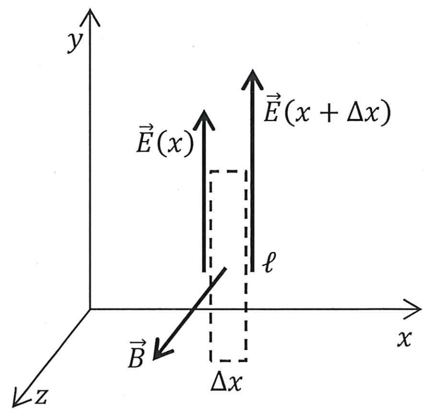
SOLUTIONS

First, let's see how a change in $\vec{B}$ will affect $\vec{E}$.

We'll make a rectangular loop Δx wide by ℓ high, and evaluate Faraday's law:



$$\oint \vec{E}\cdot\vec{ds} = -\frac{\partial \Phi_B}{\partial t}$$

Go around counter-clockwise, starting on left leg:

$$-E(x)\ell + 0\Delta x + \underbrace{E(x+\Delta x)}\,\ell + 0\Delta x = -\frac{\partial(B\Delta x\ell)}{\partial t}$$

$$E(x+\Delta x) = E(x) + \frac{\partial E}{\partial x}\Delta x \quad \text{(from your calc class)}$$

$$-E(x)\ell \quad + \overbrace{\left(E(x) + \frac{\partial E}{\partial x}\Delta x\right)}\ell \quad = -\frac{\partial(B\Delta x\ell)}{\partial t}$$

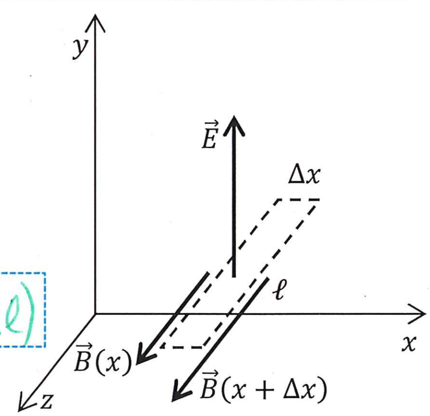
$$-E(x)\ell \quad + E(x)\ell + \frac{\partial E}{\partial x}\Delta x\ell \quad = -\Delta x\ell\frac{\partial B}{\partial t}$$

$$\boxed{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}}$$

Call this equation (1)

Now you try it! Let's see how a change in $\vec{E}$ will affect $\vec{B}$.

We'll make a rectangular loop Δx wide by ℓ deep, and evaluate Ampère's law:



$$\oint \vec{B}\cdot\vec{ds} = \mu_0 I_{ENC} + \mu_0\epsilon_0\frac{\partial \Phi_E}{\partial t}$$

Go around counter-clockwise, starting on left leg:

$$B(x)\ell + 0\Delta x + \boxed{-B(x+\Delta x)\ell + 0\Delta x} = 0 + \mu_0\epsilon_0\frac{\partial \boxed{(E\Delta x\ell)}}{\partial t}$$

$$B(x+\Delta x) = \boxed{B(x) + \frac{\partial B}{\partial x}\Delta x}$$

$$B(x)\ell - \left(B(x) + \frac{\partial B}{\partial x}\Delta x\right)\ell = \mu_0\epsilon_0\Delta x\ell\frac{\partial E}{\partial t}$$

$$\cancel{B(x)\ell} - \cancel{B(x)\ell} - \frac{\partial B}{\partial x}\cancel{\Delta x\ell} = \mu_0\epsilon_0\cancel{\Delta x\ell}\frac{\partial E}{\partial t}$$

$$\frac{\partial B}{\partial x} = \boxed{-\mu_0\epsilon_0\frac{\partial E}{\partial t}}$$

Call this equation (2)

Now we will separate $\vec{E}$ and $\vec{B}$:

- Start with equation (1):

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

- Take $\partial/\partial x$ of both sides

$$\frac{\partial}{\partial x}\frac{\partial E}{\partial x} = \frac{\partial}{\partial x}\left(-\frac{\partial B}{\partial t}\right)$$

- Switch the order of the derivatives:

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right)$$

- Substitute for $\partial B/\partial x$, from equation (2):

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial t}\overbrace{\left(-\mu_0\epsilon_0\frac{\partial E}{\partial t}\right)}$$

- Simplify:

$$\boxed{\frac{\partial^2 E}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2 E}{\partial t^2}}$$

Now you try it:

- Start with equation (2):

$$\frac{\partial B}{\partial x} = -\mu_0\epsilon_0\frac{\partial E}{\partial t}$$

- Take $\partial/\partial x$ of both sides

$$\frac{\partial}{\partial x}\frac{\partial B}{\partial x} = \frac{\partial}{\partial x}\left(-\mu_0\epsilon_0\frac{\partial E}{\partial t}\right)$$

- Switch the order of the derivatives:

$$\frac{\partial^2 B}{\partial x^2} = -\mu_0\epsilon_0\frac{\partial}{\partial t}\left(\frac{\partial E}{\partial x}\right)$$

- Substitute for $\partial E/\partial x$, from equation (1):

$$\frac{\partial^2 B}{\partial x^2} = -\mu_0\epsilon_0\frac{\partial}{\partial t}\left(-\frac{\partial B}{\partial t}\right)$$

- Simplify:

$$\boxed{\frac{\partial^2 B}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2 B}{\partial t^2}}$$