Note: Some of these problems may be adapted or copied from published sources.

Answers to problems are at the end of this document.

MC1. In class, we imagined an experiment in which Anna held out her arms with a lightbulb in each hand, and turned on the lightbulbs simultaneously. Take each of Anna's arms to be one meter long.

(a) Draw a spacetime diagram (or "Minkowski diagram") in the reference frame of Anna showing three events: each of the two lightbulbs turning on, and the photons from both bulbs arriving at the tip of Anna's nose. Take the photons arriving at Anna's nose to occur at $x = 0$ and $t = 0$. (Yes, this means that the photons were emitted at some time $t < 0$.)

(b) Anna turns on the bulbs in a train car moving at speed $v = 0.5c$ in the positive x direction relative to Bob, who is standing beside the tracks. What is Bob's velocity relative to Anna? (*Be careful with your signs here!*).

(c) Bob's nose is exactly even with Anna's nose at $t = t' = 0$ and $x = x' = 0$, so the third event (photons arriving at Anna's nose) happens at $x' = 0$ and $t' = 0$. Use the Galilean transformations to find the coordinates of the other two events in Bob's reference frame.

(d) Draw another spacetime diagram showing those same three events in the reference frame of Bob, according to the Galilean transformations you used in part (c).

(e) According to your work in the previous two parts, what are the speeds of the photons from the two lights in Bob's reference frame?

(Note: the velocities you find in part (e) are different from c, which violates the postulate of relativity. The purpose of this exercise is to point out that the Galilean transformations are simply not correct at high speeds, for precisely this reason.)

MC2. Anna is traveling on a spacecraft at a speed of $0.8c$ relative to the Earth, and is taking advantage of her travel time by working on her physics homework. According to Anna, it took her one hour to finish her homework. (a) How long does it take Anna to finish her homework according to Bob, who is sitting at his desk in Richmond? (b) From what we did in class, you know that for a problem like this, the times according to Anna and Bob will *always* differ by a factor of γ. The trick is to figure out whether to multiply or divide by γ; that is, to determine who measures the shorter time and who measures the longer time. Now that you have checked your answer to part (a) below, *how do you know for sure* whether Anna or Bob measures the shorter time? (Hint: what are the two events you are finding Δt for, and how do you determine who measures the *proper time* between those two events?)

MC3. In the "lab" we did in class, we imagined Anna in a train car, moving fast enough that her clock and Bob's clock differed by a factor of 1.5. Exactly how fast would Anna have to be going for that to happen?

MC4. A GPS satellite orbits the Earth at a speed of 14,000 km/hr. The GPS satellite carries a very accurate atomic clock on board, which can measure time very accurately. If the clock on a satellite starts out synchronized to a clock on the surface of the Earth, by how much will the clock on Earth differ from the clock on the satellite after one full day, based on the velocity of the satellite? To do this calculation, you will need to use a calculator that can keep a LOT of digits; the default calculator in Windows will work fine, as will the calculator at this link. (Also note: I have to say "based on the velocity of the satellite" because there's also an additional gravitational effect due to the altitude of the satellite, which is about 5 times larger than what you're calculating here.)

MC5. Anna travels at speed $0.7c$ by spacecraft to the nearby star Proxima Centauri, which is a distance of 4.2 light years away from Earth in the Earth's reference frame. (a) How long will the trip take her in the Earth's reference frame? (b) By how much will Anna *age* during her trip? That is, how long does the trip take in Anna's reference frame? (c) How far is Proxima Centauri from Earth in Anna's reference frame? (Hint: if you laid a 4.2 light-year-long ruler between the two stars, how long would the ruler be in Anna's frame?) (d) Bob, on Earth, calculates Anna's speed by dividing the distance she travels by the time of her trip, in his reference frame. Anna calculates her speed by dividing the distance she travels by the time of her trip, in her own reference frame. What do Anna and Bob each calculate for Anna's speed, and do they agree?

MC6. From the pages from Griffiths' book Electrodynamics that are posted under "Notes," give three examples of equations that are integrated into sentences or introduced *differently* from anything you did in your first writing assignment. For each example, write the sentence and say something intelligent about how the equation (or a part of the equation) functions in the sentence. (For instance, does the equation function as the main clause of the sentence? Or as a subordinate clause? Is one of the variables in the equation obviously identifiable as the subject or predicate nominative of the sentence? If you are unsure of these grammatical terms, you can focus instead on the ways in which variables are defined either before or after the sentence, for instance by using "where," after the equation, or in an appositive before the equation, etc.)

