mathbf{E}$ , outside of the  $^{208}\text{Pb}$  nucleus as a function of  $r$ , the distance from the center of the sphere. Show and explain each step of the calculation.

(d) Use Gauss' Law to find an expression for  $\mathbf{E}$  inside the nucleus as a function of  $r$ . Show and explain each step of the calculation.

(e) Construct a data table with the column headings  $r$  (fm),  $E$  (N/C), and  $F$  (N) in the space below. Then use the expressions you derived above to calculate the electric field for at least ten radial positions between 0 and 20 fm and enter the results in the table. Include 7.11 fm as one of your radial positions.

#### Activity 2: The force on a Proton

(a) Consider a head-on collision between a proton and the  $^{208}\text{Pb}$  nucleus. Generate an expression for the force on the proton as a function of  $r$  outside the lead nucleus.

- (b) Calculate an expression for the force on the proton as a function of  $r$  inside the lead nucleus.
- (c) Use these expressions to calculate the force at the same radial positions where you calculated  $E$  and enter the results in the table.
- (d) Graph  $F$  ( $y$  axis) vs.  $r$  and attach a copy to this unit.

**Activity 3: The Work Done by the Field**

(a) Recall that the work done by a force is  $W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{s}$  where  $r_i$  and  $r_f$  are the initial and final positions of the object in motion. Treat the  $^{208}\text{Pb}$  nucleus as a fixed target and calculate the work done by the Coulomb force on the proton if the proton approaches from infinity and reaches the surface of the lead nucleus. Show all work.

- (b) We know that the nuclear force binds particles like neutrons and protons together once they enter the nucleus. From the graph of the force as a function of position, what is the minimum strength of the nuclear force in  $^{208}\text{Pb}$  that will bind the proton in the nucleus? Clearly state your reasoning.
- (c) Consider a proton that has just enough initial kinetic energy to move from infinitely far away and just reach the surface of the lead nucleus. How is the initial kinetic energy related to the work done by the field?
- (d) How would the trajectory of the proton be affected if the kinetic energy is less?
- (e) How would the trajectory of the proton be affected if the kinetic energy is greater?
- (f) What minimum kinetic energy is needed for a proton to probe the nucleus? Explain.

## 25 The Charge Distribution of the H<sub>2</sub>O Molecule

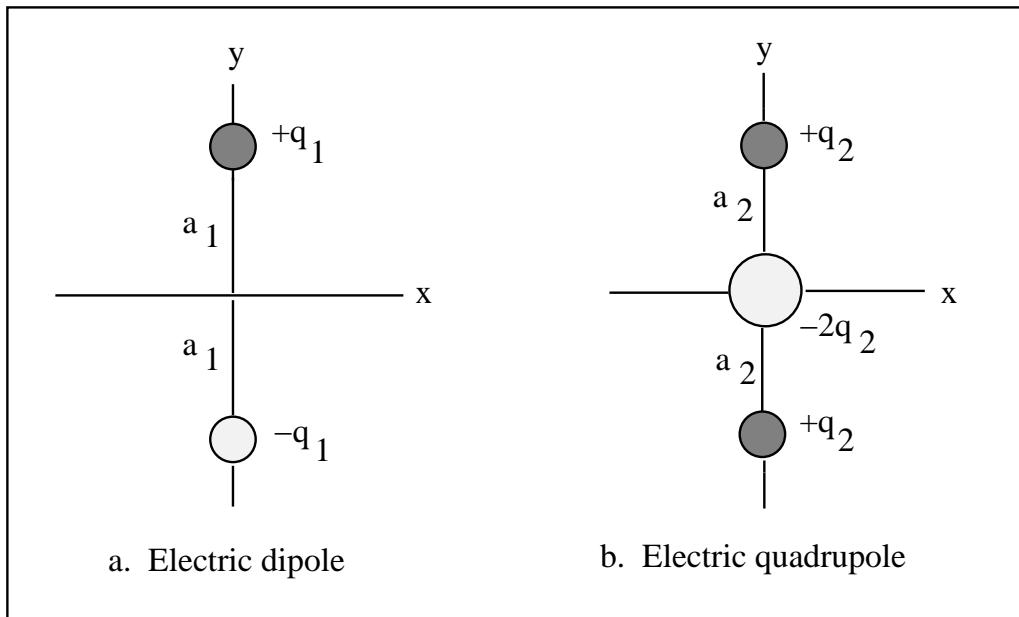
Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Introduction

In this exercise you will make a theoretical investigation of the charge distribution within a water molecule (H<sub>2</sub>O). The molecule is electrically neutral and is made of 10 positive electric charges and 10 negative electric charges (8 protons and electrons from the oxygen atom and 1 proton and electron from each of the two hydrogen atoms). Despite the net electrical neutrality of the molecule it can produce an electric field if the positive and negative charges within it have different configurations. If the most probable position of the positive charges in the molecule does not coincide with the most probable position of the negative charges as shown in Figure 1a, then we call that configuration an electric dipole. Another candidate for the charge distribution of H<sub>2</sub>O is shown in Figure 1b and is called an electric quadrupole. The most probable position of the positive charge is either above or below the x-axis while the negative charge is most likely to be found at the origin. This 'probabilistic' view of the location of the charges is the basis of quantum mechanics, the theory describing the subatomic world. Below you will calculate the electric potential for these two charge distributions and you will use your results to find the electric field for the dipole and quadrupole. You will then consider the design of an experiment to distinguish between the two different charge distributions.



### Activity 1: The Electric Dipole

- (a) Calculate the electric potential,  $V$ , for the electric dipole for a point along the y-axis, but at a position  $|y| > a_1$ . Your final expression should depend on the position,  $y$ , the charge,  $q_1$ , and the most probable location of the charges,  $a_1$ .

- (b) Generate an expression for the potential energy,  $U$ , of the charge distribution when another charge,  $q_{test}$ , placed somewhere on the y-axis with  $|y| > a_1$ .
- (c) Recall the component of the force on a particle is related to the potential energy of the particle by  $F_y = -\frac{dU}{dy}$ . Calculate the force on the test charge in terms of the position on the y-axis, the charges, and  $a_1$ .
- (d) Make the approximation that the test charge is far from the molecule, i.e.,  $|y| \gg a_1$ , to generate a new expression for the force.

**Activity 2: The Electric Quadrupole**

- (a) Calculate the electric potential,  $V$ , for the electric quadrupole for a point along the y-axis, but at a position  $|y| > a_2$ . Your final expression should depend on the position,  $y$ , the charge,  $q_2$ , and the most probable location of the charges,  $a_2$ .
- (b) Generate an expression for the potential energy,  $U$ , of the charge distribution when another charge,  $q_{test}$ , is placed somewhere on the y-axis with  $|y| > a_2$ .

- (c) Calculate the force on the test charge in terms of the position on the y-axis, the charges, and  $a_2$ .
- (d) Make the approximation that the test charge is far from the molecule, i.e.,  $|y| \gg a_2$ , to generate a new expression for the force.

**Activity 3: Distinguishing Between the Two Different Charge Distributions**

(a) Construct a table in the space below with column headings  $y$ (angstroms), Dipole Force (arbitrary units), and Quadrupole Force (arbitrary units). The units of distance known as angstroms are commonly used in atomic physics. One angstrom is equal to  $10^{-10}$  m.

(b) To calculate the force on the test charge one must know the charges and their positions for each charge distribution ( $q_1, q_2, a_1, a_2$ ). These quantities are unknown to us at this point. However, we want to compare the behavior of the force due to each as a possible probe of the molecule's charge distribution. In the expressions you generated in step (d) of Activities 1 and 2 for  $|y| \gg a_1$  or  $|y| \gg a_2$  the forces depend on some power of  $y$ . For convenience, set the constant in front of this power of  $y$  to unity. Calculate the forces for the dipole and quadrupole at distances between 1.5 angstroms and 5.0 angstroms in 0.5 angstrom steps. Enter your results in the table.

(c) Make a graph of the force due to the dipole and due to the quadrupole as a function of  $y$ . Insert a copy of the graph into your notebook.

(d) In many experiments constants like those in your expressions for the force on the test charge are unknown or poorly known. Suppose, however, that you are able to accurately measure the radial dependence of the force on  $q_{test}$  at distances far from the H<sub>2</sub>O molecule. If that is the case use the results from steps (b) and (c) to propose a technique to distinguish between the two candidates for the charge distribution of H<sub>2</sub>O. Support your proposal with a mathematical argument. (Hint: Consider a ratio involving the measured force.)

(e) Experiment has found that water behaves as an electric dipole with the electric dipole moment,  $q_1a_1$ , measured to be  $6.2 \times 10^{-30}$  C-m. If the charge that creates the dipole,  $q_1$ , is the charge associated with the protons and electrons of the molecule calculate the separation of the charges,  $a_1$ . How does it compare with the size of the H<sub>2</sub>O molecule of about 1.0 angstroms?

## 26 Capacitance Series Circuit

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To investigate the relationship among charge, potential difference, and capacitance in a series combination of capacitors.

### Introduction

In class the series combination of capacitors was studied with the assumption of a constant potential difference energizing the circuit. In practice this is difficult to reproduce in the lab because capacitors typically do not maintain a steady charge for very long. To get around this we will energize the circuit with an alternating potential difference provided by a sine wave generator. The relationships among  $Q$ ,  $V$ , and  $C$  are still the same, so we can study these circuits in the lab.

For a series combination of capacitors, the total voltage across the circuit is the sum of the voltages across the individual capacitors, that is

$$V = V_1 + V_2 + V_3$$

The equivalent capacitance of the combination is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

### Apparatus

- Sine wave generator
- Capacitors of 1.0, 4.7, and  $10\ \mu F$
- Digital multimeter
- Connecting leads

### Activity

1. Connect the three capacitors in series with the sine wave generator.
2. Set the sine wave generator to 200 Hz (not critical) and adjust the amplitude so that the output measures 5 to 6 volts as measured with the digital multimeter set for AC volts.
3. Using the multimeter, measure  $V$  for the sine wave generator and also for each of the individual capacitors (all with uncertainties) and list them here. Don't forget units.

4. Calculate the total voltage and its uncertainty from the first equation above and compare with your measured value. Do they agree?
  5. Assuming 10 percent uncertainties on each of the capacitances, calculate the equivalent capacitance and its uncertainty from the second equation above. (Note: To calculate the uncertainty we assume that if the value of each capacitor is known to 10 percent, then the value of  $1/C$  for that capacitor is also known to 10 percent.)
  6. Calculate the total charge for the circuit (and its uncertainty) from  $Q = CV$ .
  7. **Prediction:** How will the charge on the individual capacitors compare with the total charge calculated above?
  8. Calculate the charge on each capacitor (and its uncertainty) and compare with the total. Do the results agree with your prediction?

## 27 Ohm's Law

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

- To investigate the most important principle in electronics.
- To determine how resistors in series and parallel add.

### Introduction

The rate at which electric charge flows through a conductor is called the electric current. In order to have a current, a potential difference, or voltage is necessary. We first want to determine the relationship between the potential difference at two ends of a conductor and the current flowing through it.

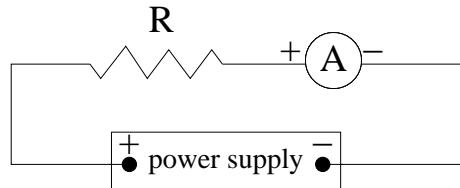
**Note:** Do not turn on a power supply until you are sure your circuit is correct. If you are at all unsure, please ask your instructor to approve your setup. Ammeters can be instantly and permanently ruined by an improper connection. Be sure to turn off the power supply before making any changes to the circuit.

### Apparatus

- Power supply
- Two resistors
- Leads and alligator clips
- DC Milliammeter
- Multimeter

### Activity 1: Ohm's Law

- With the power supply off, turn the coarse current control all the way up and both voltage controls all the way down. Connect one of the resistors in series with the milliammeter (marked "A" for "ammeter") as shown in the figure below. Be sure the polarity of the milliammeter is as shown.



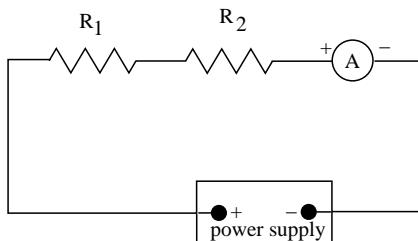
- When sure of your circuit, turn on the power supply, and turn the coarse voltage control up until the milliammeter reads 20 mA. Measure the voltage across the resistor (with the multimeter set for DC voltage) and record both voltage and current here. Include an uncertainty in the voltage measurement.

- Now turn the coarse voltage control up and record current and voltage for four more values of current: 40, 60, 80, and 100 mA. Include uncertainties in the voltage measurements.

- Turn off the power supply.
- Using *Excel*, plot your five pairs of readings with the voltage on the vertical axis and the current on the horizontal axis. Include the origin as a sixth point (why is this valid?).
- Fit a straight line to the points (including the origin). Include a trendline with equation.
- Use the LINEST function in *Excel* (see **Appendix C**) to determine the slope of the line and its uncertainty. What is the meaning of the slope? Write its value with uncertainty here (including units):
- Print the graph and include it with this unit.
- Remove the resistor from the rest of the circuit and use the ohmmeter option on the multimeter to measure the resistance of R directly. Does it fall within the range of values determined above? Can you think of a reason for any discrepancy?
- What is the general relationship between voltage, current, and resistance? This is Ohm's Law.
- Why is the origin a legitimate point on the curve?

### Activity 2: Resistors in Series

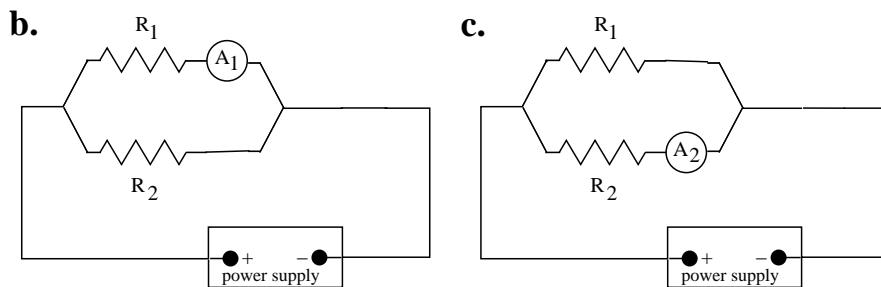
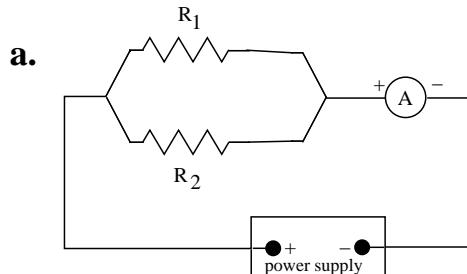
- Turn the voltage control down to zero, and connect two resistors in series as shown in the figure below.



- Turn on the power supply and turn up the coarse voltage control to approximately 10 volts. Measure the voltages across  $R_1$ ,  $R_2$ , and the milliammeter. (The voltage across the milliammeter may be in millivolts.) Also measure the total voltage across all three elements in series. Include uncertainties in each measurement. Record your results here:
  
  
  
  
- How is the last measurement related to the three individual measurements?
  
  
  
  
- What can you conclude about the voltage across resistors in series?
  
  
  
  
- Measure the current in the series circuit and record it here:
  
  
  
  
- Using Ohm's Law, determine the total resistance of the circuit from the total voltage and the current which you have measured.
  
  
  
  
- Turn the voltage control down to zero and turn off the power supply.
- Calculate  $R_1$ ,  $R_2$ , and  $R_A$  from Ohm's Law and your above readings. From these three values, determine the total resistance of the circuit. Does it agree with the value you calculated above? Record your results here:
  
  
  
  
- What can you conclude about the total resistance in a circuit containing resistors in series?

**Activity 3: Resistors in Parallel**

- Connect the two resistors in parallel as shown in figure **a** below, with the milliammeter at the point marked "A". Have circuit checked before continuing.



- When you are sure the circuit is set up correctly, turn on the power supply and turn the coarse voltage control up to about 10 volts. Record the total current through the circuit and the voltage across the parallel resistance combination. Also measure the voltage across the milliammeter (which may be in millivolts). Turn off the power supply.
- Connect the milliammeter to the point marked  $A_1$  in figure **b** above, without disturbing the rest of the circuit. Turn on the power supply and record the current through  $R_1$  and the voltage across the parallel combination. Turn off the power supply.
- Repeat the above measurements for  $R_2$ , connecting the milliammeter at  $A_2$  as in figure **c** above. Turn off the power supply.

- What is the relationship between the total current and the current in each of the branches of the parallel circuit? (This is an example of Kirchhoff's junction rule which is based on conservation of charge.)
- Using Ohm's Law and your data above, calculate the resistance of each resistor and the total resistance of the circuit.
- What is the relationship between the total resistance of the parallel circuit and the resistance of each of the branches? Using this relationship, calculate the total resistance of the circuit from the individual resistances you determined above. Does the result agree with what you calculated in the previous item? Calculate a percent difference and record it here.

## 28 Kirchhoff's Rules

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To study Kirchhoff's rules for analyzing circuits.

### Introduction

Two statements comprise Kirchhoff's rules. The first, the so-called junction rule, restates the conservation of charge; the second, the so-called loop rule, restates the conservation of energy:

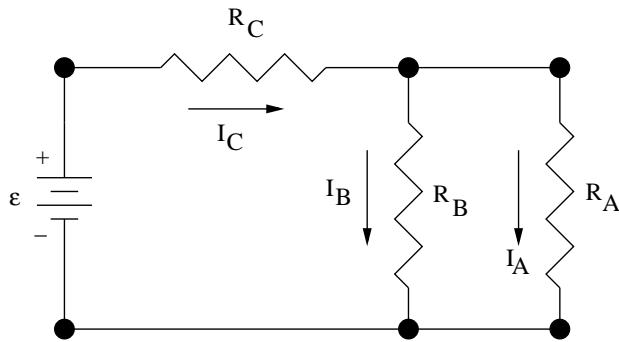
1. The sum of the currents entering any junction (or node) must equal the sum of the currents leaving that junction.
2. The algebraic sum of electrical potential changes across all the elements around any closed loop must be zero.

**Note:** Do not turn on a power supply until you are sure your circuit is correct. Please ask your instructor to approve your setup. Ammeters can be instantly and permanently ruined by an improper connection. Be sure to turn off the power supply before making any changes to the circuit.

### Apparatus

- power supply
- three-resistor board
- analog ammeter
- digital voltmeter

### Activity



1. Referring to the circuit above, choose a junction and write an equation which satisfies Kirchhoff's first rule.

2. Identify two closed loops on the circuit which contain at least one element not included in the other. Write an equation for each loop that satisfies Kirchhoff's second rule.
3. You now have three equations that can be solved for the three currents in terms of the supplied voltage (emf),  $\varepsilon$ , and the three resistors,  $R_A$ ,  $R_B$ , and  $R_C$ . Carry out the algebra to obtain expressions for the three currents in terms of these quantities.
4. Now use Ohm's law and the sum rules for resistors in series and in parallel to derive expressions for the three currents.
5. Are the results of 3 and 4 the same? They should be.
6. Set a digital multimeter to measure resistance. Determine and record values for the three resistors,  $R_A$ ,  $R_B$ , and  $R_C$ , which are mounted on the wooden block.
7. **Prediction:** if the emf,  $\varepsilon$ , were 5.0 V, what do you predict the three currents would be?

8. Configure the circuit as in the figure above, using the power supply as the source of emf. With the power supply off, set the current control at mid-range and the voltage control all the way down. Set the digital multimeter to measure voltage and connect it in parallel with the power supply to measure  $\varepsilon$ . Connect the analog ammeter in series with resistor  $R_A$  to measure the current  $I_A$ .
9. After checking with your instructor that the circuit is correct, turn on the power supply and set  $\varepsilon$  to about 5.0 V. Record  $\varepsilon$  and  $I_A$ , then turn off the power supply.
10. Set up the circuit to measure  $I_B$ . After getting assurance from you instructor that your circuit is correct, turn on the power supply and record  $\varepsilon$  and  $I_B$ . Turn off the power supply.
11. Set up, measure and record  $I_C$ .
12. Do your results agree with your prediction? How large, in terms of percentage, are the discrepancies? Speculate on what might cause any differences.

## 29 RC Circuits

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To investigate the behavior of a series RC circuit, i.e. one containing a resistor and a capacitor in series with a DC power supply of fixed emf  $V_0$ .

### Introduction

For the charging circuit, the charge on the capacitor as a function of time is given by

$$Q(t) = CV_0 \left(1 - e^{-t/RC}\right) \quad (1)$$

while for the discharging circuit (no emf) the charge is

$$Q(t) = Q_0 e^{-t/RC} \quad (2)$$

where  $Q_0$  is the initial charge on the capacitor. Since  $Q = CV$  for the capacitor, the voltage across the capacitor has the same time dependence, namely

$$V(t) = V_0 \left(1 - e^{-t/RC}\right) \quad (3)$$

for the charging circuit and

$$V(t) = V_0 e^{-t/RC} \quad (4)$$

for the discharging circuit.

We will use the discharging circuit in this experiment. For this circuit, the time required for  $Q$  (or  $V$ ) to decrease to  $1/e$  of its initial value (the time constant) is

$$\tau = RC \quad (5)$$

The time required for  $Q$  (or  $V$ ) to decrease to  $1/2$  of its initial value (the half-life) is

$$t_{1/2} = (\ln 2)\tau = .693\tau \quad (6)$$

This will be useful because in practice it is easier to measure the half-life than the time constant.

### Apparatus

- DC power supply (low voltage)
- Electrolytic capacitor (1000-5000  $\mu\text{f}$ )
- Voltmeter (0-3V)
- Stop watch
- Connecting leads
- Voltage sensor

### Activity 1: Proof of Equation (6)

Prove equation (6) above, beginning with equation (2).

**Activity 2: Measurement of Half-life**

The capacitor will be charged briefly, then allowed to discharge through the voltmeter which will act as the resistor in this case. The internal resistance of the voltmeter (0-3V) is about 3000 ohms. Assume this is known to within 10% and the capacitance as marked is known to within 10%.

1. Connect the voltmeter directly across the capacitor. Electrolytic capacitors are polarity sensitive, so be sure the terminal marked with a minus sign is connected to the negative terminal of the voltmeter.
2. Set the power supply to about 2.5 volts and connect its negative terminal to the negative terminal of the capacitor.
3. Connect the positive terminal of the power supply to the corresponding terminal of the capacitor briefly to charge it, then disconnect the positive lead and observe the exponential decay of the voltage. Use a stop watch to measure the half-life (5 trials) as follows: switch the stop watch on when the voltmeter reads 2.0 volts, and switch it off when the meter reads 1.0 volt. Record your five measurements here.

**Activity 3: Determination of Time Constant**

1. Experimental: Find the average of your five half-life measurements and its uncertainty (the standard deviation) and determine the time constant from equation (6) above, and its uncertainty. (Note: Since the time constant and the half-life are directly proportional, the fractional uncertainty in the time constant will be the same as the fractional uncertainty in the half-life.)
2. Theoretical: Calculate the time constant and its uncertainty from equation (5). Do your two results agree within experimental uncertainties?

**Activity 4: Determination of Time Constant Using Pasco Interface and Computer**

We will now look at a graph of the discharge curve using the Pasco interface and computer. Plug the voltage sensor into the input channel *A* of the Pasco interface. Then connect the red and black leads from the voltage sensor to the terminals of the capacitor (the voltmeter should still be connected to the capacitor as before), being sure to connect the red lead to positive and black lead to negative. Connect the negative lead from the power supply to the negative terminal of the capacitor.

To record voltage versus time data, go to *Start* → *Programs* → *Physics Applications* → *DataStudio*. Select the option *CreateExperiment*. You will see an image of the 750 interface box. On this image, single click on the channel *A* input. You will see a list of sensors. Scroll down to the *voltagesensor* and select by clicking *OK*. Finally, open a graph display by double clicking on the *Graph* option (on the left).

Make sure your power supply is still set at around 2.5 volts. Charge the capacitor (by touching the positive lead from the power supply to the positive capacitor terminal) and click the *Start* button to begin taking data. Disconnect the positive power supply lead and you will see the discharge curve on the screen as the capacitor discharges. When you have an acceptable curve, click *Stop*. Print the graph and attach it to this unit.

Use the *Smart* tool to determine the time when the voltage is 2.00 volts and again when the voltage is 1.00 volt. The difference is the half-life. Record it here, with an estimated uncertainty:

Now calculate the time constant and its uncertainty from equation (6) and compare with your value of  $RC$  determined from equation (5). Do they agree within experimental uncertainty (i.e. do the ranges overlap)?

## 30 Magnetism I

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

- To investigate the characteristics of magnets.
- To understand how a compass works.

### Introduction

The electric interaction, you probably know, is not the only one in which opposites attract and likes repel. Magnetic interactions have similar characteristics. All simple magnets, regardless of size, are bipolar: there are two magnetic poles. Consider this question, then: Can we talk about like and unlike as we do for electricity?

### Apparatus

- 2 bar magnets
- 2 cylindrical magnets
- rods and clamps
- wool cloth
- rubber rod
- string

### Activity 1: The Characteristics of Magnets

1. Feel the attraction between two magnets when pulled apart after having come together without effort on your part. Describe qualitatively in terms of strength and separation.
  
  
  
  
  
2. Feel the repulsion when one of them is turned around and pushed toward the other. Describe as in step 1.
  
  
  
  
  
3. Note and describe the difference in (strength and direction of) interactions between the ends and the middle.

### Activity 2: How a Compass Works

1. Identify geographic north and south.
2. Hang one of the cylindrical magnets horizontally from a horizontal rod.

3. When it comes to rest, along which geographical line does the magnet lie?
4. Which end (colored or uncolored) is the "north-seeking" end?
5. Remove the cylindrical magnet and repeat step 2 with the second cylindrical magnet. Answer, again, the questions above.
6. What happens when you bring the "north-seeking" end of the first magnet near the hanging one's north-seeking end?
7. What happens when you bring the first magnet's opposite end near the second's north-seeking end?
8. What about the first magnet's north-seeking end near the opposite end of the hanging one?
9. What happens when you bring the opposite ends near one another?
10. Define in your own words like and unlike poles?
11. What always happens between like poles?
12. What always happens between unlike poles?

13. Determine with a labelled bar magnet which end of your hanging magnet should be identified as the north pole and which the south.
14. Why do we identify one end of a magnet as the north pole and the other as the south?
15. In your own words, explain a compass.
16. In terms of magnetism, what is the earth?
17. Charge a rubber rod with the wool cloth and bring it near the ends of the suspended magnet; describe its effect on the magnet.
18. Does a south magnetic pole repel a negative electric charge?
19. Does a north magnetic pole attract a negative electric charge?

## 31 Magnetism II

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To investigate the magnetic field around a permanent magnet.

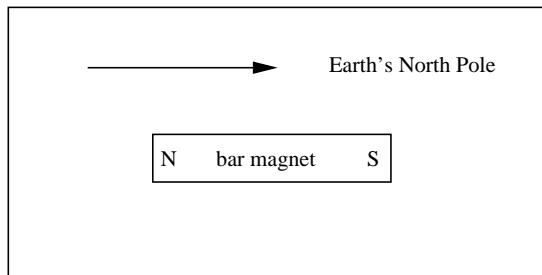
### Introduction

The magnetic field around a permanent magnet is determined by the properties of the magnetized material within the magnet. The field is a function of position relative to the magnet, and will form a pattern around the magnet. The field is a vector field, and its direction at any point is indicated by the orientation of the north pole of a compass located at the point. On earth, the field mapped out around the magnet is actually the resultant of the field due to the magnet and the field due to the earth. The purpose of this experiment is to map the magnetic field due to a permanent magnet or two magnets in close proximity.

### Apparatus

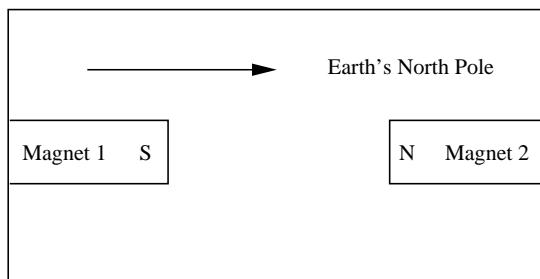
- 2 bar magnets
- small compass
- white paper

### Activity 1: A Single Bar Magnet

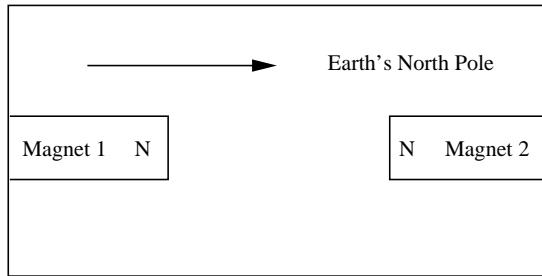


1. Trace the outline of a bar magnet on a piece of paper. Indicate the magnet's polarity on the outline.
2. Place the small compass on the side of the magnet near the north pole and make a dot at each end of the needle using a pencil not encased in metal.
3. Move the compass forward until its south pole points at the previous north pole dot, and make a new dot at the north pole (kind of a leap-frog effect).
4. Repeat 3 until the series of dots reaches the side of the magnet near the south pole or the edge of the paper.
5. In a similar manner, trace enough lines to map the magnetic field over the entire paper.
6. Do lines of force ever cross?

7. Where is the magnetic field strongest? Weakest? How do the field lines indicate this? Does a field line represent a constant field along its entire length?
8. Do the lines intersect the magnet at a particular angle (like the electric field lines near a conductor)?

**Activity 2: Two Bar Magnets—Unlike Poles Facing One Another**

1. Set up two bar magnets on a sheet of paper as shown in the figure above. The magnets should be 8-10 cm apart.
2. Repeat steps 2 through 5 from the previous activity.
3. What sort of charge configuration produces an electric field that looks similar to the magnetic field you just identified?
4. What differences can you recognize?

**Activity 3: Two Bar Magnets—Like Poles Facing One Another**

1. Set up two bar magnets on a sheet of paper as shown in the figure above. The magnets should be 8-10 cm apart.
2. Repeat steps 2 through 5 from Activity 1 of this investigation.
3. Try to identify on your map a point at which the magnetic field is zero. Explain what causes this effect.
4. What sort of electric charge configuration would produce a similar field map?

