SOLUTIONS

Finding Potential from Electric Field

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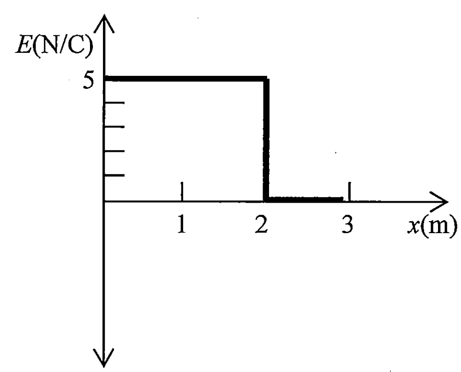
In this lab, you will calculate and graph the electric potential V from a known electric field E. Keep in mind that the relationship between these two can be written as either a *definite* or *indefinite* integral:

$$\Delta V_{AB} = -\int_A^B \vec{E} \cdot \overrightarrow{ds} \quad \text{or} \quad V = -\int \vec{E} \cdot \overrightarrow{ds}.$$

When evaluating the indefinite integral, remember that you always need to consider a constant of integration, $+C$.

1. The graph to the right shows a region of a uniform electric field $E(x)$.



(a) If a positively charged particle starts at $x = 0$ and is accelerated by the electric field to $x = 2$, would the particle's kinetic energy K *increase* or *decrease*?

Increase

(b) In part (a), would the particle's potential energy U *increase* or *decrease*?

Decrease

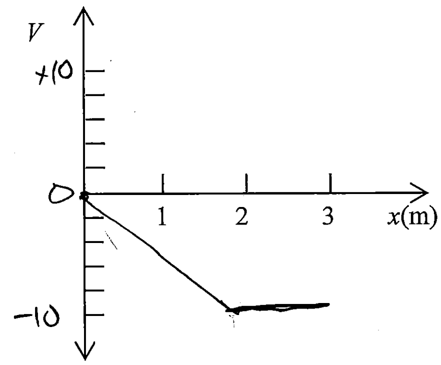
(c) Calculate the change in electric potential ΔV from $x = 0$ to $x = 2$. (Careful with your signs!)

$$\Delta V = -\int_0^2 (5\,N/C)\,dx = -10\text{ Volts}$$

(d) If the potential at the origin is defined as $V(0) = 0$ Volts (our "reference"), what is the value of the potential V at $x = 2$?

-10 Volts

(e) Draw a graph of the electric potential on the axes below. Include a scale on the vertical axis.



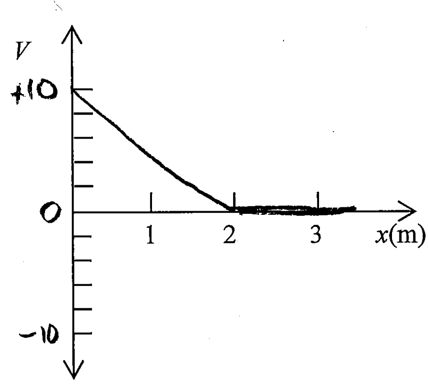
(f) Calculate the change in electric potential ΔV between $x = 2$ and $x = 3$.

$$\Delta V = -\int_2^3 E\,dx = 0 \text{ Volts}$$

(g) Recalling your answers to parts (d) and (f), what is the value of the electric potential V at $x = 3$? (After you've answered this, you may want to go back and fix up your graph in part (e) to clarify $V(x)$ for $x > 2$.

Must be -10 Volts, same as (d).

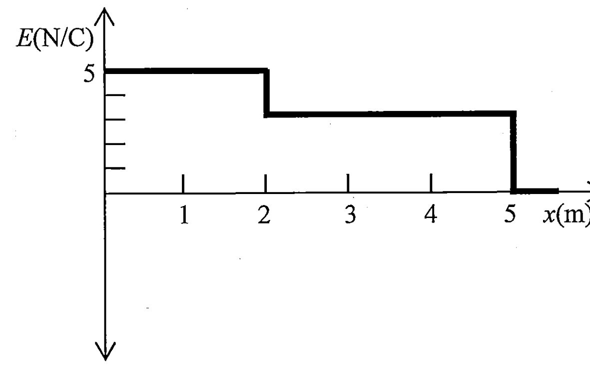
(h) Draw a graph showing the potential if we chose our reference so that $V(\infty) = 0$ instead.