MC7. Anna and Bob perform another experiment with a flashlight and a mirror, similar to what they did in Activity 1 of Lab 6. This time, Anna rides the train in the $+x$ direction with velocity $v = +0.75c$. Bob holds the flashlight at the origin, 1 m above the mirror, turning it on at time $t = t' = 0$ in both of their frames. (a) Draw Minkowski diagrams for the photon in both Anna's and Bob's reference frames, under the (incorrect) Galilean Transformations. Be careful with your signs! (b) Draw Minkowski diagrams for the photon in both reference frames under the (correct) Lorentz Transformations. (c) Using the Mathematica applet, estimate the coordinates (x', ct') when the particle is detected according to Anna in part (b). (d) Calculate the exact value of $c\Delta t$ (in meters) according to both Anna and Bob, where Δt is the time between the emission and detection of the photon. (Hint: one of them measures the proper time Δt_0.)

MC8. In reference frame S, an event occurs at coordinates $x = 2$ m, $t = 4$ m$/c$. Find the coordinates (x', t') of this event in a reference frame S' moving with respect to S at $v = -0.6c$.

MC9. Anna is riding on a train at velocity $v = +0.6c$. Bob stands on the ground beside the tracks. He snaps his fingers at position $x = 4$ m, at time $t = 8$ m$/c$. Where and when does the snap occur in Anna's reference frame? (Although the problem doesn't say it explicitly, you can always assume that Anna's and Bob's origins coincide at $t = t' = 0$.)

MC10. In class, I gave a short justification of why the Lorentz transformations require that $y' = y$ and $z' = z$. The example I used involved Anna on a train, just barely making it under a low-clearance bridge. Think of another good example to use to motivate why this must be true, and describe it here briefly. (Maybe two objects moving towards each other? Maybe something in sports—ooh, I bet either baseball or archery could work well! Remember, the motion of your two objects is along the x direction, so you can focus your argument on either the y or z axis; your example doesn't have to focus on height.)

MC11. In problem MC2, Anna traveled on a spacecraft at a speed of $0.8c$ relative to the Earth. According to Anna, it took her one hour to finish her physics homework. You should have found that according to Bob (on Earth), Anna finished her homework in 1 hour and 40 minutes. We didn't mention that Bob was also doing his physics homework that day, and according to Bob it took him 1 hour to finish it. How long does it take Bob to finish his homework according to Anna?

[Note: problems MC12 through MC15 below relate directly to the video on Blackboard in which I derive the Lorentz transformations, and the four pages of notes that go along with them.]

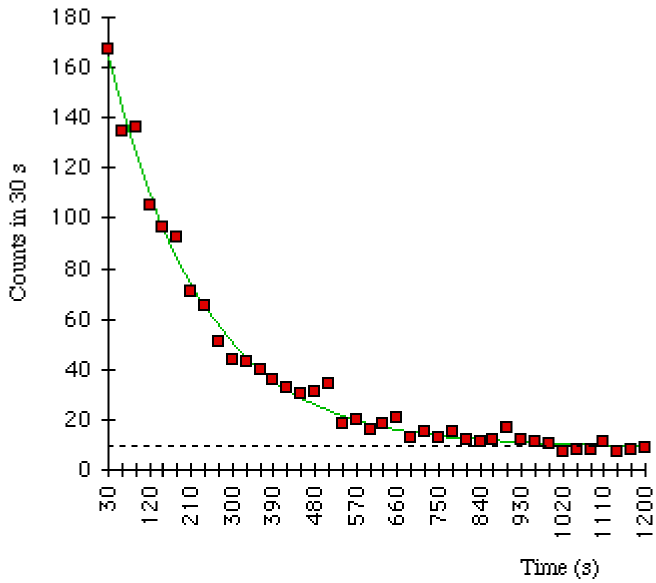
MC12. In the first page of the notes, I reason that the Lorentz transformations must be *linear* in x and t, that is, $x' = Ax + Bt$. Write one or two sentences explaining in words how we know that this must be true.