## 32 Magnetism III

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

To investigate:

- The effect of magnetic fields on moving charges.
- The effect of moving charges (currents) on magnets.

### Apparatus

- Bar magnet
- Oscilloscope
- Tangent galvanometer
- Compass
- Power supply
- Banana plug leads (2) with alligator clips

### Activity 1: Magnetic Forces on Moving Charges

1. An oscilloscope is built around the principle of the cathode ray tube. It emits electrons from its back end. These are accelerated by a series of electrodes and focused to strike a fluorescent screen at its front. The result is a visible spot (or trace) indicating voltage as a function of time.
2. **Predictions:** What, if anything, will happen to the spot on the screen if the north pole of a magnet is brought near the left side of the oscilloscope? What will happen if you do the same with the south pole? What about when each of the poles are brought near to the top? [Please do not touch the oscilloscope with the magnet.]
3. Turn on the oscilloscope by pressing the power button. Turn the TIME/DIV knob completely counter-clockwise. Adjust the INTEN (intensity) and FOCUS knobs so that a small bright spot is formed on the oscilloscope screen by the beam of electrons traveling toward the screen. Do not make the spot very bright. Adjust the ILLUM (illumination) knob so that the grid on the screen can be seen clearly. Use the horizontal and vertical POSITION controls to center the spot on the screen.
4. Bring the N-pole of a horizontal bar magnet near, but not touching, the left side of the oscilloscope case at the height of the spot. Record the direction of any deflection. Repeat with the S-pole.

5. Bring the N-pole of a vertical bar magnet near, but not touching, the top of the oscilloscope case just above the spot. Record the direction of any deflection. Repeat with the S-pole.
  
  
  
  
  
  
6. Turn off the oscilloscope.
7. Did the directions of deflections meet your expectations? Explain.

### Activity 2: The Effect of Moving Charges (Currents) on Magnets

In this investigation we will use a device known as a tangent galvanometer to make a qualitative study of the effect of current (moving charges) in a coil of wire on a compass. A sketch of the galvanometer is shown below.

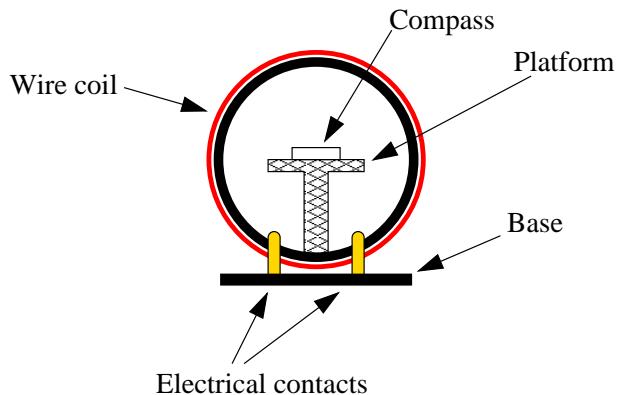


Figure 1. Tangent galvanometer and compass.

1. With the power supply off, connect the positive and negative terminals on the power supply to the two side screws on the tangent galvanometer. Turn the coarse current control on the power supply all the way up and the voltage controls all the way down.
2. Before turning the power supply on, place the compass on the platform in the center of the tangent galvanometer. Rotate the galvanometer until the compass needle is aligned with the plane of the wire coil of the galvanometer. Turn the power supply on and slowly turn up the fine voltage control. What do you observe? Make a sketch to show the orientation of the compass needle and galvanometer coil with the voltage on and off. (It may be easiest to do this with TOP views.)

3. Turn the voltage on the power supply to zero. Rotate the entire setup (galvanometer, compass, wires)  $180^\circ$  so the contacts are on the opposite side from where they were before. Make sure the compass needle is again aligned with the plane of the wire coil. Slowly turn the voltage back up. What do you observe? Make another sketch to show the orientation of the compass needle and galvanometer coil with the voltage on and off.
  4. The deflection of the compass when current flows in the tangent galvanometer implies the current creates a magnetic field. From your observations can you tell the direction of the magnetic field? Explain.
  5. Reverse the wires on the contacts of the tangent galvanometer to reverse the direction of the current in the coil of the galvanometer. We will now repeat the observations from above. With the voltage off, align the compass needle and the plane of the wire coil of the galvanometer as in number 2 above. Slowly turn up the voltage. What do you observe? Make a sketch to show the orientation of the compass and galvanometer coil with the voltage on and off.
  6. Rotate the entire setup (galvanometer, compass, wires)  $180^\circ$  so the contacts are on the opposite side from where they were before. Make sure the compass needle is again aligned with the plane of the wire coil. Slowly turn the voltage back up. What do you observe? Make another sketch.
  7. Based on your observations, what happens to the magnetic field of the tangent galvanometer when you reverse the direction of the current?

## 33 Magnetic Field of the Earth

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

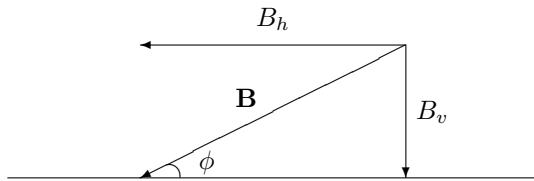
### Objective

- A measurement of the earth's magnetic field.

### Introduction

The magnetic field lines of a bar magnet, though emanating from its full length, are densest near the poles. These lines are typically not perpendicular to the faces of the magnet. The Earth, as you know, is like a giant bar magnet, with its magnetic south pole near its geographic north pole.

The field lines of the Earth's magnetic field, therefore, tend to point obliquely at any given spot on its surface. The angle (let's call it  $\phi$ ) that a line makes with a surface is known as its *dip angle* (see figure). Thus, each locality on earth has its characteristic value for  $\phi$  with regard to the terrestrial magnetic field.



The horizontal component of the Earth's magnetic field,

$$B_h = B \cos \phi, \quad (1)$$

causes the magnetized needle of a compass to align in the geographic north-south direction.

You will use a tangent galvanometer to produce an additional horizontal magnetic field perpendicular to the Earth's field (in other words, one that points east or west). A tangent galvanometer consists of a vertical, circular coil of wire with  $N$  turns, a pedestal compass at the center of the coil, and electrical contacts so that a direct current can be established in the coil. The magnetic field at the center of the galvanometer coil caused by a current  $I$  in the coil is

$$B_c = N \frac{\mu_0 I}{2R} \quad (2)$$

where  $R$  is the radius of the coil and  $\mu_0$  is the permeability of free space ( $1.25664 \times 10^{-6}$  T·m/A).  $B_c$  is perpendicular to the vertical plane of the coil. If the plane of the coil is placed in the north-south direction (that is, parallel to the compass needle when there is no current in the coil), then, when there is current in the coil, the compass will be influenced by two perpendicular fields,  $B_h$  and  $B_c$ . The compass needle will now point in the direction of the vector sum  $\mathbf{B}_h + \mathbf{B}_c$ .

Let  $\theta$  be the angle through which the compass needle turns when the current in the tangent galvanometer is turned on. In other words,  $\theta$  is the angle between  $\mathbf{B}_h$  and  $\mathbf{B}_h + \mathbf{B}_c$ . Sketch a picture of these vectors, and use it to convince yourself that

$$B_h = \frac{B_c}{\tan \theta} \quad (3)$$

By measuring the angle  $\theta$ , we can determine  $B_h$ . If we then measure the dip angle  $\phi$ , we can determine the Earth's magnetic field from equation (1) above.

### Apparatus

- tangent galvanometer
- power supply
- DC ammeter (0-1 amp)
- banana plug leads (1 red, 2 black) with alligator clips
- compass
- ruler
- dip angle compass

### Activity

1. Measure and record on the accompanying data sheet the diameter D of the galvanometer coil. Calculate and record the radius R of the coil.
2. Count and record the number of turns, N, of the coil.
3. With the power supply off, connect its positive and negative terminals to the tangent galvanometer and a DC ammeter (in series). Be sure the ammeter is connected in series with the coil. Use long wires to connect the outside terminals of the coil to the circuit so that the coil may be removed from the magnetic effects of the ammeter and power supply. Be sure no magnetic material other than the compass is in the vicinity of the coil. Turn the coarse current control on the power supply all the way up and the voltage controls all the way down.
4. Place the compass on the platform of the tangent galvanometer and rotate the galvanometer until the compass needle is aligned with the plane of the wire coil of the galvanometer. Rotate the compass body so that the needle points north and south on the compass.
5. Turn on the power supply and slowly turn up the fine voltage control until the current is about 0.2 A as measured with the orange ammeter. Note that the compass needle rotates. It might be a good idea to tap lightly the face of the compass to be sure the needle hasn't become stuck. Record the current and the displacement angles at both the north and south poles of the needle. Calculate  $B_c$  from equation (2).
6. Average the north and south angles; use this average to calculate  $B_h$  from equation (3).
7. Repeat steps 5 and 6 for currents of 0.4 A, 0.6 A, and 0.8 A.
8. Reverse the current (by switching the two leads at the galvanometer coil) and repeat steps 5, 6 and 7.
9. Determine and record the dip angle at two separate locations in the classroom (using the dip angle compasses). Average these measurements and use the result for the dip angle,  $\phi$ .
10. Using equation (1) above, calculate the Earth's magnetic field **B** for each set of data. Remember that in equation (1),  $\phi$  is the dip angle, not the displacement angle of the compass. Calculate an average value for **B** and a standard deviation. How far off, in terms of numbers of standard deviations, is your result from the accepted value for Richmond,  $5.1 \times 10^{-5}$  T?

**Data Sheet**

Diameter of Coil, D (m) \_\_\_\_\_

Radius of Coil, R (m) \_\_\_\_\_

Number of Turns of Coil, N \_\_\_\_\_

Dip Angle,  $\phi$  ( $^{\circ}$ ): reading 1 \_\_\_\_\_ reading 2 \_\_\_\_\_Average Dip Angle,  $\phi$  ( $^{\circ}$ ): \_\_\_\_\_

Current (A)	Coil Field $B_c$ (T)	North Angle $\theta_N$ ( $^{\circ}$ )	South Angle $\theta_S$ ( $^{\circ}$ )	Average Angle ( $^{\circ}$ )	Horizontal Component $B_h$ (T)	Earth's Field $\mathbf{B}$ (T)

Earth's Magnetic Field (measured),  $\langle \mathbf{B} \rangle$  (T) \_\_\_\_\_Standard Deviation on Measurement,  $\sigma_B$  (T) \_\_\_\_\_Number of Standard Deviations from Accepted Value,  $\frac{|\langle B \rangle - B_{accepted}|}{\sigma_B}$ : \_\_\_\_\_**Show calculations for one trial here:**

## 34 Centripetal Force<sup>6</sup>

Name \_\_\_\_\_

Section \_\_\_\_\_

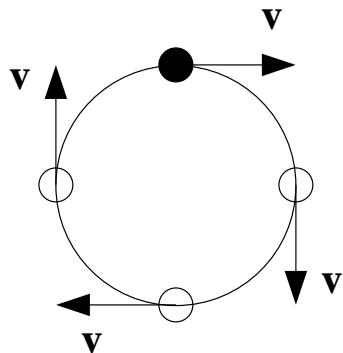
Date \_\_\_\_\_

### Objective

To explore uniform circular motion and the accelerations and forces needed to maintain it.

### Overview

In this unit you will explore uniform circular motion, in which an object moves at a constant speed in a circle. In particular, you will develop a mathematical description of centripetal acceleration and the force needed to keep an object moving in a circle.



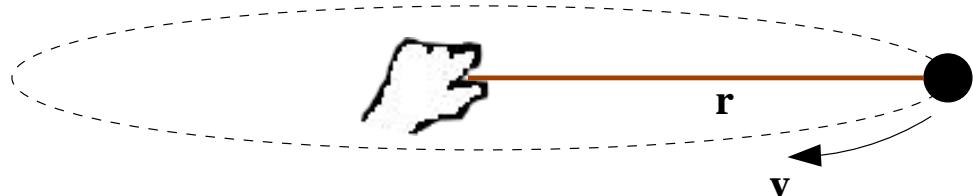
### Apparatus

- A video analysis system.
- *DataStudio*
- A ruler.
- Graphing software (*Excel*).

### Moving in a Circle at a Constant Speed

When a race car speeds around a circular track, or when David twirled a stone at the end of a rope to clobber Goliath, or when a planet like Venus orbits the sun, they undergo uniform circular motion. Understanding the forces which govern orbital motion has been vital to astronomers in their quest to understand the laws of gravitation.

But we are getting ahead of ourselves, for as we have done in the case of linear and projectile motion we will begin our study by considering situations involving external applied forces that lead to circular motion in the absence of friction.



<sup>6</sup>1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material have been modified locally and may not have been classroom tested at Dickinson College.

Let's begin our study with some very simple considerations. Suppose an astronaut goes into outer space, ties a ball to the end of a rope, and spins the ball so that it moves at a constant speed.

**Activity 1: Uniform Circular Motion**

1. Consider the figure above. What is the speed of a ball that moves in a circle of radius  $r = 2.5$  m if it takes 0.50 s to complete one revolution?
2. The speed of the ball is constant! Would you say that this is accelerated motion?
3. What is the definition of acceleration? (Remember that acceleration is a vector!)
4. Are velocity and speed the same thing? Is the velocity of the ball constant? (Hint: Velocity is a vector quantity!)
5. In light of your answers to (c) and (d), would you like to change your answer to part (b)? Explain.
6. As the ball moves on around the circle, what is the direction of its acceleration?
7. Use Newton's second law in vector form ( $\sum \mathbf{F} = m\mathbf{a}$ ) to describe the direction of the net force on the ball as it moves around the circle.
8. If the ball is being twirled around on a string, what is the source of the net force needed to keep it moving in a circle?

### Activity 2: How Does $a_c$ Depend on $v$ and $r$ ?

1. Do you expect you would need more centripetal acceleration or less centripetal acceleration to cause an object moving at a certain speed to rotate in a smaller circle? In other words, would the magnitude,  $a_c$ , have to increase or decrease as  $r$  decreases if circular motion is to be maintained? Explain.
  
  
  
  
  
  
2. Do you expect you would need more centripetal acceleration or less centripetal acceleration to cause an object to rotate at a given radius  $r$  if the speed  $v$  is increased? In other words, would the magnitude,  $a_c$ , have to increase or decrease as  $v$  increases if circular motion is to be maintained? Explain.

You may have guessed that it requires more acceleration to move an object of a certain speed in a circle of smaller radius and that it also takes more acceleration to move an object that has a higher speed in a circle of a given radius. From our past study of kinematics you might remember that the magnitude of the centripetal acceleration is

$$a_c = \frac{v^2}{r} \quad (4)$$

where  $r$  is the radius of the circular motion and  $v$  is the constant speed of the object. The direction of the acceleration is always towards the center of the circle and its magnitude is constant (the speed doesn't change).

### Experimental Test of the Centripetal Force Equation

Whenever you see an object of mass  $m$  moving in a circle of radius  $r$  at a constant speed  $v$ , it must at all times be experiencing a net centripetal force directed toward the center of the circle that has a magnitude of

$$F_c = ma_c = m\frac{v^2}{r} \quad (5)$$

Let's check this out. Does this rather odd equation really work for an external force?

To test the validity of the derivation we must compare it to experience. We will use a "toy" airplane suspended from a string and flying in a circular path. See Figure 3. We will use the video analysis system to measure the properties of the motion and determine the horizontal and vertical components of the force exerted on the airplane using Equation 5. We will compare that result with a direct measurement of the tension in the string using a force sensor.

### Activity 3: Verifying the $F_c$ Equation

1. To test the validity of Equation 5 you will analyze data that has already been collected (because of time constraints). Download the movie *airplaneMovie.wmv* from the location  
<https:// facultystaff.richmond.edu/~ggilfoyl/genphys/131/links.html>  
and store it on your computer Desktop. Download the *DataStudio* file *airplaneData.ds* from the same site and also store it on your Desktop.
2. Some of the parameters of the airplane's "flight" are

$$m_{plane} = 0.172 \text{ kg} \quad l_{plane} = 0.169 \text{ m} \quad L = 0.984 \text{ m}$$

where  $m_{plane}$  is the mass of the plane,  $l_{plane}$  is the length of the plane, and  $L$  is the distance from the center of mass of the airplane, along the string, up to the pivot where the string becomes vertical. See Figure 3.

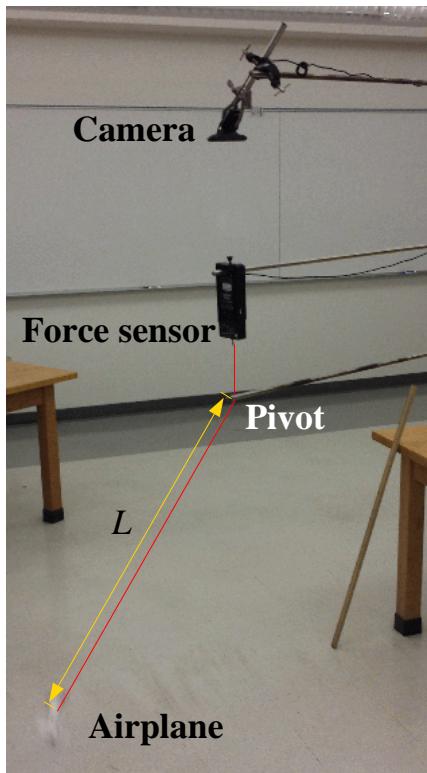


Figure 3: Setup for testing the centripetal force equation. Red line is the string from the force sensor to the airplane.

3. Determine the force exerted by the string as shown in Figure 3. Open *DataStudio* (Go to Start → All Programs → Physics Application → DataStudio and click ‘Open Activity’ on the pop-up window when *DataStudio* starts. Open the file you just downloaded. You will see a measurement of the force exerted by the string on the airplane as a function of time. The measurement was made using a force sensor. See Figure 3. A string was attached from the hook on the sensor (red line in Figure 3), ran vertically downward, passed through a small hole in a fixed steel rod (the pivot), and then was attached to the airplane. This setup allowed the string to rotate freely so the only forces acting on the airplane were gravity and the tension in the string (the friction force here is very small). There are two distinct regions on your plot; where the airplane was orbiting ( $0 < t < 40$  s) and then after it was caught and placed on the lab table so the tension in the string went to zero ( $45$  s  $< t < 65$  s). Use the *DataStudio* tools to measure the average force exerted by the string while it was orbiting. Record your answer here along with the standard deviation.

$$F_{scale} = \quad \pm$$

4. Determine the position of the airplane during at least two complete revolutions. To do this task follow the instructions in the Appendix for analyzing a movie file. Calibrate your plot with the known length of the airplane  $l_{plane}$ . When finished, you may want to export your measurements to a file for safekeeping.
5. Make a plot of all the points on the trajectory of the airplane during those two revolutions by turning on all the measured points by clicking on **Set trail length** (see **Appendix D**). Print your plot and attach it to your lab.
6. Is the motion circular? What is your evidence?

7. Use a ruler and your plot to measure the diameter of the airplane's trajectory. Make at least five separate measurements between different diameters of the trajectory and enter them in the table below. Use the known length of the airplane  $l_{plane}$  to convert those measurements to the scale of the movie and then calculate each radius.

Measured Diameter	Converted Diameter	Radius (m)

8. What is the average radius of the motion?

$$r_{ave} =$$

9. To test the validity of the expression for  $F_c$  we must know the speed. Use the measurements of the radius of the airplane's trajectory and the time for at least two complete revolutions to calculate the average speed. The time is recorded in the data table in the video analysis software.

$$v_{ave} =$$

10. Use your results for the mass, the average velocity of the airplane, and the average radius of the circular motion to calculate the centripetal force exerted by the string.

$$F_c =$$

11. We will now determine the net force exerted by the string on the airplane by using a little trigonometry. Recall that we know  $r_{ave}$ , the radius of the airplane's circular path, and  $L$ , the total length of the string that is actually rotating.

- (a) Using these two distances ( $r_{ave}$  and  $L$ ), calculate the angle the string makes with the horizontal. You may want to draw a diagram to do this.

$$\theta =$$

- (b) We determined the horizontal component of the force on the airplane  $F_c$  in part 10. Knowing the angle  $\theta$  we can determine an expression for the net force exerted by the string on the airplane,  $F_{plane}$ . Draw a vector force diagram including  $F_c$  and  $F_{plane}$ , derive an expression for  $F_{plane}$  and calculate it.

$$F_{plane} =$$

12. Compare your result for  $F_{plane}$  with the measurement of the spring scale  $F_{scale}$  from part 3. Should they be different? Should they be the same? Explain.
  13. Calculate the difference between  $F_{plane}$  and  $F_{scale}$  ( $F_{plane} - F_{scale}$ ) and record it below. Go around to the other lab groups and get their results for this quantity. Make a histogram of the results you collect and calculate the average and standard deviation. Print the histogram and attach it to this unit. For information on making histograms, see **Appendix C**. For information on calculating the average and standard deviation, see **Appendix A**. Record the average and standard deviation here.
  14. What is your expectation for the difference  $F_{plane} - F_{scale}$ ? Do the data from the class support this expectation? Use the average and standard deviation for the class to quantitatively answer this question.
  15. What does the histogram of the class data tell you? Be quantitative in your answer.
  16. Discuss the major sources of uncertainty in this experiment.

## 35 Weighing an Electron

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

To investigate the force on a charged particle due to a magnetic field and learn how the motion of the particle can be used to weigh it.

### Apparatus

- $e/m$  apparatus
- Low- and high-voltage power supplies
- Analog ammeter and digital multi-meter
- Flashlight

### Overview

Mass spectroscopy is an experimental technique that is used to determine the mass of atoms, molecules, and sub-atomic particles. The central idea is to make a version of the particle carrying an electric charge (*e.g.*, a bare electron, an ionized molecule, *etc*), accelerate it through an electric potential, and then inject it into a magnetic field. If properly oriented, the magnetic field will bend the object into a curved path. The amount of curvature depends on the mass of the object and the electric charge it is carrying so that different mass particles with the same charge will bend by different amounts. Measuring this bend is equivalent to a mass measurement. This method is widely used to do things like determine the mass of newly discovered particles, hunt for oil, and even validate the authenticity of works of art.

### Activity 1: Magnetic Force on a Charged Particle

It has been found by careful measurements that the force  $\vec{F}_B$  on a charged particle due to a magnetic field is

$$\vec{F}_B = q\vec{v} \times \vec{B} \quad (1)$$

where  $q$  is the electric charge of the particle,  $\vec{v}$  is its velocity, and  $\vec{B}$  is the local magnetic field. Find the relevant sections in your text for a discussion of the cross product.

(a) The figure below shows the relationship between  $\vec{F}_B$ ,  $\vec{v}$ , and  $\vec{B}$ . How is the magnitude of the cross product  $\vec{v} \times \vec{B}$  related to the magnitude of  $v$ ,  $B$ , and  $\theta$  the angle between  $\vec{v}$  and  $\vec{B}$ ?

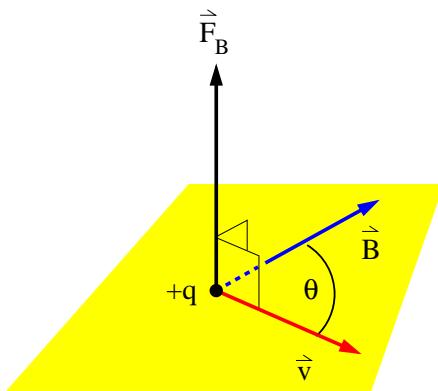


Figure 1: Vectors associated with the magnetic force.

- (b) Consider a situation where the magnetic field is uniform in space and points in the  $z$  direction  $\vec{B} = B\hat{k}$  and the velocity of the charged particle is in the  $y$  direction  $\vec{v} = v\hat{j}$ . What is  $\theta$ , the angle between the magnetic field and velocity vectors? What is  $\vec{F}_B$  in terms of the magnitudes of  $q$ ,  $v$ , and  $B$  and the appropriate unit vectors?
- (c) Describe the trajectory of the charged particle. What is the  $z$  component of its trajectory?
- (d) What is the magnitude of the magnetic force as this charged particle moves through the region of the magnetic field? Does this magnitude change as the particle moves through the field?
- (e) How are the directions of  $\vec{F}_B$  and  $\vec{v}$  related?

**Activity 2: Relating  $\vec{F}_B$  to the Kinematics**

At the end of the previous section you should have found that the magnitude of the magnetic force is  $|qvB|$  and that  $\vec{v}$  and  $\vec{B}$  are always perpendicular to one another.

- (a) What type of force have we encountered that is perpendicular to the velocity and constant in magnitude? (Hint: Recall some of the applications of Newton's Laws in your text.)
- (b) How is centripetal acceleration related to  $v$  and  $r$  (the radius of the circular motion)? From Newton's 2nd Law, what is the expression for the magnitude of the centripetal force  $|\vec{F}_c|$ ?

(c) If the magnetic force from Activity 1 is the only force acting on the particle and we know it moves in a circle, then the magnetic force must be the centripetal force causing the particle to move in a circle. Equate the expressions for the magnitudes of  $\vec{F}_B$  from 1(d) and  $\vec{F}_c$  from 2(b). Solve for the mass  $m$  in terms of the radius  $r$  of the particle's path,  $|q|$ ,  $v$ , and  $B$ .

(d) Using your result from 2(c), answer the following questions. For two particles with the same charge and velocity, but different mass, which one will have the larger path radius? How will increasing the velocity change the path radius?

(e) Remember that in a mass spectrometer, the particle is first accelerated across an electric potential difference. Typically, we know the kinetic energy  $K$  of the charged particle after this acceleration instead of the velocity. Rewrite your result in 2(c) in terms of the kinetic energy  $K$  instead of the velocity  $v$ . (Solve the kinetic energy equation for velocity and substitute in result of 2(c). Then solve for  $m$ .)

(f) The electron ‘falls’ across a potential difference created by the accelerating voltage to gain a velocity  $v$  before it enters the magnetic field region. Assuming the electron starts from rest, what is the relationship between the accelerating voltage and the kinetic energy  $K$  when it leaves the accelerating region and enters the magnetic field? (Use conservation of energy to determine this.) Combine this result with the one from part 2(e) to get an expression for the mass of the electron in terms of the accelerating voltage  $V$ , the electron charge  $e$ , the radius of the electron's path  $r$ , and the magnetic field  $B$ .

### Activity 3: Predictions for a Charged Particle in a Magnetic Field

We now have the mathematical tools to understand the motion of a charged particle in a magnetic field so we can start investigating the physics.

- (a) First make some predictions. If  $\vec{B} = B\hat{k}$  and  $\vec{v} = v\hat{j}$ , then what direction is  $\vec{v} \times \vec{B}$ ?
- (b) What direction is  $\vec{F}_B$  for a positively charge particle if  $\vec{B} = B\hat{k}$  and  $\vec{v} = v\hat{j}$ ?
- (c) What direction is  $\vec{F}_B$  for a negatively charge particle if  $\vec{B} = B\hat{k}$  and  $\vec{v} = v\hat{j}$ ?

#### Activity 4: Measuring a Charged Particle in a Magnetic Field

The last piece of the puzzle before we start measuring things is the magnetic field  $\vec{B}$ . The apparatus you are using consists of a pair of wire coils (called Helmholtz coils) and an electron gun that produces a beam of electrons. See Figure 2 and identify the Helmholtz coils on your apparatus. The magnitude of the magnetic field along the

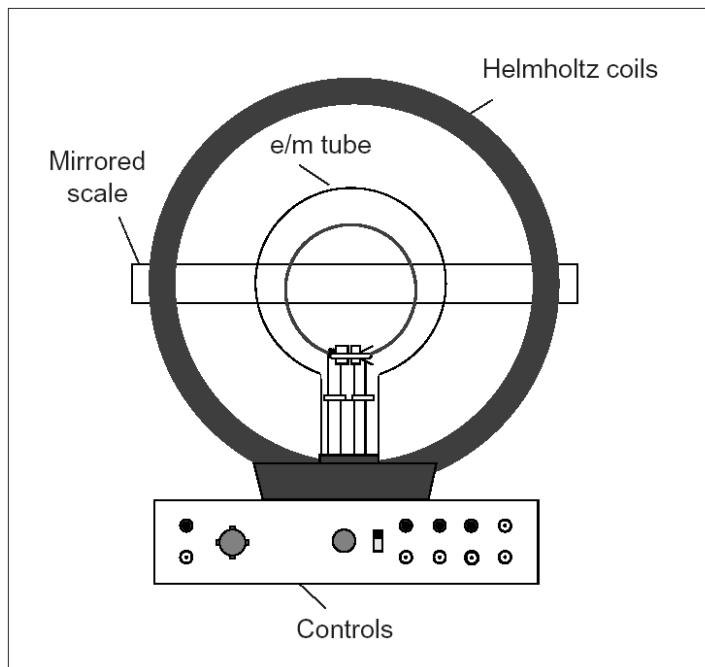


Figure 2: The  $e/m$  apparatus.

axis of the pair of coils is

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{N\mu_0 I}{a} \quad (2)$$

where  $N$  is the number of turns of wire in each coil,  $\mu_0$  is the permeability constant,  $I$  is the current in the coils, and  $a$  is the radius of the coil. The direction of the field is parallel or anti-parallel to the axis of the pair of coils depending on the direction of the current in the coils. The direction of the current is the same in each coil. The magnetic field in the region between the coils is approximately equal to the field along the axis of the coils so we will use the expression above for our magnetic field in the equation you generated in Activity 2.f.

- (a) The number of turns of wire in each Helmholtz coil is  $N = 130$ . Notice that the turns on each coil are wrapped on top of one another, so the radii of the turns are not all the same. We will approximate an average value by measuring the diameter of the coils from the center of the housing on one side to the center of the housing on the other side. Record that diameter and the value of  $a$  (half the diameter) here:

Now calculate the magnetic field  $B$  due to the Helmholtz coils using equation (2), assuming a current of 1.50 amperes in the coils. Show that the result is in *tesla*.

- (b) Identify each item on the front panel below the Helmholtz coils using Figure 3 as a guide. Check that all the connections are correct using Figure 3 as your guide.