(i) Write an equation describing $V(x)$ based on your graph above.

$$V(x) = 10V - (5\,V/m)x$$

2. The graph to the right shows the electric field $E(x)$ in some other region.



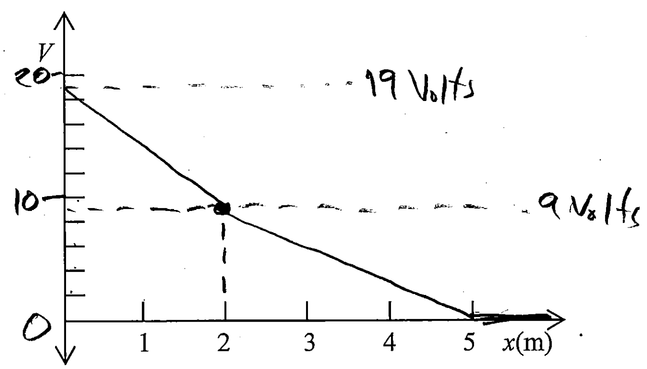
(a) Calculate the change in electric potential ΔV between $x = 0$ and $x = 2$.

$$\Delta V = -\int_0^2 \left(5\frac{V}{m}\right) dx = -10 \text{ Volts}$$

(b) Calculate the change in electric potential ΔV between $x = 2$ and $x = 5$.

$$\Delta V = -\int_2^5 (3V/m)\, dx = -9 \text{ Volts}$$

(c) Draw a graph showing the potential, where the reference is $V(\infty) = 0$.



(d) Use the indefinite integral $V = -\int \vec{E} \cdot \overrightarrow{ds}$ to write an equation for $V(x)$ in the region $2 < x < 5$. (Remember to include an integration constant, $+C$. What equation should you use to find the value of C?)

$$V = -\int E\, dx$$

$$V = (-3V/m)x + C$$

At $x = 5m$, $V = 0 \Rightarrow$

$$0 = \left(-\frac{3V}{m}\right)(5m) + C$$

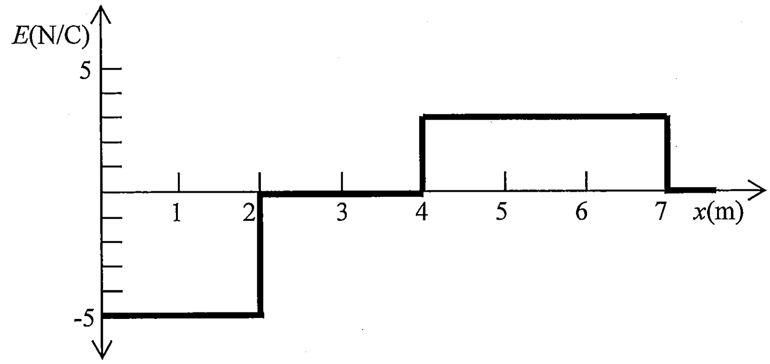
$$C = 15 \text{ Volts}$$

(e) Write an equation describing $V(x)$ for all $x > 0$. It will be of the form

$$V(x) = \begin{cases} \textit{blah blah blah}, & 0 < x < 2 \\ \textit{yada yada yada}, & 2 < x < 5 \\ \textit{something else}, & 5 < x \end{cases}$$

$$V(x) = \begin{cases} -(5V/m)x + 19 \text{ Volts} & 0 < x < 2 \\ -(3V/m)x + 15 \text{ Volts} & 2 < x < 5 \\ 0 & x > 5 \end{cases}$$

3. The graph below shows the electric field $E(x)$ in yet another region.



(a) Calculate the change in electric potential ΔV over each of the three regions shown.