MC13. On the second page of notes, I imagine *two* special cases of an object that starts at the origin at time $t = 0$. For each of these special cases, write briefly in words what the two situations are, and draw Minkowski diagrams with worldlines for the object, one in frame S, one in frame S'. (I'm asking for two descriptions and four Minkowski diagrams in total.)

MC14. At the bottom of the second page of notes, I made an algebra error in deriving Equation 4. Find the mistake, and write Equation 4 correctly.

MC15. At the top of page 3, and again at the top of page 4, I use a "trick" in which I rewrite a transformation equation after switching which frame I call S and which frame I call S'. What *two* things do we have to change in a transformation equation to account for this switch in the labeling of the two reference frames? (Hint: primes and a sign.)

MC16. The graph below shows the number of decays of the excited state of ^{137}Ba measured as a function of time, just as you did in Lab 8. (a) Estimate *by eye* the half-life $t_{1/2}$ of the excited state of ^{137}Ba from the graph below. (I don't want you actually *calculate* anything. I want you to think about what "half-life" means and give me a rough value for it.) (b) Estimate *by* eye the "mean lifetime" τ from the graph below. (c) Based on your previous answer, estimate the value of the decay constant λ, *including the correct units.* (Bear in mind that this λ has nothing to do with any wavelength, other than that they happen to use the same Greek letter.)



MC17. The radioactive isotope ^{15}C has a half-life of 2.449 seconds. (a) Calculate the mean lifetime τ and decay constant λ for ^{15}C. (b) Suppose you begin at time $t = 0$ with a sample of $N_0 = 10{,}000$ atoms of ^{15}C. How many atoms N do you expect to be left at time $t = 8$ seconds? (c) What percent of the original atoms (that is, N/N_0, expressed as a percentage) would you expect to remain at time $t = 11$ seconds?

MC18. At rest, neutrons that are not bound within atomic nuclei have a mean lifetime of $\tau = 14.692$ minutes. The fusion of deuterium and tritium nuclei (^2H and ^3H, respectively), in a laboratory or in the sun, produces helium (^4He) and a very high-energy neutron that travels at 17.3% of the speed of light. What is the mean lifetime of these fast neutrons, in seconds, in the "laboratory" reference frame (that is, the reference frame of the Earth)? *You'll need to keep five significant digits in your answer, to make it clear that your answer is different from the 14.692 minutes in the reference frame of the neutron.*

Answers:

MC1. (b) $v = -0.5c$ (e) $1.5c$ and $0.5c$.

MC2. (a) 1 hour and 40 minutes

MC3. $0.745c$

MC4. About 7 microseconds

MC5. (a) 6 years (b) 4.29 years (c) 3 light years (d) Both had better calculate $0.7c$.

MC7. (d) Bob measures $c\Delta t = 2$ m. Anna measures $c\Delta t = 3.02$ m. Incidentally, if you would like to check your other answers with me for parts (a) through (c), I'll be happy to do that during office hours. I often don't post answers here for qualitative questions (like graphs, or yes/no or multiple choice questions), because once you see those answers, even by accident, it's too easy to convince yourself that *of course you knew how to get that answer on your own* when you really didn't. But once you've given the problem a try, I'm always happy to chat with you about whether you got it right.

MC8. $x' = 5.5$ m, $t' = 6.5$ m$/c$

MC9. $x' = -1$ m, $t' = 7$ m$/c$

MC11. Also 1 hour and 40 minutes. (If you calculated 36 minutes, then you may want to review problem MB10. When two spacecraft pass each other at speed v, it's meaningless to try to discern which one is *really* moving and which one is standing still; both reference frames are equally valid. That's true for any two reference frames, even when one is based on a tiny spaceship and one is based on a large planet. Bob's planet is doubtless more spacious than Anna's spacecraft, but their two reference frames are nevertheless equally valid.)

MC16. (a) About 210 seconds (b) About 300 seconds (c) $\approx 3.3 \times 10^{-3}\ \mathrm{s}^{-1}$

MC17. (a) $\tau = 3.53$ s, $\lambda = 0.283\ \mathrm{s}^{-1}$ (b) About 1039 atoms, though due to statistical fluctuations it might be a little higher or lower, in the same way that if you flipped 10,000 coins, you might end up with results slightly different from 5000 heads and 5000 tails. (c) 4.4%

MC18. 894.99 seconds.