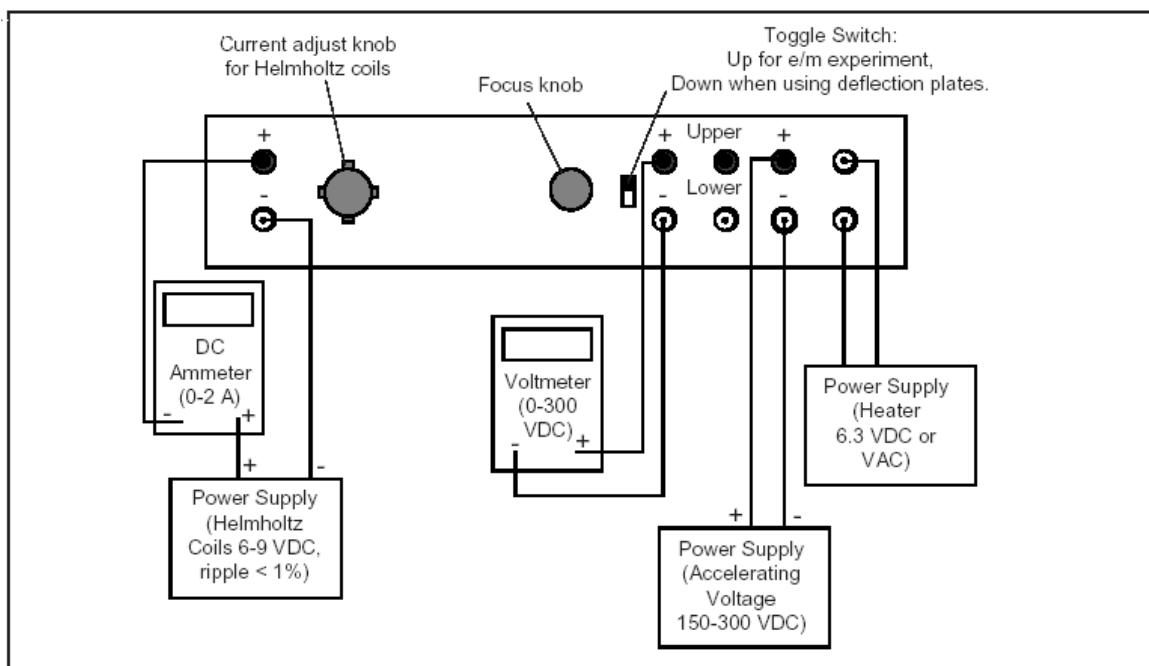


Figure 3: Instrument connections for the *e/m* experiment.

- (c) Place the hood over the *e/m* apparatus and flip the toggle switch up to the *e/m* MEASURE position. Turn the current adjust knob for the Helmholtz coils to the vertical position (white line at the 12 o'clock noon position).
- (d) Identify the ammeter used to measure the current in the Helmholtz coils using Figure 3 as a guide. Next, turn on the low-voltage power supply for the Helmholtz coils. Now watch the current in the ammeter (it should not exceed 2 A under any circumstances) and turn up the voltage on the Helmholtz coils into the range of 6 – 9 V.
- (e) As you watch the ammeter slowly turn the current adjust knob for the Helmholtz coils clockwise. Take care

that the current in the ammeter does not exceed 2 A. Set the current adjust knob for the Helmholtz coils to 1.50 A.

(f) Identify the power supply that runs the heater for the electron gun using Figure 3 as a guide and turn on the power supply. Turn the voltage up to 6 volts. **CAUTION:** Do not exceed 6 volts or you may destroy the  $e/m$  tube.

(g) Turn on the power supply for the accelerating voltage for the electron gun. Turn the voltage up so that it is in the range of 150 – 160 V.

(h) Wait several minutes for the cathode to heat up. When it does, you will see the electron beam emerge from the electron gun and it will be curved by the field from the Helmholtz coils. If the electron beam is not sharp, adjust the focus knob. Check that the electron beam is parallel to the Helmholtz coils. If it is not, consult your instructor. Carefully read the current in the Helmholtz coils from your ammeter and the accelerating voltage from your voltmeter. Record the values here.

(i) Carefully measure the radius of the electron beam. Look through the tube at the electron beam. To avoid parallax errors, move your head to align the electron beam with the reflection of the beam that you can see on the mirrored scale. Measure the radius of the beam as you see it on both sides of the scale, then average the results. Record your results here.

(j) Slowly turn the current adjust knob for the Helmholtz coils either up or down as you watch the ammeter and take care that the current does not exceed 2 A. What parameter does this action affect, the magnetic field or the accelerating voltage? What happens to the path of the electrons as the current in the Helmholtz coils changes? Does this agree with what you expected? Explain.

(k) Return the current in the Helmholtz coils to the value you recorded in part (h) (1.50 A). Slowly increase the accelerating voltage. What parameter does this voltage affect? What happens to the electron's path? For four additional values of the accelerating voltage (two lower than 150 V and two higher) record the accelerating voltage, the current in the Helmholtz coils (should be the same for all) and the average radius of the electron beam (by measuring the radius of the beam on both sides of the scale and averaging as you did in part (i) above). Create a table in the space below with the headings  $V$ ,  $I$ ,  $r_{left}$ ,  $r_{right}$ ,  $r_{ave}$ ,  $r^2$ . Your data from part (i) above will be the third line in the table (with two voltages below 150 and two voltages above 150).

- (l) When you are finished, turn down to zero the accelerating voltage, the heater voltage, the Helmholtz coil voltage, and the current adjust knob.

**Activity 5: Extracting the Electron Mass From Your Data.**

(a) Use *Excel* to plot a graph of  $r^2$  vs  $V$  for the measurements you have tabulated in part 4(k). Be sure and include the proper units on the axes. Include a trendline with a linear fit. Use the LINEST function in *Excel* to determine the slope and the uncertainty in the slope of your graph, including proper units for both. Print the graph and attach it to this unit. From your result of Activity 2(f), solve for  $r^2$  and write a formula for the slope of the  $r^2$  vs  $V$  graph.

(b) Set the slope of your graph (including units) equal to the formula you obtained in part (a), and calculate the mass of the electron. (Remember that you determined the magnetic field  $B$  due to the Helmholtz coils in Activity 4(a).) Show that the units are kilograms.

(c) Calculate the uncertainty in the mass assuming the fractional uncertainty in mass is equal to the fractional uncertainty in the slope. Write the mass as  $mass = m \pm \Delta m$ . Does the accepted value fall within your range? What are some sources of error?

## 36 Nuclear Decay and Radiocarbon Dating

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

To develop an understanding of the use of the radioactive decay of atomic nuclei to date objects like the Shroud of Turin.

### Apparatus

- Radioactive sources.
- Radiation counter.
- Jack.
- Isotope generator.
- Surgical gloves, eye protection, and lab coat for handling radioactive liquids.
- Lead and plastic sheets.

### Introduction

Atoms can be broken down into light, negatively-charged, electrons, and a small, dense, positively-charged nucleus. These atomic nuclei can spontaneously break apart into smaller nuclei in a process called radioactive decay. By measuring the rate of this decay under the appropriate circumstances one can develop a “clock” that can be used to determine how long ago in the past an event occurred. In this laboratory we will apply this notion to a particular object, the Shroud of Turin which is purported to be the burial cloth of Jesus Christ.

### Activity 1: Nuclear Terminology

Atomic nuclei can be constructed from protons and neutrons. The number of protons in a nucleus is called the atomic number and is represented by the letter Z while the number of neutrons is represented by the letter N. The protons carry a charge of  $+e$  while the neutrons are electrically neutral. The sum of these two quantities is the mass number A.

$$A = N + Z$$

Protons and neutrons are often referred to as nucleons.

Nuclei are represented using their chemical symbol (determined by the atomic number) and the mass number. For example, the most common form of carbon has six protons and six neutrons in its nucleus and is written as  $^{12}\text{C}$ . If another neutron is added to this nucleus, then one has an isotope of carbon,  $^{13}\text{C}$ . Isotopes of a chemical element have the same atomic number(Z), but have a different numbers of neutrons (N) and a different mass number (A). The difference is reflected in the value of the superscript on the chemical symbol.

(a) Consider the following list of the number of protons and neutrons that combine to form a particular nucleus. In the third column enter the chemical symbol and mass number as shown above (e.g.,  $^{12}\text{C}$ ). Use the periodic chart at the end of this unit to determine what to enter in the third column.

Number of Protons	Number of Neutrons	Nucleus
7	8	
79	118	
26	30	

(b) Consider the following list of atomic nuclei. In the second and third columns enter the number of protons and neutrons in each nucleus.

Nucleus	Number of Protons	Number of Neutrons
${}^4\text{He}$		
${}^{235}\text{U}$		
${}^{108}\text{Ag}$		

Some isotopes can spontaneously decay into other nuclei. In many of these decays the number of nucleons is conserved. This means that the number of protons and neutrons added together in the parent nucleus before the decay must be the same in the final products. Electric charge is always conserved.

(c) In the table below a nuclear decay is shown in the first column. In most cases the original nucleus (often referred to as the parent) produces two smaller nuclei (called daughters). Only one of the daughter nuclei is listed. In the adjacent column list the missing nucleus. Notice there are two emitted particles that we have not mentioned before. The  $e^-$  which is an electron and is often called a beta particle. It has almost zero mass compared to a nucleon. The other is  $\gamma$  and is called a “gamma” particle. This is photon or a particle of light or electromagnetic energy. The gamma has no mass or charge, but does carry energy and momentum.

Decay	Unknown
${}^{190}\text{Po} \rightarrow {}^4\text{He} + ?$	
${}^{210}\text{Th} \rightarrow {}^4\text{He} + ?$	
${}^{16}\text{Ne} \rightarrow \text{p} + ?$	
${}^{90}\text{Sr} \rightarrow e^- + ?$	
${}^{60}\text{Co} \rightarrow \gamma + ?$	

### Activity 2: The Properties of Radiation

An essential attribute of radiation is its ability to penetrate matter. Here we will study how three different types of radiation (alpha, beta, and gamma radiation) penetrate matter. We will do this by using a radiation counter and samples of three nuclei. The heart of the radiation counter is a gas-filled cylinder with a wire at high voltage running down its center. This cylinder is called a Geiger-Muller or G-M tube. Sub-atomic particles flying through the counter ionize atoms in the gas which are collected at the center wire producing a voltage pulse that can be measured. NOTE: Before going any further read the appendix on nuclear safety.

(a) Compare the figure below with your setup. Familiarize yourself with the different components. DO NOT TOUCH THE FACE OF THE DETECTOR INSIDE THE SNOOT UNDER ANY CIRCUMSTANCES.

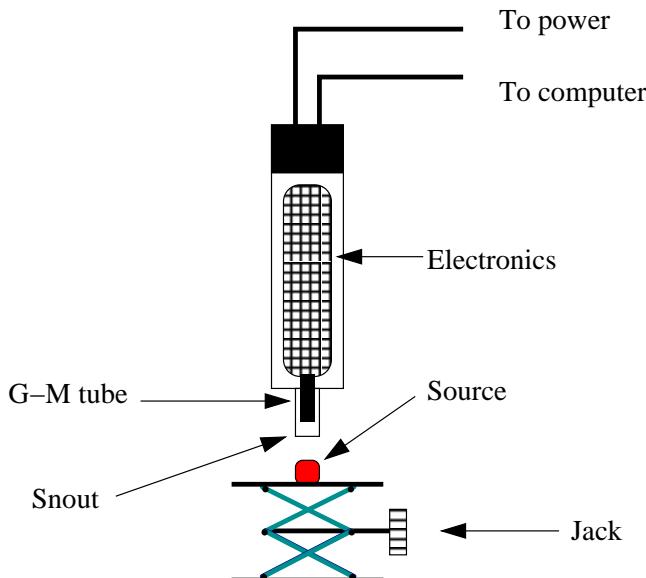


Figure 1. Schematic drawing of radiation counter and setup.

(b) Carefully remove the red, plastic cover on the snout of the radiation counter. DO NOT TOUCH THE FACE OF THE DETECTOR UNDER ANY CIRCUMSTANCES. This would likely break the window and destroy the counter.

(c) Obtain a set of radioactive sources from your instructor. Pick one of the sources, place it on the jack, and position the source about 1 cm below the snout of the radiation counter. Plug the power cord into a standard electric outlet and make sure the data cable is plugged into digital channel 1 on the *DataStudio* 750 interface. You should see the power light come on near the base of the counter (which is actually at the top). Another light near the snout of the counter blinks whenever a particle is detected. If you don't see either light, consult your instructor.

(d) Open the *Radiation Counter* activity in the **132 Workshop** menu and click the **Start** button on the *DataStudio* interface. Nothing will seem to be happening, but after 30 seconds (watch the clock at the top of the *DataStudio* interface) the number of radioactive decays detected by the radiation counter will appear. Record this result in the appropriate place in the table below.

(e) Now carefully place a piece of plastic on top of the source and run the counter for another 30 seconds. Record the result below.

(f) Remove the plastic and place a small sheet of lead on top of the source. Run the counter and record the result.

(g) Repeat steps d-f for the other two radioactive sources.

Radiation	Air	Plastic	Lead
$\gamma$			
$\beta$			
$\alpha$			

(h) Which type of radiation is most penetrating? Why?

(i) Which type of radiation is least penetrating? Why?

(j) What material provides the best shielding of radioactivity? Why?

### Activity 3: Background Radiation

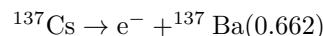
(a) Return the radioactive sources to the instructor's table.

(b) With no radioactive sources nearby, make several runs with the radiation counter. The counts you observe in the detector are due to cosmic rays, radioactive decay in the building materials surrounding you, and even your own body. Record the counts and calculate the average and standard deviation of this background radiation.

### Activity 4: Nuclear Decay

To understand the “clock” we will use to date the Shroud of Turin we will investigate how the clock “ticks”. In this activity you will use a sample of radioactive material and a nuclear detector to measure the behavior of the material as a function of time. We will then build a mathematical model of the time dependence of nuclear decay. We will apply this model to analyze the results of  $^{14}\text{C}$  measurements on the Shroud.

To obtain the radioactive material we will use a procedure known as “milking the cow”. We start with a liquid that contains the radioactive isotope  $^{137}\text{Cs}$  or cesium-137. This isotope decays very slowly; it would take about 30 years for half of a sample to decay (a bit long for an introductory physics experiment). However, when  $^{137}\text{Cs}$  does decay it usually does so in the following way.



Notice the additional number “0.662” beside the Ba-137. This number means there is still some energy (0.662 million electron-volts or MeV) stored in the Ba-137 nucleus and it has not yet reached its lowest-energy or ground state. This “excited” state of Ba-137 then emits a high-energy photon or gamma ray to reach the stable ground state of  $^{137}\text{Ba}$ . A diagram of the process is below.

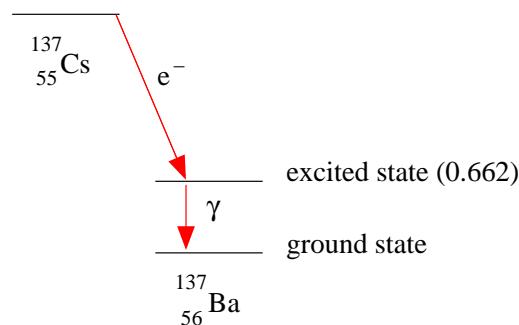


Figure 2. Decay scheme of cesium-137.

This intermediate state (labeled “0.662”) decays quickly to the ground state and it is the one we will study.

We will prepare a sample of Ba-137 in its excited, 0.662-MeV state by starting with a Cs-137 “generator”. The Cs-137 produces Ba-137 at a steady rate. We remove the Ba-137 from the “generator” by passing a hydrochloric-acid-saline solution through the generator (your instructor will do this). This is called eluting which means separate by washing. The generator is commonly referred to as the “cow” and the Ba-137 is “milked” from the cow. The Ba-137 is eluted from the generator and can then be used to study its decay.

(a) First, make a prediction of the count rate as a function of time. Sketch your prediction in the space below.

(b) What is the mathematical form of your prediction? Why did you choose it?

(c) Open the *Nuclear Decay* application in the **132 Workshop** menu. When you click **Start** it will plot the count rate in intervals of 10 seconds. Get the radioactive sources from your instructor and try this out with one of them to make sure you know how to use the hardware and software. Return the sources to your instructor when you are finished with this test.

(d) Read the rest of this procedure carefully. If you have to redo the procedure it may take a long time for the “generator” to produce enough Ba-137 for you to use.

(e) You have a small, metal disk called a planchette that sits on the jack which will be positioned close to the snout of the radiation counter. This will hold the radioactive material. Put the empty planchette in place and do a “dry run”.

(f) One team member should be responsible for positioning the planchette. That person should put on the surgical gloves, eye protection, and a lab coat. The other team member can run the data acquisition.

(g) When you are ready, alert the instructor. He or she will come over and place a few drops of the eluate containing the Ba-137 on the planchette. Immediately place this under the Geiger counter and click **Start** on the *DataStudio* interface.

**Caution:** Care should be taken in handling the sample. If any portion of the sample touches your skin immediately wash off in the sink.

(h) Let the data acquisition run for about fifteen minutes or so and then click **Stop**. Dispose of the planchette according to the guidance from the instructor.

(i) Make a plot of your results using the data in the *Counts versus Time Table*. Notice that if you click on the title of the table, all of the data will be selected. You can then paste the data into *Excel*. Make sure you subtract the background radiation from your results.

(j) Make a fit to your data. What is the best choice of function for fitting your data? How did you make your choice? Attach a copy of your plot with the fit to this unit. Record the fit equation below. Do NOT close your spreadsheet. We may use it later.

### Activity 5: Analyzing Nuclear Decay

Observation of a sample of radioactive material reveals that the decay of the atomic nuclei in the sample is determined by statistical processes. In other words, the number of nuclei  $N_{nuc}$  that decay per unit time is proportional to the number of nuclei in the sample.

$$\frac{dN_{nuc}}{dt} \propto N_{nuc}$$

This expression can be turned into an equality by inserting a constant of proportionality  $\lambda$  so

$$\frac{dN_{nuc}}{dt} = -\lambda N_{nuc}$$

where the minus sign is needed because the number of nuclei  $N_{nuc}$  decreases with time. The decay constant  $\lambda$  is a characteristic of each atomic nucleus.

(a) In the previous activity, you used a particular function to fit your data. Try to prove that you made the right choice by taking derivatives and seeing if they will satisfy the original differential equation above. Did it work?

- (b) It is claimed the solution of the differential equation above describing nuclear decay is the following expression.

$$N_{nuc}(t) = N_0 e^{-\lambda t}$$

Prove this statement by taking the derivative of  $N_{nuc}(t)$  and showing it satisfies the original differential equation. Make a sketch of the function and describe it in words. How did your fit function do?

- (c) The decay of atomic nuclei is often characterized by a quantity known as the half-life  $\tau$ . The half-life is the period of time for one-half of the original sample to disappear via radioactive decay. This statement can be expressed mathematically in the following way.

$$N_{nuc}(t = \tau) = \frac{N_0}{2}$$

Starting with the above expression show that the decay constant  $\lambda$  and the half-life are related by the following equation.

$$\tau = \frac{\ln 2}{\lambda}$$

- (d) Now return to the results of your experiment. Does your count rate fall off exponentially? Did you fit your data with an exponential? If not, go back and do so. Record the decay constant  $\lambda$ .

- (e) What is the half-life of Ba-137? Compare this with the accepted value of 2.552 minutes.

(f) Consider the following example as a warm-up. A sample of the isotope of iodine  $^{131}\text{I}$  has an initial decay rate of  $1.8 \times 10^5$  decays/s. This isotope has a half-life of 8.04 days. It is shipped to a medical diagnostic laboratory where it will be used as a radioactive tracer. When the shipment arrives at the lab the decay rate has fallen to  $1.4 \times 10^5$  decays/s. How long did it take for the shipment to reach the laboratory?

#### Activity 6: Dating the Shroud of Turin

The previous example shows how one can use the measured decay rate of an atomic nucleus as a “clock” to determine the passage of time. The same concept is used in radiocarbon dating. Carbon on the planet Earth consists largely of three isotopes with  $A = 12, 13$ , and  $14$ . The most common form is  $^{12}\text{C}$  and only a very small fraction of the carbon is  $^{14}\text{C}$ . However,  $^{14}\text{C}$  decays via



where  $\beta^-$  is an electron and  $\bar{\nu}$  is a particle known as a neutrino. Notice this decay does NOT preserve the number of protons and neutrons in the original nucleus. The ratio  $R$  of  $^{14}\text{C}$  to  $^{12}\text{C}$  on the Earth is  $1.30 \times 10^{-12}$  and is roughly constant despite the fact that the  $^{14}\text{C}$  constantly disappears. The ratio is constant because the supply of  $^{14}\text{C}$  in the atmosphere is replenished by the reaction of cosmic rays from outer space with the nitrogen in the upper atmosphere.

Living organisms contain large quantities of carbon and are constantly exchanging carbon with their surroundings. They contain the same proportion of  $^{14}\text{C}$  to  $^{12}\text{C}$  as observed in the atmosphere. However, this proportion begins to change after the organism dies. The  $^{12}\text{C}$  remains in the dead body, but the  $^{14}\text{C}$  turns into gaseous  $^{14}\text{N}$  (see decay above) and leaves the body. Hence, the proportion of  $^{14}\text{C}$  decreases with time, and one has a “clock” that can be used to determine when an organism died.

The Shroud of Turin is a piece of cloth that bears the image of a man who appears to have been crucified. It was first displayed in France in the fourteenth century and has been kept at the Royal Chapel of Turin Cathedral in a special shrine since 1694. Many believe the image on the Shroud is of Christ and the cloth is his burial wrap. In 1989, three laboratories in Arizona in the USA, Oxford in the UK, and Zurich in Switzerland used advanced methods of radiocarbon dating in an attempt to determine the age of the Shroud[1]. The Shroud is woven of cloth made from plants. Like a living organism that has died, the  $^{14}\text{C}$  in the Shroud began to gradually disappear after the plants used to make it were harvested.

(a) The half-life of  $^{14}\text{C}$  is 5730 years. What is the decay constant  $\lambda$ ?

(b) The three laboratories obtained the following results for the ratio  $R$  of  $^{14}\text{C}$  to  $^{12}\text{C}$ . The ratio of  $^{14}\text{C}$  to  $^{12}\text{C}$  in the atmosphere is  $R_i = 1.30 \times 10^{-12}$ . What is the implied age of the Shroud for each measurement? Use the space below for your calculations and enter the results in the table.

Laboratory	$R_f$	Age (years)
Arizona	$1.20 \times 10^{-12}$	
Oxford	$1.18 \times 10^{-12}$	
Zurich	$1.19 \times 10^{-12}$	

(c) What is the average age of the Shroud?

(d) The typical uncertainty in these measurements is a standard deviation of  $\pm 40$  years. Are the results of the three laboratories consistent?

(e) Is the age of the Shroud consistent with it being the burial wrap of Christ?

(f) Are there any reasons to doubt these results?

## Homework

- The half-life of a particular radioactive isotope is 6.5 h. If there are initially  $48 \times 10^{19}$  atoms of this isotope, how many atoms of this isotope remain after 26 h?
- A radioactive isotope of mercury,  $^{197}\text{Hg}$ , decays into gold,  $^{197}\text{Au}$ , with a disintegration constant of  $0.0108 \text{ h}^{-1}$ . (a) What is its half-life? (b) What fraction of the original amount will remain after three half-lives? (c) What fraction will remain after 10.0 days?
- The radionuclide  $^{64}\text{Cu}$  has a half-life of 12.7 h. How much of an initially pure, 5.50-g sample of  $^{64}\text{Cu}$  will decay during the 2.0-h period beginning 14.0 h later?

## The Periodic Chart

Element Chart																																				
1	H																2	He																		
3	Li	4	Be																																	
11	Na	12	Mg																																	
19	K	20	Ca	21	Sc	22	Ti	23	V	24	Cr	25	Mn	26	Fe	27	Co	28	Ni	29	Cu	30	Zn	31	Ga	32	Ge	33	As	34	Se	35	Br	36	Kr	
37	Rb	38	Sr	39	Y	40	Zr	41	Nb	42	Mo	43	Tc	44	Ru	45	Rh	46	Pd	47	Ag	48	Cd	49	In	50	Sn	51	Sb	52	Te	53	I	54	Xe	
55	Cs	56	Ba	57	La	72	Hf	73	Ta	74	W	75	Re	76	Os	77	Ir	78	Pt	79	Au	80	Hg	81	Tl	82	Pb	83	Bi	84	Po	85	At	86	Rn	
87	Fr	88	Ra	89	Ac	104	Ku	105	Bo																											
						58	Ge	59	Pr	60	Nd	61	Pm	62	Sm	63	Eu	64	Gd	65	Tb	66	Dy	67	Ho	68	Er	69	Tm	70	Yb	71	Lu			
						90	Th	91	Pa	92	U	93	Np	94	Pu	95	Am	96	Gm	97	Bk	98	Gf	99	Es	100	Fm	101	Md	102	No	103	Lr			

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## 37 Electromagnetic Induction

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

To investigate:

- The effect of changing magnetic fields on induced emf and current.

### Introduction

If the magnetic flux through a coil of wire changes, an emf (or voltage) will be induced in the coil. This is Faraday's Law. If the coil of wire forms a closed loop, then a current will be induced in the wire. The direction of this current is such that the magnetic field it produces opposes the change in the external field. This is known as Lenz's Law. Similarly, varying the current in one coil (the primary) produces a current in another nearby coil (the secondary) due to the varying magnetic field produced by the first coil. The current in the second coil will flow in a direction that creates a magnetic field which opposes the change in the field of the first coil (again, due to Lenz's Law). These relationships between changing fields and currents are known collectively as electromagnetic induction.

### Apparatus

- one small wire coil
- voltage sensor
- two alligator clips
- bar magnet
- Pasco 750 Interface

### Activity 1: A Moving Magnet and a Coil

1. Connect the coil to the voltage sensor using the alligator clips. Connect the voltage sensor to port A on the Pasco 750 interface.
2. Turn on the computer and launch *EM Induction* in the 132*Workshop* under *Physics Applications* in the **Start** menu.
3. Place a bar magnet vertically along the axis of the small coil with the N-pole touching the coil.
4. Start recording data and lift the bar magnet quickly straight up.
5. At the end of the data taking interval, the computer should display a value for the electromotive force (emf) induced in the small coil. Several trials may be required to get the correct timing between starting the data acquisition and removing the magnet. Note and record the sign of the induced emf.
6. **Prediction:** If you lower the magnet, N-pole down, quickly toward the coil, what will be the sign of the emf?
7. Carry out the experiment, starting the data acquisition, then lowering the magnet. Record the sign of the induced emf.

8. Did your result confirm or refute your prediction?
  
  
  
  
  
9. **Prediction:** What will happen to the emf if you perform the same pair of experiments with the S-pole toward the coil?
  
  
  
  
  
10. Perform the two experiments, lifting and lowering the magnet, with the S-pole down. Record the sign of the induced emf in each case.
  
  
  
  
  
11. How did the results compare with your predictions?

### Activity 2: Predictions About Making Waves Electromagnetically

Consider what we have observed about electricity and magnetism. A static, unchanging magnetic field does not do much to our coil of wire. A varying magnetic field creates, across space, a current in our coil. The changing magnetic field must be creating an electric field or else the electrons in the coil would not feel a force. We have also observed phenomena where a changing electric field induced a magnetic field. In other words, a changing electric field induces a magnetic field and vice versa; a changing magnetic field induces an electric field. Notice these statements don't require the presence of electrons or other material. The fields can be induced in a vacuum. We are now going to explore via a simulation, what happens when charges are wiggled (*i.e.* oscillate) up and down.

1. To begin to explore our wiggling charges, consider some questions. Suppose the motion of the charges can be described as an oscillating dipole so the dipole moment as a function of time looks like Figure 1. Assume the dipole is aligned with the  $z$ -axis. What do you expect the electric field to look like as a function of time at some arbitrary distance  $r$  away from the source in the  $x - y$  plane? What is the direction of the  $\vec{E}$  field? Make a sketch of your answer on the plot and label your curve.

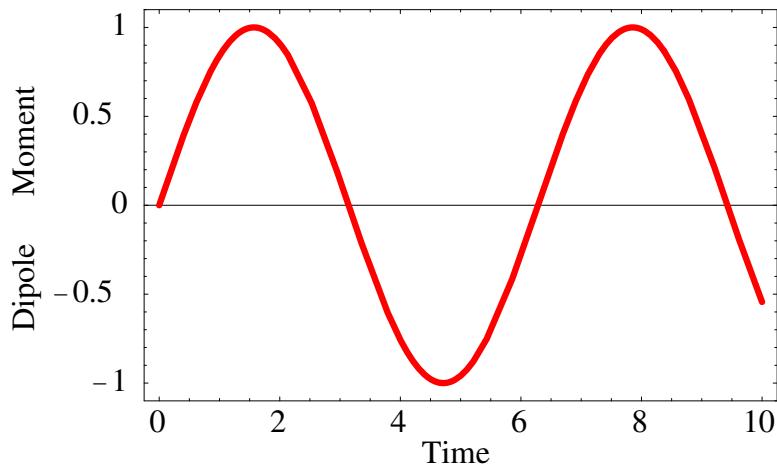


Figure 1: Time dependence of a dipole source oriented along the  $z$ -axis.

2. What would the magnetic field strength look like as a function of time at the same distance  $r$  from the source? In what direction does the  $\vec{B}$  field point? How are the directions of the  $\vec{E}$  and  $\vec{B}$  field related? Draw a dashed line on Figure 1 to represent the magnetic field strength.

### Activity 3: Simulating Electromagnetic Waves

We are now going to use a computer simulation to investigate the behavior of our oscillating dipole. The situation we are exploring is very similar to the generation of radio waves with an antenna. Charges (usually electrons) are driven up and down in the antenna and emit electromagnetic waves.

1. Open *Internet Explorer (IE)* and go to the website <http://www.falstad.com/emwave2>. A Java applet will pop up showing brightly colored waves like the ones in Figure 2 propagating outward from our oscillating electric dipole. If you don't see this window, consult your instructor.