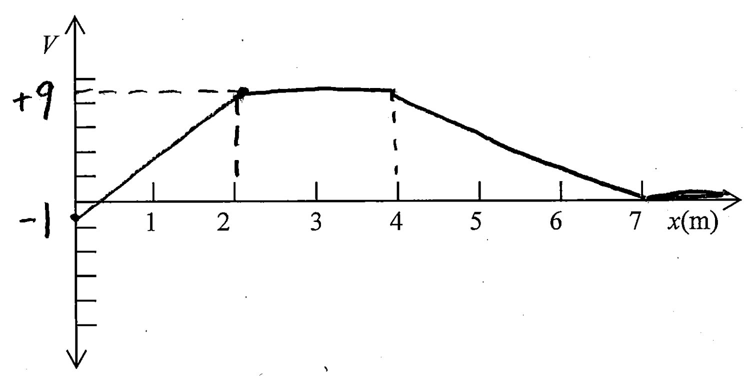
$x=0$ to $x=2$: $\Delta V = -\int_0^2 (-5N/C)\,dx = +10$ Volts

$x=2$ to $x=4$: $\Delta V = 0$

$x=4$ to $x=7$: $\Delta V = -9$ Volts

($x>7$: $\Delta V = 0$)

(b) Draw a graph of the potential $V(x)$, where the reference is $V(\infty)=0$.



(c) Is $V=0$ at $x=3$ (bearing in mind that the correct answer is "No")?

No.

(d) In general, if $E=0$, does it follow that $V=0$?

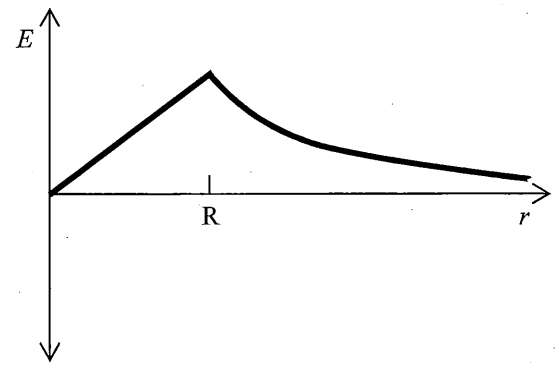
No.

(e) Where on your graph does $V=0$? In general, if $V=0$, does it follow that $E=0$?

$V=0$ at $x=0.2$ meters. No, $V=0 \not\Rightarrow E=0$

4. The graph to the right shows the electric field $E(r)$ near a uniformly charged sphere. The electric field is given by

$$E = \begin{cases} \dfrac{k_e Q r}{R^3}, & 0 < r < R \\ \dfrac{k_e Q}{r^2}, & R < r \end{cases}$$



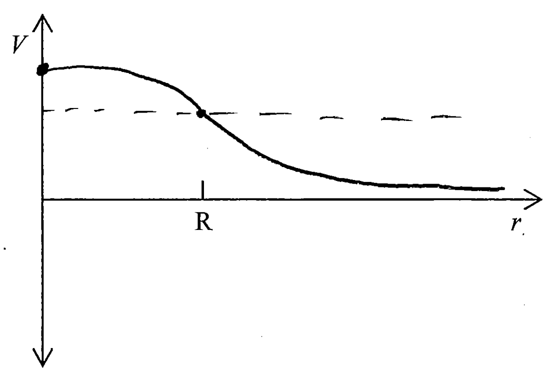
(a) Use a definite integral to calculate ΔV between $r = R$ and $r = \infty$.

$$\Delta V = -\int_R^\infty \frac{k_e Q\,dr}{r^2} = -\frac{k_e Q}{R}$$

(b) Use a definite integral to calculate ΔV between $r = 0$ and $r = R$.

$$\Delta V = -\int_0^R \frac{k_e Q r\,dr}{R^3} = -\frac{k_e Q}{R^3}\left(\frac{1}{2}r^2\right)\Big|_0^R = -\frac{1}{2}k_e Q/R$$

(c) Draw a graph of the potential $V(r)$, using $V(\infty) = 0$ as a reference.



(d) Use indefinite integrals to write equations for $V(r)$ for each of the two regions, using $V(\infty) = 0$ as your reference. Be careful with signs, and remember the integration constants!

$$r > R:\quad V = -\int \frac{k_e Q}{r^2}\,dr = \frac{k_e Q}{r} + C_1 \qquad \left(V(\infty) = 0 \Rightarrow C_1 = 0\right)$$

$$r < R:\quad V = -\int \frac{k_e Q r}{R^3}\,dr = -\frac{1}{2}\frac{k_e Q r^2}{R^3} + C_2$$

$$\text{at } r = R,\ V = \frac{k_e Q}{R} \Rightarrow -\frac{1}{2}\frac{k_e Q R^2}{R^3} + C_2 = \frac{k_e Q}{R}$$

$$\Rightarrow C_2 = \frac{3}{2}\frac{k_e Q}{R}.$$