

First, let's see how a change in  $\vec{B}$  will affect  $\vec{E}$ .

We'll make a rectangular loop  $\Delta x$  wide by  $\ell$  high, and evaluate Faraday's law:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{\partial \Phi_B}{\partial t}$$

Go around counter-clockwise, starting on left leg:

$$-E(x)\ell + 0\Delta x + E(x + \Delta x)\ell + 0\Delta x = -\frac{\partial(B\Delta x\ell)}{\partial t}$$

"E of x," not  
"E times x"

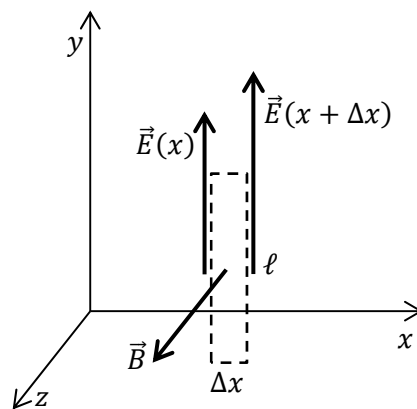
$$E(x + \Delta x) = E(x) + \frac{\partial E}{\partial x} \Delta x \quad (\text{from your calc class})$$

$$-E(x)\ell + \left(E(x) + \frac{\partial E}{\partial x} \Delta x\right)\ell = -\frac{\partial(B\Delta x\ell)}{\partial t}$$

$$-E(x)\ell + E(x)\ell + \frac{\partial E}{\partial x} \Delta x \ell = -\Delta x \ell \frac{\partial B}{\partial t}$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

Call this equation (1)



Now you try it! Let's see how a change in  $\vec{E}$  will affect  $\vec{B}$ .

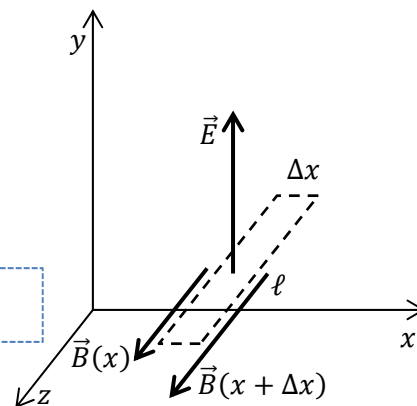
We'll make a rectangular loop  $\Delta x$  wide by  $\ell$  deep, and evaluate Ampère's law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{ENC} + \mu_0 \epsilon_0 \frac{\partial \Phi_E}{\partial t}$$

Go around counter-clockwise, starting on left leg:

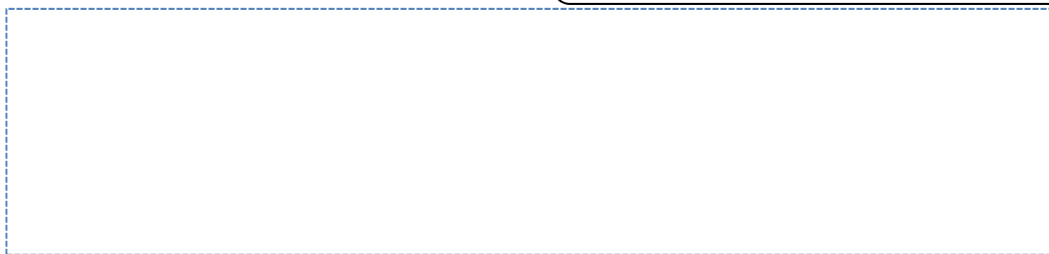
$$B(x)\ell + 0\Delta x + \boxed{\phantom{0\Delta x}} = 0 + \mu_0 \epsilon_0 \frac{\partial \boxed{\phantom{0\Delta x}}}{\partial t}$$

$$B(x + \Delta x) = \boxed{\phantom{0\Delta x}}$$



$$\frac{\partial B}{\partial x} = \boxed{\phantom{0\Delta x}}$$

Call this equation (2)



Now we will separate  $\vec{E}$  and  $\vec{B}$ :

- Start with equation (1):

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

- Take  $\partial/\partial x$  of both sides

$$\frac{\partial}{\partial x} \frac{\partial E}{\partial x} = \frac{\partial}{\partial x} \left( -\frac{\partial B}{\partial t} \right)$$

- Switch the order of the derivatives:

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial t} \left( \frac{\partial B}{\partial x} \right)$$

- Substitute for  $\partial B/\partial x$ , from equation (2):

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial t} \left( -\mu_0 \epsilon_0 \frac{\partial E}{\partial t} \right)$$

- Simplify:

$$\boxed{\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}}$$

Now you try it:

- Start with equation (2):

$$\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

- Take  $\partial/\partial x$  of both sides

- Switch the order of the derivatives:

- Substitute for  $\partial E/\partial x$ , from equation (1):

- Simplify:

$$\boxed{\frac{\partial^2 B}{\partial x^2} =}$$