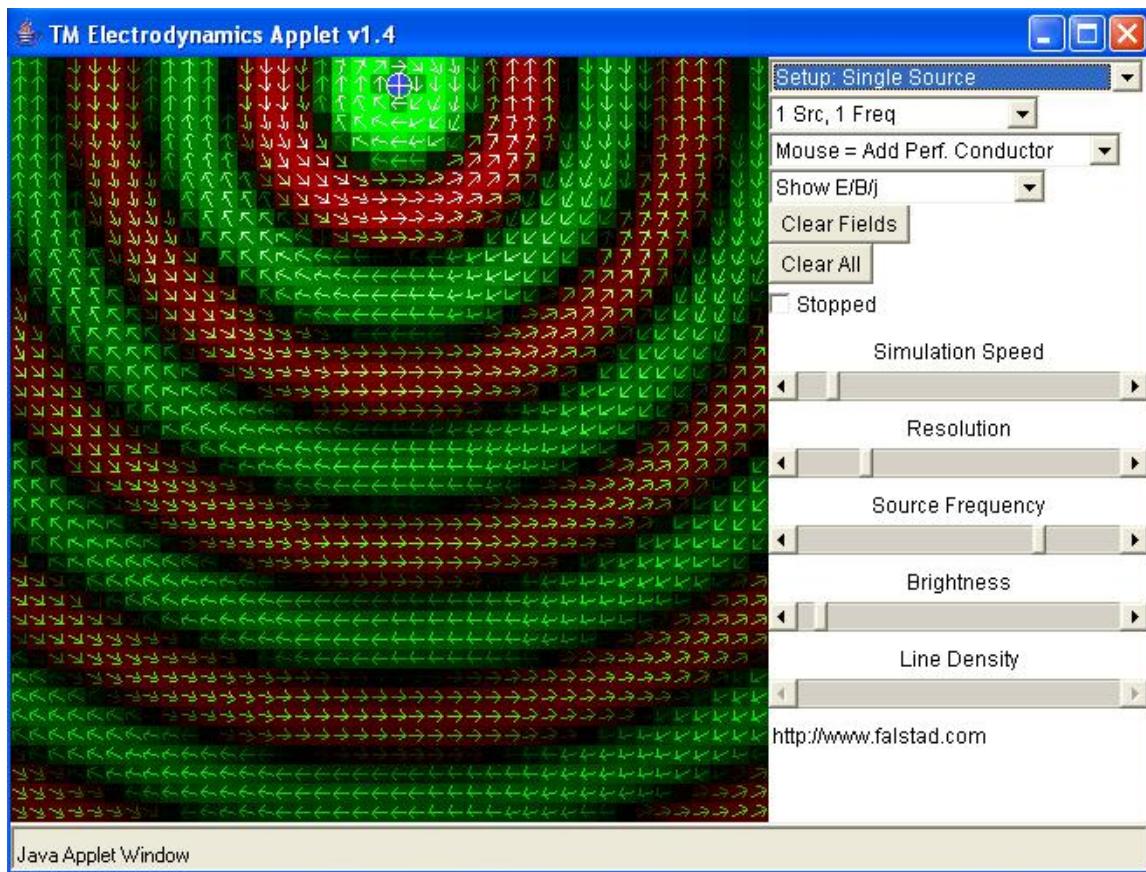


Figure 2: Applet showing electromagnetic waves.

2. It is useful to slow down the simulation speed to observe the waves more clearly. Do this using the slide labeled **Simulation Speed** on the right-hand side of the applet window. The alternating yellow and blue circle at the top of the applet is the source (the dipole) as viewed from above. The color indicates the electric field; green areas are positive ( $\vec{E}$  toward you) and the red areas are negative ( $\vec{E}$  away from you).

The electric field is always perpendicular to the plane of the screen. In addition to the red and green color, you will see arrows which indicate the direction of the magnetic field (which is always in the plane of the screen). Also, sources and conductors may show a blue or yellow color, indicating current. Yellow means that current is flowing toward you and blue means it is flowing away from you. Describe what you see in your own words.

3. Are these waves spherical or plane waves? Why?
4. What is the orientation of the  $\vec{E}$  field? What is the orientation of the  $\vec{B}$  field? How are the two related?
5. Go back to your predictions in Activity 2 about the directions of the  $\vec{E}$  and  $\vec{B}$  fields. Compare your predictions with these observations. Correct any disagreements.
6. Reduce the frequency of the oscillation of the dipole using the slide labeled **Source Frequency** on the right-hand side of the applet. You may have to increase the brightness of the applet using the slide labeled **Brightness**. What happens to the distance between equal positions on successive waves (*i.e.*, successive peaks or crests of the waves defined by the red regions in the applet)? This distance is called the wavelength and it is characteristic of a particular wave. Light, for example, is an electromagnetic wave and different wavelengths correspond to different colors.
7. One can calculate a quantity known as the Poynting vector which points in the direction of the flow of energy in an individual wave. Make the applet draw the Poynting vector by clicking on the arrow in the box with **Show E/B/j** entered in it. Scroll up or down until you find **Show Poynting vector** and highlight it. In what direction does the energy flow?
8. Go back to **Show E/B/j** mode you were in before. Just for fun, we want to introduce one of the central phenomena associated with waves known as interference. Go to the second menu from the top of the right-hand side of the applet. Click and highlight **2 Src, 1 Freq**. This will place an additional oscillating dipole at the bottom of the applet. Describe what happens. When waves add together they can cancel

one another (this is called destructive interference). Do you observe destructive interference? What do you think happens for constructive interference?

- Now use your mouse to grab one of the sources and drag it to a position near the other one. How does the interference pattern change? Try different positions for the source (move it closer or further away). Describe what you see. If these were light waves, where would you see bright light? Where would you see dark regions?

#### Activity 4: Plane Waves

You just completed a study of spherical waves emitted by a dipole source. We now want to consider another type of wave that we will study when we explore light.

- Use IE to go to the site <http://www.amanogawa.com/archive/PlaneWave/PlaneWave-2.html> and a Java applet will appear like the one in Figure 3. If you don't see this window, consult your instructor. The top panel of the applet shows a plane, electromagnetic wave. The red lines represent the direction and magnitude of the  $\vec{E}$  field and the blue ones do the same for the  $\vec{B}$  field. NOTE: The magnetic field in this applet is labeled  $\vec{H}$ . In this case, this is exactly the same as our previous  $\vec{B}$ . To eliminate some unnecessary lines, click the box beside **Phasors** so the check in the box disappears.

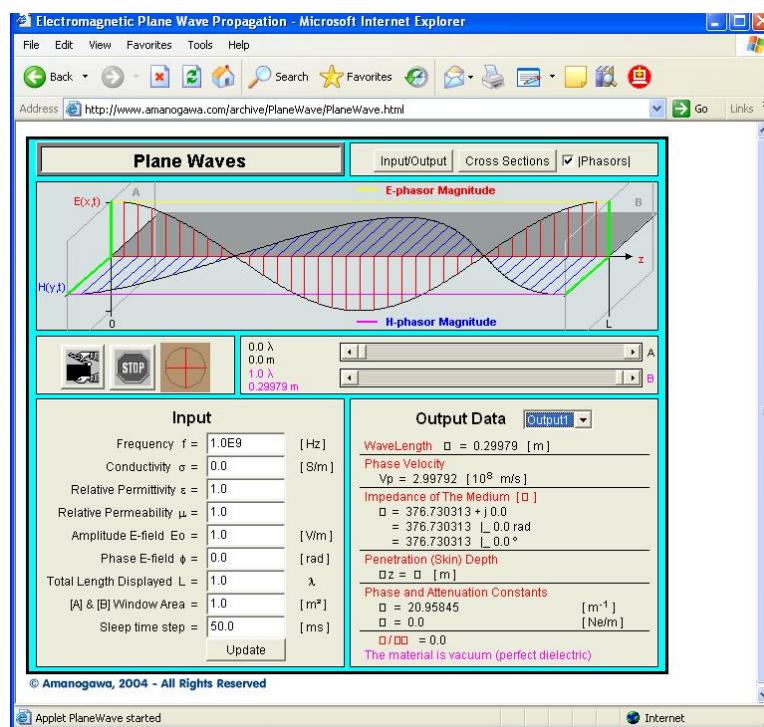


Figure 3: Applet showing plane electromagnetic waves.

2. Why do you think it's called a plane wave?
  
  
  
  
  
  
3. What is the orientation of the  $\vec{E}$  field? What is the orientation of the  $\vec{B}$  field? Compare your answer here with your predictions in Activity 2 and your observations in part 3.4. Correct any disagreements.
  
  
  
  
  
  
4. Click the start button to watch the wave move or propagate. The button is the one to the left of the STOP button on the top of the applet. Describe what happens to the electric and magnetic fields and how they are related (*i.e.* When the  $\vec{E}$  is large, what is the  $\vec{B}$  field doing?).
  
  
  
  
  
  
5. Click on the **Cross Sections** button at the top of the applet. You will see new panels that show the  $\vec{E}$  (red) and  $\vec{B}$  (blue) vectors in cross section at the planes A and B labeled in the top panel. Use these vectors to confirm your observations in parts 4.3-4.4.
  
  
  
  
  
  
6. Use the B slide at the bottom of the applet to move the B cross section position. Set it one-half wavelength from the A cross section. How are the  $\vec{E}$  and  $\vec{B}$  vectors in the A cross section related to their partners in the B cross section. What will be the total electric and magnetic fields if two waves are added that are out of line by one-half wavelength?

## 38 Electromagnetic Induction II

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

**Introduction.** In this lab, we'll take a further look at Faraday's Law, which says that a changing magnetic field can produce an emf (that is, a voltage) in a loop of wire. Current will be supplied to a large coil (which is a series of loops of wire), resulting in a magnetic field. If the current were constant in time, then a steady magnetic field would be produced. Since we want to produce a changing magnetic field, we supply the coil with *alternating current*, that is, current that flows back and forth in alternating directions, oscillating in a sine-wave fashion. The result is an ever-changing magnetic field, which induces an emf in a separate small coil.

### Apparatus

- one large wire coil
- one small wire coil
- two banana plug leads with alligator clips
- voltage sensor
- Pasco 750 Interface

**Setup.** The apparatus should be wired up to the computer interface in the following way: The two ends of the large coil should be connected to the output terminals on the right side of the interface (using the banana plug leads with alligator clips). These output terminals supply alternating current to the large coil. The two ends of the small coil should be connected to a voltage sensor, which is plugged into port A on the interface. Place the small coil in the center of the large coil. Both coils should be arranged horizontally (that is, with the axes pointing straight up and down). If you are not sure whether things are hooked up correctly, consult your instructor.

**Activity 1: Qualitative predictions.** Suppose that the top graph on the last page of this experiment represents the current flowing through the large coil as a function of time. When  $I$  is positive, that means current is flowing in one direction around the coil, and when  $I$  is negative the current is flowing in the other direction. On the axes below the top graph, sketch the shapes of the following other quantities. In each case, don't worry about the overall scale on the vertical axis; just focus on the shapes of the graphs, and particularly the locations of the peaks and valleys. Hint: some of the graphs will be exactly the same shape as the given graph of  $I$ .

1. On the first set of axes, sketch the vertical component of the magnetic field,  $B_z$ , at the center of the large coil (realizing that this field is directly proportional to the current in the coil).
2. On the next set of axes, sketch the magnetic flux  $\Phi_B$  passing through the small coil. (This flux is proportional to the magnetic field produced in the large coil).
3. On the bottom set of axes, sketch the emf  $\mathcal{E}$  induced in the small coil (which, by Farady's law, is proportional to the negative time derivative of the flux through the coil).

The computer will measure and graph  $I(t)$ , the current in the large coil, and  $\mathcal{E}(t)$ , the emf in the small coil. We want to test whether the two graphs are related to each other in the way you predicted. To be specific, make the following predictions from your graphs before trying the experiment:

1. At times when  $I(t)$  is at its maximum value, what is  $\mathcal{E}$ ?

2. At times when  $I(t)$  is at its minimum value, what is  $\mathcal{E}$ ?

3. At times when  $I(t) = 0$ , what is  $\mathcal{E}$ ?

### Activity 2: Testing the predictions.

Start the program called “induction2.” You should see two graphs, one showing the current in the large coil and one showing the emf (voltage) across the small coil. Both graphs will be blank until you start taking data, of course. There should be an additional box that allows you to control the frequency of oscillation of the alternating current. Leave this alone for now; we’ll mess around with it later.

Hit “Start” and take data for about 5 seconds. Do the shapes of the two graphs generally match your predictions from Activity 1? (To answer this, you may want to expand the scales of the horizontal axes of the graphs so you can see the shapes better, and also align the graphs so that the times are synchronized.)

If the answer to this question is “yes,” then go on. If it’s “no, the shape of the second one is exactly the opposite of what I expected,” then you just have the small coil upside-down. Turn it over and try again. If the answer is not one of those two, then consult your instructor.

Let’s be a bit more specific in comparing the predictions with the data. Examine the current graph, and record the times of the first four moments when the current reaches a maximum. Then look at the other graph, and record the emf at those same four times. Are the results consistent with your prediction number 1 in Activity 1?

Perform a similar analysis to check your predictions 2 and 3 from Activity 1. In each case, check several appropriately-chosen moments of time to see if the relationship between  $I$  and  $\mathcal{E}$  is as you predicted.

Suppose that you rotate the small coil  $90^\circ$ , so that its axis is horizontal. What do you predict about the resulting emf? Test your prediction and record the results here.

**Activity 3: A more quantitative analysis.** In Activity 1, we just looked at the shapes of the graphs, without worrying about the exact details. Let's do the same analysis a bit more precisely. The current in the coil is described by the equation

$$I(t) = I_{\max} \sin(2\pi ft).$$

Here  $I_{\max}$  is the maximum value of the current, and  $f$  is the frequency (that is, the number of cycles per second). Assuming that the large coil consists of  $N_1$  circular loops of wire, each with radius  $R$ , write down a similar expression for  $B_z(t)$ , the magnetic field at the center of the coil.

Now suppose that the small coil consists of  $N_2$  loops of wire, each having area  $A$ . Write down an expression for  $\Phi_B(t)$ , the flux through the small coil.

Using Faraday's Law, give an expression for  $\mathcal{E}(t)$ , the emf in the small coil.

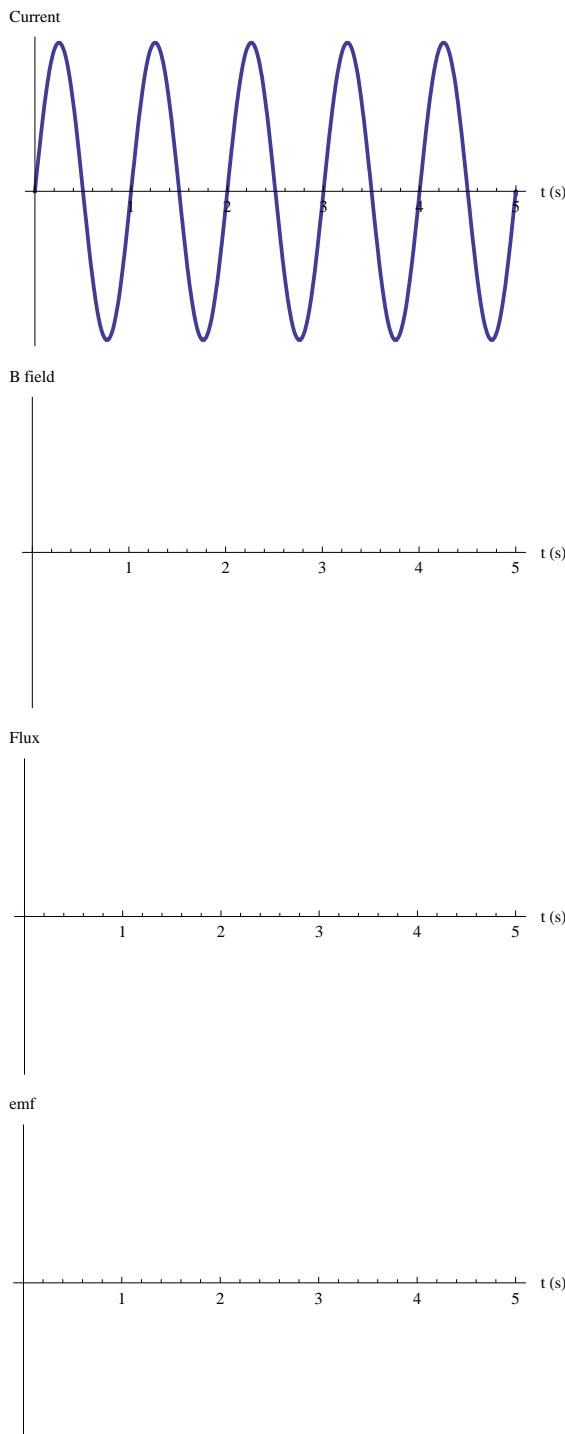
What is the maximum emf  $\mathcal{E}_{\max}$ ? Express your prediction in terms of quantities such as  $I_{\max}, f, A, N_1, N_2, R$ .

Rearrange this expression so that it has the quantities  $\mathcal{E}_{\max}$ ,  $I_{\max}$ ,  $f$  on the left-hand side, and everything else on the right.

We will now try an experiment in which we vary the frequency  $f$ . We'll find that this results in changes in both the current and the emf. But note that everything on the right-hand side of the above expression must remain constant throughout this experiment. That means that the expression on the left must also remain constant. We will test this prediction.

Record the frequency  $f$ , the maximum current  $I_{\max}$ , and the maximum emf  $\mathcal{E}_{\max}$  from your data. Then, using the box on your screen that controls the current in the coil, change the frequency to a higher value, and record these three quantities again. Repeat this a total of four times, so that you have four sets of values  $f$ ,  $I_{\max}$ ,  $\mathcal{E}_{\max}$ . Take care not to move the coils from one run to the next.

You made a prediction above that a certain combination of these three quantities would remain constant throughout this experiment. Calculate that quantity for each of the four data runs. Are the results consistent with your prediction?

**Graphs for Activity 1**

## 39 The LR Circuit

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

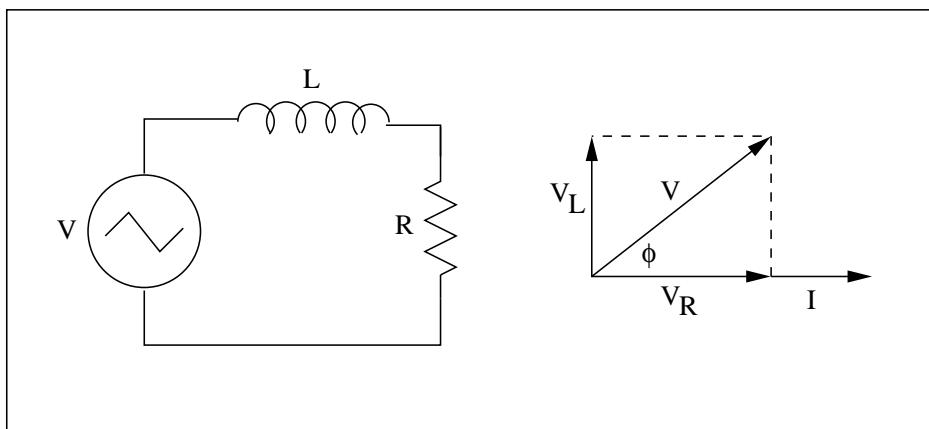
To investigate the relationships among the voltages in a series LR circuit.

### Apparatus

- DataStudio 750 Interface
- AC/DC Electronics Laboratory
- Resistor ( $5\Omega$ )
- Inductor and iron core
- Voltage sensors (2)
- Digital multimeter (DMM)
- LR Circuit activity
- Wires to complete the circuit

### Introduction

In this exercise you will study a series LR circuit. If a sinusoidally varying source of emf with a frequency  $f$  is placed in series with a resistor and a pure inductor, the current  $I$  will vary with the same frequency as the generator, but it will be shifted in phase by an angle  $\phi$  relative to the voltage of the generator. The voltage across each of the circuit elements has its own characteristic phase relationship with the current. The voltage across the resistor  $V_R$  is in phase with the current  $I$ , the voltage across the inductor  $V_L$  leads the current by  $90^\circ$ , and the voltage across the generator leads the current by the phase angle  $\phi$  whose value is dependent on the circuit parameters. Measurements of the voltage across each element in a series circuit containing an inductor, a resistor, and a sine wave generator will be used to perform a detailed study of the LR circuit.



Let's begin by considering the circuit obtained by placing a pure inductance  $L$  and a resistor  $R$  in series with a sine wave generator with a voltage amplitude  $V$  as shown in the figure above. Also shown in the figure is a phasor diagram for the circuit. Note that the voltage across the resistor  $V_R$  is in phase with the current  $I$ , the voltage across the inductor  $V_L$  is  $90^\circ$  ahead of  $I$ , and the generator voltage  $V$  is angle  $\phi$  ahead of  $I$ . From the phasor diagram we can write

$$V = \sqrt{V_L^2 + V_R^2}$$

and

$$\tan \phi = \frac{V_L}{V_R} = \frac{I\omega L}{IR} = \frac{\omega L}{R}$$

where  $\omega = 2\pi f$  is the angular frequency of the generator and we have used the inductive reactance and Ohm's Law.

Real inductors have both an inductance  $L$  and an internal resistance  $r$ . A real inductor can be represented by a pure inductance  $L$  in series with a resistance  $r$ . In the figure below a real inductor is shown in series with a resistor  $R$  and a generator of voltage amplitude  $V$ . Also shown in the figure is a phasor diagram for the circuit. The voltage across the real inductor is referred to as  $V_{ind}$ . There is, of course, some voltage  $V_L$  across  $L$  alone, and some voltage  $V_r$  across  $r$  alone. However, there can be no direct measurement of  $V_L$  or  $V_r$  since they are from the same length of wire. The only quantity which can be measured is  $V_{ind}$  which is the vector sum of  $V_L$  and  $V_r$ . Applying the law of cosines to the triangle formed by  $V$ ,  $V_R$ , and  $V_{ind}$  leads to the following equation.

$$V_{ind}^2 = V^2 + V_R^2 - 2VV_R \cos \phi$$

Solving this equation for  $\cos \phi$  we get

$$\cos \phi = \frac{V^2 + V_R^2 - V_{ind}^2}{2VV_R}$$

Further examination of the phasor diagram shows that the unknown voltages  $V_L$  and  $V_r$  can be determined from  $V$ ,  $V_R$ , and  $\phi$  by the following:

$$V_L = V \sin \phi$$

$$V_r = V \cos \phi - V_R$$

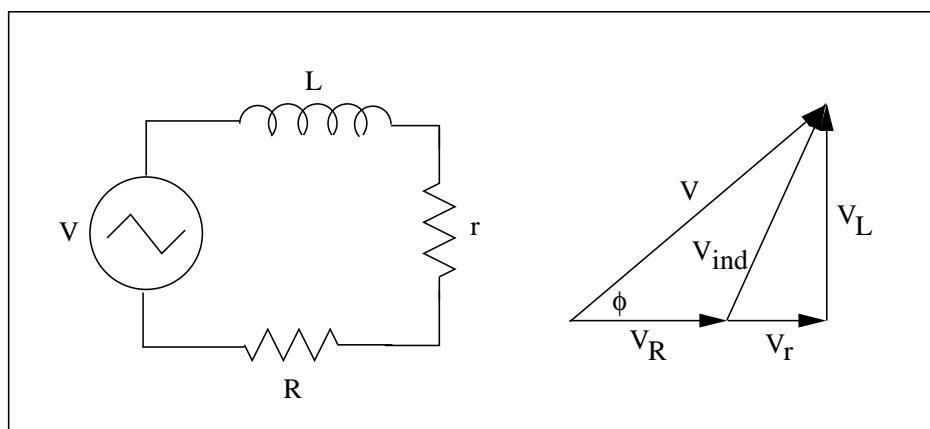
The current  $I$  can be related to the voltage across each element by the following equations:

$$V_L = I\omega L$$

$$V_R = IR$$

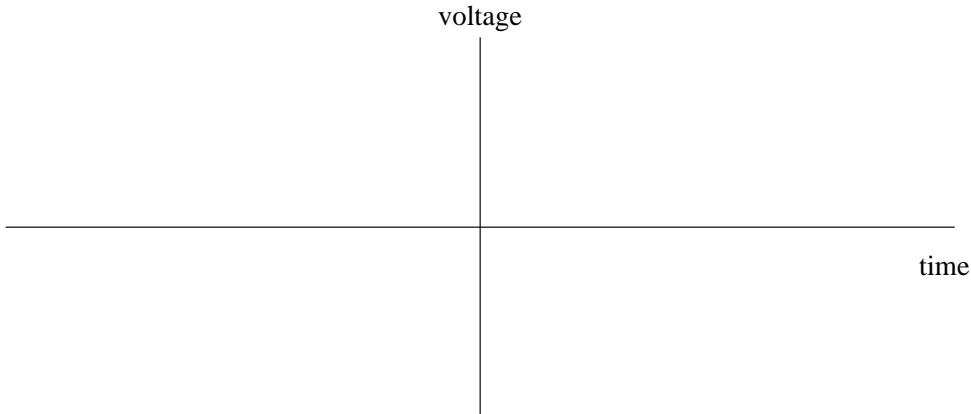
Once  $V_L$  and  $V_r$  have been determined, the equations above can be used to solve for  $\omega L$  by eliminating  $I$  to get

$$\omega L = R \frac{V_L}{V_R}$$



**Activity 1: Voltage in the LR Circuit**

In this laboratory you will be using a voltage source that varies sinusoidally in time. What will the voltage from this source look like as a function of time? In the space below sketch your answer. What is the phase relationship between the source voltage and the voltage drop across an inductor? Sketch that relationship on the same graph below. Label your curve  $V_{ind}$ . What is the phase relationship between the voltage drop across a resistor in the LR circuit and the source voltage? Sketch it below and label it  $V_R$ .

**Activity 2: Measuring Voltages**

(a) Construct a data table with 2 columns and nine rows in the space below. In the first column, label the rows  $R$  ( $\Omega$ ),  $r$  ( $\Omega$ ),  $V$  (V),  $V_R$  (V),  $V_{ind}$  (V),  $\phi$  (deg),  $V_L$  (V),  $V_r$  (V), and  $L$  (H).

(b) Open the *LR Circuit* activity in the 132 Workshop folder in the Programs menu. Go to the ‘Signal Generator’ window and set the frequency to 50 Hz, the amplitude to 3.0 V, and the wave form to ‘Sine Wave’. If you can’t find the ‘Signal Generator’ window, click the ‘setup’ button at the top of the DataStudio menu. You will see an image of the Pasco 750 interface with several connections marked. Single click on the one called ‘Output’. The ‘Signal Generator’ window should pop up. Set the values on it and return to the original voltage graph window.

(c) Set up the circuit on the AC/DC Electronics Laboratory so that it corresponds to the one shown in the second figure. The iron core (a long metal cylinder) should be inserted in the inductor. The inductor is the large open, cylinder in the upper-right-hand part of the AC/DC Electronics Laboratory. The computer is set up to measure the voltage across each of the circuit elements. The resistor in the circuit has a value  $R \approx 5\Omega$ . Check this value with the DMM and record it as  $R$  on your data sheet. Measure and record the value for  $r$  with the DMM. **NOTE: These values must be measured with  $R$  and  $L$  disconnected from the circuit.**

(d) Take data by clicking the *Start* button. The computer will display three sinusoidal voltage signals. The color scheme to identify each voltage measurement is shown on the right of the voltage graph window. Determine which trace corresponds to  $V$  (the source signal),  $V_R$  (the resistor voltage), and  $V_{ind}$  (the inductor voltage). Print a copy of this graph and attach it to this unit. Check that the phase relationships agree with the phasor diagram in the second figure above, i.e.  $V_{ind}$  leads  $V$ ,  $V_R$  lags  $V$ .

$V_{ind}$  leads  $V$ ?

$V_R$  lags  $V$ ?

(e) Measure the amplitude of each of the signals and record these quantities in your data table. This can be done using the SmartTool (see Appendix B), or read directly from the graph.

#### Activity 3: Analyzing the Circuit

(a) Calculate the remaining quantities necessary to complete your data table. Show the calculations in the space below.

(b) Determine and record the value of  $L$  for your inductor. Make careful note of the particular inductor and AC/DC Electronics Laboratory which you have used. You may need to identify it and use it again in another laboratory exercise.

(c) Construct to scale a phasor diagram like the one shown in the second figure in the space below, based on your data.

(d) From the three sinusoidal voltage signals found in 2(d) above, determine the phase difference between the resistor voltage and the output voltage, convert to degrees and compare with your calculated value of  $\phi$ . You may want to expand the time axis on the graph to do this more accurately. Also, determine the phase difference between the resistor voltage and the inductor voltage, convert to degrees and compare with the corresponding angle in your phasor diagram above. Present the results here.

#### Activity 4: Determining L from Half-life Measurement

- (a) Delete the sinusoidal graphs from Activity 2.
- (b) Go to the 'Signal Generator' window and change the wave form to 'Positive Square Wave' and the frequency to 30 Hz.
- (c) Take data by clicking the *Start* button. The computer will display a square wave for the output voltage, and rising and falling exponential curves for the voltages across the resistor and inductor, characteristic of an LR circuit energized by a constant emf.
- (d) Measure the half-life from one of the exponential curves.
- (e) Determine the time constant  $\tau$  from  $\tau = \text{half-life}/\ln 2$ .
- (f) Determine the inductance  $L$  from  $\tau = L/R$ .

Does the result agree with what you found in Activity 3? What is the percent difference?

## 40 Refraction of Light

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

To investigate the path traveled by light through a plate of plexiglass (a transparent solid material).

