Physics 132
# Biot-Savart Law Calculations

March 4, 2016

Part 1: Magnetic field above a current loop.

We have a horizontal circular current loop of radius R, carrying current I. We want to determine the magnetic field at a point a height z directly above the center of the loop.

These pictures shows the situation. In the left, the red circle is the current loop, and the red dot is the point at which we're trying to evaluate the magnetic field. The right picture shows a side view of the situation. The gray bar shows the current loop, viewed edge-on. The black rectangle at the lower left is a current element (that is, a small segment of the current loop). The current in this part of the loop is flowing out of the page towards you, and P marks the point at which we're evaluating the field..
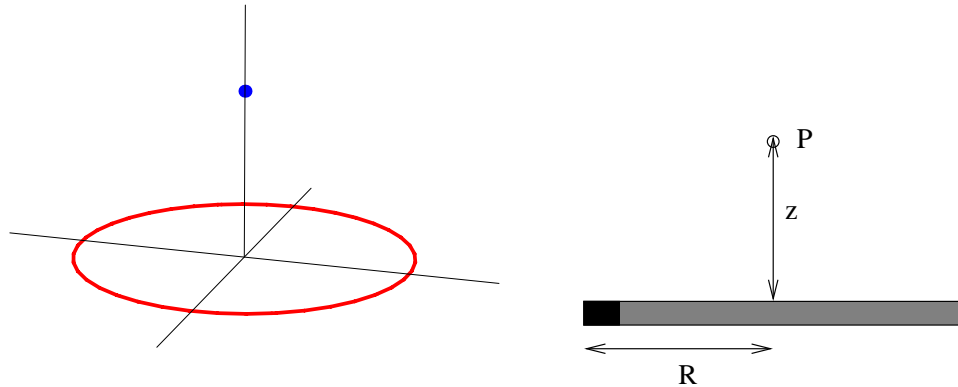

1. Sketch a copy of the right-hand picture above on your own paper, and draw the direction of the vector $d\vec{B}$, which is the magnetic field produced by that current element. Put the tail of the vector at the point P, which is the place where the field is being evaluated.

2. Use the Biot-Savart law to express the magnitude of $d\vec{B}$ in terms of the various given quantities in the problem such as ds, I, R, z.

3. Now suppose that we consider a current element on the exact opposite side of the loop. Draw the direction of the magnetic field $d\vec{B}$ caused by this current element. Label this one $d\vec{B}_2$ to distinguish it from the earlier one.

4. When we eventually add up (i.e., integrate) all the $d\vec{B}$'s from all the different current elements, what will be the direction of the resulting vector?

5. Before we do the integration, we need to pick out just that one component of $d\vec{B}$. Indicate an appropriate angle θ on your drawing, and write down an expression giving the relevent component of $d\vec{B}$ in terms of the magnitude dB and the angle θ.

6. We need to express the angle θ in terms of the given quantities in the problem. In order to do this, identify a right triangle containing θ and also the dimensions R, z. Use this triangle to express the relevant trigonometric function of θ in terms of R, z.

7. Put your answers to questions 2, 5, 6 together to give an expression giving the relevant component of $d\vec{B}$ in terms of the various given quantities I, R, z, and ds.

8. Integrate the previous expression to get the magnitude of the magnetic field produced by the entire loop of current. (Note: if you find yourself doing a hard integral, you're thinking about it the wrong way.)

9. What does your answer say about the strength of the magnetic field right at the center of the loop (that is, at the point $z = 0$)? The result should agree with the expression we got earlier for this quantity, namely $B = \mu_0 I/(2R)$.

Part 2: Magnetic field above a square loop.

Now suppose that we have a loop of wire shaped like a square as shown. The square lies in the xy plane and is centered on the origin. Each side has length $2a$, so that it extends from $-a$ to a. It carries current I going counterclockwise. As in the previous part, our goal is to calculate the magnetic field at a point located a height z above the center of the loop. (That is, we're considering a point lying above the plane of this piece of paper.)
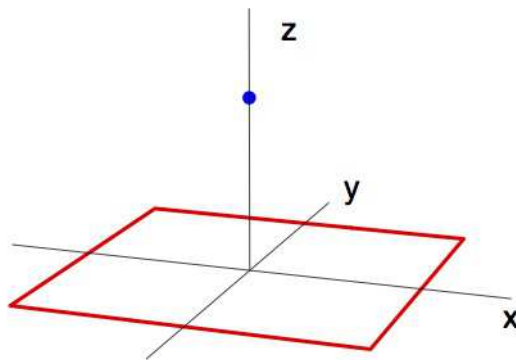
To do this, we'll have to consider one side of the square at a time. In each case, we'll want to know the magnetic field produced by a straight segment of current. Fortunately, you worked out that expression in Lab 21.

Let's start by considering the side on the left of the picture, which extends from $(x, y) = (-a, -a)$ to $(x, y) = (-a, a)$.

1. Draw an arrow indicating the direction of the magnetic field due to this segment. Call it $\vec{B}_1$.

2. The magnitude of $\vec{B}_1$ is given by the formula you derived in Lab 21. What is it? Note: the thing called y in that lab is the distance from the center of the wire to the point at which the field is being evaluated. In this case, what is that quantity (in terms of z, a)?

3. When we add up the contributions from all four sides, only one component is going to matter. Decide which component that is. Then indicate an appropriate angle θ on your drawing, and write down an expression giving the magnitude of the relevant component of $\vec{B}_1$ in terms of θ.

4. Draw a triangle showing the angle θ from the previous part, as well as lengths such as z, a. Express the relevant trigonometric function of θ in terms of these quantities.

5. Put the last three steps together to get an expression giving the relevant component of $\vec{B}_1$ in terms of the given quantities of the problem, such as I, a, z.

6. The other three sides of the square will give contributions $\vec{B}_2, \vec{B}_3, \vec{B}_4$ to the magnetic field. When you add up all four sides, what will the resulting magnetic field be? This is the final answer to the question we originally posed.

7. What is the strength of the magnetic field right at the center of the square (that is, at $z = 0$)?

Part 3: The field at very great distances.

In this part, I want to consider the results of the previous two calculations in the case where z is very large – that is, the case where we're very far away from the current loop.

1. Take your expression from question 8 of part 1, giving the magnetic field above a circular current loop. Suppose that we're extremely far away from the loop, that is, that z is much, much greater than R. In that case, we can approximate $z^2 + R^2 \approx z^2$. Write down the expression for the magnetic field strength in this approximation.

2. Your answer to the previous question should have R's in it. For reasons that we'll see in a minute, it turns out to be nice to express the answer in terms of the area A of the loop instead of the radius. Make the appropriate substitution to get an expression giving B in terms of I, A, z.

   Next, we'll play the same game with the square loop.

3. Using your expression from question 6 of Part 2, write down an approximate expression for B assuming that the distance z is very large.

4. What is the area of the square loop (in terms of a)? Express the answer to the previous question in terms of the area A instead of the length a.

You should find that the expressions are the same for the circular and square loops. If you did not find this, talk to me.

This result turns out to be true for loops of all shapes: as long as you're far away from the loop, the magnetic field depends on the area of the loop, but not on its exact shape.