First, let's see how a change in $\vec{B}$ will affect $\vec{E}$.

We'll make a rectangular loop Δx wide by ℓ high, and evaluate Faraday's law:

$$\oint \vec{E} \cdot \vec{ds} = -\frac{\partial \Phi_B}{\partial t}$$

Go around counter-clockwise, starting on left leg:

$$-\underbrace{E(x)}\, \ell + 0\Delta x + \underbrace{E(x + \Delta x)}\, \ell + 0\Delta x = -\frac{\partial(B\Delta x\ell)}{\partial t}$$
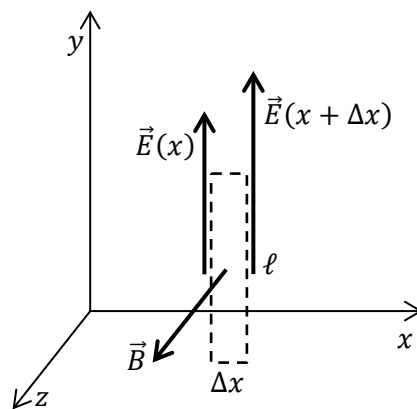
"E of x," not
"E times x"

$$E(x + \Delta x) = E(x) + \frac{\partial E}{\partial x}\Delta x \quad \text{(from your calc class)}$$

$$-E(x)\ell \quad + \left(E(x) + \frac{\partial E}{\partial x}\Delta x\right)\ell \quad = -\frac{\partial(B\Delta x\ell)}{\partial t}$$

$$-E(x)\ell \quad + E(x)\ell + \frac{\partial E}{\partial x}\Delta x\ell \quad = -\Delta x\ell\frac{\partial B}{\partial t}$$

$$\boxed{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}}$$

Call this equation (1)

Now you try it! Let's see how a change in $\vec{E}$ will affect $\vec{B}$.
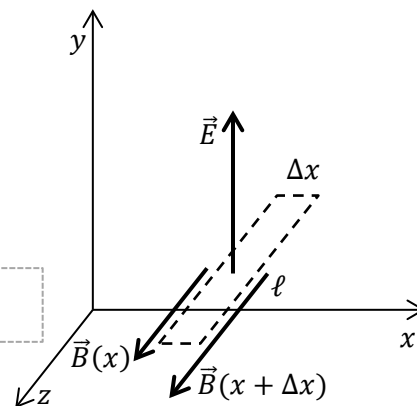
We'll make a rectangular loop Δx wide by ℓ deep, and evaluate Ampère's law:

$$\oint \vec{B} \cdot \vec{ds} = \mu_0 I_{ENC} + \mu_0 \epsilon_0 \frac{\partial \Phi_E}{\partial t}$$

Go around counter-clockwise, starting on left leg:

$$B(x)\ell + 0\Delta x + \boxed{} = 0 + \mu_0\epsilon_0 \frac{\partial}{\partial t}\boxed{}$$

$$B(x + \Delta x) = \boxed{}$$

$$\frac{\partial B}{\partial x} = \boxed{}$$

Call this equation (2)

Now we will separate $\vec{E}$ and $\vec{B}$:

- Start with equation (1):

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

- Take $\partial/\partial x$ of both sides

$$\frac{\partial}{\partial x}\frac{\partial E}{\partial x} = \frac{\partial}{\partial x}\left(-\frac{\partial B}{\partial t}\right)$$

- Switch the order of the derivatives:

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right)$$

- Substitute for $\partial B/\partial x$, from equation (2):

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial t}\left(-\mu_0\epsilon_0\frac{\partial E}{\partial t}\right)$$

- Simplify:

$$\boxed{\frac{\partial^2 E}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2 E}{\partial t^2}}$$

Now you try it:

- Start with equation (2):

$$\frac{\partial B}{\partial x} = -\mu_0\epsilon_0\frac{\partial E}{\partial t}$$

- Take $\partial/\partial x$ of both sides

- Switch the order of the derivatives:

- Substitute for $\partial E/\partial x$, from equation (1):

- Simplify:

$$\frac{\partial^2 B}{\partial x^2} =$$