### Introduction

The speed of light depends on the medium in which it travels. In passing from one medium, at least some light energy is reflected. If the second medium is transparent, most of the light will pass into and through it. If the beam is not perpendicular to the boundary between the two media, it will bend as it enters, an effect known as refraction. The direction a single ray of light travels when refracted is given by Snell's law:

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

where

$i$  = incident angle

$r$  = refracted angle

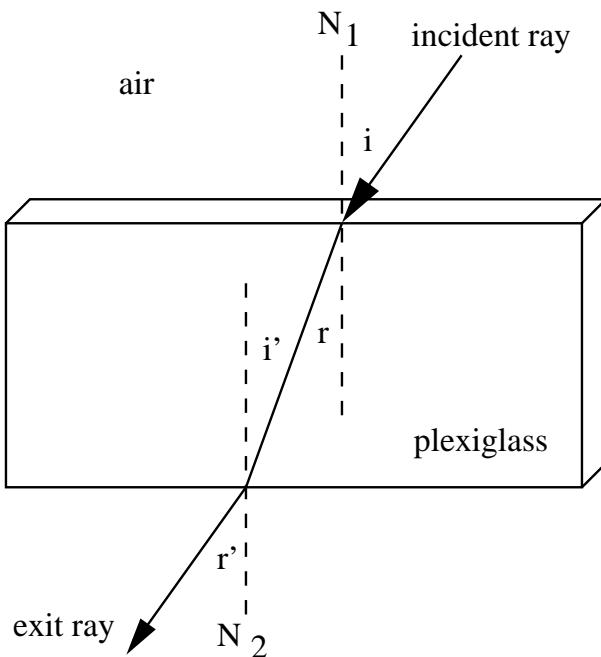
$v_1$  = light speed in medium 1

$v_2$  = light speed in medium 2

$n_1$  = index of refraction of medium 1

$n_2$  = index of refraction of medium 2

**Note:** All angles are measured from the normal to the boundary at the point the ray enters (or leaves) the medium. The index of refraction is the ratio of light's speed in a vacuum,  $c$ , to its speed in the medium,  $v$ :  $n = c/v$ . It is worth remembering that  $n_{air} \approx 1.00$ .



## Apparatus

- light fence
- plexiglass block
- white paper, pins, and wood board
- protractor

## Activity

1. Put a plexiglass plate at the center of a piece of paper. Outline its position. Identify a normal,  $N_1$ , perpendicular to an edge of the plate.
2. Arrange the light source apparatus so that the parallel rays of light cross the paper and are incident at a  $30^\circ$ - $35^\circ$  angle to the normal. Trace one of these rays.
3. Sight the corresponding ray as it emerges from the other side of the plexiglass. Trace this ray.
4. Remove the plexiglass and connect the points where the light ray entered and left the plexiglass with a straight line. This represents the path of the light ray as it passes through the plexiglass.
5. Construct the normal,  $N_2$ , and measure and record  $i$ ,  $r$ ,  $i'$ , and  $r'$ .
6. Repeat the above procedure for two more incident angles between  $40^\circ$  and  $60^\circ$ .
7. Plot a graph of  $\sin i$  (on the vertical axis) as a function of  $\sin r$  (on the horizontal axis) including the point  $(0,0)$  which corresponds to a light ray incident perpendicular to the plexiglass. Include a linear trendline and don't forget to include a title and axis labels.
8. Use the LINEST function in *Excel* (see **Appendix C**) to determine the slope of the graph ( $n_{plexiglass}$ ) and its uncertainty.
9. Does  $i = i'$ ? Explain.

10. Does  $r = r'$ ? Explain.
  11. Are the incident and exit rays parallel? Explain.
  12. What is the speed of light in the plexiglass?
  13. Under what conditions would a refraction angle be greater than an incident angle?
  14. Collect the results from the other lab groups and determine an average and standard deviation for the index of refraction of the plexiglass.

## 41 Refraction at Spherical Surfaces: Thin Lenses

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To investigate thin lenses.

### Introduction

A lens converges or diverges light rays. It is a transparent material bounded, in the case of thin lenses, by spherical edges. The line between the centers of curvature of these edges is referred to as the principal axis. The principal focus is the point on the principal axis where parallel incident rays converge. The distance from the lens to this point is known as the focal length. The relation between the focal distance,  $f$ , the object distance,  $p$ , and the image distance,  $q$ , is:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}.$$

### Apparatus

- light fence
- converging (convex) lens in holder
- converging and diverging lenses (1 each) without holders
- optical bench
- light source with 12v output power source
- optical viewing screen (white)
- plastic ruler

### Activity 1

- Arrange the light source apparatus so that the parallel rays of light cross a piece of paper.
  - Place a convex lens (without holder) on the paper perpendicular to the central ray. Outline its position and the path of the rays. Pay particular attention to the condition near the principal focus.
  - What is the focal length of this lens?
- 
- Include a sketch of the light rays in your lab book, showing the focal point and focal length of the lens.

### Activity 2

- Place the light source in its bracket at one end of the optical bench. The arrow on the light source will be the object in this investigation. Measure and record its height,  $h_0$ .

- Place a converging lens in its holder on the optical bench 70 cm from the object (this is the object distance  $p$ ). Turn on the light source and position the optical viewing screen on the optical bench so that a sharp image of the arrow appears on the screen. Measure the distance of the screen from the lens. This is the image distance  $q$ . Also measure the height of the image  $h_i$ . Record  $p$ ,  $q$ , and  $h_i$  in the first line of the following table.

$p$	$q$	$h_i$	$\frac{h_i}{h_0}$	$\frac{q}{p}$	$f$

- Move the lens to create four more object distances of 60, 50, 40, and 30 cm. In each case, measure the image distance and the height of the image and record in the above table. Calculate and record the ratios of the image and object heights,  $h_i/h_0$ , and the image and object distances,  $q/p$ . Record these values in the above table. You should now have the first five columns of the above table filled in.
- Calculate and record the focal length,  $f$ , for each observation. Show one of the calculations here:

- Determine an average focal length,  $f_{ave}$ , and a standard deviation based on your five values.
- What is the relationship between the ratio of the image to object heights and the ratio of image and object distances? The first ratio is called the magnification.
- Replace the converging lens with a diverging one. Try to obtain a real image on the viewing screen.
- Why can you not form a real image with a diverging lens?

### Activity 3

- Repeat the procedure of Activity 1 with a concave lens. Locate the principal focus by extending the refracted rays backwards.
- What is the focal length of this lens?
- Include a sketch of the light rays in your lab book, showing the focal point and focal length of the lens.

## 42 The Interference of Light

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To investigate the interference of light waves as they pass through a set of slits.

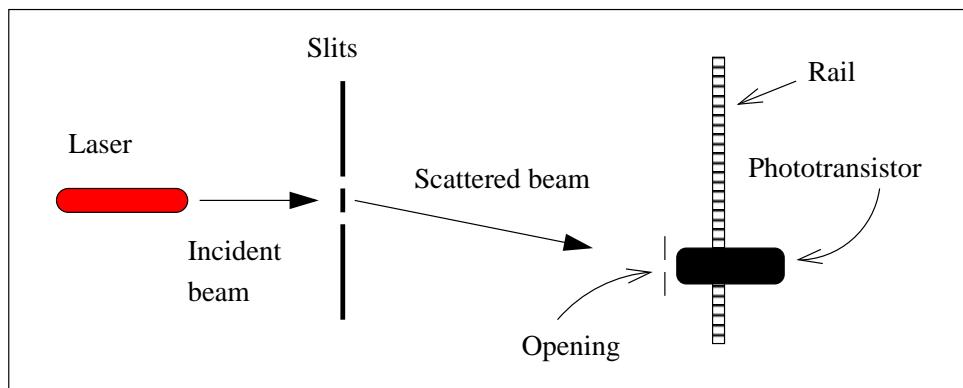
### Apparatus

- Basic optics diode laser
- Optical bench with rotary motion sensor
- Phototransistor for measuring light intensity (mounted on rotary motion sensor)
- “Multiple Slit Set” slit accessory
- Glass plate with sets of narrow slits
- DataStudio 750 Interface*

### Introduction

In this laboratory you will investigate the interference of light produced by a laser beam passing through a set of narrow, adjacent slits. When light passes the slits each opening acts as an independent source of waves that can overlap one another to produce a distinctive pattern of bright and dark spots on a screen. The position of the bright spots depends on the separation of the adjacent slits and the wavelength of the incident light.

You can measure this interference pattern with the setup shown below. (This is a top view of the set-up.) A phototransistor is seated behind the narrow opening on top of the large, metal mount sitting on a rail. The phototransistor can translate the intensity of the light falling on it into a voltage signal that can be read by the computer. In addition, the phototransistor can be moved back and forth on a rotary motion sensor that measures the position of the mount. These two signals can be combined to make a graph of the intensity as a function of position.



In this unit you will pass light from the laser through slits of known separation and use the interference pattern to determine the wavelength of the light.

### Activity 1: An Alternative View

Isaac Newton believed that light was made up of small, unseen particles that obeyed (surprisingly enough!) Newton's Laws. This view is known as the corpuscular theory. We want to consider how this model of light predicts different behavior from the wave theory.

(a) Consider a laser beam shining on a circular hole. If a beam of light consisted of small, unseen particles that behaved as tiny billiard balls what would you see on a screen that is downstream from the circular hole? A sketch might be useful here.

(b) Now consider the same laser beam shining on a pair of narrow slits. What would you see on a screen downstream from the slits if light were made of corpuscles?

For the questions above you should have predicted that the laser would form a single bright spot (for part a) or two parallel lines (for part b). The experiment you are about to perform provided compelling evidence that Newton's corpuscular theory was wrong.

#### Activity 2: The Interference of Light

(a) You are now ready to turn on the laser. DO NOT LOOK DIRECTLY INTO THE BEAM OR POINT THE LASER CARELESSLY ABOUT THE ROOM. Mount the laser on the optical bench at the opposite end from the rotary motion sensor. Turn on the laser and you should see the bright red spot of the beam striking the rotary motion sensor assembly. On the "Multiple Slit Set" accessory, select a double slit of width .04 mm and separation .125 mm. Rotate the wheel so that the double slit is at the center of the opening.

(b) Position the double slit about 70 cm from the phototransistor mount. Adjust the laser beam direction so that it falls on the double slit you have selected. (There are two adjustment screws on the back of the laser.) You should see the interference pattern on the phototransistor mount.

(c) Position the phototransistor mount so the interference pattern is at the same height as the opening in the center of the phototransistor mount. The phototransistor is mounted behind this hole. To make accurate measurements it is important to carefully determine the geometry of your setup. Check to see if the slits and the phototransistor mount are perpendicular to the incident laser beam. You want to make sure the phototransistor can "see" as many bright spots as possible. Carefully slide the phototransistor mount back and forth to make sure that it stays centered on the interference pattern. Then set it at one side of the pattern to begin the experiment. Start the "Interference" activity in the **132 Workshop** folder. When you are ready, click **Start** and slowly move the phototransistor from one side of the slide to the other by turning the wheel on the rotary motion sensor. Move carefully and take about 4-5 seconds to complete the motion. Click **Stop**. When data acquisition is complete you will see a graph representing the intensity reading versus the position reading. You should see several distinct peaks. This graph is the interference pattern. If you do not see this pattern consult your instructor. Add the title "Double Slit Interference/Diffraction Pattern" and the axis labels "Relative Intensity" (vertical) and "Position (m)" (horizontal). Make a hardcopy of this graph and attach it to this unit.

(d) In theory, the interference pattern should look like the figure at the bottom of page 915 of your textbook (also shown in Fig. 2 of the next experiment). The intensity is just a  $\cos^2$  function. Obviously, the pattern doesn't look like that. The reason is that light from each slit undergoes single slit diffraction which we will study in the next experiment. (That's why we are calling the pattern an "interference/diffraction" pattern.) But the peaks should still be equally spaced. Is the spacing between the intensity peaks constant? Is the intensity of each peak the same? Does it appear that any peaks are missing? The more peaks you see the more (and hopefully better) data you can collect. There is a button on top of the phototransistor labeled "Gain" which changes the size of the intensity signal. Try changing this setting to see if you can get more peaks in your spectrum (do this only if necessary).

(e) Now switch to 3 slits by rotating the wheel on the slit accessory appropriately, and run the pattern again. You will see that the peaks are narrower, because the pattern is not just a  $\cos^2$  function; it is more complicated. Now try 4 slits and then 5 slits. In each case the peaks are a bit narrower, although overall the pattern is similar because you haven't changed the slit width or the slit separation.

(f) Now switch to the center opening in the glass plate. This has 15 or 20 slits, so the peaks will be narrower still. In this case the slit separation is 0.03295 mm, so the maxima will be farther apart than in the previous cases. Run the pattern again. Measure the distance from the glass plate to the phototransistor as accurately as possible (the distance  $L$  below), realizing that the phototransistor lies 25.4 mm behind the opening in the mount.

(g) When you are satisfied with the quality of your spectrum record the position of each peak in the left column of the table below, with the central peak halfway down the column. Use the **Smart Tool** to accurately read off the peak positions by clicking on the appropriate button along the top of the graph. A set of cross-hairs will appear on the plot. Grab the cross-hairs by clicking on them and dragging them to the point you want to measure. The coordinates will be printed by the cross-hairs. Turn off the laser when you are finished. Print the graph with the title "Multiple slit Interference/Diffraction pattern" and the same axis labels as before.

Position Reading (m)	Change in Reading (m)

### Activity 3: Determining the Wavelength of the Laser

(a) For the data you recorded in the previous activity, calculate the difference between each pair of adjacent readings and record it in the right column of the above table (first space will be blank).

(b) Calculate the average and standard deviation of the separation between adjacent peaks (right hand column values). Don't forget units.

- (c) The position of the interference maxima can be described by (see text p. 914)

$$y_m = \frac{\lambda L}{d} m$$

where  $y_m$  is the distance of a bright spot from the central maximum (the distance along the slide in this experiment) and  $L$  is the distance from the slits to the phototransistor. The quantity  $d$  is the slit separation,  $\lambda$  is the wavelength of the light, and  $m$  is the order of the bright spot. Generate an expression for the distance  $\Delta y$  between adjacent bright spots.

- (d) Use the expression you generated in part (c) and the average separation between bright spots you determined in part (b) to calculate the wavelength of the laser light and its uncertainty (both in nm). Show how you get the result in nanometers. Compare your result with the wavelength range indicated on the laser.

- (e) Collect the results for the wavelength from the other teams in class and calculate the average and standard deviation. Record the results here. (Don't forget units.) Are your results consistent with the class results? Why or why not?

- (f) Recall the earlier discussion of Newton's corpuscular theory of light. Does your data support Newton's theory or the wave theory? Why?

## 43 Diffraction of Light

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

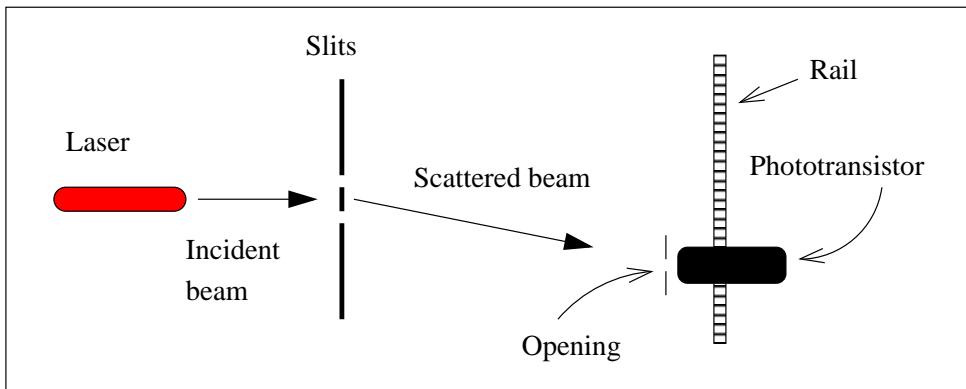
To investigate how the interference and diffraction of light waves combine to form a distinctive pattern and how that pattern can be used to measure the size of an object.

### Introduction

In this laboratory you will investigate the interference and diffraction of light produced by a laser beam passing through a set of narrow, adjacent slits. When light passes through a set of slits, each opening acts as an independent source of waves that can overlap one another to produce a distinctive pattern of bright and dark spots on a screen. The position of the bright spots depends on the separation of the adjacent slits and the wavelength of the incident light. In addition, diffraction produced by the individual slits modifies the intensity of each spot. To perform the following activities you will need:

- Laser.
- Phototransistor for measuring light intensity (mounted on rotary motion sensor).
- Set of narrow slits.
- *DataStudio 750 Interface*.
- Plumb line.

You can measure this interference/diffraction pattern with the setup shown below. A phototransistor is seated behind the narrow opening on top of the metal mount. The phototransistor can translate the intensity of the light falling on it into a voltage signal that can be read by the computer. In addition, the phototransistor can be moved back and forth on a rotary motion sensor that measures the position of the mount. These two signals can be combined to make a graph of the intensity as a function of position.



**Fig. 1.** View of diffraction apparatus from above.

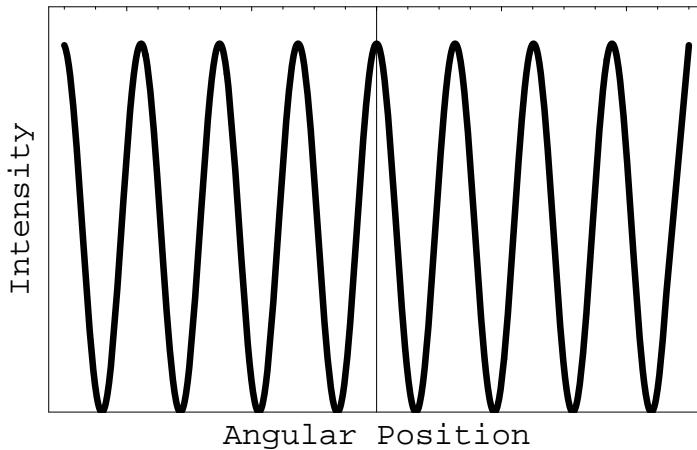
In this unit you will pass light of known wavelength through slits and use the diffraction pattern to determine the size of the individual slit through which the light passed.

### Intensity of Interference

For light that passes through two very narrow slits one can calculate a theoretical expression for the interference pattern that would be produced in such a situation. The expression is

$$I_{int} = 4I_0 \cos^2\left(\frac{\pi d}{\lambda} \sin \theta\right)$$

where  $I_{int}$  is the intensity of the light at the phototransistor,  $I_0$  is the maximum intensity of the incident light,  $d$  is the slit separation,  $\theta$  is the angular position of the scattered light relative to the incident beam, and  $\lambda$  is the wavelength of the light. This expression has a characteristic shape shown below. We will compare this prediction of “pure” interference (without diffraction effects) with the measured pattern in the next activity.



**Fig. 2.** Intensity distribution of pure interference.

#### Activity 1: The Interference of Light

(a) You are now ready to turn on the laser. DO NOT LOOK DIRECTLY INTO THE BEAM OR POINT THE LASER CARELESSLY ABOUT THE ROOM. Turn on the laser and you should see the bright red spot of the beam striking the wall. You should have a glass plate with a green border and several different slit arrangements on it. Place the opening in the center of the plate in the path of the laser beam. The adjacent slits in the center hole are 0.03295 mm apart. What do you see?

(b) Position the glass plate about 30-40 cm from the phototransistor apparatus with the beam striking the center opening. Place the apparatus so the intensity pattern is at the same height as the opening in the center of the device mounted on a movable slide. The phototransistor is mounted behind this opening. To make accurate measurements it is important to carefully determine the geometry of your setup. Check to see if the slits and the phototransistor mount are perpendicular to the incident laser beam. You also want the phototransistor to “see” as many bright spots as possible. Gently slide the mount back and forth to make sure the phototransistor stays centered on the interference pattern. You need to know the perpendicular distance from the center hole to the position of the phototransistor. Place the phototransistor mount at the position of the central maximum (the brightest spot) and measure the distance from the center hole of the slits to the front face of the phototransistor mount. You may find it useful to use the plumb line to measure this distance. Record your result. The phototransistor itself sits 25.4 mm behind the opening on the front face of the phototransistor mount.

(c) Start the “Interference” activity in the **132 Workshop** folder. When you are ready, click **Start** and slowly move the phototransistor from one side of the slide to the other by turning the wheel on the rotary motion

sensor. Move carefully and take about 4-5 seconds to complete the motion. When data acquisition is complete, click **Stop** and you will see a graph representing the intensity reading versus the position reading. You should see several distinct peaks. This graph is the interference pattern. If you do not see this pattern, consult your instructor. Make a hardcopy of this graph and attach it to this unit.

(d) Is the spacing between the intensity peaks constant? Is the intensity of each peak the same? Does it appear that any peaks are missing? How does the measured intensity spectrum compare with the one predicted for "pure" interference discussed above? What is different? What is the same?

(e) Measure and record the Position Reading and Intensity Reading of each peak in the table below. Use the **Smart Tool** to accurately read off the peak positions by clicking on the appropriate button along the top of the graph. A set of cross-hairs will appear on the plot. Grab the cross-hairs by clicking on them and dragging them to the point you want to measure. The coordinates will be printed by the cross-hairs. Turn off the laser when you are finished.

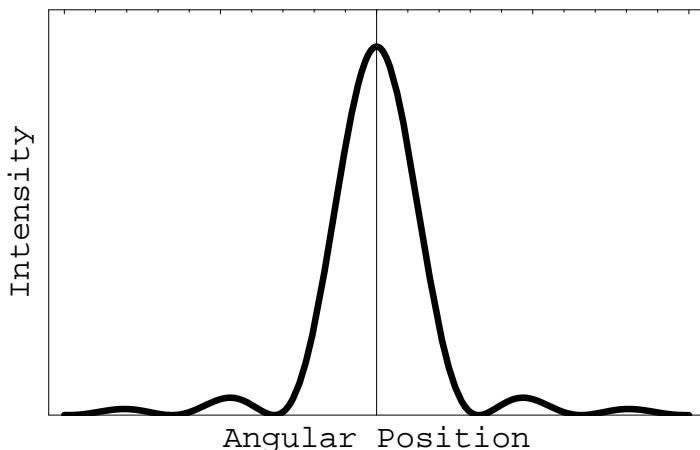
Position Reading (m)	Intensity Reading (%)

### Activity 2: Intensity of Diffraction

Applying the same theoretical techniques that were applied to interference (see equation above) one can derive a prediction for the intensity pattern due to diffraction of light passing through a single slit. The result is

$$I_{diff} = I_m \left( \frac{\sin(\frac{\pi a}{\lambda} \sin \theta)}{\frac{\pi a}{\lambda} \sin \theta} \right)^2$$

where  $a$  is the size of the single slit,  $\theta$  is the angular position of the phototransistor relative to the incident beam,  $I_m$  is the maximum intensity at the center of the diffraction pattern, and  $\lambda$  is the wavelength of the light. The shape of this intensity distribution is shown in Figure 3.



**Fig. 3.** Intensity distribution of diffraction from a single slit.

(a) The figure above shows the diffraction pattern has a central maximum with a series of points where the intensity goes to zero at positive and negative angles. When is the expression for the intensity in the equation above equal to zero?

(b) Using the result of part (a), what is the angular position of the minima on either side of the central maximum?

(c) Finally, generate an expression for the angular width of the central maximum in terms of  $a$ ,  $\lambda$ , and any other constants you need.

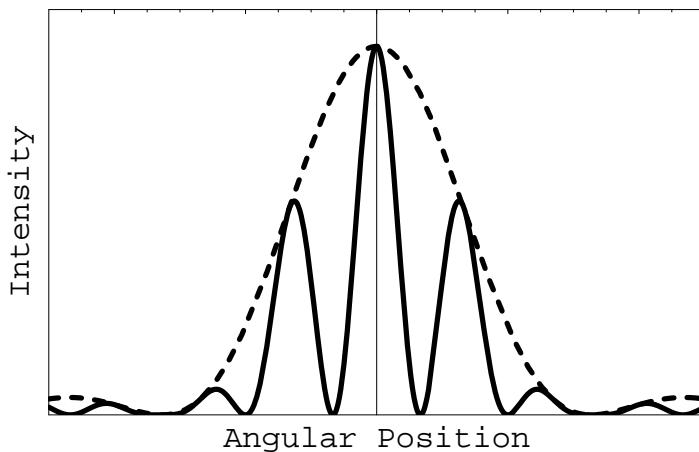
### Combining Interference and Diffraction

By now you should have realized that your measured intensity distribution does not completely agree with the distribution predicted by “pure” interference as represented by the first equation and Figure 2. When light passes through a pair of slits diffraction occurs at each individual slit and casts the characteristic pattern described by the second equation. At the same time there is interference between the light from different slits that creates an interference pattern described by the first equation. The net effect is a multiplication of these two equations to yield

$$I_{total} = I_m \cos^2\left(\frac{\pi d}{\lambda} \sin \theta\right) \left(\frac{\sin\left(\frac{\pi a}{\lambda} \sin \theta\right)}{\frac{\pi a}{\lambda} \sin \theta}\right)^2$$

where  $I_m$  is the intensity of the central maximum,  $\theta$  is the position of the phototransistor,  $d$  is the separation of

the slits,  $a$  is the size of an individual slit, and  $\lambda$  is the wavelength of the light. The shape of this distribution is shown by the solid curve in Figure 4.



**Fig. 4.** Intensity distribution of light passing through a pair of slits.

The intensity of the interference peaks is no longer constant, but is modulated by the diffraction envelope represented by the dashed curve. This dashed curve is a plot of the second equation normalized to the maximum intensity at zero degrees. The intensity of each peak in the distribution represents the intensity due to the diffraction effects. If more slits are added, then the widths of the individual peaks in Figure 4 become narrower, but their intensity remains the same. In the next Activity you will use your data to determine the diffraction pattern and the angular width of the central diffraction envelope. This width can be used to measure the size of the individual slits that produced the distribution.

### Activity 3: Measuring the Size of the Slit with the Diffraction Pattern

- (a) Does the intensity distribution you measured with the phototransistor resemble the pattern shown in Figure 4? If not, consult your instructor.

(b) In Activity 1 you recorded the position and intensity of the interference peaks you measured with the phototransistor. How would you calculate the angular position of each peak relative to the central maximum? A sketch might be helpful here.

(c) Use your expression to calculate the angular position of each interference peak that you recorded and enter your results in the table below. Plot intensity versus angular position. Does your plot resemble the shape of the diffraction pattern shown in Figure 3? If not, consult your instructor. Print the plot and attach it to this unit.

Angular Position (radians)	Intensity Reading (V)

(d) What is the angular width of the central maximum in your data? Use the expression from part 2(c) to calculate the size of the slit. The expected result is 0.015 mm. What is your percent difference? Use the wavelength for the laser light that you found in the unit on interference of light.

(e) Collect the results for the slit width from the other teams in class and calculate the average and standard deviation. Record the result here. Are your results consistent with the class results? Why or why not?

(f) Can you think of any other methods for measuring small separations like this accurately?

## 44 Diffraction 2

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To investigate the diffraction of light from a single slit and to see how this diffraction combines with interference from multiple slits.

### Apparatus

- Basic optics diode laser
- Optical bench with rotary motion sensor
- Phototransistor for measuring light intensity (mounted on rotary motion sensor)
- “Single Slit Set” slit accessory
- “Multiple Slit Set” slit accessory
- *DataStudio 750 Interface*

### Introduction

The set-up for this experiment is the same as for the Interference of Light experiment. Mount the laser on the optical bench on the opposite end from the rotary motion sensor.

#### Activity 1 - Single Slit Diffraction

(a) On the “Single Slit Set” slit accessory, select a slit of width .08 mm. Rotate the wheel so that this single slit is at the center of the opening. Position the single slit about 70 cm from the phototransistor mount. Measure the distance from the slit to the phototransistor carefully, realizing that the phototransistor lies 25.4 mm behind the opening in the mount. This is the distance  $L$  below. Record the distance here.

(b) Turn on the laser and adjust the beam direction so that it falls on the single slit you have selected. (There are two adjustment screws on the back of the laser.) You should see the diffraction pattern on the phototransistor mount.

(c) Position the phototransistor mount so the diffraction pattern is at the same height as the opening in the center of the phototransistor mount. The phototransistor is mounted behind this hole. Slide the phototransistor mount back and forth to make sure that it stays centered on the diffraction pattern. Then set it at one side of the pattern to begin the experiment. Start the “Interference” activity in the **132 Workshop** folder. When you are ready, click **Start** and move the phototransistor from one side of the slide to the other as you did in the Interference of Light experiment, taking 4-5 seconds to complete the motion. Click **Stop**. The diffraction pattern will appear on the computer screen. In theory, the pattern should look like Fig. 3 of the Diffraction of Light experiment (also see p. 921 of your textbook). Add the title “Single Slit Diffraction Pattern” and the axis labels “Relative Intensity” (vertical) and “Position (m)” (horizontal). Print the graph and attach it to this unit.

(d) Use the **Smart Tool** to measure the positions of the first minima on either side of the central maximum and record them here. Estimate an uncertainty in each measurement.

(e) Determine the difference between these two measurements (with uncertainty).

(f) The width of the central maximum is given by (see text p. 922)

$$\Delta y = \frac{2\lambda L}{a}$$

where  $\lambda$  is the wavelength of the laser beam,  $L$  is the distance from the slit to the phototransistor, and  $a$  is the slit width. Using your data from above, calculate the wavelength of the laser beam and its uncertainty.

### Activity 2 - Double Slit Interference/Diffraction Pattern

(a) Replace the “Single Slit Set” accessory with the “Multiple Slit Set” accessory that you used in the Interference of Light experiment. Select a double slit of width .04 mm and separation .125 mm. Position the double slit about 70 cm from the phototransistor, measure the distance carefully, and run the scan as you did in the Interference experiment. The result should look like Fig. 4 of the Diffraction of Light experiment. This is the same pattern you did in Activity 2 of the Interference experiment. Include an appropriate title and axis labels and print the graph. Use the **Smart Tool** to read the positions of the maxima in the left column of the table below, with the central peak half way down the column.

Position Reading (m)	Change in Reading (m)

(b) Calculate the difference between each pair of adjacent readings (as you did in the Interference experiment) and record it in the right column of the above table. Calculate the average and standard deviation of the separation between adjacent peaks and record them here.

(c) Calculate the wavelength of the laser beam as you did in the Interference experiment. Determine an uncertainty and express both in nanometers.

## 45 The Diffraction Grating

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To determine the wavelength of laser light using a diffraction grating.

### Introduction

Light bends (a bit) around corners. This phenomenon is called diffraction. Interference, or the overlap of waves, is the basis for diffraction. In a transmission grating, lines, about 4,000 to 8,000 per centimeter, are ruled onto glass. The unruled portions of the glass act as slits. The interference, and thus diffraction, which results from shining a beam of light through the grating permit the measurement of the wavelength of the light. The relevant relationship, known as the grating equation is:

$$n\lambda = d \sin \theta$$

where  $n$  is the order of the spectrum (the number of bright spots from the center),  $\lambda$  is the wavelength in nanometers ( $10^{-9}$  meters),  $d$  is the separation in nanometers between grating lines, and  $\theta$  is the angle of deviation from the light beam's original direction through the grating (the angle of diffraction).

