SOLUTIONS

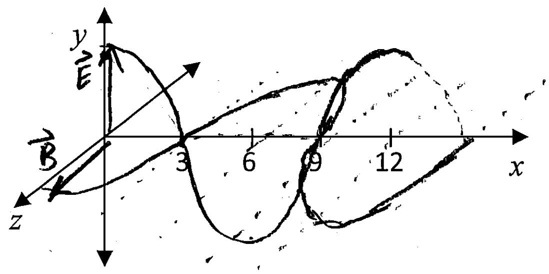
A quick exercise in visualizing electromagnetic waves

An electromagnetic **PLANE** wave is described by

$$\vec{E} = E_{MAX}\cos(kx - \omega t)\,\hat{y}$$
$$\vec{B} = B_{MAX}\cos(kx - \omega t)\,\hat{z},$$

and has a wavelength of $\lambda = 12$ meters. All (x, y, z) points in this problem are in meters.

1. Draw a sketch of the wave on the axes below, at time $t = 0$.



($\vec{E}$ is along y-axis; $\vec{B}$ along z-axis. Both are sinusoidal.)

2. At $t = 0$, find both $\vec{E}$ and $\vec{B}$ at the points (0,0,0), (3,0,0), (6,0,0) and (9,0,0). (Your answers are vectors, and should include both **magnitude** and **direction**.)

From picture: (0,0,0): $\vec{E} = E_{max}\hat{y}$; $\vec{B} = B_{max}\hat{z}$.
(3,0,0): $\vec{E} = 0$; $\vec{B} = 0$.
(6,0,0): $\vec{E} = -E_{max}\hat{y}$; $\vec{B} = -B_{max}\hat{z}$.
(9,0,0): $\vec{E} = 0$; $\vec{B} = 0$.

3. At $t = 0$, find both $\vec{E}$ and $\vec{B}$ at the points (0,0,1), (0,1,0), (0,1,1) and (3,1,1). (Remember, it's a plane wave….) Because it's a plane wave, values of $\vec{E}$ and $\vec{B}$ do NOT change with y or z. The points (0,0,1), (0,1,0) and (0,1,1) all have $\vec{E} = E_{max}\hat{y}$, $\vec{B} = B_{max}\hat{z}$, just like (0,0,0). At (3,1,1), $\vec{E} = 0$ and $\vec{B} = 0$.

4. Find the period T and frequency f of the wave. (Numerical answers, please.)

$$T = \frac{\lambda}{v} = \frac{\lambda}{c} = \frac{12\text{ meters}}{(3\times10^8\text{ m/sec})} = 40\text{ nanoseconds.}$$

$$f = 1/T = 2.5\times10^7\text{ Hz} \quad (25\text{ MHz})$$

5. At $t = 10$ nsec, find $\vec{E}$ at and $\vec{B}$ at the points (0,0,0), (3,0,0), (6,0,0), (9,0,0), and (3,1,2).

That's 1/4 of a period, so the picture in #1 is shifted right by $\frac{1}{4}\lambda$, or 3 meters.

At (0,0,0): $\vec{E} = 0$, $\vec{B} = 0$.
At (3,0,0): $\vec{E} = E_{max}\hat{y}$; $\vec{B} = B_{max}\hat{z}$.
At (6,0,0): $\vec{E} = 0$; $\vec{B} = 0$.
At (9,0,0): $\vec{E} = -E_{max}\hat{y}$; $\vec{B} = -B_{max}\hat{z}$.

At (3,1,2), same as (3,0,0): $\vec{E} = E_{max}\hat{y}$; $\vec{B} = B_{max}\hat{z}$