### Activity

1. Record the separation between grating lines:  $d =$
2. Turn on the laser, being careful to avoid looking directly into the beam or shining it at anyone. Aim the light beam through the diffraction grating so that a horizontal series of dots appears on the wall. Adjust the positions of the laser and grating until you easily see at least two dots on either side of the brightest (central) dot.
3. Are the dots of the interference/diffraction pattern the same intensity? Describe the pattern you observe.
4. Measure and record the distance from the grating to the wall,  $L$ , in the table below, as well as the distances from the central dot to the first dot to the right,  $x$ , and the first dot to the left,  $x'$ . Compute the average of these  $x$  values and record it as  $x_{ave}$ .
5. Compute and record a value for  $\theta$  by using the appropriate trigonometric relation between  $L$  and  $x_{ave}$ . Then, compute  $\sin \theta$ . Finally, compute the wavelength using the equation above.
6. Repeat the procedure for three additional values of  $L$ .
7. Compute the average of your four determinations of the laser light's wavelength. Compute the standard deviation and consider it an uncertainty in your measurement. Express the result as  $\lambda = \text{average} \pm \text{uncertainty}$ .

$L$ (cm)	$x$ (cm)	$x'$ (cm)	$x_{ave}$ (cm)	$\theta$ (deg)	$\sin \theta$	$\lambda$ (nm)

## 46 Conservation of Angular Momentum

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objectives

To test the Law of Conservation of Angular Momentum and to explore the applicability of angular momentum conservation among objects that experience no external torques.

### Apparatus

- A video analysis system

### Overview

As a consequence of Newton's laws angular momentum, like linear momentum, is conserved in isolated systems. This means that, no matter how many internal interactions occur, the total angular momentum of any system should remain constant if there are no external torques. When one of the objects gains some angular momentum another part of the system must lose the same amount. If angular momentum isn't conserved, there is some outside torque acting on the system. By expanding the boundary of the system to include the source of that torque we can always preserve the Law of Angular Momentum Conservation.

In this unit you will test the notion of the conservation of angular momentum. As in tests of the conservation of linear momentum, we will investigate what happens when two bodies undergo a "rotational" collision. You will view a movie where a large weight is dropped onto a rotating disk and determine the moment of inertia, the angular speed, and finally, the angular momentum of the rotator-disk-weight system before and after this perfectly inelastic collision.

#### Activity 1: Analyzing a Movie of the Collision

- (a) Download the movie *spinningCollision.wmv* from the location

<https:// facultystaff.richmond.edu/~ggilfoyl/genphys/132/links.html>

and store it on your computer Desktop.

- (b) Analyze the movie using the tools described in the Appendix. Start by scaling it using the ruler in the movie frame which has a length  $l_{ruler} = 0.381\text{ m}$ . Be careful to place the origin of your coordinate system on the axle of the rotator so the angular displacement you measure will be the desired one.

#### Activity 2: The Moment of Inertia Before and After the Collision

- (a) Calculate the theoretical value of the moment of inertia of the metal disk using measurements of its radius and mass. The mass of the disk is  $M_d = 5.10\text{ kg}$ . Use the video analysis tools to measure the radius  $r_d$ . Be sure to state units and show the expression you used!

$$r_d =$$

$$M_d = 5.1\text{ kg}$$

$$I_d =$$

- (b) The rotating fixture that holds the disk has a complex shape. We have determined its moment of inertia without the disk and recorded the result here.

$$I_f = 0.0020\text{ kg} - \text{m}^2$$

- (c) After dropping the weight on the rotating disk, the system will have a new moment of inertia. Derive a formula for the moment of inertia of a cylindrical-shaped weight of mass  $m_w$  and radius  $r_w$  revolving about the

origin at a distance  $r_r$ . The distance  $r_r$  is from the center of the rotator to the center of the dropped weight. (You will have to use the parallel axis theorem to do this.)

$$I_w =$$

(d) The mass of the weight is  $m_w = 1.0 \text{ kg}$ . Use the movie image to measure its diameter and record it here.

$$m_w = 1.0 \text{ kg}$$

$$r_w =$$

(e) Come up with a formula for the moment of inertia  $I$  of the whole system before and after the collision and calculate the moment of inertia before the collision only. (The moment of inertia after the collision will be determined at the end of the experiment, because you will not know  $r_r$  until then.) Don't forget to include the units in  $I_{\text{before}}$ .

$$I_{\text{before}} =$$

$$I_{\text{after}} =$$

(b) Check that you placed the origin of your coordinate system on the axle of the rotator so the angular displacement you measure will be the desired one. Use a marker at or near the edge of the disk to record position for each frame. The resulting data should contain three columns with the values of time,  $x$ -position, and  $y$ -position. Calculate the angular position for each time using the  $x$ - and  $y$ -position data, fit the results, and extract the angular speed of the rotator before the 'collision'.

$$\omega_{\text{before}} =$$

(b) Now follow a similar procedure to determine the angular speed after the collision. Fit the data and extract the angular speed of the rotator after the 'collision'.

$$\omega_{\text{after}} =$$

(c) Determine the distance of the center of the weight from the center of the rotator  $r_r$ . To do this, measure the distance from the center of the rotator to the edge of the weight  $r_{\text{edge}}$  and use the result for the radius of the weight  $r_w$ . Calculate the distance from the origin to the center of the weight  $r_r$  (it is just the sum of  $r_{\text{edge}}$  and  $r_w$ ). Use these results and those above to calculate the final moment of inertia.

$$r_{\text{edge}} =$$

$$r_w =$$

$$r_r =$$

$$I_{\text{after}} =$$

### Activity 3: Calculation of Angular Momentum

(a) Calculate the angular momentum before and after the collision, including UNITS. Calculate the average of your two measurement  $L_{\text{ave}}$  and use that result to calculate the percent difference between the two results. Record it below. Go around to the other lab groups and get their results for the percent difference between the angular momenta before and after the collision. Make a histogram of the results you collect and calculate the average and standard deviation. For information on making histograms, see the Appendix. For information on calculating the average and standard deviation, also see the Appendix. Record the average and standard deviation here. Attach the histogram to this unit.

$$L_{\text{before}} =$$

$$L_{\text{ave}} =$$

$$L_{\text{after}} =$$

$$\frac{\Delta L}{L} = \frac{L_{\text{after}} - L_{\text{before}}}{L_{\text{ave}}}$$

(b) What is your expectation for the difference between the initial and final angular momentum? Do the data from the class support this expectation? Use the average and standard deviation for the class to quantitatively answer this question.

(c) Within the limits of experimental uncertainty, is angular momentum conserved? Be quantitative.

(c) Would the procedure you followed above change if the weight was moving horizontally at a constant velocity when you dropped it? If it changed, what would be different?

## 47 The Optical Spectrum of Hydrogen

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

- To determine the wavelengths of the visible lines in the hydrogen spectrum using a spectrometer and a diffraction grating.
- To determine the value of Rydberg's constant.
- To compare the predicted energy levels with the measured ones.

### Introduction

The spectral lines of the hydrogen spectrum that fall in the visible region are designated as the  $H_\alpha$ ,  $H_\beta$ ,  $H_\gamma$ , and  $H_\delta$  lines. All (there happen to be four of them) belong to the Balmer series. In general, the spectrum of hydrogen can be represented by Rydberg's formula:

$$\frac{1}{\lambda} = R_H \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad (1)$$

where  $n_f$  can be any positive integer and  $n_i$  takes on the values of  $n_f + 1$ ,  $n_f + 2$ ,  $n_f + 3$ , and so on and  $R_H$  is the Rydberg constant for hydrogen and equals  $1.097 \times 10^7 m^{-1}$ .

### Apparatus

- spectrometer
- diffraction grating
- hydrogen gas spectral tube
- power supply for hydrogen tube

### Activity 1: Measuring Spectral Lines

Diffraction grating spacing  $d =$  Å

Using the spectrometer, measure the angular position of the direct beam,  $\theta_o$ , and record it here:

$\theta_o =$  degrees. For each spectral line, measure the angular position of the first order maximum (once on each side) and record them in columns 2 and 3 below. Then determine the diffraction angle on each side (columns 4 and 5) and an average of the two (column 6). Calculate the wavelength of each line using the relation:

$$\lambda = d \sin \theta \quad (2)$$

where  $d$  is the diffraction grating spacing.

Line	$\theta_{left}$ (degrees)	$\theta_{right}$ (degrees)	$\theta_o - \theta_{left}$ (degrees)	$\theta_{right} - \theta_o$ (degrees)	$\theta_{average}$ (degrees)	Wavelength (Å)	Color
$H_\alpha$							red
$H_\beta$							blue
$H_\gamma$							violet

**Activity 2: Calculating the Rydberg Constant**

The visible spectral lines for hydrogen correspond to transitions where  $n_f = 2$ . Using values for  $n_i$  of 3, 4, and 5 for the three lines you have measured, and the wavelengths you have determined, calculate a value of  $R_H$  for each line (from equation 1) and compare the average of these with the accepted value. Use your results to predict the wavelength of the next line in the series  $\lambda_\delta$  (using  $n_i = 6$ ). Its measured value is 4101Å. How does your prediction compare?

$$R_\alpha =$$

$$R_\beta =$$

$$R_\gamma =$$

$$R_{average} = \quad \% \text{ difference} =$$

$$\lambda_\delta = \quad \% \text{ difference} =$$

Collect values of  $R_{average}$  from the other groups in the class and calculate an average and standard deviation. Record them below. How does this result compare with the accepted value? How does it compare with your individual measurement (i.e. does your measurement fall within the range indicated by the average and standard deviation)?

**Activity 3: The Hydrogen Energy Level Diagram**

The energy levels of hydrogen can be described by the equation

$$E_n = -\frac{13.6 \text{ eV}}{n^2} \quad (3)$$

where  $n$  is called the principal quantum number. The relationship between the wavelength of the emitted light and its energy is  $E = hc/\lambda$  where  $c$  is the speed of light and  $h$  is Planck's constant. Continue on next page.

Make an energy level diagram showing the transitions you believe you have measured. Calculate the transition energies (experimental) from  $E = hc/\lambda$ , based on your measured wavelengths.

Calculate the theoretical values of these transition energies predicted by equation 3, noting that  $E = E_i - E_f$ .

How do your measured transition energies compare with the predicted ones?

## 48 A Theory for the Hydrogen Atom

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

To use a numerical technique to solve the Schroedinger equation for the hydrogen atom, compare calculations with spectroscopic measurements from hydrogen, and explore the implications of the quantum theory especially the quantization of energy in bound systems.

### Apparatus

- *Schrödinger Shooter* software
- Results from previous measurements of hydrogen energy transitions.

### Overview

Quantum mechanics is the theoretical structure used to make sense of the atomic and sub-atomic world. It is based on a small set of assumptions and a lot of associated mathematics.

#### The Postulates of Quantum Mechanics

1. The quantum state of a particle is characterized by a wave function  $\Psi(\vec{r}, t)$ , which contains all the information about the system an observer can possibly obtain. The square of the magnitude of the wave function  $|\Psi(\vec{r}, t)|^2$  is interpreted as a probability or probability density for the particle's presence.
2. The things we measure (*e.g.* energy, momentum) are called observables. Each observable has a corresponding mathematical object called an operator that does 'something' to the wave function  $\Psi(\vec{r}, t)$ . The radial dependence of the wave function  $\Psi(\vec{r}, t)$  is governed by the energy operator which generates a famous expression called the Schrödinger equation.

$$-\frac{\hbar^2}{2m} \left( \frac{d^2}{dr^2} + \frac{L^2}{2mr^2} \right) \Psi(r) + U\Psi(r) = E\Psi(r) \quad (1)$$

Ultimately, the success of this approach will depend on how the theoretical results we generate from solving the Schrödinger equation compare with our measurements.

We will focus on these postulates in this laboratory and in particular on understanding and solving Equation 1, the Schrödinger equation.

#### Activity 1: Energy in Three Dimensions

Consider a particular form of the total mechanical energy of a particle in three dimensions

$$E = \frac{1}{2}m\vec{v} \cdot \vec{v} + V(\vec{r}) = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2) + V(\vec{r}) = \frac{1}{2}mv^2 + V(\vec{r}) \quad (2)$$

where  $m$  is the particle's mass,  $\vec{v}$  is its velocity, and  $V(\vec{r})$  is the potential energy of the particle and depends only on the particle's position. We have used the definition of the momentum  $\vec{p} = m\vec{v}$ . The potential energy between the electron and proton in a hydrogen atom is the Coulomb potential so

$$V(\vec{r}) = k_e \frac{q_1 q_2}{r} = -k_e \frac{e^2}{r} \quad (3)$$

where we have used the known proton and electron charges.

If we have an electron in the vicinity of a proton (*i.e.* a hydrogen atom), then it is convenient to rewrite the kinetic energy part of Equation 2 in polar coordinates. The proton, electron and the electron's velocity vector form a plane where we can describe the velocity in terms of a radial component and an angular one

$$\vec{v} = v_r \hat{r} + v_t \hat{\theta} \quad (4)$$

where  $\hat{r}$  points radially along a line from the origin to the electron's position and  $\hat{\theta}$  is perpendicular to  $\hat{r}$  and points so counter-clockwise rotations are positive.

(a) Get an expression for  $v^2$  in terms of  $v_r$  and  $v_t$  and substitute your result in the energy equation (Equation 2).

(b) We can now define the momentum associated with the radial motion as  $\vec{p}_r = m\vec{v}_r$  and the momentum associated with the angular motion  $L = mr v_t$ . The angular momentum  $L$  is a constant of the motion so it is unchanging. Rewrite your energy equation in terms of  $p_r$ ,  $L$ ,  $r$ , and any other constants. Include the explicit version of the potential in Equation 3.

(c) You should have found in part 1.b that the three-dimensional energy of a particle in a Coulomb field can be written as

$$E = \frac{p_r^2}{2m} + \frac{L^2}{2mr^2} - k_e \frac{e^2}{r} \quad (5)$$

where  $L$  is a constant so the energy only depends on  $r$  and  $p_r$ . Note the similarities with the forms of the Schrödinger equation in Equation 1. Make a sketch of the potential energy as a function of  $r$ . What are the limiting values of the potential?

(d) The angular momentum  $L$  is a constant of the motion so the last two terms in Equation 5 can be treated as a single 'effective' potential that governs the radial motion of particles. Make a sketch of those last two terms as a function of  $r$  for constant  $L$ . How is your figure different from the one in 1.c?

(e) Since the energy is constant you can draw it on your previous sketch as a flat, straight line. We are considering bound states (the hydrogen atom) so  $E < 0$ . Draw an energy line for a bound state on your previous sketch. Does your energy ‘curve’ intersect the effective potential curve anywhere? For a classical particle like a ball rolling on a hill or a satellite orbiting the Earth, what happens at this intersection? Describe the motion of a classical particle in this potential. What restrictions are there on the energy  $E$  of a classical particle?

### Activity 2: Quantum Predictions

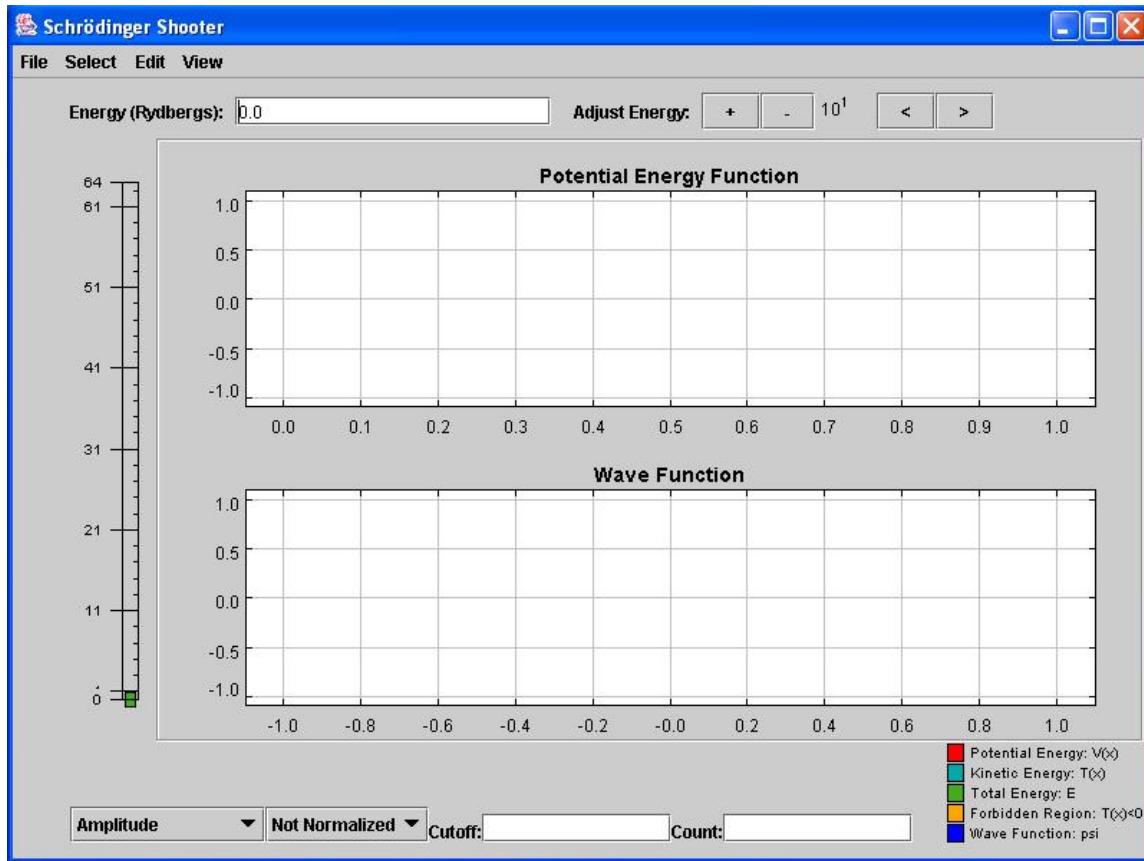
(a) What do you expect the wave function of the electron in a hydrogen atom to look like? Copy your drawing from part 1.d and add a sketch of your expectation for the hydrogen atom wave function on the same graph. Consider what happens as the effective potential changes with  $r$ .

(b) What happens to your wave function as the energy increases? Make another copy here of the graph of the effective potential energy that you made in 2.a. Draw an energy curve for a higher energy state than your previous one. Now sketch the new, higher-energy wave function. How is it different from the curve you made in 2.a?

### Activity 3: Solving the Schrödinger Equation

(a) We are now ready to start solving the Schrödinger Equation. Go first to the All Programs menu, then to Physics Applications and click on Schrödinger Shooter. You will see the Schrödinger Shooter window like the one in the next figure. If you don’t see this window, consult your instructor.

(b) The first step is to select a potential energy function. Click on File and go to New Potential Energy Function. Select the potential energy function most appropriate for the hydrogen atom. Next select the charge of the atomic nucleus by entering that value in the box labeled Z: in units of the electronic charge  $e$ . Set the quantum number of the angular momentum to zero in the box labeled L:. Use the second menu button from the left-hand side at the bottom of the Schrödinger Shooter window to choose Normalized wave functions (it should be labeled Not Normalized when you start the program). This last choice makes it easier to compare wave functions for different quantum numbers. Last, set the initial value of the energy to -1.5 in the box labeled Energy (Rydbergs): and hit return. The conversion between eV and rydbergs is

Figure 1: Initial panel for *Schrödinger Shooter*

13.6056923(12) eV/rydberg. The number in parentheses is the uncertainty on the last two digits in the experimental value. At this point you should see curves in both of the panels of the *Schrödinger Shooter* window. If you don't, consult your instructor.

(c) You should see several curves in the upper panel. What quantities do the blue, red, yellow, and green curves represent? Use the legend in the lower right portion of the *Schrödinger Shooter* window. Record them here.

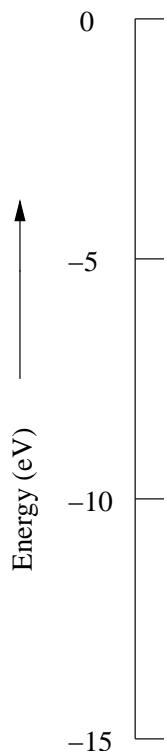
(d) You can now adjust the energy of the electron in the field of the proton to find the energy levels of hydrogen. There are three ways to change this energy. There is a slide on the left-hand side of the *Schrödinger Shooter* window. Click and drag the slide to change the energy and read the value in the box labeled **Energy(Rydbergs)** located near the top of the *Schrödinger Shooter* window. You can also enter the energy in the same box labeled **Energy(Rydbergs):** as you did in part 3.b. Last, to the right of this last box there are buttons labeled **Adjust Energy:** that will increase or decrease the energy and change the step size.

(e) Tune the energy to find lowest five energy levels in your hydrogen atom by obtaining physically acceptable wave functions. You may need to increase the horizontal range to view the full wave function. You can do this by increasing the value in the box labeled **Cutoff:** at the bottom of the *Schrödinger Shooter* window. If you increase the horizontal scale, then increase the next box labeled **Count:** (next to the **Cutoff** box) by the same ratio. In other words, if you double the cutoff, then double the count. This last parameter controls the stepsize

used in the numerical integration of the Schrödinger equation. What does the wave function look like when you have found the correct energy level? What postulate did you exploit to find the correct solutions? How does the radial wave function change as the energy increases? Record your values for each of the energy levels below.

**Activity 4: Analysis and Comparison with Previous Data**

- (a) Enter the values you found for the energies in an *Excel* spreadsheet and convert the values to eV. Use the scale below to make an energy level diagram for hydrogen.



(b) In the lab entitled *Optical Spectrum of Hydrogen* you measured and identified transitions in hydrogen between different principle quantum levels. Using the results in part 4.a calculate the energies of the same transitions and the wavelength of the emitted photon. Record your results below.

(c) Compare your measured results from the *Optical Spectrum of Hydrogen* with the predictions of your model in part 4.b. Calculate the percentage difference for each transition. Does your model agree with the data? Be quantitative in your answer.

#### Activity 5: The Meaning of Quantum Theory

(a) For one of your hydrogen energy states, change the angular momentum of the quantum state by setting  $L = 1$  in the box labeled L:. How does the radial wave function change? How much does the energy change? Do the same thing with the next higher energy state. Can you jump to any conclusions about the effect of angular momentum on the energy of a quantum state in hydrogen?

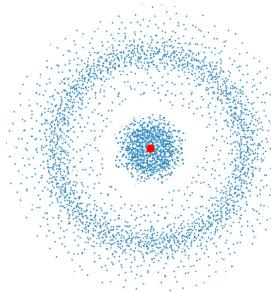
(b) Return to the questions in Activity 2 and examine your predictions. How did you do? Correct any statements that you now find are wrong.

(c) What requirement or postulate forces us to choose particular energy states (*i.e.* what causes energy quantization)?

- (d) Reconsider a variation of one of the questions in part 1.e. In that question you were asked to describe the motion of a classical particle like a satellite orbiting the Earth. Describe the motion of a quantum particle in the hydrogen atom potential.

## Homework

1. An atom (not a hydrogen atom) absorbs a photon whose frequency is  $f = 6.2 \times 10^{14} \text{ Hz}$ . How much does the energy of the atom increase?
2. What is the energy of the hydrogen atom electron whose probability density is represented by the dot plot shown below? What minimum energy is needed to remove this electron from the atom?



3. What are the energy and wavelength of a photon emitted when a hydrogen atom undergoes a transition from the  $n = 3$  state to the  $n = 1$  state?
4. What is the wavelength of the least energetic photon emitted in the Lyman series (where  $n_f = 1$ ) of the hydrogen atom spectrum lines?
5. What is the series limit for the Lyman series (where  $n_f = 1$ )?

## 49 Where's the Electron?

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

To solve the Schrödinger equation for the hydrogen atom, use the results to visualize the probability distribution of the electron, and to determine the location of the electron as a function of energy, radial position, angle, and angular momentum.

### Apparatus

- *Schrödinger Shooter* software
- *Mathematica*.

### Overview

Quantum mechanics is the theoretical structure used to understand the atomic and sub-atomic world. We start with the three-dimensional energy of a particle in a Coulomb field that can be written as

$$E = \frac{p_r^2}{2m} + \frac{L^2}{2mr^2} - k_e \frac{e^2}{r} \quad (1)$$

where  $r$  is the distance from the proton to the electron in the hydrogen atom,  $m$  is the electron mass,  $p_r$  is the radial component of the momentum,  $e$  is the electronic charge,  $k_e$  is the Coulomb constant, and  $L$  is the angular momentum. The energy only depends on  $r$  and  $p_r$ . The first two terms form the kinetic energy and the third term is the potential energy. The angular momentum  $L$  is a constant of the motion so the last two terms in Equation 1 above can be treated as a single ‘effective’ potential that governs the radial motion of particle. This form of the energy is then used to assemble a set of postulates that form the basis of the quantum theory.

### The Postulates of Quantum Mechanics

1. The quantum state of a particle is characterized by a wave function  $\Psi(\vec{r}, t)$ , which contains all the information about the system an observer can possibly obtain. The square of the magnitude of the wave function  $|\Psi(\vec{r}, t)|^2$  is interpreted as a probability or probability density of the particle’s location.
2. The things we measure (*e.g.* energy, momentum) are called observables. Each observable has a corresponding mathematical object called an operator that does ‘something’ to the wave function  $\Psi(\vec{r}, t)$ . The radial dependence of the wave function  $\Psi(\vec{r}, t)$  is governed by the energy operator which generates an expression called the Schrödinger equation.

$$-\frac{\hbar^2}{2m} \left( \frac{d^2}{dr^2} + \frac{L^2}{2mr^2} \right) \Psi(r) + U\Psi(r) = E\Psi(r) \quad (2)$$

In this laboratory, the potential energy  $U$  will be the Coulomb potential energy  $U_C = -k_e e^2 / r$  (see Equation 1).

### Activity 1: The Electron Potential Energy

- (a) Make a sketch of the potential energy as a function of  $r$ . Make sure you label your axes. What are the limiting values of the potential?

- (b) The angular momentum  $L$  is a constant of the motion so the last two terms in Equation 1 can be treated as a single ‘effective’ potential that governs the radial motion of particles. Make a sketch of those last two terms as a function of  $r$  for constant  $L$  as a dashed line on the same sketch you made in part 1.a. How is your curve different from the one in 1.a?
- (c) Make a sketch of those last two terms similar to the one in part 1.b, but with a larger value of the angular momentum  $L$ . Use a dotted line to add the curve to your previous drawing. How is your curve different from the one in 1.b?
- (d) Since the total energy is constant you can draw it on your previous sketch as a flat, straight line. We are considering bound states (the hydrogen atom) so  $E < 0$ . Draw an energy line for a bound state on your previous sketch. Does your energy ‘curve’ intersect the effective potential curve anywhere? For a classical particle like a ball rolling on a hill or a satellite orbiting the Earth, what happens at this intersection? Describe the motion of a classical particle in this potential. What restrictions are there on the energy  $E$  of a classical particle?

### Activity 2: Quantum Predictions

- (a) What do you expect the probability distribution  $|\psi|^2$  of the electron in a hydrogen atom to look like? Make a sketch of your prediction. Consider what happens as the effective potential changes with  $r$ . Be sure to label your axes.

(b) What do you expect for the probability distribution as the energy increases? Sketch the probability distribution at a higher energy as a dashed line on the figure you drew in part 2.a. How is it different from the curve you made in 2.a?

(c) What do you expect for the probability distribution if the energy is fixed, but the angular momentum increases? Sketch this probability distribution function as a dotted line on the figure you drew in part 2.a. How is it different from the curve you made in 2.b?

### Activity 3: Solving the Schrödinger Equation

(a) We are now ready to start solving the Schrödinger Equation. Go first to the **All Programs** menu, then to **Physics Applications** and click on **Shooter**. You will see the **Schrödinger Shooter** window like the one in the next figure. If you don't see this window, consult your instructor.

(b) The first step is to select a potential energy function. Click on **File** and go to **New Potential Energy Function**. Select the potential energy function most appropriate for the hydrogen atom. Next select the charge of the atomic nucleus by entering that value in the box labeled **Z**: in units of the electronic charge  $e$ . Set the quantum number of the angular momentum to zero in the box labeled **L**::. Use the first menu button on the left-hand side at the bottom of the **Schrödinger Shooter** window to choose **Amplitude Squared** so the lower plot will be  $|\psi|^2$ . Use the second menu button from the left-hand side of the window to choose **Normalized** wave functions (it should be labeled **Not Normalized** when you start the program). This last choice makes it easier to compare wave functions for different quantum numbers. Last, set the initial value of the energy to -1.5 in the box labeled **Energy (Rydbergs)**: and hit return. The conversion between eV and rydbergs is 13.6056923(12) eV/rydberg. The number in parentheses is the uncertainty on the last two digits in the experimental value. At this point you should see curves in both of the panels of the **Schrödinger Shooter** window. If you don't, consult your instructor.

(c) You should see several curves in the panels. What quantities do the blue, light blue, red, yellow, and green curves represent? Use the legend in the lower right portion of the **Schrödinger Shooter** window. Record them here.

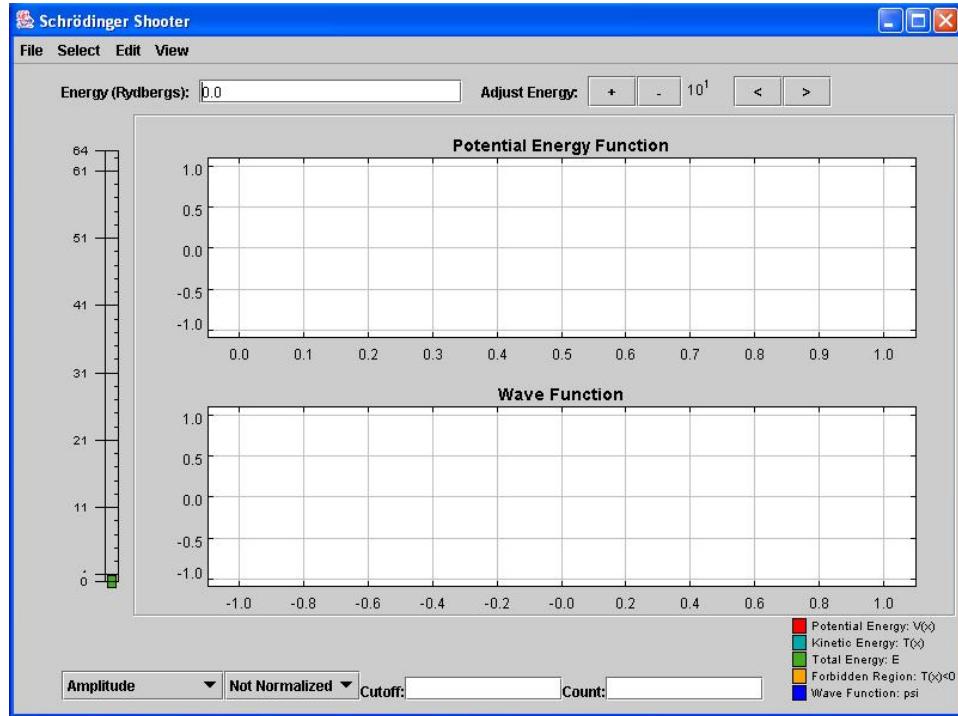


Figure 1: Initial panel for *Schrödinger Shooter*.

- (d) You can now adjust the energy of the electron in the field of the proton to find the energy levels of hydrogen. There are three ways to change this energy. There is a slide on the left-hand side of the **Schrödinger Shooter** window. Click and drag the slide to change the energy and read the value in the box labeled **Energy(Rydbergs)** located near the top of the **Schrödinger Shooter** window. You can also enter the energy in the same box labeled **Energy(Rydbergs):** as you did in part 3.b. Last, to the right of this last box there are buttons labeled **Adjust Energy:** that will increase or decrease the energy and change the step size.
- (e) Tune the energy to find lowest energy level in your hydrogen atom by obtaining a physically acceptable probability distribution. You may need to increase the horizontal range to view the full wave function. You can do this by increasing the value in the box labeled **Cutoff:** at the bottom of the **Schrödinger Shooter** window. If you increase the horizontal scale, then increase the next box labeled **Count:** (next to the **Cutoff** box) by the same ratio. In other words, if you double the cutoff, then double the count. This last parameter controls the stepsize used in the numerical integration of the Schrödinger equation. Sketch  $|\psi|^2$  here when you have found the correct energy level. What postulate did you exploit to find the correct solutions? Where is the electron most likely to be found? Where is the electron least likely to be found?

(f) Next, tune the energy to find the third energy level (second excited state) in your hydrogen atom by obtaining a physically acceptable probability distribution. Sketch  $|\psi|^2$  here when you have found the correct energy level. How does  $|\psi|^2$  change as the energy increases? Where is the electron most likely to be found? Where is the electron least likely to be found?

(g) Next, change the value of the angular momentum quantum number from  $l = 0$  to  $L = 1$ . Tune the energy to find the energy level for this state. Sketch  $|\psi|^2$  when you have found the correct energy level. How is it different from the result in part 3.f? Where is the electron most likely to be found? Where is the electron least likely to be found? Is the energy here different from the energy in part 3.f?

(h) Summarize your results here. How does the probability distribution  $|\psi|^2$  change as the energy (and the principle quantum number  $n$ ) increases? How does changing the value of the angular momentum  $L$  change the energy? How does it change  $|\psi|^2$ ?

#### Activity 4: Where is the Electron?

We will now explore the probability distribution of the electron bound to a proton as a function of the distance  $r$  from the proton to the electron and the angle  $\theta$  relative to the  $z$  axis.

(a) Download the *Mathematica* file *HydrogenOrbitals.nb* from the location

<https://facultystaff.richmond.edu/~ggilfoyl/genphys/132/links.html>

and store it on your computer Desktop.

(b) Go to the All Programs menu and click on *Mathematica*. You should see *Mathematica* go through its startup and then click Open on the *Mathematica* start-up screen. Locate the file you just downloaded and open it. Follow the instructions in the *Mathematica* notebook and you will get the interface shown below for plotting the electron probability density for the hydrogen atom. If you don't see the interface in Figure 2, consult your instructor. Click on the buttons and play with the sliders to see how they effect the probability distribution for the hydrogen orbitals.

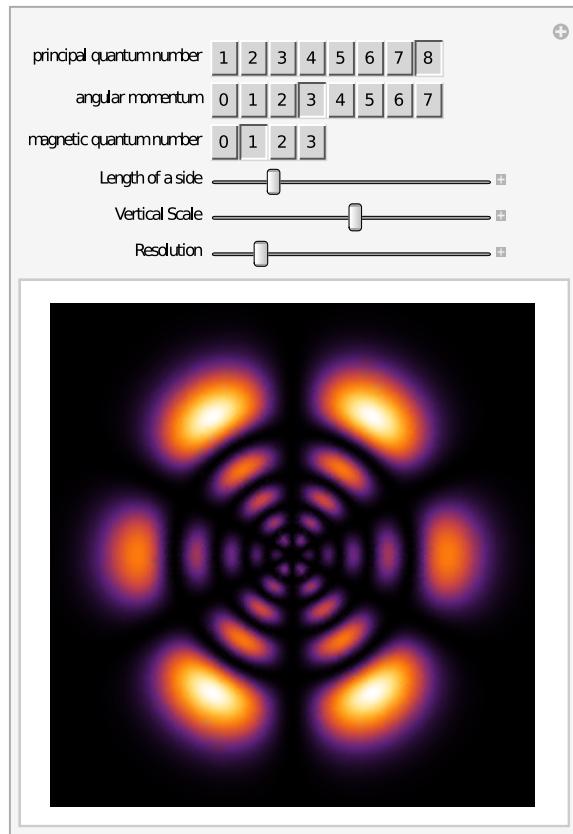


Figure 2: Panel for *HydrogenOrbitals.nb*

(c) Examine different values of the principle quantum number  $n$  for  $L = 0$ . How does  $|\psi|^2$  change as  $n$  increases? Compare these pictures with your results for  $|\psi|^2$  for increasing energy in parts 2.f and 2.g. Where is the electron most likely to be found? Where is the electron least likely to be found?

(d) Set the principle quantum number to  $n = 8$  and the angular momentum to  $L = 0$ . Increase the value of  $L$ . Describe what happens to the probability distribution. Where is the electron most likely to be found? Where is the electron least likely to be found?

(e) Return to your predictions in Activity 2 and examine your predictions. How did you do? Correct any statements that you now find are wrong.

(f) Based on what you now know can you describe the motion of the electron in the hydrogen atom? Explain.

## 50 Galilean Relativity

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

To investigate the relationship between different inertial reference frames and develop the equations of Galilean relativity.

### Overview

Before we begin our study of Einstein's special theory of relativity we will first investigate the effects of observing a phenomenon in two different inertial frames of reference. A spring-launched projectile will be "fired" from a moving platform and we will discover how the description of this phenomenon changes (or doesn't change) in a frame that moves along with the platform. You will use the video analysis software and mathematical modeling tools to find the equations that describe the horizontal motion ( $x$  vs.  $t$ ) and the vertical motion ( $y$  vs.  $t$ ) of the projectile in a stationary frame and then in an inertial frame moving at constant speed with the launcher. The set of relationships between the inertial reference frames forms Galilean relativity. These relationships provide an intuitive, "common sense" picture of the world that works well at low velocities, but fails in many surprising ways at high velocities. At high velocities we must resort to Einstein's special theory of relativity that we will discuss later. To do the activities in this unit you will need:

- A video analysis system (*VideoPoint*).
- The film Moving Launcher.
- Graphing and curve fitting software.

### Activity 1: Observing Projectile Motion From a Moving Launcher

(a) Use the *VideoPoint* package to analyze the film *Moving Launcher* and determine the position of the projectile in each frame with a fixed origin. To do this task follow the instructions of **Appendix D: Video Analysis** for calibrating the film and analyzing the data. When you are analyzing the movie, place the fixed origin in the first frame of the movie at the dark spot on the cart the launcher rides. This makes the comparison with the activities below easier. Change the origin by performing the following steps.

1. Click on the arrow icon near the top of the menu bar to the left. The cursor will be shaped like an arrow when you place it on the movie frame.
2. Click at the origin (where the axes cross) and drag the origin to the desired location.
3. Click on the circle at the top of the menu bar to the left to return to the standard cursor for marking points on the film.

(b) Collect the data for analysis by following the instructions in **Appendix D**. The data table should contain three columns with the values of time,  $x$ -position, and  $y$ -position. Print the data table and attach it to this unit.

(c) Determine the position of the launcher at the first and last frames of the movie. Using these results, what is the horizontal and vertical speed of the launcher during the movie?

$$x_0 = \quad x_1 = \quad \Delta t = \quad v_{x\text{launcher}} =$$

$$y_0 = \quad y_1 = \quad v_{y\text{launcher}} =$$

(d) Plot and fit the position versus time data for the horizontal and vertical positions of the projectile. Print each plot and attach it to this unit. Record the equation of each fit here. Be sure to properly label the units of each coefficient.

$$x(t) =$$

$$y(t) =$$

What is the horizontal speed of the projectile? How did you determine this?

### Activity 2: Changing Reference Frames

(a) We now want to consider how the phenomenon we just observed would appear to an observer that was riding along on the launcher at a constant speed. Assume the moving observer places her origin at the same place you put your origin on the first frame of the movie. Predict how each graph will change for the moving observer.

Horizontal position versus time:

Vertical position versus time:

(b) Use the *VideoPoint* package to analyze the film *Moving Launcher* again and determine the position of the projectile in each frame. However, this time you will use a moving origin that is placed at the same point on the cart on each frame. Use the same point on the launcher that you used to define the origin in the first frame during the previous activity. To change the origin from frame to frame follow these instructions.

1. Open the movie as usual and enter one object to record. First you must select the existing origin and change it from a fixed one to a moving one. Click on the arrow near to top of the menu bar to the left. The cursor will have the shape of an arrow when you place it on the movie frame. Click on the existing origin (where the axes cross) and it will be highlighted.
  2. Under the **Edit** menu drag down and highlight **Edit Selected Series**. A dialog box will appear. Click on the box labelled **Data Type** and change the selection to **Frame-by-Frame**. Click OK.
  3. Click on the circle at the top of the menu bar to the left to change the cursor back to the usual one for marking points. Go to the first frame of interest. When the cursor is placed in the movie frame it will be labelled with "Point S1". Click on the object of interest. The film will NOT advance and the label on the cursor will now be "Origin 1". Click on the desired location of the origin in that frame. The film will advance as usual. Repeat this procedure to accumulate the x- and y-positions relative to the origin you've defined in each frame.
- (c) Collect the data for analysis. The data table should contain three columns with the values of time, x-position, and y-position. Print the data table and attach it to this unit.

(d) Plot and fit the position versus time data for the horizontal and vertical positions. When you have found a good fit to the data, record your result below, print the graph, and attach a copy to this unit. Be sure to properly label the units of each coefficient in your equation.

$$x(t) =$$

$$y(t) =$$

What is the horizontal speed of the projectile? How did you determine this?

### Activity 3: Relating Different Reference Frames

(a) Compare the two plots for the vertical position as a function of time. How do they differ in appearance? Are the coefficients of the fit for each set of data different? Do these results agree with your predictions above? If not, record a corrected “prediction” here.

(b) Compare the two plots for the horizontal position as a function of time. How do they differ in appearance? Are the coefficients of the fit for each set of data different? Do these results agree with your predictions above? If not, record a corrected “prediction” here.

(c) What is the difference between the horizontal velocities in the two reference frames? How does this difference compare with the horizontal velocity of the launcher? How are the horizontal velocities of the projectile in each inertial reference frame and the velocity of the launcher that you determined above related to one another? Does this relationship make sense? Why or why not?

(d) Consider a point  $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$  on the ball’s trajectory in the stationary observer’s reference frame. If the moving observer’s time frame is moving at the speed  $v_x$  *launcher* then what would the moving observer measure for  $x$ ? Call this horizontal position of the moving observer  $x'$ .

(e) What would the moving observer measure for  $y$ ? Call this vertical position of the moving observer  $y'$ .

(f) The relationships you found above are from Galilean relativity. You should have obtained the following results.

$$\begin{aligned}x' &= x - v_{x\text{launcher}} t \\y' &= y \\v_x &= v'_x + v_{x\text{launcher}}\end{aligned}$$

The primes refer to measurements made in the moving frame of reference in this case. If you did not get these expressions consult your instructor.

#### Activity 4: Testing Galilean Relativity

(a) You can test your mathematical relationships with the spreadsheet capabilities of *Excel*. Use your data for the stationary observer and the relationship you derived to calculate what the observer moving with the launcher would measure. As an example of *Excel*'s spreadsheet functions, consider graphing a function like  $f(x) = 2x - 1$  where the values of  $x$  are entered as data in column 1 in *Excel*. To calculate  $f(x)$  and place the result for each value of  $x$  in column 2, double click on the box at the top of column 2 that has the lone “2” in it. The cursor will now appear in the box at the top of the Data window. In that window use the following syntax to calculate  $f(x) = 2x - 1$

$$=(2.0*C1)-1$$

where “C1” refers to the data in column 1 and the “\*” implies multiplication. Hit the “Enter” key when you are finished. You should see the proper series of numbers appear in column 2. If you do not, consult your instructor.

(b) Once you know how to make spreadsheet calculations use the data in *Moving Launcher Data* to calculate the horizontal position versus time for the moving observer. Make a plot of the calculated time for the moving observer and fit the result.

For the transformed stationary observer data:

$$x'(t) =$$

For the moving observer data (see Activity 3.c):

$$x(t) =$$

(c) Your “transformed” data for the stationary observer should closely resemble the results for the moving observer. Is this what you observe? If not, consult your instructor. Print and attach a copy of your plot to this unit.

## 51 The Twins Paradox

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

### Objective

To investigate some of the unusual implications of Einstein's special theory of relativity.

### Overview

Einstein's theory of special relativity leads to a variety of apparent paradoxes that depart radically from our everyday expectations. One of the most celebrated is the twins paradox, in which an identical twin makes a long interstellar journey while the other twin remains on the (roughly) stationary Earth. When the space-faring twin returns she finds her partner has aged considerably more than she has. In this unit you will explore the quantitative aspects of the paradox and some of the surprising consequences.

### Activity 1: Setting Things Up

Problems in special relativity are often very counterintuitive, so it is instructive to consider the situation non-relativistically first. Investigate this problem without applying any of the new ideas you have learned about the theory of special relativity.

One member of a pair of identical twins has decided to embark on a long space voyage. The two twins have lived their lives in close proximity to one another and are very similar in appearance. The adventurous twin boards a fast spacecraft and leaves the Earth behind at a speed of  $0.99c$  or 99% of the speed of light. The space-faring twin's itinerary is rather monotonous, and she simply travels at this constant speed for a time, turns around, and returns to the Earth at the same speed. She measures the time of her trip to be  $\Delta t_p$ . In the meantime the Earth-bound twin has seen twenty years pass by. We will refer to this time as  $\Delta t$ .

(a) In mathematical terms, what is the relationship between the times  $\Delta t_p$  and  $\Delta t$ ?

(b) Which time is associated with which twin?

(c) When the twins are reunited will their appearances differ?

### Activity 2: Applying Special Relativity

(a) Now we want to apply the lessons of special relativity. Time dilation implies that moving clocks run more slowly when observed by someone in a different inertial frame. For the twins paradox what does this imply about the time interval the space-faring twin measures during her trip? Will it be less than, equal to, or greater than the interval measured by the Earth-bound twin? Will the space-faring twin age more, less, or the same amount as the Earth-bound twin?

(b) What is the mathematical relationship between  $\Delta t_p$  and  $\Delta t$  according to the special theory of relativity?

- (c) How much time has passed on the Earth-bound twin's clock?
- (d) How much time has passed on the space-faring twin's clock?
- (e) If this result is inconsistent with your prediction above how should you resolve the contradiction?
- (f) How will the two twins' appearances differ, if at all? Is the difference only in the measurement of the time intervals or are there real physiological differences between the twins after the trip?
- (g) If the average speed of the space-faring twin was more like the typical orbital speed of the space shuttle (about 7.4 km/s) what would the time difference between the twins' clocks be? Show your calculation.

**Activity 3: Graphical Analysis**

(a) Find a mathematical relationship for the ratio of the time interval measured by the space-faring twin to the time interval measured by the Earth-bound twin. Show your work and record your result here.

(b) We want to use the plotting capabilities of *Excel* to graph the ratio you derived above as a function of  $\beta$ , the speed of the space-faring twin's spaceship as a fraction of the speed of light. As an example, to graph a

function like  $f(x) = \sqrt{x}$  we first have to prepare a list of values of  $x$  where we want to calculate  $f(x)$ . To do this in column 1 of the *Excel* data window click in cell *A1*, the first row and first column of the spreadsheet. In that cell use the following syntax to calculate  $\beta$  in steps of 0.05.

$$=0.05*\text{row}()$$

where “row()” is a function in *Excel* that enables you to use the row number in the spreadsheet for calculations. Hit the “Enter” key when you are finished. You should see zero in that top cell. Now click on the bottom, right-hand, corner of the cell and drag down as many cells as you need. You should see a series of numbers 0.0, 0.05, 0.10, 0.15, ... appear in column 1. If you do not, consult your instructor. Now we can generate the values of the function in column 2. Follow a procedure similar to the one you just performed. Click on the top cell in column 2 and use the following syntax to calculate  $\sqrt{x}$ .

$$=\text{sqrt}(A1)$$

where “A1” refers to the data in column 1 that will be used to calculate  $f(x)$ . Now you can make a line plot of column 2 versus column 1 in the usual fashion.

(c) Make a plot of the ratio of the time interval measured by the space-faring twin to the time interval measured by the Earth-bound twin. Restrict the range of  $\beta$  to 0-1. At what average speed does the effect of time dilation become significant? Is there a limit to the ratio? Is there any reason to restrict the range of  $\beta$  to 0-1? Clearly state your reasoning. Print your plot and attach a copy to this unit.

(d) Consider the following scenario. As the space-faring twin’s craft recedes from the Earth it is moving at a constant speed. Since no inertial frame can be considered “better” than any other there is nothing physically inconsistent with the view that the space-faring twin is observing the Earth recede from her at a constant velocity. Hence, the space-faring twin will observe clocks on the Earth to move slowly and the Earth-bound twin will age at a slower rate than the space-faring one. Is this reasoning flawed? How?

(e) If the scenario is not flawed how can it be that the space-faring twin was found to have aged less in the original problem?

## A Treatment of Experimental Data

### Recording Data

When performing an experiment, record all required original observations as soon as they are made. By "original observations" is meant what you actually see, not quantities found by calculation. For example, suppose you want to know the stretch of a coiled spring as caused by an added weight. You must read a scale both before and after the weight is added and then subtract one reading from the other to get the desired result. The proper scientific procedure is to record both readings as seen. Errors in calculations can be checked only if the original readings are on record.

All data should be recorded with units. If several measurements are made of the same physical quantity, the data should be recorded in a table with the units reported in the column heading.

### Significant Figures

A laboratory worker must learn to determine how many figures in any measurement or calculation are reliable, or "significant" (that is, have physical meaning), and should avoid making long calculations using figures which he/she could not possibly claim to know. *All sure figures plus one estimated figure are considered significant.*

The measured diameter of a circle, for example, might be recorded to four significant figures, the fourth figure being in doubt, since it is an estimated fraction of the smallest division on the measuring apparatus. How this doubtful fourth figure affects the accuracy of the computed area can be seen from the following example.

Assume for example that the diameter of the circle has been measured as .5264 cm, with the last digit being in doubt as indicated by the line under it. When this number is squared the result will contain eight digits, of which the last five are doubtful. Only one of the five doubtful digits should be retained, yielding a four-digit number as the final result.

In the sample calculation shown below, each doubtful figure has a short line under it. Of course, each figure obtained from the use of a doubtful figure will itself be doubtful. The result of this calculation should be recorded as  $0.2771 \text{ cm}^2$ , including the doubtful fourth figure. (The zero to the left of the decimal point is often used to emphasize that no significant figures precede the decimal point. This zero is not itself a significant figure.)

$$(.526\underline{4} \text{ cm})^2 = .2770969\underline{9} \text{ cm}^2 = 0.2771 \text{ cm}^2$$

*In multiplication and division, the rule is that a calculated result should contain the same number of significant figures as the least that were used in the calculation.*

*In addition and subtraction, do not carry a result beyond the first column that contains a doubtful figure.*

### Statistical Analysis

Any measurement is an intelligent estimation of the true value of the quantity being measured. To arrive at a "best value" we usually make several measurements of the same quantity and then analyze these measurements statistically. The results of such an analysis can be represented in several ways. Those in which we are most interested in this course are the following:

Mean - The mean is the sum of a number of measurements of a quantity divided by the number of such measurements, which is just the arithmetic mean or the so-called average. It generally represents the best estimate of true value of the measured quantity.

Standard Deviation - The standard deviation ( $\sigma$ ) is a measure of the range on either side of the mean within which approximately two-thirds of the measured values fall. For example, if the mean is  $9.75 \text{ m/s}^2$  and the standard deviation is  $0.10 \text{ m/s}^2$ , then approximately two-thirds of the measured values lie within the range  $9.65 \text{ m/s}^2$  to  $9.85 \text{ m/s}^2$ . A customary way of expressing an experimentally determined value is: Mean  $\pm \sigma$ , or  $(9.75 \pm 0.10) \text{ m/s}^2$ . Thus, the standard deviation is an indicator of the spread in the individual measurements, and a small  $\sigma$  implies high precision. Also, it means that the probability of any future measurement falling in this range is approximately two to one. The equation for calculating the standard deviation is

$$\sigma = \sqrt{\frac{\sum (x_i - \langle x \rangle)^2}{N - 1}}$$

where  $x_i$  are the individual measurements,  $\langle x \rangle$  is the mean, and N is the total number of measurements.

% Difference - Often one wishes to compare the value of a quantity determined in the laboratory with the best known or "accepted value" of the quantity obtained through repeated determinations by a number of investigators. *The % difference is calculated by subtracting the accepted value from your value, dividing by the accepted value, and multiplying by 100.* If your value is greater than the accepted value, the % difference will be positive. If your value is less than the accepted value, the % difference will be negative. The % difference between two values in a case where neither is an accepted value can be calculated by choosing one as the accepted value.

## B Introduction to DataStudio

### Quick Reference Guide

Shown below is the quick reference guide for DataStudio.

What You Want To Do	How You Do It	Button
Start recording data.	Click the 'Start' button or select 'Start Data' on the Experiment menu (or on the keyboard press CTRL - R (Windows) or Command - R (Mac))	 Start
Stop recording (or monitoring) data.	Click the 'Stop' button or select 'Stop Data' on the Experiment menu (or on the keyboard press CTRL - . (period) (Win) or Command - . (Mac))	 Stop
Start monitoring data.	Select 'Monitor Data' on the Experiment menu (or on the keyboard press CTRL - M (Win) or Command - M (Mac))	none

On the Graph Display	In the Graph Toolbar	Button
Re-scale the data so it fills the Graph display window	Click the 'Scale to Fit' button.	
Pinpoint the x- and y-coordinate values on the Graph display	Click the 'Smart Tool' button. The coordinates appear next to the 'Smart Tool'.	
'Zoom In' or 'Zoom Out'	Click the 'Zoom In' or 'Zoom Out' buttons.	
Magnify a selected portion of the plotted data	Click the 'Zoom Select' button and drag across the data section to be magnified.	
Create a Calculation	Click the 'Calculate' button	
Add a text note to the Graph	Click the 'Note' button.	
Select from the Statistics menu	Click the Statistics menu button	
Add or remove a data run	Click the 'Add/Remove Data' menu button	
Delete something	Click the 'Delete' button	
Select Graph settings	Click the 'Settings' menu button	

### Selecting a Section of Data

1. To select a data section, hold the mouse button down and move the cursor to draw a rectangle around the data of interest. The data in the region of interest will be highlighted.
2. To unselect the data, click anywhere in the graph window.

### Fitting a Section of Data

1. Select the section of data to be fitted.

2. Click on the **Fit** button on the Graph Toolbar and select a mathematical model. The results of the fit will be displayed on the graph.
3. To remove the fit, click the **Fit** button and select the checked function type.

#### Finding the Area Under a Curve

1. Use the **Zoom Select** button on the Graph Toolbar to zoom in around the region of interest in the graph. See the quick reference guide above for instructions.
2. Select the section of data that you want to integrate under.
3. Click the **Statistics** button on the Graph Toolbar and select **Area**. The results of the integration will be displayed on the graph.
4. To undo the integration, click on the **Statistics** button and select **Area**.

## C Introduction to Excel

Microsoft Excel is the spreadsheet program we will use for much of our data analysis and graphing. It is a powerful and easy-to-use application for graphing, fitting, and manipulating data. In this appendix, we will briefly describe how to use Excel to do some useful tasks. The current version is Excel 2013.

### C.1 Data and formulae

Figure 3 below shows a sample Excel spreadsheet containing data from a made-up experiment. The experimenter was trying to measure the density of a certain material by taking a set of cubes made of the material and measuring their masses and the lengths of the sides of the cubes. The first two columns contain her measured results. **Note that the top of each column contains both a description of the quantity contained in that column and its units.** You should make sure that all of the columns of your data tables do as well. You should also make sure that the whole spreadsheet has a descriptive title and your names at the top, as indicated in the sample spreadsheet below.

In the third column, the experimenter has figured out the volume of each of the cubes, by taking the cube of the length of a side. To avoid repetitious calculations, she had Excel do this automatically. She entered the formula `=B5^3` into cell C5. Note the equals sign, which indicates to Excel that a formula is coming. The `^` sign stands for raising to a power. After entering a formula into a cell, you can grab the square in the lower right corner of the cell with the mouse and drag it down the column, or you can just double-click on that square. (Either way, note that thing you're clicking on is the tiny square in the corner; clicking somewhere else in the cell won't work.) This will copy the cell, making the appropriate changes, into the rest of the column. For instance, in this case, cell C6 contains the formula `=B6^3`, and so forth.

Column D was similarly produced with a formula that divides the mass in column A by the volume in column C.

At the bottom of the spreadsheet we find the mean and standard deviation of the calculated densities (that is, of the numbers in cells D5 through D8). Those are computed using the formulae `=average(D5:D8)` and `=stdev(D5:D8)`.

### C.2 Graphs

Here's how to make graphs in Excel. First, use the mouse to select the columns of numbers you want to graph. (If the two columns aren't next to each other, rearrange them so that they are next to each other. The variable you want on the horizontal axis needs to be to the left of the variable you want on the vertical axis). Then

The screenshot shows a Microsoft Excel window with the title bar "Book1 - Microsoft Excel". The ribbon menu includes Home, Insert, Page Layout, Formulas, Data, Review, View, and Add-Ins. The Home tab is selected, showing various font and style options. The spreadsheet contains the following data:

	A	B	C	D	E	F	G	H	I
1	Density Experiment		A. Einstein, I. Newton, and M. Curie						
2									
3	Mass (kg)	Length (m)	Volume (m <sup>3</sup> )	Density (kg/m <sup>3</sup> )					
4									
5	0.00371	0.0132	2.29997E-06	743.4886051					
6	0.0158	0.0275	2.07969E-05	759.7295267					
7	0.0481	0.0402	6.49648E-05	740.4008644					
8	0.118	0.0538	0.000155721	757.7661137					
9									
10			Mean of density measurements	750.3462775					
11			Std. dev. of density measurements	9.815604026					
12									
13									
14									
15									
16									

The status bar at the bottom left says "Ready" and the bottom right shows "100%".

Figure 3: Sample Excel spreadsheet

click on the **Insert** tab at the top of the window. In the menu that shows up, there is a section called **Charts**. Almost all of the graphs we make will be scatter plots (meaning plots with one point for each row of data), so click on the **Scatter** icon, which is the lower one in the group that looks like this  . Several possible scatter plots appear. Select the first one with the same icon as before, and your graph will appear.

Next, you'll need to customize the graph in various ways, such as labeling the axes correctly.

This is done in the **Chart Elements** menu which is accessed by clicking on the plus sign (+) in a box, at the upper right corner of your graph. Click on **Axis Titles**, and places appear for axis titles. Edit the text inside of the two axis titles so that it indicates what's on the two axes of your graph, *including the appropriate units for each*.

Next, give your graph an overall title. Click on **Chart Title** and a box appears around it. Delete “Chart Title” and enter a proper title such as “Position vs Time” or whatever.

Usually you want your graph to contain a best-fit line passing through your data points. To do this, right click on one of the data points and select **Add Trendline**. A number of options appear such as “linear”, “polynomial”, “power”, and so on. Select the one you want, and also check the **Display Equation on chart** option near the bottom. You can then drag the equation to someplace else on the chart so you can read it better. Remember that Excel won't include the correct units on the numbers in this equation, but you should. Also, Excel will always call the two variables  $x$  and  $y$ , even though they might be something else entirely. Bear these points in mind when transcribing the equation into your lab notebook.

Sometimes, you may want to make a graph in Excel where the  $x$  column is to the right of the  $y$  column in your worksheet. In these cases, Excel will make the graph with the  $x$  and  $y$  axes reversed. Here's how to fix this problem: Before you make your graph, make a copy of the  $y$  column in the worksheet and paste it so that it's to the right of the  $x$  column. Then follow the above procedure and everything will be fine.

### C.3 Making Histograms

A histogram is a useful graphing tool when you want to analyze groups of data, based on the frequency at given intervals. In other words, you graph groups of numbers according to how often they appear. You start by choosing a set of ‘bins’, *i.e.*, creating a table of numbers that mark the edges of the intervals. You then go through your data, sorting the numbers into each bin or interval, and tabulating the number of data points that fall into each bin (this is the frequency). At the end, you have a visualization of the distribution of your data.

Start by entering your raw data in a column like the one shown in the left-hand panel of Figure 4. Look over your numbers to see what is the range of the data. If you have lots of values to sift through you might consider sorting your data in ascending or descending order. (You don't need to do this if you can easily see the range of data.) To do this task, choose the column containing your data by clicking on the letter at the top of the column, go to **Data** in the menubar, select **Sort**, and pick ascending or descending. The data will be rearranged in the order you've chosen and it will be easier to see the range of the data. For an example, see the middle column of data in the left-hand panel of Figure 4.

Now to create your bins pick a new column on your spreadsheet and enter the values of the bin edges. Bins should be of equal size with the bin edges given by simple numbers. Make sure the bins you choose cover the range of the data. See the left-hand panel of Figure 4 again for an example.

You now have the ingredients for making the histogram. Go to **Data** in the menubar, select **Data Analysis** (on the right) and choose **Histogram**. Click **OK**. You should see a dialog box like the one in the right-hand panel of Figure 4. Click in the box labeled **Input Range** and then highlight the column on the spreadsheet containing your data. Next, click in the box labeled **Bin Range** and highlight the column on the spreadsheet containing the bins. Under **Output Options**, select **Chart Output** and click **OK**. The histogram should come up. If it doesn't, go to **Insert** in the menubar and select the histogram icon in the **Charts** section (the first icon in that section). Change the horizontal axis label to whatever you are plotting (including units). The vertical axis should already be labeled "Frequency". Change the chart title to an appropriate title for whatever you are plotting. The result should look like the right hand panel of Figure 5.

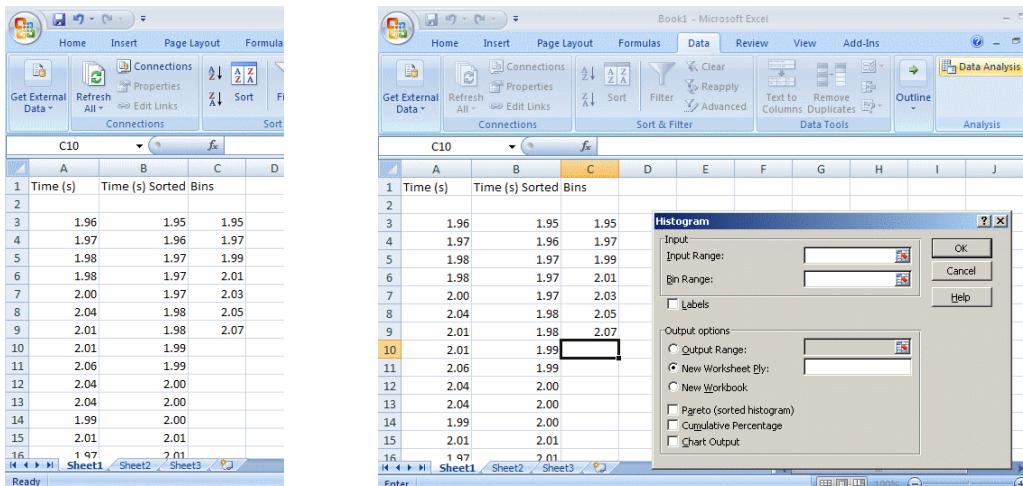


Figure 4: Column data and bins (left-hand panel) and dialog box (right-hand panel) for making a histogram in Excel.

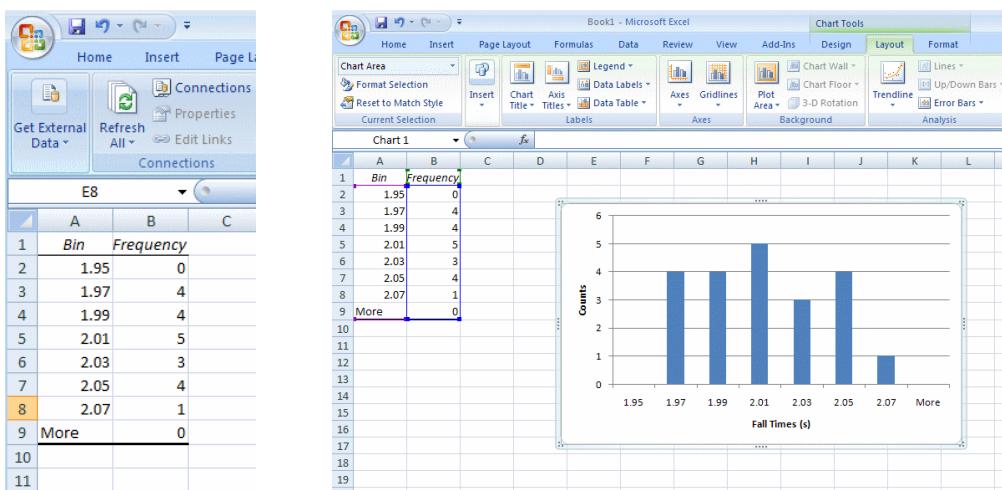


Figure 5: Newly-created worksheet (left-hand panel) and final plot (right-hand panel) for histogram worksheet in Excel.

## C.4 LINEST

LINEST is a function in Excel which gives a LINEar ESTimate of the slope and the uncertainty in the slope for any linear data set. The uncertainty is based on the scatter of the data points about a perfect straight line. To use LINEST in Excel, perform the following steps:

1. Select a  $2 \times 2$  group of cells (not including any cells with data)
2. In the function line type =LINEST(
3. Select range of  $y$  values
4. Type comma (,)
5. Select range of  $x$  values
6. Type comma (,)
7. Type 1,1)
8. Hold down control AND shift buttons, press enter
9. The result should look like this:

slope	intercept
$\Delta$ slope	$\Delta$ intercept

$\Delta$ slope represents the uncertainty in the slope value. You will still need to round off both numbers, depending on the relative magnitudes of the two numbers. It is also common practice to express both numbers to the same power of 10.

## D Video Analysis Using ViseoPoint

### Making a Movie

To make a movie, perform the following steps:

1. Start up *Videopoint Capture* by going to *Start → Programs → Physics Applications → VideoPoint → VP Capture*.
2. The program will first ask you to choose a file name and location for the video you are going to make. You should choose to put the file on the **Desktop**.
3. Click on the *Capture rate* box and set the capture rate to 30 frames per second.
4. Go to the *Size & Colors* under the *Capture Options* menu, and choose the largest available size for the video.
5. Be sure the camera is at least 1 meter from the object you will be viewing. This constraint is required to reduce the effect of perspective for objects viewed near the edge of the field of view. Set up the camera so that its field of view is centered on the expected region where you will perform the experiment.
6. Focus the camera by rotating the barrel on the outside of the lens until you have a clear picture.
7. Place a ruler or some object of known size in the field of view where it won't interfere with the experiment. The ruler should be the same distance away from the camera as the motion you are analyzing so the horizontal and vertical scales will be accurately determined. It should also be parallel to one of the sides of the movie frame.
8. One member of your group should perform the computer tasks while the others do the experiment.
9. To start recording your video, click *Record*. When you're done, click *Stop*.
10. The next step is to decide how much of the movie to save. Use the slider to step through the movie frame by frame. When you find the first frame you want to save, click *First*. When you find the last frame you want to save, click *Last*. (You may want to save the entire movie, in which case *First* and *Last* really will be the first and last frames. Often, though, there will be "dead" time either at the beginning or the end of the movie, which you might as well cut out before saving.)
11. After you've selected the range of frames you want to save, the button at the lower right should say *Keep*. Make sure that the box next to this button says *All*. (If it says *Double*, change it to *All*.) Then click *Keep* and *Save*. You will be asked to provide a file name. Pick something unique that you can easily identify. You will then see a quick replay of the movie as Videopoint converts and saves it.

### Analyzing the Movie

To determine the position of an object at different times during the motion, perform the following steps:

1. Start up *VideoPoint* by going to *Start → Programs → Physics Applications → VideoPoint → 2.5*. Click *Open Movie*. You will see a dialogue box. Set the *Files of type*: box to *All Files (\*.\*)* and select the file you created before. The file should have an extension like '.mov' or '.movvv'. Click *Open*.
2. *VideoPoint* will request the number of objects you want to track in the movie. Carefully read the instructions for the unit you are working on to find this number. Enter it in the space provided. You will now see several windows. (Note: You may have to move the movie window out of the way to see the other windows.) One contains the movie and has control buttons and a slider along the bottom of the frame to control the motion of the film. Experiment with these controls to learn their function. Another window below the movie frame (labeled *Table*) contains position and time data and a third window to the right of the frame (labeled *Coordinate Systems*) describes the coordinate system in use.

3. This is a good time to calibrate the scale. Go to a frame where an object of known size is clearly visible (see item 6 in the previous section). Under the **Movie** menu highlight **Scale Movie**. A dialog box will appear. Enter the length of the object and set **Scale Type** to **Fixed**. Click **Continue**. Move the cursor over the frame and click on the ends of scaling object.
4. You are now ready to record the position and time data. Go to the first frame of interest. Move the cursor over the frame and it will change into a small circle with an attached label. Place the circle over the object of interest in the frame and click. The *x* and *y* positions will be stored and the film advanced one frame. Move the circle over the position of the object in the frame and repeat. Continue this process until you have mapped out the motion of the object. If you entered more than one object to keep track of when you opened the movie, then you will click on all those objects in each frame before the film advances.
5. When you have entered all the points you want, go to the **File** menu and select **Export data**. This will allow you to save your data table as an Excel file. Save this file on the desktop (by clicking on the “Open” button, which actually doesn’t open anything), and double-click on the saved file to start up Excel. You will now be able to continue your data analysis in Excel.
6. Once you have looked at your data in Excel and made sure everything looks OK, you can quit Videopoint Analysis. If you are sure you have exported your data correctly to Excel, there is no need to save in Videopoint.

### Changing the Origin

To change the position of the origin take the following steps.

1. Click on the arrow icon near the top of the menu bar to the left. The cursor will be shaped like an arrow when you place it on the movie frame.
2. Click at the origin (where the axes cross) and drag the origin to the desired location.
3. Click on the circle at the top of the menu bar to the left to return to the standard cursor for marking points on the film.

### Using a Moving Coordinate System

To record the position of an object and to change the coordinate system from frame to frame take the following steps.

1. Open the movie as usual and enter one object to record. First we have to select the existing origin and change it from a fixed one to a moving one. Click on the arrow near the top of the menu bar to the left. The cursor will have the shape of an arrow when you place it on the movie frame. Click on the existing origin (where the axes cross) and it will be highlighted.
2. Under the **Edit** menu drag down and highlight **Edit Selected Series**. A dialog box will appear. Click on the box labeled **Data Type** and highlight the selection **Frame-by-Frame**. Click **OK**.
3. Click on the circle at the top of the menu bar to the left to change the cursor back to the usual one for marking points. Go to the first frame of interest. When the cursor is placed in the movie frame it will be labeled with “Point S1.” Click on the object of interest. The film will NOT advance and the label on the cursor will change to “Origin 1.” Click on the desired location of the origin in that frame. The film will advance as usual. Repeat the procedure to accumulate the *x*- and *y*-positions relative to the origin you’ve defined in each frame.

## E Video Analysis Using Tracker

### Making a Movie with “Windows Live Movie Maker”

To make a movie, perform the following steps:

1. Make sure the camera is connected to a USB port on your computer. Close all windows, applications, programs, and browsers.
2. Click the **Start** button in the lower-left corner of your screen and type ‘movie maker’ in the Search programs and files box. Once the search results appear, click on **Windows Live Movie Maker** under **Programs**. After movie maker starts, close any pop-up windows that may appear.
3. Click the **Webcam Video** button. A webcam video window opens. Enlarge the video frame by dragging the vertical line on the right side of the video frame.
4. Position the camera 2-3 meters from the object you will be viewing. Adjust the camera height and orientation so that the field of view is centered on the expected region where the object will move.
5. Place a meter stick or an object of known size in the field of view where it won’t interfere with the experiment. The meter stick should be the same distance away from the camera as the motion you are analyzing so the horizontal and vertical scales will be accurately determined. It should also be parallel to one of the sides of the movie frame. Make sure that the meter stick is not far away from the central region of field of view, and that it is perpendicular to the line of sight of camera.
6. One member of your group should perform the computer tasks while the other does the experiment.
7. To start recording your video, click **Record**. When you are done, click **Stop**.

Save the video on your Desktop with a unique name that you can easily identify. The video will be saved in Windows Media Video format, i.e. with extension wmv. (Do not save the video as a Movie Maker Project file.)

### Analyzing the Movie

To determine the position of an object at different times during the motion, perform the following steps:

1. Start up Tracker by going to **Start** → **All Programs** → **Physics Applications** → **Tracker**. When **Tracker** starts it appears as shown below. The menu icons and buttons that we will use are identified by arrows.
2. Click the **Open Video** button on the toolbar (see figure below) to import your video. After your video is imported, Tracker will warn you that the video frames don’t have the same time duration. This is okay since Windows Live Movie Maker uses a variable frame rate. Click **Close** on the warning window to ignore Tracker’s recommendation.
3. Click the **Clip Settings** button (see figure below) to identify the frames you wish to analyze. A clip settings dialog box appears. Here, you only need to identify and set the start and end frames. Leave everything else in the dialog box unchanged. To find and set the start frame, drag the player’s left slider to scan forward through the video, and stop when you get to the first frame of interest. Now, the start frame is set and the corresponding frame number should be displayed in the dialog box. If not, then click on the **Start Frame** in the dialog box, enter the number of the frame (printed in the lower right part of the Tracker window), and click outside the box. Then, click the **Play video** button to go to the last frame in the video. Next, drag the player’s right slider to scan backward through the video to find the last frame of interest. Stop when you get to the frame of interest. Now, the end frame is also set and the corresponding frame number should be displayed in the dialog box. If not, then click on the **End Frame** in the dialog box, enter the number of the frame, and click outside the box. Finally, click the **OK** button to close the dialog box, and then click the player’s **Return** button to return to the start frame.

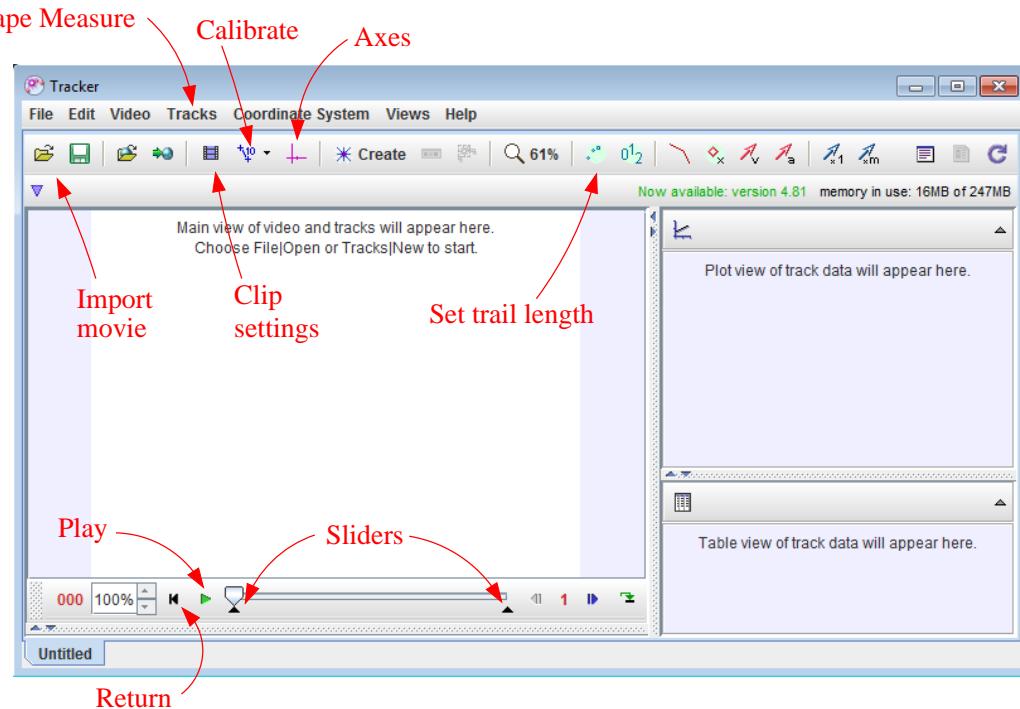


Figure 6: Initial **Tracker** window for video analysis.

4. Click the **Calibration** button (see the figure) and select the **calibration stick**. A blue calibration line appears on the video frame. Drag the ends of this blue line to the ends of your calibration meter stick. Then click the readout box on the calibration line to select it. Enter the length of the meter stick in this box (without units). For example, if your calibration meter stick is 1.00 meter long, enter 1.00 in the box and then click outside the box to accept the value or hit **Return**. At this point, you can right-click the video frame to zoom in for more accurate adjustment of the ends of the calibration stick. Right-click the video again to zoom out.
5. Click the **Axes** button (see the figure) to set the origin and orientation of the x-y coordinate axes. Drag the origin of the axes to the desired position (in most cases the initial position of the object of interest). Click the video outside the origin to fix the position of the origin. To change the orientation (angle) of the axes, drag the x axis. Click the video to fix the new orientation.
6. Click the **Create** button (see the figure) to track the object of interest in the video. From the menu of choices select **Point Mass** for the track type. Make sure the video is at the start frame, which shows the initial position of the object of interest. Mark this position by holding down the **shift key** and clicking the mouse (crosshair cursor) on the object. As the position is marked, the video automatically advances to the next frame. Similarly, mark the position of the object on this and subsequent frames by holding down the **shift key** and clicking the mouse. Do not skip any frames.  
After marking the position on the end frame, you can adjust any one of the marked positions. Advance the video to the frame where you would like to make a fine adjustment. Right-click the video frame to zoom in and drag the marked position with the mouse.  
If you would like to track additional objects, repeat the procedure outlined here for each object.
7. Click the **Set track length** button (see the figure) to get a drop-down menu that will enable you to show all of the points you marked on the frame.
8. **Plotting and Analyzing the Tracks:** The track data (position versus time) are listed in the Table View and plotted in Plot View sections of the Tracker screen. Click the vertical axis label of the plot to

change the variable plotted along that axis. To plot multiple graphs, click the **Plot** button, located above the plot, and select the desired number.

Right-click on a plot to access display and analysis options in a pop-up menu. To fit your data to a line, parabola, or other functions, select the **Analyze** option. On the Data-Tool window that opens up, click on the **Analyze** tab at the top of the plot and check the **Curve Fits** box. Select the fit type from the **Fit Name** drop-down menu. Make sure the **Auto Fit** box is checked.

Note that the curve fitter fits the selected function to the data in the two leftmost columns of the displayed data table. The leftmost column, identified by a yellow header cell, defines the independent variable, and the second leftmost column, identified by a green header cell, defines the dependent variable. So, to fit the data in other columns, their corresponding headers must be dragged to the two leftmost columns.

9. **Measuring Distances and Angles:** After scaling the movie, you can measure angles and distances on a frame. Go to the **Tracks** menu and drag down to **New**, then **Measuring Tools**, and select **Tape Measure** for distances and **Protractor** for angles.
10. **Printing and Exporting Data:** Track data can also be easily exported to Excel for further analysis by copying the data from the data table to the clipboard and pasting into Excel. Click the **Table** button, located above the data table in the table view section of the main screen, and select from the displayed list the data you would like to display in the data table. Select the desired data in the table by clicking and dragging, then right-click and choose **Copy Data** from the pop-up menu. Now, paste the data into Excel. To print out the displayed plot or data table on Tracker's screen, right-click on the plot or table and choose **Print** from the pop-up menu.

## F Instrumentation

### Introduction

Being both quantitative and experimental, physics is basically a science of measurement. A great deal of effort has been expended over the centuries improving the accuracy with which the fundamental quantities of length, mass, time, and charge can be measured.

It is important that the appropriate instrument be used when measuring. Ordinarily, a rough comparison with a numerical scale, taken at a glance and given in round numbers, is adequate. Increasing precision, though, requires a more accurate scale read to a fraction of its smallest division. The “least count” of an instrument is the smallest division that is marked on the scale. This is the smallest quantity that can be read directly without estimating fractions of a division.

Even at the limit of an instrument’s precision, however, accidental errors—which cannot be eliminated—still occur. These errors result in a distribution of results when a series of seemingly identical measurements are made. The best value, known as the most probable value, is the arithmetic mean or average of the measurements.

Other errors, characteristic of all instruments, are known as systematic errors. These can be minimized by improving the equipment and by taking precautions when using it.

### Length Measurement

Three instruments will be available in this class for length measurements: a ruler (one- or two-meter sticks, for example), the vernier caliper, and the micrometer caliper.

#### *The Meter Stick*

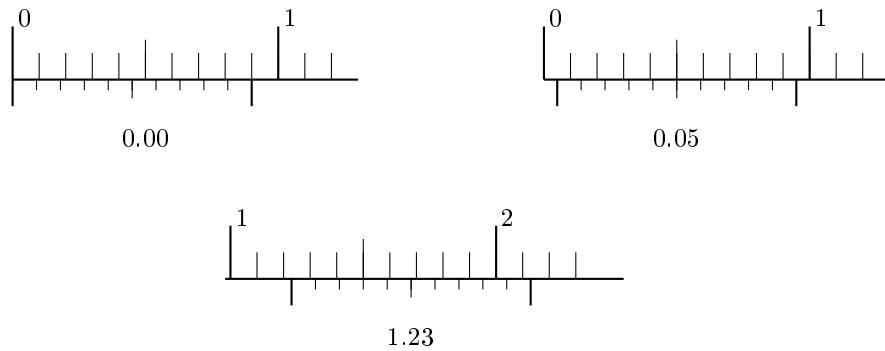
A meter stick, by definition, is 1 meter (m) long. Its scale is divided, and numbered, into 100 centimeters (cm). Each centimeter, in turn, is divided into 10 millimeters. Thus  $1\text{ cm} = 10^{-2}\text{ m}$ , and  $1\text{ mm} = 10^{-1}\text{ cm} = 10^{-3}\text{ m}$ .

When measuring a length with a meter stick, different regions along the scale should be used for the series of measurements resulting in an average value. This way, non-uniformities resulting from the meter stick manufacturing process will tend to cancel out and so reduce systematic errors. The ends of the stick, too, should be avoided, because these may be worn down and not give a true reading. Another error which arises in the reading of the scale is introduced by the positioning of the eyes, an effect known as parallax. Uncertainty due to this effect can be reduced by arranging the scale on the stick as close to the object being measured as possible.

#### *The Vernier Caliper*

A vernier is a small auxiliary scale that slides along the main scale. It allows more accurate estimates of fractional parts of the smallest division on the main scale.

On a vernier caliper, the main scale, divided into centimeters and millimeters, is engraved on the fixed part of the instrument. The vernier scale, engraved on the movable jaw, has ten divisions that cover the same spatial interval as nine divisions on the main scale: each vernier division is  $\frac{9}{10}$  the length of a main scale division. In the case of a vernier caliper, the vernier division length is 0.9 mm. [See figures below.]



Examples of vernier caliper readings

To measure length with a vernier caliper, close the jaws on the object and read the main scale at the position indicated by the zero-line of the vernier. The fractional part of a main-scale division is obtained from the first vernier division to coincide with a main scale line. [See examples above.]

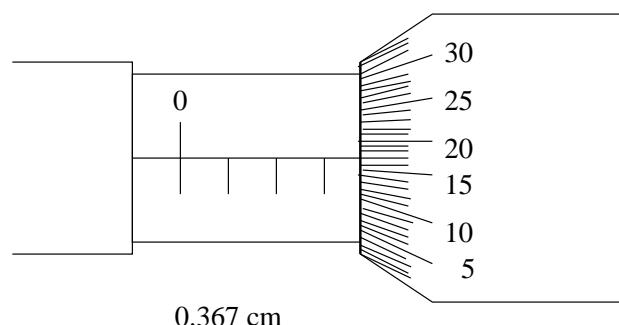
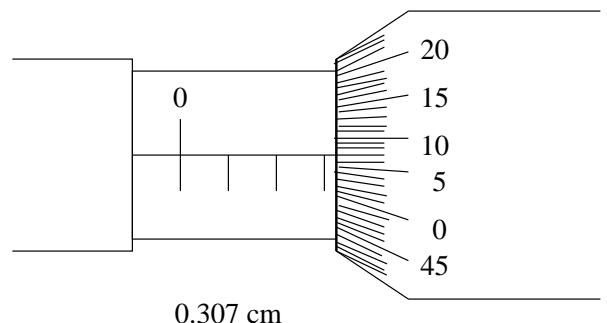
If the zero-lines of the main and vernier scales do not coincide when the jaws are closed, all measurements will be systematically shifted. The magnitude of this shift, called the zero reading or zero correction, should be noted and recorded, so that length measurements made with the vernier caliper can be corrected, thereby removing the systematic error.

#### *The Micrometer Caliper*

A micrometer caliper is an instrument that allows direct readings to one hundredth of a millimeter and estimations to one thousandth of a millimeter or one millionth of a meter (and, hence, its name). It is essentially a carefully machined screw housed in a strong frame. To measure objects, place them between the end of the screw and the projecting end of the frame (the anvil). The screw is advanced or retracting by rotating a thimble on which is engraved a circular scale. The thimble thus moves along the barrel of the frame which contains the screw and on which is engraved a longitudinal scale divided in millimeters. The pitch of the screw is 0.5 mm, so that a complete revolution of the thimble moves the screw 0.5 mm. The scale on the thimble has 50 divisions, so that a turn of one division is  $\frac{1}{50}$  of 0.5 mm, or 0.01 mm.

Advance the screw until the object is gripped gently. Do not force the screw. A micrometer caliper is a delicate instrument.

To read a micrometer caliper, note the position of the edge of the thimble along the longitudinal scale and the position of the axial line on the circular scale. The first scale gives the measurement to the nearest whole division; the second scale gives the fractional part. It takes two revolutions to advance one full millimeter, so note carefully whether you are on the first or second half of a millimeter. The result is the sum of the two scales. (See examples below).



As with the vernier caliper, the zero reading may not be exactly zero. A zero error should be checked for and recorded, and measurements should be appropriately corrected.

#### **Mass Measurement**

Three kinds of instruments will be available to determine mass: a digital scale and two types of balances. The operation of the first instrument is trivial, and so will not be explained here.

Please understand that with each of these instruments we are really comparing weights, not masses, but the proportionality of weight and mass allows the instruments to be calibrated for mass.

#### *The Equal-Arm Balance*

The equal-arm balance has two trays on opposite sides of a pivot. The total mass placed on one tray required to balance the object on the other gives the mass of the object. Most equal-arm balances have a slider, as well, that can move along a scale and allow for greater precision than the smallest calibrated mass available. Typically, this scale has 0.5 g divisions.

#### *The Triple-Beam Balance*

The triple-beam balance, so-called because of its three slider scales, can be read to 0.1 g and estimated to half that. With an object on the tray, the masses of the different scales are slid to notches until balanced. Get close with the larger masses first and then fine-adjust with the smallest slider.

### **Time Measurement**

Time measurements in this course will be made either with a computer or with a stop watch. This first is out of your control.

#### *The Stop Watch*

The stop watches you will use in class have a time range of from hours to hundredths of a second. There are two buttons at the top: a stop/start button and a reset button. The operation of these should be evident, although once the watch is reset, the reset button also starts the watch (but doesn't stop it). Please be aware of this feature.

### **Charge Measurements**

The magnitude of charge is among the most difficult measurements to make. Instead a number of indirect measurements are undertaken to understand electric phenomena. These measurements are most often carried out with a digital multimeter

#### *The Digital Multimeter*

The digital multimeters available for laboratory exercises have pushbutton control to select five ac and dc voltage ranges, five ac and dc current ranges, and six resistance ranges. The ranges of accuracy are 100 microvolts to 1200 volts ac and dc, 100 nanoamperes to 1.999 amperes ac and dc, and 100 milliohms to 19.99 megaohms.

To perform a DC voltage measurement, select the DCV function and choose a range maximum from one of 200 millivolts or 2, 20, 200, or 1200 volts. Be sure the input connections used are V- $\Omega$  and COMMON. The same is true for AC voltage, regarding range and inputs, but the ACV function button should be selected.

For DC current choose DC MA (for DC milliamperes), while for AC current choose AC MA. Your choices for largest current are 200 microamperes or 2, 20, 200, or 2000 milliamperes. Check that the input are connected to MA and COMMON.

There are two choices for resistance measurement: Kilohms ( $K\Omega$ ) and Megohms ( $20M\Omega$ ). The input connectors are the same as when measuring voltage, namely V- $\Omega$  and COMMON. The range switches do not function with the Megohm function, but one of the range buttons must be set. The maximum settings for Kilohm readings are  $200\Omega$  or 2, 20, 200, or  $2000k\Omega$ .

### Calibrating Force Sensors

1. Connect force sensor to Pasco interface (in port “A”).
2. Open whatever software application you will be using in the current experiment.
3. Click “Setup”.
4. Click “Calibrate Sensors”.

#### FOR HORIZONTAL USE:

5. Set force sensor on track, and with no force applied to the sensor, press TARE button on side of the sensor. This sets the sensor to zero. Then set 1st calibration point to 0 newtons, press upper “Read from sensor” button.
6. While holding sensor stationary (with your hand), hang 200g from sensor over a pulley at end of track, as in Figure 2 of Experiment 13. Set 2nd calibration point to 1.96 newtons, press lower “Read from sensor” button.
7. Click “OK”. Force sensor is now calibrated for the rest of your experiment. Close “Calibrate Sensors” window, close “Setup” window.
8. While still holding the force sensor still, press “Start”. Graph should now show a reading of 1.96 N. Press “Stop”.
9. Try hanging a different mass from the force sensor (over the pulley); press “Start” and check that it is reading correctly.

#### FOR VERTICAL USE:

5. Holding sensor vertically (with hook down and no force applied to it), press TARE button on side of the sensor. This sets the sensor to zero. Then set 1st calibration point to 0 newtons, press upper “Read from sensor” button.
6. Hang 200 g from sensor, set 2nd calibration point to 1.96 newtons, press lower “Read from sensor” button.
7. Click “OK”. Force sensor is now calibrated for the rest of your experiment. Close “Calibrate Sensors” window, close “Setup” window.
8. While still holding the force sensor still, press “Start”. Graph should now show a reading of 1.96 N. Press “Stop”.
9. Try hanging a different mass from the force sensor; press “Start” and check that it is reading correctly.

## G Nuclear Safety

All of the radioactive sources we will use in class are very low-level isotopes referred to as “license-free” sources. The following guidelines should be followed for handling radioactive materials in the classroom.

1. Eating, drinking, and application of cosmetics in the laboratory are not permitted.
2. Pipetting by mouth is never permitted. Use suction devices such as pipette filters.
3. Gloves and lab coats should be worn when working with all liquid isotopes.
4. Before leaving the lab, wash your hands thoroughly and check for possible contamination with a survey instrument.
5. All radioactive liquid wastes are to be poured into the liquid waste container, NEVER into a sink.
6. Report all spills, wounds, or other emergencies to your instructor.
7. Maintain good housekeeping at all times in the lab.
8. Store radioactive material only in the designated storage area. Do not remove sources from the lab.